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Abstract
We investigate whether the fluctuations of agricultural output prices may explain the low level of input use in Sub-Saharan Africa. We combine data on local maize prices and data on farmers' fertilizer use over the 2009-2011 period in Burkina-Faso to estimate a panel-tobit model of fertilizer use. We separate the predictable and unpredictable components of maize price fluctuations and find that fertilizer use decreases when maize price fluctuations increase, and more specifically when unpredictable price fluctuations increase.

Keywords : Fertilizer use, price risk, maize, intensification, Sub-Saharan Africa

Utilisation d'intrants et risques de prix : le cas du maïs au Burkina Faso

Résumé
Nous analysons si les fluctuations des prix des produits agricoles peuvent expliquer le faible niveau d'utilisation des intrants en Afrique Sub Saharienne. Nous combinons des données sur les prix locaux du maïs et des données sur l'utilisation des engrais chimiques sur la période 2009-2011 au Burkina-Faso pour estimer un modèle tobit en panel d'utilisation d'intrants. Nous distinguons les composants prévisibles et imprévisibles des fluctuations des prix du mais et établissons que l'utilisation d'intrants diminue quand les fluctuations des prix du maïs augmentent, et que cet effet est lié à la composante imprévisible des fluctuations des prix.

Mots-clés : Utilisation d'intrants, risques de prix, maïs, intensification, Afrique Sub Saharienne

JEL: Q12, Q13

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1. Introduction

Cereal yields have risen significantly in most developing countries in the last forty years, but West Africa is lagging behind (World Bank, 2007, Wiebe et al 2017). Cereal yields have reached an average of almost six tons per hectare in East Asia while they hardly exceed one ton per hectare in West Africa[1]. Although fertilizer use is not uniformly low across West Africa and across farmers (Sheahan and Barrett, 2014), the low use of chemical fertilizers is considered as a major cause of yield stagnation in Africa (Morris et al., 2007). Indeed, the average Nitrogen, Phosphorus and Potassium (NPK) nutrient uses are around 10 kg per hectare of arable land in West Africa, while they exceed 500 kg per hectare in East Asia[2].

Many studies have investigated the reasons why chemical fertilizers are under-used in Africa. On the supply side, fertilizer distribution may be discouraged by unfavorable business conditions characterized by market segmentation, and high transportation costs (Morris et al., 2007). On the demand side, fertilizer use can be hindered by high fertilizer prices (Liverpool-Tasie, 2014), especially for farmers facing liquidity constraints and imperfect credit markets (Minot, Kherallah and Berry, 2000). Other authors have also raised the issue of farmers' lack of awareness or technical skills with regard to fertilizer use (Feder and Slade, 1984), and of farmers' risk aversion that may explain, at least partly, low demand for fertilizers that are considered risk-increasing inputs (Binswanger and Sillers, 1983; Duflo, Kremer and Robinson, 2009).

Another strand of the literature focuses on the link between output prices and input use. It has been theoretically and empirically demonstrated that output price levels influence positively the use of yield-increasing inputs such as fertilizers (Alene et al., 2008). The effect of output price risks on input demand is also well described theoretically (Batra and Ullah, 1974; Hartman, 1976; Isik, 2002), but the empirical magnitude of this effect is much less documented. It is indeed difficult to disentangle price risk from other risks, or from imperfectly observed correlated variables, like market segmentation, weather uncertainties, or soil suitability. In spite of existing evidence that cereal price fluctuations tend to be greater in Africa than in other regions (Minot, 2014), there is very little empirical evidence on how farmers' input use decisions respond to output price instability in Africa (Jayne, 2012). The purpose of this article is to fill this gap, both with a theoretical model adapted from Isik (2002) and with an empirical investigation focusing on the use of fertilizer by maize producers in Burkina-Faso. We use an original database, that combines farm level information on input use and local market information on monthly maize prices. This database allows us to build a balanced panel for a representative sample of 1632 rural households that grow maize and covers the 2009-2011 period[3]. To highlight the respective impact of predictable and unpredictable price output price fluctuations on input use, we estimate both types of

[1] Latest available data for 2018 indicate average cereal yields of 1.4 ton per hectare in West Africa, and 5.8 tons per hectare in East Asia). In Burkina Faso, cereal yields are of 1.1 ton per hectare in 2018 (FAOSTAT data downloaded on http://www.fao.org/faostat/en/#data on April 2020, 17th)

[2] Latest data available for 2017 indicate average NPK (nitrogen, phosphorus, and potassium) nutrients of 15kg/ ha of arable land in Western Africa and 427 kg/ha in Eastern Asia. In Burkina Faso, average NPK nutrients are of 18 kg/ ha of arable land (FAOSTAT data downloaded on http://www.fao.org/faostat/en/#data on April 2020, 17th):

[3] This data set may seem a bit outdated, but it is the period we found with reliable data both on prices and fertilizers than can be merged and representative at the country level.

fluctuations and we establish that maize unpredictable price fluctuations significantly decrease fertilizer use at farm level in Burkina Faso while maize predictable price fluctuations have no significant effect. These results can help formulate policy recommendations to encourage the use of farm inputs in countries with high agricultural price volatility.

The paper is structured as follows. In section 2, we isolate the expected effects of maize predictable and unpredictable price fluctuations on fertilizer use decisions by adapting the Isik's seminal model (2002). We establish theoretically, and under the assumption of farmer's risk aversion, that only unpredictable fluctuations (volatility) of output prices are expected to exert a negative effect on fertilizer use. In Section 3, we describe our empirical strategy to assess the magnitude and significance of such effects: we present the price and rural household datasets used and we detail the fixed-effect panel tobit models used to estimate price risk measures and distinguish predictable from non-predictable price fluctuations. In Section 4, we deliver and discuss our empirical results. Section 5 concludes.

2. Price risks and input use

The effect of output price risks on aggregate agricultural supply has been extensively studied at macroeconomic level (Combes et al., 2014 ; Subervie, 2008; Haile et al, 2015). The indicators of price risk used in these studies encompass all price fluctuations, including fluctuations relating to the market fundamentals (supply and demand dynamics), regular fluctuations (seasonality, trends), and "unpredictable" fluctuations related to unexpected shocks. Farmers adopt risk-mitigating strategies to deal with output price fluctuations but they may differ for "predictable price fluctuations" , on which it is assumed that they can form expectations, and for "unpredictable price fluctuations ", also called volatility. Researchers have suggested and tested methods to analyse the impact of unpredictable output price fluctuations on household decisions at farm level, notably Holt and Moschini (1992) on sow farrowing decisions in the US, and Rezitis and Stavropoulos (2009) on pork supply in Greece. They both show a negative impact of price volatility on farm supply. Our paper uses similar methods to estimate the predictable and unpredictable components of maize price fluctuations in Burkina Faso.

The economic literature uses various indicators to measure price risks. The most common one is the coefficient of variation of past prices. However, such indicator overestimates the risk, as it encompasses both predictable and unpredictable price fluctuations (Aizenman and Pinto 2005). It is therefore more accurate to regard total variability as the sum of predictable variability and pure risk (Wolf 2005), and to rely on empirical approaches decomposing price series between deterministic and stochastic parts (Dehn 2000). In this paper, we present two options for computing price risk. The first option uses the variance of observed prices as a price risk indicator. It comes from the direct application of Isik's modeling framework (2002), and is akin to "total price variability". It should be seen as the upper bound of price risk, more than the price risk per se. The second option relies on a decomposition of prices series between a deterministic part ("predictable price variability") measured as the variance of predicted prices estimated with a price autoregressive conditional heteroscedastic model, and a stochastic part (unpredictable price fluctuations, ie "pure price risk" or volatility) measured as the variance of the residuals obtained from this model.

We modify the price risk specification used in Isik's model (2002) to distinguish between "total price variability", and "predictable price variability versus pure price risk". Whereas in Isik (2002), the price is the sum of a constant and an error term, we include the possibility that not only stochastic fluctuations can affect input but also deterministic fluctuations, related to past prices, trends or seasonality. Indeed we argue that farmers, at time of input use decision, do not know the time at which they will sell and cannot therefore know the price they will get on the market. Thus uncertainty about the future selling price remains partially unresolved, even for the deterministic part of price fluctuations. Therefore, the variance of deterministic price fluctuations may affect input choice (predictable price variability), just as the variance of stochastic price fluctuations (pure price risk).

We additively decompose monthly price P_t into a deterministic part, $\widehat{P_t}$ and a stochastic part θ_t, writing $P_t = \widehat{P_t} + \theta_t$. The deterministic part is a typical autoregressive price formation model with trend and seasonality. In equation (1), β_0 is the constant term, $\beta_1$is the autoregressive coefficient, β_2 is the trend coefficient, γ_i is the coefficient associated with season i and S_i are dummy variables taking 1 for month t and 0 for all others.

$$P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 t + \sum_{i=1}^{11} \gamma_i S_i + \theta_t \quad (1)$$

This price formation process produces a series of predicted prices $\widehat{P_t}$ with variance σ_V (predictable price variability). The remaining risk – unpredictable - is encapsulated in σ_θ, the variance of θ_t (pure price risk).

The optimal input use x is given by the following expected utility (EU) maximization program, conditional on price information until one month before harvest.

$$\max_{x_{t-1}} \text{EU}\ (\pi_t(x_{t-1})|P_{t-1}) = \text{EU}\left[(\widehat{P_t} + \theta_t)[f(x_{t-1}) + h(x_{t-1})\varepsilon_t] - w x_{t-1}|P_{t-1}\right] \quad (2)$$

Where w is the input price, assumed constant[4], $f(.)$ is the deterministic part of the production function and h(.) is the stochastic part, (f'(x) +h'(.)ε_t>0 and f''(.) +h''(.)θ_t<0) , and π_t is the profit after sale at time t. Fertilizer is a risk increasing input in the sense of Just and Pope (1979), and $h_{x_{t-1}} > 0$. The decision on input use takes place before the decision to sell, and the optimization program is solved with respect to x_{t-1}.

The first order condition with respect to x_{t-1} is given by:

$$\frac{d\text{EU}(\pi_t|P_{t-1})}{dx_{t-1}} = \frac{\partial \text{U}(\pi_t|P_{t-1})}{\partial \pi_t} * \frac{\partial \pi_t}{\partial x_{t-1}} = \text{E}\left[\text{U}_{\pi_t}(\pi_t)\left((\widehat{P_t} + \theta_t\right)(f' + h'\varepsilon_t) - w)\right]|P_{t-1} = 0 \quad (3)$$

Where U_{π_t} denotes the derivative of U with regard to profit. Following Isik (2002), using a Taylor development series around $\widehat{P_t}$ and around f(x_{t-1}), and assuming that $\text{cov}(\theta_t, \varepsilon_t) = 0$, i.e. that output price volatility and production risks are uncorrelated, we retrieve

$$\widehat{P_t} f' = \frac{\Phi h h_{x_{t-1}} \sigma_\varepsilon (\widehat{P_t}^2 + \sigma_\theta^2) + w}{1 - \Phi \frac{f\sigma_\theta}{\widehat{P_t}}} \quad (4)$$

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[4] We explain in section 3 why this assumption is valid in Burkina Faso

where $\Phi = -\frac{U_{\pi\pi}(\pi_{P_t})}{U_{\pi}(\pi_{P_t})}$ is the Arrow-Pratt measure of absolute risk aversion.

The detailed resolution of our model is given in Appendix 5.

This expression predicts that an increase in the unexpected price variance σ_θ or an increase in the production variance ε_t generates a decrease in fertilizer use. The difference between this first order condition (defining implicitly the optimum fertilizer use with risk, x_{t-1}^*) and the similar problem without risk (defining the optimal fertilizer use without risk x^o) reflects the marginal risk premium due to risk aversion. The numerator of the right hand side term in expression (4) is positive because all its components are positive and the denominator has to be positive because f' is positive. Thus, $0 \leq 1 - \Phi \frac{f\sigma_\theta}{\widehat{P_t}} \leq 1$ which implies that $\widehat{P_t} f' \geq w$ and thus $x_{t-1}^* \leq x^0$. When σ_θ increases or when ε_t increases, the right hand side of expression (4) increases, meaning an increase in the risk premium and a decrease in x_{t-1}.

3. Empirical Strategy

Maize production has an increasing role in agricultural development in Burkina Faso. It accounts for 35% of total cereal production, while it was only 7% of the cereal production in 1984[5]. From data collected by the Ministry of Agriculture of Burkina Faso, we estimate that more than 75% of farmers grow maize, both for household consumption and for selling on the market. 15% of maize production is sold on domestic markets, representing an important source of income for farmers.

Maize is the most cultivated staple in the southern and western regions of Burkina Faso, while it is scarcer in the northern region of the country because of unfavourable dry climatic conditions. Yields are highly dependent upon fertilization (Koussoubé and Nauges, 2015). The three main components of fertilizers, Nitrogen (N), Phosphorus (P) and Potassium (K), can be brought by manure, crop residues or by chemical fertilizers (mostly NPK blends and Urea). Recommendations for an optimal application of chemical fertilizers onto maize crops vary according to soil quality, weather conditions and the use of organic manure or compost, but are on average of 150 kg/ha of NPK blends and 50 kg of urea. Yet the average amounts of NPK fertilizers used on maize plots are much lower than these recommendations, ranging from 10 kg of NPK/ha in northern regions to 100 kg/ha in south-western regions.

3.1 Price data

We use maize price data collected by the SONAGESS (Société Nationale de Gestion du Stock de Sécurité Sociale- National society for security stock management) in Burkina Faso. Since 1992, SONAGESS has set into place a Market Information System: Prices of main agricultural products are collected on a monthly basis for 48 local markets in the whole country.

Because of missing values, we used a subset of 39 markets for which maize prices were available for the 2006-11 period. Monthly maize price series are expressed in local currency

[5] FAOSTAT data http://www.fao.org/faostat/en/#data/ consulted on April 2020, 17th

per kilogram (FCFA/kg), and then deflated by the monthly Consumer Price Index (CPI) to generate real price series.

Descriptive statistics of average real maize prices by regions (each region includes between 2 to 4 markets) over the 2006-11 period are provided in Table 1 (no correction has been applied to weight average prices according to the volumes of maize sold on those markets). Regions are either maize-surplus regions (maize production exceeds maize consumption and, on average, rural households are net sellers), maize-deficit regions (maize production is lower than maize consumption and, on average, rural households are net buyers), or regions where maize production can be above or below maize consumption, depending on yearly rainfall conditions.

Table 1. Descriptive statistics of average real maize prices by regions from 2006 to 2011 (FCFA/kg)

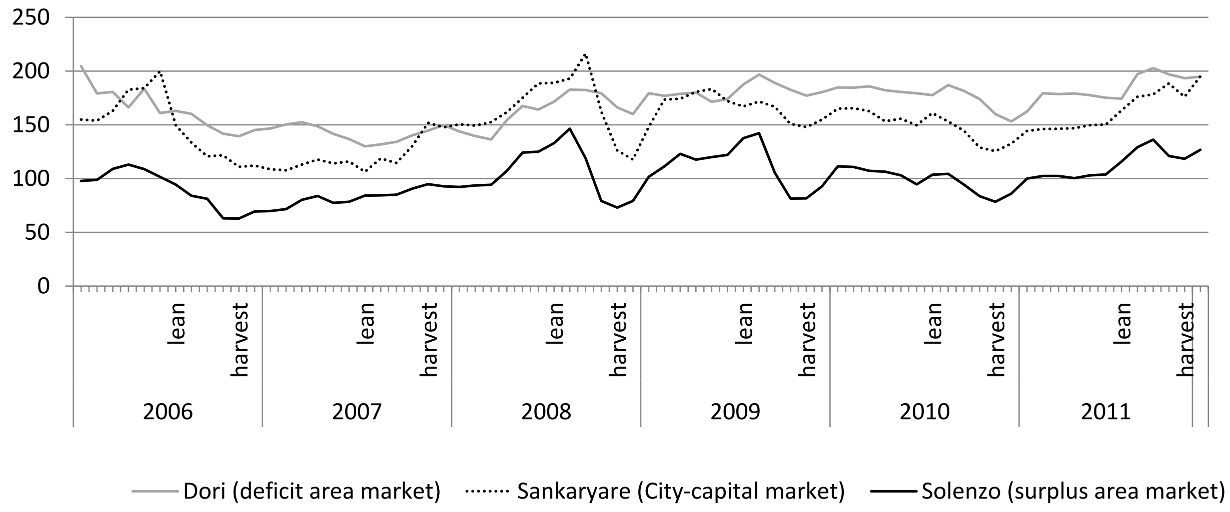
	Surplus region	Mean	Std.Dev.	Min	Max
Boucle du Mouhoun	Yes	119	21	98	157
Cascades	Yes	103	12	88	119
Centre	No	152	3	148	155
Centre-Est	Yes/No	139	14	116	157
Centre-Nord	No	145	5	138	151
Centre-Ouest	Yes	131	6	123	141
Centre-Sud	Yes/No	149	6	139	159
Est	Yes/No	129	12	112	143
Hauts-Bassins	Yes	92	9	74	104
Nord	No	153	6	143	166
Plateau central	No	153	3	148	158
Sahel	No	143	20	124	169
Sud-Ouest	Yes	138	13	118	159

(from SONAGESS data)

Agricultural markets are poorly connected, and, as a consequence, maize price levels and dynamics tend to vary greatly across markets, remote markets exhibiting higher levels of price fluctuations (Le Cotty et al 2017). This is confirmed by our data. Markets located in northern regions (*Nord, Sahel, Centre-Nord, Centre and Plateau central*) correspond to deficit regions and register the highest levels of maize price while markets located in southern-western regions (*Boucle du Mouhoun, Cascades, Sud-Ouest and Hauts-Bassins*), where maize production exceeds maize consumptions, display lower maize price levels.

Figure 1 displays the changes in maize real prices between 2006 and 2011 for three markets Sankaryare, a capital-city market (region of *Plateau Central*); Solenzo market located in the "*Boucle du Mouhoun*" surplus region market and the Dori market located in the "*Sahel*" deficit region.

Figure 1. Changes in monthly maize real prices (FCFA/kg) in three markets



(from SONAGESS data)

In these three markets -and more generally in the 39 markets studied- the price patterns present similarities. First, prices are affected by seasonal patterns: they are lower in the harvest season that begins in September/October and higher in the lean period from June to August. Deflating by the consumer price index (CPI) does remove the annual time trend associated with inflation but does not affect the seasonality pattern displayed by maize prices. Secondly, inter-annual variations also have common features. For instance, the 2008 spike is observed everywhere in the country: it is mainly due to a severe drought that occurred during the 2007 crop campaign and, to a lesser extent, to the transmission of the international 2007/2008 price spike linked to the food crisis.

3.2 Farm household data

We merged price data with rural household data collected yearly by the Ministry of Agriculture of Burkina Faso on a representative sample of around 4000 rural households. This annual survey collects information, through direct interviews of farmers, on their agricultural activities and their socio-economic characteristics. We used a balanced panel of households over the 2009-2011 period to analyze farm household strategies. We assumed that rural households sell their products on the nearest (distance as the crow flies) market[6], and thus pay or receive the observed prices on this market. When price data were missing in the nearest market, we used the second nearest market. The combination of available market prices (39 markets) and continuously available household data over the 2009-2011 period led to a population of 2190 households, from which we extracted our sample of the 1632 households who grow maize. Table 2 presents the summary statistics on this sample of maize growers, both on household characteristics and on agricultural decisions, over the 2009-2011 period.

[6] Which may indeed not be the case, for example if the nearest market is reachable by an unpaved road whereas an alternative market, located further away is reachable trough a paved road or better connects with collective transports.

Table 2. Descriptive statistics of our sample of 1632 maize growers

	Mean	Std. Dev	Min	Max
Age (head of household)	44.8	11.8	16	96
Number of household members	8.8	4.5	1	66
Total cultivated area (ha)	3.7	4.2	.01	73.9
Maize area (ha)	1.0	1.8	0.1	33.8
Cotton area (ha)	0.6	2.1	0	69.2
Cattle size (number of cows)	3.5	15.1	0	537
NPK use on maize (kg)	60.3	142.6	0	2300
Urea use on maize (kg)	29.8	113.5	0	6100
Intensity of NPK use on maize plots(kg/ha)	62.8	136.3	0	497*
Intensity of urea use on maize plots (kg/ha)	31.0	60.9	0	240*

(from Ministry of Agriculture of Burkina Faso data)

Average household size is almost 9 people, while average total cultivated area is 3,7 hectares and average cattle herd size is 3,5 cows. Maize production represents an average of 1.0 hectare per farm and cotton production an average of 0.6 hectare. Other crops include sorghum, millet and cash crops, notably cowpea and groundnut.

In terms of the use of chemical fertilizers for maize production, the sample displays a large heterogeneity. Almost two third (64%) of maize producers do not apply fertilizers at all on their maize plots. The average fertilizer use observed for our sample is close to national averages: 63 kg of NPK and 31 kg of urea per hectare of maize, below the technical recommendations of 150 kg of NPK and 50 kg of urea.

21% of the surveyed farmers grow cotton. Cotton is a cash crop. It facilitates access to chemical fertilizers. Indeed, most farmers growing cotton are under contract with the cotton company. They get bags of fertilizers at the planting time, and the corresponding costs are deducted from their payments at harvest time. It is common for some of the fertilizer provided to farmers for their cotton to be used for other crops, mainly maize.

3.3 Estimation of output price risks

Following the theoretical model described in section 2, we measure maize price risks with three distinct variance indicators: (1) the variance of observed past prices (total price variability), (2) the variance of expected prices (predicted price variability) and (3) the variance of the error term of an expected price process (pure price risk). The first indicator corresponds to Isik's model $P_t = \beta_0 + \varepsilon_t$ where the variance of observed prices is equal to the variance of ε_t. The second and third indicators are derived from an adapted model accounting for both predictable and unpredictable price fluctuations. We use an autoregressive price formation model of order 1, where present monthly prices depend upon lagged prices, a time trend and monthly dummies capturing seasonality (see equation 5).

$$P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 t + \sum_{i=1}^{11} \gamma_i S_i + \varepsilon_t = \widehat{P_t} + \varepsilon_t. \quad (5)$$

This price formation model produces a series of predicted prices $\widehat{P_t}$ with a variance σ_V (predictable price fluctuation) and the variance of ε_t is σ_ε (unpredictable price fluctuation). We derive the last two indicators from autoregressive price formation models, estimated for each of the 39 markets of our sample and for each of the three years of our panel, 2009, 2010 and 2011.

Maize production is highly correlated with the rainy season in Burkina Faso: land preparation starts in June, sowing in July, weeding and fertilizing between July and September, and harvesting between October and November. Thus, maize produced during the growing season is sold on the markets from December onwards and usually farmers have sold out their stocks by the beginning of the next agricultural campaign, in June. Table 3 represents a calendar of agricultural activities on three consecutive crop years. On the right side of the table, t corresponds to the start of the fourth crop year (June).

Table 3. Agricultural activity and memory of prices fluctuations

CROP YEAR 1		CROP YEAR 2		CROP YEAR 3		
t-36	t-30	t-24	t-18	t-12	t-6	t
Maize production	Maize selling	Maize production	Maize selling	Maize production	Maize selling	
	←-------------------------------Memory of prices fluctuations----------------------------→					

We consider that farmers when planting (which is also the time when they apply fertilizers) take into account the price fluctuations that occurred during the last three growing seasons, thus during the 30 months preceding the planting season. This assumption reflects also discussions with farmers on the way they remember prices and the fact that sales tend to be concentrated in the six months period after maize has been harvested.

Total price variability

The first indicator is the variance of monthly maize prices on market i at month t, $y_{i,t}$, reflecting the average price fluctuations observed by the farmer over the 30 months preceding his decision to till, sow and fertilize. The variance of observed prices in each of the 39 markets can be expressed as follows:

$$TPV_{i,t} = \text{Total Price Variability } y_{i,t} = \frac{\sum_{k=t-30}^{t}(P_{i,k} - \bar{P}_i)^2}{30}, \quad (6)$$

where $P_{i,k}$ is the observed maize price on market i at month k and $\bar{P}_i$ is the average maize price in real terms of market i from $t - 30$ to t.

Predicted price variability and pure price risk

To measure the two other price risk indicators, we estimate a monthly price formation process on each market separately, that accounts for what a farmer can and cannot predict.

$$P_{it} = \alpha_0 + \alpha_1 P_{it-1} + \alpha_2 t + \alpha_t S_t + \varepsilon_{it} \quad (7)$$

Let P_{it} be the observed monthly maize price, in real term, on market i for month (t), P_{it-1} denotes a one-order autoregressive term, t captures the time trend and S_t is a seasonal dummy variable - equal to 1 for month t, 0 for all other months, ε_{it} is the random error term. The monthly expected price P_{it} , resulting from the estimation of equation (7) is additively decomposed into a predictable part, $\widehat{P}_{it} = \alpha_0 + \alpha_1 P_{it-1} + \alpha_2 t + \alpha_t S_t$ and an unpredictable part $\hat{\varepsilon}_{it} = P_{it} - \widehat{P}_{i,t}$, $\widehat{P}_{it}$ being the predicted price with one- month lag with perfect information available at t on P_{it-1}.

Modeling the price formation process as indicated above, we use the variance of expected prices as an indicator of expected price fluctuations. For each 30-month sub-period preceding the planting season and for each market, we measure the variance of expected prices

$$PPV_{i,t} = \text{Predicted Price Variability}_{i,t} = \frac{\sum_{k=t-30}^{t}(\widehat{P}_k - \overline{\widehat{P}}_i)^2}{30}. \quad (8)$$

With $\overline{\widehat{P}}_i$ the average of predicted maize price of market i from month (t-30) to month (t).

Although this type of price fluctuation could theoretically be perfectly anticipated with a one-month lag, it is empirically worth checking whether it weighs as a risk on farmers' decisions, i.e. whether fertilizer use decreases after a period of greater variance of predictable prices.

Finally, our third indicator is the variance of the residual term of the above autoregressive price formation model, which corresponds to the unpredictable component of price fluctuations over each 30-month sub-period preceding the start of the planting season for each market. This indicator is a proxy for pure price risk definition, as it encompasses only unpredictable price fluctuations.

$$PPR_{it} = \text{Pure Price Risk}_{i,t} = \frac{\sum_{k=t-30}^{t}(\hat{\varepsilon}_{ik} - \overline{\hat{\varepsilon}}_i)^2}{30} \quad (9)$$

With $\overline{\hat{\varepsilon}}_i$ the average residual term of estimated maize prices on market i from month (t-30) to month (t).

Summary statistics of the three measures of maize price risk used in the analysis are presented in Table 4 by regions, and in Table 5 by surplus and deficit regions.

Table 4. Mean total price variability, predicted price variability and pure price risk by regions for the 2009-2011 period

Indicator	Total prices variability	Predicted prices variability	Pure price risk
Measure	Variance of observed prices	Variance of predicted prices by an autoregressive model	Variance of the residuals of a price prediction autoregressive model
Boucle du Mouhoun	365	212	142
Cascades	428	246	178
Centre	556	372	178
Centre – Est	581	397	170
Centre – Nord	330	194	124
Centre – Ouest	548	296	227
Centre – Sud	372	366	134
Est	607	380	215
Hauts – Bassins	430	240	174
Nord	295	194	96
Plateau central	381	277	129
Sahel	241	322	73
Sud ouest	556	334	213

(authors' calculation from SONAGESS data)

Table 5. Mean prices and mean total prices coefficient of variation*, predicted prices coefficient of variation* and pure price risk coefficient of variation*

	Mean price (FCFA/kg)	Normalized total prices variability	Normalized predicted prices variability	Normalized pure price risk*
Surplus regions	116	4.00	2.29	1.61
Deficit regions	149	2.42	1.83	0.80
National	130	3.51	2.23	1.29

*to avoid size effects when comparing price variability in surplus and deficit regions, we calculate the coefficients of variations, i.e. the variance over the price mean

Surplus regions display higher total price variability than deficit regions both because of local climatic supply shocks and seasonal price fluctuations. Deficit regions are partially fed by imported maize whose price and availability are more stable, thus smoothing out local market price fluctuations. The level of predicted prices variability is similar between surplus and deficit regions: overall, the predictable part of price fluctuations, due to seasonal effects is similar. Pure price risk is higher in surplus than in deficit regions, because of the existence of climatic supply price shocks that cannot be anticipated.

3.4 Estimation of the effect of output price risks on input use

Although maize yields are highly dependent upon the use of chemical fertilizers, two third of surveyed farmers do not apply chemical fertilizers in their maize plots. We interpret the observed zero values as censored observations. This censoring occurs if the marginal net profitability of fertilizer is negative even for the first unit used. In other words, farmers would use "negative quantities of fertilizers" if they could choose to do so, i.e. in case they had fertilizers at hand, they would rather sell it on the market to get cash than apply it on their maize crops. This could be explained by situations of low and/ or volatile maize prices, that discourages surveyed farmers either to produce maize or to apply fertilizers on their maize plots. Under these circumstances, an ordinary least squares regression may produce biased estimates because the relation between explanatory variables and the dependent variable registered at zero value is not the same as the relation between the explanatory variables and the dependent variable taking non-zero values. This suggests that a Tobit regression should be used instead (Tobin, 1958). We subsequently resort to the panel estimation of a Tobit model.

We build a panel to establish the effect of maize price risks on individual fertilizer use, so as to estimate individual and province fixed effects, and thus we correct for unobservable variables potentially correlated with price risks. Tobit models of fertilizer use with individual and province fixed effect are estimated. We use both total price variance and unpredictable price variation as price risks measures. As detailed in the former subsection, the unpredictable price variation is measured after estimating an auto-regressive conditional heteroscedastic model of price, and extracting the province-level conditional variance series.

In the following sections, we estimate for each measure of price variability its effect on two categories of dependent variables: (a) the total use of NPK fertilizer on maize plots at farm level; (b) the intensity of fertilizer use on maize plots measured as the quantity of NPK used on maize divided by the number of hectares of maize. We present below the structure of the generic model: the dependent variable $Y_{j,T}$ can either represent total quantity or quantity per hectare of fertilizer.

The observed dependent variable Y_{jT}, the fertilizer use by household j at time period T, is related to the latent variable ${Y_{jT}}^*$ which is the optimal fertilizer use without non-negativity constraint:

$$Y_{jT} = \begin{cases} {Y_{jT}}^* & \text{if } {Y_{jT}}^* > 0 \\ 0 & \text{if } {Y_{jT}}^* \leq 0 \end{cases} \tag{10}$$

Equation (10) represents a censored distribution of quantity of fertilizers use since the value of Y_{jT} for all non-fertilizers use equals zero. The estimation of a Tobit panel model specifies that:

$${Y_{jT}}^* = X'_{jT}\beta + \alpha_j + \varepsilon_{jT} \tag{11}$$

With the vector of regressors X_{jT} (including a measure of maize price fluctuation), β is the column vector of estimated parameters, α_j is the random individual-specific effect, and ε_{jT} is an idiosyncratic error. With $\alpha_j \approx N(0,\sigma_\alpha^2)$ and $\varepsilon_{jT} \approx N(0, \sigma_\varepsilon^2)$.

Total prices variability (Isik model)

The first model corresponds to our first indicator of price risk, namely the variance of observed maize prices. It describes the effect of observed maize price and observed price variances on the level of NPK use for maize production

$$Y_{jiT} = \theta_1 P_{iT} + \delta_1 \mathrm{TOTAL\_PRICE\_VARIABILITY}_{iT} + \gamma_1 \mathrm{PRODUCTION\_RISK}_{iT} + X'_{jiT}\beta + \alpha_j + \upsilon_i + T + \varepsilon_{jiT} \quad (12)$$

With j, i and T denote the household, the market and the year, spanning from 2009 to 2011, respectively, Y_{jiT} denotes the level of NPK used by household j , related to market i in year T. P_{iT} denotes the average maize price over the last 30 months preceding the start of the planting season in year T (June of year T actually); $\mathrm{Total\_Price\_Variability}_{iT}$ denotes the variance of observed prices over the 30-month sub-period preceding June of year T and $\mathrm{PRODUCTION\_RISK}_{iT}$ is the production risk approximated by the variance of the millet yield in the province in year T, that is highly correlated with rain. X is the vector of control variables. α_j represents household random effects, υ_i the market dummy and T is a time trend, while ε_{jit} is an error term with zero mean and variance σ_ε^2.

Two hypotheses are tested: $\theta_1 > 0$ and $\delta_1 < 0$. The level of maize price increases the use of NPK (resp. intensity of fertilizer use) farm, whereas the variance of observed prices decreases it.

Predicted prices variability and pure price risk (adapted model from Isik)

The second model assesses the effect of maize unpredictable and predictable price fluctuations on NPK use for maize production.

$$Y_{jiT} = \theta_2 P_{iT} + \delta_2 \mathrm{PREDICTED\_\widehat{PRICE}\_VARIABILITY}_{iT} + \phi_2 \mathrm{PURE\_PRICE\_RISK}_{iT} + \gamma_1 \mathrm{PRODUCTION\_RISK}_{iT} + X'_{jiT}\beta + \alpha_j + \upsilon_i + T + \varepsilon_{jiT} \quad (13)$$

With $\mathrm{PREDICTED\_PRICE\_VARIABILITY}_{iT} = PPV_{i,T}$ of equation 8, and $\mathrm{PURE\_PRICE\_RISK}_{it}$ is $PPR_{i,T}$ of equation 9 is The main hypotheses tested are: $\theta_2 > 0$, $\delta_3 < 0$ and $\phi_2 < 0$. In accordance with the theoretical model in equations 9 and 10, we expect both the variance of expected prices and the variance of the error term derived from an expected price model to have negative effects on fertilizer use.

Control variables

The choice of control variables used is based on Feder, Just and Zilberman, (1985) who reviewed factors affecting fertilizer use in the case of developing countries. They identified the following variables: land, human capital, household assets, financial liquidity, fertilizer supply constraints and price of agricultural outputs and fertilizers. Fertilizer prices were not available at the local level: we considered that a unique import price of fertilizer determines the market price to a large extent and we included province and time dummies to account for fertilizer price heterogeneity at local level (related to differences in transport costs and marketing facilities) and to adjust to changes in time. The tobit model on panel data could not be run with individual fixed factor. To account for other production factors, we included in our basic set of control variables the following variables: cattle size (number of heads), total cultivated area (hectares) and the number of household members, which is a proxy for labor force.

The livestock variable can have two effects on fertilizer use. On the one hand, cattle indicates that manure can be used as a substitute for chemical, and can thus reduce the need for fertilizer. On the other hand, cattle can be sold on the market when cash is needed and can relieve the liquidity constraint that could otherwise limit fertilizer use. The effect of the livestock variable is therefore undetermined.

The influence of farm size on the intensity of fertilizer use is also not easily predicted. According to empirical regularities coined in developing countries as the "the inverse farm size- productivity relationship" (Bardhan 1973, Carter 1984, Feder 1985, Barret et al 2010), we could expect farm size to influence negatively fertilizer use per hectare. On the other hand, the imperfection on the credit market means that small farms may have difficulties to find the necessary cash to buy fertilizers whereas larger farmers may have greater financial resources, access to information, as well as a facilitated access to input.

We also include a dummy to indicate whether the household grows cotton. Indeed, fertilizers are supplied to cotton growers in a contractual arrangement with the cotton marketing board, but it is frequent to observe that farmers divert them for other crops.This dummy is therefore a proxy for farmers' access to fertilizers.

Finally, we include in our models a proxy for production risk to account for random shocks which farmers cannot control for: we use the provincial standard deviation of millet yields. As there is hardly any chemical fertilizer applied on millet plots, millet yields depend mostly on rainfall levels and pest attacks, and thus variance of millet yields can be attributed to natural variations.

4. Empirical Results

4.1 Total prices variability and fertilizer use (Test of Isik's model)

We estimate five Tobit[7] random effects[8] models. Table 6 presents the results on the total use of NPK at farm level (kg), while Table 7 presents the results on the intensity of NPK fertilizers used in maize crops (kg/ha).

[7] The Wald test of the hypothesis that all regression coefficients are jointly equal to zero is rejected with a high level of significance.

[8] The random effect parameter is highly statistically significant. The quantity labeled rho measures the fraction of the total variance that is due to the random effect parameter. Thus 23% of the dependent variable is explained by the individual specific effect.

Table 6. Effect of total price variability and production risk on total fertilizer use (kg)

	(1)	(2)	(3)	(4)	(5)
Prices	1.078**	1.051**	1.149**	1.028**	0.988**
	[2.22]	[2.16]	[2.36]	[2.11]	[2.07]
Total price variability	-0.0562***	-0.0569***	-0.0532***	-0.0473**	-0.0466**
	[-2.87]	[-2.90]	[-2.72]	[-2.42]	[-2.44]
Production risk	0.00184	0.00257	0.00498	0.00535	0.00663
	[0.18]	[0.25]	[0.49]	[0.53]	[0.67]
Age of head of household (HH)		-1.278***	-1.281***	-1.114***	-0.937***
		[-5.60]	[-5.70]	[-5.16]	[-4.48]
Education of HH		5.890**	6.488***	6.563***	6.937***
		[2.40]	[2.69]	[2.81]	[3.07]
Number of household members			3.975***	3.525***	1.609***
			[7.07]	[6.42]	[2.88]
Cattle size (heads)			0.419**	0.465**	0.314*
			[2.12]	[2.45]	[1.67]
Total cultivated area (*) (ha)				64.27***	52.68***
				[11.18]	[9.39]
Cotton dummy (1 if cotton is cultivated					6.869***
					[11.08]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	-186.5**	-133.0*	-184.3**	-168.9**	-166.9**
	[-2.48]	[-1.75]	[-2.41]	[-2.21]	[-2.24]
sigma_u_cons	89.57***	87.89***	83.51***	74.54***	69.91***
	[30.03]	[29.73]	[28.52]	[25.83]	[25.14]
sigma_e_cons	82.02***	82.05***	82.42***	83.55***	82.20***
	[46.38]	[46.29]	[46.29]	[46.37]	[46.85]
N	4186	4176	4175	4175	4175
Ll	-12480.9	-12441.9	-12412.9	-12352.0	-12292.6
Rho	0.544	0.534	0.507	0.443	0.420
chi2	983.3	1026.2	1120.6	1339.1	1526.6

*Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$*
The total number of observations accounts for the 3 years observation period and the households in our sample with non missing variables
Reported p-values are based on robust standard errors.
(*) We do not include the maize area in the estimations to avoid endogeneity issues.

Table 7. Effect of total price variability and production risk on the intensity of fertilizer use (kg/ /ha)

	(1)	(2)	(3)	(4)	(5)
Prices	1.577*	1.597*	1.699*	1.686*	1.564*
	[1.69]	[1.70]	[1.81]	[1.79]	[1.66]
Total price variability	-0.0996***	-0.101***	-0.0960**	-0.0960**	-0.0899**
	[-2.67]	[-2.69]	[-2.56]	[-2.55]	[-2.40]
Production risk	-0.00726	-0.00641	-0.00442	-0.00399	-0.00254
	[-0.39]	[-0.34]	[-0.23]	[-0.21]	[-0.14]
Age of head of household (HH)		-1.186***	-1.109***	-1.068***	-0.919**
		[-3.24]	[-3.03]	[-2.91]	[-2.52]
Education of HH		8.918**	9.602**	9.738**	9.558**
		[2.17]	[2.34]	[2.37]	[2.34]
Number of household members			3.479***	2.962***	2.738***
			[3.58]	[2.85]	[2.65]
Cattle size (heads)			-0.762*	-0.822*	-0.698
			[-1.77]	[-1.87]	[-1.62]
Total cultivated area (*) (ha)				1.633	-0.211
				[1.42]	[-0.18]
Cotton dummy (1 if cotton is cultivated					69.78***
					[6.83]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	-182.3	-142.9	-193.2	-191.9	-174.6
	[-1.27]	[-0.99]	[-1.33]	[-1.32]	[-1.20]
sigma_u_cons	102.3***	100.5***	98.90***	98.73***	95.46***
	[18.15]	[17.71]	[17.34]	[17.29]	[16.64]
sigma_e_cons	169.1***	169.7***	170.0***	170.1***	170.1***
	[49.70]	[49.56]	[49.50]	[49.43]	[49.42]
N	4186	4176	4175	4175	4175
Ll	-13564.9	-13536.1	-13528.8	-13527.7	-13504.5
Rho	0.268	0.260	0.253	0.252	0.240
chi2	567.5	580.5	594.3	595.5	642.4

*Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$*
Reported p-values are based on robust standard errors.
(*) We do not include the maize area in the estimations to avoid endogeneity issues.

Observed maize price level has a positive effect on the quantity of fertilizers used (Table 6) and on the intensity of fertilizer use (Table 7), indicating that farmers who get better maize output prices tend to increase the overall quantity of fertilizer used (because of higher income from maize) and the quantity of fertilizer used per hectare. Production risk has no impact either on total fertilizer used nor on the level of fertilizer used per hectare. This suggests that rainfall variations and pest attacks are not a major impediment to fertilizer use in maize production. This result is consistent with the findings of Smith and Umali (1985) who suggest that the level of fertilizer use on rainfed rice in the Philippines cannot be attributed to production risk.

The variance of observed prices has a negative and robust effect, both on the quantity and on the intensity of fertilizer use with a 5% level of statistical significance. This effect holds for all sets of control variables. This is consistent with the theoretical framework developed by Isik (2002), in which variance is used as a risk indicator, and which suggests that output price risk deters fertilizer use.

Age has a negative effect for on the intensity of fertilizer use, while education has a positive effect on it. This is consistent with empirical findings that younger and more educated farmers are more likely to adopt modern agricultural technologies (Feder et al. 1985; Nkonya et al., 1997; Foster and Rosenzweig 2010).

The number of household members, which is a proxy for the working force, has a positive and statistically significant impact on fertilizer use, suggesting that the use of fertilizers and working force are complementary inputs. Similar results were found by Croppenstedt and Demeke (1996) and Minot, Kherallah and Berry (2000) in their studies in different developing countries. This may be explained by the labor requirements of fertilizer application and the increased weeding need associated with fertilizer use. The magnitude of the effect is quite large: each additional household member raises the quantity of fertilizer use by 2.7 Kg per hectare.

Cattle size significantly decreases the intensity of fertilizer used. This result tends to indicate that farmer with more cattle are less dependent on mineral fertilizers. It also indicates that farmer treat organic and mineral fertilizers as substitutable.

The market dummy variables indicate that there are important regional effects, which are not taken into account by the other variables. This result confirms evidence of spatial difference in the fertilizer use in Burkina Faso, resulting from our earlier descriptive analysis.

4.2 Predicted price variability and pure price risk (adapted model from Isik)

We conduct the same type of statistical analysis by replacing the variance of observed prices by the predicted prices variability and the pure price risk. Table 8 presents the results on the quantity of fertilizer used and Table 9 presents the results on the intensity of fertilizers used in maize crops (kg/ha).

Table 8. Effect of predicted prices variability and pure price risk on total fertilizer used (kg)

	(1)	(2)	(3)	(4)	(5)
Prices	1.118**	1.098**	1.206**	1.115**	1.080**
	[2.28]	[2.24]	[2.46]	[2.26]	[2.24]
Predicted prices variability	0.000416	0.000155	0.00513	0.00795	0.00539
	[0.03]	[0.01]	[0.38]	[0.58]	[0.41]
Pure price risk	-0.119**	-0.127***	-0.115**	-0.113**	-0.120**
	[-2.45]	[-2.60]	[-2.37]	[-2.32]	[-2.51]
Production risk	0.00181	0.00256	0.00472	0.00506	0.00645
	[0.17]	[0.25]	[0.46]	[0.50]	[0.65]
Age of HH		-1.272***	-1.275***	-1.107***	-0.929***
		[-5.58]	[-5.67]	[-5.12]	[-4.44]
Education of HH		5.951**	6.530***	6.613***	7.012***
		[2.43]	[2.70]	[2.83]	[3.10]
Household members			3.990***	3.538***	1.610***
			[7.09]	[6.44]	[2.88]
Cattle size (heads)			0.410**	0.457**	0.307
			[2.08]	[2.41]	[1.64]
Cotton dummy				64.57***	52.92***
				[11.24]	[9.44]
Total cultivated area					6.896***
					[11.13]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	-195.3**	-143.2*	-196.2**	-184.8**	-183.4**
	[-2.57]	[-1.86]	[-2.54]	[-2.40]	[-2.44]
sigma_u_cons	89.70***	88.01***	83.63***	74.65***	70.02***
	[30.04]	[29.74]	[28.53]	[25.86]	[25.18]
sigma_e_cons	81.98***	82.00***	82.37***	83.48***	82.11***
	[46.34]	[46.25]	[46.25]	[46.33]	[46.81]
N	4178	4168	4167	4167	4167
Ll	-12467.6	-12428.7	-12399.7	-12338.1	-12278.3
rho	0.545	0.535	0.508	0.444	0.421
chi2	979.5	1022.6	1116.8	1336.3	1524.5

*Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$*
Reported p-values are based on robust standard errors.

Table 9. Effect of predicted prices variability and pure price risk on the intensity of fertilizer use (kg/ /ha)

	(1)	(2)	(3)	(4)	(5)
Prices	1.656*	1.689*	1.805*	1.792*	1.702*
	[1.75]	[1.78]	[1.90]	[1.88]	[1.79]
Predicted prices variability	-0.004	-0.00434	-0.000199	-0.000860	0.00302
	[-0.15]	[-0.17]	[-0.01]	[-0.03]	[0.12]
Pure price risk	-0.214**	-0.226**	-0.216**	-0.217**	-0.214**
	[-2.26]	[-2.37]	[-2.25]	[-2.27]	[-2.23]
Production risk	-0.0072	-0.00634	-0.00452	-0.00405	-0.00266
	[-0.38]	[-0.34]	[-0.24]	[-0.21]	[-0.14]
Age of HH		-1.174***	-1.096***	-1.053***	-0.903**
		[-3.21]	[-2.99]	[-2.86]	[-2.48]
Education of HH		9.049**	9.731**	9.876**	9.693**
		[2.20]	[2.37]	[2.40]	[2.38]
Household members			3.507***	2.970***	2.749***
			[3.61]	[2.86]	[2.66]
Cattle size (heads)			-0.768*	-0.829*	-0.705
			[-1.79]	[-1.89]	[-1.63]
Total cultivated area				1.694	-0.163
				[1.48]	[-0.14]
Cotton dummy					70.16***
					[6.87]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	-198.91	-162.1	-214.7	-213.4	-200.4
	[-1.37]	[-1.11]	[-1.46]	[-1.45]	[-1.36]
sigma_u_cons	102.4***	100.7***	99.06***	98.90***	95.65***
	[18.19]	[17.74]	[17.38]	[17.33]	[16.69]
sigma_e_cons	168.9***	169.5***	169.8***	170.0***	169.9***
	[49.66]	[49.52]	[49.46]	[49.39]	[49.38]
N	4178	4168	4167	4167	4167
Ll	-13549.2	-13520.5	-13513.0	-13511.9	-13488.4
Rho	0.269	0.261	0.254	0.253	0.241
Chi2	566.35	579.4	593.2	594.5	641.8

Notes: t statistics in brackets * *p < 0.10,* ** *p < 0.05,* *** *p < 0.01*
Reported p-values are based on robust standard errors.

The estimates associated with predicted price variability are not significant, indicating that price fluctuations that are predictable do not affect fertilizer use decisions. However, pure price risk has a negative and significant impact both on total fertilizer use and on the intensity of fertilizer use. These results confirm theoretical results regarding the effect of pure price risk, although they do not confirm theoretical predictions regarding production risk (equation 4). Our empirical estimations indicate that farmers tend to reduce fertilizer use when facing unpredictable output price fluctuations.

Our results also suggest that the control variables introduced do not add something new to the analysis: the negative effect of pure price risk holds whatever the situation. This outcome echoes findings in the literature on how food price risk affects decisions made by farmers (Holt and Moschini, 1992, Combes et al., 2014, Rezitis et al. 2009).

4.3 Robustness tests

We test the sensitivity of our two empirical main results –namely the negative effect of total prices variability and the negative effect of pure price risk on fertilizer use- by considering different time spans for the calculation of price variability indicators. Appendix 3A (resp. Appendix 3B) presents estimations with measures of total price variability (resp. predicted price variability and pure price risks) calculated with a price memory of 12 to 48 months before the planting month of years 2009, 2010 and 2011.

We also test the sensitivity of our results to linear panel models with time-invariant regressors and error-components structure. (Mundlak, 1978; Chamberlain, 1982) instead of a Tobit random-effect model that relies on strong exogeneity assumptions (See Appendix 4). Main conclusions are unchanged.

5. Conclusion

This paper investigates whether maize price risks explain the low use of chemical fertilizers on maize in Burkina Faso. The originality of the analysis lies in the fact that price risks have been separated into a partly predictable part, calculated through an autoregressive price formation model, and a purely unpredictable part measured as the variance of the error terms of this estimated price formation model (pure price risk).

We used a panel sample of 1600 maize growers, whose agricultural decisions have been surveyed over the 2009-2011 period. We find that the predictable component of maize price fluctuations does not have any significant effect on the quantity and the intensity of fertilizers used, while an increase in the pure price risk does reduce fertilizer use significantly.

This empirical evidence that unpredictable prices fluctuations particularly affects maize farmers' demand for NPK fertilizers has policy implications. Donors and policymakers should be aware that episodes of massive and unanticipated price fluctuations could be a major issue by inducing farmers to lower their use of fertilizer. In the case of Burkina Faso, it could put at risk national food security. Indeed, chemical fertilizers are essential to increase maize yields and therefore to improve staple food supply. Consequently, reducing unpredictable prices fluctuations is one of the options that should be considered to reduce the maize yield gap in the country. It has been showed that remote markets are particularly vulnerable to those unpredictable price fluctuations (Le Cotty and al, 2017). Thus, one way to deal with the issue of low fertilizer use would be to connect rural markets to major consumption centers across the country as well as in neighboring countries, by funding road infrastructures and information technologies. This will be key to activate marketing dynamics and to contain unpredictable prices fluctuations contribute to slow down the intensification process in Burkina Faso.

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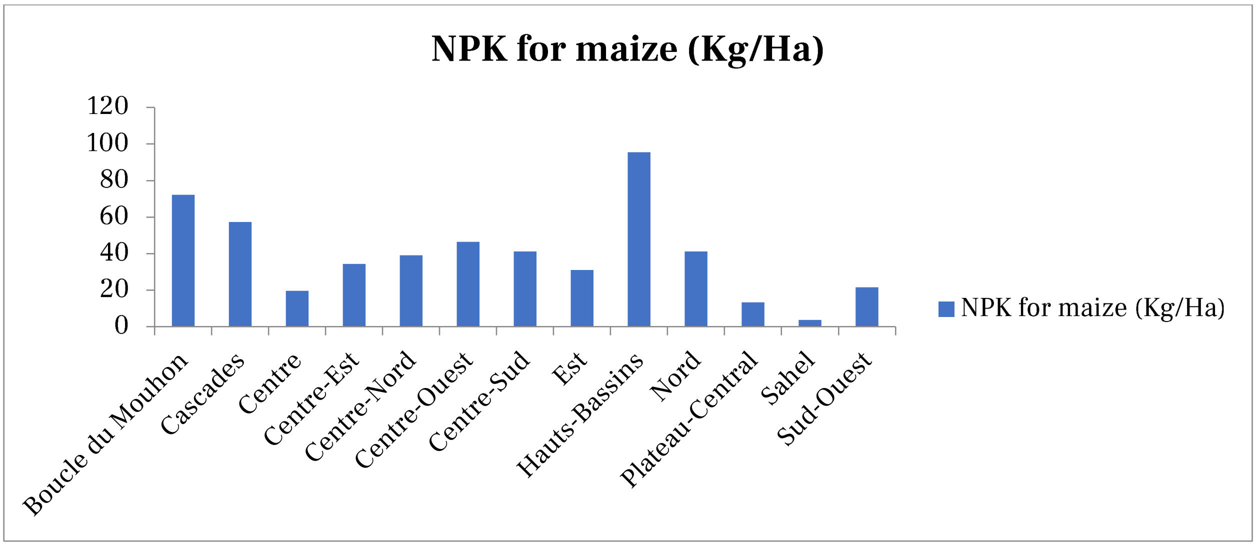
Appendix 1. NPK Fertilizers used (Kg/ha) for maize production by region (average 2009-2011)



Appendix 2. NPK Fertilizers used (Kg/ha) for maize production by year

	mean	Sd	Min	Max
2009				
NPK	29.8	59.0	0	491.2
2010				
NPK	35.6	64.7	0	497.5
2011				
NPK	39.9	69.8	0	497.5
Total				
NPK	35.1	64.7	0	497.5

Appendix 3. Robustness analysis: results' sensitivity of our results to different time spans

The sensitivity of our previous results is by considering different time periods for the measure of price variability. We test the sensitivity of our two empirical main results –namely the negative effect of total prices variability and the negative effect of pure price risk on fertilizer use- by considering different time spans for the calculation of price variability indicators. Appendix 3A (resp. Appendix 3B) presents estimations with measures of total price variability (resp. predicted price variability and pure price risks) calculated with a price memory of 12 to 48 months before the planting month of years 2009, 2010 and 2011. We find that the negative impact of price variability on fertilizer use is all the greater that the memory of price variations extends over a longer period. We could expect intuitively that pure price risks have a larger impact in the longer run, as more price shocks are memorized and built into the indicator of risks. This is observed with the estimated parameter for the pure price risk variable. It is greater when pure price risk is calculated with a 36 month-long memory than with only a 12-month long memory. However, it decreases for the 48 month-long memory.

Appendix 3A. Effect of total prices variability on fertilizer use intensity for different time spans

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Period	12 months	18 months	24 months	30 months	36 months	42 months	48 months
Prices	1.106	1.262	1.329	1.564*	1.472	1.436	1.518
	[1.17]	[1.36]	[1.43]	[1.66]	[1.55]	[1.54]	[1.62]
Expected price variability	0.0293	0.0106	-0.0958**	-0.0899**	-0.0616	-0.112**	-0.0536**
	[1.00]	[0.30]	[-2.28]	[-2.40]	[-1.35]	[-2.08]	[-2.25]
Production risk	-0.00405	-0.00375	-0.00272	-0.00254	-0.00280	-0.00224	-0.00166
	[-0.22]	[-0.20]	[-0.14]	[-0.14]	[-0.15]	[-0.12]	[-0.09]
Age	-0.906**	-0.902**	-0.900**	-0.919**	-0.923**	-0.926**	-0.918**
	[-2.48]	[-2.47]	[-2.47]	[-2.52]	[-2.53]	[-2.54]	[-2.52]
Education	9.324**	9.341**	9.607**	9.558**	9.326**	9.203**	9.612**
	[2.29]	[2.29]	[2.36]	[2.34]	[2.29]	[2.26]	[2.36]
Household members	2.805***	2.812***	2.836***	2.738***	2.755***	2.768***	2.616**
	[2.71]	[2.72]	[2.75]	[2.65]	[2.67]	[2.68]	[2.52]
Cattle size	-0.729*	-0.723*	-0.701	-0.698	-0.714*	-0.706	-0.718*
	[-1.67]	[-1.66]	[-1.62]	[-1.62]	[-1.64]	[-1.63]	[-1.66]
Total cultivated area	-0.205	-0.196	-0.141	-0.211	-0.254	-0.262	-0.202
	[-0.17]	[-0.17]	[-0.12]	[-0.18]	[-0.22]	[-0.22]	[-0.17]
Cotton dummy	70.68***	70.44***	69.33***	69.78***	70.61***	70.65***	70.42***
	[6.91]	[6.88]	[6.78]	[6.83]	[6.91]	[6.92]	[6.89]
Province dummies	YES	YES	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES	YES	YES
_cons	-113.7	-139.6	-141.7	-174.6	-153.2	-133.8	-162.8
	[-0.78]	[-0.97]	[-0.98]	[-1.20]	[-1.06]	[-0.93]	[-1.13]
sigma_u cons	95.43***	95.41***	95.60***	95.46***	95.13***	95.43***	95.41***
	[16.60]	[16.59]	[16.66]	[16.64]	[16.54]	[16.60]	[16.59]
sigma_e_cons	170.2***	170.1***	170.1***	169.9***	169.7***	170.2***	170.1***
	[49.40]	[49.39]	[49.39]	[49.38]	[49.41]	[49.40]	[49.39]
N	4167	4167	4167	4175	4175	4175	4175
Ll	-13490.4	-13490.8	-13488.3	-13504.5	-13506.4	-13505.2	-13504.8
Rho	0.239	0.239	0.240	0.240	0.238	0.239	0.240
chi2	638.0	637.7	639.6	642.4	640.7	640.9	643.5

Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Reported p-values are based on robust standard errors.

Appendix 3.B. Effect of the expected prices variability and pure price risk on fertilizer use intensity for different time spans

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Period	12 months	18 months	24 months	30 months	36 months	42 months	48 months
Prices	1.129	1.761*	1.525	1.702*	2.020**	1.585*	1.243
	[1.19]	[1.79]	[1.58]	[1.79]	[2.11]	[1.67]	[1.32]
Expected price variability	0.0396	0.0807	-0.0195	0.00302	0.0146	0.00896	0.00190
	[1.00]	[1.54]	[-0.56]	[0.12]	[0.77]	[0.65]	[0.18]
Pure price risk	0.00419	-0.0514	-0.135*	-0.214**	-0.507***	-0.278*	-0.0181
	[0.08]	[-0.72]	[-1.68]	[-2.23]	[-3.23]	[-1.88]	[-0.26]
Production risk	-0.00361	-0.00209	-0.00121	-0.00266	-0.000930	-0.00228	-0.00335
	[-0.19]	[-0.11]	[-0.06]	[-0.14]	[-0.05]	[-0.12]	[-0.18]
Age	-0.910**	-0.907**	-0.902**	-0.903**	-0.926**	-0.921**	-0.929**
	[-2.49]	[-2.49]	[-2.47]	[-2.48]	[-2.54]	[-2.53]	[-2.55]
Education	9.307**	9.304**	9.629**	9.693**	9.751**	9.386**	9.339**
	[2.28]	[2.28]	[2.36]	[2.38]	[2.40]	[2.30]	[2.29]
Household members	2.801***	2.819***	2.763***	2.749***	2.708***	2.706***	2.786***
	[2.71]	[2.73]	[2.68]	[2.66]	[2.62]	[2.62]	[2.68]
Cattle size	-0.723*	-0.719*	-0.710	-0.705	-0.709*	-0.710	-0.719*
	[-1.66]	[-1.66]	[-1.64]	[-1.63]	[-1.65]	[-1.64]	[-1.65]
Total cultivated area	-0.199	-0.213	-0.130	-0.163	-0.269	-0.259	-0.223
	[-0.17]	[-0.18]	[-0.11]	[-0.14]	[-0.23]	[-0.22]	[-0.19]
Cotton dummy	70.86***	70.94***	69.94***	70.16***	71.09***	70.90***	70.43***
	[6.92]	[6.93]	[6.84]	[6.87]	[6.96]	[6.94]	[6.89]
Province dummies	YES	YES	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES	YES	YES
_cons	-117.0	-215.1	-174.9	-200.4	-200.3	-158.1	-134.3
	[-0.80]	[-1.41]	[-1.17]	[-1.36]	[-1.37]	[-1.08]	[-0.92]
sigma_u cons	95.37***	95.49***	95.54***	95.65***	95.68***	95.49***	95.31***
	[16.59]	[16.62]	[16.65]	[16.69]	[16.73]	[16.65]	[16.58]
sigma_e_cons	170.2***	170.1***	170.1***	169.9***	169.7***	170.2***	170.1***
	[49.40]	[49.39]	[49.39]	[49.38]	[49.41]	[49.40]	[49.39]
N	4167	4167	4167	4167	4175	4175	4175
Ll	-13490.3	-13489.6	-13489.4	-13488.4	-13501.3	-13505.1	-13507.3
Rho	0.239	0.240	0.240	0.241	0.241	0.240	0.239
chi2	638.7	639.8	639.6	641.8	648.6	642.8	640.3

Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Reported p-values are based on robust standard errors.

Appendix 4. Robustness tests: Results' sensitivity to linear panel models with time-invariant regressors and error-components structure

The application of a Tobit random-effect estimators relies on exogeneity assumptions that are often too strong. Several solutions include the correlated random-effects model (Mundlak, 1978; Chamberlain, 1982). Thus, we use a correlated random-effects model which can provide partial robustness in this case of time-invariant variables from our baseline model.

Appendix 4A.A time-invariant regressors and error-components structure panel model

	(1)	(2)	(3)	(4)	(5)
	1	2	3	4	5
Prices	0.730**	0.742**	0.763**	0.776**	0.734**
	[2.18]	[2.21]	[2.27]	[2.30]	[2.19]
Total price variability	-0.0231*	-0.0237*	-0.0221*	-0.0220*	-0.0198
	[-1.93]	[-1.95]	[-1.83]	[-1.82]	[-1.64]
Production risk	0.00146	0.00166	0.00244	0.00211	0.00271
	[0.15]	[0.17]	[0.24]	[0.21]	[0.27]
Age of head of household		-0.153	-0.109	-0.133	-0.0765
		[-0.94]	[-0.65]	[-0.79]	[-0.45]
Education of HH		3.110	3.371	3.270	3.072
		[1.51]	[1.62]	[1.56]	[1.48]
Household members			1.015***	1.372***	1.348***
			[2.65]	[3.16]	[3.10]
Cattle size			-0.332***	-0.317***	-0.272***
			[-3.11]	[-3.06]	[-2.85]
Total cultivated area				-1.092**	-2.015***
				[-2.17]	[-3.52]
Cotton					32.33***
					[6.15]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	0	0	-139.9**	0	39.58***
	[.]	[.]	[-2.17]	[.]	[17.55]
N	4186	4176	4175	4175	4175
N_clust	1881	1878	1878	1878	1878
Instruments	40	43	45	46	47

*Notes: t statistics in brackets * p < 0.10, ** p < 0.05, *** p < 0.01*

Reported p-values are based on robust standard errors.

Appendix 4.B time-invariant regressors and error-components structure panel model (Chamberlain model

	(1) 1	(2) 2	(3) 3	(4) 4	(5) 5
Prices	0.881**	0.898**	0.919***	0.930***	0.895**
	[2.52]	[2.55]	[2.61]	[2.63]	[2.55]
Predicted price variability	0.00374	0.00341	0.00447	0.00499	0.00659
	[0.44]	[0.40]	[0.52]	[0.58]	[0.77]
Pure price risk	-0.123***	-0.127***	-0.122***	-0.121***	-0.118***
	[-3.06]	[-3.12]	[-3.05]	[-3.03]	[-2.95]
Production risk	0.00174	0.00194	0.00269	0.00236	0.00294
	[0.18]	[0.19]	[0.27]	[0.23]	[0.29]
Age		-0.151	-0.108	-0.131	-0.0740
		[-0.93]	[-0.64]	[-0.78]	[-0.44]
Education		3.208	3.464*	3.362	3.160
		[1.55]	[1.65]	[1.60]	[1.52]
Household members			1.010***	1.363***	1.340***
			[2.66]	[3.16]	[3.10]
Cattle size			-0.331***	-0.316***	-0.271***
			[-3.14]	[-3.08]	[-2.87]
Total cultivated area				-1.077**	-2.004***
				[-2.15]	[-3.51]
Cotton dummy					32.37***
					[6.16]
Province dummies	YES	YES	YES	YES	YES
Time dummies	YES	YES	YES	YES	YES
_cons	42.94***	0	0	0	0
	[13.54]	[.]	[.]	[.]	[.]
N	4178	4168	4167	4167	4167
N_clust	1876	1873	1873	1873	1873
Instrument	42	44	46	47	48

*Notes: t statistics in brackets * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$*
Reported p-values are based on robust standard errors.

Appendix 5. Model resolution

First order condition $\quad E\left[U_{\pi_t}(\pi_t)[\,(\widehat{P_t}\ +\ \theta_t)(f' + h'\varepsilon_t) - w]|P_{t-1}\right.$=0

Arrow Pratt risk aversion $\quad \Phi = -\,\frac{U_{\pi\pi}(\pi_{P_t})}{U_{\pi}(\pi_{P_t})}$

Exact profit $\quad \pi_t = (\widehat{P_t}\ +\ \theta_t)[f(x_{t-1}) + h(x_{t-1})\varepsilon_t] - w$

$$\widehat{\pi_t} = \widehat{P_t}\ f(x_{t-1}) - w$$

Taylor development of U_{π_t} around $\widehat{\pi_t}$

$$U_{\pi_t}(\pi_t) = U_{\pi_t}(\hat{\pi}_t) + [\widehat{P}_t h(x_{t-1})\varepsilon_t +\ \theta_t[f(x_{t-1}) + h(x_{t-1})\varepsilon_t].\,U_{\pi_t\pi_t}(\hat{\pi}_t)$$

We introduce this development in the First order condition and get

$$E\left[\left[1 - \Phi\left[\widehat{P}_t h(x_{t-1})\varepsilon_t +\ \theta_t[f(x_{t-1}) + h(x_{t-1})\varepsilon_t]\right]\right].\left[(\widehat{P_t}\ +\ \theta_t)(f' + h'\varepsilon_t) - w\right]\right] = 0$$

$$E\left[(\widehat{P_t}\ +\ \theta_t)(f' + h'\varepsilon_t) - w - \Phi\left[\widehat{P}_t h(x_{t-1})\varepsilon_t +\ \theta_t[f(x_{t-1}) + h(x_{t-1})\varepsilon_t]\right].\left[(\widehat{P_t}\ +\ \theta_t)(f' + h'\varepsilon_t) - w\right]\right] = 0$$

$$\widehat{P_t}\ f' - w\ - \Phi E\left[\left[\widehat{P}_t h(x_{t-1})\varepsilon_t +\ \theta_t[f(x_{t-1}) + h(x_{t-1})\varepsilon_t]\right].\left[(\widehat{P_t}\ +\ \theta_t)(f' + h'^{\varepsilon_t}) - w\right]\right] = 0$$

Dropping subscripts and arguments for readibility

$$\widehat{P_t}\ f' - w\ - \Phi E\left[\widehat{P}h\varepsilon +\ \theta f + \theta h\varepsilon\right].\left[(\widehat{P}f' + \widehat{P}h'\,\varepsilon +\ \theta f' + \theta h'\varepsilon - w\right] = 0$$

$$\widehat{P_t}\ f' - w \quad - \Phi E\left[\widehat{P^2}hf'\varepsilon + \widehat{P^2}hh'\varepsilon^2 + \widehat{P}h\varepsilon\theta f' +\ \widehat{P}h\theta h'\varepsilon^2 - w\theta h'\varepsilon +\ \theta f\widehat{P}f' + \theta f\widehat{P}h'\varepsilon +\ \theta^2 ff' + \theta^2 fh'\varepsilon - w\theta f + \theta h\varepsilon\,\widehat{P}f' +\ \theta h\widehat{P}h'\,\varepsilon^2 +\ \theta^2 h\varepsilon f' + h\theta^2 h'\varepsilon^2 - w\theta h\varepsilon\right] = 0$$

$$\widehat{P_t}\ f' - w\ - \Phi E\left[\widehat{P^2}hh'\varepsilon^2 +\ \theta^2 ff' + \theta^2 fh'\varepsilon + h\theta^2 h'\varepsilon^2\right]=0$$

$$\widehat{P_t}\ f' - w\ - \Phi\left[\widehat{P^2}hh'{\sigma_\varepsilon}^2 +\ ff'{\sigma_\theta}^2 + hh'{\sigma_\theta}^2{\sigma_\varepsilon}^2\right]=0$$

$$(\widehat{P}_t - \Phi f\sigma_\theta^2)f' - w\ - \Phi\left[\widehat{P^2}hh'{\sigma_\varepsilon}^2 + hh'{\sigma_\theta}^2{\sigma_\varepsilon}^2\right]=0$$

$$\widehat{P}_t\left(1 - \frac{\Phi f\sigma_\theta^2}{\widehat{P_t}}\right)f' = w\ + \Phi hh'{\sigma_\varepsilon}^2\left[\widehat{P^2} + {\sigma_\theta}^2\right]$$

$$\widehat{P_t}\,f' = \frac{\Phi h h_{x_{t-1}}{\sigma_\varepsilon}^2(\widehat{P_t}^{\,2} + {\sigma_\theta}^2) + w}{1 - \Phi\,\frac{f{\sigma_\theta}^2}{\widehat{P_t}}}$$