Equal division among the few: an experiment about a coalition formation game

Yukihiko Funaki, Emmanuel Sol, Marc Willinger

Equal division among the few: an experiment about a coalition formation game

Yukihiko Funaki,
Emmanuel Sol
&
Marc Willinger

CEE-M Working Paper 2021-10

Equal division among the few:
an experiment about a coalition formation game.

Yukihiko Funaki, Emmanuel Sol, Marc Willinger

Abstract

We study experimentally a three player sequential and symmetric coalition formation game with empty core. In each round a randomly chosen proposer must choose between a two players coalition or a three players coalition and decide about the payoff division among the coalition members. Players who receive a proposition can accept or reject it. In case of acceptance the game ends. If it is rejected, a new proposer is randomly selected. The game was played repeatedly, with randomly rematched groups. We observe that over 86% of the realized coalitions are two-players coalitions. Three players coalitions are often observed in early rounds but are frequently rejected. Equal splits are the most frequently observed divisions among coalition members, and their frequency increases sharply over time. We propose an extension of von Neumann and Morgenstern (1944)'s notion of stable set to account for our results.

1 Introduction

A key issue in cooperative game theory is that players can form coalitions and make binding agreements on how to distribute the proceeds of these coalitions (see Peters (2008)). So far, the question has remained unanswered. Instead, most of the literature in game theory focused on the related issue of surplus division, under the assumption that the grand coalition will be formed.

But how the grand coalition is eventually reached remains an open question, and the emergence of alternative coalition structures cannot be precluded. From this viewpoint, the procedure by which members of a group can form coalitions becomes central. In contrast to non-cooperative game theory which to a large extent is grounded on procedures that are clearly and well defined, cooperative game theory is much less precise about the procedures by which agents may eventually reach an agreement. This can be seen as a merit as well as a demerit.

In contrast to non-cooperative extensive form games, where the order of moves is given, the choice sets are clearly defined and the outcomes are well-defined by a payoff function, in cooperative games the order of players' moves is not predetermined, and there is no final step on which they can rely to start a backward induction reasoning process.

However, the outcomes of non-cooperative games are often sensitive to small changes in procedures, strategy sets or payoff functions.[1] In contrast, in cooperative game theory where procedures are undefined, the outcome of the negotiation or bargaining process among humans might have some pattern predicted by the theory.

Since cooperative game theory assumes the grand coalition without any theoretical justification, it is worth exploring how likely the grand coalition emerges in a coalition formation game. The purpose of our paper is therefore to provide some insights into the coalition formation game based on a laboratory experiment. Laboratory experiments can be useful to provide evidence about regularities in the outcomes of the bargaining process among subjects. In particular is the theoretical assumption that the grand coalition will always form a reasonable assumption with respect to the experimental evidence? What type of division do the subjects choose?

We consider a game involving 3 players who can either form a three-player (*3P*) coalition or a two-player (*2P*) coalition from which one group member is excluded. The total amount to be divided is larger for the *3P* coalition than for the *2P* coalition. But each member of a *2P* coalition can potentially earn more than in a *3P* coalition. In particular this is the case if the cake is divided equally in both coalitions.

Our setting, i.e. a three player sharing game, has been studied in several experiments, e.g. Güth and van Damme (1998), Bolton and Ockenfels (1998), Bolton et al. (2003), Riedl and Vyrastekova (2003), Okada & Riedl (2005) and Güth et al. (2010). We adopt however, a fundamentally different approach by considering a bargaining game instead of the non-cooperative division game studied by previous literature. Previous experiments relied on the ultimatum bargaining game. Since the game is non-cooperative, proposers are under the threat of a reject, in which case the game ends with all players earning zero. In our game, such an issue cannot arise since (non-excluded) players can make counter-offers as long as no agreement is reached. Furthermore, our game has no stable solution and might therefore go on endlessly.

The data of our three player experimental game clearly reject the prediction that the grand coalition is always formed. In contrast, our main findings are as follows : (i) *2P* coalitions are the most frequently observed agreement, (ii) over rounds the frequency of *2P* coalitions increases, (iii) the equal split is the most frequently chosen division both in *2P* and *3P* coalitions, (iv) over time the payoff difference in *2P* coalitions drops sharply and tends towards equal division. We show that an extension of the notion of *stable set* (von Neumann and Morgenstern, 1944), taking into account bargaining costs, nicely accounts for our main findings.

The remainder of the paper is organized as follows. In section 2 we define the notion of stable set and section 3 presents the experimental design. The results are presented in section 4 and section 5 provides a discussion of our findings.

[1]For example, in extensive form games, if the order of the moves is slightly changed, the equilibrium can change drastically. If two alternatives induce two payoffs 1 and $1+\epsilon$, the best choice differs depending on the sign of ϵ and this affects the equilibrium strongly.

2 Stable set

In cooperative game theory, it is usually assumed that the efficient grand coalition is always formed in super-additive games. The core is one of the most important concepts for justifying such an outcome. However, if the core is empty or does not exist, the outcome of the bargaining process becomes less predictable. Our experiment was precisely designed to address this issue: what are the more likely outcomes of the bargaining process whenever the core is empty? Restricting to TU games, a possible candidate is provided by von Neumann and Morgenstern (1944)'s notion of stable set (vNM hereafter). As a first step, we extend the concept of stable set to account for other possible outcomes than the grand coalition. We discuss this issue in subsection 2.2.

2.1 vNM stable set

Let us introduce a formal definition of the vNM stable set, shortly stable set. A TU game is a pair (N, v), where $N = \{1, 2, ..., n\}$ is a set of players, and v is a characteristic function which assigns the worth of a coalition $v(S)$ to each coalition $S \subseteq N$. We assume that the game (N, v) satisfies super-additivity: $v(S \cup T) \geq v(S) + v(T)$ $\forall S, T$ with $S \cap T = \emptyset$. The vNM notion of stable set assumes a domination relation over allocations.

Let I be the set of all allocations, called "imputation" set:

$$I = \{x \in \Re^N | \sum_{i \in N} x_i = v(N), x_i \geq v(\{i\}) \quad \forall i \in N\}$$

For $x, y \in I$, we define a domination relation: x dom $y \iff$ there is a coalition S such that $\sum_{i \in S} x_i \leq v(S)$ and $x_i > y_i$ $\forall i \in S$.

In words, since all players in S prefer imputation x to y they can threaten to leave the grand coalition if the imputation y is implemented instead of x because they can credibly achieve a higher value $v(S)$ by forming coalition S. In this case, we say S is effective for x, and x dominates y via S.

The core C is defined as a set of imputations which cannot be dominated by any other imputation. Under super-additivity, the core is equivalent to $\{x \in I | \sum_{i \in S} x_i \geq v(S) \ \ \forall S \subset N\}$.

The stable set K is defined as a set of imputations which satisfy:

(1) External stability: for any $y \notin K, \exists x \in K$ such that x *dom* y,

(2) Internal stability: for any $x \in K, \nexists\, z \in K$ such that z *dom* x.

External stability means that any imputation that it outside the stable set is dominated by at least one imputation in the stable set. Internal stability means that imputations that belong to K cannot dominate each other. See von-Neumann and Morgenstern (1944) and Peters (2015).

For three person games, it is known that the vNM stable set always exists, although not uniquely, and includes the core.

An example is provided by the following typical three person super-additive symmetric game (N, v) with empty core: $(N, v) : N = \{1, 2, 3\}, v(N) = v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1, v(\{i\}) = 0 \ \ \forall i \in N$. One of the stable sets is given by:

$$K = \{(1/2, 1/2, 0), (1/2, 0, 1/2), (0, 1/2, 1/2)\}.$$

This is the only symmetric stable set, where all the permuted allocations of K belong to K.

Consider now the following class of three person super-additive symmetric games with empty core $\Gamma = (N, v) : N = \{1, 2, 3\}, v(N) = 1, v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = a, v(\{i\}) = 0 \ \forall i \in N$, with $2/3 < a \leq 1$. Although the core is empty, there exists many stable sets. The symmetric stable set K_s is given by

$$K_s = \{(\frac{b}{2}, \frac{b}{2}, 1 - b), (\frac{b}{2}, 1 - b, \frac{b}{2}), (1 - b, \frac{b}{2}, \frac{b}{2})|\ a \leq b \leq 1\}.$$

In one of our treatments we have: $(N, v) : N = \{1, 2, 3\}, v(N) = 300, v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 260, v(\{i\}) = 0 \ \ \forall i \in N$. The corresponding symmetric stable set is therefore given by: $K = \{(150 - \frac{b}{2}, 150 - \frac{b}{2}, b), (150 - \frac{b}{2}, b, 150 - \frac{b}{2}), (b, 150 - \frac{b}{2}, 150 - \frac{b}{2})\}$ with $0 \leq b \leq 40$.

This can be understood as follows: a *2P* gets 260. If its members share equally each one receives 130. They could get more by sharing together the 300 of the grand coalition and giving b to the third player. This solution dominates the equal sharing of the *2P* coalition if $b \leq 40$. Furthermore, the third player could reject such a proposal because he has a low share, but since there is no dominant allocation outside the stable set he is under the threat of being excluded. It is therefore an empirical issue whether such outcome is likely to arise. Our experiment was designed to investigate whether subjects apply such type of solution, or if they adopt some alternative pattern of behavior.

2.2 An extension of vNM stable set

How can we improve the notion of stable set to account for other possible outcomes than the grand coalition? To provide a sensible answer we consider the case of a three persons game, allowing for the possibility that a *2P* coalition is formed as a result of the bargaining process.

First we modify the definition of the imputation set to allow the consideration of other coalition structures than the grand coalition. Let π be a partition of a player set N, that is, $\pi = \{S_1, S_2, ..., S_m | \cup_{k=1}^{m} S_k = N$, and $S_k \cap S_j = \emptyset$ for $k \neq j\}$. We extend I to I^e: $I^e = \cup_{\pi \in \Pi} I^\pi$, where

$$I^\pi = \{x \in \Re^N | \sum_{i \in S} x_i = v(S), \text{ for all } S \in \pi\},$$

and Π is the set of all partitions of N.

For a three person game, I^e is given by:

$$\begin{aligned} I^e = I &\cup \{x \in \Re^N | x_1 + x_2 = v(1,2), x_3 = v(3)\} \\ &\cup \{x \in \Re^N | x_1 + x_3 = v(1,3), x_2 = v(2)\} \\ &\cup \{x \in \Re^N | x_2 + x_3 = v(2,3), x_1 = v(1)\} \cup \{(v(1), v(2), v(3))\}. \end{aligned}$$

Our modification of the definition of the imputation set does not affect the stable set. It can be easily checked that any extended imputation $x \in I^e$ which satisfies $\sum_{i \in N} x_i < v(N)$ is dominated by some imputation $y \in I$. If we want to allow for other coalition structures we need therefore to adopt a broader definition for the domination relation. We do this by assuming that bargaining is more costly when a larger number of players is involved in the negotiation process. For instance, finding an agreement requires more discussions and more time as the number of players increases. The presence of bargaining costs can prevent the formation of the grand coalition. In our example of a three persons coalition formation game, we expect therefore that costly negotiation will lead to more frequent deviations from the 3-persons agreement than from 2-persons agreements.

We introduce the bargaining cost to our domination relations as follows. Let $\epsilon > 0$ represent the cost of negotiating a 3-players agreement. The modified definition of domination taking into account ϵ for the 3-persons coalitions is : for $x, y \in I^e$, x dom $y \iff$ there exists a two person coalition S such that $\sum_{i \in S} x_i \leq v(S)$ and $x_i > y_i \ \forall i \in S$, or for the three person grand coalition N, $\sum_{i \in N} x_i \leq v(N) - \epsilon$ and $x_i > y_i \ \forall i \in N$. $\epsilon > 0$ can be interpreted as an additional cost for forming the three person coalition compared to the two-person coalition. Our new definition of the domination relations affects the content of the stable set K^e.

Consider again the three person game Γ introduced in the previous subsection. Recall that the symmetric stable set $K_s \subseteq I$ is given by:

$$K_s = \{(\frac{b}{2}, \frac{b}{2}, 1-b), (\frac{b}{2}, 1-b, \frac{b}{2}), (1-b, \frac{b}{2}, \frac{b}{2}) | \ a \leq b \leq 1\}$$

On the other hand, the modified symmetric stable set $K_s^e \subseteq I^e$ is given by $K_s^e = K_s \cup \{(a/2, a/2, 0),\ (a/2, 0, a/2), (0, a/2, a/2)\}$ when the cost ϵ for a domination by *3P* coalition coincides with a. This is because each allocation in $K_s^e \setminus K_s = \{(a/2, a/2, 0),\ (a/2, 0, a/2), (0, a/2, a/2)\}$ cannot be dominated by any allocation in K_s due to the domination cost ϵ.

3 Experimental design

We consider a three player bargaining game as introduced in the previous section. The experiment is based on a specific coalition formation procedure, that involves a sequence of offers and counter-offers in order to reach an agreement.

The coalition formation game was divided into a sequence of rounds. In each round, participants could earn points by reaching an agreement. At the beginning of each round, one of the members in each three-players group was randomly selected to be the proposer. The proposer's task consisted in two successive decision stages. The stage 1 decision was to choose the size of the coalition: precisely, the proposer had to choose between three options : "sharing between two players", "sharing between three players", or "pass". The first option corresponds to a *2P* and the second option to the *3P*. If the proposer decided to choose "pass" another proposer was randomly chosen among the two other members of the group to act as the next proposer. If the proposer chose either a *3P* coalition or a *2P* coalition, in stage 2, he had to choose a distribution of the available points, i.e. decide about the amount to be allocated to each coalition member. In stage 3 the coalition member(s) had to decide whether to accept or to reject the proposer's offer. If one of the group members who received an offer rejected it, a new proposer was randomly selected among players who did not act as a proposer in the current round. If the proposal was accepted payoffs were implemented and the experiment moved to the next round. At the end of each round, new groups were randomly formed.

Our experiment involves two treatments which differ by the stakes to be shared among coalition members (low (L) or high (H) amount m). Formally, let $v(N) = h$, $v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = l$ and $v(\{1\}) = v(\{2\}) = v(\{3\}) = i$. In the L-treatment, $h = 300$, $l = 260$ and $i = 0$, while in the H-treatment, $h' = 600$, $l' = 460$ and $i' = 100$. For the L-treatment (low stakes), the available amount to be divided was 260 points for a *2P* coalition and 300 points for a *3P* coalition. Any distribution in integer amounts (including zero) of the available amount was feasible. In theory, the two treatment are equivalent because the H-treatment is obtained by adding constant 100 for each individual since $h' = h + 300, l' = l + 200$, and $i' = i + 100$. The difference is that each individual gets 100 more in any cases.

In the H-treatment the amounts to be shared were determined by shifting the amounts of the L-treatment by 300 points, i.e. 600 points for a *3P* coalition and 560 points in case a *2P* coalition was formed: 460 points to be shared by the members of the *2P* coalition and 100 points for the non-member. By shifting in this way the amounts of the L-treatment by 300 points, the value difference between a *3P* and a *2P* coalition is kept constant across treatments as well as the average payoff difference between the insiders and the outsider for *2P* coalitions. For instance, in case on an equal division, the members of the *2P* coalition earned 130 points more than the outsider, in both treatments. We also conjectured that the H-treatment would be favorable to the formation of *2P* coalitions because the stand-alone player earns a positive amount compared to the L-treatment. Guilt-aversion (Battigalli and Dufwenberg (2007)), and inequality-aversion (Fehr and Schmidt (1999), Bolton and Ockenfels (1998, 2000)) are two reason that could favor the formation of *3P* to avoid a null payoff for the outsider. The instructions (see appendix) detail the rules by which participants could earn points.

In order to collect two independent observations per session, each session involved 18 participants which were split into two subsets of 9 players. In each round subjects interacted in groups of 3 players, selected among the 9 participants of their subset. New groups of 3 players were randomly formed after each round. Neither the number of periods, nor the duration of a period was announced to the participants. They were only told that there was a time-limit for reaching an agreement, but that they disposed of at least 2 minutes of time for reaching an agreement. If ever no agreement was reached by the time-limit for a period, each group-member earned zero points for that period. This happened only once out of 720 cases.

We collected 12 independent observations, 6 for each treatment. An independent observation is a subset of 9 subjects who interact in groups of 3 players over 20 rounds. Therefore in each subset we observed 60 coalition structures, which provides a data set of 720 coalitions in total.

4 Results

We start with a presentation of the observed coalitions sizes (4.1) before presenting the associated payoff distributions (4.2).

4.1 Coalitions sizes

Result 1
2P coalitions are the most frequently observed agreement, and are more frequent in the H-treatment than in L-treatment.

Support for result 1

Table 1 shows the frequencies of the various coalition sizes for each independent group. We observed 720 coalitions in total. The null coalition was observed only once. Overall more than 86% of the coalitions are *2P* coalitions, which are significantly more frequent than *3P* coalitions (Wilcoxon signed-rank test, 1%). There is some variance across groups : the lowest frequency is 63% and the highest 98%. Two player coalitions are more frequent in the H-treatment (94.72%) than in the L-treatment (78.33 %), a significant difference (Mann-Whitney, 1%).

[INSERT TABLE 1 ABOUT HERE]

Let us call initial proposal, the proposal made by the first proposer in a three-player group at the beginning of a round.

Result 2
Initial proposals within groups are mostly 2P coalitions, and are relatively less frequently rejected than 3P initial proposals.

Support for result 2

2P coalitions were proposed in the beginning of rounds in 73.3% of the cases of the L-treatment and 88.1% of the cases of the H-treatment. 22.1% and 17.0% were respectively rejected. In comparison the rejection rates for the initial proposals of *3P* coalitions are much higher: 38.1% and 65.1% respectively. Table 2 provides detailed data about rejection frequencies of initial proposals. *3P* coalitions are clearly relatively more frequently rejected than *2P* coalitions (Wilcoxon signed-ranked test, 5%). Furthermore, after a rejection of a *3P* coalition, the outcome was more frequently a *2P* coalition. As shown by table 3, the frequency of agreed *2P* coalitions increased significantly (Wilcoxon signed-ranked test, 5%) with respect to the beginning of round proposals, at the expense of *3P* coalition proposals.

[INSERT TABLE 2 and TABLE 3 ABOUT HERE]

Result 3
Over rounds the frequency of 2P coalitions increases steadily in each independent group

[INSERT FIGURE 1 ABOUT HERE]

Support for result 3

Figure 1 reports the evolution of coalition sizes over time. Clearly the frequency of *2P* coalitions increases steadily over time, leading to a sharp decline of the frequency of *3P* coalitions. In the H-treatment there is already a huge gap between the frequency of *2P* and *3P* in the first round. Knowing that the outsider has 100 points instead of 0 points seem to have affected subjects' behaviour. Plausibly, they felt less guilty to let down one of their group members than in the L-treatment. In contrast in the L-treatment, the round 1 frequency of *3P* coalitions is larger than for *2P* coalitions. But by round 2 the frequencies are reversed, as one can observe a sharp decline of *3P* coalitions and symmetrically a sharp increase of *2P* coalitions. In table 4 we grouped the rounds into sequences of 5 consecutive rounds, to highlight the evolution of the frequency of *2P* coalitions. The frequency of *2P* coalitions increases over sequences in all groups and in both treatments. Remarkably, in the H-treatment nearly all groups choose the *2P* coalition after sequence 1-5 for all remaining rounds. The frequency of *2P* coalitions is larger in sequence 16-20 than in sequence 1-5 for both treatments (Wilcoxon signed-rank test, 1%). Note that for the L-treatment the frequency of *2P* coalitions increases between any two subsequent sequences.

[INSERT TABLE 4 ABOUT HERE]

4.2 Payoff distributions

Result 4
Equal splits are the most frequently chosen divisions, both in 2P and in 3P coalitions, but equal divisions are more frequent in 3P coalitions than in 2P coalitions.

Support for result 4

Table 5 compares the frequency of equal splits in case of *2P* coalitions and *3P* coalitions for each group and for the two treatments. Equal splits are relatively more frequent in *3P* coalitions than in *2P* coalitions. However, the difference is significant only for the L-treatment(χ^2-test, 5% and Wilcoxon Mann-Whitney, 5%). For the H-treatment, we cannot reject the null hypothesis that frequencies of equal splits are equal for the two types of coalitions (χ^2-test, 5% and Wilcoxon Mann-Whitney, 5%). In figure 2 we report the frequencies of observed splits in *2P* coalitions, which is the most frequent type of coalition. Equal splits are very frequent, in particular in the H-treatment (over 75%).

[INSERT TABLE 5 ABOUT HERE]

[INSERT FIGURE 2 ABOUT HERE]

Result 5
Over time the payoff difference in 2P coalitions drops sharply and tends towards equal division.

Support for result 5

Figure 3 shows the evolution over time of the average payoff difference in *2P* coalitions. This average was computed as follows: first, for each independent group of 9 players we measure the average payoff difference of the realized *2P* coalitions round by round, and second we average the former result over all groups. Clearly, the average payoff difference drops sharply for both treatments and ends up nearly at zero. Actually in most groups of the H-treatment, the average payoff difference was null in the end periods as can be seen from table 6. This table also reveals that in each group of each treatment the average payoff difference is lower in the last sequence compared to the first sequence, a significant difference (Wilcoxon signed-ranked test, 5%). Finally, although the average payoff difference is smaller in the H-treatment than in the L-treatment, the difference is not significant (Mann-Whitney, 5%).

[INSERT FIGURE 3 ABOUT HERE]

Figure 4 compares the frequencies of average splits in *2P* coalitions between the first round and the last round. Clearly equal splits are the most frequent distribution in the last round: over 80% in the L-treatment and reaching 100% in the H-treatment. However, in the first round equal split are rare in the L-treatment, and only slightly above 50% in the H-treatment. This observation suggest that most *2P* coalitions start with unequal payoff distributions in early rounds and move towards more equal distribution over time.[2]

[INSERT FIGURE 4 ABOUT HERE]

We summarize our key findings as follows: (i) subjects are more likely to propose and accept a *2P* coalition than a *3P* coalition. This tendency, (ii) is stronger when stakes are high (H-treatment) and, (iii) becomes reinforced in each treatment as players gain experience, (iv) equal splits are the most frequently observed allocation in *2P* and *3P* coalitions, and when inequalities are observed they tend to vanish over rounds.The high frequency of equal divisions observed in *2P* agrees with our extended notion of the stable set outlined in section 2. In the next section we discuss the behavioral patterns underlying our observations and show how our extended definition of the stable set accounts for these findings.

5 Discussion

In our coalition formation game players have to bargain to reach an agreement. In each step of the bargaining process, one player which was not the

[2]We observe the same pattern as in table 5, if we consider the average of the n beginning and the n terminal rounds, for each value n$\leq$ 5.

proposer in the previous step, is randomly chosen to make an offer to the other players. Our experimental results clearly show that the usual assumption that the grand coalition will be formed is rejected. Instead, most of the time subjects proposed and accepted *2P* coalitions and very frequently with an equal division. In other words, players are more likely to form a *2P* coalition than a *3P* coalition.

Before we discuss how our extended notion of stable set fits this data, let us provide some intuition about how subjects likely behaved in our game. Consider the initial proposer. At first sight his choice depends on whether he is a fair-minded or selfish person. Let us assume that a fair person prefers that all members of the group get an equal share, while the selfish person aims at maximizing her payoff. Assume now that a fair player is selected to be the initial proposer and that he chooses the grand coalition and proposes to share equally. A selfish responder will reject such an offer because he has the possibility to increase her payoff by choosing (or by being offered) a two player coalition from which the initial proposer might be excluded. If the two other players are selfishly oriented, the initial proposer will actually stand alone at the end, because he always rejects the *2P* coalition when it is offered to him. If the initial proposer cares about his material payoff, he may therefore propose a *2P* coalition with equal division rather than a *3P*.

Consider now that the initial proposer is a selfish player who proposes a *2P* coalition to the other player with at least her fair share of the grand coalition. This offer will be rejected if the selected player is fair-minded. Assume that only one of the other players is fair-minded. The proposer's offer will be rejected but he will receive a proposition for a *2P* coalition or a *3P* coalition depending on which of the other two members becomes the new proposer. In any case, the fair-minded player will be excluded (i.e. stand alone) while the two other players will form a *2P* coalition. Therefore, in a population of two selfish players and one fair minded player, the fair minded always ends up standing alone. In a population of three selfish players a *2P* coalition is always formed and in a group of three fair-minded players the grand coalition is always formed. Finally in a population with two fair minded persons, either the grand coalition is reached or no agreement is reached. Therefore in our three-players example the assumption that the grand coalition will be formed implicitly requires that at least two players are fair-minded. If three players are fair-minded the grand coalition will be formed with certainty but the issue of division is obvious in this case: they will share equally. This discussion highlights a key component of the bargaining process: the cost of reaching an agreement. Assume that there is a fixed cost $\epsilon > 0$ per iteration in the bargaining process. Depending on the number of fair-minded/selfish players in the group, trying to reach the *3P* coalition is almost always more costly than trying to form the *2P* coalition. Taking into account this observation as a hypothesis, we now discuss the predictive power of our modified definition of the stable set.

As already discussed in section 2, our extended notion of stable set provides a theoretical background for one of our experimental results for selfish players: the equal division of the worth observed for *2P* coalitions. We now show that this framework can also account for the higher frequency of *2P* coalitions in the experiments. Then we introduce a concept of *dominated region* to eliminate all allocations in the set $\{(\frac{b}{2}, \frac{b}{2}, 1-b), (\frac{b}{2}, 1-b, \frac{b}{2}), (1-b, \frac{b}{2}, \frac{b}{2})|\ a < b \leq 1\}$ from $K_s = \{(\frac{b}{2}, \frac{b}{2}, 1-b), (\frac{b}{2}, 1-b, \frac{b}{2}), (1-b, \frac{b}{2}, \frac{b}{2})|\ a \leq b \leq 1\}$.This is shown in figure 6 after we provide and illustration of the dominated region in figure 5.

For any $x \in I^e$, the dominated region $Dom\{x\} \subseteq I$ of x is given by:

$$Dom\{x\} = \{y \in I | x\ dom\ y\}$$

[INSERT FIGURE 5 ABOUT HERE]

The region $Dom\{x\}$ is illustrated in Figure 5. Here (A), (B), (C) are the dominated region via coalitions $\{1,2\}, \{1,3\}, \{2,3\}$, respectively. It is important to remark that x is above the line of $x_1 = v(N) - v(23)$ when $\{2,3\}$ is effective for x. We denote the size of this region by $|Dom\{x\}|$.[3] [4] It is not difficult to compute that for each b ($a < b \leq 1$), $|Dom\{(\frac{b}{2}, \frac{b}{2}, 1-b)\}| = (1-b)\frac{2b}{\sqrt{3}}$. and $|Dom\{(\frac{a}{2}, \frac{a}{2}, 1-a)\}| = (1-a)\frac{2a}{\sqrt{3}} + \frac{\sqrt{3}}{2}a^2$. Notice that coalition $\{1,2\}$ is effective only for $(\frac{a}{2}, \frac{a}{2}, 1-a)$. Thus, the allocation can dominate other imputations via $\{1,2\}$. Other two coalitions $\{1,3\}, \{2,3\}$ are always effective for all allocations $(\frac{b}{2}, \frac{b}{2}, 1-b), (a \leq b \leq 1)$. When we compare the area of those two regions, the latter one is larger than the former one because $(1-b)\frac{2b}{\sqrt{3}}$ is decreasing for $a \leq b \leq 1$ and $\frac{\sqrt{3}}{2}a^2 > 0$.

[INSERT FIGURE 6 ABOUT HERE]

Consider now that the negotiation process has the following property: agents can only propose a new allocation which dominates the current one. This implies that the size of the dominated region reflects the effect of the proposed allocation. We can therefore consider that $\{(\frac{a}{2}, \frac{a}{2}, 1-a), (\frac{a}{2}, 1-a, \frac{a}{2}), (1-a, \frac{a}{2}, \frac{a}{2})\}$ occurs more often than $\{(\frac{b}{2}, \frac{b}{2}, 1-b), (\frac{b}{2}, 1-b, \frac{b}{2}), (1-b, \frac{b}{2}, \frac{b}{2})|\ a < b \leq 1\}$.

We are now able to compare the following two possible sets of symmetric allocations: $\{(\frac{a}{2}, \frac{a}{2}, 1-a), (\frac{a}{2}, 1-a, \frac{a}{2}), (1-a, \frac{a}{2}, \frac{a}{2})\}$ and $\{(\frac{a}{2}, \frac{a}{2}, 0), (\frac{a}{2}, 0, \frac{a}{2}), (0, \frac{a}{2}, \frac{a}{2})\}$. The first set contains allocations of the worth of the grand coalition $v(N)$ and the second set contains allocations of the worth of *2P* coalitions. Both of these sets give the same payoff to two of the players while the remaining player receives a lower payoff in the set corresponding to a *2P* coalition. Of course the dominated region of the second set ($\frac{\sqrt{3}}{2}a^2$) is smaller than for the first set. Why should therefore the second set be "more stable" than the first one?

[3] For a set $M \subset \Re^2$, $|M|$ shows an area of M.

[4] If we define $Dom^e\{x\} = \{y \in I^e | x\ dom\ y\} \subseteq I^e$, then $|Dom^e\{x\}| = |Dom\{x\}|$ because $I^e \setminus I$ is an union of line segments and a point.

In the original domination relation, when x dom y via coalition S, we only require $x \in I$ and $x_i > y_i$ for $i \in S$ and $\sum_{j \in S} x_j \leq v(S)$. Here $x \in I$ means that for $j \in N \setminus S$ agree with the allocation of $v(N)$ even j is not a member of S. Indeed allocation $(\frac{a}{2}, \frac{a}{2}, 1-a) \in I$ dominates other allocations in I via the *2P* coalition $\{1,2\}$ but not the grand coalition. However we have to suppose the formation of N to have an allocation in I in theory.

In our experiment, the outside player $j \in N \setminus S$ of a *2P* coalition S has a power to refuse an allocation of $v(N)$. It is therefore more natural to consider the following stronger domination dom': For $x, y \in I^e$,

$$x \; dom' \; y \iff$$

$$\exists S \text{ s.t. } (1)\ x_i > y_i \ \forall i \in S;\ (2) \sum_{i \in S} x_i \leq v(S); (3)\ x \in I^{\pi} \ \text{ for some } \pi \ni S$$

This means that when allocation x dominates other allocations, x should be consistent with the partition π containing the effective coalition S, that is $x \in I^{\pi}, \pi \ni S$ [5].

Under this stronger domination, each allocation in I, $(\frac{a}{2}, \frac{a}{2}, 1-a), (\frac{a}{2}, 1-a, \frac{a}{2}), (1-a, \frac{a}{2}, \frac{a}{2})$ cannot dominate any allocation in I, that is, $Dom'\{(\frac{a}{2}, \frac{a}{2}, 1-a)\} = \emptyset$. However each allocation $(\frac{a}{2}, \frac{a}{2}, 0), (\frac{a}{2}, 0, \frac{a}{2}), (0, \frac{a}{2}, \frac{a}{2})$ dominates many allocations in I, that is, $|Dom'\{(\frac{a}{2}, \frac{a}{2}, 0)\}| = \frac{2}{\sqrt{3}}a^2$. The dominated region of these three allocations is most of I. Indeed the dominated region in I is given by the white region in Figure 6. By these arguments, we state $\{(\frac{a}{2}, \frac{a}{2}, 0), (\frac{a}{2}, 0, \frac{a}{2}), (0, \frac{a}{2}, \frac{a}{2})\}$ is the most natural and attractive set of allocations in the extended stable set. This discussion is consistent with the data of our experiment.

6 Conclusion

In our three player coalition formation experiment, we found that (i) two player coalitions are the most frequently observed type of coalition, (ii) over rounds the frequency of two player coalition increases, (iii) the equal split is the most frequently chosen division both in two player and three players coalitions, (iv) over time the payoff difference in two player coalitions drops sharply and tends towards equal division. These findings are consistent with the vNM stable set solution of cooperative game.

We also found that two player coalitions are more frequent in the H-treatment than in L-treatment. This finding suggests that the payoff of the left-alone player affects the likelihood of the grand coalition: when the left-alone player's payoff is larger, all other things equal, the grand coalition is less likely to be formed.

[5] For (3), we can consider "for any π" instead "for some π". This is more pessimistic about the outsiders and corresponds to a treatment of the externalities. See Funaki and Yamato (1999) for a detailed discussion of this point. In our model the both concepts coincide.

This result could be explained by lower guilt aversion (Battigali and Dufwenberg (2007)) of the coalition members towards the stand-alone party, when the latter's payoff is larger.

To account for our findings, we proposed new two notions of domination relation, "domination with cost" which induces a modified stable set K_s^e and a stronger domination dom' that is consistent with a partition function. These concepts explains the experimental outcomes well and can be applied to other theoretical and experimental models. However there are other possibilities to consider a modification of a domination in consistent with a partition function theoretically (see footnote 5, for example).Thus we have to consider which notion is more concrete and reasonable. To consider theoretical results, it is worth to mention that since our new concept of a stronger domination is more restrictive than the original concept of a domination, the core, the famous concept of a cooperative game and given by the set of non-dominated imputations, becomes larger than the original one. But it is not easy to find such a property for the vNM stable set respect to this new domination.

References

[1] Battigalli P, Dufwenberg M (2007). Guilt in games. *Amer. Econom. Rev. Papers Proc.* 97:170–176.

[2] Bolton G., Ockenfels A. (1998). Strategy and Equity : An ERC-analysis of the Güth-van Damme Game. *Journal of Mathematical* Psychology,42, 215-226.

[3] Bolton, G. E., and Ockenfels, A. (2000). ERC : A Theory of Equity, Reciprocity, and Competition. *American Economic Review*, Vol. 90, n° 1, p. 166–193.

[4] Bolton G., Chatterjee K., McGinn K. (2003). How Communication Links Influence Coalition Bargaining: A Laboratory Investigation. *Management Science*, Vol. 49, No. 5, May 2003, pp. 583–598

[5] Fehr E., Schmidt K. (1999). A theory of fairness, competition and cooperation. *Quarterly Journal of Economics*, 114, 817-868.

[6] Güth W., Van Damme E. (1998). Information, Strategic Behavior, and Fairness in Ultimatum Bargaining: An Experimental Study. *Journal of Mathematical Psychology* 42, 227-247.

[7] Güth, W., Pull, K., Stadler, M., Stribeck, A. Equity versus Efficiency? Evidence from Three-Person Generosity Experiments. *Games* 2010, 1, 89-102.

[8] von Neumann, J., Morgenstern, O. (1944 1st ed., 1947 2nd ed., 1953 3rd ed., 2020 60th Anniversary Commemorative Ed.) *Theory of Games and Economic Behavior*, Princeton University Press.

[9] Okada, A., Riedl, A., (2005). Inefficiency and social exclusion in a coalition formation game : experimental evidence. *Games and Economic Behavior*, 50, 278–311.

[10] Peters, H. (2015), *Game Theory: A Multi-Leveled Approach, 2nd edition*, Springer-Verlag, Berlin Heidelberg.

[11] Riedl, A. and Vyrastekova, J. (2003). Responder Behavior in Three-Person Ultimatum Game Experiments. working paper.

APPENDIX

Instructions (translated from French)

Welcome,

The experiment you are going to participate in is intended for the study of decisions. The instructions are simple. If you follow them carefully and make the right decisions, you can earn a fair amount of money. All your responses will be treated anonymously and will be collected through a computer network. You will indicate your choices to the computer in which you are seated, and it will communicate to you the gains that you make as the experiment unfolds.

The total amount of money earned during the experiment will be paid to you, in cash, at the end of the experiment.

GENERAL FRAMEWORK OF THE EXPERIMENT

18 people are participating in this experiment (including you). It involves about 20 periods. At each period you will be assigned to a group of 3 people. So you will only interact with 2 other people in a period. The 2 people with whom you will interact will be chosen at random among the participants present in this room. In total there are therefore 6 groups of 3 people in the room. At the end of each period 6 new groups of 3 people will be formed randomly.

At each period, each member of your group will receive an identifier. There are three possible identities: "player X", "player Y" and "player Z". Once each member of your group is aware of his identity, one of the identities will be drawn to take on the role of Proposer. The Proposer's task is to make a proposal to the other two members of his group.

PROPOSAL

The proposal has two stages:

Stage 1: the proposer chooses one of the following 3 options:
- "distribution between 2 players"
- "distribution between 3 players"
- " skip the turn "

If he chooses the option "skip the turn", one of the other two members of his group will then be randomly selected to become a Proposer and make a choice between the 3 options.

If he chooses the option "distribution between 2 players" he will have to choose the identity of the player with whom he will make this distribution. For example, if the Proposer is player X, he will have to choose whether to make the proposal to player Y or to player Z.

If one of the two options "distribution between 2 players" or "distribution between 3 players" is chosen, the computer will go to step 2 of the proposal.

Stage 2: the proposer must choose a distribution of the amount of points available. The amount of points to be distributed depends on the option that was chosen in step 1:
- "distribution between 2 players": 260 points are to be divided between the 2 players
- "distribution between 3 players": 300 points are to be distributed among the 3 players.

If the "distribution between 2 players" option is chosen, the proposer may offer the other player any amount between 0 and 260 points. It will suffice for the proposer to indicate the amount he is offering to the other player. The computer will automatically calculate the amount remaining for him.

If the “distribution between 3 players” option is chosen, the proposer may offer any amount between 0 and 300 to each of the other two players, provided that the sum does not exceed 300. It will suffice for the proposer to indicate the amount he offers to each. The computer will automatically calculate the amount remaining for him. Of course, the sum of the three amounts will necessarily equal 300.

Each player who receives a proposal has to decide to accept it or to reject it. If the option "split between 2 players" is chosen, only the player concerned by the offer can decide to accept or reject the offer made by the Proposer. If the option "split between 3 players" is chosen, each of the two players receives a proposal and will have to decide whether to accept or reject it. If a player refuses the Proposer's offer, the proposal is canceled. In this case, one of the two players who was not the Proposer will be drawn at random to be the new Proposer for that period. The new Proposer has exactly the same choice options as at the start of the period:
- "distribution between 2 players"
- "distribution between 3 players"
- " skip the turn "

Each time a proposal is refused during a period, a new proposer will be chosen from the group. The period will end when a proposal is accepted by all players who have received a proposal or if the time limit is exceeded. When a proposal is accepted, each player in the group receives the amount corresponding to that offer. If the time limit is reached during a period, each player will receive 0 points for that period. Your group will have at least 2 minutes for each period.

At the end of each period, 6 new groups will be formed, so that you will interact with two other people. The same rules will apply for each of the periods.

At the end of the session, you will receive a capital in points, the calculation of which is as follows: the points earned during all the periods (approximately 20 periods) will be added together and the total will be divided by the number of periods in order to calculate your average gain over all the periods. This amount will then be converted into Euros according to the following rule:

1 Euro = 10 points.

Good luck !

TABLES AND FIGURES

Group	2P	3P	0
1.1	47	12	1
1.2	38	22	0
1.3	44	16	0
1.4	58	2	0
1.5	42	18	0
1.6	53	7	0
2.1	54	6	0
2.2	57	3	0
2.3	58	2	0
2.4	55	5	0
2.5	58	2	0
2.6	59	1	0
Total	623	96	1

Table 1 : Frequencies of *2P*, *3P* and no coalition per group

	2P			**3P**		
Group	**Initial**	#	%	**Initial**	#	%
	Proposed	Rejected	Rejected %	Proposed	Rejected	Rejected %
1.1	39	7	17.95%	21	10	47.62%
1.2	33	4	12.12%	27	8	29.63%
1.3	44	7	15.91%	16	3	18.75%
1.4	54	14	25.93%	6	6	100.00%
1.5	43	16	37.21%	17	6	35.29%
1.6	50	10	20.00%	10	4	40.00%
Total	263	58	22.05%	97	37	38.14%
2.1	53	12	22.64%	7	3	42.86%
2.2	54	7	12.96%	6	3	50.00%
2.3	54	9	16.67%	6	4	66.67%
2.4	44	11	25.00%	16	10	62.50%
2.5	58	7	12.07%	2	2	100.00%
2.6	54	8	14.81%	6	6	100.00%
Total	317	54	17.03%	43	28	65.12%

Table 2 : Rejection frequencies of initial proposals

	2P		3P	
Group	**Initial**	**Final**	**Initial**	**Final**
1.1	39	47	21	12
1.2	33	38	27	22
1.3	44	44	16	16
1.4	54	58	6	2
1.5	43	42	17	18
1.6	50	53	10	7
2.1	53	54	7	6
2.2	54	57	6	3
2.3	54	58	6	2
2.4	44	55	16	5
2.5	58	58	2	2
2.6	54	59	6	1
Total	580	623	140	96

Table 3: Starting and ending coalitions

Group	**1 - 5**	**6 - 10**	**11 - 15**	**16 - 20**
1.1	7	12	14	14
1.2	6	7	11	14
1.3	7	7	15	15
1.4	13	15	15	15
1.5	9	9	11	13
1.6	12	13	14	14
Total	**54**	**63**	**80**	**85**
%	*60.00*	*70.00*	*88.89*	*94.44*

(L-Treatment)

Group	**1 - 5**	**6 - 10**	**11 - 15**	**16 - 20**
2.1	9	15	15	15
2.2	13	14	15	15
2.3	13	14	15	15
2.4	11	14	15	15
2.5	14	14	15	15
2.6	14	15	15	15
Total	**74**	**86**	**90**	**90**
%	*82.22*	*95.56*	*100.00*	*100.00*

(H-Treatment)

Table 4: Evolution of the frequency of 2 player coalitions per group (blocks of 5 periods)

L-Treatment			H-Treatment		
Group	**2 P**	**3P**	**Group**	**2 P**	**3P**
1.1	87.23	100	2.1	68.52	100
1.2	65.58	95.45	2.2	87.72	100
1.3	61.36	93.75	2.3	77.19	100
1.4	36.20	100	2.4	32.73	40
1.5	54.76	100	2.5	98.27	50
1.6	15.09	57.14	2.6	89.65	100

Table 5: Frequencies of equal division per group in *3P* coalitions and *2P* coalitions

Sequence	1.1	1.2	1.3	1.4	1.5	1.6
1-5	2.50	3.75	33.75	30.67	29.00	34.67
6-10	4.00	4.00	24.00	24.67	17.00	44.00
11-15	1.33	5.00	10.00	11.33	8.67	51.00
16-20	2.00	4.67	1.73	5.33	4.67	28.67

[L-Treatment]

Sequence	2.1	22	2.3	2.4	2.5	2.6
1-5	18.00	17.47	14.67	52.00	2.00	9.33
6-10	14.00	0.67	8.00	30.67	0.00	1.33
11-15	2.00	1.33	4.67	18.00	0.00	0.00
16-20	0.00	0.00	2.00	10.00	0.00	0.00

[H-Treatment]

Table 5: Average payoff difference in *2P* coalitions per 5 periods sequences

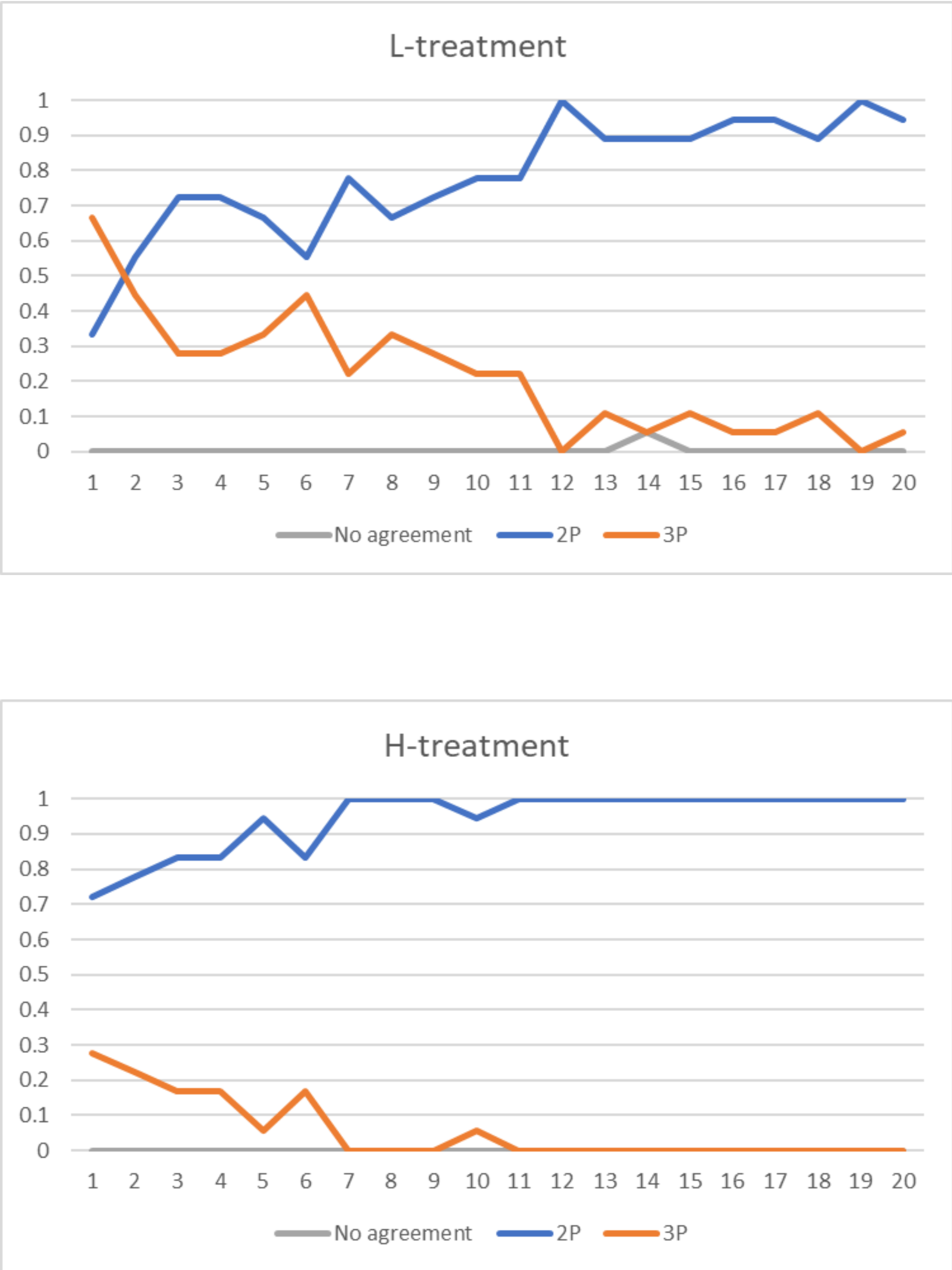
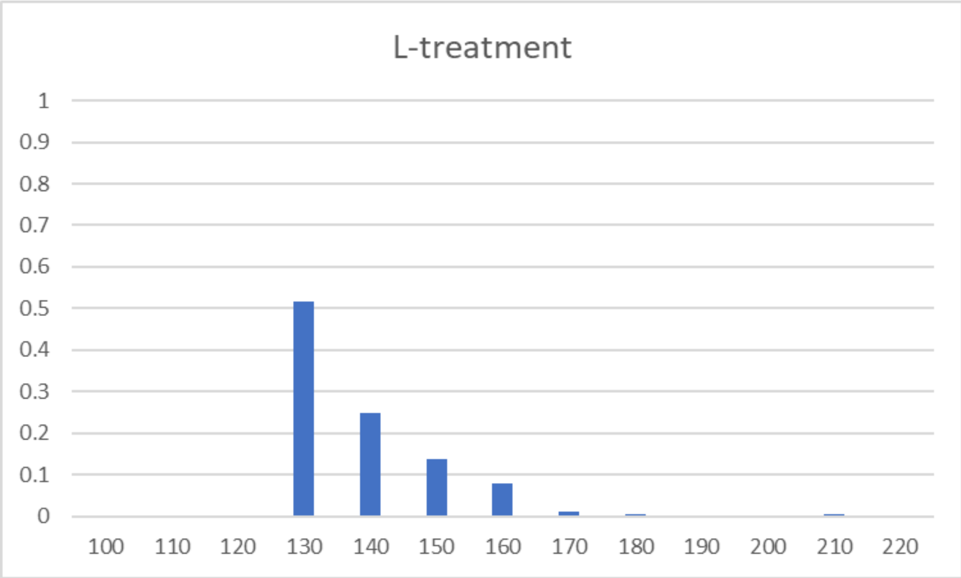


Figure 1 : Frequencies of 2P and 3P coalitions over time



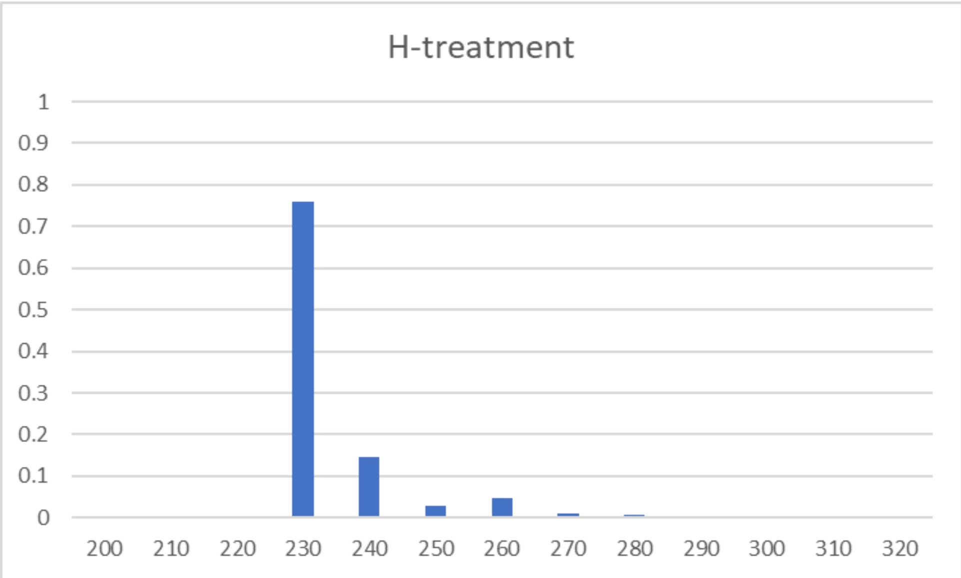


Figure 2: Frequency distribution of payoff divisions in 2P coalitions

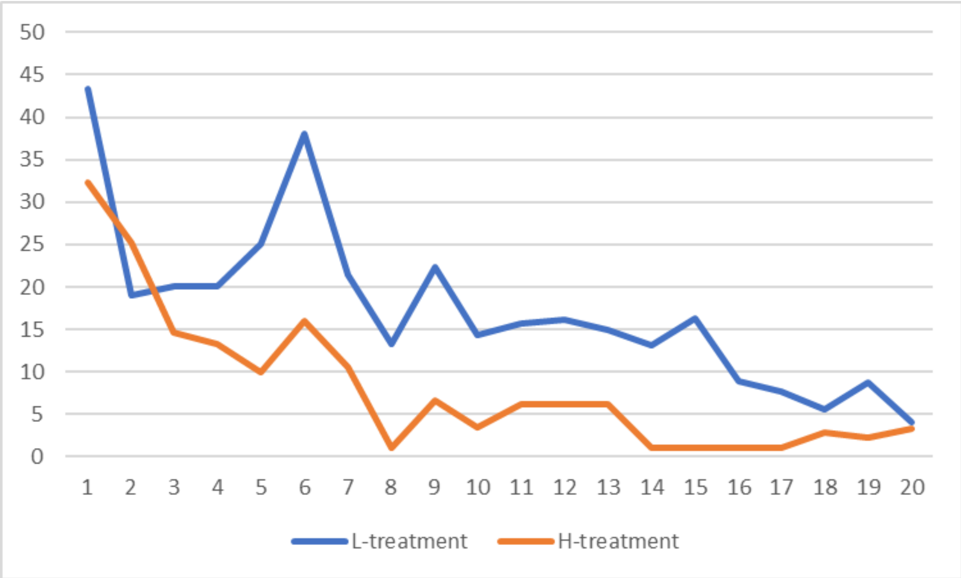


Figure 3: Average payoff differences in 2P coalitions over time

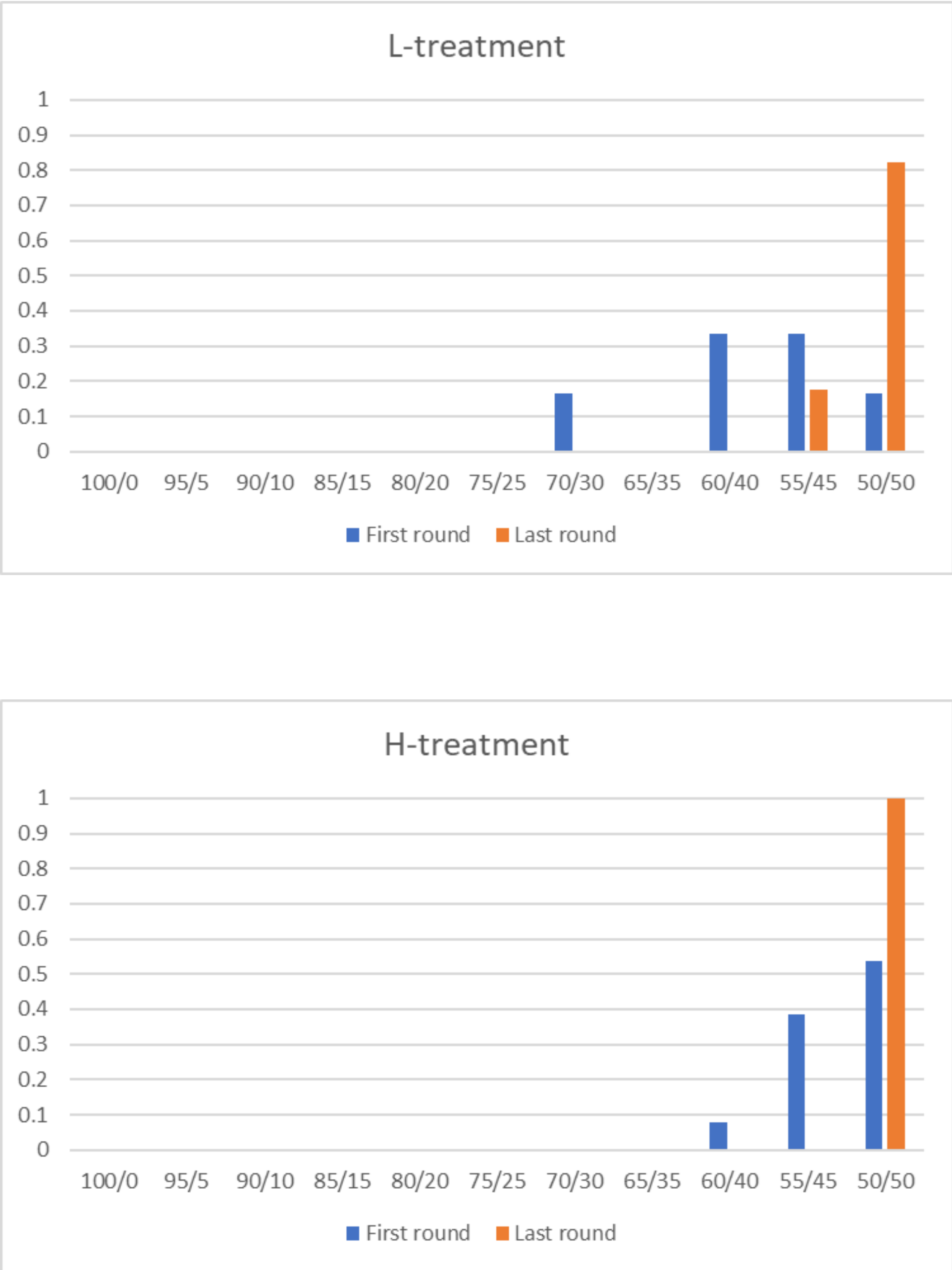


Figure 4: Average payoff distributions in 2P in the first and last rounds

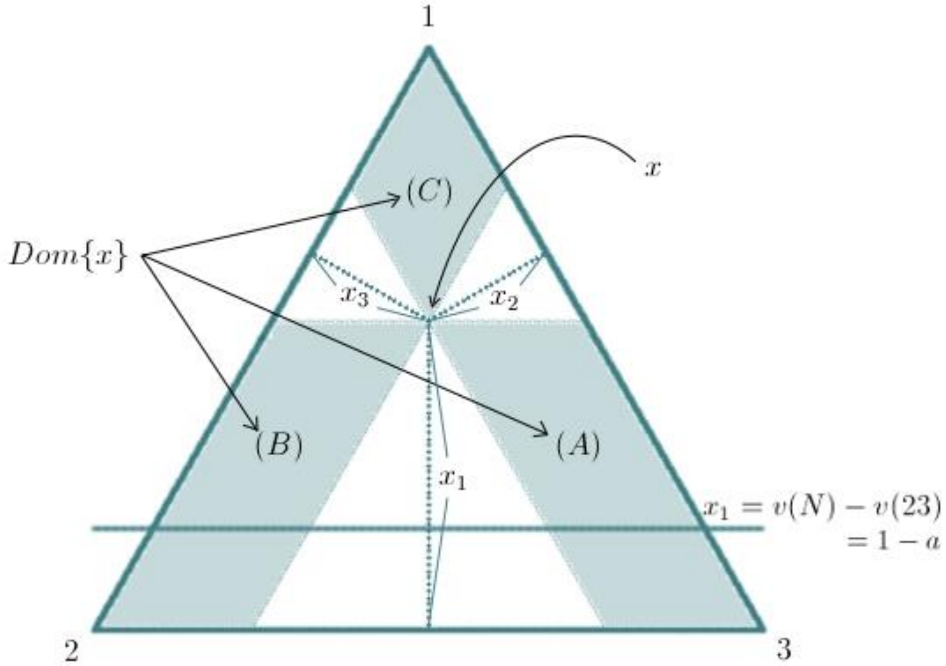


Figure 5: Dominated region of x

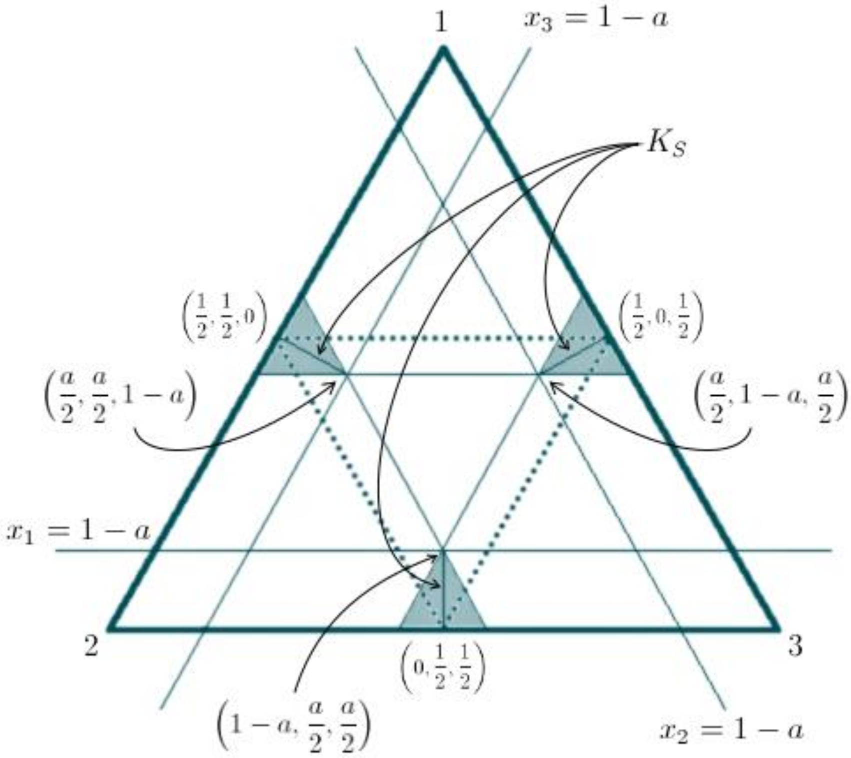


Figure 6: Symmetric stable set.

CEE-M Working Papers[1] - 2021

WP 2021-01	Philippe Mahenc & **Alexandre Volle** « Price Signaling and Quality Monitoring in Markets for Credence Goods »
WP 2021-02	Mamadou Gueye, **Nicolas Quérou**, & **Raphael Soubeyran** « Inequality Aversion and the Distribution of Rewards in Organizations »
WP 2021-03	Lesly Cassin, Paolo Melindi-Ghidi & **Fabien Prieur** « Voting for environmental policy with green consumers: the impact of income inequality »
WP 2021-04	**Marion Davin** & **Emmanuelle Lavaine** « The Role Of Health At Birth And Parental Investment In Early Child Development. Evidence From The French ELFE Cohort »
WP 2021-05	**Ko Serge William Yao**, **Emmanuelle Lavaine & Marc Willinger** « Does the approval mechanism induce the effcient extraction in Common Pool Resource games? »
WP 2021-06	**Raphaël Soubeyran** « Pro-social Motivations, Externalities and Incentives »
WP 2021-07	**Murielle Djiguemde, Dimitri Dubois, Alexandre Sauquet & Mabel Tidball** « Continuous versus Discrete Time in Dynamic Common Pool Resource Game Experiments »
WP 2021-08	**Marion Davin**, Mouez Fodha & Thomas Seegmuller « Environment, public debt and epidemics »
WP 2021-09	**Marc Willinger**, Oussama Rhouma & Klarizze Anne Puzon « Veto power and coalition formation in the commons: an experiment »
WP 2021-10	Yukihiko Funaki, **Emmanuel Sol**, **Marc Willinger** « Equal division among the few: an experiment about a coalition formation game »

[1] **CEE-M Working Papers / Contact** : laurent.garnier@inra.fr

- RePEc https://ideas.repec.org/s/hal/wpceem.html
- HAL https://halshs.archives-ouvertes.fr/CEE-M-WP/