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# An exponential analysis of total factor productivity

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## Abstract

As an important economic measure, the total factor productivity indices can be employed to evaluate the performance of decision-making units over time. These indices are usually based on multiplicative or additive distance functions estimated by either parametric or non parametric approaches. Under a non parametric analytic framework, this paper introduces a multiplicative directional productivity measure: the Directional Hicks-Moorsteen (DHM) indicator. A dynamical combination of multiplicative directional distance functions is introduced and non convex production technologies are assumed to estimate the distance functions. In an empirical illustration, the proposed model is applied to display productivity changes among Chinese provincial public hospitals over the period 2014-2018. The results indicate that a consistent outcome is obtained under the multiplicative technology and the production technology of free disposal hull while more productivity gains are observed under free disposal hull technology.

**Keywords:** Exponential Efficiency Index, Non Convexity, Total Factor Productivity .

**JEL:** C61, D24

# 1 Introduction

The scarcity of economic factors and the technological limitations narrow the production possibility set of the firms. In the context of multiple inputs-outputs production processes, additive and multiplicative distance functions have become standard tools to analyze technical efficiency of the firms (Briec, 1997; Chambers et al., 1996, 1998; Debreu, 1951; Färe et al., 1985; Farrell, 1957; Luenberger, 1992ab; Shephard, 1970). These efficiency indices completely characterize the production technology<sup>1</sup>. Unlike usual approaches estimating production technology by Shephard (Shephard, 1970) or directional distance function (Chambers et al., 1996), we focus on the Multiplicative Directional Distance Function (MDDF) as functional characterization of the production process (Mehdiloozad et al., 2014; Peyrache and Coelli, 2009). The MDDF efficiency index encompasses usual multiplicative distance functions (Debreu, 1951; Färe et al., 1985; Farrell, 1957; Shephard, 1970) as special cases (Peyrache and Coelli, 2009). In addition, the logarithmic transformation of the MDDF inherits the basic structure of the directional distance function (Briec, 1997; Chambers et al., 1996, 1998; Luenberger, 1992ab)<sup>2</sup>. Moreover, the MDDF permits to estimate efficiency of firms under non convex technologies allowing to take into account of strictly increasing marginal products.

The combination of distance functions over periods allows to define Total Factor Productivity (TFP) change. TFP indicators have been widely studied in the literature (Hulten, 2001). According to the functional characterization of the production process, two main approaches hold. First, in the context of continuous-time, derivatives of the production functions display Solow's (1957) productivity change. Second, difference and ratio of distance functions define additive and multiplicative TFP measures over consecutive time periods (Bjurek, 1996; Briec and Kerstens, 2004; Chambers, 2002; Färe et al., 1994)<sup>3</sup>. In this paper, we estimate a directional TFP index through a combination of multiplicative directional distance function. Specifically, the proposed Directional Hicks-Moorsteen (DHM) productivity index inherits the structure of the multiplicative Hicks-Moorsteen productivity measure (Bjurek, 1996; Diewert, 1992ab). Moreover, the DHM productivity index is defined in dynamical context, displaying adjustment paths of TFP variation. Hence, this paper proposes to evaluate the productivity change over periods through a non linear efficiency measure allowing to take into account for strictly increasing marginal products. In addition, the DHM avoids infeasibilities as it inherits the structure of the Hicks-Moorsteen productivity index (Briec and Kerstens, 2011). Through a non parametric framework using both data envelopment analysis (DEA) and free disposal hull (FDH) methods, this paper provides an empirical illustration on the productivity of medical institutions. The dynamic DHM productivity is estimated

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<sup>1</sup>In the case of multiple inputs-outputs, additive and multiplicative distance functions replace production functions as functional characterisation of the production technology (Chambers and Färe, 2020).

<sup>2</sup>Remark that the directional distance function (Chambers et al., 1996) is the benefit function translated in the production economics scheme. The benefit function was introduced to evaluate social welfare (Luenberger, 1992b; Briec et al., 2021).

<sup>3</sup>Comparisons of difference- and ratio-based TFP indicators are defined in Chambers (1998, 2002) and Diewert (2005).

for 31 Chinese provincial hospitals, through linear and non linear production processes.

The remainder of the paper consists of the following contents. Section 2 introduces some theoretical preliminaries. The production technology along with the efficiency and productivity indices are displayed in this section. The dynamical multiplicative directional productivity measure is introduced in the Section 3. Section 4 defines non parametric estimation procedures for the dynamical efficiency and productivity indices under linear and non linear production processes. The empirical illustration allows to estimate the dynamical Hicks-Moorsteen productivity measures for a sample of 31 provincial Chinese hospitals over the period 2014-2018. The main empirical outcomes are presented in the Section 5. Finally, Section 6 discusses and concludes.

## 2 Background

This section presents the axioms of the production set and the efficiency measure considered throughout the paper.

### 2.1 Production technology: definition and properties

Let  $x_t \in \mathbb{R}_+^n$  be the input vector allowing to produce the output vector  $y_t \in \mathbb{R}_+^m$  such that the input vector is composed of  $i \in [n]$  elements and the output vector contains  $j \in [m]$  elements with  $[n] = \text{Card}(x_t)$  and  $[m] = \text{Card}(y_t)$ . The definition and properties of the production set are introduced below.

The production technology is defined as,

$$T_t = \{(x_t, y_t) \in \mathbb{R}_+^{n+m} : x_t \text{ can produce } y_t\}. \quad (2.1)$$

Notice that the production set (2.1) can be characterized by the output  $P_t : \mathbb{R}_+^n \mapsto 2^{\mathbb{R}_+^m}$  or the input  $L_t : \mathbb{R}_+^m \mapsto 2^{\mathbb{R}_+^n}$ , correspondences such that:

$$P_t(x_t) = \{y_t \in \mathbb{R}_+^m : (x_t, y_t) \in T_t\}, \quad (2.2)$$

and

$$L_t(y_t) = \{x_t \in \mathbb{R}_+^n : (x_t, y_t) \in T_t\}. \quad (2.3)$$

In such case, the following statement holds:

$$x_t \in L_t(y_t) \Leftrightarrow (x_t, y_t) \in T_t \Leftrightarrow y_t \in P_t(x_t). \quad (2.4)$$

Assume that the production technology satisfies the following regular properties (Färe et al., 1985):

*T1:*  $(0, 0) \in T_t$ ,  $(0, y_t) \in T_t \Rightarrow y_t = 0$ .

*T2:*  $T_t(y_t) = \{(u_t, v_t) \in T_t : v_t \leq y_t\}$  is bounded for all  $y_t \in \mathbb{R}_+^m$ .

*T3:*  $T_t$  is closed.

*T4:*  $\forall (x_t, y_t) \in T_t \wedge \forall (u_t, v_t) \in \mathbb{R}_+^n \times \mathbb{R}_+^m$  if  $(x_t, -y_t) \leq (u_t, -v_t)$  then  $(u_t, v_t) \in T_t$ .

The above axioms define a free disposal production set such that (T1) means that there is no free lunch, (T2) and (T3) postulate that the production technology is compact and (T4) notifies that the production set is free disposable in both input and output dimensions. Remark that the convexity property is not imposed.

## 2.2 Multiplicative efficiency measure

The next result presents the efficiency measure used throughout this paper namely, the multiplicative directional distance function (Mehdiloozad et al., 2014; Peyrache and Coelli, 2009)

**Definition 2.1** For any  $(x_t, y_t) \in \mathbb{R}_+^{n+m}$  and any directional vector  $g_t = (h_t, k_t) \in \mathbb{R}_+^{n+m}$ , the function

$$M(x_t, y_t; h_t, k_t) = \sup_{\delta} \left\{ \delta : (\delta^{-h_t} x_t, \delta^{k_t} y_t) \in T_t \right\} \quad (2.5)$$

is the multiplicative directional distance function.

Remark that this efficiency measure has a multiplicative formulation and is non linear.

Assume that the Multiplicative Directional Distance Function (MDDF) satisfies the properties below (Peyrache and Coelli, 2009):

D1:  $M(x_t, y_t; h_t, k_t)$  fully characterizes the production set.

D2:  $M(x_t, y_t; h_t, k_t)$  is equal to 1 when the observation belongs to the efficient frontier.

D3:  $M(x_t, y_t; h_t, k_t)$  is almost homogeneous of degree  $(-1)$  in  $(x_t, y_t)$ .

D4:  $M(x_t, y_t; h_t, k_t)$  is homogeneous of degree  $(-1)$  in  $g_t$ .

D5:  $M(x_t, y_t; h_t, k_t)$  is invariant with respect to the unit of measurement.

Also consider that there exists a strictly positive production set  $T_t^{++}$  defined as:

$$T_t^{++} = \{(x_t, y_t) \in \mathbb{R}_+^{n+m} : x_t \text{ can produce } y_t\}. \quad (2.6)$$

Under this production technology, a logarithmic formulation of the MDDF is provided as below:

$$\ln(M(x_t, y_t; h_t, k_t)) = \sup_{\delta} \left\{ \delta : (\ln(x_t) - h_t \ln(\delta), \ln(y_t) + k_t \ln(\delta)) \in \ln(T_t^{++}) \right\}. \quad (2.7)$$

Note that the logarithmic transformation of the MDDF is an additive graph efficiency measure. Moreover, this functional representation of the MDDF is structurally similar to the directional distance function (Luenberger, 1992ab; Chambers et al., 1996). Indeed, through the logarithmic formulation, the MDDF becomes a log-additive and log-linear distance function.

The MDDF distance function encompasses as special cases the traditional input and output distance functions (Peyrache and Coelli, 2009). Indeed, the input oriented Debreu-Farrell efficiency measure (Debreu, 1951; Farrell, 1957; Shephard, 1970) can be retrieved when  $g_t = (1, 0)$  and, the output oriented one when  $g_t = (0, 1)$ . In addition, the hyperbolic distance function (Färe et al., 1985) is obtained when the directional vector is  $g_t = (1, 1)$ . In such case, the MDDF distance function represents a generalized shape for radial measures allowing to fully characterize the production process.

### 2.3 Productivity index

The next statement displays the Hicks-Moorsteen productivity index (Bjurek, 1996; Diewert, 1992ab).

**Definition 2.2** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any directional vector  $(g_t, g_{t+1})$ , the function

$$HM_{t,t+1}(x_t, y_t, x_{t+1}, y_{t+1}) = \left[ HM_t(x_t, y_t, x_{t+1}, y_{t+1}) \times HM_{t+1}(x_t, y_t, x_{t+1}, y_{t+1}) \right]^{\frac{1}{2}} \quad (2.8)$$

is called the Hicks-Moorsteen productivity index.

This productivity index estimates the productivity change between two consecutive time periods. To avoid an arbitrary choice of a base period, the global productivity index is the geometric mean of the productivity measures of the time periods ( $t$ ) and ( $t + 1$ ). Notify that these Hicks-Moorsteen indices are the ratio between output and input quantity indices such that:

$$HM_t(x_t, y_t, x_{t+1}, y_{t+1}) = \frac{MO_t(x_t, y_t, y_{t+1})}{MI_t(x_t, x_{t+1}, y_t)} \quad (2.9)$$

$$\text{and} \quad HM_{t+1}(x_t, y_t, x_{t+1}, y_{t+1}) = \frac{MO_{t+1}(x_{t+1}, y_t, y_{t+1})}{MI_{t+1}(x_t, x_{t+1}, y_{t+1})}, \quad (2.10)$$

where  $MO$  and  $MI$  denote the output and input Malmquist quantity indices respectively. Moreover, these quantity indices are based upon radial measures as the Shephard distance function (Shephard, 1970) or the Debreu-Farrell efficiency measure (Debreu, 1951; Farrell, 1957).

Remark that when  $HM$  is greater than 1 (lesser than 1) then the considered observation has a productivity growth (loss). In addition, when  $MO$  is greater than 1 (lesser than 1) then more (less) outputs are produced for the same input quantity between the consecutive periods. Finally, when  $MI$  is greater than 1 (lesser than 1) then more (less) inputs are needed to produce the same output quantity over periods. Hence,  $MO \geq 1$  and  $MI \leq 1$  contribute to a productivity gain.

### 3 Methodology

This section aims to present some theoretical results. A directional Hicks-Moorsteen productivity index is proposed, and further dynamical formulation of this productivity measure is presented.

#### 3.1 Directional productivity measure

The directional Hicks-Moorsteen productivity index is defined below.

**Definition 3.1** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any directional vectors  $(g_t, g_{t+1}) \in \mathbb{R}_+^{n+m}$ , the function :

$$DHM_{t,t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \left[ DHM_t(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) \times DHM_{t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) \right]^{\frac{1}{2}} \quad (3.1)$$

is called the *Directional Hicks-Moorsteen productivity index*.

This directional Hicks-Moorsteen index is a global measure of the productivity between two time periods since it is defined as the geometric mean of the productivity measures for the periods  $(t)$  and  $(t + 1)$ . The proposition below presents the definition of these productivity indices.

**Proposition 3.2** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any directional vectors  $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}_+^{n+m}$  with  $g_{t,t+1}^o = (0, k_{t,t+1}) \in \mathbb{R}_+^m$  and  $g_{t,t+1}^i = (h_{t,t+1}, 0) \in \mathbb{R}_+^n$ ,

$$DHM_t(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \frac{DMO_t(x_t, y_t, y_{t+1}; g_t^o, g_{t+1}^o)}{DMI_t(x_t, x_{t+1}, y_t; g_t^i, g_{t+1}^i)} \quad (3.2)$$

and

$$DHM_{t+1}(x_t, x_{t+1}, y_t, y_{t+1}; g_t, g_{t+1}) = \frac{DMO_{t+1}(x_{t+1}, y_t, y_{t+1}; g_t^o, g_{t+1}^o)}{DMI_{t+1}(x_t, x_{t+1}, y_{t+1}; g_t^i, g_{t+1}^i)} \quad (3.3)$$

are respectively the *directional Hicks-Moorsteen productivity measures for periods  $(t)$  and  $(t + 1)$*  such that  $DMO_{t,t+1}$  and  $DMI_{t,t+1}$  are respectively the *output and input oriented directional Malmquist quantity indices*.

The oriented directional Malmquist quantity indices of the period  $(t)$  are defined as:

$$DMO_t(x_t, y_t, y_{t+1}; g_t^o, g_{t+1}^o) = \frac{M_t(x_t, y_t; 0, k_t)}{M_t(x_t, y_{t+1}; 0, k_{t+1})}, \quad (3.4)$$

$$\text{and} \quad DMI_t(x_t, x_{t+1}, y_t; g_t^i, g_{t+1}^i) = \frac{M_t(x_{t+1}, y_t; h_{t+1}, 0)}{M_t(x_t, y_t; h_t, 0)}. \quad (3.5)$$

Moreover, the output and input directional Malmquist quantity indices of the period  $(t + 1)$  are as follows:

$$DMO_{t+1}(x_{t+1}, y_t, y_{t+1}; g_t^o, g_{t+1}^o) = \frac{M_{t+1}(x_{t+1}, y_t; 0, k_t)}{M_{t+1}(x_{t+1}, y_{t+1}; 0, k_{t+1})}, \quad (3.6)$$

$$\text{and } DMI_{t+1}(x_t, x_{t+1}, y_t; g_t^i, g_{t+1}^i) = \frac{M_{t+1}(x_{t+1}, y_{t+1}; h_{t+1}, 0)}{M_{t+1}(x_t, y_{t+1}; h_t, 0)}. \quad (3.7)$$

Remark that these quantity indices involve some shadow points as  $(x_t, y_{t+1})$  and  $(x_{t+1}, y_t)$ . And, the efficiency measure of these points with respect to the production technology of the period  $(t)$  are as follows:

$$M_t(x_t, y_{t+1}; h_t, k_{t+1}) = \sup_{\delta} \left\{ \delta : (\delta^{-h_t} x_t, \delta^{k_{t+1}} y_{t+1}) \in T_t \right\}, \quad (3.8)$$

$$\text{and } M_t(x_{t+1}, y_t; h_{t+1}, k_t) = \sup_{\delta} \left\{ \delta : (\delta^{-h_{t+1}} x_{t+1}, \delta^{k_t} y_t) \in T_t \right\}. \quad (3.9)$$

Note that when  $DHM$  is greater than 1 (lesser than 1) then the considered observation has a productivity gain (loss). In addition, when  $DMO$  is greater than 1 (lesser than 1) then more (less) outputs are produced for the same input quantity between the assessed periods. Finally, when  $DMI$  is greater than 1 (lesser than 1) then more (less) inputs are needed to produce the same output quantity over periods. Consequently,  $DMO \geq 1$  and  $DMI \leq 1$  contribute to a productivity growth.

## 3.2 Dynamical context

This subsection defines the dynamical formulation of the cross-period multiplicative directional distance function.

**Definition 3.3** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any  $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}_+^{n+m}$ , the cross-period efficiency measures are defined respectively as follows:

$$M_{t+1(t)}^{i,o} = \left( \frac{x_t}{x_{t+1}} \right)^{1/\rho_{t+1(t)}^i h_t} = \left( \frac{y_{t+1}}{y_t} \right)^{1/\rho_{t+1(t)}^o k_t} \quad (3.10)$$

$$M_{t(t+1)}^{i,o} = \left( \frac{x_t}{x_{t+1}} \right)^{1/\rho_{t(t+1)}^i h_{t+1}} = \left( \frac{y_{t+1}}{y_t} \right)^{1/\rho_{t(t+1)}^o k_{t+1}} \quad (3.11)$$

such that  $M^{i,o}$  displays the input and output sub-vectors  $MDDF$  where,  $\rho^i$  and  $\rho^o$  are the dynamical adjustment parameters in input and output dimensions, respectively.

Remark that the subscript  $t + 1(t)$  means that the projection is made onto the efficient frontier of the period  $(t + 1)$  whereas  $t(t + 1)$  refers to the efficient frontier of the period  $(t)$  (Abad and Ravelojaona, 2017).

The dynamical adjustment parameters  $\rho$  represent the impacts of internal and external factors on the efficiency between two consecutive time periods. These parameters are defined in the following proposition.



**Proposition 3.4** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any  $g_{t,t+1} = (h_{t,t+1}, k_{t,t+1}) \in \mathbb{R}_+^{n+m}$ , the dynamical adjustment parameters  $\rho$  are defined below:

$$\left\{ \begin{array}{l} \rho_{t+1(t)}^i = \frac{\ln(x_t) - \ln(x_{t+1})}{h_t \ln(M_{t+1(t)}^i)} \\ \rho_{t+1(t)}^o = \frac{\ln(y_{t+1}) - \ln(y_t)}{k_t \ln(M_{t+1(t)}^o)} \end{array} \right. \quad (3.12)$$

$$\left\{ \begin{array}{l} \rho_{t(t+1)}^i = \frac{\ln(x_t) - \ln(x_{t+1})}{h_{t+1} \ln(M_{t(t+1)}^i)} \\ \rho_{t(t+1)}^o = \frac{\ln(y_{t+1}) - \ln(y_t)}{k_{t+1} \ln(M_{t(t+1)}^o)} \end{array} \right. \quad (3.13)$$

The dynamical directional quantity indices are defined in the next statement.

**Proposition 3.5** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$  and any directional vectors  $g_{t,t+1} = (g_{t,t+1}^i, g_{t,t+1}^o) \in \mathbb{R}_+^{n+m}$  where  $g_{t,t+1}^o = (0, k_{t,t+1}) \in \mathbb{R}_+^m$  and  $g_{t,t+1}^i = (h_{t,t+1}, 0) \in \mathbb{R}_+^n$ , the dynamical output and input directional quantity indices for the periods  $(t)$  and  $(t+1)$  are defined as:

$$DMO_t = M_t(x_t, y_t; 0, k_t) \times \left( \frac{y_t}{y_{t+1}} \right)^{1/\rho_{t(t+1)}^o k_{t+1}} \quad (3.14)$$

$$DMI_t = M_t(x_t, y_t; h_t, 0)^{-1} \times \left( \frac{x_t}{x_{t+1}} \right)^{1/\rho_{t(t+1)}^i h_{t+1}}$$

and

$$DMO_{t+1} = M_{t+1}(x_{t+1}, y_{t+1}; 0, k_{t+1})^{-1} \times \left( \frac{y_{t+1}}{y_t} \right)^{1/\rho_{t+1(t)}^o k_t} \quad (3.15)$$

$$DMI_{t+1} = M_{t+1}(x_{t+1}, y_{t+1}; h_{t+1}, 0) \times \left( \frac{x_{t+1}}{x_t} \right)^{1/\rho_{t+1(t)}^i h_t}.$$

Following the aforementioned results (3.14) and (3.15), the definition of the dynamical DHM productivity index is immediate.

The *DMI* and the *DMO* are global indices indicating global increase (decrease) of inputs utilization and outputs production. Thus, these quantity indices do not allow to have a detailed knowledge of the inputs and outputs that are growing or diminishing. Nonetheless, the dynamical productivity measures allow to go further in details by identifying specifically increasing or decreasing inputs and outputs by means of the dynamical adjustment parameters and the dynamical cross-period efficiency measures (Proposition 3.6).

**Proposition 3.6** For any  $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ , any dynamical adjustment parameters  $(\rho_{t(t+1)}^i, \rho_{t+1(t)}^i, \rho_{t(t+1)}^o, \rho_{t+1(t)}^o) \in \mathbb{R}^4$  and any cross-period distance functions  $(M_{t+1(t)}^i, M_{t(t+1)}^i, M_{t+1(t)}^o, M_{t(t+1)}^o) \in \mathbb{R}^4$ , the following statements hold:

- i)  $x_{t+1} > x_t$  if one of the following cases occurs:
- |   |   |
|---|---|
| <p><b>1i.</b> <math>\rho_{t(t+1)}^i &lt; 0</math> and <math>M_{t(t+1)}^i &gt; 1</math>;</p> <p><b>2i.</b> <math>\rho_{t(t+1)}^i &gt; 0</math> and <math>M_{t(t+1)}^i &lt; 1</math>;</p> | <p><b>3i.</b> <math>\rho_{t+1(t)}^i &lt; 0</math> and <math>M_{t+1(t)}^i &gt; 1</math>;</p> <p><b>4i.</b> <math>\rho_{t+1(t)}^i &gt; 0</math> and <math>M_{t+1(t)}^i &lt; 1</math>;</p> |
|---|---|
- ii)  $y_{t+1} > y_t$  if one of the assertions below takes place:
- |   |   |
|---|---|
| <p><b>1ii.</b> <math>\rho_{t(t+1)}^o &lt; 0</math> and <math>M_{t(t+1)}^o &lt; 1</math>;</p> <p><b>2ii.</b> <math>\rho_{t(t+1)}^o &gt; 0</math> and <math>M_{t(t+1)}^o &gt; 1</math>;</p> | <p><b>3ii.</b> <math>\rho_{t+1(t)}^o &lt; 0</math> and <math>M_{t+1(t)}^o &lt; 1</math>;</p> <p><b>4ii.</b> <math>\rho_{t+1(t)}^o &gt; 0</math> and <math>M_{t+1(t)}^o &gt; 1</math>;</p> |
|---|---|

## 4 Non parametric specifications

The proposed directional Hicks-Moorsteen productivity index is estimated through a non parametric framework using non convex models, namely the multiplicative technology and the Free Disposal Hull (FDH) production process, respectively. Remark that the hypothesis of convexity is relaxed to take into account for strictly increasing marginal products of potentially efficient production units.

### 4.1 Non parametric production sets

To evaluate the directional Hicks-Moorsteen productivity measure, the MDDF is estimated under a multiplicative technology and a Free-Disposal Hull production set.

The multiplicative production technology has a non linear structure allowing to consider strictly increasing marginal products (Banker and Maindiratta, 1986). For any set of observations  $\mathcal{Z} = \{0, \dots, Z\}$  with  $z \in \mathcal{Z}$  and through the Data Envelopment Analysis (DEA) method, the multiplicative production technology is defined as:

$$T_t^M = \left\{ (x_t, y_t) \in \mathbb{R}_+^{n+m} : x_t^i \geq \prod_{z \in \mathcal{Z}} (x_t^{i,z})^{\lambda_z}, y_t^j \leq \prod_{z \in \mathcal{Z}} (y_t^{j,z})^{\lambda_z}, \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0, i \in [n], j \in [m] \right\}. \quad (4.1)$$

This production set satisfies axioms  $T1 - T4$  and, is a log-convex production set. In such case, it satisfies a “geometric” convexity.

Notice that this production technology becomes linear through a natural logarithmic transformation. Indeed, assuming strictly positive inputs and outputs yields the

following result:

$$\ln(T_t^M) = \left\{ (x_t, y_t) \in \mathbb{R}_{++}^{n+m} : \ln(x_t^i) \geq \sum_{z \in \mathcal{Z}} \lambda_z \ln(x_t^{i,z}), \ln(y_t^j) \leq \sum_{z \in \mathcal{Z}} \lambda_z \ln(y_t^{j,z}), \right. \\ \left. \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0, i \in [n], j \in [m] \right\}. \quad (4.2)$$

The Free Disposal Hull (FDH) production model is the smallest production set satisfying the axioms  $T1-T4$  (Tulkens, 1993). In such case, this production technology does not require the convexity property. For any set of observations  $\mathcal{Z}$  with  $z \in \mathcal{Z}$ , the FDH production technology is defined as follows:

$$T_t^{FDH} = \left\{ (x_t, y_t) \in \mathbb{R}_+^{n+m} : (x_t, y_t) \in \bigcup_{z \in \mathcal{Z}} S_t(x_t^z, y_t^z) \right\}. \quad (4.3)$$

The above definition means that the FDH production set is the union of individual production sets  $S_t(x_t^z, y_t^z)$  such that

$$S_t(x_t^z, y_t^z) = \left\{ (x_t, y_t) \in \mathbb{R}_+^{n+m} : x_t^i \geq x_t^{i,z}, y_t^j \leq y_t^{j,z}, i \in [n], j \in [m], z \in \mathcal{Z} \right\}. \quad (4.4)$$

As the FDH production set is the smallest non convex set allowing for free disposability of inputs and outputs then, it is logical that the number of efficient observations under a FDH technology is greater than or equal to those under a multiplicative production set. However, the FDH technology enables for substantial finite sample error (Post, 2001; Jeong and Simar, 2006). Hence, when the number of input-output variables is higher than the number of observations then, the efficiency estimates are biased. Consequently, the multiplicative production set can overcome this limitation since it is a non convex possibility set allowing for increasing marginal products. Moreover, the multiplicative technology is piecewise log-linear and hence, can be estimated through the DEA model.

## 4.2 Non parametric efficiency measure

This subsection aims to define the multiplicative directional distance function through the DEA model with respect to both the multiplicative production technology and the FDH production set.

Consider the multiplicative production set as defined in (4.1). For any set of observations  $z \in \mathcal{Z}$ , the non parametric MDFF of the observation  $(x_t^0, y_t^0)$  is defined as

follows:

$$M_t(x_t, y_t; g_t) = \sup_{\delta} \left\{ \delta : \delta^{-h_t^i} x_t^{i,0} \geq \prod_{z \in \mathcal{Z}} (x_t^{i,z})^{\lambda_z}, \delta^{k_t^j} y_t^{j,0} \leq \prod_{z \in \mathcal{Z}} (y_t^{j,z})^{\lambda_z}, \right. \\ \left. \sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0, i \in [n], j \in [m] \right\} \quad (4.5)$$

The above specification is non linear. However, through a logarithmic transformation, a linear program can be provided to estimate this efficiency measure. Since this paper focuses on either the input- or the output- oriented efficiency measures, the linear programs of these efficiency indices are presented below:

$$\begin{aligned} \ln [M_t(x_t, y_t; g_t^i)] &= \max \ln(\delta) \\ \text{s.t} \quad &\ln(x_t^{i,0}) - h_t^i \ln(\delta) \geq \sum_{z \in \mathcal{Z}} \lambda_z \ln(x_t^{i,z}) \quad i \in [n] \\ &\ln(y_t^{j,0}) \leq \sum_{z \in \mathcal{Z}} \lambda_z \ln(y_t^{j,z}) \quad j \in [m] \\ &\sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0. \end{aligned} \quad (4.6)$$

and

$$\begin{aligned} \ln [M_t(x_t, y_t; g_t^o)] &= \max \ln(\delta) \\ \text{s.t} \quad &\ln(x_t^{i,0}) \geq \sum_{z \in \mathcal{Z}} \lambda_z \ln(x_t^{i,z}) \quad i \in [n] \\ &\ln(y_t^{j,0}) + k_t^j \ln(\delta) \leq \sum_{z \in \mathcal{Z}} \lambda_z \ln(y_t^{j,z}) \quad j \in [m] \\ &\sum_{z \in \mathcal{Z}} \lambda_z = 1, \lambda_z \geq 0. \end{aligned} \quad (4.7)$$

Obviously, equations (4.6) and (4.7) are the input and the output oriented multiplicative directional efficiency measures respectively, with  $g_t^i = (h_t, 0)$  and  $g_t^o = (0, k_t)$ .

Now, assume that the observation  $(x_t^0, y_t^0)$  operates under a FDH production technology such that the MDDF efficiency measure is defined as follows:

$$M_t(x_t, y_t; g_t) = \sup_{\delta} \left\{ \delta : \delta^{-h_t^i} x_t^{i,0} \geq x_t^{i,z}, \delta^{k_t^j} y_t^{j,0} \leq y_t^{j,z}, i \in [n], j \in [m] \right\}. \quad (4.8)$$

Based upon this non parametric definition of the graph efficiency measure, the input-

and output-oriented multiplicative directional distance functions are given as below:

$$M_t(x_t, y_t; g_t^i) = \max_{z \in \mathcal{Z}} \left[ \min_{i \in [n]} \left( \frac{x_t^{i,0}}{x_t^{i,z}} \right)^{1/h_t^i} \right]; \quad (4.9)$$

$$M_t(x_t, y_t; g_t^o) = \max_{z \in \mathcal{Z}} \left[ \min_{j \in [m]} \left( \frac{y_t^{j,z}}{y_t^{j,0}} \right)^{1/k_t^j} \right]. \quad (4.10)$$

## 5 Empirical illustration

### 5.1 Data in brief

We follow the data setting in Shen and Valdmanis (2019), and Boussemart et al. (2020). The production technology of Chinese medical institutions is defined with four inputs and four outputs. Specifically, the inputs of medical institutions include **[I1]** number of licensed doctors (10000 persons); **[I2]** number of registered nurse (10000 persons); **[I3]** other technical staff (10000 persons); **[I4]** number of beds in health care institutions (10000 units). The outputs produced by inputs, are : **[O1]** emergency treatment in health institutions (million person-times); **[O2]** number of outpatients visits (million persons); **[O3]** number of inpatients visits (million persons); **[O4]** operation of hospitalized in health institutions (namely surgery, million persons-times). Due to data availability, a balanced provincial data is selected for 31 main provinces (municipalities) during the period over 2014-2018. The recent reform of Chinese medical and health system is covered in the sample period. The data is from the National Bureau of Statistics of China (2020)<sup>4</sup>.

The statistical description of the sample data is displayed in Table 1. One can note a significant variation in the sample according to values of the standard deviation (S.D.) that implies unbalanced development levels among provincial medical institutions over 2014-2018. The annual growth rates of inputs and outputs are denoted as the trend. It suggests that the fastest expansion of outputs is the number of surgery while the lowest growth is the number of outpatients visits. In addition, the increase of outputs is mainly motivated by inputs expansion, especially by utilizing more nurses and beds in health care institutions.

### 5.2 Results

In this subsection, we focus on the results for the period 2017-2018. We estimate the multiplicative directional Hicks-Moorsteen productivity index with respect to two different production sets namely the Free Disposal Hull production technology and the Multiplicative production technology. All the results are displayed in Appendix.

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<sup>4</sup>National Bureau of Statistics of China, (2021). Chinese Statistic Yearbook. Accessed on 1 August 2021, Available from: <http://www.stats.gov.cn/english/>

Indicator		Mean	Max	Min	S.D.	Trend
<b>Inputs</b>	Doctors	10.40	29.04	0.56	6.71	5.75%
	Nurses	11.39	33.46	0.27	7.35	8.41%
	Other staffs	5.64	14.43	0.44	3.42	2.11%
	Beds	24.11	60.85	1.19	15.28	6.04%
<b>Outputs</b>	Treatments	245.49	825.89	12.35	191.48	2.39%
	Outpatients	10.75	38.58	0.64	8.83	0.84%
	Inpatients	7.36	19.16	0.23	4.96	5.66%
	Surgery	1.66	7.35	0.04	1.26	8.52%

Table 1: Statistical description of variables (2014-2018)

### Multiplicative technology (MT)

In Table 2, the second column from the right shows that Liaoning and Heilongjiang are the only regions having a directional Hicks-Moorsteen productivity index equal to and greater than one, respectively. Hence, the remaining 29 observations present a loss of productivity over the period 2017-2018. Table 3 enables to see that all observations increase the use of their inputs between the two periods (input directional malmquist quantity index - DMI MT greater than 1). Besides, all observations produce either more or the same quantity of outputs between 2017 and 2018 (output directional malmquist quantity index - DMO MT greater than 1). However, the production growth does not compensate the rise of input utilization. Thus, the observations suffer from productivity loss between the two periods except Liaoning and Heilongjiang. Table 5 allows to go bit further in details. Indeed, we can notice that all observations increase the use of inputs (1, 2) between 2017 and 2018. Moreover, except Inner Mongolia, Heilongjiang, Shandong, Hubei, Chongqing and Xinjiang, the other provinces also improve input (3). Finally, except Tianjing, all the remaining provinces rise input (4) utilization. These situations are shown by one of the following cases: [i]  $\rho_{t(t+1)}^i < 0$  and  $M_{t(t+1)}^i > 1$ ; [ii]  $\rho_{t+1(t)}^i < 0$  and  $M_{t+1(t)}^i > 1$ ; [iii]  $\rho_{t(t+1)}^i > 0$  and  $M_{t(t+1)}^i < 1$  or; [iv]  $\rho_{t+1(t)}^i > 0$  and  $M_{t+1(t)}^i < 1$ . Still concerning Table 5, the adjustment parameters of outputs show that two-thirds of observations (except Tianjin, Shanxi, Liaoning, Heilongjiang, Jiangxi, Hubei, Guangxi, Tibet, Gansu, Xinjiang) increase the production of output 1. In addition, 20 DMUs<sup>5</sup> (excluding Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Tibet, Gansu, Ningxia, Xinjiang) reduce the output (2). Moreover, excluding Heilongjiang, Tibet and Xinjiang, all the left observations rise the production of output 3. Finally, all the DMUs unless Qinghai expand their output (4). This situation is pointed out by one of the following cases: [1]  $\rho_{t(t+1)}^o < 0$  and  $M_{t(t+1)}^o < 1$ ; [2]  $\rho_{t+1(t)}^o < 0$  and  $M_{t+1(t)}^o < 1$ ; [3]  $\rho_{t(t+1)}^o > 0$  and  $M_{t(t+1)}^o > 1$  or; [4]  $\rho_{t+1(t)}^o > 0$  and  $M_{t+1(t)}^o > 1$ .

Consequently, the increase in all inputs has been counterbalanced by the production growth of outputs (1, 3, 4) for Liaoning. For this reason, this DMU has neither gain nor loss of productivity between 2017 and 2018. Regarding Heilongjiang, the increase in inputs (1, 2, 4) and the decrease in outputs (1, 3) has been compensated by the

<sup>5</sup>DMU: decision making unit.

decrease in input (3) and the increase in outputs (2, 4). It results that Heilongjiang gains in productivity over the period 2017-2018.

From a dynamical standpoint, the adjustment parameters proposed in Table 5 allow to compute the dynamical directional Hicks-Moorsteen index as well as the dynamical Malmquist quantity indices as shown in Table 3. The dynamical productivity index (ratio between DMO and DMI) provides the same result that in Table 2.

### **Free Disposal Hull technology (FDH)**

Regarding the directional Hicks-Moorsteen productivity scores under a FDH technology, the first column from the right of Table 2 shows that 13 observations (Inner Mongolia, Liaoning, Heilongjiang, Zhejiang, Fujian, Hubei, Guangdong, Guangxi, Sichuan, Yunnan, Tibet, Ningxia, Xinjiang) present productivity improvement or stagnation. Table 3 allows to deduce that all observations except Heilongjiang use more inputs between 2017 and 2018. Moreover, 25 DMUs, not including Beijing, Jilin, Shandong, Hunan, Hainan and Qinghai, produce more outputs between the two time periods. Hence, the outputs growth counterbalances the increase in inputs for the 13 DMUs presenting positive productivity variation. From Table 6, we can see that all observations increase their four inputs except Tianjin, Inner Mongolia, Heilongjiang, Shandong, Hubei, Chongqing and Xinjiang. Indeed, Tianjin rises the utilization of inputs (1, 2, 3) and reduces input (4) whereas the six remaining observations increase inputs (1, 2, 4) while diminishing input (3). Concerning the output part, Table 6 allows to deduce that 8 observations (Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Ningxia) increase all of their outputs. Besides, Gansu gets its outputs (2, 3, 4) growing while output (1) diminishing. In addition, 12 DMUs (Beijing, Hebei, Jilin, Shanghai, Anhui, Shandong, Henan, Hunan, Hainan, Chongqing, Yunnan, Shaanxi) expand their outputs (1, 3, 4) and decrease their output (2). Moreover, 6 DMUs (Tianjin, Shanxi, Liaoning, Jiangxi, Hubei, Guangxi) increase outputs (3,4) and reduce outputs (1,2). Tibet and Xinjiang have growing outputs (2, 4) and reducing outputs (1, 3) whereas Qinghai experiences the opposite. Finally, Heilongjiang has the output (4) as the single increasing output while the three other outputs are reducing. These results are consistent with those obtained under a multiplicative production set.

Regarding the dynamical viewpoint, the dynamical deviations in Table 6 enable to estimate the dynamical directional Hicks-Moorsteen index and the dynamical Malmquist quantity indices (Table 3). The results highlight that the dynamical directional Hicks-Moorsteen is equal to the standard Hicks-Moorsteen measure as presented in Table 2.

To summarize, regarding the two production sets and the directional Hicks-Moorsteen productivity scores, most of observations increase all their inputs (I1. licensed doctors; I2. registered nurse; I3. other technical staff; I4. beds) except Tianjin whose input (4) decreases and, Inner Mongolia, Heilongjiang, Shandong, Hubei, Chongqing and Xinjiang that diminish their input (3). Concerning the output part, all DMUs improve the production of outputs 3 (inpatients) and 4 (operation of hospitalized) except Heilongjiang, Tibet and Xinjiang having a decreasing output (3) and, Heilongjiang presenting a reduced output (4). In addition, 21 DMUs (excluding Tianjian, Shanxi,

Liaoning, Heilongjiang, Jiangxi, Hubei, Guangxi, Tibet, Gansu and Xinjiang) have growing output 1 (emergency treatment). Finally, most of the observations (except Inner Mongolia, Jiangsu, Zhejiang, Fujian, Guangdong, Sichuan, Guizhou, Tibet, Gansu, Ningxia and Xinjiang) have decreasing output 2 (other outpatients visits). Hence, almost all of DMUs increase all of their inputs and most of them decrease their output (2). This fact is mostly supposed to be counterbalanced with the increase in outputs (3, 4) for almost all DMUs and in output (1) for some of them. Concerning the productivity variation, Liaoning and Heilongjiang are presenting positive productivity change in both multiplicative and FDH production sets. Moreover, there are more observations having gain of productivity under a FDH technology than under a multiplicative technology. It is logical since the FDH is the smallest non convex production possibility set satisfying the free disposability in inputs and outputs. Thus, there could be more efficient DMUs under a FDH production set than under a multiplicative technology.

Globally, from the results over year 2014 to year 2018 (Appendix), we can notice that most of observations have a directional input Malmquist quantity index greater than one. This means that there is an increasing utilization of inputs over the analyzed period. This result is strengthened by the ones about dynamical deviations. Indeed, we can see that inputs (1, 2, 4) increase for almost all observations whereas there is a decrease in input (3) for a quarter of observations. Besides, all observations have a directional output Malmquist quantity index greater than one meaning that more outputs are produced between the time periods. The dynamical deviations show that from 2014 to 2018, there is a production growth of outputs (1, 3, 4) for all observations. In addition, there is a gradual decrease of output (2) for around a half of the observations each year. Nonetheless, the increase in outputs does not compensate the rising input utilization. Hence, most of observations present a loss of productivity from year to year. Not surprisingly, the FDH production set presents more observations having a gain of productivity. However, the directional Hicks-Moorsteen scores withing FDH technology can be biased by slacks but also by the small sample size.

## 6 Concluding comments

This paper is the first to define a directional Hicks-Moorsteen productivity index. The directional Hicks-Moorsteen productivity measure is estimated using the multiplicative directional distance function through non convex production possibility sets, namely the multiplicative technology and the free disposal hull production set. As the directional Hicks-Moorsteen productivity index is evaluated through the multiplicative directional distance functions, it allows to analyze the productivity variation taking into account of strictly increasing marginal products. Besides, the directional Hicks-Moorsteen productivity measure inherits the structure of the Hicks-Moorsteen index such that it does not present infeasibilities.

These theoretical backgrounds are illustrated by considering 31 Chinese provincial hospitals over the period 2014-2018. The data includes four inputs and four outputs and the results are obtained through the data envelopment analysis framework. The



empirical illustration indicates that there are more production units presenting productivity gain under a FDH technology. This is not surprising since the FDH production set is smaller than the multiplicative one.

In further works, the decomposition of the proposed TFP measure could be investigated. For instance, the proposed productivity index could be separated into components such as efficiency change and technological progress. Alternatively, the distance functions could be estimated by both parametric and non parametric approaches to compare their results.

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# Appendix

	2014-2015		2015-2016		2016-2017		2017-2018	
	DHM-MT	DHM-FDH	DHM-MT	DHM-FDH	DHM-MT	DHM-FDH	DHM-MT	DHM-FDH
Beijing	0.998	0.898	0.971	1.069	0.956	1.247	0.991	0.709
Tianjin	0.942	1.018	0.966	0.968	0.945	1.036	0.978	0.980
Hebei	0.951	0.911	0.966	1.012	0.936	0.928	0.940	0.945
Shanxi	1.017	0.959	0.960	1.015	1.021	1.047	0.982	0.990
Inner Mongolia	0.949	0.928	0.978	1.041	0.934	1.062	0.960	1.017
Liaoning	1.029	0.978	0.997	1.021	1.019	1.002	1.000	1.058
Jilin	0.976	0.935	0.978	1.010	1.006	1.039	0.965	0.927
Heilongjiang	0.992	0.961	0.976	1.008	0.926	0.932	1.007	1.074
Shanghai	0.966	0.901	0.956	0.939	0.954	0.998	0.961	0.985
Jiangsu	0.962	0.944	1.000	1.023	0.959	1.011	0.955	0.986
Zhejiang	0.911	0.972	0.933	1.018	0.937	1.094	0.962	1.005
Anhui	0.975	0.985	0.968	0.997	0.984	1.056	0.950	0.979
Fujian	0.956	0.972	1.004	1.028	0.959	0.961	0.956	1.052
Jiangxi	0.955	0.979	0.955	0.995	0.927	0.983	0.956	0.977
Shandong	0.983	0.948	0.967	1.052	0.949	0.987	0.973	0.955
Henan	0.958	0.966	0.952	0.978	0.952	0.995	0.944	0.980
Hubei	0.951	0.977	0.967	1.056	0.975	0.947	0.979	1.014
Hunan	0.948	0.993	0.964	1.018	0.967	1.008	0.954	0.919
Guangdong	0.949	0.917	0.936	0.980	0.948	1.022	0.951	1.031
Guangxi	0.939	0.933	0.952	0.995	0.940	0.968	0.946	1.014
Hainan	0.927	0.863	0.971	0.978	0.974	1.122	0.954	0.907
Chongqing	0.922	0.941	0.934	0.931	0.942	1.022	0.950	0.997
Sichuan	0.954	0.940	0.948	1.013	0.934	1.002	0.947	1.085
Guizhou	0.908	0.909	0.918	0.996	0.901	0.964	0.926	0.989
Yunnan	0.917	0.881	0.910	1.010	0.886	0.927	0.941	1.000
Tibet	0.890	1.018	0.958	0.902	0.929	1.113	0.880	1.044
Shaanxi	0.964	0.972	0.938	0.998	0.942	1.023	0.950	0.988
Gansu	0.971	1.028	0.956	1.046	0.918	0.971	0.931	0.975
Qinghai	0.954	0.997	0.971	1.026	0.902	1.030	0.955	0.888
Ningxia	0.980	0.929	0.954	1.035	0.923	0.788	0.952	1.205
Xinjiang	0.964	1.011	0.953	1.040	0.947	1.042	0.955	1.003

Table 2: Directional Hicks-Moorsteen index under FDH (DHM-FDH) and multiplicative (DHM-MT) technologies

	2017-2018						2016-2017					
	Multiplicative			Free Disposal Hull			Multiplicative			Free Disposal Hull		
	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM
Beijing	1.025	1.015	0.991	1.035	0.734	0.709	1.030	0.985	0.956	1.043	1.301	1.247
Tianjin	1.023	1.000	0.978	1.023	1.002	0.980	1.058	1.000	0.945	1.058	1.096	1.036
Hebei	1.063	1.000	0.940	1.063	1.005	0.945	1.068	1.000	0.936	1.068	0.991	0.928
Shanxi	1.039	1.021	0.982	1.046	1.036	0.990	1.015	1.036	1.021	1.027	1.075	1.047
Inner Mongolia	1.058	1.015	0.960	1.031	1.048	1.017	1.078	1.008	0.934	1.049	1.114	1.062
Liaoning	1.015	1.015	1.000	1.029	1.088	1.058	0.996	1.015	1.019	1.029	1.032	1.002
Jilin	1.036	1.000	0.965	1.068	0.990	0.927	0.994	1.000	1.006	0.996	1.034	1.039
Heilongjiang	1.026	1.034	1.007	0.994	1.068	1.074	1.056	0.977	0.926	1.038	0.967	0.932
Shanghai	1.041	1.000	0.961	1.041	1.025	0.985	1.048	1.000	0.954	1.048	1.046	0.998
Jiangsu	1.047	1.000	0.955	1.063	1.048	0.986	1.042	1.000	0.959	1.049	1.061	1.011
Zhejiang	1.040	1.000	0.962	1.052	1.057	1.005	1.067	1.000	0.937	1.054	1.153	1.094
Anhui	1.052	1.000	0.950	1.064	1.042	0.979	1.029	1.012	0.984	1.045	1.103	1.056
Fujian	1.056	1.010	0.956	1.050	1.105	1.052	1.044	1.001	0.959	1.051	1.010	0.961
Jiangxi	1.046	1.000	0.956	1.044	1.021	0.977	1.078	1.000	0.927	1.078	1.059	0.983
Shandong	1.028	1.000	0.973	1.028	0.981	0.955	1.054	1.000	0.949	1.054	1.040	0.987
Henan	1.059	1.000	0.944	1.059	1.038	0.980	1.050	1.000	0.952	1.047	1.042	0.995
Hubei	1.022	1.000	0.979	1.023	1.037	1.014	1.025	1.000	0.975	1.028	0.973	0.947
Hunan	1.048	1.000	0.954	1.048	0.964	0.919	1.034	1.000	0.967	1.035	1.043	1.008
Guangdong	1.052	1.000	0.951	1.052	1.084	1.031	1.055	1.000	0.948	1.055	1.078	1.022
Guangxi	1.058	1.000	0.946	1.047	1.062	1.014	1.064	1.000	0.940	1.050	1.017	0.968
Hainan	1.048	1.000	0.954	1.046	0.949	0.907	1.029	1.002	0.974	1.038	1.164	1.122
Chongqing	1.052	1.000	0.950	1.052	1.050	0.997	1.061	1.000	0.942	1.061	1.085	1.022
Sichuan	1.056	1.000	0.947	1.056	1.146	1.085	1.070	1.000	0.934	1.070	1.072	1.002
Guizhou	1.091	1.010	0.926	1.080	1.068	0.989	1.099	0.990	0.901	1.097	1.057	0.964
Yunnan	1.063	1.000	0.941	1.000	1.000	1.000	1.129	1.000	0.886	1.136	1.053	0.927
Tibet	1.137	1.000	0.880	1.137	1.187	1.044	1.077	1.000	0.929	1.077	1.199	1.113
Shaanxi	1.055	1.002	0.950	1.048	1.035	0.988	1.077	1.014	0.942	1.066	1.090	1.023
Gansu	1.074	1.000	0.931	1.071	1.044	0.975	1.089	1.000	0.918	1.093	1.062	0.971
Qinghai	1.047	1.000	0.955	1.069	0.949	0.888	1.109	1.000	0.902	1.114	1.147	1.030
Ningxia	1.051	1.000	0.952	1.054	1.269	1.205	1.084	1.000	0.923	1.113	0.877	0.788
Xinjiang	1.047	1.000	0.955	1.027	1.030	1.003	1.056	1.000	0.947	1.011	1.053	1.042

Table 3: Dynamical DHM-MT and DHM-FDH (2016-2018)

	2015-2016						2014-2015					
	Multiplicative technology			FDH technology			Multiplicative technology			FDH technology		
	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM	DMI	DMO	DHM
Beijing	1.049	1.019	0.971	1.034	1.106	1.069	1.016	1.014	0.998	1.042	0.936	0.898
Tianjin	1.035	1.000	0.966	1.035	1.001	0.968	1.062	1.000	0.942	1.062	1.081	1.018
Hebei	1.036	1.000	0.966	1.036	1.049	1.012	1.051	1.000	0.951	1.051	0.958	0.911
Shanxi	1.039	0.997	0.960	1.060	1.076	1.015	0.998	1.015	1.017	1.025	0.983	0.959
Inner Mongolia	1.040	1.017	0.978	1.051	1.094	1.041	1.038	0.985	0.949	1.055	0.979	0.928
Liaoning	1.000	0.998	0.997	1.033	1.055	1.021	0.998	1.027	1.029	1.023	1.000	0.978
Jilin	1.023	1.000	0.978	1.042	1.052	1.010	1.024	1.000	0.976	1.036	0.968	0.935
Heilongjiang	1.036	1.011	0.976	1.026	1.034	1.008	1.022	1.014	0.992	1.013	0.974	0.961
Shanghai	1.046	1.000	0.956	1.046	0.982	0.939	1.036	1.000	0.966	1.036	0.933	0.901
Jiangsu	1.000	1.000	1.000	1.027	1.050	1.023	1.040	1.000	0.962	1.055	0.996	0.944
Zhejiang	1.072	1.000	0.933	1.055	1.074	1.018	1.097	1.000	0.911	1.055	1.025	0.972
Anhui	1.025	0.992	0.968	1.038	1.035	0.997	1.029	1.003	0.975	1.043	1.027	0.985
Fujian	1.009	1.014	1.004	1.026	1.055	1.028	1.038	0.992	0.956	1.015	0.987	0.972
Jiangxi	1.047	1.000	0.955	1.048	1.043	0.995	1.047	1.000	0.955	1.047	1.026	0.979
Shandong	1.034	1.000	0.967	1.034	1.087	1.052	1.017	1.000	0.983	1.017	0.964	0.948
Henan	1.050	1.000	0.952	1.050	1.026	0.978	1.044	1.000	0.958	1.044	1.008	0.966
Hubei	1.034	1.000	0.967	1.038	1.096	1.056	1.052	1.000	0.951	1.082	1.057	0.977
Hunan	1.038	1.000	0.964	1.038	1.057	1.018	1.055	1.000	0.948	1.055	1.047	0.993
Guangdong	1.068	1.000	0.936	1.068	1.046	0.980	1.054	1.000	0.949	1.054	0.966	0.917
Guangxi	1.050	1.000	0.952	1.046	1.041	0.995	1.065	1.000	0.939	1.057	0.987	0.933
Hainan	1.028	0.998	0.971	1.043	1.020	0.978	1.079	1.000	0.927	1.079	0.931	0.863
Chongqing	1.071	1.000	0.934	1.074	1.000	0.931	1.084	1.000	0.922	1.083	1.018	0.941
Sichuan	1.055	1.000	0.948	1.055	1.069	1.013	1.048	1.000	0.954	1.048	0.985	0.940
Guizhou	1.089	1.000	0.918	1.083	1.078	0.996	1.101	1.000	0.908	1.097	0.997	0.909
Yunnan	1.099	1.000	0.910	1.092	1.103	1.010	1.091	1.000	0.917	1.091	0.961	0.881
Tibet	1.043	1.000	0.958	1.043	0.941	0.902	1.124	1.000	0.890	1.124	1.144	1.018
Shaanxi	1.069	1.003	0.938	1.087	1.085	0.998	1.053	1.015	0.964	1.053	1.024	0.972
Gansu	1.046	1.000	0.956	1.019	1.066	1.046	1.030	1.000	0.971	1.008	1.036	1.028
Qinghai	1.030	1.000	0.971	1.038	1.066	1.026	1.048	1.000	0.954	1.046	1.044	0.997
Ningxia	1.048	1.000	0.954	1.063	1.100	1.035	1.020	1.000	0.980	1.022	0.950	0.929
Xinjiang	1.050	1.000	0.953	1.052	1.095	1.040	1.057	1.019	0.964	1.057	1.069	1.011

Table 4: Dynamical DHM-MT and DHM-FDH (2014-2016)

2017-2018										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-1.139	-0.747	-0.284	-0.502	1.0501	-1.701	32.708	-2.350	-2.737	0.9693
Tianjin	16.266	10.363	6.160	-1.000	0.9971	0.026	1.000	-0.207	-1.025	0.8752
Hebei	-3.690	-3.334	-1.000	-2.514	1.0266	-0.079	1.453	-0.878	-1.689	0.9626
Shanxi	-0.276	-0.363	-0.099	-0.274	1.2143	-0.333	-0.442	0.717	1.032	1.1263
Inner Mongolia	-0.162	-0.219	0.002	-0.201	1.3222	0.053	0.461	0.310	0.378	1.2016
Liaoning	-1.420	-1.919	-0.086	-1.827	1.0286	0.091	0.299	-0.143	-3.015	0.9396
Jilin	-23.688	-34.310	-1.000	-22.295	1.0037	-1.402	3.112	-2.246	-1.994	0.9764
Heilongjiang	-0.071	-0.186	0.290	-0.218	1.1694	-0.914	-0.780	-0.549	3.119	1.0617
Shanghai	-1.966	-1.779	-1.000	-1.198	1.0272	-0.150	0.557	-0.602	-1.000	0.8946
Jiangsu	-2.549	-3.361	-1.000	-1.648	1.0286	-0.379	-1.195	-0.497	-1.773	0.9569
Zhejiang	-10.023	-10.877	-1.000	-8.830	1.0065	-0.789	-1.036	-1.067	-0.589	0.9351
Anhui	-1.124	-1.866	-1.000	-1.633	1.0442	-1.346	0.548	-0.319	-2.307	0.9543
Fujian	-0.688	-0.654	-0.140	-0.456	1.1254	2.198	12.709	3.067	3.308	1.0135
Jiangxi	-1.637	-2.373	-0.855	-2.421	1.0268	0.225	1.100	-0.688	-1.734	0.9372
Shandong	2.365	2.404	-1.000	1.009	0.9614	-1.565	4.734	-0.631	-2.026	0.9862
Henan	-2.238	-2.833	-1.000	-2.819	1.0306	-0.021	0.221	-1.104	-0.834	0.9188
Hubei	79.869	88.172	-1.000	113.377	0.9996	0.117	0.510	-0.312	-1.255	0.9072
Hunan	-1.473	-2.116	-1.000	-2.130	1.0307	-0.072	2.515	-0.704	-1.289	0.9416
Guangdong	-3.995	-4.862	-1.000	-2.857	1.0174	-0.071	-0.143	-0.298	-1.000	0.8596
Guangxi	-0.929	-1.263	-0.543	-1.175	1.0520	0.328	0.344	-0.546	-2.281	0.9399
Hainan	-2.121	-1.639	-0.613	-1.979	1.0331	-0.585	6.561	-0.733	-2.962	0.9665
Chongqing	9.663	10.078	-1.000	5.704	0.9888	-0.299	0.542	-0.302	-1.652	0.9164
Sichuan	-1.626	-2.551	-1.000	-1.969	1.0314	-0.340	-1.717	-0.037	-0.814	0.8563
Guizhou	-0.781	-1.103	-0.469	-0.540	1.1024	-0.861	-0.174	-1.211	-1.325	0.9157
Yunnan	-1.020	-1.081	-1.012	-0.992	1.0602	-0.208	1.668	-0.962	-1.021	0.9253
Tibet	-2.132	-5.025	-3.844	-1.000	1.0435	0.016	-1.000	0.163	-0.095	0.6645
Shaanxi	-0.416	-0.570	-0.068	-0.343	1.1573	0.858	-3.033	2.085	3.368	1.0294
Gansu	-1.579	-2.529	-0.847	-2.749	1.0386	0.230	-0.145	-1.158	-0.192	0.9112
Qinghai	-1.996	-3.059	-5.464	-1.000	1.0209	-1.830	9.515	-0.898	2.664	0.9865
Ningxia	-2.210	-2.517	-1.509	-1.000	1.0302	-0.167	-3.404	-0.200	-0.278	0.8749
Xinjiang	-0.333	-0.506	0.255	-1.349	1.0495	4.783	-10.211	0.835	-5.563	0.9892

  

2017-2018										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	2.219	1.456	0.553	0.979	0.9752	1.151	-22.139	1.590	1.853	1.0471
Tianjin	1.000	0.637	0.379	-0.061	0.9535	0.036	1.410	-0.291	-1.446	0.9098
Hebei	1.000	0.903	0.271	0.681	0.9078	-0.055	1.000	-0.604	-1.162	0.9461
Shanxi	-0.640	-0.841	-0.229	-0.636	1.0873	-0.223	-0.297	0.481	0.693	1.1939
Inner Mongolia	-0.306	-0.412	0.004	-0.379	1.1600	0.034	0.300	0.201	0.246	1.3262
Liaoning	1.416	1.914	0.086	1.822	0.9721	-0.154	-0.506	0.241	5.095	1.0375
Jilin	1.307	1.893	0.055	1.230	0.9348	-0.474	1.052	-0.760	-0.675	0.9319
Heilongjiang	-0.294	-0.770	1.204	-0.905	1.0385	-0.409	-0.349	-0.246	1.396	1.1431
Shanghai	1.000	0.905	0.509	0.609	0.9486	-1.668	6.200	-6.700	-11.135	0.9900
Jiangsu	1.122	1.479	0.440	0.726	0.9380	0.941	2.967	1.234	4.402	1.0179
Zhejiang	0.921	1.000	0.092	0.812	0.9315	1.080	1.419	1.462	0.807	1.0502
Anhui	0.824	1.369	0.733	1.198	0.9427	4.486	-1.826	1.063	7.689	1.0141
Fujian	10.698	10.174	2.185	7.096	0.9924	0.265	1.534	0.370	0.399	1.1171
Jiangxi	0.676	0.980	0.353	1.000	0.9379	0.840	4.098	-2.564	-6.458	0.9827
Shandong	0.983	1.000	-0.416	0.420	0.9097	-0.498	1.505	-0.201	-0.644	0.9571
Henan	0.790	1.000	0.353	0.995	0.9183	-0.123	1.300	-6.484	-4.894	0.9857
Hubei	0.731	0.807	-0.009	1.038	0.9576	3.253	14.220	-8.709	-35.013	0.9965
Hunan	0.692	0.995	0.470	1.001	0.9377	-0.120	4.225	-1.183	-2.165	0.9648
Guangdong	0.822	1.000	0.206	0.588	0.9197	1.000	2.007	4.174	13.988	1.0109
Guangxi	0.768	1.044	0.449	0.971	0.9405	-1.530	-1.604	2.547	10.646	1.0134
Hainan	1.127	0.871	0.326	1.052	0.9405	3.981	-44.637	4.987	20.155	1.0050
Chongqing	0.959	1.000	-0.099	0.566	0.8927	2.872	-5.212	2.906	15.876	1.0091
Sichuan	0.638	1.000	0.392	0.772	0.9241	9.132	46.189	1.000	21.894	1.0058
Guizhou	0.779	1.100	0.468	0.539	0.9069	0.782	0.158	1.101	1.205	1.1017
Yunnan	0.946	1.003	0.938	0.919	0.9389	0.515	-4.122	2.377	2.523	1.0319
Tibet	0.424	1.000	0.765	0.199	0.8075	0.097	-6.126	1.000	-0.584	0.9355
Shaanxi	-1.782	-2.441	-0.294	-1.470	1.0347	0.153	-0.540	0.371	0.600	1.1764
Gansu	0.574	0.920	0.308	1.000	0.9010	2.703	-1.702	-13.632	-2.254	0.9921
Qinghai	0.576	0.883	1.577	0.289	0.9309	-0.495	2.576	-0.243	0.721	0.9511
Ningxia	0.942	1.073	0.643	0.426	0.9327	0.954	19.429	1.141	1.587	1.0237
Xinjiang	0.375	0.569	-0.287	1.520	0.9580	6.279	-13.405	1.096	-7.304	0.9918

Table 5: Dynamical Deviation under the multiplicative production set

2017-2018										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-4.014	-2.633	-1.000	-1.771	1.0140	-0.621	11.951	-0.859	-1.000	0.9183
Tianjin	16.266	10.363	6.160	-1.000	0.9971	0.025	0.975	-0.201	-1.000	0.8722
Hebei	-3.690	-3.334	-1.000	-2.514	1.0266	-0.047	0.860	-0.520	-1.000	0.9376
Shanxi	-2.794	-3.671	-1.000	-2.777	1.0194	0.322	0.428	-0.695	-1.000	0.8845
Inner Mongolia	69.303	93.429	-1.000	85.954	0.9993	-0.114	-1.000	-0.672	-0.821	0.9188
Liaoning	-16.496	-22.298	-1.000	-21.223	1.0024	0.030	0.099	-0.047	-1.000	0.8288
Jilin	-23.688	-34.310	-1.000	-22.295	1.0037	-0.624	1.385	-1.000	-0.888	0.9478
Heilongjiang	0.244	0.640	-1.000	0.752	0.9556	0.293	0.250	0.176	-1.000	0.8297
Shanghai	-1.966	-1.779	-1.000	-1.198	1.0272	-0.150	0.557	-0.602	-1.000	0.8946
Jiangsu	-2.549	-3.361	-1.000	-1.648	1.0286	-0.214	-0.674	-0.280	-1.000	0.9249
Zhejiang	-10.023	-10.877	-1.000	-8.830	1.0065	-0.739	-0.971	-1.000	-0.552	0.9309
Anhui	-1.124	-1.866	-1.000	-1.633	1.0442	-0.583	0.237	-0.138	-1.000	0.8977
Fujian	-4.897	-4.657	-1.000	-3.248	1.0167	-0.173	-1.000	-0.241	-0.260	0.8438
Jiangxi	-1.913	-2.775	-1.000	-2.831	1.0229	0.130	0.635	-0.397	-1.000	0.8937
Shandong	2.365	2.404	-1.000	1.009	0.9614	-0.773	2.337	-0.311	-1.000	0.9722
Henan	-2.238	-2.833	-1.000	-2.819	1.0306	-0.019	0.200	-1.000	-0.755	0.9107
Hubei	79.869	88.172	-1.000	113.377	0.9996	0.093	0.406	-0.249	-1.000	0.8849
Hunan	-1.473	-2.116	-1.000	-2.130	1.0307	-0.056	1.952	-0.547	-1.000	0.9254
Guangdong	-3.995	-4.862	-1.000	-2.857	1.0174	-0.071	-0.143	-0.298	-1.000	0.8596
Guangxi	-1.712	-2.326	-1.000	-2.164	1.0279	0.144	0.151	-0.239	-1.000	0.8682
Hainan	-3.461	-2.675	-1.000	-3.231	1.0202	-0.198	2.215	-0.247	-1.000	0.9041
Chongqing	9.663	10.078	-1.000	5.704	0.9888	-0.181	0.328	-0.183	-1.000	0.8657
Sichuan	-1.626	-2.551	-1.000	-1.969	1.0314	-0.198	-1.000	-0.022	-0.474	0.7662
Guizhou	-1.664	-2.350	-1.000	-1.151	1.0468	-0.649	-0.131	-0.914	-1.000	0.8899
Yunnan	-1.029	-1.091	-1.021	-1.000	1.0597	-0.204	1.634	-0.942	-1.000	0.9238
Tibet	-2.132	-5.025	-3.844	-1.000	1.0435	0.016	-1.000	0.163	-0.095	0.6645
Shaanxi	-6.070	-8.317	-1.000	-5.008	1.0101	-0.255	0.901	-0.619	-1.000	0.9072
Gansu	-1.864	-2.986	-1.000	-3.246	1.0326	0.198	-0.125	-1.000	-0.165	0.8979
Qinghai	-1.996	-3.059	-5.464	-1.000	1.0209	-1.000	5.200	-0.491	1.456	0.9755
Ningxia	-2.210	-2.517	-1.509	-1.000	1.0302	-0.049	-1.000	-0.059	-0.082	0.6346
Xinjiang	1.309	1.986	-1.000	5.301	0.9878	0.468	-1.000	0.082	-0.545	0.8955

  

2017-2018										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.656	0.249	0.441	0.9458	-0.075	1.447	-0.104	-0.121	0.4945
Tianjin	1.000	0.637	0.379	-0.061	0.9535	0.026	1.000	-0.207	-1.025	0.8752
Hebei	1.000	0.903	0.271	0.681	0.9078	-0.055	1.000	-0.604	-1.162	0.9461
Shanxi	0.761	1.000	0.272	0.756	0.9320	0.753	1.000	-1.623	-2.336	0.9488
Inner Mongolia	0.742	1.000	-0.011	0.920	0.9407	1.000	8.764	5.888	7.199	1.0097
Liaoning	0.740	1.000	0.045	0.952	0.9473	0.304	1.000	-0.476	-10.069	0.9815
Jilin	0.690	1.000	0.029	0.650	0.8801	-0.451	1.000	-0.722	-0.641	0.9285
Heilongjiang	0.325	0.851	-1.330	1.000	0.9664	1.000	0.854	0.601	-3.413	0.9468
Shanghai	1.000	0.905	0.509	0.609	0.9486	-0.269	1.000	-1.081	-1.796	0.9398
Jiangsu	0.759	1.000	0.298	0.490	0.9097	1.000	3.152	1.311	4.676	1.0168
Zhejiang	0.691	0.750	0.069	0.609	0.9097	1.339	1.759	1.812	1.000	1.0403
Anhui	0.602	1.000	0.536	0.875	0.9224	-2.457	1.000	-0.582	-4.211	0.9747
Fujian	1.000	0.951	0.204	0.663	0.9220	1.000	5.783	1.395	1.505	1.0298
Jiangxi	0.676	0.980	0.353	1.000	0.9379	0.205	1.000	-0.626	-1.576	0.9312
Shandong	0.983	1.000	-0.416	0.420	0.9097	-0.331	1.000	-0.133	-0.428	0.9362
Henan	0.790	1.000	0.353	0.995	0.9183	-0.095	1.000	-4.989	-3.765	0.9814
Hubei	0.704	0.778	-0.009	1.000	0.9560	0.229	1.000	-0.612	-2.462	0.9515
Hunan	0.692	0.994	0.470	1.000	0.9376	-0.028	1.000	-0.280	-0.512	0.8596
Guangdong	0.822	1.000	0.206	0.588	0.9197	1.000	2.007	4.174	13.988	1.0109
Guangxi	0.736	1.000	0.430	0.930	0.9380	0.954	1.000	-1.588	-6.637	0.9789
Hainan	1.000	0.773	0.289	0.933	0.9332	-0.097	1.084	-0.121	-0.490	0.8139
Chongqing	0.959	1.000	-0.099	0.566	0.8927	-0.551	1.000	-0.558	-3.046	0.9538
Sichuan	0.638	1.000	0.392	0.772	0.9241	9.132	46.189	1.000	21.894	1.0058
Guizhou	0.708	1.000	0.426	0.490	0.8980	4.952	1.000	6.969	7.625	1.0154
Yunnan	-1.029	-1.091	-1.021	-1.000	1.0597	-0.204	1.634	-0.942	-1.000	0.9238
Tibet	0.424	1.000	0.765	0.199	0.8075	0.097	-6.126	1.000	-0.584	0.9355
Shaanxi	0.730	1.000	0.120	0.602	0.9201	-0.848	2.996	-2.060	-3.327	0.9711
Gansu	0.574	0.920	0.308	1.000	0.9010	1.000	-0.630	-5.044	-0.834	0.9789
Qinghai	0.365	0.560	1.000	0.183	0.8932	-0.192	1.000	-0.094	0.280	0.8789
Ningxia	0.878	1.000	0.600	0.397	0.9280	1.000	20.361	1.196	1.663	1.0226
Xinjiang	0.247	0.375	-0.189	1.000	0.9368	1.000	-2.135	0.175	-1.163	0.9496

Table 6: Dynamical Deviation under FDH production technology



2016-2017										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-1.813	-1.795	-1.158	-1.000	1.030	-8.137	124.191	11.548	9.059	1.005
Tianjin	-2.792	-1.900	-1.000	-1.278	1.030	-0.001	-1.085	0.130	-0.154	0.825
Hebei	-2.444	-3.031	-1.000	-2.792	1.033	0.031	1.567	-1.150	-1.146	0.957
Shanxi	-0.135	-0.239	-0.015	-0.194	1.231	0.243	0.348	0.342	0.613	1.183
Inner Mongolia	-0.184	-0.254	-0.054	-0.246	1.364	0.074	-0.678	0.465	0.689	1.236
Liaoning	-1.311	-1.677	0.232	-1.222	1.041	3.403	-2.031	4.997	7.271	1.012
Jilin	0.460	0.703	-1.000	0.568	0.971	-0.140	0.297	-0.960	-1.969	0.961
Heilongjiang	-0.340	-0.408	0.144	-0.676	1.149	0.021	-1.241	0.636	-0.126	1.115
Shanghai	-1.000	-1.470	-1.287	-1.092	1.038	-0.393	-0.677	-1.015	-0.933	0.938
Jiangsu	-2.159	-2.511	-1.000	-2.095	1.028	-0.785	-0.517	-1.102	-0.907	0.930
Zhejiang	-1.188	-1.425	-0.718	-1.535	1.052	-0.721	-1.223	-0.905	-2.282	0.910
Anhui	-2.988	-3.848	0.032	-3.535	1.023	-0.844	-1.964	-1.491	-1.119	0.932
Fujian	-0.481	-0.515	-0.397	-0.390	1.116	0.668	-0.443	0.624	0.856	1.049
Jiangxi	-1.411	-2.239	-0.958	-2.928	1.039	-0.104	-0.357	-1.000	-0.824	0.901
Shandong	-5.062	-5.876	-1.000	-5.094	1.015	-0.460	0.711	-0.865	-1.603	0.916
Henan	-4.723	-6.252	-0.608	-5.158	1.014	-0.169	0.034	-1.094	-0.719	0.925
Hubei	-11.522	-15.648	-1.000	-12.719	1.003	-0.118	1.762	-0.875	-1.054	0.927
Hunan	13.390	11.960	-1.000	10.913	0.994	-0.388	-1.089	-0.887	-1.041	0.945
Guangdong	-2.320	-3.181	-1.000	-2.216	1.026	-0.231	-0.185	-0.437	-1.000	0.881
Guangxi	-0.861	-1.376	-0.511	-1.374	1.053	-0.472	1.058	-0.754	-1.603	0.941
Hainan	-2.680	-4.086	-0.247	-2.397	1.017	-0.526	-11.478	-1.473	-4.139	0.962
Chongqing	-2.108	-3.394	-1.000	-2.900	1.027	-0.418	-0.964	-0.815	-1.151	0.901
Sichuan	-1.246	-2.391	-1.000	-2.044	1.041	-0.431	-0.949	-1.000	-0.998	0.908
Guizhou	-1.018	-1.480	-0.617	-1.162	1.092	-0.977	0.139	-1.013	-1.247	0.905
Yunnan	-1.159	-2.501	-0.797	-1.044	1.080	-0.403	0.139	-0.797	-1.108	0.899
Tibet	12.289	13.152	-1.000	8.830	0.988	-0.308	-1.000	0.088	-0.330	0.672
Shaanxi	-0.445	-0.443	-0.233	-0.361	1.208	2.073	2.237	5.528	7.613	1.018
Gansu	-1.427	-3.397	-0.796	-2.049	1.044	-0.363	-0.864	-1.022	-0.829	0.917
Qinghai	-1.620	-1.787	-1.000	-1.254	1.081	-0.212	-1.070	-0.305	-0.458	0.807
Ningxia	-1.840	-5.123	-1.000	-2.632	1.035	-0.572	3.695	-1.000	-0.123	0.907
Xinjiang	-0.640	-0.846	0.867	-1.294	1.052	-0.565	0.395	-0.501	-2.551	0.953

  

2016-2017										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	9.680	9.586	6.185	5.340	0.994	10.830	-165.306	-15.371	-12.058	0.997
Tianjin	1.000	0.680	0.358	0.458	0.920	-0.019	-19.045	2.275	-2.698	0.989
Hebei	0.806	1.000	0.330	0.921	0.906	0.020	1.000	-0.734	-0.731	0.934
Shanxi	-0.215	-0.382	-0.024	-0.310	1.139	0.165	0.236	0.232	0.416	1.281
Inner Mongolia	-0.390	-0.538	-0.113	-0.522	1.158	0.054	-0.496	0.340	0.504	1.336
Liaoning	-2.153	-2.755	0.381	-2.007	1.025	0.542	-0.324	0.796	1.158	1.078
Jilin	0.817	1.247	-1.776	1.008	0.984	-0.469	1.000	-3.230	-6.623	0.988
Heilongjiang	-0.615	-0.739	0.261	-1.223	1.080	0.019	-1.106	0.567	-0.112	1.130
Shanghai	0.680	1.000	0.875	0.743	0.946	1.000	1.721	2.583	2.373	1.025
Jiangsu	1.061	1.234	0.491	1.029	0.946	1.057	0.697	1.485	1.221	1.055
Zhejiang	0.774	0.929	0.468	1.000	0.925	1.000	1.696	1.255	3.164	1.071
Anhui	1.276	1.644	-0.014	1.510	0.947	0.722	1.680	1.275	0.957	1.085
Fujian	-2.318	-2.484	-1.915	-1.879	1.023	0.338	-0.224	0.316	0.433	1.098
Jiangxi	0.482	0.765	0.327	1.000	0.894	0.349	1.206	3.375	2.782	1.032
Shandong	0.861	1.000	0.170	0.867	0.914	-1.024	1.584	-1.927	-3.571	0.961
Henan	0.755	1.000	0.097	0.825	0.919	5.180	-1.040	33.516	22.026	1.003
Hubei	0.819	1.112	0.071	0.904	0.954	-0.705	10.487	-5.208	-6.276	0.987
Hunan	1.024	0.915	-0.076	0.835	0.930	0.539	1.513	1.232	1.446	1.042
Guangdong	0.729	1.000	0.314	0.697	0.922	1.106	0.886	2.086	4.779	1.027
Guangxi	0.626	1.001	0.372	0.999	0.931	1.173	-2.630	1.875	3.984	1.025
Hainan	1.057	1.612	0.098	0.946	0.958	0.569	12.402	1.592	4.472	1.036
Chongqing	0.621	1.000	0.295	0.854	0.912	0.663	1.528	1.292	1.825	1.068
Sichuan	0.521	1.000	0.418	0.855	0.909	0.642	1.414	1.490	1.487	1.067
Guizhou	1.146	1.666	0.694	1.308	0.925	0.834	-0.119	0.865	1.065	1.125
Yunnan	0.538	1.162	0.370	0.485	0.847	0.764	-0.264	1.511	2.100	1.058
Tibet	0.934	1.000	-0.076	0.671	0.852	-3.508	-11.377	1.000	-3.753	0.966
Shaanxi	-4.471	-4.449	-2.341	-3.623	1.019	0.179	0.194	0.478	0.659	1.229
Gansu	0.472	1.125	0.263	0.678	0.879	0.636	1.512	1.788	1.451	1.051
Qinghai	0.975	1.076	0.602	0.755	0.879	0.721	3.629	1.034	1.552	1.065
Ningxia	0.509	1.417	0.277	0.728	0.882	-1.184	7.653	-2.071	-0.254	0.954
Xinjiang	0.558	0.737	-0.756	1.128	0.943	0.746	-0.522	0.661	3.370	1.037

Table 7: Dynamical deviation under Multiplicative technology (2016-2017)

2016-2017										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-1.813	-1.795	-1.158	-1.000	1.030	0.066	-1.000	-0.093	-0.073	0.570
Tianjin	-2.792	-1.900	-1.000	-1.278	1.030	-0.001	-1.000	0.119	-0.142	0.812
Hebei	-2.444	-3.031	-1.000	-2.792	1.033	0.027	1.362	-1.000	-0.996	0.951
Shanxi	-8.996	-15.967	-1.000	-12.979	1.003	-0.396	-0.567	-0.557	-1.000	0.902
Inner Mongolia	-3.440	-4.745	-1.000	-4.598	1.017	-0.107	0.985	-0.675	-1.000	0.864
Liaoning	5.658	7.240	-1.000	5.273	0.991	-0.468	0.279	-0.687	-1.000	0.917
Jilin	0.460	0.703	-1.000	0.568	0.971	-0.071	0.151	-0.488	-1.000	0.924
Heilongjiang	2.355	2.829	-1.000	4.683	0.980	-0.033	1.951	-1.000	0.198	0.933
Shanghai	-1.000	-1.470	-1.287	-1.092	1.038	-0.387	-0.667	-1.000	-0.919	0.937
Jiangsu	-2.159	-2.511	-1.000	-2.095	1.028	-0.712	-0.469	-1.000	-0.822	0.923
Zhejiang	-1.655	-1.985	-1.000	-2.137	1.037	-0.316	-0.536	-0.397	-1.000	0.806
Anhui	91.980	118.463	-1.000	108.829	0.999	-0.430	-1.000	-0.759	-0.570	0.871
Fujian	-1.234	-1.322	-1.019	-1.000	1.044	-0.781	0.517	-0.729	-1.000	0.960
Jiangxi	-1.474	-2.338	-1.000	-3.058	1.037	-0.104	-0.357	-1.000	-0.824	0.901
Shandong	-5.062	-5.876	-1.000	-5.094	1.015	-0.287	0.444	-0.539	-1.000	0.869
Henan	-7.772	-10.289	-1.000	-8.489	1.008	-0.155	0.031	-1.000	-0.657	0.918
Hubei	-11.522	-15.648	-1.000	-12.719	1.003	-0.112	1.671	-0.830	-1.000	0.923
Hunan	13.390	11.960	-1.000	10.913	0.994	-0.356	-1.000	-0.814	-0.956	0.940
Guangdong	-2.320	-3.181	-1.000	-2.216	1.026	-0.231	-0.185	-0.437	-1.000	0.881
Guangxi	-1.683	-2.691	-1.000	-2.687	1.027	-0.294	0.660	-0.471	-1.000	0.907
Hainan	-10.829	-16.511	-1.000	-9.684	1.004	-0.072	-1.559	-0.200	-0.562	0.753
Chongqing	-2.108	-3.394	-1.000	-2.900	1.027	-0.363	-0.838	-0.708	-1.000	0.887
Sichuan	-1.246	-2.391	-1.000	-2.044	1.041	-0.431	-0.949	-1.000	-0.998	0.908
Guizhou	-1.650	-2.399	-1.000	-1.883	1.056	-0.784	0.112	-0.812	-1.000	0.882
Yunnan	-1.454	-3.138	-1.000	-1.310	1.063	-0.364	0.126	-0.719	-1.000	0.888
Tibet	12.289	13.152	-1.000	8.830	0.988	-0.308	-1.000	0.088	-0.330	0.672
Shaanxi	-1.910	-1.901	-1.000	-1.548	1.045	-0.272	-0.294	-0.726	-1.000	0.873
Gansu	-1.793	-4.270	-1.000	-2.575	1.034	-0.356	-0.846	-1.000	-0.812	0.915
Qinghai	-1.620	-1.787	-1.000	-1.254	1.081	-0.199	-1.000	-0.285	-0.428	0.795
Ningxia	-1.840	-5.123	-1.000	-2.632	1.035	-0.572	3.695	-1.000	-0.123	0.907
Xinjiang	0.738	0.975	-1.000	1.492	0.957	-0.221	0.155	-0.196	-1.000	0.885

  

2016-2017										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.990	0.639	0.552	0.947	1.000	-15.263	-1.419	-1.113	0.964
Tianjin	1.000	0.680	0.358	0.458	0.920	-0.008	-8.372	1.000	-1.186	0.975
Hebei	0.806	1.000	0.330	0.921	0.906	0.020	1.000	-0.734	-0.731	0.934
Shanxi	0.563	1.000	0.063	0.813	0.952	1.000	1.433	1.407	2.526	1.042
Inner Mongolia	0.725	1.000	0.211	0.969	0.924	0.224	-2.051	1.405	2.083	1.073
Liaoning	0.782	1.000	-0.138	0.728	0.935	-1.675	1.000	-2.460	-3.579	0.976
Jilin	0.655	1.000	-1.423	0.808	0.980	-0.469	1.000	-3.230	-6.623	0.988
Heilongjiang	0.503	0.604	-0.214	1.000	0.910	-0.017	1.000	-0.512	0.101	0.873
Shanghai	0.680	1.000	0.875	0.743	0.946	1.000	1.721	2.583	2.373	1.025
Jiangsu	0.860	1.000	0.398	0.834	0.934	1.518	1.000	2.132	1.753	1.038
Zhejiang	0.883	1.059	0.534	1.140	0.934	1.000	1.696	1.255	3.164	1.071
Anhui	0.776	1.000	-0.008	0.919	0.915	1.000	2.327	1.766	1.325	1.061
Fujian	0.933	1.000	0.771	0.757	0.945	-1.509	1.000	-1.410	-1.934	0.979
Jiangxi	0.482	0.765	0.327	1.000	0.894	1.000	3.450	9.659	7.962	1.011
Shandong	0.861	1.000	0.170	0.867	0.914	-0.646	1.000	-1.216	-2.254	0.939
Henan	0.755	1.000	0.097	0.825	0.919	-4.979	1.000	-32.220	-21.174	0.997
Hubei	0.736	1.000	0.064	0.813	0.949	-0.067	1.000	-0.497	-0.598	0.875
Hunan	1.000	0.893	-0.075	0.815	0.928	1.000	2.806	2.286	2.682	1.022
Guangdong	0.729	1.000	0.314	0.697	0.922	1.249	1.000	2.355	5.395	1.024
Guangxi	0.626	1.000	0.372	0.998	0.931	-0.446	1.000	-0.713	-1.515	0.937
Hainan	0.656	1.000	0.061	0.587	0.933	1.000	21.802	2.799	7.862	1.021
Chongqing	0.621	1.000	0.295	0.854	0.912	1.000	2.305	1.947	2.752	1.044
Sichuan	0.521	1.000	0.418	0.855	0.909	1.000	2.203	2.321	2.317	1.043
Guizhou	0.688	1.000	0.417	0.785	0.877	-7.011	1.000	-7.269	-8.947	0.986
Yunnan	0.463	1.000	0.319	0.417	0.825	-2.896	1.000	-5.727	-7.962	0.985
Tibet	0.934	1.000	-0.076	0.671	0.852	-3.508	-11.377	1.000	-3.753	0.966
Shaanxi	1.000	0.995	0.523	0.810	0.919	1.000	1.079	2.666	3.672	1.038
Gansu	0.420	1.000	0.234	0.603	0.865	1.000	2.378	2.812	2.283	1.032
Qinghai	0.906	1.000	0.560	0.702	0.871	1.000	5.033	1.434	2.153	1.047
Ningxia	0.359	1.000	0.195	0.514	0.837	-0.155	1.000	-0.271	-0.033	0.698
Xinjiang	0.495	0.654	-0.670	1.000	0.936	-1.429	1.000	-1.267	-6.455	0.981

Table 8: Dynamical deviation under FDH technology (2016-2017)

2015-2016										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-0.617	-0.462	-0.239	-0.609	1.081	-1.726	-0.534	-3.275	-4.919	0.964
Tianjin	-18.113	-22.958	-1.000	-11.539	1.003	-0.146	-1.102	-0.711	-1.127	0.903
Hebei	8.444	10.921	-1.000	7.423	0.993	-0.364	0.248	-1.177	-0.920	0.903
Shanxi	-0.071	-0.433	-0.164	-0.150	1.261	0.216	0.165	0.768	0.598	1.169
Inner Mongolia	-0.110	-0.271	-0.059	-0.129	1.355	0.148	0.123	0.507	0.406	1.236
Liaoning	-1.227	-1.792	0.131	-1.596	1.040	-7.314	-1.836	-12.131	-16.850	0.994
Jilin	-12.472	-27.650	-1.000	-15.673	1.003	-0.387	-0.769	-1.206	-0.480	0.938
Heilongjiang	-0.149	-0.383	0.011	-0.249	1.149	0.701	-0.414	1.569	1.226	1.060
Shanghai	-1.000	-1.381	-1.435	-1.360	1.038	-0.105	1.239	-0.607	-1.000	0.862
Jiangsu	2.761	2.842	-1.000	2.420	0.972	-0.203	-1.079	-1.461	-1.763	0.952
Zhejiang	-1.194	-1.692	-0.510	-1.194	1.053	-0.708	-1.035	-1.467	-1.435	0.936
Anhui	-1.880	-2.408	-0.729	-2.188	1.024	-0.182	-0.485	-1.814	-1.055	0.966
Fujian	-0.220	-0.570	0.039	-0.105	1.102	0.555	-0.658	0.345	2.276	1.068
Jiangxi	-1.000	-2.090	-1.272	-1.815	1.031	-0.406	0.285	-0.720	-1.620	0.938
Shandong	-2.715	-4.553	-1.000	-3.417	1.012	-0.060	-0.678	-0.928	-1.404	0.892
Henan	-2.109	-4.105	-1.000	-3.315	1.019	-0.676	0.641	-0.995	-1.446	0.937
Hubei	-2.595	-3.625	-1.000	-3.115	1.016	-0.115	-0.602	-0.580	-1.238	0.874
Hunan	14.320	17.861	-1.000	15.863	0.996	-0.327	-0.759	-0.909	-1.069	0.924
Guangdong	-3.008	-5.324	-1.000	-3.150	1.021	-0.308	0.204	-0.624	-1.000	0.893
Guangxi	-1.505	-2.227	-0.277	-1.268	1.036	-0.171	0.706	-0.539	-1.963	0.938
Hainan	-3.256	-5.079	-1.000	-2.976	1.014	-0.926	1.324	-0.956	-2.083	0.948
Chongqing	-1.451	-2.491	-1.000	-1.922	1.042	-0.475	1.009	-1.000	-0.985	0.936
Sichuan	-1.000	-4.012	-1.203	-2.828	1.022	-0.317	-0.633	-0.763	-1.176	0.915
Guizhou	-1.109	-1.616	-0.462	-0.893	1.079	-0.588	-0.643	-0.537	-1.323	0.923
Yunnan	-1.216	-2.040	-0.776	-1.042	1.064	-0.587	-0.713	-0.774	-1.092	0.890
Tibet	0.563	1.891	-1.000	0.336	0.909	-0.151	1.502	-0.705	-1.000	0.785
Shaanxi	-0.392	-0.592	-0.282	-0.323	1.211	1.746	-2.066	2.500	3.779	1.034
Gansu	-2.191	-1.952	0.864	-1.783	1.029	-0.502	0.005	-1.508	-1.100	0.918
Qinghai	-1.000	8.929	7.007	0.760	0.991	-0.324	-0.867	-0.897	-1.093	0.914
Ningxia	-11.093	-18.026	-1.000	-11.205	1.006	-0.559	-0.633	-0.697	-1.000	0.885
Xinjiang	-1.087	-1.205	-1.252	-0.916	1.048	-1.097	-2.909	-0.930	-1.520	0.954

  

2015-2016										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.013	0.759	0.392	1.000	0.953	1.010	0.313	1.917	2.879	1.065
Tianjin	0.789	1.000	0.044	0.503	0.937	-0.155	1.170	-0.755	-1.197	0.908
Hebei	0.773	1.000	-0.092	0.680	0.926	-1.470	1.000	-4.748	-3.709	0.975
Shanxi	-0.099	-0.606	-0.229	-0.210	1.180	0.107	0.082	0.381	0.297	1.370
Inner Mongolia	-0.170	-0.416	-0.091	-0.198	1.219	0.089	0.075	0.306	0.245	1.420
Liaoning	-1.118	-1.632	0.120	-1.453	1.044	0.336	0.084	0.557	0.774	1.132
Jilin	0.862	1.910	0.069	1.083	0.959	0.645	1.283	2.013	0.800	1.039
Heilongjiang	-0.434	-1.117	0.032	-0.725	1.049	0.286	-0.169	0.639	0.500	1.153
Shanghai	0.697	0.963	1.000	0.947	0.948	-0.358	4.240	-2.078	-3.421	0.958
Jiangsu	2.802	2.885	-1.015	2.457	0.972	0.503	2.681	3.629	4.379	1.020
Zhejiang	0.706	1.000	0.301	0.706	0.916	1.000	1.462	2.072	2.028	1.048
Anhui	4.005	5.131	1.552	4.662	0.989	0.098	0.262	0.980	0.570	1.066
Fujian	-0.379	-0.980	0.068	-0.180	1.058	0.402	-0.477	0.250	1.650	1.095
Jiangxi	0.495	1.036	0.630	0.899	0.940	1.612	-1.129	2.856	6.427	1.016
Shandong	0.596	1.000	0.220	0.750	0.947	0.234	2.635	3.606	5.457	1.030
Henan	0.514	1.000	0.244	0.808	0.925	64.481	-61.194	94.917	138.037	1.001
Hubei	0.808	1.129	0.312	0.970	0.950	0.367	1.922	1.850	3.951	1.043
Hunan	0.802	1.000	-0.056	0.888	0.924	0.450	1.043	1.247	1.468	1.059
Guangdong	0.565	1.000	0.188	0.592	0.895	3.473	-2.305	7.034	11.275	1.010
Guangxi	0.861	1.274	0.158	0.726	0.939	0.698	-2.874	2.195	7.994	1.016
Hainan	1.200	1.871	0.368	1.097	0.963	0.903	-1.291	0.932	2.031	1.056
Chongqing	0.617	1.060	0.425	0.818	0.909	0.902	-1.917	1.901	1.873	1.035
Sichuan	0.249	1.000	0.300	0.705	0.918	0.610	1.220	1.470	2.264	1.048
Guizhou	0.904	1.317	0.376	0.728	0.910	0.901	0.986	0.823	2.030	1.054
Yunnan	0.596	1.000	0.381	0.511	0.880	0.893	1.085	1.177	1.662	1.079
Tibet	0.297	1.000	-0.529	0.178	0.835	-0.100	1.000	-0.470	-0.666	0.696
Shaanxi	-1.579	-2.388	-1.138	-1.303	1.049	0.265	-0.314	0.380	0.574	1.249
Gansu	1.000	0.891	-0.395	0.814	0.940	11.435	-0.124	34.386	25.074	1.004
Qinghai	-0.140	1.249	0.981	0.106	0.935	0.414	1.107	1.146	1.396	1.073
Ningxia	0.807	1.312	0.073	0.815	0.916	0.935	1.059	1.167	1.673	1.076
Xinjiang	1.022	1.134	1.178	0.862	0.951	1.179	3.127	1.000	1.634	1.045

Table 9: Dynamical deviation under Multiplicative technology (2015-2016)

2015-2016										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-2.582	-1.935	-1.000	-2.548	1.019	-0.351	-0.109	-0.666	-1.000	0.834
Tianjin	-18.113	-22.958	-1.000	-11.539	1.003	-0.129	0.978	-0.631	-1.000	0.891
Hebei	8.444	10.921	-1.000	7.423	0.993	-0.310	0.211	-1.000	-0.781	0.887
Shanxi	-1.000	-6.101	-2.306	-2.112	1.017	-0.281	-0.215	-1.000	-0.779	0.887
Inner Mongolia	-1.865	-4.570	-1.000	-2.172	1.018	-0.326	-0.273	-1.119	-0.896	0.908
Liaoning	9.334	13.626	-1.000	12.135	0.995	-0.434	-0.109	-0.720	-1.000	0.909
Jilin	-12.472	-27.650	-1.000	-15.673	1.003	-0.321	-0.637	-1.000	-0.398	0.926
Heilongjiang	13.623	35.106	-1.000	22.780	0.998	-0.447	0.264	-1.000	-0.782	0.913
Shanghai	-1.000	-1.381	-1.435	-1.360	1.038	-0.105	1.239	-0.607	-1.000	0.862
Jiangsu	2.761	2.842	-1.000	2.420	0.972	-0.115	-0.612	-0.829	-1.000	0.916
Zhejiang	-2.344	-3.319	-1.000	-2.344	1.027	-0.483	-0.706	-1.000	-0.979	0.908
Anhui	-2.580	-3.305	-1.000	-3.003	1.018	-0.100	-0.267	-1.000	-0.582	0.939
Fujian	5.604	14.488	-1.000	2.665	0.996	-0.244	0.289	-0.152	-1.000	0.861
Jiangxi	-1.000	-2.090	-1.272	-1.815	1.031	-0.251	0.176	-0.444	-1.000	0.902
Shandong	-2.715	-4.553	-1.000	-3.417	1.012	-0.043	-0.483	-0.661	-1.000	0.852
Henan	-2.109	-4.105	-1.000	-3.315	1.019	-0.467	0.443	-0.688	-1.000	0.910
Hubei	-2.595	-3.625	-1.000	-3.115	1.016	-0.093	-0.486	-0.468	-1.000	0.846
Hunan	14.320	17.861	-1.000	15.863	0.996	-0.306	-0.710	-0.850	-1.000	0.919
Guangdong	-3.008	-5.324	-1.000	-3.150	1.021	-0.308	0.204	-0.624	-1.000	0.893
Guangxi	-5.433	-8.041	-1.000	-4.579	1.010	-0.087	0.360	-0.275	-1.000	0.882
Hainan	-3.256	-5.079	-1.000	-2.976	1.014	-0.445	0.635	-0.459	-1.000	0.896
Chongqing	-1.451	-2.491	-1.000	-1.922	1.042	-0.475	1.009	-1.000	-0.985	0.936
Sichuan	-1.000	-4.012	-1.203	-2.828	1.022	-0.269	-0.539	-0.649	-1.000	0.900
Guizhou	-2.401	-3.500	-1.000	-1.934	1.036	-0.444	-0.486	-0.406	-1.000	0.899
Yunnan	-1.567	-2.628	-1.000	-1.342	1.050	-0.538	-0.653	-0.709	-1.000	0.881
Tibet	0.563	1.891	-1.000	0.336	0.909	-0.151	1.502	-0.705	-1.000	0.785
Shaanxi	-1.388	-2.099	-1.000	-1.145	1.055	-0.462	0.547	-0.662	-1.000	0.880
Gansu	2.535	2.258	-1.000	2.062	0.976	-0.333	0.004	-1.000	-0.729	0.879
Qinghai	-1.000	8.929	7.007	0.760	0.991	-0.297	-0.793	-0.821	-1.000	0.907
Ningxia	-11.093	-18.026	-1.000	-11.205	1.006	-0.559	-0.633	-0.697	-1.000	0.885
Xinjiang	-1.186	-1.316	-1.367	-1.000	1.044	-0.377	-1.000	-0.320	-0.522	0.872

  

2015-2016										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	1.000	0.749	0.387	0.987	0.953	3.231	1.000	6.132	9.208	1.020
Tianjin	0.789	1.000	0.044	0.503	0.937	-0.132	1.000	-0.645	-1.023	0.894
Hebei	0.773	1.000	-0.092	0.680	0.926	-1.470	1.000	-4.748	-3.709	0.975
Shanxi	0.164	1.000	0.378	0.346	0.904	1.308	1.000	4.660	3.630	1.026
Inner Mongolia	0.408	1.000	0.219	0.475	0.921	0.373	0.312	1.280	1.026	1.088
Liaoning	0.685	1.000	-0.073	0.891	0.932	3.984	1.000	6.609	9.180	1.011
Jilin	0.451	1.000	0.036	0.567	0.923	1.000	1.988	3.119	1.240	1.025
Heilongjiang	0.388	1.000	-0.028	0.649	0.948	-1.691	1.000	-3.786	-2.959	0.976
Shanghai	0.697	0.963	1.000	0.947	0.948	-0.084	1.000	-0.490	-0.807	0.832
Jiangsu	0.971	1.000	-0.352	0.852	0.922	1.000	5.328	7.214	8.704	1.010
Zhejiang	0.765	1.083	0.326	0.765	0.922	1.000	1.462	2.072	2.028	1.048
Anhui	0.781	1.000	0.303	0.909	0.944	1.000	2.661	9.953	5.790	1.006
Fujian	0.387	1.000	-0.069	0.184	0.946	-0.843	1.000	-0.524	-3.458	0.958
Jiangxi	0.478	1.000	0.608	0.868	0.938	-1.428	1.000	-2.530	-5.693	0.982
Shandong	0.596	1.000	0.220	0.750	0.947	1.000	11.264	15.415	23.325	1.007
Henan	0.514	1.000	0.244	0.808	0.925	-1.054	1.000	-1.551	-2.256	0.959
Hubei	0.716	1.000	0.276	0.859	0.944	1.000	5.230	5.034	10.753	1.016
Hunan	0.802	1.000	-0.056	0.888	0.924	1.000	2.320	2.775	3.266	1.026
Guangdong	0.565	1.000	0.188	0.592	0.895	-1.507	1.000	-3.051	-4.891	0.977
Guangxi	0.676	1.000	0.124	0.569	0.923	-0.243	1.000	-0.764	-2.782	0.956
Hainan	0.641	1.000	0.197	0.586	0.932	-0.700	1.000	-0.722	-1.574	0.932
Chongqing	0.583	1.000	0.402	0.772	0.904	-0.471	1.000	-0.992	-0.977	0.936
Sichuan	0.249	1.000	0.300	0.705	0.918	1.000	2.000	2.410	3.712	1.029
Guizhou	0.686	1.000	0.286	0.553	0.884	1.094	1.198	1.000	2.465	1.044
Yunnan	0.596	1.000	0.381	0.511	0.880	1.000	1.215	1.318	1.860	1.071
Tibet	0.297	1.000	-0.529	0.178	0.835	-0.100	1.000	-0.470	-0.666	0.696
Shaanxi	0.661	1.000	0.476	0.546	0.893	1.644	-1.945	2.354	3.558	1.037
Gansu	1.000	0.891	-0.395	0.814	0.940	-91.996	1.000	-276.647	-201.728	0.999
Qinghai	-0.112	1.000	0.785	0.085	0.919	1.000	2.674	2.769	3.371	1.030
Ningxia	0.615	1.000	0.055	0.622	0.891	1.000	1.133	1.248	1.790	1.071
Xinjiang	0.868	0.963	1.000	0.732	0.943	1.179	3.127	1.000	1.634	1.045

Table 10: Dynamical deviation under FDH technology (2015-2016)

2014-2015										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-0.904	-0.938	-0.249	-0.229	1.074	0.473	-3.514	0.542	-0.262	1.045
Tianjin	-1.672	-1.497	-1.085	-1.000	1.046	-0.205	-1.364	-0.133	-0.120	0.901
Hebei	-4.111	-6.269	-1.000	-4.188	1.014	-1.150	1.479	-0.487	4.985	0.978
Shanxi	-0.021	-0.245	0.012	-0.152	1.236	-0.054	-0.214	-0.088	0.098	1.337
Inner Mongolia	-0.118	-0.284	-0.133	-0.139	1.309	0.023	-0.346	-0.050	-0.129	1.358
Liaoning	-0.344	-0.566	0.024	-0.520	1.088	0.069	-0.157	0.166	0.035	1.111
Jilin	-6.331	-5.767	-1.000	-2.530	1.010	-0.318	-3.621	0.774	-0.635	1.023
Heilongjiang	-0.113	-0.297	0.205	-0.391	1.150	-0.263	-0.579	0.283	-0.244	1.196
Shanghai	-1.298	-2.129	-1.000	-1.976	1.023	-0.416	3.159	-0.792	-1.000	0.938
Jiangsu	-1.974	-2.651	-1.000	-1.810	1.030	-0.840	1.334	-1.154	-1.111	0.953
Zhejiang	-0.986	-1.173	-0.350	-1.245	1.086	-1.000	0.022	-0.896	-0.660	0.951
Anhui	-1.353	-2.360	-0.590	-2.070	1.029	0.182	1.520	-1.034	-3.642	0.975
Fujian	-0.327	-0.505	0.227	-0.450	1.114	0.011	-0.691	-0.207	0.417	1.102
Jiangxi	-1.000	-2.180	-1.421	-1.987	1.029	0.324	0.415	-0.606	-1.815	0.964
Shandong	11.222	14.403	-1.000	15.613	0.998	1.781	6.764	-1.038	0.134	0.987
Henan	-3.682	-5.540	-1.000	-4.904	1.013	-0.405	1.347	-1.049	-1.804	0.965
Hubei	-3.568	-6.423	-1.000	-3.661	1.021	-0.128	-0.016	-0.589	-1.283	0.919
Hunan	7.941	5.948	-1.000	7.151	0.985	-0.346	-0.965	-0.935	-1.063	0.937
Guangdong	-2.609	-4.197	-1.000	-3.530	1.020	-0.160	2.230	-0.614	-1.000	0.946
Guangxi	-1.137	-1.683	-0.518	-1.231	1.052	-0.869	4.824	0.132	-3.003	0.985
Hainan	-2.066	-2.639	-1.000	-3.101	1.038	-0.458	4.230	-1.177	-1.769	0.939
Chongqing	-0.941	-2.128	-1.229	-1.824	1.053	-0.765	0.475	-1.000	-0.972	0.933
Sichuan	-1.000	-7.449	-2.932	-5.559	1.011	-0.425	2.056	-0.657	-1.233	0.963
Guizhou	-1.209	-1.609	-0.816	-0.981	1.080	-0.537	1.731	0.302	-1.565	0.968
Yunnan	-1.000	-2.203	-1.535	-1.013	1.056	-0.723	2.350	-0.416	-1.159	0.936
Tibet	-1.611	-2.689	-1.000	-2.572	1.065	-0.150	-0.923	-1.000	-0.144	0.791
Shaanxi	-0.189	-0.346	-0.165	-0.299	1.226	-0.004	-0.004	0.412	0.313	1.126
Gansu	-2.398	-3.027	2.068	-2.602	1.016	-0.544	0.045	-1.190	-2.245	0.968
Qinghai	-1.785	-0.920	-1.087	-1.329	1.034	-0.090	-1.708	0.417	0.065	0.936
Ningxia	2.797	3.078	-1.000	1.883	0.979	-0.282	0.143	-1.000	7.580	0.984
Xinjiang	-0.514	-0.763	-0.522	-0.573	1.091	-6.937	-21.825	-9.213	-13.218	0.995

  

2014-2015										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	-4.471	-4.640	-1.232	-1.132	1.014	0.363	-2.699	0.417	-0.201	1.060
Tianjin	1.000	0.895	0.649	0.598	0.928	1.544	10.263	1.002	0.907	1.014
Hebei	0.656	1.000	0.160	0.668	0.917	-0.279	0.358	-0.118	1.208	0.912
Shanxi	-0.023	-0.263	0.013	-0.164	1.217	-0.070	-0.279	-0.115	0.127	1.249
Inner Mongolia	-0.139	-0.336	-0.158	-0.164	1.255	0.028	-0.416	-0.060	-0.155	1.290
Liaoning	-0.622	-1.026	0.044	-0.941	1.048	0.122	-0.280	0.295	0.062	1.061
Jilin	1.597	1.455	0.252	0.638	0.962	0.093	1.060	-0.226	0.186	0.926
Heilongjiang	-0.204	-0.535	0.369	-0.703	1.081	-0.672	-1.481	0.722	-0.623	1.072
Shanghai	0.610	1.000	0.470	0.928	0.954	-0.378	2.866	-0.718	-0.907	0.932
Jiangsu	1.172	1.574	0.594	1.075	0.952	1.080	-1.715	1.484	1.429	1.038
Zhejiang	0.792	0.942	0.281	1.000	0.902	1.242	-0.027	1.113	0.819	1.041
Anhui	1.159	2.021	0.505	1.773	0.967	-0.149	-1.244	0.846	2.981	1.031
Fujian	-0.722	-1.117	0.502	-0.995	1.050	0.013	-0.819	-0.246	0.495	1.086
Jiangxi	0.468	1.020	0.665	0.930	0.940	1.714	2.199	-3.206	-9.609	0.993
Shandong	0.719	0.923	-0.064	1.000	0.964	0.357	1.356	-0.208	0.027	0.939
Henan	0.665	1.000	0.181	0.885	0.930	-0.567	1.887	-1.470	-2.527	0.975
Hubei	0.945	1.701	0.265	0.969	0.923	0.448	0.057	2.060	4.484	1.025
Hunan	1.000	0.749	-0.126	0.901	0.885	0.465	1.297	1.256	1.429	1.050
Guangdong	0.622	1.000	0.238	0.841	0.919	-0.174	2.421	-0.666	-1.086	0.950
Guangxi	0.750	1.109	0.341	0.812	0.926	-0.815	4.526	0.124	-2.817	0.984
Hainan	0.671	0.856	0.325	1.006	0.892	0.980	-9.044	2.517	3.782	1.030
Chongqing	0.443	1.001	0.578	0.858	0.896	1.683	-1.046	2.200	2.139	1.032
Sichuan	0.134	1.000	0.394	0.746	0.921	1.610	-7.780	2.486	4.668	1.010
Guizhou	0.800	1.065	0.540	0.649	0.891	-0.788	2.540	0.443	-2.297	0.978
Yunnan	0.454	1.000	0.697	0.460	0.887	11.858	-38.518	6.824	18.993	1.004
Tibet	0.599	1.000	0.372	0.957	0.844	1.043	6.414	6.952	1.000	1.034
Shaanxi	-0.535	-0.981	-0.468	-0.846	1.075	-0.002	-0.003	0.275	0.209	1.194
Gansu	0.917	1.158	-0.791	0.995	0.958	1.440	-0.118	3.149	5.942	1.012
Qinghai	1.000	0.515	0.609	0.744	0.942	-0.215	-4.095	1.000	0.156	0.973
Ningxia	0.954	1.050	-0.341	0.642	0.941	-0.037	0.019	-0.132	1.000	0.888
Xinjiang	0.792	1.178	0.806	0.884	0.945	0.755	2.377	1.003	1.440	1.043

Table 11: Dynamical deviation under Multiplicative technology (2014-2015)

2014-2015										
$\rho_{t(t+1)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_t(x_{t+1}, y_t; h_{t+1}, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_t(x_t, y_{t+1}; 0, k_{t+1})$
Beijing	-3.950	-4.099	-1.089	-1.000	1.016	-0.871	6.478	-1.000	0.483	0.976
Tianjin	-1.672	-1.497	-1.085	-1.000	1.046	-0.150	-1.000	-0.098	-0.088	0.867
Hebei	-4.111	-6.269	-1.000	-4.188	1.014	-1.000	1.286	-0.424	4.334	0.975
Shanxi	1.802	20.979	-1.000	13.045	0.998	0.552	2.192	0.901	-1.000	0.972
Inner Mongolia	-1.000	-2.413	-1.133	-1.178	1.032	-1.000	14.811	2.138	5.537	0.993
Liaoning	14.215	23.438	-1.000	21.507	0.998	-0.414	0.948	-1.000	-0.212	0.983
Jilin	-6.331	-5.767	-1.000	-2.530	1.010	0.411	4.680	-1.000	0.821	0.983
Heilongjiang	0.553	1.452	-1.000	1.906	0.972	0.930	2.051	-1.000	0.863	0.951
Shanghai	-1.298	-2.129	-1.000	-1.976	1.023	-0.416	3.159	-0.792	-1.000	0.938
Jiangsu	-1.974	-2.651	-1.000	-1.810	1.030	-0.728	1.156	-1.000	-0.963	0.946
Zhejiang	-2.821	-3.354	-1.000	-3.562	1.029	-1.000	0.022	-0.896	-0.660	0.951
Anhui	-2.294	-4.001	-1.000	-3.509	1.017	0.050	0.417	-0.284	-1.000	0.912
Fujian	1.440	2.227	-1.000	1.984	0.976	-0.027	1.656	0.497	-1.000	0.960
Jiangxi	-1.000	-2.180	-1.421	-1.987	1.029	0.178	0.229	-0.334	-1.000	0.936
Shandong	11.222	14.403	-1.000	15.613	0.998	1.716	6.515	-1.000	0.129	0.987
Henan	-3.682	-5.540	-1.000	-4.904	1.013	-0.224	0.747	-0.582	-1.000	0.938
Hubei	-3.568	-6.423	-1.000	-3.661	1.021	-0.100	-0.013	-0.459	-1.000	0.897
Hunan	7.941	5.948	-1.000	7.151	0.985	-0.326	-0.908	-0.879	-1.000	0.933
Guangdong	-2.609	-4.197	-1.000	-3.530	1.020	-0.160	2.230	-0.614	-1.000	0.946
Guangxi	-2.196	-3.249	-1.000	-2.377	1.026	-0.289	1.606	0.044	-1.000	0.956
Hainan	-2.066	-2.639	-1.000	-3.101	1.038	-0.259	2.391	-0.665	-1.000	0.894
Chongqing	-1.000	-2.261	-1.306	-1.938	1.050	-0.765	0.475	-1.000	-0.972	0.933
Sichuan	-1.000	-7.449	-2.932	-5.559	1.011	-0.345	1.667	-0.533	-1.000	0.954
Guizhou	-1.481	-1.970	-1.000	-1.202	1.064	-0.343	1.106	0.193	-1.000	0.950
Yunnan	-1.000	-2.203	-1.535	-1.013	1.056	-0.624	2.028	-0.359	-1.000	0.926
Tibet	-1.611	-2.689	-1.000	-2.572	1.065	-0.150	-0.923	-1.000	-0.144	0.791
Shaanxi	-1.143	-2.095	-1.000	-1.807	1.034	0.009	0.010	-1.000	-0.759	0.952
Gansu	1.159	1.464	-1.000	1.258	0.967	-0.242	0.020	-0.530	-1.000	0.930
Qinghai	-1.941	-1.000	-1.182	-1.445	1.031	-0.053	-1.000	0.244	0.038	0.893
Ningxia	2.797	3.078	-1.000	1.883	0.979	-0.282	0.143	-1.000	7.580	0.984
Xinjiang	-1.000	-1.487	-1.017	-1.116	1.046	-0.318	-1.000	-0.422	-0.606	0.904

  

2014-2015										
$\rho_{t+1(t)}$	$x_1$	$x_2$	$x_3$	$x_4$	$M_{t+1}(x_t, y_{t+1}; h_t, 0)$	$y_1$	$y_2$	$y_3$	$y_4$	$M_{t+1}(x_{t+1}, y_t; 0, k_t)$
Beijing	0.964	1.000	0.266	0.244	0.936	-0.134	1.000	-0.154	0.075	0.856
Tianjin	1.000	0.895	0.649	0.598	0.928	1.703	11.321	1.106	1.000	1.013
Hebei	0.656	1.000	0.160	0.668	0.917	-0.231	0.297	-0.098	1.000	0.895
Shanxi	0.086	1.000	-0.048	0.622	0.950	0.252	1.000	0.411	-0.456	0.940
Inner Mongolia	0.414	1.000	0.470	0.488	0.926	-0.144	2.137	0.308	0.799	0.952
Liaoning	0.607	1.000	-0.043	0.918	0.953	-0.436	1.000	-1.054	-0.223	0.984
Jilin	1.000	0.911	0.158	0.400	0.940	0.088	1.000	-0.214	0.175	0.921
Heilongjiang	0.290	0.762	-0.525	1.000	0.947	0.454	1.000	-0.488	0.421	0.902
Shanghai	0.610	1.000	0.470	0.928	0.954	-0.132	1.000	-0.251	-0.317	0.817
Jiangsu	0.745	1.000	0.377	0.683	0.925	-0.630	1.000	-0.865	-0.833	0.938
Zhejiang	1.055	1.254	0.374	1.332	0.925	-45.394	1.000	-40.690	-29.949	0.999
Anhui	0.573	1.000	0.250	0.877	0.935	0.120	1.000	-0.680	-2.396	0.963
Fujian	0.646	1.000	-0.449	0.891	0.947	-0.016	1.000	0.300	-0.604	0.935
Jiangxi	0.459	1.000	0.652	0.912	0.939	0.780	1.000	-1.458	-4.370	0.985
Shandong	0.719	0.923	-0.064	1.000	0.964	0.263	1.000	-0.153	0.020	0.918
Henan	0.665	1.000	0.181	0.885	0.930	-0.301	1.000	-0.779	-1.339	0.954
Hubei	0.556	1.000	0.156	0.570	0.873	7.820	1.000	35.959	78.278	1.001
Hunan	1.000	0.749	-0.126	0.901	0.885	1.000	2.786	2.699	3.070	1.023
Guangdong	0.622	1.000	0.238	0.841	0.919	-0.072	1.000	-0.275	-0.448	0.883
Guangxi	0.676	1.000	0.308	0.732	0.919	-0.180	1.000	0.027	-0.623	0.931
Hainan	0.666	0.851	0.322	1.000	0.891	-0.114	1.054	-0.293	-0.441	0.776
Chongqing	0.442	1.000	0.578	0.857	0.896	-1.609	1.000	-2.104	-2.045	0.967
Sichuan	0.134	1.000	0.394	0.746	0.921	-0.207	1.000	-0.320	-0.600	0.925
Guizhou	0.751	1.000	0.508	0.610	0.884	-0.310	1.000	0.174	-0.904	0.945
Yunnan	0.454	1.000	0.697	0.460	0.887	-0.308	1.000	-0.177	-0.493	0.856
Tibet	0.599	1.000	0.372	0.957	0.844	1.043	6.414	6.952	1.000	1.034
Shaanxi	0.546	1.000	0.477	0.862	0.932	0.841	1.000	-98.347	-74.682	0.999
Gansu	0.792	1.000	-0.683	0.860	0.952	-12.184	1.000	-26.646	-50.282	0.999
Qinghai	1.000	0.515	0.609	0.744	0.942	-0.215	-4.095	1.000	0.156	0.973
Ningxia	0.909	1.000	-0.325	0.612	0.938	-0.037	0.019	-0.132	1.000	0.888
Xinjiang	0.673	1.000	0.684	0.751	0.936	1.000	3.146	1.328	1.906	1.033

Table 12: Dynamical deviation under FDH technology (2014-2015)