



**HAL**  
open science

# The evolutionary stability of in-group altruism in productive and destructive group contests

Guillaume Cheikbossian

► **To cite this version:**

Guillaume Cheikbossian. The evolutionary stability of in-group altruism in productive and destructive group contests. *Journal of Economic Behavior and Organization*, 2021, 188, pp.236-252. 10.1016/j.jebo.2021.05.017 . hal-03233980

**HAL Id: hal-03233980**

**<https://hal.inrae.fr/hal-03233980>**

Submitted on 13 Jun 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

# The evolutionary stability of in-group altruism in productive and destructive group contests\*

Guillaume Cheikbossian, CEE-M (UNIVERSITY OF MONTPELLIER - INRAE- CNRS - SUPAGRO)

May 2021

**Abstract:** This article examines the evolutionary stability of other-regarding preferences in a group contest for a prize, which is endogenously determined. In a destructive contest, such as war, contest efforts of all groups decrease the value of the prize. In contrast, in a productive contest, such as a patent race, contest efforts of all groups increase the value of the prize. The indirect evolutionary approach allows to endogenize players' preferences, that is, the utility weights given by a group member, in her subjective utility function, to the material payoffs of in-group and out-group members. After characterized the set of evolutionarily stable preference types, I show that the evolutionary stable degree of in-group altruism is always stronger when the group contest is destructive than when it is productive. Moreover, when the group contest is *strongly* productive, preference evolution leads to in-group spite. However, a smaller group size and a larger number of competing groups makes this outcome less likely.

*Keywords:* Group Contests; Endogenous Prize; In-group Altruism; Evolutionary Stability; Indirect Evolutionary Approach

*JEL classification:* D72; D74

\*I am very grateful to two reviewers whose detailed and extremely helpful comments helped me to clarify several arguments, and to greatly improve the paper. I would also like to thank an anonymous Associate Editor for his/her recommendations. The usual disclaimer applies.

# 1 Introduction

From ancient times, intergroup competition and conflict have been ubiquitous in human societies. Thus, they have been extensively studied within different disciplines, including social psychology (Böhm et al., 2020), political science (Lopez and Johnson, 2020), and economics (Kimbrough et al., 2020). In particular, early studies in social psychology have pointed out that in-group favoritism can emerge in intergroup conflict even though groups are not formed according to some intrinsic characteristics, but by random assignment (Tajfel and Turner, 1979; Tajfel, 1982). More recent experimental works by social psychologists show, in simple team games, that intergroup competition improves coordination within the group and, hence, enhances group effort (see, e.g., Rapoport and Bornstein, 1987, 1989; Bornstein, 1992, 2003; Bornstein et al., 2002). Economists have also conducted experimental studies with monetary incentives, and generally conclude that subjects over-contribute to group effort in group contests, compared to the predictions implied by the *Homo economicus* paradigm (see, e.g., Abbink et al., 2010, 2012; and for a survey, see Sheremeta, 2018).

While there is an important literature in human evolutionary biology that explains how genetic relatedness, or kinship, can result in "parochial altruism" – the combination of in-group altruism and out-group hostility – in the context of warfare, there are very few theoretical analyses of the evolutionary stability of *non-kin* altruism within competing groups.<sup>1</sup> Eaton et al. (2011) analyze in-group favoritism and out-group spite by using a "guns versus butter" model, where groups compete for a common access resource that is processed to produce a consumption good. Players' preferences are observable and population is infinite. They show (numerically) that intergroup hostility falls, while intragroup solidarity rises as the resource becomes more abundant. Konrad and Morath (2012) analyze the evolutionary stability of in-group favoritism and out-group spite in a model of group contest for an exogenous prize. The population is finite, and players' preferences are private information. Thus, they introduce the concept of "robust beliefs" such that any player with a certain preference type believes that all other players are of the same type (and have the same "robust beliefs"). In this context, they characterize the set of evolutionarily stable preference types, which involve a linear combination of the two parameters reflecting in-group altruism and out-group spite, with the two traits being perfect substitutes.

In this paper, I also analyze the evolutionary stability of non-kin altruism in group contests, but I consider that the value of the contested prize changes with the aggregate effort of all groups.<sup>2</sup> Indeed, although the fixed-value assumption appears to be reasonable in some specific contexts – such as in the case of lobbying competition for quotas – it is much more questionable in a variety of real-world situations. On the one hand, intergroup conflict can reduce the value of the contested prize. The group contest is thus *destructive*. One can

---

<sup>1</sup>For studies on "parochial altruism", see Choi and Bowles (2007), Lehmann and Feldman (2008), and for a survey, see Rusch (2014), Glowacki et al. (2020), and Rusch and Gravilets (2020). There are also few theoretical analyses of "gene-culture coevolutionary" processes leading to human altruism in group conflicts (see, e.g., Bowles, 2006; Alger et al., 2020).

<sup>2</sup>In another paper (Cheikbossian, 2021), I investigate the evolutionary stability of in-group altruism in group contests for an exogenous prize. As in Eaton et al. (2011), I consider an infinite population whose preferences are observable, so that a change in a player's preference type induces changes in other players' equilibrium actions. I show that group members are more likely to be altruistic towards each other under preference evolution when the number of competing groups, and the degree of complementarity between individual efforts, are both relatively large.

think, for example, of a tribal war for land and natural resources that are damaged by the conflict, thus diminishing their economic value for the victorious tribe. On the other hand, intergroup competition can result in wealth creation, thus increasing the value of the prize. The group contest is thus *productive*. One can think, for example, of R&D investments to obtain a monopoly right, or even to the competition in medical sciences and biology between research teams for finding a vaccine. Research efforts by different teams produce knowledge externalities, that can ultimately benefit the research team(s) that successfully developed the vaccine.

The equilibrium concept is that of *Evolutionarily Stable Strategies*, introduced by evolutionary biologists Maynard Smith and Price (1973), and adapted to the case of finite population by Schaffer (1988). An ESS is such that it cannot be invaded by another strategy in the sense that if the whole population of players adopts it, then, there is no other strategy that a player could use for obtaining a higher *relative* payoff or *fitness*.

In group contests, the evolutionary success of an individual player depends on his material payoff relative, not only to the material payoffs of his teammates, but also relative to the material payoffs of out-group members. If, for instance, a player increases his contribution to collective effort above the level corresponding to selfish behavior, it increases the probability of success of his group to the detriment of the rival groups and, hence, it can increase his fitness relative to that of out-group members. However, it also decreases the player's fitness relative to that of his teammates because they benefit from the increase in the probability of success of the group, without having exerted additional effort. Overall, as shown by Konrad and Morath (2012), group members expend more effort in the evolutionarily stable equilibrium than in the standard Nash equilibrium when the value of the prize is constant. I demonstrate that this result still holds when the group contest is destructive or when it is productive, provided the elasticity of the prize, with respect to change in the aggregate effort of all groups, is sufficiently small. If, however, the group contest is quite productive, in that the elasticity of the prize is relatively large, group members exert less effort in the evolutionarily stable equilibrium, than if they were to maximize their material payoff only.

Next, I use the *indirect evolutionary approach*, pioneered by Güth and Yaari (1992), for endogenizing players' preferences. In other words, evolution does not play directly at the level of strategies, but indirectly at the level of preferences, which determine players' actions and, in turn, individual material payoff.<sup>3</sup> Following Konrad and Morath (2012), I also consider that players have "robust beliefs" about others' preferences, and then determine which objective function(s) can achieve the equilibrium level of effort in evolutionarily stable strategies. This approach has the advantage of interpreting evolutionarily stable behaviors as the result of evolutionarily stable interdependent preferences of conscious and rational players.

Specifically, the preference type of a player is a couple of parameters, corresponding to the utility weights given by the player, in his *subjective utility*, to the material payoffs of his teammates and to the material payoffs of the members of the rival groups. I first characterize the set of evolutionarily stable preference types without restricting the signs of the two preference parameters in any way. I then show that several combinations of in-group altru-

---

<sup>3</sup>For early analyses of preference evolution in games between single players, see Bester and Güth (1998), Sethi and Somanathan (2001), Ok and Vega-Redondo, (2001), Dekel et al. (2007), Heifetz et al. (2007), and for a recent survey, see Alger and Weibull (2019). For the evolutionary stability of strategies and preferences in contests between single players, see Eaton and Eswaran (2003), Hehenkamp et al. (2004), and Leiminger (2009).

ism/spite and out-group altruism/spite can constitute an equilibrium in evolutionarily stable preferences, whether the contest is destructive or productive. Interestingly, however, when the elasticity of the prize is sufficiently large, but not too large relative to the ‘decisiveness parameter’ of the contest success function, parochial altruism – i.e. in-group altruism *and* out-group spite – cannot be an evolutionarily stable preference type.

Next, I assume that the subjective utility function of each player is based on his own material payoff, and on the material payoffs of *either* his teammates, *or* his opponents. When preferences towards the out-group members are ignored, the evolutionarily stable utility weight given by a group member to the material payoffs of his teammates is decreasing in the elasticity of the prize with respect to change in the aggregate effort of all groups. If this elasticity is negative, the group contest is destructive and preference evolution leads to a positive degree of *in-group altruism*, that induces a higher level of effort than in the symmetric Nash equilibrium. This, in turn, aggravates the destructiveness of the conflict between groups. When the elasticity of the prize is positive, but sufficiently small, group members are still altruistic towards each other under preference evolution. However, the evolutionarily stable degree of in-group altruism is lower than when the group contest is destructive. Finally, when the elasticity of the prize is relatively large, then *in-group spite* emerges in the equilibrium in evolutionarily stable preferences, thus resulting in a lower level of aggregate effort than in a monomorphic population of selfish players. I also investigate the symmetric situation where in-group preferences are ignored, and explain that an equilibrium in evolutionarily stable out-group preferences may fail to exist in this case.

Finally, to obtain additional results on the impact of the ‘economic environment’ on the evolutionarily stable equilibrium, I consider that the prize function is quasi-linear in the initial value of the prize, and focus on evolutionarily stable in-group preferences in the case of productive contests. I then show that a larger group size makes in-group spite more likely to emerge under preference evolution. In contrast, a stronger rivalry in the environment, as measured by the number of competing groups, or a larger initial prize value, makes in-group altruism more likely to be evolutionarily stable.

The basic framework is a simple group contest game (see Katz et al., 1990; Nitzan, 1991; and for a survey, see Konrad, 2009). The case where the value of the prize is endogenous has been investigated in contests between single players. Chung (1996) considers the case of productive contests, and shows that the amount of aggregate effort can be larger in the Nash equilibrium than in the social optimum. Shaffer (2006) considers both productive and destructive contests with symmetric players, while Chowdhury and Sheremeta (2011), Hirai and Szidarovszky (2013), and Damianov et al. (2018), extend the analysis to the case of asymmetric (productive and destructive) contests between single players. In the present analysis, I consider contests, not between single agents, but between groups of players. More importantly, I analyze the evolutionary stability of non-kin altruism within groups in group contests with endogenous prizes.

This article is organized as follows. Section 2 presents the model of a group contest with an endogenous prize and derives the Nash solution. Section 3 characterizes evolutionarily stable strategies in the group contest. In Section 4, I introduce the indirect evolutionary approach for characterizing evolutionarily stable preferences. In Section 5, I suppose that the value of the prize is a quasi-linear function, and analyze the impact of the ‘economic environment’ on the evolutionarily stable equilibrium. Finally, Section 6 concludes.

## 2 The group contest game

I consider a finite number of players  $N$  who are randomly partitioned into  $m$  groups of  $n$  players (with  $n \geq 2$  and  $m \geq 2$ ). The groups compete for a prize  $G$ . Let  $(i, j)$  for  $i \in I = \{1, 2, \dots, n\}$  and  $j \in J = \{1, 2, \dots, m\}$ , denote member  $i$  of group  $j$ .  $x_{ij} \geq 0$  is the amount of effort expended by player  $(i, j)$ , and I assume that each player has the same finite endowment  $w$ , so that  $x_{ij} \leq w$ . Let  $W = Nw$  be the aggregate endowment, and  $X_j = \sum_{i=1}^n x_{ij} \in \mathbb{R}_+$  be the total effort of group  $j$ . The probability of winning the prize for each group depends on its collective effort relative to the collective efforts of competing groups according to a Contest Success Function (CSF), which has the ratio-form. Specifically, given the vector of group efforts  $\mathbf{X} \equiv (X_1, \dots, X_m) \in \mathbb{R}_+^m$ , the winning probability of group  $j$ ,  $p_j : \mathbb{R}_+^m \rightarrow [0, 1]$ , is

$$p_j(\mathbf{X}) = \begin{cases} \frac{X_j^r}{X_j^r + \sum_{k \neq j}^m X_k^r} & \text{if } X_j^r + \sum_{k \neq j}^m X_k^r \neq 0, \\ \frac{1}{m} & \text{otherwise,} \end{cases} \quad (1)$$

where  $r \in (0, 1]$  is a ‘decisiveness parameter’ that measures the responsiveness of the probability of winning to the ratio of group efforts.<sup>4</sup>

In case of winning, each member receives an equal amount of the prize  $G$  that depends on the aggregate effort  $X = \sum_{j=1}^m X_j$ . I assume that  $G(X)$  satisfies the following properties.

### Assumption 1.

- (i)  $G(0)$  is strictly positive and bounded;
- (ii)  $G(X)$  is twice continuously differentiable for all  $X \in (0, W]$ ;
- (iii) For all  $X \in [0, W]$ ,  $G(X) > 0$  and  $G(X)$  satisfies **either** (iiia) **or** (iiib) where:
  - (iiia)  $G'(X) \geq 0$  and  $G''(X) \leq 0$ ;
  - (iiib)  $G'(X) \leq 0$ ,  $\epsilon_X > \eta$  and  $\sigma_X > \eta$ , where  $\epsilon_X = G'(X)X/G(X)$ ,  $\sigma_X = G''(X)X/G'(X)$  and  $\eta \equiv -r(m-1)$ ;
- (iv) There exists  $\bar{X} \in (0, W]$  such that  $G(\bar{X}) - \bar{X} = 0$ .

Property (iii) states that  $G(X)$  is always strictly positive, and allows for both productive and destructive contests, that is, for situations where the value of the prize increases or decreases with the aggregate effort of all groups. If the group contest is productive – i.e.  $G'(X) \geq 0$  – it is assumed that the value of the prize increases with  $X$  at a decreasing rate – i.e.  $G''(X) \leq 0$ . This implies that the elasticity of the prize with respect to change in the aggregate effort – denoted by  $\epsilon_X$  – is *strictly* lower than 1, for any  $X \in [0, W]$ .<sup>5</sup>

When the group contest is destructive – i.e.  $G'(X) \leq 0$  – the elasticity  $\epsilon_X$  is negative, and the existence of a symmetric interior Nash equilibrium in contest efforts, and of an

<sup>4</sup>For an axiomatization of group contest success functions, see Munster (2009).

<sup>5</sup>The sign of  $\epsilon_X - 1 = [G'(X)X - G(X)]/G(X)$  is the same as the sign of its numerator, which is decreasing in  $X$ . Indeed, the derivative of the numerator with respect to  $X$  is given by  $G''(X)X$ , which is negative, under the weak concavity assumption of the  $G(\cdot)$  function in the case of a productive contest. The numerator of  $\epsilon_X - 1$  thus reaches a maximum in  $X = 0$ , in which case it becomes  $-G(0) < 0$ , under property (i). This implies that  $\epsilon_X - 1 < 0$ .

evolutionarily stable equilibrium, requires that this elasticity is not "too" small (or "too" large in absolute value) – i.e.  $\epsilon_X > \eta$  (with  $\eta < 0$ ). However, unlike the case of a productive contest, we do not restrict the  $G(\cdot)$  function to be always a concave function. It can be a convex function as well – i.e.  $G''(X) \geq 0$  – which may be more plausible in the case of destructive conflicts about natural resources. Yet, it must not be "too convex" for equilibrium existence. It boils down to a lower bound, which is negative, on the elasticity of the  $G(\cdot)$  function with respect to change in the aggregate effort. Specifically, this elasticity – denoted  $\sigma_X$  – must have the same lower bound than the elasticity of the  $G(\cdot)$  function itself with respect to  $X$  – i.e.  $\sigma_X > \eta$  (with  $\eta < 0$ ).<sup>6</sup>

Finally, property (iv) is similar to Assumption 3 in Chung (1996), and is also required for the existence of a symmetric interior Nash equilibrium. It simply states that there exists a finite and strictly positive level of aggregate effort  $\bar{X}$ , such that social welfare is equal to 0. Together with (iii), it implies that social welfare is strictly positive for any level of aggregate effort below  $\bar{X}$  – i.e.  $G(X) - X > 0$  for  $X \in [0, \bar{X})$ .

Note also that  $G(X)$  depends on the number of competing groups  $m$  and on group size  $n$ , because the aggregate effort  $X$  depends on the total number of players involved in the contest. Furthermore, in group contests, a distinction must be made between the total value of the prize and the per-capita value of the prize for the winning group. The per-capita value of the prize can also depend *directly* – and negatively – on group size according to the degree of rivalry of the prize. In order to save on notation, I shall simply write  $G(X)$  for the value of the prize received by *each* member of the winning group, with the understanding that this per-capita value can also, in general, depend directly on group size  $n$ .

Finally, I assume that all players incur a linear cost function of effort. As a result, the expected *material payoff* of player  $(i, j) \in I \times J$  is given by  $\Pi_{ij} : \mathbb{R}_+ \times [0, 1] \times [0, w] \rightarrow \mathbb{R}_+$ , that is,

$$\Pi_{ij} = p_j(\mathbf{X})G(X) - x_{ij}. \quad (2)$$

Let us first characterize the equilibrium induced by Nash behavior. One can establish the following Proposition.

**Proposition 1.** *Under Assumption 1, there exists a unique symmetric pure-strategy Nash equilibrium. The equilibrium aggregate effort  $X^N$  must satisfy*

$$(m - 1) [r\Phi(X^N) - 1] + [G'(X^N) - 1] = 0, \quad (3)$$

where  $\Phi(X^N) \equiv G(X^N)/X^N$ .<sup>7</sup>

**Proof.** Appendix A1.  $\square$

It is worth emphasizing that, due to the public-good nature of group contests, there exists a continuum of (asymmetric) pure-strategy Nash equilibria. However, we consider the unique symmetric equilibrium such that all members of all groups exert the same level of effort.

<sup>6</sup>This condition is always satisfied when the  $G(\cdot)$  function is decreasing and concave since then  $\sigma_X > 0$ .

<sup>7</sup> $\Phi(X)$  is a decreasing function. We have  $\Phi'(X) = [G'(X)X - G(X)]/X^2$ , which is always negative. Indeed, when the contest is destructive, we have  $G'(X) < 0$ . When the contest is productive, the numerator of  $\Phi'(X)$  is also negative under the concavity assumption of the  $G(\cdot)$  function (see Footnote 5).

Also, the Nash equilibrium is, in general, inefficient. This is obvious when the group contest is destructive. When the contest is productive, the (in-)efficiency of contest efforts is more ambiguous because, as first pointed out by Chung (1996), there are two opposing effects. On the one hand, there is a negative rent-seeking – or unproductive – effect in that all groups expend resources to increase their probability of winning the prize, but neutralize each other in the symmetric equilibrium. On the other hand, there is a positive – or productive – effect in that the value of the prize increases with the aggregate effort of all groups. Consequently, in productive contests, the Nash equilibrium can generate too much or too low amounts of aggregate effort compared to the social optimum.<sup>8</sup>

### 3 Evolutionarily stable strategies

Let us now determine which level of individual effort is an *evolutionarily stable strategy* (ESS) by using Schaffer (1988)'s definition of ESS for finite populations, and adapted to group contests by Konrad and Morath (2012). Suppose that, initially, all players adopt the same strategy by expending the same amount of effort  $x$ . A *mutant* strategy  $x_M \neq x$  adopted by a single player can *invade*  $x$ , if the material payoff of the mutant player using  $x_M$  is *strictly* larger than that of the other players using  $x$ . A strategy  $x^E \in [0, w]$  is an ESS, if *it cannot be invaded* by any other strategy. As already mentioned, this equilibrium concept originates from theoretical biology. The underlying justification is that, in the dynamic process of mutations and of adjustments to ESS (not modeled here), more successful strategies have an advantage in reproducing, thus leading to the extinction of less successful strategies. And, in an economic context, success is simply identified with material payoff.

Let  $\Pi_M(x_M, \mathbf{x}_{-M}^E)$  be the expected material payoff of the mutant player who exerts an amount of effort  $x_M \in [0, w]$ , given that all other players exert the effort level  $x^E$ . We have

$$\Pi_M(x_M, \mathbf{x}_{-M}^E) = \frac{[x_M + (n-1)x^E]^r}{[x_M + (n-1)x^E]^r + (m-1)[nx^E]^r} G(x_M, \mathbf{x}_{-M}^E) - x_M. \quad (4)$$

Similarly, let  $\Pi_{-M}(x_M, \mathbf{x}_{-M}^E)$  be the expected material payoff of a player exerting the effort level  $x^E$ , given that  $mn - 2$  other players also spend  $x^E$ , and that there is one mutant player expending an amount of effort  $x_M$ . Each player expending  $x^E$  has a probability  $(n-1)/(mn-1)$  of being matched with the mutant player in the same group, and a probability  $n(m-1)/(mn-1)$  of belonging to a group with all players expending  $x^E$ . Therefore, we have

$$\Pi_{-M}(x_M, \mathbf{x}_{-M}^E) = \frac{n-1}{mn-1} \left[ \frac{[x_M + (n-1)x^E]^r}{[x_M + (n-1)x^E]^r + (m-1)[nx^E]^r} G(x_M, \mathbf{x}_{-M}^E) - x^E \right]$$

---

<sup>8</sup>Specifically, the socially efficient level of aggregate effort is  $X^* \equiv \arg \max_X G(X) - X$ , and thus satisfies  $G'(X^*) = 1$ . Using (3), the Nash equilibrium is efficient – i.e.  $X^N = X^*$  – when  $r = r^*$  with  $r^* = 1/\Phi(X^*) = X^*/G(X^*) \in (0, 1)$ . Furthermore, as shown in the proof of Proposition 1, the left-hand term of Eq. (3) is decreasing in  $X$ , and is equal to 0 for  $X = X^N$ . Hence, if  $r > r^*$ , the left-hand term of Eq. (3) becomes strictly positive – i.e.  $(m-1)[r\Phi(X^*) - 1] > 0$  – which implies that  $X^* < X^N$ , because  $\Phi(\cdot)$  and  $G'(\cdot)$  are decreasing functions. A symmetric reasoning applies for  $r < r^*$ . In conclusion, if  $r > r^*$  ( $r < r^*$ ) the aggregate effort in the Nash equilibrium is greater (lower) than the efficient level in the case of a productive contest (see also Chung, 1996).



$$+ \frac{n(m-1)}{mn-1} \left[ \frac{[nx^E]^r}{[x_M + (n-1)x^E]^r + (m-1)[nx^E]^r} G(x_M, \mathbf{x}_{-M}^E) - x^E \right]. \quad (5)$$

For  $x^E$  to be an ESS, one must have  $\Pi_M(x_M, \mathbf{x}_{-M}^E) \leq \Pi_{-M}(x_M, \mathbf{x}_{-M}^E)$  for all  $x_M \in [0, w]$ . An alternative equivalent definition of ESS is as follows:

**Definition 1.** Let  $\mathbf{x}_{-M}^E$  be the vector of efforts of all  $mn-1$  players – other than the mutant player  $M$  – who all expend the effort level  $x^E \in [0, w]$ . Then,  $x^E$  is an evolutionarily stable strategy (ESS) if it is a solution to:

$$\underset{x_M}{\text{Max}} \Pi_M(x_M, \mathbf{x}_{-M}^E) - \Pi_{-M}(x_M, \mathbf{x}_{-M}^E), \quad (6)$$

for  $x_M \in [0, w]$ .

Thus, if all players choose  $x^E$ , a mutant player cannot obtain a strictly higher *relative* payoff than ESS players by choosing a different strategy  $x_M \neq x^E$ . In other words, we search for an evolutionarily stable strategy  $x^E$  that is a solution of (6) for all  $x_M \in [0, w]$ , in which case (6) as a function of  $x_M$  would reach a global maximum at  $x_M = x^E$ . When the prize is exogenous and is independent of players' efforts, such an ESS exists.<sup>9</sup> When the prize is endogenous and depends on the aggregate effort of all groups, we can show that an ESS still exists on the condition that we use the restriction of *local* stability, as introduced by Alós-Ferrer and Ania (2001). Thus, let us introduce the following definition:

**Definition 2.**  $x^E$  is a locally evolutionarily stable strategy (ESS) if it is a solution of (6) for  $x_M$  in some neighborhood of  $x^E$ .

In other words,  $x^E$  is *locally* evolutionarily stable if (6) as a function of  $x_M$  reaches a *local* maximum at  $x_M = x^E$ . This definition of local ESS differs from what Schaffer (1988) calls a 'playing the field ESS'. The requirement of local stability is however sufficient if we consider that the evolutionary process is such that mutating players enter the game with similar – but slightly deviant – behaviors compared to the incumbent behavior. One can indeed imagine that behaviors are driven by social (or group) norms that change gradually, thus preventing too deviant behaviors.

I can now state the following result.

**Proposition 2.** Under Assumption 1, there exists a unique locally ESS  $x^E$ , and it is characterized by

$$\Phi(X^E) = \frac{mn-1}{rn(m-1)}, \quad (7)$$

where  $\Phi(X^E) \equiv G(X^E)/X^E$ , with  $X^E = mnx^E$ .

**Proof.** Appendix A2.  $\square$

---

<sup>9</sup>When  $G'(X) = 0$ , it can be verified that the second derivative of  $\Pi_M(x_M, \mathbf{x}_{-M}^E) - \Pi_{-M}(x_M, \mathbf{x}_{-M}^E)$  with respect to  $x_M$  is strictly negative for any  $x^E > 0$  (see the proof of Proposition 2). Furthermore,  $x^E = 0$  cannot be an ESS since then  $\Pi_M(x_M, \mathbf{x}_{-M}^E) - \Pi_{-M}(x_M, \mathbf{x}_{-M}^E)$  becomes strictly positive by expending an arbitrarily small amount  $x_M$ .

Under Assumption 1,  $\Phi(X)$  is decreasing in  $X$ , whether the contest is productive or destructive (see Footnote 7). Consequently,  $x^E$  and  $X^E$  are both increasing in  $r$ , because the right-hand term of (7) is decreasing in  $r$ . Again, this parameter is a measure of the sensitivity of the probability of winning the contest to the ratio of group efforts. As  $r$  increases, the contest technology becomes more responsive to any differences in group efforts. Thus, the result that the evolutionarily stable equilibrium level of effort increases with  $r$  is very intuitive.

One can also observe that the right-hand-term of (7) is decreasing in the number of competing groups  $m$ , implying that  $X^E$  is increasing in  $m$ . The individual level of effort does not, however, necessarily increase in  $m$ , because  $x^E = X^E/mn$ . Note, also, that the right-hand term of (7) is increasing in group size  $n$ . Furthermore, recall that the per-capita value of the prize  $G(\cdot)$  corresponds to the total value of the prize when it has the characteristics of a pure public good, while it decreases directly with  $n$  when it is (at least partially) private. Therefore,  $\Phi(\cdot)$  can be a decreasing function of  $n$ . Since it is also decreasing in  $X$ , it must be the case that the aggregate effort  $X^E$ , in evolutionarily stable strategies, decreases as group size  $n$  increases (while the aggregate effort is independent of group size in the symmetric Nash equilibrium, as shown by (3)). Obviously, it implies that the individual level of effort  $x^E$  is also decreasing in  $n$ . I shall return to the effects of the number of competing groups and group size in Section 5.

The next Proposition compares the aggregate level of effort induced by ESS with that when players adopt the Nash standard of behavior.

**Proposition 3.** *Let  $\kappa = r(m - 1)/(mn - 1) < r$  and let  $\epsilon_X = G'(X^N)X^N/G(X^N)$ , with  $X^N$  implicitly given by (3). If  $\epsilon_X < \kappa$  ( $\epsilon_X > \kappa$ ), then  $X^E$ , implicitly given by (7), is strictly higher (lower) than  $X^N$ , and  $X^E = X^N$  if  $\epsilon_X = \kappa$ .*

**Proof.** (3) can be rewritten as  $\Phi(X^N) = m/[r(m - 1) + \epsilon_X]$  and  $\Phi(X^E)$  is given by (7). We have that  $\Phi(X^N) \gtrless \Phi(X^E)$  as  $\epsilon_X \lesseqgtr \kappa$ .  $\Phi(\cdot)$  being a decreasing function, we thus have  $X^E \gtrless X^N$  as  $\epsilon_X \lesseqgtr \kappa$ .  $\square$

The reason for which behaviors, induced by the ESS, deviate to those in the Nash equilibrium is that a player's fitness is based on relative payoffs. If starting with a monomorphic population of players exerting the Nash equilibrium level of effort, a mutation occurs with one player exerting a (slightly) higher level of effort, it increases the winning probability of his group and induces a direct fitness cost to the player. The higher probability of success also benefits the mutant's teammates, although they did not bear the cost of the additional effort. Consequently, it increases their fitness relative to that of the mutant player, independently of whether the group contest is destructive or productive.

However, the additional contribution of the mutant player also decreases the probability of success for the competing groups. If, the contest is destructive, it decreases the value of the prize for all players, including out-group members. In this case, the additional contribution of the mutant player decreases both the probability of success and the value of the prize for the members of the rival groups, thus decreasing their expected material payoff. Therefore, it increases the mutant player's fitness relative to that of out-group members. In sum, if starting with a population of selfish players, one player exerts a higher level of effort, it makes the player worse off relative to his teammates, but better off relative to his opponents in the

group contest. Overall, efforts are higher in the ESS than in the symmetric Nash equilibrium when the group contest is destructive. In other words, relative payoff considerations across groups dominate those within groups.

Now, consider that the group contest is productive. In this case, the higher level of effort exerted by the mutant player has two countervailing effects on the expected material payoff of the members of the rival groups. On the one hand, it decreases their probability of winning the contest but, on the other hand, it increases the value of the prize if they win the prize. However, according to Proposition 3, if the group contest is *moderately* productive – that is if  $\epsilon_X \leq \kappa$  – evolutionary pressure still leads to higher levels of effort than in the symmetric Nash equilibrium. This is because a higher level of effort still improves one’s fitness relative to out-group members by more than it decreases one’s fitness relative to in-group members.

If, however, the group contest is *quite* productive – that is if  $\epsilon_X \geq \kappa$  – group members exert less effort in the (locally) evolutionarily stable equilibrium than if they were to maximize material payoff only. If starting with a monomorphic population of Nash players, a mutation occurs with one player exerting a (slightly) *lower* level of effort, it decreases the probability of success of his own group. The mutant player is now better off relative to his teammates because he bears a lower effort cost. The lower level of effort exerted by the mutant player also increases the winning probability of the rival groups. However, the members of the rival groups also suffer from a decrease in the value of the prize if they win the contest, all the more so as the elasticity of the prize with respect to change in the aggregate effort, is relatively large. Consequently, if this elasticity is sufficiently large – that is if  $\epsilon_X \geq \kappa$  – evolutionary pressure leads to lower levels of effort than in the symmetric Nash equilibrium.

In fact, it might be possible that decreasing one’s own level of effort decreases, *in absolute terms*, the expected material payoff of the out-group members, despite their greater probability of winning the contest.<sup>10</sup> This depends on how sensitive is the outcome of the contest with respect to differences in group effort, as measured by the ‘decisiveness parameter’  $r$ , relative to the elasticity of the prize with respect to change in the aggregate effort of all groups. Specifically, let evaluate the marginal material payoff of player  $(k, l)$  with respect to player  $(i, j)$ ’s effort, with  $j \neq l$ . Using (2), we have  $\Pi_{kl}/\partial x_{ij} = [\partial p_l(\mathbf{X})/\partial x_{ij}] G(X) + p_l(\mathbf{X}) G'(X)$ . Starting with a monomorphic population of players who exert the same level of effort  $x$ , we have  $(\partial \Pi_{kl}/\partial x_{ij})|_{x_{ij}=x} = -[r/(mX)] G(X) + [1/m] G'(X)$ . This expression can be rewritten as a function of the elasticity  $\epsilon_X$  to obtain  $(\partial \Pi_{kl}/\partial x_{ij})|_{x_{ij}=x} = [\Phi(X)/m] (\epsilon_X - r)$ , which is positive for  $\epsilon_X \geq r$ , and negative for  $\epsilon_X \leq r$ . In other words, the *lower* level of effort exerted by the mutant player can *decrease* the expected material payoff of out-group members, despite a higher probability of success in the group contest. This happens if the contest is *strongly* productive – that is if  $\epsilon_X \geq r$  – otherwise, the members of the rival groups enjoy a higher expected material payoff.

To conclude, if the group contest is *quite*, but not too, productive – that is, if  $\epsilon_X \in [\kappa, r]$  – the lower level of effort exerted by the mutant player decreases his fitness relative to that of his opponents in the rival groups, but this is more than offset by the increase in his fitness relative to that of his teammates. Relative payoff considerations within groups dominate relative payoff considerations across groups, which leads to lower levels of effort in the (locally) evolutionarily stable equilibrium than in the symmetric Nash equilibrium. It is

---

<sup>10</sup>I am indebted to an anonymous reviewer for calling my attention to this possibility. Also, this paragraph directly follows from the detailed comments of the reviewer in his/her report.

even more valid when the group contest is *strongly* productive, that is when  $\epsilon_X \in [r, 1)$ . In this case, exerting a lower level of effort not only allows increasing one's fitness relative to one's teammates, but also reduces the expected material payoff of the members of the rival groups.<sup>11</sup>

## 4 Evolutionarily stable preferences

In economics, it is typically assumed that effort choices result from a conscious and rational process that consists in maximizing a utility function. It is thus only natural to ask which utility function of players induces the ESS characterized in the previous section. To this end, I use the *indirect evolutionary approach* pioneered by Güth and Yaari (1992). The players are randomly matched into the  $m$  groups of size  $n$ , and each of them has a *subjective utility* function that depends on her own material payoff, and on the material payoffs of both in-group and out-group members.<sup>12</sup> Specifically, let the utility of player  $(i, j)$  – member  $i$  of group  $j$  – be given by

$$V_{ij}(\alpha_{ij}, \beta_{ij}) = \Pi_{ij} + \frac{\alpha_{ij}}{n-1} \sum_{k \neq i}^n \Pi_{kj} + \frac{\beta_{ij}}{n(m-1)} \sum_{l \neq j}^m \sum_{i=1}^n \Pi_{il}, \quad (8)$$

where the pair  $(\alpha_{ij}, \beta_{ij})$  is the *preference type* of player  $(i, j)$ , that is, the utility weights given by this group member, in her subjective utility, to the material payoffs of her teammates in group  $j$ , and to the material payoffs of her opponents in all groups  $l \neq j$ . It would be natural to suppose, as in Konrad and Morath (2012), that  $\alpha_{ij} \geq 0$  and  $\beta_{ij} \leq 0$  for all  $(i, j) \in I \times J$ , or that players have altruistic preferences towards their teammates and spiteful preferences towards the members of the rival groups. However, at this stage, let us remain agnostic as to whether these parameters are positive or negative. Thus, consider that  $\alpha_{ij} \in (-1, K]$  – with  $K \geq 1$  – and that  $\beta_{ij} \in (\underline{L}, \bar{L})$  with  $\underline{L} < 0$  and  $\bar{L} > 0$ .<sup>13</sup> Also, note that the valuations of the material payoffs of both in-group members and out-group members are weighted by the inverse of their respective numbers, so that the relative weight of player  $(i, j)$ 's payoff does not change with either group size or the number of competing groups.

Following Konrad and Morath (2012), I assume that players have "robust beliefs" in the sense that a player endowed with a certain preference type believes that all other players – including out-group members – are of the same type and have the same robust beliefs. The assumption of unobservability of types is not only more plausible than the opposite. It also

---

<sup>11</sup>However, it does not necessarily reduce their fitness relative to that of the mutant player. This is because a lower level of effort also decreases the expected material payoff of the mutant player in *absolute terms* (simply because he does not play his best response to other players' strategies). We must remember that an EES-player does not maximize his material payoff, but the difference between his material payoff and the average material payoff of the other players.

<sup>12</sup>In a previous version of this article, I ignored preferences towards the out-group members. The general analysis of the evolutionary stability of both in-group and out-group preferences in the group contest follows the detailed comments of an anonymous reviewer. Following this general analysis, I will restrict the set of admissible preference types that are subject to evolutionary pressure.

<sup>13</sup>We exclude the case  $\alpha_{ij} \leq -1$  since then it would prevent positive levels of efforts in a symmetric equilibrium if  $\beta_{ij} = 0$ . However, degrees of altruism  $\alpha_{ij} \geq 1$  may exist, for instance, between partners. Also, at this stage, we do not impose restrictions on  $\underline{L}$  or  $\bar{L}$  (other than their signs).

drastically simplifies the analysis of preference evolution, because it eliminates all strategic effects of a change in a player's type on other players' equilibrium actions.

Suppose that  $mn-1$  players are of type  $(\alpha^E, \beta^E)$ , while one player – the mutant – is of type  $(\alpha_M, \beta_M) \neq (\alpha^E, \beta^E)$ . All players choose their effort level to maximize their subjective utility given by (8). Let  $\Pi_M((\alpha_M, \beta_M), (\alpha^E, \beta^E))$  and  $\Pi_{-M}((\alpha_M, \beta_M), (\alpha^E, \beta^E))$  be, respectively, the expected material payoff obtained by the mutant player and by a player of type  $(\alpha^E, \beta^E)$  in the Nash equilibrium with robust beliefs, where  $(\alpha^E, \beta^E)$  is the vector of  $mn-1$  copies of  $(\alpha^E, \beta^E)$ .

Let us define *locally evolutionarily stable preference types* as follows:

**Definition 3.** Let  $(\alpha^E, \beta^E) \in (-1, K]^{mn-1} \times (\underline{L}, \overline{L})^{mn-1}$  be the vector of preference types of all  $mn-1$  players – other than the mutant player  $M$  – all with the same pair of preference parameters  $(\alpha^E, \beta^E) \in (-1, K] \times (\underline{L}, \overline{L})$ . Then,  $(\alpha^E, \beta^E)$  is a *locally evolutionarily stable preference type (ESP)*, with robust beliefs, if it is a solution to:

$$\underset{\alpha_M, \beta_M}{\text{Max}} \Pi_M((\alpha_M, \beta_M), (\alpha^E, \beta^E)) - \Pi_{-M}((\alpha_M, \beta_M), (\alpha^E, \beta^E)), \quad (9)$$

for  $(\alpha_M, \beta_M)$  in some neighborhood of  $(\alpha^E, \beta^E)$ .

The notion of local stability at the level of preferences can be justified if we consider that the evolutionary process exhibits what Alger and Weibull (2010) calls a "cultural drift", with the mutant player endowed with preferences slightly different from those of the incumbent players.

The next Proposition characterizes the set of (locally) evolutionarily stable preference types.

**Proposition 4.** Let  $\kappa = r(m-1)/(mn-1) < r$ . If Assumption 1 holds with  $G'''(X) \leq 0$  for all  $X \in (0, W]$ , and if  $(\underline{L}, \overline{L}) \subseteq (-n\kappa/r, n\kappa(m-1))$ , then there exists a non-empty set of locally ESP  $(\alpha^E, \beta^E) \in (-1, K] \times (\underline{L}, \overline{L})$  with robust beliefs, that must satisfy

$$1 + \alpha^E = \frac{mn\kappa + \beta^E(r - \epsilon_X)}{r(m-1) + \epsilon_X}, \quad (10)$$

where  $\epsilon_X = G'(X^E)X^E/G(X^E)$ , with  $X^E$  implicitly given by (7).

**Proof.** Appendix A3.  $\square$

The proof of this result follows Konrad and Morath (2012) and can be summarized as follows. When players have robust beliefs, we can characterize the "symmetric robust-beliefs Nash equilibrium" in efforts of each player type. For a certain preference type, it would correspond to the symmetric Nash equilibrium under complete information when all players are effectively of the type in question. When players have different preferences and cannot observe others' preferences, their choices do not correspond to a set of mutually best replies. But these choices only occur *off* the equilibrium with evolutionarily stable preferences. Indeed, in such an equilibrium, all players have the same preferences and according to Definition 3, a mutant

player with different preference parameters than  $(\alpha^E, \beta^E)$  cannot obtain a strictly higher relative payoff.<sup>14</sup>

Proposition 2 describes the evolutionarily stable strategy, that is the effort choice that cannot be invaded by another effort choice for maximizing relative payoff. Since under the indirect evolutionary approach, preferences determine behaviors, evolutionarily stable preferences, as characterized by Proposition 4, must implement evolutionarily stable strategies, as characterized by Proposition 2.

Also, the set of evolutionarily stable preference types involves a linear combination of the two parameters reflecting in-group bias and out-group bias. The reason is that the subjective utility of a player is itself given by a linear combination of the material payoffs of the player's teammates, and of the material payoffs of the out-group members. However, each player exerts just one level of effort and, hence, there exists an infinity of preference types  $(\alpha^E, \beta^E)$  that implement evolutionarily stable strategies as characterized by Proposition 2. In other words, the system is under-determined and, consequently, several combinations of in-group altruism/spite and out-group altruism/spite can constitute a (local) equilibrium in evolutionarily stable preferences, whether the contest is destructive or productive.<sup>15</sup>

However, the next Proposition states that one can always exclude a specific combination of preference parameters.

**Proposition 5.** *Let  $\kappa = r(m - 1)/(mn - 1) < r$  and  $\epsilon_X = G'(X^E)X^E/G(X^E)$ , then all combinations of in-group altruism/spite and out-group altruism/spite can constitute a (local) equilibrium in evolutionarily stable preferences with robust beliefs, **except**:*

- (i) *In-group spite and out-group altruism – i.e.  $\alpha^E < 0$  and  $\beta^E > 0$  – if  $\epsilon_X \leq \kappa < r$ ;*
- (ii) *In-group altruism and out-group spite – i.e.  $\alpha^E > 0$  and  $\beta^E < 0$  – if  $\epsilon_X \in [\kappa, r)$ ;*
- (iii) *In-group altruism and out-group altruism – i.e.  $\alpha^E > 0$  and  $\beta^E > 0$  – if  $\epsilon_X > r$ ;*

**Proof.** Let  $\omega = (\epsilon_X - \kappa)/(r - \epsilon_X)$ . (i) If  $\epsilon_X \leq \kappa < r$ , then  $\omega \leq 0$ . Suppose first that  $\alpha^E > 0$  (or that  $1 + \alpha^E > 1$ ). Using (10), this implies that  $\beta^E > \omega$ , which can be verified with both  $\beta^E \geq 0$  and  $\beta^E \leq 0$ . Symmetrically, suppose that  $\alpha^E < 0$ . This implies that  $\beta^E < \omega$ , so that one must have  $\beta^E < 0$ . (ii) If  $\epsilon_X \in [\kappa, r)$ , then  $\omega \geq 0$ .  $\alpha^E > 0$  still implies that  $\beta^E > \omega$ , so that one must have  $\beta^E > 0$ .  $\alpha^E < 0$  implies that  $\beta^E < \omega$ , which can be verified with both  $\beta^E \geq 0$  and  $\beta^E \leq 0$ . (iii) If  $\epsilon_X > r > \kappa$ , then  $\omega < 0$ . Using (10),  $\alpha^E > 0$  now implies that  $\beta^E < \omega$ , so that one must have  $\beta^E < 0$ .  $\alpha^E < 0$  implies  $\beta^E > \omega$ , which can be verified with both  $\beta^E \geq 0$  and  $\beta^E \leq 0$ .  $\square$

Interestingly, the combination of in-group altruism and out-group spite – or "parochial altruism" – cannot always achieve evolutionarily stable effort choices under preference evolution. This happens when the group contest is relatively productive, but neither too much nor too little, that is when  $\epsilon_X \in [\kappa, r)$ . In this case, group members exert a lower level of effort in the ESS than in the symmetric Nash equilibrium, and at the same time, exerting less effort than

<sup>14</sup>For a detailed discussion of the properties of evolutionarily stable preferences with robust beliefs, see Konrad and Morath (2012).

<sup>15</sup>This problem of underdetermination does not occur in Eaton et al. (2011) because they consider a "production and conflict" model, where each player exerts two types of effort: a processing effort and an appropriative effort. However, they do not obtain closed-form solutions for their model, and rely on numerical simulations for endogenizing the utility weights.

all other players increases the expected material payoff of the members of the rival groups. Therefore, if preference evolution leads to in-group altruism, it must be accompanied by out-group altruism. But, a lower level of effort in the ESS when  $\epsilon_X \in [\kappa, r)$  can also be achieved with out-group altruism and in-group spite, or spiteful preferences towards both in-group and out-group members. Also, when  $\epsilon_X > r$ , exerting a lower level of effort decreases the expected material payoff of out-group members, because it decreases the value of the prize by more than it increases their probability of winning the prize. Consequently, if the lower ESS effort level (relative to the Nash equilibrium effort) is induced by in-group altruism, it must be accompanied by out-group spite under preference evolution. But, as for  $\epsilon_X \in [\kappa, r)$ , in-group spite together with out-group altruism, or fully spiteful preferences towards both in-group and out-group members, are also (locally) evolutionarily stable.

To move forward, let us ignore *either* in-group preferences *or* out-group preferences by setting  $\alpha_{ij} = 0$  or  $\beta_{ij} = 0$  for all  $(i, j) \in I \times J$ . We can obtain the following Proposition.

**Proposition 6.** *Let  $\kappa = r(m - 1)/(mn - 1) < r$  and  $\epsilon_X = G'(X^E)X^E/G(X^E)$ .*

(i) *Suppose that  $\beta_{ij} = 0$  for all  $(i, j) \in I \times J$ .*

*Under Assumption 1, there exists a unique locally ESP  $\alpha^E \in (-1, K]$ , and it is characterized by:*

$$1 + \alpha^E = \frac{mn\kappa}{r(m - 1) + \epsilon_X}, \quad (11)$$

*which is strictly greater (lower) than 1 for  $\epsilon_X < \kappa$  ( $\epsilon_X > \kappa$ ), and equal to 1 for  $\epsilon_X = \kappa$ .*

(ii) *Suppose that  $\alpha_{ij} = 0$  for all  $(i, j) \in I \times J$ .*

(iia) *If Assumption 1 holds with  $G''(X) \leq 0$  for all  $X \in (0, W]$ , and if  $\epsilon_X < n\kappa < r$ , then there exists a unique locally ESP  $\beta^E \in (-n\kappa/r, (m - 1))$ , and it is characterized by*

$$\beta^E = \frac{\epsilon_X - \kappa}{r - \epsilon_X}, \quad (12)$$

*which is strictly negative (positive) for  $\epsilon_X < \kappa$  ( $\epsilon_X \in (\kappa, n\kappa)$ ), and equal to 0 for  $\epsilon_X = \kappa$ ; if  $\epsilon_X \geq n\kappa$ , a locally ESP may fail to exist.*

(iib) *If Assumption 1 holds with  $G'''(X) = 0$  for all  $X \in (0, W]$ —implying that  $G(X)$  is affine—then  $\epsilon_X < r$ , and there always exists a unique locally ESP  $\beta^E \in (-1, (m - 1))$  given by (12), which is strictly negative (positive) for  $\epsilon_X < \kappa$  ( $\epsilon_X > \kappa$ ), and equal to 0 for  $\epsilon_X = \kappa$ .*

**Proof.** Appendix A4.  $\square$

Before interpreting these results, some technical remarks have to be made. First, if we ignore out-group preferences, one can demonstrate the existence of a unique (locally) evolutionarily stable in-group preference parameter, without assuming that the prize function is always concave. When the contest is destructive, it can be convex as well under the restriction that property (iiib) of Assumption 1 is satisfied. However, if we ignore in-group preferences, the existence of a unique locally out-group preference parameter is not always guaranteed, except if the value of the prize is affine in the level of aggregate effort. Otherwise, we need to assume

that the prize function is concave, and that the elasticity  $\epsilon_X$  is lower than a certain threshold given by  $n\kappa$ .

In general, if the group contest is quite or strongly productive – i.e. if  $\epsilon_X \geq n\kappa$  – a locally ESP may fail to exist when ignoring in-group preferences. This would be the case when the elasticity of the prize  $\epsilon_X$  is "close" to the ‘decisiveness parameter’  $r$  of the contest success function. To understand it, recall that evolutionarily stable preferences must implement evolutionarily stable strategies, and that these strategies induce behaviors that maximize relative payoff. Now, consider the polar case where  $\epsilon_X$  is equal to  $r$ , so that a player’s effort has no effect on the material payoffs of the members of the rival groups. Indeed, in this case, if a group member exerts a higher level of effort, the increase in the value of the prize just counterbalances the decrease in the probability of success for the members of the rival groups, thus leaving unchanged their expected material payoffs. Thus, if players’ subjective utilities do not depend on the payoffs of in-group members and if  $\epsilon_X = r$ , players can only behave as if they were to maximize their absolute material payoff (which is evolutionarily stable only in the special case where  $\epsilon_X = \kappa$ ). Consequently, an equilibrium in evolutionarily stable preferences fails to exist in this case.

Also, ignoring out-group preferences, one can observe that the condition determining the sign of the evolutionarily stable in-group preference parameter is the same than that obtained in Proposition 3 for the comparison of efforts between ESS and Nash equilibria. Proposition 3 and property (i) of Proposition 6 together state that if the elasticity of the prize  $\epsilon_X$  is lower (larger) than the threshold value  $\kappa$ , then effort levels are higher (lower) in the ESS than in the symmetric Nash equilibrium, and it corresponds to in-group altruism (spite) under preference evolution. However, the elasticity of the prize is evaluated at  $X^E$  in Proposition 6, while it is evaluated at  $X^N$  in Proposition 3. Notwithstanding this, a higher (lower) ESS effort level, relative to the Nash equilibrium level, must correspond to a positive (negative) value for  $\alpha^E$  under preference evolution. This is obvious when the group contest is destructive because, in this case, the elasticity of the prize is always negative (and thus, always, lower than  $\kappa$ ). When the contest is productive, it must also be the case that the two (positive) elasticities evaluated at  $X^N$  and  $X^E$  are either both lower or both larger than the threshold value  $\kappa$ .

First, the two elasticities are equal if and only if  $X^E = X^N$ , in which case they are both equal to  $\kappa$ . Thus, if out-group preference are ignored and if  $\epsilon_X = \kappa$ , we have  $\alpha^E = 0$  and absolute payoff maximization is evolutionarily stable. Suppose now that  $X^N \neq X^E$  and that  $\epsilon_X$  evaluated at  $X^N$  is slightly lower (higher) than  $\kappa$ . According to Proposition 3, this implies that  $X^E > X^N$  ( $X^E < X^N$ ). Players expend more (less) effort in the evolutionarily stable equilibrium than if they were absolute-payoff maximizers. This necessarily reflects a positive (negative) degree of in-group altruism under preference evolution, so that  $\epsilon_X$  evaluated at  $X^E$  must also be slightly lower (higher) than  $\kappa$ . The reason is that a player’s equilibrium effort level depends positively on his own degree of in-group altruism. Indeed, a positive degree of in-group altruism increases the marginal benefit of exerting effort and has no effect on the marginal cost.<sup>16</sup> Consequently, mutating towards in-group altruism (spite), under preference

---

<sup>16</sup>Formally, suppose that player  $(i, j)$  has robust beliefs and anticipates that the  $mn - 1$  other players exert the effort level  $x$ . Then, player  $(i, j)$ ’s subjective utility is  $V_{ij}(\alpha_{ij}) = (1 + \alpha_{ij})p_j(\mathbf{X})G(X) - (x_{ij} + \alpha_{ij}x)$ , which is continuous and locally concave in  $x_{ij}$  (see the proof of Proposition 6). Hence, the first-order condition is necessary and sufficient for local maximization. Consequently, the impact of a marginal increase in  $\alpha_{ij}$  on the player  $(i, j)$ ’s optimal effort level is given by the cross-derivative of  $V_{ij}(\alpha_{ij})$  with respect to  $x_{ij}$  and  $\alpha_{ij}$ . We have  $\partial[\partial V_{ij}(\alpha_{ij})/\partial x_{ij}]/\partial \alpha_{ij} = [\partial p_j(\mathbf{X})/\partial x_{ij}]G(X) + p_j(\mathbf{X})G'(X)$ , which is clearly strictly positive



evolution, must correspond to a higher (lower) level of effort than in the symmetric Nash equilibrium, under strategy evolution.

To summarize, when the group contest is destructive or moderately productive – that is when  $\epsilon_X \leq \kappa$  – group members are more aggressive than if they were absolute-payoff maximizers. When out-group preferences are ignored, this can be viewed as *in-group altruism* under preference evolution. If a player exerts a higher level of effort, it increases the probability of success of his group to the detriment of the rival groups. If the group contest is destructive, it also decreases the value of the prize for the player and for her teammates. But, it also decreases it for all the members of the rival groups, in addition to decreasing their probability of winning the prize. If the group contest is *moderately* productive – i.e. if  $\epsilon_X \leq \kappa$  – preference evolution still leads to in-group altruism, but to a lesser extent than when the group contest is destructive. The reason is that a player’s contribution increases the value of the prize for all players, including the members of the rival groups, which counteracts the decrease in their probability of winning the prize.

Symmetrically, the lower ESS effort level when the group contest is quite productive – that is when  $\epsilon_X \geq \kappa$  – can be viewed as induced by *in-group spite*. Indeed, in this case, if a player exerts a lower level of effort, it increases his fitness relative to that of his teammates, and it also increases the winning probabilities of the rival groups. According to property (iia) of Proposition 6, the lower ESS effort level can also be interpreted as *out-group altruism* when in-group preferences are ignored, under the condition that  $\epsilon_X < n\kappa < r$ . In this case, if a group member expends less effort, the positive effect on the probability of success dominates the negative effect on the value of the prize for the members of the rival groups. Consequently, the out-group members obtain a higher expected material payoff. The same would apply if  $\epsilon_X \in [n\kappa, r)$  but, again, we cannot demonstrate that an equilibrium in evolutionarily stable preferences always exists in this case, except if the value of the prize is an affine function. If it is the case, then  $\epsilon_X < r$ , and preference evolution leads to out-group altruism for any  $\epsilon_X \geq \kappa$  (property (iib) of Proposition 6).

In general, however, one can have  $\epsilon_X > r$ , in which case a lower level of effort decreases the expected material payoffs of the out-group members despite a higher probability of success. Again, equilibrium existence is not always guaranteed in this case if we ignore in-group preferences. Allowing for both in-group and out-group preferences, all preference combinations can be evolutionarily stable when  $\epsilon_X > r$ , except fully altruistic preferences towards both in-group members and out-group members (property (iii) of Proposition 5).

## 5 The impact of the ‘economic environment’<sup>17</sup>

The results about the emergence of evolutionarily stable preference types were framed in terms of the elasticity of the prize with respect to change in the aggregate level of effort, which is itself a function of this level at the equilibrium in evolutionarily stable strategies. Without additional constraints on the  $G(\cdot)$  function, we cannot evaluate the impact of a change in the parameters reflecting the ‘economic environment’ on the evolutionarily stable equilibrium.

---

when the group contest is productive.

<sup>17</sup>This Section builds on the detailed comments of an anonymous reviewer. I am indebted to him/her for the results, presented in this Section.

Thus, suppose that the prize function  $G(X)$  is quasilinear and that, consistently with Assumption 1, it satisfies the following properties.

**Assumption 2.** Let  $G(X) = R + F(X)$  with:

- (i)  $R > 0$  and  $F(0) = 0$ ;
- (ii)  $F(X)$  is twice continuously differentiable for all  $X \in (0, W]$ ;
- (iii) For all  $X \in [0, W]$ ,  $F(X)$  satisfies **either** (iiia) **or** (iiib) where:
  - (iiia)  $F'(X) \geq 0$  and  $F''(X) \leq 0$ ;
  - (iiib)  $F'(X) \leq 0$ ,  $F'(X)X > \eta[R + F(X)]$ , and  $\zeta_X > \eta$ , where  $\zeta_X = F''(X)X/F'(X)$  and  $\eta \equiv -r(m - 1)$ ;
- (iv)  $R \in (\max\{0; -F(W)\}, W - F(W))$ .<sup>18</sup>

We can now investigate the impact of the initial value of the prize –  $G(0) = R$  – that of group size  $n$ , and that of the number of groups  $m$ , on the evolutionarily stable level of aggregate effort with the help of figures.<sup>19</sup> Figure 1 represents the case of a productive contest, with  $F(X)$  satisfying (iiia) of Assumption 2 (together with (i)-(ii)-(iv)). Figure 2 represents the case of a destructive contest, with  $F(X)$  being convex and satisfying (iiib) of Assumption 2 (together with (i)-(ii)-(iv)).

## INSERT FIGURES 1 & 2

Suppose that players' subjective utilities are based on their own material payoffs and on those of their teammates only. Figure 1 also shows whether in-group altruism or out-group spite is evolutionarily stable according to values of the parameters reflecting the 'economic environment'. Indeed, observe that (7) can be rewritten as  $\Phi(X^E) = 1/n\kappa$ , while the elasticity of the prize with respect to change in  $X$  at  $X = X^E$  can be rewritten as  $\epsilon_X = G'(X^E)/\Phi(X^E) = n\kappa G'(X^E)$ , with  $\kappa = r(m - 1)/(mn - 1)$ . Hence, using (i) of Proposition 6, evolutionary pressure on preferences leads to in-group altruism if  $\epsilon_X \leq \kappa$  or  $G'(X^E) \leq 1/n$ , and to in-group spite if  $\epsilon_X \geq \kappa$  or  $G'(X^E) \geq 1/n$ . Since  $G'(X) = F'(X)$ , the vertical dashed line corresponding to  $X = F'^{-1}(1/n)$  splits the figure into two regions: one corresponding to in-group spite on the left, and the other corresponding to in-group altruism on the right. When the initial value of the prize is equal to  $R_1 > 0$ , the evolutionarily stable level of aggregate effort  $X^E(R_1, n)$  is, from (7), given by the intersection between  $G_1(X)$  and the dashed line passing through the origin with slope  $(mn - 1)/[rn(m - 1)]$ . Preference evolution

<sup>18</sup>Property (iv) ensures that the final value of the prize is always strictly positive, for any  $X \in [0, W]$ , and in particular when  $F(\cdot)$  is a decreasing function and  $X = W$ . This requires a lower bound for  $R$ . Property (iv) of Assumption 2 also requires an upper bound for  $R$ , which directly follows from property (iv) of Assumption 1. In an additional appendix, I provide closed form solutions for  $\alpha^E$  and  $X^E$  by using the functional form  $F(X) = \theta\sqrt{X}$  with  $\theta \in [-1, 1]$ . When  $\theta \geq 0$ , the group contest is productive and  $F(\cdot)$  is concave, while when  $\theta \leq 0$ , the group contest is destructive and  $F(\cdot)$  is convex. In order to satisfy (iii) and (iv) of Assumption 2 for all  $\theta \in [-1, 1]$ , one must have  $R \in ((1 + 2r)\sqrt{W}/2r, W - \sqrt{W})$  with  $W > [(1 + 4r)/2r]^2$ , and  $r > 1/[2(m - 1)]$ . This appendix is available upon request.

<sup>19</sup>Figure 1 for a productive contest has been provided by a reviewer in his/her report. I am also providing the figure for a destructive contest, which is actually simpler because, in this case, evolutionary pressure always leads to in-group altruism. Thus, the figures must be credited to the reviewer.

leads to in-group spite in this case. If the initial value of the prize increases from  $R_1$  to  $R_2$ , then it shifts the curve of the prize function  $G_2(X)$  upward. The evolutionarily stable level of aggregate effort is now given by  $X^E(R_2, n)$ , which falls into the region of in-group altruism.

Now, suppose that group size increases from  $n$  to  $n'$ , then the vertical dashed line delimiting the two regions of altruism and spite within groups moves to the right. At the same time, the slope of the dashed line passing through the origin increases and intersects with  $G_2(X)$  at  $X^E(R_2, n')$ . The level of aggregate effort in evolutionarily stable strategies is thus lower when group size becomes larger, and this level  $X^E(R_2, n')$  falls, once again, in the (new) region of in-group spite. In contrast, an increase in the number of competing group  $m$  (not represented in the figure) reduces the slope of the dashed line passing through the origin. It increases the equilibrium level of aggregate effort in evolutionarily stable strategies, which can counteract the increase in group size for restoring in-group altruism under preference evolution.

When the contest is destructive, and as shown by figure 2, the aggregate level of effort in evolutionarily stable strategies is also increasing in the initial value of the prize, decreasing in group size, and increasing in the number of competing groups. The difference is that preference evolution always leads to in-group altruism in this case.

In sum, the evolutionarily stable level of aggregate effort is always increasing in the initial value of the prize, which is rather intuitive because an increase in  $R$  automatically increases the stake of the contest, independently of (the impact of) the level of aggregate effort. Contest efforts would also increase with  $R$  in the symmetric Nash equilibrium. Thus, the (less intuitive) result that an increase in the initial value of the prize can change the evolutionarily stable equilibrium from in-group spite to in-group altruism, when the group contest is productive, reflects the fact that the ESS effort level increases more rapidly with increasing  $R$  than the Nash equilibrium effort. Overall, even in the case of a productive contest, in-group altruism can be always evolutionarily stable if the initial value of the prize is sufficiently large.

Also, an increase in group size of all groups (for a given number of competing groups) always leads to a decrease in the aggregate and individual levels of effort in evolutionarily stable strategies. The intuition is the following. A larger group size reduces the marginal impact of one's contribution on the probability of success of the group and, hence, on the possibility to increase one's fitness relative to that of out-group members. As group size increases, relative payoff considerations within groups become increasingly important for individual fitness. Thus, the evolutionary pressure towards free-riding on others' efforts within the group is increased with the number of teammates. And if group size is relatively large, then preference evolution can lead to in-group spite in productive contests, which exacerbates the free-riding incentives within groups, by comparison to what would be expected if group members were purely egoistic.

In contrast, an increase in the number of competing groups (for a given group size) always leads to an increase in the aggregate level of effort in evolutionarily stable strategies, but not necessarily to an increase in the individual effort level. Yet, an increase in the number of groups makes in-group altruism more likely to be evolutionarily stable under preference evolution when the group contest is productive.<sup>20</sup> Indeed, a player's fitness becomes in-

---

<sup>20</sup>A decrease in the *individual* level of effort in evolutionarily stable strategies with an increase in  $m$  is not incompatible with a shift from in-group spite to in-group altruism under preference evolution. This is

creasingly directly connected to the success of his group as the number of competing group increases. Thus, when rivalry between groups becomes more intense, relative payoff considerations across groups "take over", thus reducing free-riding incentives as a result of altruistic attitudes towards the members of the same group.<sup>21</sup>

## 6 Conclusion

A large number of experimental studies show that group members expend more effort in group contests compared to what one would expect if group members were adopting the Nash standard of behavior (see, again, Sheremeta, 2018, for a recent survey on experimental research on group contests). One explanation of high collective action in group contests is that group members display parochial altruism. As shown by Konrad and Morath (2012), the combination of in-group altruism and out-group spite in group contests can be obtained as the result of evolutionary pressure on preferences. In this paper, we generalize their analysis by considering that the total value of the prize can increase as well as decrease with the aggregate effort of all groups. The main result is that evolutionarily stable in-group altruism is stronger when the contest is destructive, as in the case of an armed conflict, than when it is productive, as in the case of a patent race. Moreover, when the contest is strongly productive, in-group spite is evolutionarily stable under preference evolution. Finally, the combination of in-group altruism and out-group spite cannot always constitute an equilibrium in evolutionarily stable preferences, when the contest is productive.

This opens new prospects for experimental research on group contests. Indeed, the theoretical analysis provides clear predictions on evolutionarily stable behaviors, by comparison to the predictions implied by the *Homo economicus* paradigm, whether the prize is exogenous, or endogenous with its value increasing or decreasing with contest efforts of all groups. Group members exert more effort than in the Nash equilibrium when the group contest is destructive or moderately productive, while they under-invest in collective action when the group contest is strongly productive. Furthermore, the notion of ‘productiveness’ of contest efforts is relative to other important properties of the group contest. The larger the ‘decisiveness parameter’ of the contest success function, or the larger the number of competing groups relative to their size, the larger must be the elasticity of the prize with respect to change in the aggregate effort, for making under-investment in collective action (relative to the Nash equilibrium), and in-group spite, evolutionarily stable. However, it is worth recalling that

---

because the Nash equilibrium level of individual effort is decreasing in  $m$  (and in  $n$ ).

<sup>21</sup>It is also worth emphasizing out that the evolutionarily stable degree of in-group altruism (or spite)  $\alpha^E$  does not necessarily move in the same direction as the equilibrium level of aggregate effort  $X^E$  with a change in the parameters reflecting the ‘economic environment’. This is because  $\alpha^E$  depends on the elasticity of the prize with respect to change in the aggregate effort, and thus on whether the *marginal impact* of a player’s effort on the value of the prize is, itself, a decreasing or an increasing function. One can show, that in the case of a productive contest,  $\alpha^E$  and  $X^E$  are both increasing in  $R$  if  $F(\cdot)$  is concave. However, when the contest is destructive,  $\alpha^E$  is decreasing in  $R$  if  $F(\cdot)$  is convex (while  $X^E$  is still increasing in  $R$ ). Under the condition that the total number of players  $N = mn$  remains constant,  $X^E$  and  $\alpha^E$  also move in the same direction with an increase in group size  $n$ , or in the number of competing groups  $m$ , when  $F(\cdot)$  is increasing and concave, and move in opposite directions, when  $F(\cdot)$  is decreasing and convex. This is what I demonstrate in the additional appendix, where I also provide closed form solutions for  $\alpha^E$  and  $X^E$  when  $G(X) = R + \theta\sqrt{X}$ , with  $\theta \in [-1, 1]$ . Again, this additional appendix is available upon request.

the Nash equilibrium can lead to over-exertion of effort in productive contests, so that in-group spite can lead to behaviors that are actually closer to the social optimum. Again, this depends on the specifications of the contest success function and of the prize function.<sup>22</sup>

## Appendix

### A.1 Proof of Proposition 1

In a Nash equilibrium, each player  $(i, j)$  chooses her effort  $x_{ij}$  to maximize her material payoff  $\Pi_{ij}$  given by (2). The first-order condition for maximizing (2) with respect to  $x_{ij}$ , for  $i \in I = \{1, 2, \dots, n\}$  and  $j \in J = \{1, 2, \dots, m\}$ , is

$$\frac{rX_j^{r-1} \sum_{k \neq j}^m X_k^r}{\left[X_j^r + \sum_{k \neq j}^m X_k^r\right]^2} G(X) + \frac{X_j^r}{X_j^r + \sum_{k \neq j}^m X_k^r} G'(X) - 1 = 0. \quad (\text{A1})$$

The first-order conditions only determine group expenditures so that there can exist multiple asymmetric equilibria with players expending different levels of effort. We focus, however, on a symmetric equilibrium such that all members of all groups exert the same level of effort.

In a symmetric equilibrium, (A1) becomes

$$\frac{r(m-1)G(X)}{mX} + \frac{G'(X)}{m} = 1. \quad (\text{A2})$$

This condition can be rewritten as  $r(m-1)G(X) + XG'(X) - mX = 0$ . Rearranging yields  $r(m-1)[G(X) - X] + X[G'(X) - 1] - X(m-1)(1-r) = 0$ . Dividing by  $X$  and rearranging again, we obtain (3).

Now, differentiating (A2) with respect to  $X$ , we obtain

$$\frac{r(m-1)G(X)[\epsilon_X - 1]}{mX^2} + \frac{G''(X)}{m}, \quad (\text{A3})$$

where  $\epsilon_X = G'(X)X/G(X)$ .

When  $G''(X) \leq 0$ , the above expression is always strictly negative, because  $\epsilon_X < 1$  and  $G(X) > 0$  for all  $X \in [0, W]$ . Now, suppose that the contest is destructive – i.e.  $\epsilon_X \leq 0$  – and that  $G''(X) \geq 0$ . Then, the expression in (A3) is strictly negative if

$$-r(m-1) + \epsilon_X [r(m-1) + \sigma_X] < 0, \quad (\text{A4})$$

where  $\sigma_X = G''(X)X/G'(X)$ . A sufficient condition for inequality (A4) to be satisfied, for  $G'(X) \leq 0$  (implying  $\epsilon_X \leq 0$ ) and  $G''(X) \geq 0$ , is that  $\sigma_X > -r(m-1)$ , as stated in property (iiib) of Assumption 1.

Thus, under Assumption 1, the expression in (A3) is always strictly negative. It follows that the left-hand term of (A2) is a decreasing function of  $X$ . This term can be rewritten as

---

<sup>22</sup>For example, using the standard Tullock (1980) lottery contest, with  $r = 1$ , there is always over-exertion of effort in the Nash equilibrium, for any prize function satisfying Assumption 1 (see Footnote 8).

$$\frac{G(X) [r(m-1) + \epsilon_X]}{mX}. \quad (\text{A5})$$

In  $X = 0$ , it approaches infinity since, under Assumption 1, the numerator is a strictly positive finite number. Indeed, property (i) of Assumption 1 states that  $G(0) > 0$ , and according to property (iii) the term in  $[\cdot]$  is a strictly positive finite number as well, whether the contest is productive (and  $G(X)$  satisfies (iiia)) or destructive (and  $G(X)$  satisfies (iiib)). In  $X = W$ , the term in (A5) is also strictly lower than 1 because  $[r(m-1) + \epsilon_X] < mW/G(W)$ . The left-hand term of this inequality is strictly lower than  $m$  for  $r \leq 1$  (given that  $\epsilon_X < 1$ ), while the right-hand term is greater than  $m$  since  $W/G(W) \geq 1$ . Indeed  $\Phi(X) = G(X)/X$  is decreasing in  $X$ , and thus  $1/\Phi(X) = X/G(X)$  is increasing in  $X$ . From property (iv) of Assumption 1,  $\bar{X} \in (0, W]$  is such that  $\Phi(\bar{X}) = 1/\Phi(\bar{X}) = 1$ , and we have  $1/\Phi(W) \geq 1$  because  $W \geq \bar{X}$ . In conclusion, there exists a unique value of aggregate effort that satisfies (A2).

## A.2 Proof of Proposition 2

Calculating the first derivative of (6) with respect to  $x_M$ , we obtain

$$\begin{aligned} & \frac{r [x_M + (n-1)x^E]^{r-1} (m-1) [nx^E]^r G(\cdot)}{[[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]^2} + \frac{[x_M + (n-1)x^E]^r G'(\cdot)}{[x_M + (n-1)x^E]^r + (m-1) [nx^E]^r} - 1 \\ & \frac{n-1}{mn-1} \left\{ \frac{r [x_M + (n-1)x^E]^{r-1} (m-1) [nx^E]^r G(\cdot)}{[[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]^2} + \frac{[x_M + (n-1)x^E]^r G'(\cdot)}{[x_M + (n-1)x^E]^r + (m-1) [nx^E]^r} \right\} \\ & \frac{n(m-1)}{mn-1} \left\{ - \frac{r [x_M + (n-1)x^E]^{r-1} [nx^E]^r G(\cdot)}{[[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]^2} + \frac{[nx^E]^r G'(\cdot)}{[x_M + (n-1)x^E]^r + (m-1) [nx^E]^r} \right\}. \quad (\text{A6}) \end{aligned}$$

Rearranging this expression gives

$$\begin{aligned} & \frac{mn(m-1)r [x_M + (n-1)x^E]^{r-1} [nx^E]^r G(\cdot)}{(mn-1) [[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]^2} \\ & + \frac{n(m-1) [[x_M + (n-1)x^E]^r - [nx^E]^r] G'(\cdot)}{(mn-1) [[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]} - 1. \quad (\text{A7}) \end{aligned}$$

ESS requires identical behavior of contestants. Thus, evaluating this expression at  $x_M = x^E$ , and setting it to 0, yields (7), which is the only interior solution candidate.

Differentiating the above expression with respect to  $x_M$ , we find

$$\begin{aligned} & \frac{rmn(m-1) [nx^E]^r [x_M + (n-1)x^E]^{r-2} [(1+r) [x_M + (n-1)x^E]^r + (1-r)(m-1) [nx^E]^r] G(\cdot)}{(mn-1) [[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]^3} \\ & + \frac{2rmn(m-1) [nx^E]^r [x_M + (n-1)x^E]^{r-1} G'(\cdot)}{(mn-1) [[x_M + (n-1)x^E]^r + (m-1) [nx^E]^{r^2}]} \end{aligned}$$

$$+ \frac{n(m-1) \left[ [x_M + (n-1)x^E]^r - [nx^E]^r \right] G''(\cdot)}{(mn-1) \left[ [x_M + (n-1)x^E]^r + (m-1)[nx^E]^r \right]}. \quad (\text{A8})$$

When the prize is independent of  $X$  – implying  $G'(\cdot) = G''(\cdot) = 0$  – the second derivative of (6) with respect to  $x_M$  is given by the first term of this expression, which is strictly negative for any  $x^E > 0$ . When the prize is endogenous, we need to rely on the weaker condition of local stability. At the symmetric solution  $x_M = x^E$ , the above expression becomes

$$- \frac{rn(m-1) [m(1-r) + 2r] G(\cdot)}{(mn-1)(X^E)^2} + \frac{2rn(m-1)G'(\cdot)}{(mn-1)X^E}, \quad (\text{A9})$$

where  $X^E = mnx^E$ .

A sufficient condition for the existence of a local equilibrium in evolutionarily stable strategies is that (A9) is strictly negative or

$$- [m(1-r) + 2r] G(\cdot) + 2X^E G'(\cdot) < 0, \quad (\text{A10})$$

which is always satisfied for  $G'(\cdot) \leq 0$ . If, however  $G'(\cdot) \geq 0$ , one must have

$$\epsilon_X < \frac{[m(1-r) + 2r]}{2}, \quad (\text{A11})$$

which is always verified because the right-hand term of this inequality is always greater than 1 for any  $m \geq 2$ , and because  $\epsilon_X$  is always strictly lower than 1.

### A.3 Proof of Proposition 4

Suppose that the mutant player  $M$  believes that all other players choose  $x_i = x$ . Then, player  $M$ , endowed with the preference parameters  $(\alpha_M, \beta_M)$ , chooses  $x_M$  to maximize

$$\begin{aligned} V_M(a_M, \beta_M) &= (1 + \alpha_M) \frac{[x_M + (n-1)x]^r}{[x_M + (n-1)x]^r + (m-1)[nx]^r} G(\cdot) - x_M - \alpha_M x \\ &+ \beta_M \frac{[nx]^r}{[x_M + (n-1)x]^r + (m-1)[nx]^r} G(\cdot) - \beta_M x. \end{aligned} \quad (\text{A12})$$

The first derivative of  $V_M(a_M, \beta_M)$  with respect to  $x_M$  is given by

$$\begin{aligned} (1 + \alpha_M) &\left[ \frac{r [x_M + (n-1)x]^{r-1} (m-1) [nx]^r G(\cdot)}{[x_M + (n-1)x]^r + (m-1)[nx]^r} + \frac{[x_M + (n-1)x]^r G'(\cdot)}{[x_M + (n-1)x]^r + (m-1)[nx]^r} \right] - 1 \\ &+ \beta_M \left[ - \frac{r [x_M + (n-1)x]^{r-1} [nx]^r G(\cdot)}{[x_M + (n-1)x]^r + (m-1)[nx]^r} + \frac{[nx]^r G'(\cdot)}{[x_M + (n-1)x]^r + (m-1)[nx]^r} \right]. \end{aligned} \quad (\text{A13})$$

Evaluating this expression at  $x_M = x$ , and setting it to 0, we obtain

$$\Phi(X) = \frac{m}{(1 + \alpha_M) [r(m-1) + \epsilon_X] + \beta_M (\epsilon_X - r)}, \quad (\text{A14})$$

where  $X = mnx$ ,  $\Phi(X) = G(X)/X$ , and  $\epsilon_X = G'(X)X/G(X)$ .

This is the solution candidate for a local Nash equilibrium with robust beliefs. Solving in  $(\alpha_M, \beta_M)$  for the equality between  $\Phi(X)$  given by (A14) and  $\Phi(X^E)$  given by (7), we obtain the set of  $(\alpha^E, \beta^E)$ , as characterized by (10).

Now, we verify equilibrium existence. The second derivative of  $V_M(\alpha_M, \beta_M)$  with respect to  $x_M$  is given by

$$\begin{aligned} & \frac{r[(1 + \alpha_M)(m - 1) - \beta_M][nx]^r [x_M + (n - 1)x]^{r-2} \left[ \begin{array}{l} (1 + r)[x_M + (n - 1)x]^r \\ +(1 - r)(m - 1)[nx]^r \end{array} \right] G(\cdot)}{[x_M + (n - 1)x]^r + (m - 1)[nx]^r} \\ & + \frac{2r[(1 + \alpha_M)(m - 1) - \beta_M][nx]^r [x_M + (n - 1)x]^{r-1} G'(\cdot)}{[x_M + (n - 1)x]^r + (m - 1)[nx]^r} \\ & + \frac{[(1 + \alpha_M)[x_M + (n - 1)x]^r + \beta_M[nx]^r] G''(\cdot)}{[x_M + (n - 1)x]^r + (m - 1)[nx]^r}. \end{aligned} \quad (\text{A15})$$

Evaluating this last expression at  $x_M = x$  with  $X = mnx$ , and rearranging the first two terms, gives

$$\frac{r[(1 + \alpha_M)(m - 1) - \beta_M] [-[m(1 - r) + 2r] + 2\epsilon_X] G(\cdot)}{mX^2} + \frac{(1 + \alpha_M + \beta_M) G''(\cdot)}{m}. \quad (\text{A16})$$

Evaluating the above expression at  $(\alpha_M, \beta_M) = (\alpha^E, \beta^E)$ , as given by (10), yields

$$\frac{r[n\kappa(m - 1) - \beta^E \epsilon_X] [-[m(1 - r) + 2r] + 2\epsilon_X] G(\cdot)}{[r(m - 1) + \epsilon_X] X^2} + \frac{[n\kappa + r\beta^E] G''(\cdot)}{r(m - 1) + \epsilon_X}, \quad (\text{A17})$$

where  $\kappa = r(m - 1)/(mn - 1)$ .

A sufficient condition for the existence of a local equilibrium in evolutionarily stable preferences with robust beliefs is that the expression in (A17) is strictly negative. The second term is negative if  $G''(\cdot) \leq 0$  and  $\beta^E > -n\kappa/r$ , hence the assumption on  $\underline{L}$ . Again, we also have  $[-[m(1 - r) + 2r] + 2\epsilon_X] < 0$  (see (A11)). Thus, the first term of (A17) is strictly negative if  $n\kappa(m - 1) - \beta^E \epsilon_X > 0$  or  $\beta^E \epsilon_X < n\kappa(m - 1)$  (by recalling that  $G(\cdot)$  is always strictly positive). This inequality is always verified if either  $\epsilon_X$  or  $\beta^E$  is negative. If  $\epsilon_X$  and  $\beta^E$  are both negative, then the inequality is also always satisfied for  $\beta^E > -n\kappa/r$  since  $\epsilon_X > -r(m - 1)$  from property (iiib) of Assumption 1. If  $\epsilon_X$  and  $\beta^E$  are both positive, then  $\beta^E \epsilon_X$  reaches a maximum when  $\epsilon_X$  tends to 1. It follows that the first term of (A17) is strictly negative if  $\beta^E < n\kappa(m - 1)$ , hence the assumption on  $\bar{L}$ .

## A.4 Proof of Proposition 6

(i) Suppose first that  $\beta_{ij} = 0$  for all  $(i, j) \in I \times J$ . In this case, (A14) becomes

$$\Phi(X) = \frac{m}{(1 + \alpha_M)[r(m - 1) + \epsilon_X]}. \quad (\text{A18})$$



Solving in  $1 + \alpha_M$  for the equality between  $\Phi(X)$  given by (A18) and  $\Phi(X^E)$  given by (7), we obtain  $1 + \alpha^E$  given by (11).

Using (A16), the second-order condition for local maximization becomes

$$\frac{r(1 + \alpha_M)(m - 1)[- [m(1 - r) + 2r] + 2\epsilon_X]G(.)}{mX^2} + \frac{(1 + \alpha_M)G''(.)}{m} < 0. \quad (\text{A19})$$

This condition is always satisfied for  $G''(.) \leq 0$  because  $[- [m(1 - r) + 2r] + 2\epsilon_X] < 0$ , and  $G(.) > 0$ .

Suppose now that the group contest is destructive and  $G''(.) \geq 0$ . In this case, the first term is negative, while the second term is positive. (A19) can be rewritten equivalently as

$$-r(m - 1)[m(1 - r) + 2r] + \epsilon_X [2r(m - 1) + \sigma_X] < 0, \quad (\text{A20})$$

where  $\sigma_X = G'''(X)X/G'(X)$ . Under (iiib) of Assumption 1, we have  $\sigma_X > -r(m - 1)$  and thus (A20) is always verified for  $\epsilon_X \leq 0$ .

(iia) Suppose now that  $\alpha_{ij} = 0$  for all  $(i, j) \in I \times J$ . In this case, (A14) becomes

$$\Phi(X) = \frac{m}{[r(m - 1) + \epsilon_X] + \beta_M(\epsilon_X - r)}. \quad (\text{A21})$$

Solving in  $\beta_M$  for the equality between  $\Phi(X)$  given by (A21) and  $\Phi(X^E)$  given by (7), we obtain  $\beta^E$  given by (12).

Using (A16), the second-order condition for local maximization becomes

$$\frac{r[(m - 1) - \beta_M][- [m(1 - r) + 2r] + 2\epsilon_X]G(.)}{mX^2} + \frac{(1 + \beta_M)G''(.)}{m} < 0. \quad (\text{A22})$$

Evaluating the above expression at  $\beta_M = \beta^E = (\epsilon_X - \kappa)/(r - \epsilon_X)$ , the condition is

$$\frac{r[n\kappa - \epsilon_X][- [m(1 - r) + 2r] + 2\epsilon_X]G(.)}{(r - \epsilon_X)(X^E)^2} + \frac{(r - \kappa)G''(.)}{r - \epsilon_X} < 0. \quad (\text{A23})$$

We have  $[- [m(1 - r) + 2r] + 2\epsilon_X] < 0$  and  $(r - \kappa) > 0$ . If the contest is destructive implying that  $\epsilon_X \leq 0$ , and if  $G''(.) \leq 0$ , the inequality is clearly satisfied. Now, consider that the contest is productive, implying that  $\epsilon_X \geq 0$  and  $G''(.) \leq 0$ . If  $\epsilon_X < n\kappa < r$ , the above inequality is also clearly satisfied. If  $\epsilon_X \in [n\kappa, r)$ , then the first term of (A23) is positive, while the second term is negative. If  $\epsilon_X > r$ , then the first term is negative, while the second term is positive. If  $\epsilon_X = r$ , the left-hand-term of inequality (A23) is not defined. Thus, we cannot guarantee the existence of a (local) equilibrium in evolutionarily stable preferences with robust beliefs for  $\epsilon_X \geq n\kappa$ . Finally, note that  $\beta^E$ , given by (12), is increasing in  $\epsilon_X$ , and that  $\beta^E \in (-n\kappa/r, (m - 1))$  for  $\epsilon_X \in (-r(m - 1), n\kappa)$ .

(iib) If Assumption 1 holds with  $G'''(X) = 0$ , then  $G(X)$  is an affine function of the following form  $G(X) = R + \theta X$ , with  $\theta \in [\underline{\theta}, \bar{\theta}]$ ,  $\underline{\theta} < 0$ , and  $\bar{\theta} > 0$ . In order to satisfy (i) of Assumption 1, one must have  $R > 0$ . According to (iii) of Assumption 1, one must also have  $G(X) > 0$  for all  $X \in [0, W]$ , which implies that  $\theta > \underline{\theta} \equiv -R/W$ . Also, according to (iv) of Assumption 1, one must have  $\bar{X} = R/(1 - \theta) \in (0, W]$ , which implies that  $\bar{\theta} < 1$  and  $W > R$ , and hence  $\underline{\theta} > -1$ . Using (7), we have  $X^E = n\kappa R/[1 - \theta n\kappa]$ , and the equilibrium

value of the prize is  $G(X^E) = R/[1 - \theta n\kappa]$ . The elasticity of the prize, evaluated at  $X^E$  is  $\epsilon_X = \theta n\kappa$ , which is strictly lower than  $r$ , because  $n\kappa < r$  and  $\theta < 1$ . In this case, the second-order condition for local maximization given by (A23) is always satisfied because  $G''(\cdot) = 0$ ,  $\epsilon_X = \theta n\kappa < n\kappa < r$ , and  $[-[m(1-r) + 2r] + 2\epsilon_X] < 0$ . Finally, note that the evolutionarily stable out-group preference parameter is  $\beta^E = [(\theta n - 1)(m - 1)] / [(mn - 1) - \theta n(m - 1)]$ , which is increasing in  $\theta$ . When  $\theta$  tends to 1,  $\beta^E$  tends to  $(m - 1)$ . When  $\theta$  tends to  $-1$ , we have  $\beta^E = -[(n + 1)(m - 1)] / [2mn - (n + 1)] > -1$ .

## References

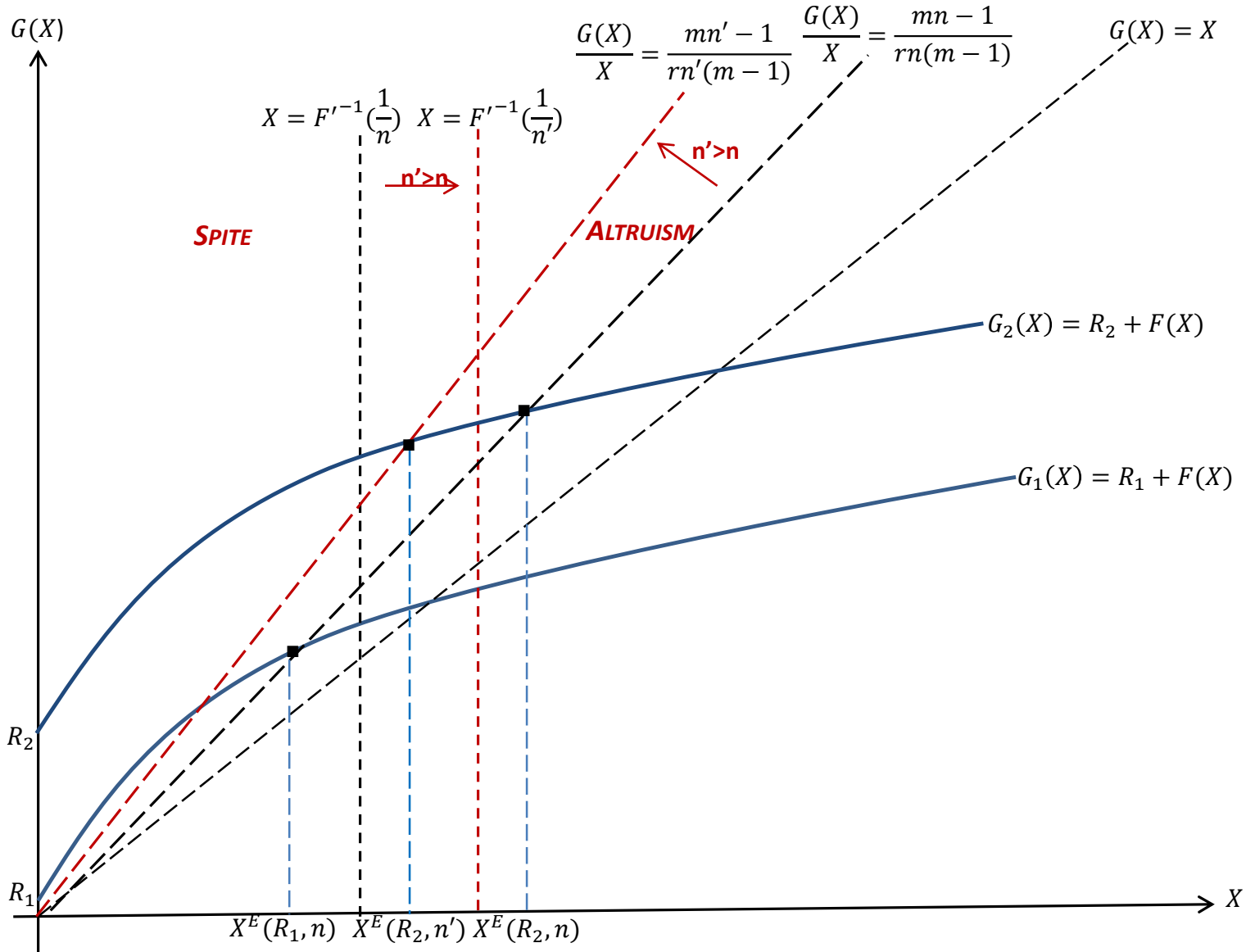
- [1] Abbink, K., Brandts, J., Herrmann, B., Orzen, H., 2010. Intergroup conflict and intra-group punishment in an experimental contest game. *American Economic Review* 100, 420–447.
- [2] Abbink, K., Brandts, J., Herrmann, B., Orzen, H., 2012. Parochial altruism in intergroup conflicts. *Economic Letters* 117, 45–48.
- [3] Alger, I., Weibull, J., 2010. Kinship, incentives and evolution. *American Economic Review* 100, 1725–58.
- [4] Alger, I., Weibull, J., 2019. Evolutionary models of preference formation. *Annual Review of Economics* 11, 329–54.
- [5] Alger, I., Weibull, J., Lehmann, L., 2020. Evolution of preferences in structured populations: Genes, guns and culture. *Journal of Economic Theory* 185, 104951.
- [6] Alós-Ferrer, C., Ania, A.B., 2001. Local equilibria in economic games. *Economic Letters* 70, 165–73.
- [7] Bester, H., Güth, W., 1998. Is altruism evolutionarily stable? *Journal of Economic Behavior and Organization* 34, 193–209.
- [8] Böhm, R., Rusch, H., Baron, J., 2020. The psychology of intergroup conflict: a review of theories and measures. *Journal of Economic Behavior and Organization* 178, 947–962.
- [9] Bowles, S., 2006. Group competition, reproductive leveling, and the evolution of human altruism. *Science* 314, 1569–1572.
- [10] Bonstein, G., 1992. The free-rider problem in intergroup conflicts over step-level and continuous public goods. *Journal of Personality and Social Psychology* 62, 597–606.
- [11] Bornstein, G., 2003. Intergroup conflict: individual, group and collective interests. *Personality and Social Psychology Review* 7(2), 129–145.
- [12] Bornstein, G., Gneezy, U., Nagel, R., 2002. The effect of intergroup competition on group coordination: an experimental study. *Games and Economic Behavior* 41, 1–25.
- [13] Cheikbossian, G., 2021. Evolutionarily stable in-group altruism in intergroup conflict over (local) public goods. *Games and Economic Behavior* 127, 206–226.

- [14] Chowdhury, S.M., Sheremeta, M.R., 2011. A generalized Tullock contest. *Public Choice* 147, 413-20.
- [15] Choi, J.K., Bowles, S., 2007. The coevolution of parochial altruism and war. *Science* 318, 636-40.
- [16] Chung, T-Y., 1996. Rent-seeking contests when the prize increases with aggregate effort. *Public Choice* 87, 55-66.
- [17] Damianov, D.S., Sanders, S., Yildizparlak, A., 2018. Asymmetric endogenous prize contests. *Theory and Decision* 85, 435-53.
- [18] Dekel, E., Ely, J.C., Yilankaya, O., 2007. Evolution of preferences. *Review of Economic Studies* 74, 685-704.
- [19] Eaton, B.C., Eswaran, M., 2003. The evolution of preferences and competition: a rationalization of Veblen's theory of invidious comparisons. *Canadian Journal of Economics* 36(4), 832-859.
- [20] Eaton, C.B., Eswaran, M., Oxoby, R.J., 2011. 'Us' and 'Them': the origin of identity, and its economic implications. *Canadian Journal of Economics* 44(3), 719-748.
- [21] Glowacki, L., Wilson, M.L. Wrangham, R.W., 2020. The evolutionary anthropology of war. *Journal of Economic Behavior and Organization* 178, 963-982.
- [22] Güth, W., Yaari, M.E., 1992. Explaining reciprocal behavior in simple strategic games. In: Witt, U. (Ed.). *Explaining Process and Change: Approaches to Evolutionary Economics*. University of Michigan Press, Ann Arbor, 23-34.
- [23] Hehenkamp, B., Leininger, W., Possajennikov, A., 2004. Evolutionary equilibrium in Tullock contests: spite and overdissipation. *European Journal of Political Economy* 20, 1045-57.
- [24] Heifetz, A., Shannon, C., Spiegel, Y., 2007. What to maximize if you must. *Journal of Economic Theory* 133, 31-57.
- [25] Hirai, S., Szidarovszky, F., 2013. Existence and uniqueness of equilibrium in asymmetric contests with endogenous prizes. *International Game Theory Review* 15(1), 1-9.
- [26] Katz, E., Nitzan, S., Rosenberg, J., 1990. Rent-seeking for pure public goods. *Public Choice* 65, 49-60.
- [27] Kimbrough, E.O., Laughren, K., Sheremeta, R., 2020. War and conflict in economics: theories, applications, and recent trends. *Journal of Economic Behavior and Organization* 178, 998-1013.
- [28] Konrad, K., 2009. *Strategy and Dynamics in Contests*. New York, NY: Oxford University Press.
- [29] Konrad, K.A., Morath, F., 2012. Evolutionarily stable in-group favoritism and out-group spite in intergroup conflict. *Journal of Theoretical Biology* 306, 61-67.

- [30] Lehmann, L., Feldman, M., 2008. War and the evolution of belligerence and bravery. *Proceedings of the Royal Society, Biological Sciences*.
- [31] Leininger, W., 2009. Evolutionarily stable preferences in contests. *Public Choice* 140, 341-356.
- [32] Lopez, A.C., Johnson, D.P., 2020. The determinants of war in international relations. *Journal of Economic Behavior and Organization* 178, 983-997.
- [33] Maynard Smith, J., Price, G.R., 1973. The logic of animal conflict. *Nature* 246, 15-18.
- [34] Münster, J., 2009. Group contest success functions. *Economic Theory* 41, 345-357.
- [35] Nitzan, S., 1991. Collective rent dissipation. *Economic Journal* 101, 1522-34.
- [36] Ok, E.A., Vega-Redondo, F., 2001. On the evolution of individualistic preferences: an incomplete information scenario. *Journal of Economic Theory* 97, 231-254.
- [37] Rapoport, A., Bornstein, G., 1987. Intergroup competition and the provision of binary public goods. *Psychological Review* 94, 291-299.
- [38] Rapoport, A., Bornstein, G., 1989. Solving public good problems in competition between equal and unequal size groups. *Journal of Conflict Resolution* 33, 460-479.
- [39] Rusch, H., 2014. The evolutionary interplay of intergroup conflict and altruism in humans: a review of parochial altruism theory and prospects for its extension. *Proceedings of the Royal Society B: Biological Sciences* 281.
- [40] Rusch, H., Gravilets, S., 2020. The logic of animal intergroup conflict: a review. *Journal of Economic Behavior and Organization* 178, 1014-1030.
- [41] Sethi, R., Somanathan, E., 2001. Preference evolution and reciprocity. *Journal of Economic Theory* 97, 273-97.
- [42] Schaffer, M.E., 1988. Evolutionary stable strategies for a finite population and a variable contest size. *Journal of Theoretical Biology* 132, 469-478.
- [43] Shaffer, S., 2006. War, labor tournaments, and contest payoffs. *Economic Letters* 92, 250-55.
- [44] Sheremeta, R.M., 2018. Behavior in group contests: a review of experimental research. *Journal of Economic Surveys* 32(3), 683-704.
- [45] Tajfel, H., 1982. Social psychology of intergroup relations. *Annual Review of Psychology* 33, 1-39.
- [46] Tajfel, H., Turner, J.C., 1979. An integrative theory of intergroup conflict". In: Worchel, S., Austin, W., (Eds). *The Social Psychology of Intergroup Relations*. Monterey, CA: Brooks/Cole, 33-47.
- [47] Tullock, G., 1980. Efficient rent-seeking. In: Buchanan J.M., Tollison, R.D., Tullock, G. (Eds). *Toward a Theory of the Rent-Seeking Society*. College Station, TX: Texas A&M University Press, 97-112.

# Figure 1: Productive contest

$R_2 > R_1$  and  $n' > n$



## Figure 2: Destructive contest

$R_2 > R_1$  and  $n' > n$

