An exponential analysis of total factor productivity: the case of evaluating Chinese public hospitals

A. Abad[⊥], P. Ravelojaona[+], and Z. Shen[*]

[⊥]University of Lorraine, University of Strasbourg, AgroParisTech, CNRS, INRAE, BETA, 54000 Nancy, France.
[+]University of Perpignan, 52 avenue Paul Alduy 66000 Perpignan, France.
[*]Beijing Institute of Technology, 100081 Beijing, China

Abstract

As an important economic measure, the total factor productivity indices can be employed to evaluate the performance of decision-making units over time. These indices are usually based on multiplicative or additive distance functions estimated by both parametric and non parametric approaches. Under a non parametric analytic framework, this paper introduces a multiplicative directional productivity measure: the Directional Hicks-Moorstens (DHM) indicator. A dynamical combination of multiplicative directional distance functions is introduced and non convex production technologies are assumed to estimate distance functions. In empirical analysis, the proposed model is applied to investigate productivity changes among Chinese provincial public hospitals over the period 2014-2018. The results indicate that a consistent outcome is obtained under the multiplicative technology and the production technology of free disposal hull while more productivity gains are observed under free disposal hull technology.

Keywords: Chinese medical institutions, Dynamical Deviation, Efficiency Index, Non Convexity, Total Factor Productivity.

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1 Introduction

Scarcity of the economic factors and technological limitations narrow the production possibility set of the firms. In the context of multiple inputs-outputs production processes, additive and multiplicative distance functions have become standard tools to analyse technical efficiency of the firms (Chambers et al., 1996, 1998; Briec, 1997; Luenberger, 1992ab; Färe et al., 1985; Shephard, 1970; Farrell, 1957; Debreu, 1951). These efficiency indices completely characterize the production technology[1]. Unlike usual approaches estimating production technology by Shephard (Shephard, 1970) or directional distance function (Chambers et al., 1996), we focus on the Multiplicative Directional Distance Function (MDDF) as functional characterization of the production process (Peyrache and Coelli, 2009; Mehdiloozad et al., 2014). The MDDF efficiency index encompasses usual multiplicative distance functions (Färe et al., 1985; Shephard, 1970; Farrell, 1957; Debreu, 1951) as special cases (Peyrache and Coelli, 2009). In addition, the logarithmic transformation of the MDDF inherits the basic structure of the directional distance function (Chambers et al., 1996, 1998; Briec, 1997; Luenberger, 1992ab). Moreover, the MDDF distance function permits to estimate efficiency of firms under non convex technologies[2] allowing to take into account of strictly increasing marginal products.

Knowing efficiency scores variation over time allows to define the Total Factor Productivity (TFP) change based upon the combination of distance functions over periods, namely TFP indices. TFP indicators have been widely studied in the literature (Hulten, 2001). According to the functional characterization of the production process, two main approaches hold. In the context of continuous-time, derivatives of the production functions display Solow's (1957) productivity change. Otherwise, difference and ratio of distance functions define additive and multiplicative TFP measures over consecutive time periods (Briec and Kerstens, 2004; Chambers, 2002; Bjurek, 1996; Färe et al., 1984)[3]. In this paper, we estimate TFP variation through a multiplicative directional productivity measure. Specifically, the proposed Directional Hicks-Moorsteen (DHM) productivity index inherits the structure of the multiplicative Hicks-Moorsteen productivity measure (Bjurek, 1996; Diewert, 1992ab). Moreover, the DHM productivity index is defined in dynamical context, displaying adjustment paths of TFP variation. Hence, this paper proposes to evaluate the productivity change over periods from a non linear efficiency measure allowing to take into account for strictly increasing marginal products. In addition, the DHM avoids infeasibilities as it is in the line of the Hicks-Moorsteen productivity index.

By constructing a non parametric framework and using both DEA and FDH methods, this paper provides an empirical analysis on the efficiency and productivity of medical institutions and estimates the dynamic DHM productivity of 31 Chinese provincial

[1]In the case of multiple inputs-outputs, additive and multiplicative distance functions replace production functions as functional forms of the production technology.

[2]The empirical part of the paper introduces non parametric specifications of the MDDF efficiency index.

[3]Comparisons of difference and ratio-based TFP indicators are defined in Chambers (1998, 2002) and Diewert (2005).

hospitals, through linear and non linear production processes. The results would provide meaningful information for understanding the use of scarce resources, promoting service quality and efficiency in hospitals, and improving the medical service and health system in China.

The remainder of the paper consists of the following contents. Section 2 introduces some theoretical preliminaries. The production technology along with the efficiency and productivity indices are displayed in this section. The dynamical multiplicative directional productivity measure is introduced in the Section 3. Section 4 defines non parametric estimation procedures for the dynamical efficiency and productivity indices under linear and non linear production processes. The empirical investigation allows to estimate the dynamical efficiency and productivity measures for a sample of 31 provincial Chinese hospitals over the periods 2014-2018. The main empirical findings are presented in the Section 5. Finaly, Section 6 discusses and concludes.

2 Background

This section presents the axioms of the production set and the efficiency measure considered throughout the paper.

2.1 Production technology: definition and properties

Let $x_t \in \mathbb{R}^n_+$ be the input vector allowing to produce the $y_t \in \mathbb{R}^m_+$ output vector such that the input vector is composed of $i \in [n]$ elements and the output vector contains $j \in [m]$ elements with $[n] = \text{Card}(x_t)$ and $[m] = \text{Card}(y_t)$. The definition and properties of the production set are introduced below.

The production technology is defined as,

$$T_t = \left\{(x_t, y_t) \in \mathbb{R}^{n+m}_+ : x_t \text{ can produce } y_t\right\}. \tag{2.1}$$

Notice that the production set (2.1) can be characterized by the output $P_t : \mathbb{R}^n_+ \mapsto 2^{\mathbb{R}^m_+}$ or the input, $L_t : \mathbb{R}^m_+ \mapsto 2^{\mathbb{R}^n_+}$, correspondences such that:

$$P_t(x_t) = \left\{y_t \in \mathbb{R}^m_+ : (x_t, y_t) \in T_t\right\}, \tag{2.2}$$

and

$$L_t(y_t) = \left\{x_t \in \mathbb{R}^n_+ : (x_t, y_t) \in T_t\right\}. \tag{2.3}$$

In such case, the following statement holds:

$$x_t \in L_t(y_t) \Leftrightarrow (x_t, y_t) \in T_t \Leftrightarrow y_t \in P_t(x_t). \tag{2.4}$$

Assume that the production technology satisfies the following regular properties (Färe et al., 1985):

$T1$: $(0, 0) \in T_t$, $(0, y_t) \in T_t \Rightarrow y_t = 0$.
$T2$: $T_t(y_t) = \{(u_t, v_t) \in T_t : v_t \leq y_t\}$ is bounded for all $y_t \in \mathbb{R}^m_+$.

$T3$: T_t is closed.
$T4$: $\forall (x_t, y_t) \in T_t \wedge \forall (u_t, v_t) \in \mathbb{R}_+^n \times \mathbb{R}_+^m$ if $(x_t, -y_t) \leq (u_t, -v_t)$ then $(u_t, v_t) \in T_t$.

The above axioms define a free disposal production set such that ($T1$) means that there is no free lunch, ($T2$) and ($T3$) postulate that the production technology is compact and ($T4$) notifies that the production set is free disposable in both inputs and outputs dimensions. Remark that the convexity property is not imposed.

2.2 Multiplicative efficiency measure

Following Peyrache and Coelli (2009) and Mehdiloozad et al. (2014), consider the efficiency measure defined below.

Definition 2.1 *For any* $(x_t, y_t) \in \mathbb{R}_+^{n+m}$ *and any directional vector* $g_t = (h_t, k_t) \in \mathbb{R}_+^{n+m}$*, the function*

$$M(x_t, y_t; h_t, k_t) = \sup_{\delta} \left\{ \delta : \left(\delta^{-h_t} x_t, \delta^{k_t} y_t \right) \in T_t \right\} \tag{2.5}$$

is the multiplicative directional distance function.

It is obvious that this efficiency measure has a multiplicative formulation and is non linear.

As postulated in Peyrache and Coelli (2009), assume that this multiplicative directional distance function (MDDF) satisfies the properties below:

$D1$: $M(x_t, y_t; h_t, k_t)$ fully characterises the production set.

$D2$: $M(x_t, y_t; h_t, k_t)$ is equal to 1 when the observation belongs to the efficient frontier.

$D3$: $M(x_t, y_t; h_t, k_t)$ is almost homogeneous of degree (-1) in (x_t, y_t).

$D4$: $M(x_t, y_t; h_t, k_t)$ is homogeneous of degree (-1) in g_t.

$D5$: $M(x_t, y_t; h_t, k_t)$ is invariant with respect to the unit of measurement.

Also consider that there exists a strictly positive production set T_t^{++} defined as:

$$T_t^{++} = \left\{ (x_t, y_t) \in \mathbb{R}_{++}^{n+m} : x_t \text{ can produce } y_t \right\}. \tag{2.6}$$

Under this production technology, a logarithmic formulation of the MDDF is provided as below:

$$\ln\left(M(x_t, y_t; h_t, k_t) \right) = \sup_{\delta} \left\{ \delta : \left(\ln(x_t) - h_t \ln(\delta), \ln(y_t) + k_t \ln(\delta) \right) \in \ln\left(T_t^{++} \right) \right\}. \tag{2.7}$$

Obviously this function is an additive graph efficiency measure. Moreover, the logarithmic transformation of the MDDF is structurally similar to the directional distance function (Luenberger, 1992ab; Chambers et al, 1996). Indeed, through the logarithmic transformation, the MDDF becomes a log-additive and log-linear distance function.

Peyrache and Coelli (2009) introduce equivalence conditions for the MDDF and, the traditional input and output distance functions (Shephard, 1970; Debreu, 1951; Farrell, 1957). Indeed, the input oriented Debreu-Farrell efficiency measure can be retrieved when $g_t = (1, 0)$ and, the output oriented one when $g_t = (0, 1)$. In addition, the hyperbolic distance function (Färe et al., 1985) is obtained when the directional vector is $g_t = (1, 1)$. In such case, the MDDF represents a generalized shape for radial measures.

2.3 Productivity index

Based upon the works of Diewert (1992ab), Bjurek (1996) proposes a multiplicative productivity index avoiding infeasibility issues. This productivity measure is defined below.

Definition 2.2 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ and any directional vector (g_t, g_{t+1}), the function*

$$HM_{t,t+1}(x_t, y_t, x_{t+1}, y_{t+1}) = \Big[HM_t(x_t, y_t, x_{t+1}, y_{t+1}) \times HM_{t+1}(x_t, y_t, x_{t+1}, y_{t+1})\Big]^{\frac{1}{2}} \quad (2.8)$$

is called the Hicks-Moorsteen productivity index.

This productivity index estimates the productivity change between two consecutive time periods. To avoid an arbitrary choice of a base period, the global productivity index is the geometric mean of the productivity measures of the time periods (t) and $(t + 1)$. Notify that these Hicks-Moorsteen indices are the ratio between output and input quantity indices such that:

$$HM_t(x_t, y_t, x_{t+1}, y_{t+1}) = \frac{MO_t(x_t, y_t, y_{t+1})}{MI_t(x_t, x_{t+1}, y_t)} \quad (2.9)$$

and $$HM_{t+1}(x_t, y_t, x_{t+1}, y_{t+1}) = \frac{MO_{t+1}(x_{t+1}, y_t, y_{t+1})}{MI_{t+1}(x_t, x_{t+1}, y_{t+1})}, \quad (2.10)$$

where MO and MI denote the output and input Malmquist quantity indices respectively. Moreover, these quantity indices are based upon radial measures as the Shephard distance function (Shephard, 1970) or the Debreu-Farrell efficiency measure (Debreu, 1951; Farrell, 1957).

Remark that when HM is greater than 1 (lesser than 1) then the considered observation has a productivity growth (loss). In addition, when MO is greater than 1 (lesser than 1) then more (less) outputs are produced for the same input quantity between the consecutive periods. Finally, when MI is greater than 1 (lesser than 1) then more (less) inputs are needed to produce the same output quantity over periods. Hence, $MO \geq 1$ and $MI \leq 1$ contribute to a productivity gain.

3 Methodology

This section aims to present some theoretical results based upon the basic concepts introduced in the previous section. In such case, a modified productivity index is proposed.

3.1 Directional productivity measure

Based upon the multiplicative directional distance function, a directional Hicks-Moorsteen productivity index is presented below.

Definition 3.1 *For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ *and any directional vectors* $(g_t, g_{t+1}) \in \mathbb{R}_+^{n+m}$*, the function :*

$$DHM_{t,t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \Big[DHM_t(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) \times DHM_{t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) \Big]^{\frac{1}{2}} \quad (3.1)$$

is called the Directional Hicks-Moorsteen productivity index.

This directional Hicks-Moorsteen index is a global measure of the productivity between two time periods since it is defined as the geometric mean of the productivity measures for the periods (t) and $(t+1)$. The proposition below presents the definition of these productivity indices.

Proposition 3.2 *For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ *and any directional vectors* $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}_+^{n+m}$ *with* $g^o_{t,t+1} = (0, k_{t,t+1}) \in \mathbb{R}_+^m$ *and* $g^i_{t,t+1} = (h_{t,t+1}, 0) \in \mathbb{R}_+^n$*,*

$$DHM_t(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \frac{DMO_t(x_t, y_t, y_{t+1}; g^o_t, g^o_{t+1})}{DMI_t(x_t, x_{t+1}, y_t; g^i_t, g^i_{t+1})} \quad (3.2)$$

and

$$DHM_{t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \frac{DMO_{t+1}(x_{t+1}, y_t, y_{t+1}; g^o_t, g^o_{t+1})}{DMI_{t+1}(x_t, x_{t+1}, y_{t+1}; g^i_t, g^i_{t+1})} \quad (3.3)$$

are respectively the directional Hicks-Moorsteen productivity measures for periods (t) *and* $(t+1)$ *such that* $DMO_{t,t+1}$ *and* $DMI_{t,t+1}$ *are respectively the output and input oriented directional Malmquist quantity indices.*

The oriented directional Malmquist quantity indices of the period (t) are defined as:

$$DMO_t(x_t, y_t, y_{t+1}; g^o_t, g^o_{t+1}) = \frac{M_t(x_t, y_t; 0, k_t)}{M_t(x_t, y_{t+1}; 0, k_{t+1})}, \quad (3.4)$$

and

$$DMI_t(x_t, x_{t+1}, y_t; g^i_t, g^i_{t+1}) = \frac{M_t(x_{t+1}, y_t; h_{t+1}, 0)}{M_t(x_t, y_t; h_t, 0)}. \quad (3.5)$$

Moreover, the output and input directional Malmquist quantity indices of the period $(t+1)$ are as follows:

$$DMO_{t+1}(x_{t+1}, y_t, y_{t+1}; g_t^o, g_{t+1}^o) = \frac{M_{t+1}(x_{t+1}, y_t; 0, k_t)}{M_{t+1}(x_{t+1}, y_{t+1}; 0, k_{t+1})}, \tag{3.6}$$

and $$DMI_{t+1}(x_t, x_{t+1}, y_t; g_t^i, g_{t+1}^i) = \frac{M_{t+1}(x_{t+1}, y_{t+1}; h_{t+1}, 0)}{M_{t+1}(x_t, y_{t+1}; h_t, 0)}. \tag{3.7}$$

Remark that these quantity indices involve some shadow points as (x_t, y_{t+1}) and (x_{t+1}, y_t). And, the efficiency measure of these points with respect to the production technology of the period (t) are as follows:

$$M_t(x_t, y_{t+1}; h_t, k_{t+1}) = \sup_\delta \Big\{\delta : \big(\delta^{-h_t} x_t, \delta^{k_{t+1}} y_{t+1}\big) \in T_t\Big\}, \tag{3.8}$$

and $$M_t(x_{t+1}, y_t; h_{t+1}, k_t) = \sup_\delta \Big\{\delta : \big(\delta^{-h_{t+1}} x_{t+1}, \delta^{k_t} y_t\big) \in T_t\Big\}. \tag{3.9}$$

Note that when DHM is greater than 1 (lesser than 1) then the considered observation has a productivity gain (loss). In addition, when DMO is greater than 1 (lesser than 1) then more (less) outputs are produced for the same input quantity between the assessed periods. Finally, when DMI is greater than 1 (lesser than 1) then more (less) inputs are needed to produce the same output quantity over periods. Consequently, $DMO \geq 1$ and $DMI \leq 1$ contribute to a productivity growth.

3.2 Dynamical context

From the definition of cross-period efficiency measures and following the work of Abad and Ravelojaona (2017), this subsection presents the dynamical formulation of the cross-period multiplicative directional distance function.

Definition 3.3 *For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ *and any* $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}_+^{n+m}$, *the cross-period efficiency measures are defined respectively as follows:*

$$M_{t+1(t)}^{i,o} = \left(\frac{x_t}{x_{t+1}}\right)^{1/\rho^i_{t+1(t)} h_t} = \left(\frac{y_{t+1}}{y_t}\right)^{1/\rho^o_{t+1(t)} k_t} \tag{3.10}$$

$$M_{t(t+1)}^{i,o} = \left(\frac{x_t}{x_{t+1}}\right)^{1/\rho^i_{t(t+1)} h_{t+1}} = \left(\frac{y_{t+1}}{y_t}\right)^{1/\rho^o_{t(t+1)} k_{t+1}} \tag{3.11}$$

such that $M^{i,o}$ *displays the input and output sub-vectors MDDF where,* ρ^i *and* ρ^o *are respectively the dynamical adjustment parameters in input and output dimensions.*

Remark that the subscript $t+1(t)$ means that the projection is made onto the efficient frontier of the period $(t+1)$ whereas $t(t+1)$ refers to the efficient frontier of the period (t).

The dynamical adjustment parameters ρ represent the impacts of internal and external factors on the efficiency between two consecutive time periods. These parameters are defined in the following proposition.

Proposition 3.4 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$ and any $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}^{n+m}_+$, the dynamical adjustment parameters ρ are defined below:*

$$\begin{cases} \rho^i_{t+1(t)} = \dfrac{\ln(x_t) - \ln(x_{t+1})}{h_t \ln(M^i_{t+1(t)})} \\ \\ \rho^o_{t+1(t)} = \dfrac{\ln(y_{t+1}) - \ln(y_t)}{k_t \ln(M^o_{t+1(t)})} \end{cases} \quad (3.12) \qquad \begin{cases} \rho^i_{t(t+1)} = \dfrac{\ln(x_t) - \ln(x_{t+1})}{h_{t+1} \ln(M^i_{t(t+1)})} \\ \\ \rho^o_{t(t+1)} = \dfrac{\ln(y_{t+1}) - \ln(y_t)}{k_{t+1} \ln(M^o_{t(t+1)})}. \end{cases} \quad (3.13)$$

The proofs of these results are similar to those presented by Abad and Ravelojaona (2017) and thus, are omitted.

Corollary 3.5 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$, any dynamical adjustment parameters $(\rho^i_{t(t+1)}, \rho^i_{t+1(t)}, \rho^o_{t(t+1)}, \rho^o_{t+1(t)}) \in \mathbb{R}^4$ and any cross-period distance functions $(M^i_{t+1(t)}, M^i_{t(t+1)}, M^o_{t+1(t)}, M^o_{t(t+1)}) \in \mathbb{R}^4$, the following statements hold:*

i) *$x_{t+1} > x_t$ if one of the following cases occurs:*
- **1i.** *$\rho^i_{t(t+1)} < 0$ and $M^i_{t(t+1)} > 1$;*
- **2i.** *$\rho^i_{t(t+1)} > 0$ and $M^i_{t(t+1)} < 1$;*
- **3i.** *$\rho^i_{t+1(t)} < 0$ and $M^i_{t+1(t)} > 1$;*
- **4i.** *$\rho^i_{t+1(t)} > 0$ and $M^i_{t+1(t)} < 1$.*

ii) *$y_{t+1} > y_t$ if one of the assertions below takes place:*
- **1ii.** *$\rho^o_{t(t+1)} < 0$ and $M^o_{t(t+1)} < 1$;*
- **2ii.** *$\rho^o_{t(t+1)} > 0$ and $M^o_{t(t+1)} > 1$;*
- **3ii.** *$\rho^o_{t+1(t)} < 0$ and $M^o_{t+1(t)} < 1$;*
- **4ii.** *$\rho^o_{t+1(t)} > 0$ and $M^o_{t+1(t)} > 1$.*

The dynamical directional quantity indices are defined in the next statement.

Proposition 3.6 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$ and any directional vectors $g_{t,t+1} = (g^i_{t,t+1}, g^o_{t,t+1}) \in \mathbb{R}^{n+m}_+$ where $g^o_{t,t+1} = (0, k_{t,t+1}) \in \mathbb{R}^m_+$ and $g^i_{t,t+1} = (h_{t,t+1}, 0) \in \mathbb{R}^n_+$, the dynamical output and input directional quantity indices for the periods (t) and $(t+1)$ are defined as:*

$$\begin{aligned} DMO_t &= M_t(x_t, y_t; 0, k_t) \times \left(\frac{y_t}{y_{t+1}}\right)^{1/\rho^o_{t(t+1)} k_{t+1}} \\ DMI_t &= M_t(x_t, y_t; h_t, 0)^{-1} \times \left(\frac{x_t}{x_{t+1}}\right)^{1/\rho^i_{t(t+1)} h_{t+1}} \end{aligned} \quad (3.14)$$

and

$$
\begin{aligned}
DMO_{t+1} &= M_{t+1}(x_{t+1}, y_{t+1}; 0, k_{t+1})^{-1} \times \left(\frac{y_{t+1}}{y_t}\right)^{1/\rho^o_{t+1(t)}k_t} \\
DMI_{t+1} &= M_{t+1}(x_{t+1}, y_{t+1}; h_{t+1}, 0) \times \left(\frac{x_{t+1}}{x_t}\right)^{1/\rho^i_{t+1(t)}h_t}.
\end{aligned}
\tag{3.15}
$$

Following the aforementioned results (3.14) and (3.15) the definition of the dynamical DHM productivity index is immediate.

The DMI and the DMO are global indices indicating global increasing (decreasing) of inputs utilization and outputs production. Thus, these quantity indices do not allow to know in detail which input and which output are growing or diminishing. Nonetheless, the dynamical productivity measures allow to go further in details by identifying specifically increasing or decreasing inputs and outputs by means of the dynamical adjustment parameters and the dynamical cross-period efficiency measures (Corollary 3.5).

4 Non parametric specifications

Based upon the proposed TFP measure, this paper investigates productivity variation among Chinese provincial public hospitals under a non parametric framework using non convex models, namely the multiplicative technology and the Free Disposal Hull (FDH) production process, respectively. Thus, an efficiency comparison is set up by the means of these two different non convex production technologies. Remark that the hypothesis of convexity is relaxed to take into account for strictly increasing marginal products of potentially efficient production units.

4.1 Non parametric production sets

To set the efficiency comparison, the performance measure is estimated under a multiplicative technology and a Free-Disposal Hull production set.

Banker and Maindiratta (1986) propose a non convex and multiplicative production technology that has a non linear structure. This production set was introduced by the authors to allow taking into account of strictly increasing marginal products. For any set of observations $\mathcal{Z} = \{0, \cdots, \mathrm{Z}\}$ with $z \in \mathcal{Z}$ and through the DEA model, the multiplicative production technology is defined as:

$$
T^M_t = \left\{ (x_t, y_t) \in \mathbb{R}^{n+m}_+ : x^i_t \geq \prod_{z \in \mathcal{Z}} (x^{i,z}_t)^{\lambda_z},\ y^j_t \leq \prod_{z \in \mathcal{Z}} (y^{j,z}_t)^{\lambda_z},\ \sum_{z \in \mathcal{Z}} \lambda_z = 1, \right.
$$
$$
\left. \lambda_z \geq 0,\ i \in [n],\ j \in [m] \right\}. \tag{4.1}
$$

This production set satisfies axioms $T1 - T4$ and, is a log-convex production set and then, satisfies a "geometric" convexity. For more details, see Banker and Maindiratta (1986) and Mehdiloozad et al. (2014).

Notice that this production technology becomes linear through a natural logarithmic transformation. Indeed, assuming strictly positive inputs and outputs yields the following result:

$$\ln\left(T_t^M\right) = \Bigg\{(x_t, y_t) \in \mathbb{R}_{++}^{n+m} : \ln(x_t^i) \geq \sum_{z\in\mathcal{Z}} \lambda_z \ln(x_t^{i,z}),\ \ln(y_t^j) \leq \sum_{z\in\mathcal{Z}} \lambda_z \ln(y_t^{j,z}),$$
$$\sum_{z\in\mathcal{Z}} \lambda_z = 1,\ \lambda_z \geq 0,\ i \in [n],\ j \in [m]\Bigg\}. \quad (4.2)$$

Tulkens (1993) introduces the Free Disposal Hull (FDH) production set that satisfies axioms $T1 - T4$. Indeed, this technology is the smallest production set satisfying the free disposability assumption. In such case, this production technology does not require the convexity property. For any set of observations $\mathcal{Z}$ with $z \in \mathcal{Z}$, the FDH production technology is defined as follows:

$$T_t^{FDH} = \Bigg\{(x_t, y_t) \in \mathbb{R}_{+}^{n+m} : (x_t, y_t) \in \bigcup_{z\in\mathcal{Z}} S_t(x_t^z, y_t^z)\Bigg\}. \quad (4.3)$$

The above definition means that the FDH production set is the union of individual production sets $S_t(x_t^z, y_t^z)$ such that

$$S_t(x_t^z, y_t^z) = \Big\{(x_t, y_t) \in \mathbb{R}_{+}^{n+m} : x_t^i \geq x_t^{i,z}, y_t^j \leq y_t^{j,z}, i \in [n], j \in [m], z \in \mathcal{Z}\Big\}. \quad (4.4)$$

As the FDH production set is the smallest non convex set allowing for free disposability of inputs and outputs then, it is logical that the number of efficient observations under a FDH technology is greater than or equal to those under a multiplicative production set. However, the FDH technology enables for substantial finite sample error (Post, 2001; Jeong and Simar, 2006). Hence, when the number of input-output variables is higher than the number of observations then, the efficiency estimates are biased. Consequently, the multiplicative production set can overcome this limitation since it is a non convex possibility set allowing for increasing marginal products. Moreover, the multiplicative technology is piecewise log-linear and hence, can be estimated through the DEA model.

4.2 Non parametric efficiency measure

This subsection aims to define the multiplicative directional distance function through the DEA model with respect to both the multiplicative production technology and the FDH production set.

Consider a multiplicative production set as defined by Banker and Maindiratta (1986). For any set of observations $z \in \mathcal{Z}$, the non parametric efficiency measure of the observation (x_t^0, y_t^0) is defined as follows:

$$M_t(x_t, y_t; g_t) = \sup_{\delta} \Bigg\{ \delta : \delta^{-h_t^i} x_t^{i,0} \geq \prod_{z \in \mathcal{Z}} (x_t^{i,z})^{\lambda_z}, \delta^{k_t^j} y_t^{j,0} \leq \prod_{z \in \mathcal{Z}} (y_t^{j,z})^{\lambda_z}, \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0, i \in [n], j \in [m] \Bigg\} \quad (4.5)$$

The above specification is non linear. However, through a logarithmic transformation, a linear program can be provided to estimate this efficiency measure. Since this paper focuses on either the input- or the output- oriented efficiency measures, the linear programs of these efficiency indices are presented below:

$$\begin{aligned} \ln\left[M_t(x_t, y_t; g_t^i)\right] &= \max \ln(\delta) \\ \text{s.t} \quad & \ln(x_t^{i,0}) - h_t^i \ln(\delta) \geq \sum_{z \in \mathcal{Z}} \lambda_z \ln(x_t^{i,z}) \quad i \in [n] \\ & \ln(y_t^{j,0}) \leq \sum_{z \in \mathcal{Z}} \lambda_z \ln(y_t^{j,z}) \quad j \in [m] \\ & \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0. \end{aligned} \quad (4.6)$$

and

$$\begin{aligned} \ln\left[M_t(x_t, y_t; g_t^o)\right] &= \max \ln(\delta) \\ \text{s.t} \quad & \ln(x_t^{i,0}) \geq \sum_{z \in \mathcal{Z}} \lambda_z \ln(x_t^{i,z}) \quad i \in [n] \\ & \ln(y_t^{j,0}) + k_t^j \ln(\delta) \leq \sum_{z \in \mathcal{Z}} \lambda_z \ln(y_t^{j,z}) \quad j \in [m] \\ & \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0. \end{aligned} \quad (4.7)$$

Obviously, equations (4.6) and (4.7) are respectively the input and the output oriented multiplicative directional efficiency measures with $g_t^i = (h_t, 0)$ and $g_t^o = (0, k_t)$.

Now, assume that the observation (x_t^0, y_t^0) operates under a FDH production technology such that the multiplicative efficiency measure is defined as follows:

$$M_t(x_t, y_t; g_t) = \sup_{\substack{\delta \\ z \in \mathcal{Z}}} \left\{ \delta : \delta^{-h_t^i} x_t^{i,0} \geq x_t^{i,z}, \delta^{k_t^j} y_t^{j,0} \leq y_t^{j,z}, i \in [n], j \in [m] \right\}. \quad (4.8)$$

Based upon this non parametric definition of the graph efficiency measure, the input-and output-oriented multiplicative directional distance functions are given as below:

$$M_t(x_t, y_t; g_t^i) = \max_{z \in \mathcal{Z}} \left[\min_{i \in [n]} \left(\frac{x_t^{i,0}}{x_t^{i,z}} \right)^{1/h_t^i} \right]; \tag{4.9}$$

$$M_t(x_t, y_t; g_t^o) = \max_{z \in \mathcal{Z}} \left[\min_{j \in [m]} \left(\frac{y_t^{i,z}}{y_t^{i,0}} \right)^{1/k_t^j} \right]. \tag{4.10}$$

5 Empirical analysis

5.1 Literature review

Regarding the literatures on hospital performance, existing studies based on non parametric estimation mainly use DEA and FDH analysis. Due to its flexibility, DEA is widely used to measure medical efficiency and productivity, which does not need to set the production function in advance and collect relative price data (O'Neill et al., 2008). In addition, it can be used for the production process with multiple inputs and outputs, and it is easy to implement in calculations. Using DEA, Grosskopf et al. (2004) measured the technical efficiency and scale efficiency of 254 teaching hospitals in the United States, and found that the most teaching hospitals are inefficient in the use of inputs, with diminishing returns-to-scale. Lee et al. (2008) applied input-oriented DEA model under constant returns-to-scale (CRS) technology to evaluate the efficiency of 106 emergency hospitals in Seoul and showed that hospital efficiency is related to inputs such as the number of beds, doctors and nurses. Chowdhury and Zelenyuk (2016) used DEA based on CRS to calculate the medical efficiency of 113 acute-care hospitals in Ontario in 2003 and 2006. The results showed that the efficiency of teaching hospitals, large hospitals, and urban hospitals is higher than that of non-teaching hospitals, small hospitals, and rural hospitals, respectively. Based on the Malmquist index, Anthun et al. (2017) analysed the productivity growth of the Norwegian hospital sector from 1999 to 2014, and further decomposed it into efficiency changes and technological changes. They argued that the hospital ownership reform in 2002 does not significantly improve productivity. FDH is another alternative non parametric analysis, which relaxes the convexity assumption when setting the production technology (Shiraz et al., 2015). The researches with FDH model in the medical field is rare. Using a linear FDH model, Arfa et al. (2020) estimated the input-oriented directional distance function, and calculated the efficiency scores of five cardiology wards in Tunisian hospitals. In addition, the literatures that use both DEA and FDH model to analyze the efficiency and productivity of the healthcare system are rarely found.

For the existing literatures on efficiency and productivity growth, most of the selected samples are concentrated in developed countries while less scholars focus on developing countries, such as China. Moreover, they mostly use the Malmquist index. Based on input-oriented DEA and Malmquist index, Ng (2011) evaluated the efficiency and productivity of 463 hospitals in Guangdong Province, and further decomposed the efficiency into technical and scale efficiency. Treating patient mortality as an undesirable output, Hu et al. (2012) applied DEA to assess the efficiency of hospitals in 30 provinces in China from 2002 to 2008, and proposed that the implementation of the new rural cooperative medical system in 2003 improved efficiency, especially in non-coastal areas. Using Malmquist index, Li et al. (2014) calculated the total factor productivity (TFP) of 12 Third-grade hospitals in Beijing from 2006 to 2009, and further decomposed TFP into technological change and technical efficiency change. Kao et al. (2021) layout a meta-frontier based on 6899 Chinese hospitals, classified the hospitals according to the type of hospital departments to construct a group frontier, and then used DEA to calculate the efficiency and productivity under the group-specific frontiers and the meta-frontier. After assessing the efficiency of different types of hospitals with DEA, Kao et al. (2021b) further explored whether there are economies of scope when hospitals with a limited number of departments expand the scope through adding new departments.

In the researches on China's medical efficiency and productivity, a few scholars explore the efficiency and productivity after China's 2009 medical reform. China has a very large population, and the development of China's medical and health services is not enough to meet the needs for medical security to people and coordinated development to social. The exposed issues such as the unbalanced medical development of urban and rural, unreasonable resource allocation, and excessive medical expenses are not conducive to the improvement of China's medical care. In order to solve the problem of tough and expensive medical treatment for the people, Chinese government proposed a medical reform in 2009. According to the Statistical Bulletin of China's Health Service Development, China's total health expenditure rose from 145.354 billion yuan in 2008 to 658.414 billion yuan in 2019, accounting for 6.64% of GDP. Since the new medical reform in 2009, the medical and health system has been continuously improved, and residents' demand for medical services has expanded rapidly. However, the pressure of medical service supply is correspondingly growing. It is significant to measure hospital efficiency and total factor productivity to evaluate the hospital effectiveness in medical reform. Jiang et al. (2016) calculated the efficiency of public hospitals in Guangxi County from 2010 to 2012. The results showed that after the medical reform, the operating efficiency of hospitals in Guangxi Province did not improve significantly. Based on meta-frontier model and Malmquist productivity index, See and Ng (2021) analyzed the overall factor productivity growth of hospitals in Shenzhen during the implementation of the hospital reform from 2005 to 2013, and found that the TFP of hospital has increased slightly after the medical reform.

5.2 Data in brief

We follow the data setting in Shen and Valdmanis (2019), and Boussemart et al. (2020). The production technology of Chinese medical institutions is defined with four inputs and four outputs. Specifically, the inputs of medical institutions include i1) number of licensed doctors (10000 persons); i2) number of registered nurse (10000 persons); i3) other technical staff (10000 persons); i4) number of beds in health care institutions (10000 units). The outputs produced by inputs, are : o1) emergency treatment in health institutions (million person-times); o2) number of outpatients visits (million persons); o3) number of inpatients visits (million persons); o4) operation of hospitalized in health institutions (namely surgery, million persons-times). Due to data availability, a balanced provincial data is selected for 31 main provinces (municipalities) during the period over 2014-2018. The recent reform of Chinese medical and health system is covered in the sample period. The data is from National Bureau of Statistics of China (2020)[4].

The statistical description of sample data is displayed in Table 1. One can note a significant variation in sample according to values of the standard deviation (S.D.) that implies unbalanced development levels among provincial medical institutions over 2014-2018. The annual growth rates of inputs and outputs are denoted as the trend that suggest the fastest expansion of outputs is number of surgery while the lowest growth is in number of outpatients visits. In addition, the increase of outputs is mainly motivated by inputs expansion, especially by utilizing more nurses and beds in health care institutions.

Indicator		**Mean**	**Max**	**Min**	**S.D.**	**Trend**
Inputs	Doctors	10.40	29.04	0.56	6.71	5.75%
	Nurses	11.39	33.46	0.27	7.35	8.41%
	Other staffs	5.64	14.43	0.44	3.42	2.11%
	Beds	24.11	60.85	1.19	15.28	6.04%
Outputs	Treatments	245.49	825.89	12.35	191.48	2.39%
	Outpatients	10.75	38.58	0.64	8.83	0.84%
	Inpatients	7.36	19.16	0.23	4.96	5.66%
	Surgery	1.66	7.35	0.04	1.26	8.52%

Table 1: Statistical description of variables (2014-2018)

5.3 Results

In this subsection, we focus on the results for the period 2017-2018. We estimate the productivity change with respect to two different production sets namely the Free Disposal Hull production technology and the Multiplicative production technology.

Multiplicative technology (MT)

The second column from the right of Table 2 shows that Liaoning and Heilongjiang have a directional Hicks-Moorsteen productivity index equal to and greater than one respectively. Hence, the remaining 29 observations present loss of productivity over the period 2017-2018. Table 5 enables to see that all observations increase the use of their inputs between the two periods (input directional malmquist quantity index - DMI MT greater than 1). Besides, all observations produce either more or the same

[4]National Bureau of Statistics of China, (2021). Chinese Statistic Yearbook. Accessed on 1 August 2021, Available from: `http://www.stats.gov.cn/english/`

quantity of outputs between 2017 and 2018 (output directional malmquist quantity index - DMO MT greater than 1). However, the production growth does not compensate the rise of input utilization. Thus, the observations suffer of productivity loss between the two periods except Liaoning and Heilongjiang. Table 3 allows to go bit further in details. Indeed, we can notice that all observations increase the use of inputs 1 and 2 between 2017 and 2018. This situation is shown by either $\left(\rho^i_{t(t+1)}, \rho^i_{t+1(t)}\right) < 0$ with $\left(M^i_{t(t+1)}, M^i_{t+1(t)}\right) > 1$ or $\left(\rho^i_{t(t+1)}, \rho^i_{t+1(t)}\right) > 0$ with $\left(M^i_{t(t+1)}, M^i_{t+1(t)}\right) < 1$. Moreover, except Inner Mongolia, Heilongjiang, Shandong, Hubei, Chongqing and Xinjiang, the other provinces also improve input 3. Finally, except Tianjing, all the remaining provinces rise input 4 utilization. Still concerning Table 3, the adjustment parameters of outputs show that two-thirds of observations (except Tianjin, Shanxi, Liaoning, Heilongjiang, Jiangxi, Hubei, Guangxi, Tibet, Gansu, Xinjiang) increase the production of output 1. This situation is pointed out by either $\left(\rho^o_{t(t+1)}, \rho^o_{t+1(t)}\right) < 0$ with $\left(M^o_{t(t+1)}, M^o_{t+1(t)}\right) < 1$ or $\left(\rho^o_{t(t+1)}, \rho^o_{t+1(t)}\right) > 0$ with $\left(M^o_{t(t+1)}, M^o_{t+1(t)}\right) > 1$. In addition, 20 DMUs[5] (excluding Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Tibet, Gansu, Ningxia, Xinjiang) reduce the output 2. Moreover, excluding Heilongjiang, Tibet and Xinjiang, all the left observations rise the production of output 3. Finally, all DMUs unless Qinghai expand their output 4.

Consequently, the increase in all inputs has been counterbalanced by the production growth of outputs 1/3/4 for Liaoning. For this reason, this DMU has neither gain nor loss of productivity between periods 2017 and 2018. Regarding Heilongjiang, the increase in inputs 1/2/4 and the decrease in outputs 1/3 has been compensated by the decrease in input 3 and the increase in outputs 2/4. It results that Heilongjiang gains in productivity over the two periods.

From a dynamical standpoint, the adjustment parameters proposed in Table 3 allow to compute dynamical directional Hicks-Moorsteen index and also dynamical Malmquist quantity indices as in Table 5. The results show that the ratio between DMO and DMI yields the productivity index and this productivity measure is equal to the result in Table 2.

[5]DMU: decision making unit.

	2014-2015		2015-2016		2016-2017		2017-2018	
	DHM-MT	**DHM-FDH**	**DHM-MT**	**DHM-FDH**	**DHM-MT**	**DHM-FDH**	**DHM-MT**	**DHM-FDH**
Beijing	0.998	0.898	0.971	1.069	0.956	1.247	0.991	0.709
Tianjin	0.942	1.018	0.966	0.968	0.945	1.036	0.978	0.980
Hebei	0.951	0.911	0.966	1.012	0.936	0.928	0.940	0.945
Shanxi	1.017	0.959	0.960	1.015	1.021	1.047	0.982	0.990
Inner Mongolia	0.949	0.928	0.978	1.041	0.934	1.062	0.960	1.017
Liaoning	1.029	0.978	0.997	1.021	1.019	1.002	1.000	1.058
Jilin	0.976	0.935	0.978	1.010	1.006	1.039	0.965	0.927
Heilongjiang	0.992	0.961	0.976	1.008	0.926	0.932	1.007	1.074
Shanghai	0.966	0.901	0.956	0.939	0.954	0.998	0.961	0.985
Jiangsu	0.962	0.944	1.000	1.023	0.959	1.011	0.955	0.986
Zhejiang	0.911	0.972	0.933	1.018	0.937	1.094	0.962	1.005
Anhui	0.975	0.985	0.968	0.997	0.984	1.056	0.950	0.979
Fujian	0.956	0.972	1.004	1.028	0.959	0.961	0.956	1.052
Jiangxi	0.955	0.979	0.955	0.995	0.927	0.983	0.956	0.977
Shandong	0.983	0.948	0.967	1.052	0.949	0.987	0.973	0.955
Henan	0.958	0.966	0.952	0.978	0.952	0.995	0.944	0.980
Hubei	0.951	0.977	0.967	1.056	0.975	0.947	0.979	1.014
Hunan	0.948	0.993	0.964	1.018	0.967	1.008	0.954	0.919
Guangdong	0.949	0.917	0.936	0.980	0.948	1.022	0.951	1.031
Guangxi	0.939	0.933	0.952	0.995	0.940	0.968	0.946	1.014
Hainan	0.927	0.863	0.971	0.978	0.974	1.122	0.954	0.907
Chongqing	0.922	0.941	0.934	0.931	0.942	1.022	0.950	0.997
Sichuan	0.954	0.940	0.948	1.013	0.934	1.002	0.947	1.085
Guizhou	0.908	0.909	0.918	0.996	0.901	0.964	0.926	0.989
Yunnan	0.917	0.881	0.910	1.010	0.886	0.927	0.941	1.000
Tibet	0.890	1.018	0.958	0.902	0.929	1.113	0.880	1.044
Shaanxi	0.964	0.972	0.938	0.998	0.942	1.023	0.950	0.988
Gansu	0.971	1.028	0.956	1.046	0.918	0.971	0.931	0.975
Qinghai	0.954	0.997	0.971	1.026	0.902	1.030	0.955	0.888
Ningxia	0.980	0.929	0.954	1.035	0.923	0.788	0.952	1.205
Xinjiang	0.964	1.011	0.953	1.040	0.947	1.042	0.955	1.003

Table 2: Directional Hicks-Moorsteen index under FDH (DHM-FDH) and multiplicative (DHM-MT) technologies

$\rho_{t(t+1)}$	2017-2018									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-1.139	-0.747	-0.284	-0.502	1.0501	-1.701	32.708	-2.350	-2.737	0.9693
Tianjin	16.266	10.363	6.160	-1.000	0.9971	0.026	1.000	-0.207	-1.025	0.8752
Hebei	-3.690	-3.334	-1.000	-2.514	1.0266	-0.079	1.453	-0.878	-1.689	0.9626
Shanxi	-0.276	-0.363	-0.099	-0.274	1.2143	-0.333	-0.442	0.717	1.032	1.1263
Inner Mongolia	-0.162	-0.219	0.002	-0.201	1.3222	0.053	0.461	0.310	0.378	1.2016
Liaoning	-1.420	-1.919	-0.086	-1.827	1.0286	0.091	0.299	-0.143	-3.015	0.9396
Jilin	-23.688	-34.310	-1.000	-22.295	1.0037	-1.402	3.112	-2.246	-1.994	0.9764
Heilongjiang	-0.071	-0.186	0.290	-0.218	1.1694	-0.914	-0.780	-0.549	3.119	1.0617
Shanghai	-1.966	-1.779	-1.000	-1.198	1.0272	-0.150	0.557	-0.602	-1.000	0.8946
Jiangsu	-2.549	-3.361	-1.000	-1.648	1.0286	-0.379	-1.195	-0.497	-1.773	0.9569
Zhejiang	-10.023	-10.877	-1.000	-8.830	1.0065	-0.789	-1.036	-1.067	-0.589	0.9351
Anhui	-1.124	-1.866	-1.000	-1.633	1.0442	-1.346	0.548	-0.319	-2.307	0.9543
Fujian	-0.688	-0.654	-0.140	-0.456	1.1254	2.198	12.709	3.067	3.308	1.0135
Jiangxi	-1.637	-2.373	-0.855	-2.421	1.0268	0.225	1.100	-0.688	-1.734	0.9372
Shandong	2.365	2.404	-1.000	1.009	0.9614	-1.565	4.734	-0.631	-2.026	0.9862
Henan	-2.238	-2.833	-1.000	-2.819	1.0306	-0.021	0.221	-1.104	-0.834	0.9188
Hubei	79.869	88.172	-1.000	113.377	0.9996	0.117	0.510	-0.312	-1.255	0.9072
Hunan	-1.473	-2.116	-1.000	-2.130	1.0307	-0.072	2.515	-0.704	-1.289	0.9416
Guangdong	-3.995	-4.862	-1.000	-2.857	1.0174	-0.071	-0.143	-0.298	-1.000	0.8596
Guangxi	-0.929	-1.263	-0.543	-1.175	1.0520	0.328	0.344	-0.546	-2.281	0.9399
Hainan	-2.121	-1.639	-0.613	-1.979	1.0331	-0.585	6.561	-0.733	-2.962	0.9665
Chongqing	9.663	10.078	-1.000	5.704	0.9888	-0.299	0.542	-0.302	-1.652	0.9164
Sichuan	-1.626	-2.551	-1.000	-1.969	1.0314	-0.340	-1.717	-0.037	-0.814	0.8563
Guizhou	-0.781	-1.103	-0.469	-0.540	1.1024	-0.861	-0.174	-1.211	-1.325	0.9157
Yunnan	-1.020	-1.081	-1.012	-0.992	1.0602	-0.208	1.668	-0.962	-1.021	0.9253
Tibet	-2.132	-5.025	-3.844	-1.000	1.0435	0.016	-1.000	0.163	-0.095	0.6645
Shaanxi	-0.416	-0.570	-0.068	-0.343	1.1573	0.858	-3.033	2.085	3.368	1.0294
Gansu	-1.579	-2.529	-0.847	-2.749	1.0386	0.230	-0.145	-1.158	-0.192	0.9112
Qinghai	-1.996	-3.059	-5.464	-1.000	1.0209	-1.830	9.515	-0.898	2.664	0.9865
Ningxia	-2.210	-2.517	-1.509	-1.000	1.0302	-0.167	-3.404	-0.200	-0.278	0.8749
Xinjiang	-0.333	-0.506	0.255	-1.349	1.0495	4.783	-10.211	0.835	-5.563	0.9892

$\rho_{t+1(t)}$	2017-2018									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	2.219	1.456	0.553	0.979	0.9752	1.151	-22.139	1.590	1.853	1.0471
Tianjin	1.000	0.637	0.379	-0.061	0.9535	0.036	1.410	-0.291	-1.446	0.9098
Hebei	1.000	0.903	0.271	0.681	0.9078	-0.055	1.000	-0.604	-1.162	0.9461
Shanxi	-0.640	-0.841	-0.229	-0.636	1.0873	-0.223	-0.297	0.481	0.693	1.1939
Inner Mongolia	-0.306	-0.412	0.004	-0.379	1.1600	0.034	0.300	0.201	0.246	1.3262
Liaoning	1.416	1.914	0.086	1.822	0.9721	-0.154	-0.506	0.241	5.095	1.0375
Jilin	1.307	1.893	0.055	1.230	0.9348	-0.474	1.052	-0.760	-0.675	0.9319
Heilongjiang	-0.294	-0.770	1.204	-0.905	1.0385	-0.409	-0.349	-0.246	1.396	1.1431
Shanghai	1.000	0.905	0.509	0.609	0.9486	-1.668	6.200	-6.700	-11.135	0.9900
Jiangsu	1.122	1.479	0.440	0.726	0.9380	0.941	2.967	1.234	4.402	1.0179
Zhejiang	0.921	1.000	0.092	0.812	0.9315	1.080	1.419	1.462	0.807	1.0502
Anhui	0.824	1.369	0.733	1.198	0.9427	4.486	-1.826	1.063	7.689	1.0141
Fujian	10.698	10.174	2.185	7.096	0.9924	0.265	1.534	0.370	0.399	1.1171
Jiangxi	0.676	0.980	0.353	1.000	0.9379	0.840	4.098	-2.564	-6.458	0.9827
Shandong	0.983	1.000	-0.416	0.420	0.9097	-0.498	1.505	-0.201	-0.644	0.9571
Henan	0.790	1.000	0.353	0.995	0.9183	-0.123	1.300	-6.484	-4.894	0.9857
Hubei	0.731	0.807	-0.009	1.038	0.9576	3.253	14.220	-8.709	-35.013	0.9965
Hunan	0.692	0.995	0.470	1.001	0.9377	-0.120	4.225	-1.183	-2.165	0.9648
Guangdong	0.822	1.000	0.206	0.588	0.9197	1.000	2.007	4.174	13.988	1.0109
Guangxi	0.768	1.044	0.449	0.971	0.9405	-1.530	-1.604	2.547	10.646	1.0134
Hainan	1.127	0.871	0.326	1.052	0.9405	3.981	-44.637	4.987	20.155	1.0050
Chongqing	0.959	1.000	-0.099	0.566	0.8927	2.872	-5.212	2.906	15.876	1.0091
Sichuan	0.638	1.000	0.392	0.772	0.9241	9.132	46.189	1.000	21.894	1.0058
Guizhou	0.779	1.100	0.468	0.539	0.9069	0.782	0.158	1.101	1.205	1.1017
Yunnan	0.946	1.003	0.938	0.919	0.9389	0.515	-4.122	2.377	2.523	1.0319
Tibet	0.424	1.000	0.765	0.199	0.8075	0.097	-6.126	1.000	-0.584	0.9355
Shaanxi	-1.782	-2.441	-0.294	-1.470	1.0347	0.153	-0.540	0.371	0.600	1.1764
Gansu	0.574	0.920	0.308	1.000	0.9010	2.703	-1.702	-13.632	-2.254	0.9921
Qinghai	0.576	0.883	1.577	0.289	0.9309	-0.495	2.576	-0.243	0.721	0.9511
Ningxia	0.942	1.073	0.643	0.426	0.9327	0.954	19.429	1.141	1.587	1.0237
Xinjiang	0.375	0.569	-0.287	1.520	0.9580	6.279	-13.405	1.096	-7.304	0.9918

Table 3: Dynamical Deviation under the multiplicative production set

$\rho_{t(t+1)}$	2017-2018									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-4.014	-2.633	-1.000	-1.771	1.0140	-0.621	11.951	-0.859	-1.000	0.9183
Tianjin	16.266	10.363	6.160	-1.000	0.9971	0.025	0.975	-0.201	-1.000	0.8722
Hebei	-3.690	-3.334	-1.000	-2.514	1.0266	-0.047	0.860	-0.520	-1.000	0.9376
Shanxi	-2.794	-3.671	-1.000	-2.777	1.0194	0.322	0.428	-0.695	-1.000	0.8845
Inner Mongolia	69.303	93.429	-1.000	85.954	0.9993	-0.114	-1.000	-0.672	-0.821	0.9188
Liaoning	-16.496	-22.298	-1.000	-21.223	1.0024	0.030	0.099	-0.047	-1.000	0.8288
Jilin	-23.688	-34.310	-1.000	-22.295	1.0037	-0.624	1.385	-1.000	-0.888	0.9478
Heilongjiang	0.244	0.640	-1.000	0.752	0.9556	0.293	0.250	0.176	-1.000	0.8297
Shanghai	-1.966	-1.779	-1.000	-1.198	1.0272	-0.150	0.557	-0.602	-1.000	0.8946
Jiangsu	-2.549	-3.361	-1.000	-1.648	1.0286	-0.214	-0.674	-0.280	-1.000	0.9249
Zhejiang	-10.023	-10.877	-1.000	-8.830	1.0065	-0.739	-0.971	-1.000	-0.552	0.9309
Anhui	-1.124	-1.866	-1.000	-1.633	1.0442	-0.583	0.237	-0.138	-1.000	0.8977
Fujian	-4.897	-4.657	-1.000	-3.248	1.0167	-0.173	-1.000	-0.241	-0.260	0.8438
Jiangxi	-1.913	-2.775	-1.000	-2.831	1.0229	0.130	0.635	-0.397	-1.000	0.8937
Shandong	2.365	2.404	-1.000	1.009	0.9614	-0.773	2.337	-0.311	-1.000	0.9722
Henan	-2.238	-2.833	-1.000	-2.819	1.0306	-0.019	0.200	-1.000	-0.755	0.9107
Hubei	79.869	88.172	-1.000	113.377	0.9996	0.093	0.406	-0.249	-1.000	0.8849
Hunan	-1.473	-2.116	-1.000	-2.130	1.0307	-0.056	1.952	-0.547	-1.000	0.9254
Guangdong	-3.995	-4.862	-1.000	-2.857	1.0174	-0.071	-0.143	-0.298	-1.000	0.8596
Guangxi	-1.712	-2.326	-1.000	-2.164	1.0279	0.144	0.151	-0.239	-1.000	0.8682
Hainan	-3.461	-2.675	-1.000	-3.231	1.0202	-0.198	2.215	-0.247	-1.000	0.9041
Chongqing	9.663	10.078	-1.000	5.704	0.9888	-0.181	0.328	-0.183	-1.000	0.8657
Sichuan	-1.626	-2.551	-1.000	-1.969	1.0314	-0.198	-1.000	-0.022	-0.474	0.7662
Guizhou	-1.664	-2.350	-1.000	-1.151	1.0468	-0.649	-0.131	-0.914	-1.000	0.8899
Yunnan	-1.029	-1.091	-1.021	-1.000	1.0597	-0.204	1.634	-0.942	-1.000	0.9238
Tibet	-2.132	-5.025	-3.844	-1.000	1.0435	0.016	-1.000	0.163	-0.095	0.6645
Shaanxi	-6.070	-8.317	-1.000	-5.008	1.0101	-0.255	0.901	-0.619	-1.000	0.9072
Gansu	-1.864	-2.986	-1.000	-3.246	1.0326	0.198	-0.125	-1.000	-0.165	0.8979
Qinghai	-1.996	-3.059	-5.464	-1.000	1.0209	-1.000	5.200	-0.491	1.456	0.9755
Ningxia	-2.210	-2.517	-1.509	-1.000	1.0302	-0.049	-1.000	-0.059	-0.082	0.6346
Xinjiang	1.309	1.986	-1.000	5.301	0.9878	0.468	-1.000	0.082	-0.545	0.8955

$\rho_{t+1(t)}$	2017-2018									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.656	0.249	0.441	0.9458	-0.075	1.447	-0.104	-0.121	0.4945
Tianjin	1.000	0.637	0.379	-0.061	0.9535	0.026	1.000	-0.207	-1.025	0.8752
Hebei	1.000	0.903	0.271	0.681	0.9078	-0.055	1.000	-0.604	-1.162	0.9461
Shanxi	0.761	1.000	0.272	0.756	0.9320	0.753	1.000	-1.623	-2.336	0.9488
Inner Mongolia	0.742	1.000	-0.011	0.920	0.9407	1.000	8.764	5.888	7.199	1.0097
Liaoning	0.740	1.000	0.045	0.952	0.9473	0.304	1.000	-0.476	-10.069	0.9815
Jilin	0.690	1.000	0.029	0.650	0.8801	-0.451	1.000	-0.722	-0.641	0.9285
Heilongjiang	0.325	0.851	-1.330	1.000	0.9664	1.000	0.854	0.601	-3.413	0.9468
Shanghai	1.000	0.905	0.509	0.609	0.9486	-0.269	1.000	-1.081	-1.796	0.9398
Jiangsu	0.759	1.000	0.298	0.490	0.9097	1.000	3.152	1.311	4.676	1.0168
Zhejiang	0.691	0.750	0.069	0.609	0.9097	1.339	1.759	1.812	1.000	1.0403
Anhui	0.602	1.000	0.536	0.875	0.9224	-2.457	1.000	-0.582	-4.211	0.9747
Fujian	1.000	0.951	0.204	0.663	0.9220	1.000	5.783	1.395	1.505	1.0298
Jiangxi	0.676	0.980	0.353	1.000	0.9379	0.205	1.000	-0.626	-1.576	0.9312
Shandong	0.983	1.000	-0.416	0.420	0.9097	-0.331	1.000	-0.133	-0.428	0.9362
Henan	0.790	1.000	0.353	0.995	0.9183	-0.095	1.000	-4.989	-3.765	0.9814
Hubei	0.704	0.778	-0.009	1.000	0.9560	0.229	1.000	-0.612	-2.462	0.9515
Hunan	0.692	0.994	0.470	1.000	0.9376	-0.028	1.000	-0.280	-0.512	0.8596
Guangdong	0.822	1.000	0.206	0.588	0.9197	1.000	2.007	4.174	13.988	1.0109
Guangxi	0.736	1.000	0.430	0.930	0.9380	0.954	1.000	-1.588	-6.637	0.9789
Hainan	1.000	0.773	0.289	0.933	0.9332	-0.097	1.084	-0.121	-0.490	0.8139
Chongqing	0.959	1.000	-0.099	0.566	0.8927	-0.551	1.000	-0.558	-3.046	0.9538
Sichuan	0.638	1.000	0.392	0.772	0.9241	9.132	46.189	1.000	21.894	1.0058
Guizhou	0.708	1.000	0.426	0.490	0.8980	4.952	1.000	6.969	7.625	1.0154
Yunnan	-1.029	-1.091	-1.021	-1.000	1.0597	-0.204	1.634	-0.942	-1.000	0.9238
Tibet	0.424	1.000	0.765	0.199	0.8075	0.097	-6.126	1.000	-0.584	0.9355
Shaanxi	0.730	1.000	0.120	0.602	0.9201	-0.848	2.996	-2.060	-3.327	0.9711
Gansu	0.574	0.920	0.308	1.000	0.9010	1.000	-0.630	-5.044	-0.834	0.9789
Qinghai	0.365	0.560	1.000	0.183	0.8932	-0.192	1.000	-0.094	0.280	0.8789
Ningxia	0.878	1.000	0.600	0.397	0.9280	1.000	20.361	1.196	1.663	1.0226
Xinjiang	0.247	0.375	-0.189	1.000	0.9368	1.000	-2.135	0.175	-1.163	0.9496

Table 4: Dynamical Deviation under FDH production technology

	2017-2018						2016-2017					
	Multiplicative			Free Disposal Hull			Multiplicative			Free Disposal Hull		
	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM
Beijing	1.025	1.015	0.991	1.035	0.734	0.709	1.030	0.985	0.956	1.043	1.301	1.247
Tianjin	1.023	1.000	0.978	1.023	1.002	0.980	1.058	1.000	0.945	1.058	1.096	1.036
Hebei	1.063	1.000	0.940	1.063	1.005	0.945	1.068	1.000	0.936	1.068	0.991	0.928
Shanxi	1.039	1.021	0.982	1.046	1.036	0.990	1.015	1.036	1.021	1.027	1.075	1.047
Inner Mongolia	1.058	1.015	0.960	1.031	1.048	1.017	1.078	1.008	0.934	1.049	1.114	1.062
Liaoning	1.015	1.015	1.000	1.029	1.088	1.058	0.996	1.015	1.019	1.029	1.032	1.002
Jilin	1.036	1.000	0.965	1.068	0.990	0.927	0.994	1.000	1.006	0.996	1.034	1.039
Heilongjiang	1.026	1.034	1.007	0.994	1.068	1.074	1.056	0.977	0.926	1.038	0.967	0.932
Shanghai	1.041	1.000	0.961	1.041	1.025	0.985	1.048	1.000	0.954	1.048	1.046	0.998
Jiangsu	1.047	1.000	0.955	1.063	1.048	0.986	1.042	1.000	0.959	1.049	1.061	1.011
Zhejiang	1.040	1.000	0.962	1.052	1.057	1.005	1.067	1.000	0.937	1.054	1.153	1.094
Anhui	1.052	1.000	0.950	1.064	1.042	0.979	1.029	1.012	0.984	1.045	1.103	1.056
Fujian	1.056	1.010	0.956	1.050	1.105	1.052	1.044	1.001	0.959	1.051	1.010	0.961
Jiangxi	1.046	1.000	0.956	1.044	1.021	0.977	1.078	1.000	0.927	1.078	1.059	0.983
Shandong	1.028	1.000	0.973	1.028	0.981	0.955	1.054	1.000	0.949	1.054	1.040	0.987
Henan	1.059	1.000	0.944	1.059	1.038	0.980	1.050	1.000	0.952	1.047	1.042	0.995
Hubei	1.022	1.000	0.979	1.023	1.037	1.014	1.025	1.000	0.975	1.028	0.973	0.947
Hunan	1.048	1.000	0.954	1.048	0.964	0.919	1.034	1.000	0.967	1.035	1.043	1.008
Guangdong	1.052	1.000	0.951	1.052	1.084	1.031	1.055	1.000	0.948	1.055	1.078	1.022
Guangxi	1.058	1.000	0.946	1.047	1.062	1.014	1.064	1.000	0.940	1.050	1.017	0.968
Hainan	1.048	1.000	0.954	1.046	0.949	0.907	1.029	1.002	0.974	1.038	1.164	1.122
Chongqing	1.052	1.000	0.950	1.052	1.050	0.997	1.061	1.000	0.942	1.061	1.085	1.022
Sichuan	1.056	1.000	0.947	1.056	1.146	1.085	1.070	1.000	0.934	1.070	1.072	1.002
Guizhou	1.091	1.010	0.926	1.080	1.068	0.989	1.099	0.990	0.901	1.097	1.057	0.964
Yunnan	1.063	1.000	0.941	1.000	1.000	1.000	1.129	1.000	0.886	1.136	1.053	0.927
Tibet	1.137	1.000	0.880	1.137	1.187	1.044	1.077	1.000	0.929	1.077	1.199	1.113
Shaanxi	1.055	1.002	0.950	1.048	1.035	0.988	1.077	1.014	0.942	1.066	1.090	1.023
Gansu	1.074	1.000	0.931	1.071	1.044	0.975	1.089	1.000	0.918	1.093	1.062	0.971
Qinghai	1.047	1.000	0.955	1.069	0.949	0.888	1.109	1.000	0.902	1.114	1.147	1.030
Ningxia	1.051	1.000	0.952	1.054	1.269	1.205	1.084	1.000	0.923	1.113	0.877	0.788
Xinjiang	1.047	1.000	0.955	1.027	1.030	1.003	1.056	1.000	0.947	1.011	1.053	1.042

Table 5: Dynamical DHM-MT and DHM-FDH (2016-2018)

Free Disposal Hull technology (FDH)
Regarding productivity change under FDH technology, the first column from the right of Table 2 shows that 13 observations (Inner Mongolia, Liaoning, Heilongjiang, Zhejiang, Fujian, Hubei, Guangdong, Guangxi, Sichuan, Yunnan, Tibet, Ningxia, Xinjiang) present productivity improvement or stagnation. Table 5 allows to deduce that all observations except Heilongjiang use more inputs between 2017 and 2018. Moreover, DMUs not including Beijing, Jilin, Shandong, Hunan, Hainan and Qinghai produce more outputs between the two time periods. Hence, the outputs growth counterbalances the increase in inputs for the 13 DMUs presenting positive productivity variation. From Table 4, we can see that all observations increase their four inputs except Tianjin, Inner Mongolia, Heilonjiang, Shandong, Hubei, Chongqing and Xinjiang. Indeed, Tianjin rises the utilization of inputs 1/2/3 and reduces input 4 whereas the six remaining observations increase inputs 1/2/4 while diminishing input 3. Concerning the output part, Table 4 allows to deduce that 8 observations (Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Ningxia) increase all of their outputs. Besides, Gansu gets its outputs 2/3/4 growing while output 1 diminishing. In addition, 12 DMUs (Beijing, Hebei, Jilin, Shanghai, Anhui, Shandong, Henan, Hunan, Hainan, Chongqing, Yunnan, Shaanxi) expand their outputs 1/3/4 and decrease their output 2. Moreover, six DMUs (Tianjin, Shanxi, Liaoning, Jiangxi, Hubei, Guangxi) increase outputs 3/4 and reduce outputs 1/2. Tibet and Xinjiang have growing outputs 2/4 and reducing outputs 1/3 whereas Qinghai experiences the opposite. Finally, Heilongjiang has the output 4 as the single increasing output while the three other outputs are reducing. These results are consistent with those obtained under a multiplicative production set.

Regarding the dynamical viewpoint, the dynamical deviations in Table 4 enable to yield the dynamical directional Hicks-Moorsteen index and the dynamical Malmquist quantity indices (Table 5). The results highlight that the ratio between DMO and DMI provides the same productivity measure as presented in Table 2.

To summarize regarding the two production sets, all observations increase all their inputs (1. licensed doctors; 2. registered nurse; 3. other technical staff; 4. beds) except Tianjin whose input 4 decreases and, Inner Mongolia, Heilongjiang, Shandong, Hubei, Chongqing and Xinjiang that diminish their input 3. Concerning the output part, all DMUs improve the production of outputs 3 (inpatients) and 4 (operation of hospitalized) except Heilongjiang, Tibet and Xinjiang with a decreasing output 3 and, Heilongjiang presenting a reduced output 4. In addition, 21 DMUs (excluding Tianjian, Shanxi, Liaoning, Heilongjiang, Jiangxi, Hubei, Guangxi, Tibet, Gansu and Xinjiang) have growing output 1 (emergency treatment). Finally, most of the observations (except Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Tibet, Gansu, Ningxia and Xinjiang) have decreasing output 2 (other outpatients visits). Hence, almost all of DMUs increase all of their inputs and most of them decrease their output 2. This fact is mostly supposed to be counterbalanced with the increase in outputs 3 and 4 for almost all DMUs and in output 1 for some of them. Concerning the productivity variation, Liaoning and Heilongjiang are presenting positive productivity change in both multiplicative and FDH production sets. Moreover, there are more observations having gain of productivity under a FDH technology than under a multiplicative technology. It is logical since the FDH is the smallest non convex production possibility set satisfying the free disposability in inputs and outputs. Thus, there could be more efficient DMUs under a FDH production set than under a multiplicative technology.

Globally, from the results over year 2014 to year 2018 (Appendix), we can notice that most of observations have a directional input Malmquist quantity index greater than one. This means that there is an increasing utilization of inputs between the time periods. This result is strengthened by the ones about dynamical deviations. Indeed, we can see that input 1,2 and 4 increase for almost all observations whereas there is a decrease in input 3 for a quarter of observations. These results are in the line of the Chinese healthcare reform[6]. Indeed, this reform includes an expansion of human resources in medical institutions. Besides, all observations have a directional output Malmquist quantity index greater than one meaning that more outputs are produced between the time periods. The dynamical deviations show that from 2014 to 2018, there is a production growth of outputs 1,3 and 4 for all observations. In addition, there is a gradual decrease of output 2 for around a half of the observations each year. Nonetheless, the increase in outputs does not compensate the rising input utilization. Hence, most of observations present a loss of productivity from year to year. The increase in outputs (1,3,4) might be the consequence of the Chinese healthcare reform through the improvement of the health insurance system[7], and the drug supply and security system[8]. Moreover, the decrease in output 2 might be the result of the improvement of the public health service system such as the implementation of telemedicine.
Not surprisingly, a FDH production set presents more observations having a gain of productivity. However, the FDH results can be biased by slacks but also the small sample size.

6 Concluding comments

This paper aims to analyse the productivity through non convex production possibility set. To drive this investigation, the Hicks-Moorsteen productivity index is estimated using the multiplicative directional distance function. This efficiency measure allows to characterize non convex production technology that takes into account for efficient production units presenting strictly increasing marginal products. As the Hicks-Moorsteen productivity index is evaluated through the multiplicative directional distance function, we refer to this productivity measure as the Directional Hicks-Moorsteen. Moreover the Directional Hicks-Moorsteen productivity measure avoids infeasibilities.

[6]See Tao et al. (2020) for a clear overview of the Chinese healthcare reform.

[7]The health insurance system includes an expansion of the basic insurance coverage as payment mechanisms, increasing financial level, fiscal subsidies and reimbursement rates.

[8]The drug supply and security system consist of the selection, production and distribution of essential medicines; quality assurance; reasonable pricing; tendering and procurement; a zero mark-up policy on sales; rational use and reimbursement.

These theoretical backgrounds are used to set an empirical analysis about Chinese medical institutions over the periods 2014-2018. These periods encompass the healthcare reform initiated by the Chinese government in 2009. The data includes four inputs (i1. Doctors; i2. Nurses; i3. Other technical staff; i4. Beds) and four outputs (o1. Treatments ; o2. Outpatients ; o3. Inpatients; o4. Surgery). The efficiency and productivity measures have been estimated through the data envelopment analysis framework in two different production processes: the multiplicative technology and the free disposal hull (FDH) production process. The productivity evaluation demonstrates that there are more production units presenting productivity gain under a FDH technology. This is not surprising since the FDH production set is smaller than the multiplicative one. However, the two production sets provide consistent results. The findings show that all production units (Chinese provinces) globally exhibit increasing inputs and outputs over years. The increase in inputs results from a reform of the Chinese healthcare system by improving human resource management. Besides, some outputs decrease which might also be a consequence of the reform since the telemedicine has been implemented. However, the improvement in outputs does not compensate the growth in inputs. Hence, most of the considered provinces show productivity loss.

In further works, the decomposition of the proposed TFP measure could be investigated. For instance, the proposed productivity index could be separated into components of efficiency change and technological progress similar to other productivity measures. Alternatively, the distance functions could be estimated by both parametric and non parametric approaches to compare their results.

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Appendix

	2015-2016						2014-2015					
	Multiplicative technology			FDH technology			Multiplicative technology			FDH technology		
	DMI	**DMO**	**DHM**	**DMI**	**DMO**	**DHM**	**DMI**	**DMO**	**DHM**	**DMI**	**DMO**	**DHM**
Beijing	1.049	1.019	0.971	1.034	1.106	1.069	1.016	1.014	0.998	1.042	0.936	0.898
Tianjin	1.035	1.000	0.966	1.035	1.001	0.968	1.062	1.000	0.942	1.062	1.081	1.018
Hebei	1.036	1.000	0.966	1.036	1.049	1.012	1.051	1.000	0.951	1.051	0.958	0.911
Shanxi	1.039	0.997	0.960	1.060	1.076	1.015	0.998	1.015	1.017	1.025	0.983	0.959
Inner Mongolia	1.040	1.017	0.978	1.051	1.094	1.041	1.038	0.985	0.949	1.055	0.979	0.928
Liaoning	1.000	0.998	0.997	1.033	1.055	1.021	0.998	1.027	1.029	1.023	1.000	0.978
Jilin	1.023	1.000	0.978	1.042	1.052	1.010	1.024	1.000	0.976	1.036	0.968	0.935
Heilongjiang	1.036	1.011	0.976	1.026	1.034	1.008	1.022	1.014	0.992	1.013	0.974	0.961
Shanghai	1.046	1.000	0.956	1.046	0.982	0.939	1.036	1.000	0.966	1.036	0.933	0.901
Jiangsu	1.000	1.000	1.000	1.027	1.050	1.023	1.040	1.000	0.962	1.055	0.996	0.944
Zhejiang	1.072	1.000	0.933	1.055	1.074	1.018	1.097	1.000	0.911	1.055	1.025	0.972
Anhui	1.025	0.992	0.968	1.038	1.035	0.997	1.029	1.003	0.975	1.043	1.027	0.985
Fujian	1.009	1.014	1.004	1.026	1.055	1.028	1.038	0.992	0.956	1.015	0.987	0.972
Jiangxi	1.047	1.000	0.955	1.048	1.043	0.995	1.047	1.000	0.955	1.047	1.026	0.979
Shandong	1.034	1.000	0.967	1.034	1.087	1.052	1.017	1.000	0.983	1.017	0.964	0.948
Henan	1.050	1.000	0.952	1.050	1.026	0.978	1.044	1.000	0.958	1.044	1.008	0.966
Hubei	1.034	1.000	0.967	1.038	1.096	1.056	1.052	1.000	0.951	1.082	1.057	0.977
Hunan	1.038	1.000	0.964	1.038	1.057	1.018	1.055	1.000	0.948	1.055	1.047	0.993
Guangdong	1.068	1.000	0.936	1.068	1.046	0.980	1.054	1.000	0.949	1.054	0.966	0.917
Guangxi	1.050	1.000	0.952	1.046	1.041	0.995	1.065	1.000	0.939	1.057	0.987	0.933
Hainan	1.028	0.998	0.971	1.043	1.020	0.978	1.079	1.000	0.927	1.079	0.931	0.863
Chongqing	1.071	1.000	0.934	1.074	1.000	0.931	1.084	1.000	0.922	1.083	1.018	0.941
Sichuan	1.055	1.000	0.948	1.055	1.069	1.013	1.048	1.000	0.954	1.048	0.985	0.940
Guizhou	1.089	1.000	0.918	1.083	1.078	0.996	1.101	1.000	0.908	1.097	0.997	0.909
Yunnan	1.099	1.000	0.910	1.092	1.103	1.010	1.091	1.000	0.917	1.091	0.961	0.881
Tibet	1.043	1.000	0.958	1.043	0.941	0.902	1.124	1.000	0.890	1.124	1.144	1.018
Shaanxi	1.069	1.003	0.938	1.087	1.085	0.998	1.053	1.015	0.964	1.053	1.024	0.972
Gansu	1.046	1.000	0.956	1.019	1.066	1.046	1.030	1.000	0.971	1.008	1.036	1.028
Qinghai	1.030	1.000	0.971	1.038	1.066	1.026	1.048	1.000	0.954	1.046	1.044	0.997
Ningxia	1.048	1.000	0.954	1.063	1.100	1.035	1.020	1.000	0.980	1.022	0.950	0.929
Xinjiang	1.050	1.000	0.953	1.052	1.095	1.040	1.057	1.019	0.964	1.057	1.069	1.011

Table 6: Dynamical DHM-MT and DHM-FDH (2014-2016)

$\rho_{t(t+1)}$	2016-2017									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-1.813	-1.795	-1.158	-1.000	1.030	-8.137	124.191	11.548	9.059	1.005
Tianjin	-2.792	-1.900	-1.000	-1.278	1.030	-0.001	-1.085	0.130	-0.154	0.825
Hebei	-2.444	-3.031	-1.000	-2.792	1.033	0.031	1.567	-1.150	-1.146	0.957
Shanxi	-0.135	-0.239	-0.015	-0.194	1.231	0.243	0.348	0.342	0.613	1.183
Inner Mongolia	-0.184	-0.254	-0.054	-0.246	1.364	0.074	-0.678	0.465	0.689	1.236
Liaoning	-1.311	-1.677	0.232	-1.222	1.041	3.403	-2.031	4.997	7.271	1.012
Jilin	0.460	0.703	-1.000	0.568	0.971	-0.140	0.297	-0.960	-1.969	0.961
Heilongjiang	-0.340	-0.408	0.144	-0.676	1.149	0.021	-1.241	0.636	-0.126	1.115
Shanghai	-1.000	-1.470	-1.287	-1.092	1.038	-0.393	-0.677	-1.015	-0.933	0.938
Jiangsu	-2.159	-2.511	-1.000	-2.095	1.028	-0.785	-0.517	-1.102	-0.907	0.930
Zhejiang	-1.188	-1.425	-0.718	-1.535	1.052	-0.721	-1.223	-0.905	-2.282	0.910
Anhui	-2.988	-3.848	0.032	-3.535	1.023	-0.844	-1.964	-1.491	-1.119	0.932
Fujian	-0.481	-0.515	-0.397	-0.390	1.116	0.668	-0.443	0.624	0.856	1.049
Jiangxi	-1.411	-2.239	-0.958	-2.928	1.039	-0.104	-0.357	-1.000	-0.824	0.901
Shandong	-5.062	-5.876	-1.000	-5.094	1.015	-0.460	0.711	-0.865	-1.603	0.916
Henan	-4.723	-6.252	-0.608	-5.158	1.014	-0.169	0.034	-1.094	-0.719	0.925
Hubei	-11.522	-15.648	-1.000	-12.719	1.003	-0.118	1.762	-0.875	-1.054	0.927
Hunan	13.390	11.960	-1.000	10.913	0.994	-0.388	-1.089	-0.887	-1.041	0.945
Guangdong	-2.320	-3.181	-1.000	-2.216	1.026	-0.231	-0.185	-0.437	-1.000	0.881
Guangxi	-0.861	-1.376	-0.511	-1.374	1.053	-0.472	1.058	-0.754	-1.603	0.941
Hainan	-2.680	-4.086	-0.247	-2.397	1.017	-0.526	-11.478	-1.473	-4.139	0.962
Chongqing	-2.108	-3.394	-1.000	-2.900	1.027	-0.418	-0.964	-0.815	-1.151	0.901
Sichuan	-1.246	-2.391	-1.000	-2.044	1.041	-0.431	-0.949	-1.000	-0.998	0.908
Guizhou	-1.018	-1.480	-0.617	-1.162	1.092	-0.977	0.139	-1.013	-1.247	0.905
Yunnan	-1.159	-2.501	-0.797	-1.044	1.080	-0.403	0.139	-0.797	-1.108	0.899
Tibet	12.289	13.152	-1.000	8.830	0.988	-0.308	-1.000	0.088	-0.330	0.672
Shaanxi	-0.445	-0.443	-0.233	-0.361	1.208	2.073	2.237	5.528	7.613	1.018
Gansu	-1.427	-3.397	-0.796	-2.049	1.044	-0.363	-0.864	-1.022	-0.829	0.917
Qinghai	-1.620	-1.787	-1.000	-1.254	1.081	-0.212	-1.070	-0.305	-0.458	0.807
Ningxia	-1.840	-5.123	-1.000	-2.632	1.035	-0.572	3.695	-1.000	-0.123	0.907
Xinjiang	-0.640	-0.846	0.867	-1.294	1.052	-0.565	0.395	-0.501	-2.551	0.953

$\rho_{t+1(t)}$	2016-2017									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	9.680	9.586	6.185	5.340	0.994	10.830	-165.306	-15.371	-12.058	0.997
Tianjin	1.000	0.680	0.358	0.458	0.920	-0.019	-19.045	2.275	-2.698	0.989
Hebei	0.806	1.000	0.330	0.921	0.906	0.020	1.000	-0.734	-0.731	0.934
Shanxi	-0.215	-0.382	-0.024	-0.310	1.139	0.165	0.236	0.232	0.416	1.281
Inner Mongolia	-0.390	-0.538	-0.113	-0.522	1.158	0.054	-0.496	0.340	0.504	1.336
Liaoning	-2.153	-2.755	0.381	-2.007	1.025	0.542	-0.324	0.796	1.158	1.078
Jilin	0.817	1.247	-1.776	1.008	0.984	-0.469	1.000	-3.230	-6.623	0.988
Heilongjiang	-0.615	-0.739	0.261	-1.223	1.080	0.019	-1.106	0.567	-0.112	1.130
Shanghai	0.680	1.000	0.875	0.743	0.946	1.000	1.721	2.583	2.373	1.025
Jiangsu	1.061	1.234	0.491	1.029	0.946	1.057	0.697	1.485	1.221	1.055
Zhejiang	0.774	0.929	0.468	1.000	0.925	1.000	1.696	1.255	3.164	1.071
Anhui	1.276	1.644	-0.014	1.510	0.947	0.722	1.680	1.275	0.957	1.085
Fujian	-2.318	-2.484	-1.915	-1.879	1.023	0.338	-0.224	0.316	0.433	1.098
Jiangxi	0.482	0.765	0.327	1.000	0.894	0.349	1.206	3.375	2.782	1.032
Shandong	0.861	1.000	0.170	0.867	0.914	-1.024	1.584	-1.927	-3.571	0.961
Henan	0.755	1.000	0.097	0.825	0.919	5.180	-1.040	33.516	22.026	1.003
Hubei	0.819	1.112	0.071	0.904	0.954	-0.705	10.487	-5.208	-6.276	0.987
Hunan	1.024	0.915	-0.076	0.835	0.930	0.539	1.513	1.232	1.446	1.042
Guangdong	0.729	1.000	0.314	0.697	0.922	1.106	0.886	2.086	4.779	1.027
Guangxi	0.626	1.001	0.372	0.999	0.931	1.173	-2.630	1.875	3.984	1.025
Hainan	1.057	1.612	0.098	0.946	0.958	0.569	12.402	1.592	4.472	1.036
Chongqing	0.621	1.000	0.295	0.854	0.912	0.663	1.528	1.292	1.825	1.068
Sichuan	0.521	1.000	0.418	0.855	0.909	0.642	1.414	1.490	1.487	1.067
Guizhou	1.146	1.666	0.694	1.308	0.925	0.834	-0.119	0.865	1.065	1.125
Yunnan	0.538	1.162	0.370	0.485	0.847	0.764	-0.264	1.511	2.100	1.058
Tibet	0.934	1.000	-0.076	0.671	0.852	-3.508	-11.377	1.000	-3.753	0.966
Shaanxi	-4.471	-4.449	-2.341	-3.623	1.019	0.179	0.194	0.478	0.659	1.229
Gansu	0.472	1.125	0.263	0.678	0.879	0.636	1.512	1.788	1.451	1.051
Qinghai	0.975	1.076	0.602	0.755	0.879	0.721	3.629	1.034	1.552	1.065
Ningxia	0.509	1.417	0.277	0.728	0.882	-1.184	7.653	-2.071	-0.254	0.954
Xinjiang	0.558	0.737	-0.756	1.128	0.943	0.746	-0.522	0.661	3.370	1.037

Table 7: Dynamical deviation under Multiplicative technology (2016-2017)

$\rho_{t(t+1)}$	2016-2017									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+}1; 0, k_{t+1})$
Beijing	-1.813	-1.795	-1.158	-1.000	1.030	0.066	-1.000	-0.093	-0.073	0.570
Tianjin	-2.792	-1.900	-1.000	-1.278	1.030	-0.001	-1.000	0.119	-0.142	0.812
Hebei	-2.444	-3.031	-1.000	-2.792	1.033	0.027	1.362	-1.000	-0.996	0.951
Shanxi	-8.996	-15.967	-1.000	-12.979	1.003	-0.396	-0.567	-0.557	-1.000	0.902
Inner Mongolia	-3.440	-4.745	-1.000	-4.598	1.017	-0.107	0.985	-0.675	-1.000	0.864
Liaoning	5.658	7.240	-1.000	5.273	0.991	-0.468	0.279	-0.687	-1.000	0.917
Jilin	0.460	0.703	-1.000	0.568	0.971	-0.071	0.151	-0.488	-1.000	0.924
Heilongjiang	2.355	2.829	-1.000	4.683	0.980	-0.033	1.951	-1.000	0.198	0.933
Shanghai	-1.000	-1.470	-1.287	-1.092	1.038	-0.387	-0.667	-1.000	-0.919	0.937
Jiangsu	-2.159	-2.511	-1.000	-2.095	1.028	-0.712	-0.469	-1.000	-0.822	0.923
Zhejiang	-1.655	-1.985	-1.000	-2.137	1.037	-0.316	-0.536	-0.397	-1.000	0.806
Anhui	91.980	118.463	-1.000	108.829	0.999	-0.430	-1.000	-0.759	-0.570	0.871
Fujian	-1.234	-1.322	-1.019	-1.000	1.044	-0.781	0.517	-0.729	-1.000	0.960
Jiangxi	-1.474	-2.338	-1.000	-3.058	1.037	-0.104	-0.357	-1.000	-0.824	0.901
Shandong	-5.062	-5.876	-1.000	-5.094	1.015	-0.287	0.444	-0.539	-1.000	0.869
Henan	-7.772	-10.289	-1.000	-8.489	1.008	-0.155	0.031	-1.000	-0.657	0.918
Hubei	-11.522	-15.648	-1.000	-12.719	1.003	-0.112	1.671	-0.830	-1.000	0.923
Hunan	13.390	11.960	-1.000	10.913	0.994	-0.356	-1.000	-0.814	-0.956	0.940
Guangdong	-2.320	-3.181	-1.000	-2.216	1.026	-0.231	-0.185	-0.437	-1.000	0.881
Guangxi	-1.683	-2.691	-1.000	-2.687	1.027	-0.294	0.660	-0.471	-1.000	0.907
Hainan	-10.829	-16.511	-1.000	-9.684	1.004	-0.072	-1.559	-0.200	-0.562	0.753
Chongqing	-2.108	-3.394	-1.000	-2.900	1.027	-0.363	-0.838	-0.708	-1.000	0.887
Sichuan	-1.246	-2.391	-1.000	-2.044	1.041	-0.431	-0.949	-1.000	-0.998	0.908
Guizhou	-1.650	-2.399	-1.000	-1.883	1.056	-0.784	0.112	-0.812	-1.000	0.882
Yunnan	-1.454	-3.138	-1.000	-1.310	1.063	-0.364	0.126	-0.719	-1.000	0.888
Tibet	12.289	13.152	-1.000	8.830	0.988	-0.308	-1.000	0.088	-0.330	0.672
Shaanxi	-1.910	-1.901	-1.000	-1.548	1.045	-0.272	-0.294	-0.726	-1.000	0.873
Gansu	-1.793	-4.270	-1.000	-2.575	1.034	-0.356	-0.846	-1.000	-0.812	0.915
Qinghai	-1.620	-1.787	-1.000	-1.254	1.081	-0.199	-1.000	-0.285	-0.428	0.795
Ningxia	-1.840	-5.123	-1.000	-2.632	1.035	-0.572	3.695	-1.000	-0.123	0.907
Xinjiang	0.738	0.975	-1.000	1.492	0.957	-0.221	0.155	-0.196	-1.000	0.885

$\rho_{t+1(t)}$	2016-2017									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.990	0.639	0.552	0.947	1.000	-15.263	-1.419	-1.113	0.964
Tianjin	1.000	0.680	0.358	0.458	0.920	-0.008	-8.372	1.000	-1.186	0.975
Hebei	0.806	1.000	0.330	0.921	0.906	0.020	1.000	-0.734	-0.731	0.934
Shanxi	0.563	1.000	0.063	0.813	0.952	1.000	1.433	1.407	2.526	1.042
Inner Mongolia	0.725	1.000	0.211	0.969	0.924	0.224	-2.051	1.405	2.083	1.073
Liaoning	0.782	1.000	-0.138	0.728	0.935	-1.675	1.000	-2.460	-3.579	0.976
Jilin	0.655	1.000	-1.423	0.808	0.980	-0.469	1.000	-3.230	-6.623	0.988
Heilongjiang	0.503	0.604	-0.214	1.000	0.910	-0.017	1.000	-0.512	0.101	0.873
Shanghai	0.680	1.000	0.875	0.743	0.946	1.000	1.721	2.583	2.373	1.025
Jiangsu	0.860	1.000	0.398	0.834	0.934	1.518	1.000	2.132	1.753	1.038
Zhejiang	0.883	1.059	0.534	1.140	0.934	1.000	1.696	1.255	3.164	1.071
Anhui	0.776	1.000	-0.008	0.919	0.915	1.000	2.327	1.766	1.325	1.061
Fujian	0.933	1.000	0.771	0.757	0.945	-1.509	1.000	-1.410	-1.934	0.979
Jiangxi	0.482	0.765	0.327	1.000	0.894	1.000	3.450	9.659	7.962	1.011
Shandong	0.861	1.000	0.170	0.867	0.914	-0.646	1.000	-1.216	-2.254	0.939
Henan	0.755	1.000	0.097	0.825	0.919	-4.979	1.000	-32.220	-21.174	0.997
Hubei	0.736	1.000	0.064	0.813	0.949	-0.067	1.000	-0.497	-0.598	0.875
Hunan	1.000	0.893	-0.075	0.815	0.928	1.000	2.806	2.286	2.682	1.022
Guangdong	0.729	1.000	0.314	0.697	0.922	1.249	1.000	2.355	5.395	1.024
Guangxi	0.626	1.000	0.372	0.998	0.931	-0.446	1.000	-0.713	-1.515	0.937
Hainan	0.656	1.000	0.061	0.587	0.933	1.000	21.802	2.799	7.862	1.021
Chongqing	0.621	1.000	0.295	0.854	0.912	1.000	2.305	1.947	2.752	1.044
Sichuan	0.521	1.000	0.418	0.855	0.909	1.000	2.203	2.321	2.317	1.043
Guizhou	0.688	1.000	0.417	0.785	0.877	-7.011	1.000	-7.269	-8.947	0.986
Yunnan	0.463	1.000	0.319	0.417	0.825	-2.896	1.000	-5.727	-7.962	0.985
Tibet	0.934	1.000	-0.076	0.671	0.852	-3.508	-11.377	1.000	-3.753	0.966
Shaanxi	1.000	0.995	0.523	0.810	0.919	1.000	1.079	2.666	3.672	1.038
Gansu	0.420	1.000	0.234	0.603	0.865	1.000	2.378	2.812	2.283	1.032
Qinghai	0.906	1.000	0.560	0.702	0.871	1.000	5.033	1.434	2.153	1.047
Ningxia	0.359	1.000	0.195	0.514	0.837	-0.155	1.000	-0.271	-0.033	0.698
Xinjiang	0.495	0.654	-0.670	1.000	0.936	-1.429	1.000	-1.267	-6.455	0.981

Table 8: Dynamical deviation under FDH technology (2016-2017)

$\rho_{t(t+1)}$	2015-2016									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+}1; 0, k_{t+1})$
Beijing	-0.617	-0.462	-0.239	-0.609	1.081	-1.726	-0.534	-3.275	-4.919	0.964
Tianjin	-18.113	-22.958	-1.000	-11.539	1.003	-0.146	1.102	-0.711	-1.127	0.903
Hebei	8.444	10.921	-1.000	7.423	0.993	-0.364	0.248	-1.177	-0.920	0.903
Shanxi	-0.071	-0.433	-0.164	-0.150	1.261	0.216	0.165	0.768	0.598	1.169
Inner Mongolia	-0.110	-0.271	-0.059	-0.129	1.355	0.148	0.123	0.507	0.406	1.236
Liaoning	-1.227	-1.792	0.131	-1.596	1.040	-7.314	-1.836	-12.131	-16.850	0.994
Jilin	-12.472	-27.650	-1.000	-15.673	1.003	-0.387	-0.769	-1.206	-0.480	0.938
Heilongjiang	-0.149	-0.383	0.011	-0.249	1.149	0.701	-0.414	1.569	1.226	1.060
Shanghai	-1.000	-1.381	-1.435	-1.360	1.038	-0.105	1.239	-0.607	-1.000	0.862
Jiangsu	2.761	2.842	-1.000	2.420	0.972	-0.203	-1.079	-1.461	-1.763	0.952
Zhejiang	-1.194	-1.692	-0.510	-1.194	1.053	-0.708	-1.035	-1.467	-1.435	0.936
Anhui	-1.880	-2.408	-0.729	-2.188	1.024	-0.182	-0.485	-1.814	-1.055	0.966
Fujian	-0.220	-0.570	0.039	-0.105	1.102	0.555	-0.658	0.345	2.276	1.068
Jiangxi	-1.000	-2.090	-1.272	-1.815	1.031	-0.406	0.285	-0.720	-1.620	0.938
Shandong	-2.715	-4.553	-1.000	-3.417	1.012	-0.060	-0.678	-0.928	-1.404	0.892
Henan	-2.109	-4.105	-1.000	-3.315	1.019	-0.676	0.641	-0.995	-1.446	0.937
Hubei	-2.595	-3.625	-1.000	-3.115	1.016	-0.115	-0.602	-0.580	-1.238	0.874
Hunan	14.320	17.861	-1.000	15.863	0.996	-0.327	-0.759	-0.909	-1.069	0.924
Guangdong	-3.008	-5.324	-1.000	-3.150	1.021	-0.308	0.204	-0.624	-1.000	0.893
Guangxi	-1.505	-2.227	-0.277	-1.268	1.036	-0.171	0.706	-0.539	-1.963	0.938
Hainan	-3.256	-5.079	-1.000	-2.976	1.014	-0.926	1.324	-0.956	-2.083	0.948
Chongqing	-1.451	-2.491	-1.000	-1.922	1.042	-0.475	1.009	-1.000	-0.985	0.936
Sichuan	-1.000	-4.012	-1.203	-2.828	1.022	-0.317	-0.633	-0.763	-1.176	0.915
Guizhou	-1.109	-1.616	-0.462	-0.893	1.079	-0.588	-0.643	-0.537	-1.323	0.923
Yunnan	-1.216	-2.040	-0.776	-1.042	1.064	-0.587	-0.713	-0.774	-1.092	0.890
Tibet	0.563	1.891	-1.000	0.336	0.909	-0.151	1.502	-0.705	-1.000	0.785
Shaanxi	-0.392	-0.592	-0.282	-0.323	1.211	1.746	-2.066	2.500	3.779	1.034
Gansu	-2.191	-1.952	0.864	-1.783	1.029	-0.502	0.005	-1.508	-1.100	0.918
Qinghai	-1.000	8.929	7.007	0.760	0.991	-0.324	-0.867	-0.897	-1.093	0.914
Ningxia	-11.093	-18.026	-1.000	-11.205	1.006	-0.559	-0.633	-0.697	-1.000	0.885
Xinjiang	-1.087	-1.205	-1.252	-0.916	1.048	-1.097	-2.909	-0.930	-1.520	0.954

$\rho_{t+1(t)}$	2015-2016									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.013	0.759	0.392	1.000	0.953	1.010	0.313	1.917	2.879	1.065
Tianjin	0.789	1.000	0.044	0.503	0.937	-0.155	1.170	-0.755	-1.197	0.908
Hebei	0.773	1.000	-0.092	0.680	0.926	-1.470	1.000	-4.748	-3.709	0.975
Shanxi	-0.099	-0.606	-0.229	-0.210	1.180	0.107	0.082	0.381	0.297	1.370
Inner Mongolia	-0.170	-0.416	-0.091	-0.198	1.219	0.089	0.075	0.306	0.245	1.420
Liaoning	-1.118	-1.632	0.120	-1.453	1.044	0.336	0.084	0.557	0.774	1.132
Jilin	0.862	1.910	0.069	1.083	0.959	0.645	1.283	2.013	0.800	1.039
Heilongjiang	-0.434	-1.117	0.032	-0.725	1.049	0.286	-0.169	0.639	0.500	1.153
Shanghai	0.697	0.963	1.000	0.947	0.948	-0.358	4.240	-2.078	-3.421	0.958
Jiangsu	2.802	2.885	-1.015	2.457	0.972	0.503	2.681	3.629	4.379	1.020
Zhejiang	0.706	1.000	0.301	0.706	0.916	1.000	1.462	2.072	2.028	1.048
Anhui	4.005	5.131	1.552	4.662	0.989	0.098	0.262	0.980	0.570	1.066
Fujian	-0.379	-0.980	0.068	-0.180	1.058	0.402	-0.477	0.250	1.650	1.095
Jiangxi	0.495	1.036	0.630	0.899	0.940	1.612	-1.129	2.856	6.427	1.016
Shandong	0.596	1.000	0.220	0.750	0.947	0.234	2.635	3.606	5.457	1.030
Henan	0.514	1.000	0.244	0.808	0.925	64.481	-61.194	94.917	138.037	1.001
Hubei	0.808	1.129	0.312	0.970	0.950	0.367	1.922	1.850	3.951	1.043
Hunan	0.802	1.000	-0.056	0.888	0.924	0.450	1.043	1.247	1.468	1.059
Guangdong	0.565	1.000	0.188	0.592	0.895	3.473	-2.305	7.034	11.275	1.010
Guangxi	0.861	1.274	0.158	0.726	0.939	0.698	-2.874	2.195	7.994	1.016
Hainan	1.200	1.871	0.368	1.097	0.963	0.903	-1.291	0.932	2.031	1.056
Chongqing	0.617	1.060	0.425	0.818	0.909	0.902	-1.917	1.901	1.873	1.035
Sichuan	0.249	1.000	0.300	0.705	0.918	0.610	1.220	1.470	2.264	1.048
Guizhou	0.904	1.317	0.376	0.728	0.910	0.901	0.986	0.823	2.030	1.054
Yunnan	0.596	1.000	0.381	0.511	0.880	0.893	1.085	1.177	1.662	1.079
Tibet	0.297	1.000	-0.529	0.178	0.835	-0.100	1.000	-0.470	-0.666	0.696
Shaanxi	-1.579	-2.388	-1.138	-1.303	1.049	0.265	-0.314	0.380	0.574	1.249
Gansu	1.000	0.891	-0.395	0.814	0.940	11.435	-0.124	34.386	25.074	1.004
Qinghai	-0.140	1.249	0.981	0.106	0.935	0.414	1.107	1.146	1.396	1.073
Ningxia	0.807	1.312	0.073	0.815	0.916	0.935	1.059	1.167	1.673	1.076
Xinjiang	1.022	1.134	1.178	0.862	0.951	1.179	3.127	1.000	1.634	1.045

Table 9: Dynamical deviation under Multiplicative technology (2015-2016)

$\rho_{t(t+1)}$	2015-2016									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+}1; 0, k_{t+1})$
Beijing	-2.582	-1.935	-1.000	-2.548	1.019	-0.351	-0.109	-0.666	-1.000	0.834
Tianjin	-18.113	-22.958	-1.000	-11.539	1.003	-0.129	0.978	-0.631	-1.000	0.891
Hebei	8.444	10.921	-1.000	7.423	0.993	-0.310	0.211	-1.000	-0.781	0.887
Shanxi	-1.000	-6.101	-2.306	-2.112	1.017	-0.281	-0.215	-1.000	-0.779	0.887
Inner Mongolia	-1.865	-4.570	-1.000	-2.172	1.018	-0.326	-0.273	-1.119	-0.896	0.908
Liaoning	9.334	13.626	-1.000	12.135	0.995	-0.434	-0.109	-0.720	-1.000	0.909
Jilin	-12.472	-27.650	-1.000	-15.673	1.003	-0.321	-0.637	-1.000	-0.398	0.926
Heilongjiang	13.623	35.106	-1.000	22.780	0.998	-0.447	0.264	-1.000	-0.782	0.913
Shanghai	-1.000	-1.381	-1.435	-1.360	1.038	-0.105	1.239	-0.607	-1.000	0.862
Jiangsu	2.761	2.842	-1.000	2.420	0.972	-0.115	-0.612	-0.829	-1.000	0.916
Zhejiang	-2.344	-3.319	-1.000	-2.344	1.027	-0.483	-0.706	-1.000	-0.979	0.908
Anhui	-2.580	-3.305	-1.000	-3.003	1.018	-0.100	-0.267	-1.000	-0.582	0.939
Fujian	5.604	14.488	-1.000	2.665	0.996	-0.244	0.289	-0.152	-1.000	0.861
Jiangxi	-1.000	-2.090	-1.272	-1.815	1.031	-0.251	0.176	-0.444	-1.000	0.902
Shandong	-2.715	-4.553	-1.000	-3.417	1.012	-0.043	-0.483	-0.661	-1.000	0.852
Henan	-2.109	-4.105	-1.000	-3.315	1.019	-0.467	0.443	-0.688	-1.000	0.910
Hubei	-2.595	-3.625	-1.000	-3.115	1.016	-0.093	-0.486	-0.468	-1.000	0.846
Hunan	14.320	17.861	-1.000	15.863	0.996	-0.306	-0.710	-0.850	-1.000	0.919
Guangdong	-3.008	-5.324	-1.000	-3.150	1.021	-0.308	0.204	-0.624	-1.000	0.893
Guangxi	-5.433	-8.041	-1.000	-4.579	1.010	-0.087	0.360	-0.275	-1.000	0.882
Hainan	-3.256	-5.079	-1.000	-2.976	1.014	-0.445	0.635	-0.459	-1.000	0.896
Chongqing	-1.451	-2.491	-1.000	-1.922	1.042	-0.475	1.009	-1.000	-0.985	0.936
Sichuan	-1.000	-4.012	-1.203	-2.828	1.022	-0.269	-0.539	-0.649	-1.000	0.900
Guizhou	-2.401	-3.500	-1.000	-1.934	1.036	-0.444	-0.486	-0.406	-1.000	0.899
Yunnan	-1.567	-2.628	-1.000	-1.342	1.050	-0.538	-0.653	-0.709	-1.000	0.881
Tibet	0.563	1.891	-1.000	0.336	0.909	-0.151	1.502	-0.705	-1.000	0.785
Shaanxi	-1.388	-2.099	-1.000	-1.145	1.055	-0.462	0.547	-0.662	-1.000	0.880
Gansu	2.535	2.258	-1.000	2.062	0.976	-0.333	0.004	-1.000	-0.729	0.879
Qinghai	-1.000	8.929	7.007	0.760	0.991	-0.297	-0.793	-0.821	-1.000	0.907
Ningxia	-11.093	-18.026	-1.000	-11.205	1.006	-0.559	-0.633	-0.697	-1.000	0.885
Xinjiang	-1.186	-1.316	-1.367	-1.000	1.044	-0.377	-1.000	-0.320	-0.522	0.872

$\rho_{t+1(t)}$	2015-2016									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.749	0.387	0.987	0.953	3.231	1.000	6.132	9.208	1.020
Tianjin	0.789	1.000	0.044	0.503	0.937	-0.132	1.000	-0.645	-1.023	0.894
Hebei	0.773	1.000	-0.092	0.680	0.926	-1.470	1.000	-4.748	-3.709	0.975
Shanxi	0.164	1.000	0.378	0.346	0.904	1.308	1.000	4.660	3.630	1.026
Inner Mongolia	0.408	1.000	0.219	0.475	0.921	0.373	0.312	1.280	1.026	1.088
Liaoning	0.685	1.000	-0.073	0.891	0.932	3.984	1.000	6.609	9.180	1.011
Jilin	0.451	1.000	0.036	0.567	0.923	1.000	1.988	3.119	1.240	1.025
Heilongjiang	0.388	1.000	-0.028	0.649	0.948	-1.691	1.000	-3.786	-2.959	0.976
Shanghai	0.697	0.963	1.000	0.947	0.948	-0.084	1.000	-0.490	-0.807	0.832
Jiangsu	0.971	1.000	-0.352	0.852	0.922	1.000	5.328	7.214	8.704	1.010
Zhejiang	0.765	1.083	0.326	0.765	0.922	1.000	1.462	2.072	2.028	1.048
Anhui	0.781	1.000	0.303	0.909	0.944	1.000	2.661	9.953	5.790	1.006
Fujian	0.387	1.000	-0.069	0.184	0.946	-0.843	1.000	-0.524	-3.458	0.958
Jiangxi	0.478	1.000	0.608	0.868	0.938	-1.428	1.000	-2.530	-5.693	0.982
Shandong	0.596	1.000	0.220	0.750	0.947	1.000	11.264	15.415	23.325	1.007
Henan	0.514	1.000	0.244	0.808	0.925	-1.054	1.000	-1.551	-2.256	0.959
Hubei	0.716	1.000	0.276	0.859	0.944	1.000	5.230	5.034	10.753	1.016
Hunan	0.802	1.000	-0.056	0.888	0.924	1.000	2.320	2.775	3.266	1.026
Guangdong	0.565	1.000	0.188	0.592	0.895	-1.507	1.000	-3.051	-4.891	0.977
Guangxi	0.676	1.000	0.124	0.569	0.923	-0.243	1.000	-0.764	-2.782	0.956
Hainan	0.641	1.000	0.197	0.586	0.932	-0.700	1.000	-0.722	-1.574	0.932
Chongqing	0.583	1.000	0.402	0.772	0.904	-0.471	1.000	-0.992	-0.977	0.936
Sichuan	0.249	1.000	0.300	0.705	0.918	1.000	2.000	2.410	3.712	1.029
Guizhou	0.686	1.000	0.286	0.553	0.884	1.094	1.198	1.000	2.465	1.044
Yunnan	0.596	1.000	0.381	0.511	0.880	1.000	1.215	1.318	1.860	1.071
Tibet	0.297	1.000	-0.529	0.178	0.835	-0.100	1.000	-0.470	-0.666	0.696
Shaanxi	0.661	1.000	0.476	0.546	0.893	1.644	-1.945	2.354	3.558	1.037
Gansu	1.000	0.891	-0.395	0.814	0.940	-91.996	1.000	-276.647	-201.728	0.999
Qinghai	-0.112	1.000	0.785	0.085	0.919	1.000	2.674	2.769	3.371	1.030
Ningxia	0.615	1.000	0.055	0.622	0.891	1.000	1.133	1.248	1.790	1.071
Xinjiang	0.868	0.963	1.000	0.732	0.943	1.179	3.127	1.000	1.634	1.045

Table 10: Dynamical deviation under FDH technology (2015-2016)

$\rho_{t(t+1)}$	2014-2015									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+}1; 0, k_{t+1})$
Beijing	-0.904	-0.938	-0.249	-0.229	1.074	0.473	-3.514	0.542	-0.262	1.045
Tianjin	-1.672	-1.497	-1.085	-1.000	1.046	-0.205	-1.364	-0.133	-0.120	0.901
Hebei	-4.111	-6.269	-1.000	-4.188	1.014	-1.150	1.479	-0.487	4.985	0.978
Shanxi	-0.021	-0.245	0.012	-0.152	1.236	-0.054	-0.214	-0.088	0.098	1.337
Inner Mongolia	-0.118	-0.284	-0.133	-0.139	1.309	0.023	-0.346	-0.050	-0.129	1.358
Liaoning	-0.344	-0.566	0.024	-0.520	1.088	0.069	-0.157	0.166	0.035	1.111
Jilin	-6.331	-5.767	-1.000	-2.530	1.010	-0.318	-3.621	0.774	-0.635	1.023
Heilongjiang	-0.113	-0.297	0.205	-0.391	1.150	-0.263	-0.579	0.283	-0.244	1.196
Shanghai	-1.298	-2.129	-1.000	-1.976	1.023	-0.416	3.159	-0.792	-1.000	0.938
Jiangsu	-1.974	-2.651	-1.000	-1.810	1.030	-0.840	1.334	-1.154	-1.111	0.953
Zhejiang	-0.986	-1.173	-0.350	-1.245	1.086	-1.000	0.022	-0.896	-0.660	0.951
Anhui	-1.353	-2.360	-0.590	-2.070	1.029	0.182	1.520	-1.034	-3.642	0.975
Fujian	-0.327	-0.505	0.227	-0.450	1.114	0.011	-0.691	-0.207	0.417	1.102
Jiangxi	-1.000	-2.180	-1.421	-1.987	1.029	0.324	0.415	-0.606	-1.815	0.964
Shandong	11.222	14.403	-1.000	15.613	0.998	1.781	6.764	-1.038	0.134	0.987
Henan	-3.682	-5.540	-1.000	-4.904	1.013	-0.405	1.347	-1.049	-1.804	0.965
Hubei	-3.568	-6.423	-1.000	-3.661	1.021	-0.128	-0.016	-0.589	-1.283	0.919
Hunan	7.941	5.948	-1.000	7.151	0.985	-0.346	-0.965	-0.935	-1.063	0.937
Guangdong	-2.609	-4.197	-1.000	-3.530	1.020	-0.160	2.230	-0.614	-1.000	0.946
Guangxi	-1.137	-1.683	-0.518	-1.231	1.052	-0.869	4.824	0.132	-3.003	0.985
Hainan	-2.066	-2.639	-1.000	-3.101	1.038	-0.458	4.230	-1.177	-1.769	0.939
Chongqing	-0.941	-2.128	-1.229	-1.824	1.053	-0.765	0.475	-1.000	-0.972	0.933
Sichuan	-1.000	-7.449	-2.932	-5.559	1.011	-0.425	2.056	-0.657	-1.233	0.963
Guizhou	-1.209	-1.609	-0.816	-0.981	1.080	-0.537	1.731	0.302	-1.565	0.968
Yunnan	-1.000	-2.203	-1.535	-1.013	1.056	-0.723	2.350	-0.416	-1.159	0.936
Tibet	-1.611	-2.689	-1.000	-2.572	1.065	-0.150	-0.923	-1.000	-0.144	0.791
Shaanxi	-0.189	-0.346	-0.165	-0.299	1.226	-0.004	-0.004	0.412	0.313	1.126
Gansu	-2.398	-3.027	2.068	-2.602	1.016	-0.544	0.045	-1.190	-2.245	0.968
Qinghai	-1.785	-0.920	-1.087	-1.329	1.034	-0.090	-1.708	0.417	0.065	0.936
Ningxia	2.797	3.078	-1.000	1.883	0.979	-0.282	0.143	-1.000	7.580	0.984
Xinjiang	-0.514	-0.763	-0.522	-0.573	1.091	-6.937	-21.825	-9.213	-13.218	0.995

$\rho_{t+1(t)}$	2014-2015									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	-4.471	-4.640	-1.232	-1.132	1.014	0.363	-2.699	0.417	-0.201	1.060
Tianjin	1.000	0.895	0.649	0.598	0.928	1.544	10.263	1.002	0.907	1.014
Hebei	0.656	1.000	0.160	0.668	0.917	-0.279	0.358	-0.118	1.208	0.912
Shanxi	-0.023	-0.263	0.013	-0.164	1.217	-0.070	-0.279	-0.115	0.127	1.249
Inner Mongolia	-0.139	-0.336	-0.158	-0.164	1.255	0.028	-0.416	-0.060	-0.155	1.290
Liaoning	-0.622	-1.026	0.044	-0.941	1.048	0.122	-0.280	0.295	0.062	1.061
Jilin	1.597	1.455	0.252	0.638	0.962	0.093	1.060	-0.226	0.186	0.926
Heilongjiang	-0.204	-0.535	0.369	-0.703	1.081	-0.672	-1.481	0.722	-0.623	1.072
Shanghai	0.610	1.000	0.470	0.928	0.954	-0.378	2.866	-0.718	-0.907	0.932
Jiangsu	1.172	1.574	0.594	1.075	0.952	1.080	-1.715	1.484	1.429	1.038
Zhejiang	0.792	0.942	0.281	1.000	0.902	1.242	-0.027	1.113	0.819	1.041
Anhui	1.159	2.021	0.505	1.773	0.967	-0.149	-1.244	0.846	2.981	1.031
Fujian	-0.722	-1.117	0.502	-0.995	1.050	0.013	-0.819	-0.246	0.495	1.086
Jiangxi	0.468	1.020	0.665	0.930	0.940	1.714	2.199	-3.206	-9.609	0.993
Shandong	0.719	0.923	-0.064	1.000	0.964	0.357	1.356	-0.208	0.027	0.939
Henan	0.665	1.000	0.181	0.885	0.930	-0.567	1.887	-1.470	-2.527	0.975
Hubei	0.945	1.701	0.265	0.969	0.923	0.448	0.057	2.060	4.484	1.025
Hunan	1.000	0.749	-0.126	0.901	0.885	0.465	1.297	1.256	1.429	1.050
Guangdong	0.622	1.000	0.238	0.841	0.919	-0.174	2.421	-0.666	-1.086	0.950
Guangxi	0.750	1.109	0.341	0.812	0.926	-0.815	4.526	0.124	-2.817	0.984
Hainan	0.671	0.856	0.325	1.006	0.892	0.980	-9.044	2.517	3.782	1.030
Chongqing	0.443	1.001	0.578	0.858	0.896	1.683	-1.046	2.200	2.139	1.032
Sichuan	0.134	1.000	0.394	0.746	0.921	1.610	-7.780	2.486	4.668	1.010
Guizhou	0.800	1.065	0.540	0.649	0.891	-0.788	2.540	0.443	-2.297	0.978
Yunnan	0.454	1.000	0.697	0.460	0.887	11.858	-38.518	6.824	18.993	1.004
Tibet	0.599	1.000	0.372	0.957	0.844	1.043	6.414	6.952	1.000	1.034
Shaanxi	-0.535	-0.981	-0.468	-0.846	1.075	-0.002	-0.003	0.275	0.209	1.194
Gansu	0.917	1.158	-0.791	0.995	0.958	1.440	-0.118	3.149	5.942	1.012
Qinghai	1.000	0.515	0.609	0.744	0.942	-0.215	-4.095	1.000	0.156	0.973
Ningxia	0.954	1.050	-0.341	0.642	0.941	-0.037	0.019	-0.132	1.000	0.888
Xinjiang	0.792	1.178	0.806	0.884	0.945	0.755	2.377	1.003	1.440	1.043

Table 11: Dynamical deviation under Multiplicative technology (2014-2015)

$\rho_{t(t+1)}$	2014-2015									
	x_1	x_2	x_3	x_4	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	y_1	y_2	y_3	y_4	$M_t(x_t, y_{t+}1; 0, k_{t+1})$
Beijing	-3.950	-4.099	-1.089	-1.000	1.016	-0.871	6.478	-1.000	0.483	0.976
Tianjin	-1.672	-1.497	-1.085	-1.000	1.046	-0.150	-1.000	-0.098	-0.088	0.867
Hebei	-4.111	-6.269	-1.000	-4.188	1.014	-1.000	1.286	-0.424	4.334	0.975
Shanxi	1.802	20.979	-1.000	13.045	0.998	0.552	2.192	0.901	-1.000	0.972
Inner Mongolia	-1.000	-2.413	-1.133	-1.178	1.032	-1.000	14.811	2.138	5.537	0.993
Liaoning	14.215	23.438	-1.000	21.507	0.998	-0.414	0.948	-1.000	-0.212	0.983
Jilin	-6.331	-5.767	-1.000	-2.530	1.010	0.411	4.680	-1.000	0.821	0.983
Heilongjiang	0.553	1.452	-1.000	1.906	0.972	0.930	2.051	-1.000	0.863	0.951
Shanghai	-1.298	-2.129	-1.000	-1.976	1.023	-0.416	3.159	-0.792	-1.000	0.938
Jiangsu	-1.974	-2.651	-1.000	-1.810	1.030	-0.728	1.156	-1.000	-0.963	0.946
Zhejiang	-2.821	-3.354	-1.000	-3.562	1.029	-1.000	0.022	-0.896	-0.660	0.951
Anhui	-2.294	-4.001	-1.000	-3.509	1.017	0.050	0.417	-0.284	-1.000	0.912
Fujian	1.440	2.227	-1.000	1.984	0.976	-0.027	1.656	0.497	-1.000	0.960
Jiangxi	-1.000	-2.180	-1.421	-1.987	1.029	0.178	0.229	-0.334	-1.000	0.936
Shandong	11.222	14.403	-1.000	15.613	0.998	1.716	6.515	-1.000	0.129	0.987
Henan	-3.682	-5.540	-1.000	-4.904	1.013	-0.224	0.747	-0.582	-1.000	0.938
Hubei	-3.568	-6.423	-1.000	-3.661	1.021	-0.100	-0.013	-0.459	-1.000	0.897
Hunan	7.941	5.948	-1.000	7.151	0.985	-0.326	-0.908	-0.879	-1.000	0.933
Guangdong	-2.609	-4.197	-1.000	-3.530	1.020	-0.160	2.230	-0.614	-1.000	0.946
Guangxi	-2.196	-3.249	-1.000	-2.377	1.026	-0.289	1.606	0.044	-1.000	0.956
Hainan	-2.066	-2.639	-1.000	-3.101	1.038	-0.259	2.391	-0.665	-1.000	0.894
Chongqing	-1.000	-2.261	-1.306	-1.938	1.050	-0.765	0.475	-1.000	-0.972	0.933
Sichuan	-1.000	-7.449	-2.932	-5.559	1.011	-0.345	1.667	-0.533	-1.000	0.954
Guizhou	-1.481	-1.970	-1.000	-1.202	1.064	-0.343	1.106	0.193	-1.000	0.950
Yunnan	-1.000	-2.203	-1.535	-1.013	1.056	-0.624	2.028	-0.359	-1.000	0.926
Tibet	-1.611	-2.689	-1.000	-2.572	1.065	-0.150	-0.923	-1.000	-0.144	0.791
Shaanxi	-1.143	-2.095	-1.000	-1.807	1.034	0.009	0.010	-1.000	-0.759	0.952
Gansu	1.159	1.464	-1.000	1.258	0.967	-0.242	0.020	-0.530	-1.000	0.930
Qinghai	-1.941	-1.000	-1.182	-1.445	1.031	-0.053	-1.000	0.244	0.038	0.893
Ningxia	2.797	3.078	-1.000	1.883	0.979	-0.282	0.143	-1.000	7.580	0.984
Xinjiang	-1.000	-1.487	-1.017	-1.116	1.046	-0.318	-1.000	-0.422	-0.606	0.904

$\rho_{t+1(t)}$	2014-2015									
	x_1	x_2	x_3	x_4	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	y_1	y_2	y_3	y_4	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	0.964	1.000	0.266	0.244	0.936	-0.134	1.000	-0.154	0.075	0.856
Tianjin	1.000	0.895	0.649	0.598	0.928	1.703	11.321	1.106	1.000	1.013
Hebei	0.656	1.000	0.160	0.668	0.917	-0.231	0.297	-0.098	1.000	0.895
Shanxi	0.086	1.000	-0.048	0.622	0.950	0.252	1.000	0.411	-0.456	0.940
Inner Mongolia	0.414	1.000	0.470	0.488	0.926	-0.144	2.137	0.308	0.799	0.952
Liaoning	0.607	1.000	-0.043	0.918	0.953	-0.436	1.000	-1.054	-0.223	0.984
Jilin	1.000	0.911	0.158	0.400	0.940	0.088	1.000	-0.214	0.175	0.921
Heilongjiang	0.290	0.762	-0.525	1.000	0.947	0.454	1.000	-0.488	0.421	0.902
Shanghai	0.610	1.000	0.470	0.928	0.954	-0.132	1.000	-0.251	-0.317	0.817
Jiangsu	0.745	1.000	0.377	0.683	0.925	-0.630	1.000	-0.865	-0.833	0.938
Zhejiang	1.055	1.254	0.374	1.332	0.925	-45.394	1.000	-40.690	-29.949	0.999
Anhui	0.573	1.000	0.250	0.877	0.935	0.120	1.000	-0.680	-2.396	0.963
Fujian	0.646	1.000	-0.449	0.891	0.947	-0.016	1.000	0.300	-0.604	0.935
Jiangxi	0.459	1.000	0.652	0.912	0.939	0.780	1.000	-1.458	-4.370	0.985
Shandong	0.719	0.923	-0.064	1.000	0.964	0.263	1.000	-0.153	0.020	0.918
Henan	0.665	1.000	0.181	0.885	0.930	-0.301	1.000	-0.779	-1.339	0.954
Hubei	0.556	1.000	0.156	0.570	0.873	7.820	1.000	35.959	78.278	1.001
Hunan	1.000	0.749	-0.126	0.901	0.885	1.000	2.786	2.699	3.070	1.023
Guangdong	0.622	1.000	0.238	0.841	0.919	-0.072	1.000	-0.275	-0.448	0.883
Guangxi	0.676	1.000	0.308	0.732	0.919	-0.180	1.000	0.027	-0.623	0.931
Hainan	0.666	0.851	0.322	1.000	0.891	-0.114	1.054	-0.293	-0.441	0.776
Chongqing	0.442	1.000	0.578	0.857	0.896	-1.609	1.000	-2.104	-2.045	0.967
Sichuan	0.134	1.000	0.394	0.746	0.921	-0.207	1.000	-0.320	-0.600	0.925
Guizhou	0.751	1.000	0.508	0.610	0.884	-0.310	1.000	0.174	-0.904	0.945
Yunnan	0.454	1.000	0.697	0.460	0.887	-0.308	1.000	-0.177	-0.493	0.856
Tibet	0.599	1.000	0.372	0.957	0.844	1.043	6.414	6.952	1.000	1.034
Shaanxi	0.546	1.000	0.477	0.862	0.932	0.841	1.000	-98.347	-74.682	0.999
Gansu	0.792	1.000	-0.683	0.860	0.952	-12.184	1.000	-26.646	-50.282	0.999
Qinghai	1.000	0.515	0.609	0.744	0.942	-0.215	-4.095	1.000	0.156	0.973
Ningxia	0.909	1.000	-0.325	0.612	0.938	-0.037	0.019	-0.132	1.000	0.888
Xinjiang	0.673	1.000	0.684	0.751	0.936	1.000	3.146	1.328	1.906	1.033

Table 12: Dynamical deviation under FDH technology (2014-2015)