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CEE-M Working Paper 2021-15

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November 5, 2021

Abstract

Renewable energy generation and storage requires specialized capital goods, embedding critical raw materials (CRM). The scarcity of CRM therefore affects the transition from a fossil based energy system to one based on renewables, necessary to cope with climate change. We consider the issue in a theoretical model, where we allow for a very costly potential substitute, reflecting a backstop technology, and for partial and costly recycling of materials in capital goods. We characterize the main features of the efficient energy transition, and their dependence on the relative abundance of CRM and on the recycling technology. Recycling reduces the cost of the transition. It also calls for having a large stock of recyclable CRM embedded in specialized capital at the time of adoption of the backstop technology. Moreover, we consider constraints on policy tools and myopic regulation, and show how abstracting from the scarcity of CRM, or tightly linking subsidies for renewables to the carbon tax revenue, is misleading in designing climate policy.

JEL codes: Q42, Q53, Q38, Q48, E61

Key words: material scarcity, recycling, energy transition, policy acceptability, myopia.

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1 Introduction

The high technological and economic importance of Critical Raw Materials (CRM)[1], combined with concerns on their future availability hinging on geopolitical and geological factors, has led to increasing attention for CRM used for energy production from renewable sources. Indeed, to build the energy infrastructure essential to achieving greenhouse gas emission reduction targets, substantial amounts of mineral resources need to be mobilized (Vidal *et al.*, 2013, Hertwich *et al.*, 2015, Ali *et al.*, 2017, Luderer *et al.*, 2019). The latter observation is now widely shared, as evidenced by the works of the UN International Resource Panel (UNEP, 2020) or the World Bank (Arrobas *et al.*, 2017). The International Energy Agency (IEA, 2021) also points out that an energy system powered by low-carbon energy technologies needs significantly more minerals, notably copper, silicon and silver for solar PV.[2] There is currently no shortage of these mineral resources, but recent price rises for cobalt, copper, lithium and nickel highlight how supply could struggle to keep pace with world's climate ambitions. The European Commission has recently moved from concerns to policy with its Action Plan on Critical Raw Materials.[3] The criticality of these materials calls for recycling them and improving technologies that do not rely on CRM, which are the two pillars of a broad approach to minerals security.[4]

In a context where the vast majority of the existing theoretical literature ignores this issue, we investigate in this paper the role played by CRM in the energy transition. We consider in a theoretical model the scarcity of CRM, but allow for a very costly potential substitute, reflecting a backstop technology. These materials can be recycled to a certain extent and at a cost. First, we characterize the main features of the optimal energy transition, and their dependence on the relative abundance of CRM and on the recycling technology. Next, beyond optimal climate policies, we consider political economy constraints on social acceptability, then on the possibility of an imperfect information of the regulator, leading her to ignore the role of CRM when designing the climate policy.

On these issues, an example is instructive on how the topic is apprehended by public admin-

[1] Distinction must be made between CRM and rare earth elements. The European Commission (2017) defines CRM as "raw materials of high importance for the EU economy and whose supply is associated with a high risk". The same study defines them as "a set of 15 elements of the lanthanide series and two other elements: scandium and yttrium". Despite their name, rare earths are relatively abundant in the earth's crust. However, because of their geochemical properties, rare earths are generally dispersed and are not often concentrated in minerals.

[2] Lithium and rare earth elements are required for wind turbines and electric vehicle motors, copper and aluminium are necessary for electricity networks.

[3] See the report accompanying the presentation of the Action plan (European Commission, 2020) and the September 2020 *Communication from the Commission to the European parliament, the council, the European economic and social committee of the regions. Critical Raw Materials Resilience: Charting a Path towards greater Security and Sustainability.*

[4] The IEA (2021) identifies six pillars of a broad approach to minerals security, complementing countries' existing initiatives: (i) ensuring adequate investment in diversified sources of new supply; (ii) promoting technology innovation at all points along the value chain; (iii) scaling up recycling; (iv) enhancing supply chain resilience and market transparency; (v) mainstreaming higher environmental and social standards; and (vi) strengthening international collaboration between producers and consumers.

istrations. The French Low Carbon Transition Mineral Resources Programming Plan[5] considered four major families of low carbon technologies: solar photovoltaic (PV), stationary storage and networks (including smart grids), low carbon mobility (EV) and wind power. Within each family, this work compared the technologies potentially mature within 10 years, in terms of the mineral resources they use. The conclusions suggest that silver and cobalt are both crucial for the energy transition and characterized by geological scarcity. Silver is significantly mobilized in PV crystalline technologies, and its substitution possibilities are limited in the near term. Its supply can hardly be increased, while the fast growing activity in PV manufacturing is already the third largest user of silver. Cobalt is considered unavoidable for permanent magnets and in mainstream batteries.[6] Demand for cobalt increased sharply, with batteries accounting for less than 30% of total demand in 2000, but 60% in 2019. Two-thirds of the known resources are in the Copperbelt intra-sedimentary copper deposits, in the Democratic Republic of Congo and Zambia, where the ore grade is 10 times higher than elsewhere. In contrast with these conclusions, the 2020 French framework law for long term energy policy does not explicitly account for CRM embodied in the equipment and infrastructures for renewable electricity generation and storage. It only mentions that recycling should be considered.[7]

Our analytical approach links this article to the vast literature that considers the problem of intertemporal allocation of scarce natural resources (Hotelling, 1931), in particular fossil energies, in a macroeconomic framework (Dasgupta and Heal, 1974, Stiglitz, 1974, Solow, 1974). This literature has been extended to take into account an important change in perspective: the problem posed by fossil resources is not their physical scarcity but their carbon content and the temperature increase their combustion causes. The seminal contributions to the economics of climate change (Nordhaus, 1992, Ulph and Ulph, 1994) link the increase of atmospheric carbon concentration due to carbon emissions to economic damages. Chakravorty *et al.* (2006) instead consider as policy objective a ceiling on carbon concentration: in order to avoid climate-related catastrophic outcomes, atmospheric carbon concentration shall stay below a ceiling. However, insights in atmospheric physics (see Matthews *et al.*, 2009, Allen *et al.*, 2009, and Meinshausen *et al.*, 2009) indicate that the best predictor of the temperature anomaly is not the atmospheric carbon concentration but cumulative carbon emissions.[8] Therefore we frame the climate policy as an objective chosen by society in terms of a global cumulative carbon emission budget –the carbon budget–, to avoid excessive temperature increase (as in van der Ploeg and Rezai, 2020).

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[5] See `https://www.ecologie.gouv.fr/productivite-des-ressources#scroll-nav__3`

[6] See BRGM (2018). Permanent magnets are used in wind turbines and EV. Alternative designs for EV batteries are studied. For instance, Tesla announced the production of cobalt-free lithium iron phosphate batteries (Reuters, 2020).

[7] See p.185 of the consultation document "French strategy for energy and climate: multi annual energy plan" from the Ministère de la transition écologique et solidaire, 2020. Link to the source.

[8] The atmospheric carbon concentration is equal to cumulative emissions net of carbon absorption by natural sinks, forests and oceans. Mattauch *et al.* (2020) provides an explanation directed to economists of the physical phenomena explaining why there exists an almost linear relationship between cumulative emissions and the temperature increase.

Similarly to much of the literature,[9] we model the energy transition as a shift from an energy system based on fossil resources to one ultimately based on renewable sources of energy. The latter is usually modelled as a backstop technology, consisting in an abundant and steady energy flow available at a unit cost higher than the unit extraction cost of fossil energy. However we take an original stance. First, we argue, following Tsur and Zemel (2011), Amigues *et al.* (2015) and Kollenbach (2017), that capturing the flow of solar radiation, of wind or of water motion, requires dedicated capital infrastructures.[10] The accumulation of such a capital stock must be planned and smoothed over time to the extent that there are capital adjustment costs (as in Amigues *et al.*, 2015, Pommeret and Schubert, 2021). Moreover, following Fabre *et al.* (2020), we consider the case where the distinguishing feature of this specialized capital is its intensity in mineral inputs. Although any energy production actually relies on capital equipment composed of materials and mineral inputs, we assume abundant capital for the use of fossil resources to stress the asymmetry in mineral intensity across the technologies. Differently from Fabre *et al.* (2020), we suppose the existence of a backstop technology allowing to maintain a sustainable consumption level. In this respect, this paper is in line with the optimistic view of the literature discussed above considering that neither fossil fuels' exhaustibility nor climate change constitute physical limits to consumption growth because an abundant backstop technology –arguably not yet discovered– will allow the economy to overcome scarcity.

The implications of the relative material intensity of renewable energy production for climate policy are also studied in Chazel *et al.* (2020). They adapt the simplified integrated assessment model in Golosov *et al.* (2014) and apply it to the case of copper. Our analysis differs substantially from theirs because our focus is on policy design. First, we adopt a carbon budget approach whereas they introduce a damage function. Second, while they study only the optimal policy, we are interested in exploring the consequences of close-to-real-world constraints on policy making. Hence, we introduce in addition acceptability constraints on policy making, then consider myopic regulation (as Gerlagh and Okullo, 2020).[11]

Considering CRM in the energy transition introduces an additional exhaustibility constraint on top of the carbon budget, but it also allows us to put forward non trivial and policy relevant implications for the energy transition. As argued in Fabre *et al.* (2020), to the extent that the production of energy from renewable sources relies on the use of a bounded stock of non renewable mineral resources, it is crucial to consider the following asymmetry between these two resources:

[9] For instance Davison (1978) Dasgupta, Gilbert and Stiglitz (1982), Hoel and Kverndokk (1996), Tahvonen (1997), Tahvonen and Salo (2001) Smulders and de Nooij (2003), Pittel and Bretschger (2010), Hart (2019).

[10] The role of capacity constraints in the dynamics of the production of non renewable resources has been extensively studied, and improves the empirical relevance of the theory of optimal resource management. See, among others, Gaudet (1983), Lasserre (1985), Cairns (1998) and (2001), Anderson *et al.* (2018).

[11] Another important difference from Chazel *et al.* (2020) lies in our assumptions that fossil and renewable energy are perfect substitutes, and that there is a backstop technology. Our choice allows us to precisely characterize the energy transition as a sequence of different regimes (as in Boucekkine *et al.*, 2013a and 2013b, Henriet and Schubert, 2019).

minerals can be recycled, while fossil resources are lost after use.[12] As shown in their paper, this asymmetry implies that investment in specialized equipment to generate renewable electricity should be brought forward in time, in order to boost the flow of secondary resources to be recycled. While they consider an exogenous and constant recycling effort, we characterize the optimal dynamic choice of the rate at which to recycle the depreciated share of green capital. The dynamics of recycling activities have been widely studied, as, for instance, in the context of waste accumulation (Smith, 1972), of resource depletion (Weinstein and Zeckhauser, 1973), of capital accumulation (Boucekkine and El Ouardighi, 2016) or of sector-specific technological progress (Lafforgue and Rouge, 2019). As far as we know this is the first paper to consider endogenous recycling in the context of the energy transition.[13] In doing so we highlight an original mechanism affecting the timing of investment in green capital: piling it up right before switching to the backstop technology allows to reduce the cost of using the latter.

We extend the model of Pommeret and Schubert (2021) to integrate the use of scarce resources in the energy transition. They propose a stylized dynamic model of the optimal choice of the electricity mix (fossil and renewable), where fossil energy is abundant and CO_2-emitting, while renewable energy is intermittent and clean. To focus on the consequences of the scarcity of mineral resources, we suppose here that the intermittency problem is solved thanks to the existence of storage devices, for instance batteries. Green capital is therefore composed of solar panels, wind turbines and the batteries necessary to make renewable energy dispatchable.

Fossil, hydro, wind and solar energy are assumed to be available at zero variable cost, in order to focus on the issue of critical resources. It is also assumed that, at the beginning of the planning horizon, fossil-fired plants already exist, so that there is no capacity constraint on carbon intensive electricity generation. On the contrary, existing solar capacity is small and requires investment in green capital that embeds a given amount of critical resources. Alternatively, a relatively expensive backstop technology can be used to build-up green capital, which represents the CRM-free alternative.

The centralized program is solved under the constraint of a carbon budget that cannot be exceeded. There is therefore a trade-off between, on the one hand, fossil energy which is expensive to use in terms of CO_2 emissions and, on the other hand, renewable energy which is expensive because it requires costly investment in green capital. The latter relies on a critical mineral resource that is being depleted or a relatively costly backstop technology. We analyze the optimal trajectories

[12] In our context, fossil resources are abundant, because the carbon budget is relatively stringent. The later is modeled as a non renewable resource: when using part of the carbon budget, it is definitely lost. Instead, when exploiting part of the minerals' stock, these minerals will be embedded in green capital for some time, then a share of the original amount of minerals can be reused through recycling. We abstract from climate engineering and from carbon capture and sequestration (CCS), which would qualify this difference, to the extent that they allow to stabilize the cumulative carbon emissions net of sequestration, thus making to the size of the fossil resource stock relevant too.

[13] Chazel *et al.* (2020) assume that complete recycling is feasible (and indeed optimal in the long-run), and that there is not definite waste, in as much the whole cumulative production of primary minerals can be ultimately recovered and recycled.

considering different potential successions of phases: minerals may be exhausted before or after the carbon budget is reached, depending on the relative size of the two stocks. In any case, it is optimal not to use any fossil fuels at all once the carbon budget is exhausted. We study how the criticality constraint affects the dynamics of the energy transition through the accumulation of green capital. The speed of this transition undoubtedly depends on the relative strength of the two constraints -carbon budget and minerals stock- weighing on the economy. Using comparative dynamics, we are also able to analyze the consequences of a more stringent climate policy and a lower stock of CRM, on investment decisions and the energy mix, hence on the phasing out of fossil resources.

Turning to policy design, we consider the decentralized version of the economy, where the only market failure is climate change. The policy maker can implement the optimal policy, by levying a specific carbon tax. This policy may not be feasible if the regulator is constrained in the policy tools she can use. We consider the case where the regulator is only able to charge a constant carbon tax, insufficiently high to ensure, on its own, that the carbon budget is respected. She can use the carbon tax revenue to subsidize the production of renewable energy, through demand-pull subsidies, such as feed-in tariffs and feed-in premiums. We analyze the case where the latter are used, to meet the carbon budget. Given that policy makers typically ignore potential scarcity of minerals embedded in green capital, we then move on to study the case of a regulator who sets a constant carbon tax in the aim of respecting the carbon budget, and does so as if minerals were abundant. In doing so, we can see by how much cumulative emissions overshoot the carbon budget and the expected date of fossil phase-out is mistaken. Finally, the model is extended to allow for recycling and to analyze its effect on the dynamics of the energy transition. We derive results on how recycling affects the accumulation of green capital, and the dates when the exhaustible resources (minerals and carbon budget) are exhausted.

Section 2 presents the model of optimal energy transition under a scarcity constraint on minerals. Section 3 considers the decentralized equilibrium and analyzes first best policies, constrained policies and myopic regulation. In section 4 the model is extended to consider endogenous recycling of the minerals.

2 Optimal energy transition under minerals' scarcity

In this section we present the main assumptions on preferences, technology and resource constraints, then characterize the optimal paths for the economy when the production capacity of electricity from renewable sources is initially low.

Let us represent a closed economy in continuous time, with a representative household. We focus on the production (and consumption) of power, $e(t)$. We assume that electricity is produced with a linear technology, either from the combustion of a flow of fossil resources, $x(t)$, or from the use of "green" capital, $K(t)$, a stock of specialized equipment for renewable energy generation and storage. The two sources are perfect substitutes in terms of provided energy services:

$$e(t) = x(t) + \phi K(t) \tag{1}$$

with $\phi > 0$ a measure of the efficiency of green electricity generation.

While the production of fossil resources entails no direct cost, their use implies carbon emissions, which accumulate in the atmosphere:

$$\dot{X}(t) = \varepsilon x(t) \tag{2}$$

where ε is the emission coefficient and $X(t)$ denote cumulative carbon emissions. Climate policy aims at limiting the temperature increase, compared to the pre-industrial equilibrium temperature. It is represented by a targeted carbon budget $\overline{X}$, i.e. the maximal amount of cumulative emissions compatible with the chosen temperature objective.

The stock of green capital depreciates at the constant rate $\delta \in (0, 1)$. It evolves with specific investment, $I(t)$, as follows:

$$\dot{K}(t) = I(t) - \delta K(t) \tag{3}$$

Investment entails costs, $C(I(t))$, assumed increasing and strictly convex to reflect adjustment costs (i.e. C', $C'' > 0$). Investment relies either on a mineral input, $m(t)$, or on the use of a backstop input, $b(t)$. These inputs are assumed to be perfect substitutes:

$$I(t) = m(t) + b(t) \tag{4}$$

The backstop input is available anytime and in any amount, at a constant unit cost ν. Minerals are produced at no cost, but they are available in a stock, $M(t)$, that depletes:[14]

$$\dot{M}(t) = -m(t) \tag{5}$$

The economy is subject to two non-renewable resource constraints: an initial endowment of minerals, M_0, and the initial carbon budget, $\overline{X}$.

Denoting by K_0 the initial endowment in green capital, we can write the set of constraints that apply to the flow and stock variables as

$$X(t) \leq \overline{X}, \quad M(t) \geq 0, \quad x(t) \geq 0, \quad m(t) \geq 0, \quad b(t) \geq 0 \tag{6}$$

[14] For much of the literature, a backstop technology for the fossil input allows for the production of a flow of renewable energy at a constant positive cost. Here instead, the backstop technology replaces the exhaustible mineral input that allows for the production of an investment input in green capital. It can be interpreted as an abundant material input with a constant positive cost, higher than that of minerals (here nil) and also higher than the initial scarcity rent on minerals. An example is copper, more abundant than silver, but still facing technical hurdles to using it cost-effectively as a replacement for silver in the photovoltaic cell-manufacturing process.

$$X(0) = 0, \quad K(0) = K_0 \geq 0 \text{ and } M(0) = M_0 \geq 0 \text{ given} \tag{7}$$

The representative household is characterized by an instantaneous utility function that is assumed to be concave in electricity consumption, according to $u(e(t))$, with $u' > 0$ and $u'' < 0$, and quasi-linear in the remaining generic consumption good. He applies a constant discount rate, $\rho > 0$.

The benevolent social planner seeks to maximize society's net surplus, generated by power consumption net of the costs due to investment and to the use of the backstop input for investing:[15]

$$\max \int_0^\infty e^{-\rho t}[u(e(t)) - C(I(t)) - \nu b(t)]dt \tag{8}$$

subject to the technology and resource constraints (1)-(5), as well as the non-negativity and carbon budget constraints (6), with the initial conditions (7).

In order to be able to characterize the solution to this optimization problem, we define the costate variables associated to each stock as: $\mu(t)$ the value of the green capital, $\zeta(t)$ the value of mineral resources, and $\lambda(t)$ the carbon value.

The solution is a trajectory through different phases, during which different sets of the non-negativity constraints in (6) are binding. In the rest of this Section, we first present a set of results holding during the optimal energy transition, then characterize the optimal phases through which the transition runs, and finally present a few numerical illustrations of the energy transition and study how it is affected by the relative stringency of the climate constraint and the endowment of minerals. All proofs are in Appendix A.1.

Definition 1. Inputs use: *Fossil resources, mineral resources and the backstop input are each used over a continuous time interval, defined with the following notations:*

$$x(t) > 0 \ \forall t \in [T_x, T_X), \quad m(t) > 0 \ \forall t \in [T_m, T_M), \quad b(t) > 0 \ \forall t \in [T_b, T_B)$$

Continuity of the intervals follows from the application of the Bellman principle of optimality.[16]

Lemma 1. Declining consumption in the fossil economy: *When fossil resources are used, the carbon value increases at the social discount rate and electricity consumption declines:*

$$\forall t \in [T_x, T_X), \quad \frac{\dot{\lambda}(t)}{\lambda(t)} = \rho \quad \textit{and} \quad \frac{\dot{e}(t)}{e(t)} = \frac{u'(e(t))}{e(t)u''(e(t))}\rho < 0$$

[15] As shown in Sections 3.1 and 3.2 this social welfare function is consistent with preferences of the representative household that are quasi linear in the consumption of goods and services other than electricity.

[16] Discontinuity would arise if it were optimal to use again the fossil resources (mineral resources and backstop) at time t while it was optimal not to use it at time $(t - dt)$, despite its earlier use. It only makes sense not to use it at time $(t - dt)$ if the accumulation of X (K) had been excessive at time $(t - dt)$ which cannot be the case since $X(t - dt)$ $(K(t - dt))$ results of optimally-planned previous accumulation choices. A formal proof would follow the lines of Appendix B1 in Fabre *et al.* (2020).

In our approach, the social planner's climate problem is represented as that of choosing when and how much to use of the finite and non-renewable carbon budget. The solution is the one in Hotelling (1931). The use of the carbon budget shall be such that its value increases at the social discount rate. Since any additional use of the carbon budget translates into $1/\varepsilon$ marginal units of consumption, the marginal utility of consumption should also increase at the social discount rate. Hence, consumption must decline.

LEMMA 2. Investment: *At any given date, investment in green capital relies either on mineral resource inputs or on the backstop input. Its level is determined by the equality between the marginal investment cost and the value of green capital net of the scarcity rent of minerals or the cost of the backstop input:*

$$I(t) = \begin{cases} m(t) = C'^{-1}\left(\mu(t) - \zeta(t)\right) & \forall t \in [T_m, T_M) \\ b(t) = C'^{-1}\left(\mu(t) - \nu\right) & \forall t \in [T_b, T_B) \end{cases} \tag{9}$$

Mineral inputs for investment are used before the backstop input:

$$0 \leq T_M = T_b \tag{10}$$

That minerals and the backstop technology are not simultaneously used to invest in green capital follows from the assumption that the two inputs are perfect substitutes, and from the fact that their costs can be equal only at one instant of time. The scarcity rent of minerals $\zeta(t)$ measures the opportunity cost of using minerals, resulting of their exhaustible and non-renewable nature. For the same argument following Lemma 1, $\zeta(t)$ increases exponentially at the social discount rate. Hence it starts below ν and reaches it in finite time.

LEMMA 3. Resource exhaustion: *The carbon budget and the mineral stock are exhausted in finite time:*

$$T_X < \infty \quad \textit{and} \quad T_M < \infty \tag{11}$$

Indeed, an asymptotic exhaustion of the carbon budget would imply that in the long run the marginal utility of energy consumption becomes infinite, which cannot be the case when electricity can alternatively be produced with renewable means. Moreover, there is no reason to leave any unused mineral resource after date T_M, since there is no direct cost associated to its use.

Once the carbon budget and the mineral stock are exhausted, consumption relies exclusively on the electricity from renewable sources, and investment in green capital uses only the backstop technology. The economy follows then a saddle path where the stock of green capital increases towards its steady-state, at a level balancing marginal benefits (depending on preference parameters for electricity consumption and efficiency of green electricity generation) and marginal costs (the cost of the backstop, adjustment costs, discounting and depreciation) of green capital.

Lemma 4. Steady state: *The economy asymptotically converges toward a steady state equilibrium where green capital is composed of the backstop input, defined by* (K^*, μ^*)*:*

$$K^* \quad s.t. \quad \nu + C'(\delta K^*) = \frac{\phi}{\rho+\delta} u'(\phi K^*) \quad and \quad \mu^* = \nu + C'(\delta K^*) \tag{12}$$

This unique and saddle-path stable steady state exists if $u'(0) > \frac{\rho+\delta}{\phi}(\nu + C'(0))$*. Hence* $T_B = \infty$*.*

Conversely, at the beginning of the planning horizon a variety of patterns could arise. We restrict our analysis to the most relevant case, by considering the conditions characterized in the following,

Definition 2. Case with early resource use: *For* $K_0 < K^*$*, the fossil resources are used from the start:* $T_x = 0$*. Moreover, the use of mineral resources begins as early as possible, i.e.* $T_m = 0$*, if the initial value of green capital and of minerals satisfy* $C'(0) < \mu(0) - \zeta(0)$ *(see (9)).*

In the rest of the paper we focus on this case. It prevails for relevant configurations of the initial stocks K_0, M_0 and $\overline{X}$, which determine the initial shadow value of solar panels, $\mu(0)$, and the initial value of the mineral stock, $\zeta(0)$.[17]

The following Proposition characterizes the optimal energy transition.

Proposition 1. Optimal energy transition. *In the case with early resource use, the optimal path of the economy, solving the program given by the objective function (8), under the technology and resource constraints (1)-(5), the non-negativity and carbon ceiling constraints (6), and the initial conditions (7), is determined by the vector* $\{\mu^\circ(0), \zeta^\circ(0), \lambda^\circ(0), T_X^\circ, T_M^\circ\}$*, such that the economy converges towards the steady state in Lemma 4, following the system of differential equations:*

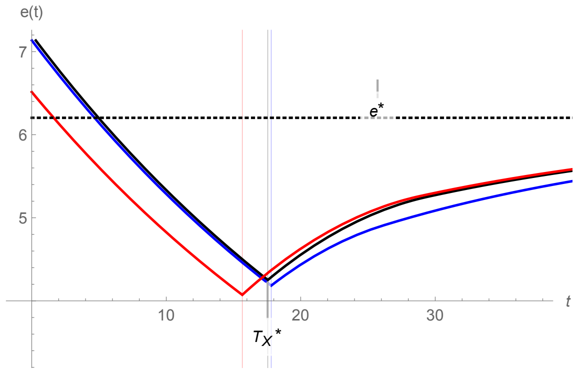
$$\dot{K}^\circ(t) = \begin{cases} C'^{-1}\left(\mu^\circ(t) - \zeta^\circ(t)\right) - \delta K^\circ(t) & \forall t < T_M^\circ \\ C'^{-1}\left(\mu^\circ(t) - \nu\right) - \delta K^\circ(t) & \forall t \geq T_M^\circ \end{cases} \tag{13}$$

$$\dot{\mu}^\circ(t) = \begin{cases} (\rho+\delta)\mu^\circ(t) - \phi\varepsilon\lambda^\circ(t) & \forall t < T_X^\circ \\ (\rho+\delta)\mu^\circ(t) - \phi u'(\phi K^\circ(t)) & \forall t \geq T_X^\circ \end{cases} \tag{14}$$

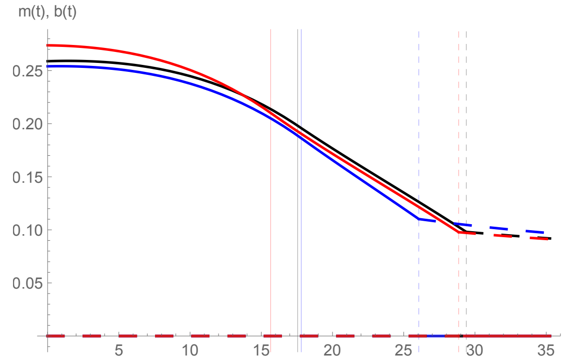
$$\dot{\zeta}^\circ(t) = \begin{cases} \rho\zeta^\circ(t) & \forall t < T_M^\circ \\ 0 & \forall t \geq T_M^\circ \end{cases} \tag{15}$$

$$\dot{\lambda}^\circ(t) = \begin{cases} \rho\lambda^\circ(t) & \forall t < T_X^\circ \\ 0 & \forall t \geq T_X^\circ \end{cases} \tag{16}$$

[17] Computing numerically the paths of endogenous variables, we have studied the relationship between the condition $C'(0) < \mu(0) - \zeta(0)$ in Definition 2 and the exogenous variables $\{K_0, M_0, \overline{X}\}$. For a given K_0, we can determine the threshold frontier $M_0 = h(\overline{X})$, such that $T_m > 0$ for $M_0 < \overline{X}$, but $T_m = 0$ for $M_0 \geq \overline{X}$. Moreover, we notice that even in the case with early resource use, it is possible that initially $\dot{K}(0) < 0$, to the extent that investment does not compensate for depreciation, i.e. if $C'(0) < \mu(0) - \zeta(0) < C'(\delta K_0)$.



(a) Energy consumption



(b) Use of minerals and backstop

Figure 1: Comparative dynamics (reference in black, lower M_0 in blue, lower $\overline{X}$ in red; vertical lines: T_X in plain lines, T_M in dotted lines; panel (b): $m(t)$ in plain lines, $b(t)$ in dashed lines)

where T_X° *and* T_M° *are the dates of exhaustion of* $\overline{X}$ *and* M_0 *respectively, and satisfy*

$$\lambda^\circ(T_X^\circ) = u'\left(\phi K^\circ\left(T_X^\circ\right)\right)/\varepsilon \quad \textit{and} \quad \zeta^\circ(T_M^\circ) = \nu \tag{17}$$

While the non-renewable nature of the two resources allows us to state that the backstop regime prevails in the long run, we may wonder what determines the ordering of the two first phases. Only two optimal sequences are possible. They both start with fossil and mineral resource use, and end in the backstop regime. However in one case, the carbon budget is exhausted before minerals, while in the other the opposite is true. The optimal sequence of regimes switches from the former to the latter, as the size of the carbon budget relative to the minerals stock overcomes a threshold.[18]

Our simulations of the dynamic system consider the case with quadratic costs and logarithmic utility, where

$$u(e(t)) \equiv \gamma \ln e(t) \tag{18}$$

$$C(I(t)) \equiv c_1 I(t) + \frac{c_2}{2}(I(t))^2 \quad \Rightarrow \quad C'(I(t)) = c_1 + c_2 I(t), \quad C'(0) = c_1 \tag{19}$$

The explicit expressions are detailed in Appendix A.1. The system of ordinary differential equations is solved using the backward shooting procedure from $\{\mu^*, K^*\}$ until $K(0) = K_0$ (Brunner and Strulik, 2002).[19]

For illustrative purposes, we present here three simulations: the baseline situation with parameters' values reported in Table 3 in Appendix A.1; the case of a 20% lower carbon budget; and the case of a 10% lower stock of minerals. Under these configurations of parameters, the initial endowment of green capital is below its steady state desired level, and the carbon budget binds first.

[18] See Appendix A.1.

[19] See Appendix A.1 for detailed explanations.

The optimal path of energy consumption is V-shaped, as illustrated on the left-hand panel of Figure 1. We can interpret such a path as a compromise between those that would be chosen if the economy were endowed with only one of the two non-renewable resources. In the absence of minerals and the backstop technology, the economy, endowed with only the non-renewable carbon budget, chooses a decreasing path of consumption, in order to ensure that its marginal utility grows at the social discount rate. The initial declining profile of energy consumption in Figure 1 reflects this choice, which according to Lemma 1, prevails also when minerals and the backstop are available, until the fossil resource is exhausted. On the other hand, if initially no fossil resource is available, the optimal path of energy consumption results of the chosen accumulation of green capital only. Overall, the V-shaped optimal path of consumption is conditional on the relative abundance of the three given stocks, K_0, $\overline{X}$ and M_0.[20] Let us now turn to comparative dynamics.

A smaller initial stock of minerals delays the fossil phase-out, but speeds up minerals exhaustion (compare blue paths to black ones in Figure 1). Having less cheap resource for building renewable energy infrastructure induces both to spread the use of the fossil on a longer time period and to anticipate the switch to the expensive backstop. Strengthening the constraint on the economy, a 10% smaller stock of minerals leads to a welfare loss which is equivalent to a permanent reduction of the electricity consumption at -2% below its initial path.

In the same spirit, a lower carbon budget constrains further the economy thus reducing welfare. A 20% smaller carbon budget generates a welfare loss which is equivalent to a permanent reduction of the electricity consumption at -10.9% below its initial path. A reduction in the fossil stock that can be used induces a quicker use of minerals (to replace fossil generated electricity with renewable electricity) which in turn implies an earlier exhaustion of mineral resources. Hence, a more stringent climate policy fastens the whole process as fossil fuels are abandoned sooner, and the switch to the backstop occurs earlier (compare red paths to black ones in Figure 1).

A key difference between the two cases lies in the optimal adjustment of the intertemporal profile of consumption, as illustrated in Figure 1(a). If the stock of mineral resources is reduced, consumption falls in particular over the later phase (i.e. when using the backstop). If the carbon budget is reduced, consumption falls relatively more over the first two phases. In fact, in the latter case, while the economy loses part of the cheapest resource to produce electricity, it maintains the possibility to build up its stock of green capital.

3 Public policy for the energy transition under mineral scarcity

In this section we consider the decentralized counterpart of our economy, to analyze the role of public policy. We proceed in four steps. First, we present the economy and characterize the

[20] The consumption path would monotonically decline for a sufficiently abundant carbon budget, or for an endowment of green capital above its desired steady state level.

equilibrium trajectory for a specific set of public policy instruments. Second, we derive the social welfare function and identify the policy that allows the equilibrium trajectory to coincide with the optimal one. Third, we study a sub-optimal policy that the regulator can implement, under the assumption that her policy instruments are constrained in a specific way, reminiscent of widespread policies implemented in the world to stimulate the fossil phase-out and the production of electricity from renewable sources of energy. Fourth, we consider a myopic regulator who sets, once and for all, these policy instruments without taking into account the scarcity of minerals, and show how the equilibrium trajectory of the economy differs from the one that the regulator was targeting. This last step gives substance to a major message of the article: it may be severely misleading to ignore the scarcity of minerals that are critical to investment in specific infrastructure and capital for the energy transition.

3.1 The decentralized economy

In the decentralized economy, the government handles a set of policy instruments, households demand electricity, power companies supply electricity, and to do so they invest in green capital and demand natural resource inputs, and resource managers supply the latter. Here we characterize the behaviors of these agents for given policy tools, and then present the main features of the general dynamic equilibrium. Most of the mathematical analysis is relegated to Appendix A.2.

The regulator has several policy tools at her disposal: she may tax carbon emissions, subsidize investment in green capital, set a feed-in-premium (FIP) for electricity produced from green capital, tax electricity consumption, give lump sum transfers to households, etc. We restrict ourselves to three instruments: a tax on carbon emissions, τ, a FIP, σ, and lump sum transfers, $\mathcal{T}$.[21]

The representative household maximizes his intertemporal utility. Utility is derived from the consumption of a generic good z and electricity services, e. The utility function is quasi-linear in good z, taken as numeraire. Hence, the program of the representative consumer is $\forall t \geq 0$:[22]

$$\begin{aligned} \max_{e(t),z(t)} \quad & \int_0^\infty e^{-\rho t} \left[z(t) + u\left(e(t)\right) \right] dt \\ \text{s.t.} \quad & \dot{a}(t) = r(t)a(t) + \pi_x(t) + \pi_m(t) + \pi_e(t) + \mathcal{T}(t) - z(t) - P_e(t)e(t) \\ & a(0) = a_0 \text{ given} \end{aligned} \tag{20}$$

where a is the household's financial wealth and a_0 its endowment, r is the real rate of return on financial wealth as well as the interest rate at which the household can borrow, while π_x, π_m and π_e are the profits of the fossil and the mineral resource producers, and of the electricity producers.

[21] The model can be extended to consider also a levy on electricity consumption, reminiscent of the EEG Umlage in Germany, or of the CSPE in France. Here, we consider the restricted set of instruments, because it allows us to point out in a clear-cut way some original trade-offs.

[22] As shown below, in section 3.2, this representation of household's preferences and problem is coherent with the objective function of the benevolent social planner (8) assumed in Section 2.

The households' optimal saving behavior ensures that $\forall t \geq 0$ $r(t) = \rho$, while its behavior on the electricity market is characterized by the inverse demand schedule:

$$P_e(t) = u'(e(t)) \tag{21}$$

Perfectly competitive utilities produce electricity from either fossil resources or from a specific green capital. Investment in green capital by utilities mobilizes either mineral resources or a backstop technology, available in-house at a constant (high) cost, ν. The representative power utility acts under perfect competition on the electricity market and on the markets for natural resources, where it takes prices as given. The firm seeks to maximize the present value of its profits, taking into account that building up its green capital to produce renewable energy requires specific inputs and implies adjustment costs. It therefore solves an intertemporal program, based on the expected evolution of the electricity price P_e, the prices of the inputs, fossil resources P_x and minerals P_m, as well as policy tools, namely, the carbon tax $\tau(t)$ and the FIP $\sigma(t)$. Defining $\Omega(t) \equiv \int_0^t r(s)ds$, the program of the power producer is:

$$\begin{aligned} \max_{x(t),m(t),b(t)} \quad & \int_0^\infty e^{-\Omega(t)} \left[P_e(t)x(t) + (P_e(t) + \sigma(t))\, \phi K(t) \right. \\ & \left. \qquad - (P_x(t) + \varepsilon\tau(t))\, x(t) - C(I(t)) - P_m(t)m(t) - \nu b(t) \right] dt \\ \text{s.t.} \quad & (3),\ (4),\ x(t) \geq 0,\ m(t) \geq 0,\ b(t) \geq 0,\ \text{and } K(0) = K_0 \ \text{ given.} \end{aligned} \tag{22}$$

The optimal investment in green capital is equivalent to that characterized in Lemma 2, with the price of minerals replacing their social value, $\zeta(t)$. The firm does not use simultaneously the two inputs, it uses minerals if their price is below the cost of the backstop, otherwise it relies on the backstop: $m(t) > 0$ and $b(t) = 0$ if $P_m(t) < \nu$, but $m(t) = 0$ and $b(t) > 0$ if $P_m(t) > \nu$. Moreover, the amount invested is such that the value of a marginal unit of green capital for the power utility, denoted $\mu_d(t)$, equals the full cost of investment: $C'(I(t)) + P_m(t)$ or $C'(I(t)) + \nu$.

When some of the electricity is optimally produced using fossil inputs, the seller price of electricity equals the cost of fossil resource use, comprehensive of the price of the resource and of the carbon tax:

$$x(t) > 0 \quad \Leftrightarrow \quad P_e(t) = P_x(t) + \varepsilon\tau(t) \tag{23}$$

On the mineral resources market, suppliers do not have any production cost, but have a limited stock of resource to sell. They are therefore willing to sell the resource at a pace that depends on the dynamics of the selling price, hence the scarcity rent denoted by ζ_d: $P_m(t) = \zeta_d(t)$.

Our focus being on an effective climate policy, which makes the environmental problem more stringent than resource scarcity, the scarcity of the natural resource stock is not binding at equilibrium on the market for fossil resources. In effect, firms act as if their resources were abundant, so that, rather than solving an intertemporal optimization problem, they maximize their current profit at each date. Given that the production costs are assumed nil and that the market is perfectly competitive, the supply adjusts to the demand originating from power suppliers.

We are ready to characterize the dynamic general equilibrium, for a given set of policy tools. The equilibrium is a sequence of regimes, with different prevailing patterns of binding non-negativity constraints on fossil resource use, on minerals use and on the use of the backstop technology.

Definition 3. Regular climate policy. *The regulator implements the policy announced at date* $t = 0$*, and relies on a smooth non-decreasing carbon tax* $\tau^{\diamond}(t)$ *and on a smooth non-increasing FIP* $\sigma^{\diamond}(t)$*, converging to a constant* $\sigma^{\diamond}_{\infty}$*, with lump-sum transfers to households to balance the budget at each date.*

Proposition 2. Equilibrium dynamics. *In the case with early resource use, under a regular climate policy the equilibrium variables satisfy* $P^e_x(t) = 0$ *(superscript "e" stands for "equilibrium") and*

$$\dot{P}^e_m(t) = \rho P^e_m(t) \qquad \forall t \in [0, T^e_M) \tag{24}$$

$$\dot{K}^e(t) = \begin{cases} C'^{-1}\left(\mu^e_d(t) - P^e_m(t)\right) - \delta K^e(t) & \forall t \in [0, T^e_M) \\ C'^{-1}\left(\mu^e_d(t) - \nu\right) - \delta K^e(t) & \forall t \geq T^e_M \end{cases} \tag{25}$$

$$\dot{\mu}^e_d(t) = \begin{cases} (\rho+\delta)\mu^e_d(t) - \phi(\varepsilon\tau^{\diamond}(t) + \sigma^{\diamond}(t)) & \forall t \in [0, T^e_X) \\ (\rho+\delta)\,\mu^e_d(t) - \phi\left(u'(\phi K^e(t)) + \sigma^{\diamond}(t)\right) & \forall t \geq T^e_X \end{cases} \tag{26}$$

starting from the equilibrium initial values $P^e_m(0)$ *and* $\mu^e_d(0)$*, and converging to a unique saddle path stable steady state* $\{K^{\diamond}_d, \mu^{\diamond}_d\}$*, solving (25)-(26) with* $\dot{K}^e(t) = \dot{\mu}^e_d(t) = 0$ *and* $\sigma^{\diamond}(t) = \sigma^{\diamond}_{\infty}$ *for* $t > \max\{T^{\diamond}_X, T^{\diamond}_M\}$*. The dates* $\{T^e_X, T^e_M\}$ *are such that the stock of minerals is exhausted at date* T^e_M*, when* $P^e_m(T^e_M) = \nu$*, and that the marginal utility of consuming only the electricity from the green capital stock equals the pass-through of the carbon tax on the electricity price for the first time at date* T^e_X*:*

$$u'(\phi K^e(T^e_X)) = \varepsilon\tau^{\diamond}(T^e_X) \tag{27}$$

with $K^e(T^e_X) = K_0 + \int_0^{T^e_X} \dot{K}^e(t)dt$ *from (25).*

Notice first that when fossil resources are used, the carbon tax effectively controls the level of electricity consumption (eq. (23) for $P^e_x(t) = 0$ in eq. (21)). Second, the subsidy in the form of the FIP affects the value of green capital, μ according to (26), by increasing the marginal revenue it generates, and thereby influences its accumulation rate, in (25). Third, for a non-decreasing carbon tax and a non-negative and non-increasing FIP, the equilibrium trajectory is a sequence of regimes, where initially both fossil and minerals resources are used, but they are not used in the long run. The economy converges to a steady state, that is affected by the asymptotic value of the FIP, and equals the one in Lemma 4 if the FIP becomes nil. The economy undergoes a temporary regime when, either at equilibrium $T^e_X < T^e_M$ and all consumption relies on green capital while investment relies on minerals, or, if at equilibrium $T_M^e < T_X^e$, fossil resources are still used to produce electricity and investment relies on the backstop technology.

3.2 Optimal policy

The government aims at maximizing the social welfare function, which can be written as follows

$$\int_0^\infty e^{-\rho t} \left[z(t) + u\left(e(t)\right)\right] dt = a_0 + \int_0^\infty e^{-\rho t} \left[u(e(t)) - \left[C(m(t) + b(t)) + \nu b(t)\right]\right] dt \tag{28}$$

This is the same function (8) considered in Section 2, apart for the given endowment a_0. As shown in Appendix A.2, this equivalence results of the definitions of profits of resource owners and of the power utility company, as well as of the current public budget constraint below, and of the fact that any difference between the present value of the flows of the household's expenditure and of income reflects the wealth endowment.

The current public budget constraint is:

$$\mathcal{T}(t) = \tau(t)\varepsilon x(t) - \sigma(t)\phi K(t) \tag{29}$$

According to it, any surplus (deficit) of carbon tax revenues over (below) expenditures for subsidies to power production originating from green capital, is redistributed to (levied from) the household.

The climate policy objective is to keep cumulative carbon emissions at or below the carbon budget $\overline{X}$. Since the use of fossil resources allows to produce valuable energy services, at no direct cost, it is not optimal to leave the carbon stock permanently below the ceiling. As a consequence, the optimal policy is such that the carbon budget is exhausted upon fossil phase out, at date T_X (i.e. (47) in Appendix A.1 holds).

To sum up, the objective of the government is to select the optimal trajectories of the two policy instruments $\tau(t)$ and $\sigma(t)$ in order to maximize this social welfare function, taking into account the climate constraint $\int_0^{T_X} \varepsilon x(t) \leq \overline{X}$.

Proposition 3. First best policy. *In the case of early resource use, the optimal policy can be implemented by levying a carbon tax equal to the optimal carbon value $\tau(t) = \lambda^\circ(t)$ and providing no FIP, with the tax revenue transferred lump-sum at each date to households.*

Since the only market failure hinges on the limited carbon budget, the policy intervention optimally prices the use of fossil resources. Once the regulator sets the initial carbon tax at $\tau(0) = \lambda^\circ(0)$ and its constant growth rate at ρ, the economy sets at equilibrium along the optimal trajectory. This result applies to all the optimal sequences of regimes. Alternatively, the regulator could set up a carbon market and allocate an amount $\overline{X}$ of allowances.

3.3 Constrained policy

In order to represent real world policy making, we assume in this section that the government can commit to levy a constant carbon tax, but not an increasing one. Indeed, in several countries the announcement of climate change mitigation policies, including an increasing carbon tax, have received strong political opposition. This opposition is weaker when the revenue from the tax is used to finance subsidies to renewable energy production (Douenne and Fabre, 2020, Klenert *et al.*, 2018). In most cases mitigation policies do not rely exclusively on pricing carbon emissions. They typically include interventions in the power market with demand-pull instruments targeted at electricity from low-carbon energy sources. Governments might combine these tools to target specific objectives, such as limiting emissions within a carbon budget, or ensuring fossil phase out by some date. We therefore consider the following case.

Definition 4. Constrained policy. *A regular climate policy, where the regulator aims at respecting the carbon budget, $X(t) \leq \overline{X}$, by charging a constant carbon tax $\tau(t) = \tau$ and by paying a FIP with the resulting tax revenue, i.e. $\mathcal{T}(t) = 0$ in (29).*

Under such a policy, the economy evolves along the following equilibrium trajectory.

Proposition 4. Equilibrium dynamics under constrained policy. *In the case with early resource use, under the constrained policy at equilibrium:*

a) *The economy converges to the steady state in Lemma 4;*

b) *The FIP is $\sigma^c(t) = \frac{\varepsilon\tau u'^{-1}(\varepsilon\tau)}{\phi K^c(t)} - \varepsilon\tau > 0 \ \forall t \leq T_X^c$, it decreases and then is nil from T_X^c onward;*

c) *Consumption is first constant at $e^c(t) = u'^{-1}(\varepsilon\tau) \ \forall t \leq T_X^c$, then increasing $e^c(t) = \phi K^c(t)$ for $t \geq T_X^c$;*

d) *Fossil resource use decreases and equals $x^c(t) = u'^{-1}(\varepsilon\tau) - \phi K^c(t) \ \forall t \leq T_X^c$, and then is nil from T_X^c onward;*

e) *The paths of the price of minerals, of the stock and of the value of green capital follow (24)-(25) and:*

$$\dot{\mu}_d^c(t) = \begin{cases} (\rho+\delta)\mu_d^c(t) - \frac{\varepsilon\tau u'^{-1}(\varepsilon\tau)}{K^c(t)} & \forall t \in [0, T_X^c) \\ (\rho+\delta)\,\mu_d^c(t) - \phi u'(\phi K^c(t)) & \forall t \geq T_X^c \end{cases} \tag{30}$$

for the equilibrium pair $\{\mu_d^c(0), P_m^c(0)\}$;

f) *Date T_X^c satisfies $K^c(T_X^c) = u'^{-1}(\varepsilon\tau)/\phi$, with $K^c(T_X^c) = K_0 + \int_0^{T_X^c} \dot{K}^c(t)dt$ from (25), and $\overline{X} = \int_0^{T_X^c} x^c(t)dt$;*

g) *At date T_M^c the stock of minerals is exhausted, with $P_m^c(T_M^c) = \nu$.*

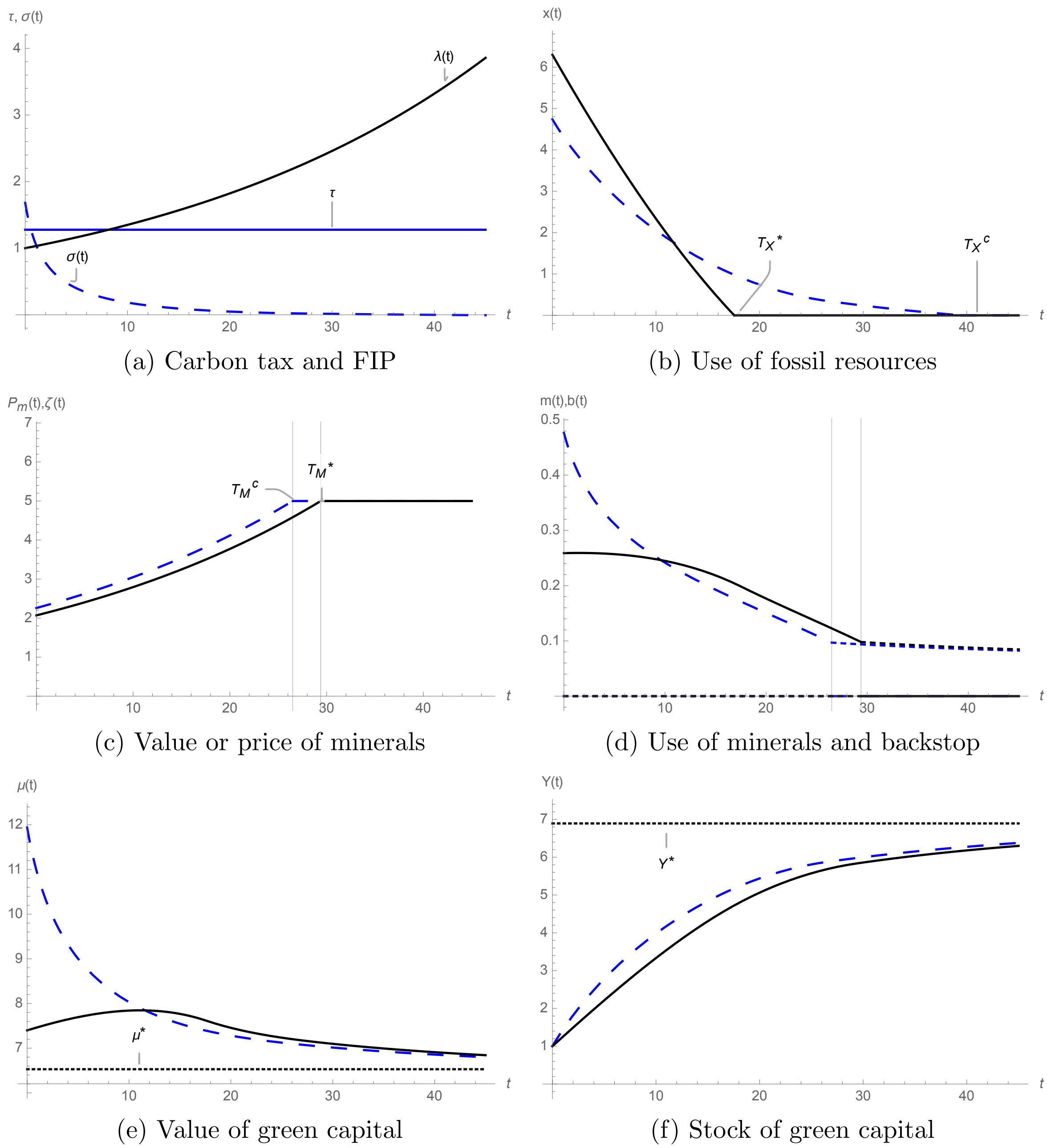
The regulator, constrained to put in place a constant carbon tax and to use the revenue to finance the FIP, only has one instrument at her disposal, the level of the tax, uniquely determined by the carbon budget. She is unable to implement the optimal trajectory, since consumption is constant instead of declining for $t < T_X$ (use eq. (23) for $P_x(t) = 0$ and $\tau(t) = \tau$ in eq. (21)). The policy directly affects the use of the fossil resource, and also influences the accumulation of green capital by impacting its value (equation (30)). Interestingly, if the utility function is isoelastic, the impact of the policy on green capital goes in opposite directions according to whether the elasticity of intertemporal substitution of electricity consumption is larger or smaller than unity, and disappears if the utility function is logarithmic, as in our numerical exercise.

The trajectories prevailing for the optimal and the constrained policy cases are compared in columns two and three of Table 1 and in Figure 2 (for the functional specifications (18)-(19) as characterized in Appendix A.2 and parameters in Table 3 in Appendix A.1). The constraint on policy tools causes a welfare loss, which is equivalent to a permanent reduction of the electricity consumption at -8.4% below its optimal path.

According to (b) in Proposition 4, in the case under analysis, the regulator effectively manages only one instrument, say the level of the carbon tax. Cumulative carbon emissions, as well as the date of fossil phase-out, decrease monotonically with the level of the carbon tax. If the objective of the regulator is expressed in terms of respecting a carbon budget, or if instead it is to phase-out fossil resources by a specific exogenous date, the policy can be designed to attain the objective. However, the goal will not be reached at the minimum social cost. Moreover, if the policy maker targets two variables, such as exhausting the carbon budget by a specific date, then the regulator is in general unable to implement it with the policy instruments considered in this subsection. In particular, it is impossible to implement the optimal trajectory of the economy.

We show here a case where the carbon tax in the sub-optimal situation is initially higher than the optimal carbon tax, which makes us wonder whether acting sub-optimally actually solves the acceptability issue. Here, a large tax is necessary to raise enough financial resources to subsidize investment, because the initial green capital is small. This observation points to the fact that limiting subsidies to the investment in green capital to be exclusively financed out of carbon tax revenues does not seem to be a wise policy option.

While, in this case of logarithmic utility, mineral resources' use is independent of the policy, the opposite is not true: if mineral resources are relatively abundant, investment in green capital proceeds faster (i.e. P_m^e in (25)), pushing fossils out of the market. Hence, abstracting from mineral resources scarcity will most likely be misleading for the policy design by the regulator. We next consider this issue.



(a) Carbon tax and FIP

(b) Use of fossil resources

(c) Value or price of minerals

(d) Use of minerals and backstop

(e) Value of green capital

(f) Stock of green capital

Figure 2: Optimal versus constrained policy (reference in black, constrained policy case in dashed blue lines)

	optimal policy	constrained policy	myopic forecast	myopic regulation
Welfare	39.8	37.1	44.8	37.7†
Initial carbon tax, $\tau(0)$	1	1.28	1.25	1.25
Carbon tax growth rate, g_τ	0.03	0	0	0
Initial FIP, $\sigma(0)$	0	1.68	1.74	1.69
Steady state green capital, K^*	6.9	6.9	10.1	6.9
Fossil phase-out date, T_X	17	39	27	47
Minerals exhaustion date, T_M	29	26	-	26
Initial price of minerals, $P_m(0)$	2.1	2.3	2.3	2.3
Initial value of green capital, $\mu_d(0)$	7.4	11.9	11.6	11.9

†: Carbon emissions overshoot the carbon budget at no social cost.

Table 1: Comparison of optimal, constrained and myopic policies

3.4 Myopic constrained policy

Suppose that the regulator implements a constrained policy, but is not aware that minerals used for investment in green capital are scarce, and that instead she believes that the supply of minerals is perfectly elastic and constant at an exogenous price P_m^f. The regulator computes the constant carbon tax she will put in place, having in mind a model of the economy where the scarcity of minerals is not taken into account by any agent, in particular by electricity producers. This tax is a function of the initial observed price of minerals $P_m^f = P_m(0)$. However, ex post, agents do acknowledge mineral resource scarcity and behave accordingly.

In order to make sense of the misperception of the regulator, we now assume that the latter interprets the initial observed scarcity rent, $P_m(0) = \zeta_d(0)$, as the constant marginal production cost, which she considers exogenous. We denote such perceived marginal and unit production cost by $\underline{\zeta}$. Hence the regulator forecasts a perfectly elastic inverse supply function of minerals $P_m(t) = \underline{\zeta} \equiv P_m^f$ $\forall m(t) \in (0, \infty)$.

The regulator observes that initially minerals are used to invest in green capital. She deduces that $\underline{\zeta} < \nu$ and expects that the power companies will never want to use the backstop technology. As a consequence, the regulator's forecasting model relies on the following law of motion of the stock of green capital

$$\forall t \geq 0 \qquad \dot{K}(t) = C'^{-1}(\mu_d(t) - P_m^f) - \delta K(t) \tag{31}$$

Notice that, for any given $\{K(t), \mu_d(t)\}$, the forecast of capital accumulation is excessive (compare to (25) knowing that $P_m^f = P_m^e(0) = \underline{\zeta} < P_m^e(t) \leq \nu \ \forall t > 0$). According to the regulator's forecast the economic dynamics is governed by (30)–(31). She therefore expects the economy to converge to a steady state $\{K^f, \mu_d^f\}$ with a larger stock of green capital, $K^f > K^*$, and a lower value, $\mu^f < \mu^*$, than they will actually be.[23]

[23]The system (30)-(31) admits a unique saddle path steady state (K^f, μ^f), solution of: $P_m^f + C'(\delta K^f) = \frac{\phi}{\rho+\delta} u'(\phi K^f)$ and $\mu^f = P_m^f + C'(\delta K^f)$. As $P_m^f < \nu$ and $u'(\phi K) - C'(\delta K)$ is a decreasing function of K, direct comparison with equation (12) shows that $K^f > K^*$, which implies $\mu^f < \mu^*$.

To analyze the consequence of myopic constrained regulation, we compare it numerically to the trajectories prevailing under constrained regulation, for the specifications (18)-(19) and for parameters in Table 3 of Appendix A.1. The myopic regulator targets the carbon budget, considering that the price of minerals will always be $P_m^f = P_m(0)$. She chooses the level of the constant carbon tax $\hat{\tau}$ that would allow to meet the target $\overline{X}$, if the economy was correctly described by the model of this subsection (i.e. characterized by an infinitely elastic supply of minerals at price P_m^f). This carbon tax is 2.3% below the one chosen when taking into account mineral scarcity. Then, the regulator commits to charge the carbon tax $\hat{\tau}$, and resources' owners, power producers and electricity consumers make their choices at every period. The resulting equilibrium differs from the one planned by the regulator, aside from the initial price of minerals which is indeed equal to the one used by the regulator to model the impact of its policy.

We find that the trajectory of mineral use, and of green capital are unchanged, but that fossil resource use is affected by myopia (see Table 1). In fact, $x(t)$ is always larger under myopic regulation. As a consequence, cumulative carbon emissions overshoot by 11% the target. This comes from the fact that ignoring minerals' exhaustibility boils down to under-estimating their real cost, hence that of the energy transition. As a result, it leads to undersizing environmental policy. Other policy characteristics are not attained: the stock of green capital at steady state is 32% lower than expected; and the fossil phase-out date is $T_X = 47$ as compared to the myopic policy forecast of 27. The latter is even farther than the one prevailing under constrained regulation, because the constant carbon tax is lower when minerals scarcity is not correctly taken into account in planning climate policy. In our baseline simulation, mineral scarcity constrains the economy substantially, welfare is much lower than expected by the regulator: the unexpected cost of mineral scarcity is equivalent to a permanent fall of the electricity consumption path by 23.7%. Overall, public intervention implies a lower carbon tax and a higher FIP; it lasts longer and is less effective than it could (without myopia).

4 Recycling minerals

If CRM play an important role for the energy transition, recycling them represents a potentially interesting opportunity. In practice, the recycling of specific elements from scrapped equipment is a complex activity, involving several sequential decisions by multiple agents. When designing a capital good, firms choose its chemical composition and the degree of concentration (dispersion) of each element in the mass of the equipment. Choosing a design with few and concentrated elements reduces *ceteris paribus* their recycling costs. Then regulation and firms' decisions affect the recovery rate of end-of-life equipment (i.e. the mass of materials to be treated at recycling facilities). Firms in charge of the recycling process decide which elements to recover, and which to leave in the final unrecoverable waste. This choice is similar to the one done by producers of primary elements at the mine: it depends on both the concentration of the elements in the mass, and the unit value of the element (Fizaine, 2020). Moreover, the technologies at each of these stages evolve over time, in response to economic incentives, learning by doing and technological spillovers. Finally, recycled elements are not always equivalent to those in the primary output.

In what follows we abstract from most of these considerations. In particular, we assume that the concentration of elements in end-of-life equipment is homogeneous and constant, and that primary and secondary materials are perfect substitutes. Our aim is in fact to study how the possibility of recycling affects the timing of the energy transition. In this respect, our simplified framework allows us to obtain answers that we can interpret.

Denote by $\alpha(t) \in [0, 1]$ the rate of recycling of the depreciated green capital $\delta K(t)$. Non-recycled minerals accumulate in an unrecoverable stock, that is assumed not to induce any social damage. Recycling is costly. The associated cost function is $R(\alpha(t), \delta K(t))$, assumed increasing and convex in the first argument, and increasing in the scale of the treated secondary resources $\delta K(t)$. We further assume away any scale (dis)economies in recycling:

$$R(\alpha, \delta K) \equiv \eta(\alpha)\delta K \quad \text{ with } \eta(\alpha) \geq 0,\ \eta(0) = 0,\ \eta'(\alpha) \geq 0 \text{ and } \eta''(\alpha) \geq 0 \tag{32}$$

Moreover, since we want to focus on plausible cases where recycling takes place but it is limited, we also assume that the marginal cost of perfect recycling (i.e. $\alpha(t) = 1$) is larger than the cost of using the backstop technology instead of recycled minerals to invest in green capital:[24]

$$\eta'(0) < \nu < \eta'(1) \tag{33}$$

Notice that we consider the case where all the depreciated green capital can be recycled, independently of its composition in terms of minerals or backstop. This is coherent with our assumption that the backstop technology is a perfect substitute for mineral inputs, and thus that the stock of green capital is homogeneous. As later explained, this assumption has some implications on the results.

In this section we study the case of optimal regulation. The planner's problem (8) is modified in its objective function

$$\max \int_0^\infty e^{-\rho t}[u(e(t)) - C(I(t)) - \nu b(t) - \eta(\alpha(t))\delta K(t)]dt \tag{34}$$

in the potential inputs to green capital investment

$$I(t) = m(t) + b(t) + \alpha(t)\delta K(t) \tag{35}$$

[24] This condition states that investing using at least some recycled green capital is cheaper than using the backstop technology. It also rules out the case of a steady state based on complete recycling of minerals.

and takes into account the additional static constraints: $0 \le \alpha(t) \le 1$. Let T_α denote the first date at which recycling is active.

Next we characterize the optimal path, summarizing results derived in Appendix A.3.[25]

Proposition 5. Optimal energy transition with recycling. *In the case with early resource use and (33) holding, the optimal path of the economy, solving the program given by the objective function (34), under the technology and resource constraints (1)–(3), (35), (5), the non-negativity and carbon ceiling constraints (6), and the initial conditions (7), is determined by the vector* $\{\mu^\diamond(0), \zeta^\diamond(0), \lambda^\diamond(0), T_X^\diamond, T_\alpha^\diamond, T_M^\diamond\}$*, such that the economy converges towards the steady state* $\{K^\star, \mu^\star, \alpha^\star\}$ *defined by:*

$$\begin{aligned}
&\alpha^\star = \eta'^{-1}(\nu) \\
&\nu + C'(\delta K^\star) - \tfrac{\delta}{\rho+\delta}\alpha^\star \left(\nu - \tfrac{\eta(\alpha^\star)}{\alpha^\star}\right) = \tfrac{\phi}{\rho+\delta} u'(\phi K^\star) \\
&\mu^\star = \nu + C'(\delta K^\star)
\end{aligned} \tag{36}$$

following the system of differential equations given by (13), (15), (16) and

$$\begin{aligned}
\dot{\mu}^\diamond(t) = \;& (\rho+\delta)\mu^\diamond(t) - \left[I_{t<T_X^\star}\phi\varepsilon\lambda^\diamond(t) + I_{t\ge T_X^\star}\phi u'\left(\phi K^\diamond(t)\right)\right] \\
& -\delta\left[I_{t\in[T_\alpha^\star, T_M^\star)}\left[\eta'^{-1}(\zeta^\diamond(t))\zeta^\diamond(t) - \eta(\eta'^{-1}(\zeta^\diamond(t)))\right] + I_{t\ge T_M^\star}\left(\alpha^\star\eta'(\alpha^\star) - \eta(\alpha^\star)\right)\right]
\end{aligned} \tag{37}$$

with $I_{t\in\mathbb{T}}$ being dummy variables equal to 1 for $t \in \mathbb{T}$ and zero otherwise. The optimal recycling rate is

$$\alpha^\diamond(t) = \begin{cases} \eta'^{-1}\left(\zeta^\diamond(t)\right) & \forall t \in [T_\alpha^\star, T_M^\star) \\ \alpha^\star & \forall t \ge T_M^\star \end{cases} \tag{38}$$

$T_X^\star$ and $T_M^\star$ imply full exhaustion of $\overline{X}$ and M_0, and satisfy (17), while

$$T_\alpha^\star = \max\left\{0, \frac{1}{\rho}\ln\left(\frac{\eta'(0)}{\zeta^\diamond(0)}\right)\right\} < T_M^\star \ . \tag{39}$$

The possibility to recycle green capital affects the steady state. Comparing the equations determining the optimal level of green capital in (36) and (12), we notice that the average cost of investment is reduced from ν (when using the backstop alone) to the weighted average of recycling and backstop production costs.[26] As a result, a larger stock of green capital is optimally chosen, and its steady state value is thus lower. It is possible to permanently benefit from investment at a lower average cost because of our assumption according to which all green capital can be recycled,

[25] See Appendix A.3 for an explicit representation of the differential equation (37).

[26] The cost of investment at steady state is the sum of the capital adjustment cost $C(\delta K^\star)$ and of the input cost. The latter depends on the composition of investment between the backstop and recycling. The average cost of the former is ν, while it equals $\frac{R(\alpha,\delta K)}{\alpha\delta K} = \frac{\eta(\alpha)}{\alpha}$, from (32), for recycled inputs. Hence, at steady state the average input cost is $(1-\alpha^\star)\nu + \alpha^\star\frac{\eta(\alpha^\star)}{\alpha^\star} = \nu - \alpha^\star\left(\nu - \frac{\eta(\alpha^\star)}{\alpha^\star}\right)$. The second term of this expression of the average cost of investment inputs is included in the last term on the left-hand-side of the second equation in (36). There, this reduction of the average input cost is weighted more the more frequent is recycling (δ) and the lower is impatience (ρ).

even if it results from investment relying on the backstop technology.[27]

When investment in green capital relies on both exhaustible minerals and recycling, the value of green capital should equalize the full marginal cost of each investment option. Both options imply the marginal capital-adjustment cost $C'(I(t))$. The mineral option implies also the value of minerals $\zeta(t)$, while recycling implies a marginal recycling cost $\eta'(\alpha(t))$. If $[T_\alpha^\star, T_M^\star)$ is not empty, i.e. (33) holds, during this time interval both the primary and the secondary resources are used, because their costs are equal (i.e. (38) holds), and the recycling rate increases along with the rising value of minerals.

Eventually the recycling rate stabilizes at its steady state level. This happens when investment starts relying on the backstop technology rather than on minerals. For the same logic used above, the marginal cost of recycling should be equal to that of the backstop technology, and it shall therefore be constant, implying a constant recycling rate at $\alpha^\star$ (see (36), (38)).

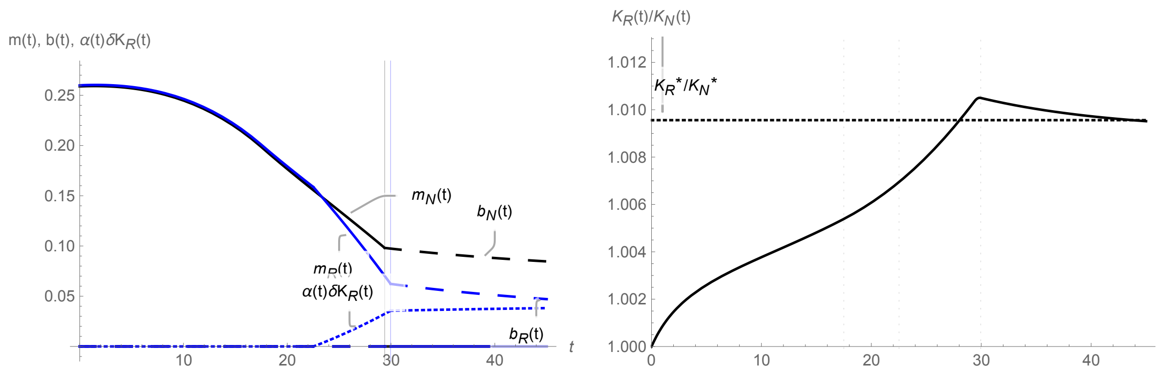
Moreover, it is optimal to recycle but not from the beginning of the planning horizon, *i.e.* $T_\alpha^\star > 0$, if recycling is costly and minerals are initially abundant, in the sense that even the very first unit recycled costs more than the value of minerals: $\zeta^\diamond(0) < \eta'(0)$ in (39) (with (33) holding).

Finally, once minerals are exhausted, the dynamic system is affected by recycling only through the differential equation of the value of green capital (37). It shows that the value of green capital takes into account the fact that investment boosts the future recycling possibilities, limiting the costly use of the backstop technology. As a consequence recycling affects the optimal trajectories of all variables, leading, for instance, to a lower initial value of minerals $\zeta(0)$.

To illustrate how recycling affects the optimal energy transition under scarcity of critical raw materials, let us perform a comparative dynamics exercise comparing the trajectory of an economy endowed with an efficient recycling technology to the one prevailing in an economy without recycling. We use the version of the model with logarithmic utility function and quadratic costs for investment (see Appendix A.3). We present the case of one optimal sequence of phases, where fossil and mineral resources are used from the start, but not recycling, and fossil phase out happens before mineral exhaustion, while recycling starts after fossil phase out but before mineral exhaustion: $T_x = T_m = 0, \quad T_X < T_\alpha < T_M = T_b$. Such a sequence prevails for initial stock of minerals M_0 sufficiently large and of a carbon budget $\overline{X}$ sufficiently small, and that $\zeta(T_X) < \zeta(T_\alpha) = \eta'(0)$. Table 2 summarizes the impact of recycling on the optimal trajectories.[28] Figure 3 illustrates the evolution over time of investment and green capital.

[27]The backstop technology can be interpreted as the availability of an abundant input at a constant, but high, marginal cost, a sort of non-critical raw materials. As an alternative, we could consider two distinct stocks of green capital embedding either minerals or the input from the backstop technology. This approach adds complexity without altering the main messages put forward in this article.

[28]We use parameters of Table 3, together with $\eta_1 = 4$ and $\eta_2 = 5/6$ in (80) of Appendix A.3 (the case without recycling prevails for $\eta_1 > \nu$).



(a) Recycling, use of minerals and backstop (b) Relative stock of green capital

Figure 3: The impact of recycling on investment and green capital (without recycling in black, indexed N, with recycling in blue, indexed R)

The stock of steady-state green capital is larger under recycling than without recycling, as already shown analytically. As this increase remains moderate while a large share (60%) of depreciated capital is recycled, a smaller steady flow of backstop is used (see panel (a) in Figure 3).

The initial value of the stock of green capital, the initial value of fossil resources and the initial value of minerals are all lower in the case with recycling. In fact, the possibility to recycle relaxes the constraints imposed by natural resources. The economy is globally better off, and the positive income effect calls for higher energy consumption at all dates. To achieve this goal, the path of the stock of green capital is always chosen larger when recycling is possible (see panel(b) in Figure 3), taking advantage of lower input costs for investment. As shown on panel (a) in Figure 3, in the recycling economy there is more investment and a larger use of minerals up to date T_α (though, under the chosen set of parameters, the difference in the figure is mild). The income effect, together with the exogeneity of the initial endowment in green capital, and the fixed total amount of fossil resources used, gives rise to a sort of weak green paradox: carbon emissions increase at early stage, and fossil resources are exhausted sooner (van der Ploeg and Withagen, 2012). However this time reallocation of fossil resource does not entail any climate related damage, and indeed improves welfare.

K^*	μ^*	b^*	λ_0	μ_0	ζ_0	$m(0)$	$x(0)$	T_X	T_M
$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$

Table 2: The impact of recycling

In our context, recycling affects the energy transition through additional non trivial channels. Let us here underscore three mechanisms at work, influencing the optimal timing of the use of mineral resources and of the backstop input.[29] The economy where recycling is feasible can reuse the mineral resource several times. It is as if it had a larger endowment of non-renewable resources (Schulze, 1974) and as shown in section 2, a larger stock of such resources favors early use of

[29]The magnitude of these effects depends in particular on the size of the adjustment costs and recycling costs.

minerals, and postpones the date at which it switches to the costly backstop technology. However, there are additional effects due to the asymmetric nature of recycling activity through time: one first needs to exploit the primary resource, to be able to then have a flow of secondary resource to recycle. This has two consequences. First, as in Fabre *et al.* (2020) this generates an additional incentive for earlier use of the mineral resources. Second, one would like to build up a large stock of green capital by date T_b, to benefit of a large flow of secondary resources (δK), from T_b onward, when using the backstop technology, in order to reduce the costly use of the latter. Panel (b) in Figure 3 illustrates this point. It depicts the evolution of the ratio of the stock of green capital in the economy with recycling to the one without recycling. We see that the stock of green capital is always larger in the former case. Moreover we see that the discrepancy reaches its highest level when the backstop is used for the first time, then falls toward its steady state level $K^\star/K^*$. This holds although the green capital increases monotonically in both economies.[30] We thus establish a novel result, according to which it is optimal to intensively use minerals for building up a large green capital stock just before switching to the backstop input.

5 Concluding remarks

This article presents a model of the energy transition, where the policy is chosen to maximize social welfare under a climate constraint. Specific equipment in the energy transition -or green capital- embeds either scarce minerals or a relatively expensive backstop technology, which can be recycled. We introduce some features of political economy and study the case where the optimal carbon tax is not feasible, so that the regulator has to turn to an acceptable sub-optimal policy. In addition, we account for the regulator's myopia as the latter takes his decisions as if minerals were abundant.

First, beyond improving the representation of the energy transition, adding the plausible role of mineral natural resources affects its qualitative features. As compared to the literature, here the energy transition encompasses the exhaustion of two stocks of non renewable resources: the carbon budget and then minerals stock. This improves welfare, since, without minerals, the economy would build its stock of green capital relying exclusively on the expensive backstop input. Of course, the opposite conclusion can be drawn if one compares to the fictitious situation with an inexhaustible abundant mineral resource, as in our case of the myopic regulator's forecast. Yet, despite the analogy, mineral scarcity has different implications from the stringency of the carbon budget. In fact, more abundant minerals contribute to increasing energy consumption mostly in the future, while a less stringent carbon budget tends to increase relatively more consumption early on. Moreover, the two scarce resources differ in their technological nature: minerals embedded in green

[30]Notice that this result differs from the one in Proposition 6 of Kollenbach (2017), where the stock of specialized capital for the production of renewable energy overshoots its long run use, resulting in excess capacity at steady state. In fact, in our case the green capital is always used at full capacity.

capital can be partially reused through recycling, but that is not the case for used carbon budget.[31] We have shown that recycling has implications for several features of the energy transition, including the timing of the adoption of the backstop solution, and a momentum in investment in green capital right before this time.

Second, we show that a close-to-real-world climate policy, consisting in a fixed carbon tax whose proceeds fund a feed-in-premium for electricity produced from renewable sources, may result in delayed fossil phasing out, and reduces welfare. Our analysis underscores the need to ensure the coherence of climate policy design, when internalizing the constraint of its social acceptability. In fact, we have shown that, if the initial desirable investment in green capital is large, the above mentioned constrained policy calls for a relatively high initial level of the carbon tax, in order to generate sufficient tax revenues to finance the subsidies. This example points to the difficulty of squaring the circle when pursuing the objectives of climate mitigation and policy acceptability.

Third, if the regulator does not take into account the need for non renewable mineral resources to build up the infrastructure for the energy transition, the carbon budget will not be satisfied, and the date of fossil phase out is farther than planned. While the role of materials for the energy transition has first been pointed out in academics, the topic has increasingly attracted the attention in public administration. Applied models used for policy decision making and design should therefore integrate this feature.[32]

Finally, do note that a material can be labeled as critical not only on the basis of geological scarcity but also on the basis of economic factors. Criticality of a raw material can be driven by its economic importance and the supply risk. Economic importance refers to how crucial a material is in terms of end applications and added value of the corresponding manufacturing sectors. Supply risk is measured in terms of the concentration of primary supply from commodity producing countries, taking into account their governance performance and trade aspects. Moreover, recycling could drastically change the picture with a different concentration of secondary supply of resources. Extensions could therefore consider market failures on the primary and secondary resource markets.

[31] The absence of CCS or climate engineering from our analysis emphasizes this difference (see footnote 12).

[32] Luderer *et al.* (2019) use integrated assessment models to obtain scenarios of the evolution of energy production systems, then apply coefficients in mineral requirements per technology from the literature on life-cycle impact assessment, to estimate the prospective demand for minerals. In the case of copper, Seck *et al.* (2020) model also the primary supply and recycling. An alternative approach relies on input-output tables for comparative statics (e.g. Beylot *et al.*, 2019).

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A Appendix

A.1 Detailed derivation of results in Section 2

The current value Hamiltonian associated to the social planner's program (8) is:

$$\mathcal{H}(.) = u(x(t) + \phi K(t)) - C(m(t) + b(t)) - \nu b(t) - \lambda(t)\varepsilon x(t) + \mu(t)(m(t) + b(t) - \delta K(t)) + \zeta(t)(-m(t))$$

Denoting ω_i the Lagrange multipliers for the inequality constraints (6), the Lagrangian is:

$$\mathcal{L}(.) = \mathcal{H}(.) + \omega_x x(t) + \omega_m m(t) + \omega_b b(t) + \omega_X(\overline{X} - X(t)) + \omega_M M(t)$$

The first order and Euler optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \iff u'(e(t)) - \lambda(t)\varepsilon + \omega_x = 0 \iff u'(e(t)) = \varepsilon\lambda(t) - \omega_x \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial m} = 0 \iff -C'(I(t)) + \mu(t) - \zeta(t) + \omega_m = 0 \iff C'(I(t)) + \zeta(t) = \mu(t) + \omega_m \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial b} = 0 \iff -C'(I(t)) - \nu + \mu(t) + \omega_b = 0 \iff C'(I(t)) + \nu = \mu(t) + \omega_b \tag{42}$$

$$-\frac{\partial \mathcal{L}}{\partial X} = \omega_X = -\dot{\lambda}(t) + \rho\lambda(t) \iff \dot{\lambda}(t) = \rho\lambda(t) - \omega_X \tag{43}$$

$$-\frac{\partial \mathcal{L}}{\partial K} = -\phi u'(e(t)) + \delta\mu(t) = \dot{\mu}(t) - \rho\mu(t) \iff \dot{\mu}(t) = (\rho + \delta)\mu(t) - \phi u'(e(t)) \tag{44}$$

$$-\frac{\partial \mathcal{L}}{\partial M} = -\omega_M = \dot{\zeta}(t) - \rho\zeta(t) \iff \dot{\zeta}(t) = \rho\zeta(t) - \omega_M \tag{45}$$

Proof of Lemma 1. Taking into account that $X(t) < \overline{X}$ implies $\omega_X = 0$ in (43), proving the first result. Given that $x(t) > 0$ implies $\omega_x = 0$ in (40), taking logs and differentiating the latter with respect to time, then using (43) with $\omega_X = 0$, one gets the second result. □

Proof of Lemma 2. The system (9) results of (41) for $m(t) > 0$ and $b(t) = 0$ (thus $\omega_m = 0$ and $\omega_b > 0$), and of (42) for $m(t) = 0$ and $b(t) > 0$ (thus $\omega_b = 0$ and $\omega_m > 0$). The assumptions on the fact that the two inputs are perfect substitutes in investment, and that the same investment cost function applies to them, implies that it could be optimal to simultaneously use both inputs only if they have the same social cost: $\zeta(t)$ for minerals, ν for the backstop technology. Minerals use $m(t) > 0$ is possible only when their stock is not exhausted, i.e. is $M(t) > 0$, thus when $\omega_M = 0$ in (45), implying that the Hotelling principle of optimal resource management applies to minerals too, so that $\zeta(t)$ increases at the social discount rate. Once the scarcity rent of minerals equals the cost of the backstop (i.e. when $\zeta(t) = \nu$), and forever thereafter, only the backstop input will be used ($T_M = T_b$). Investment switches instantly from minerals to the backstop input. □

Proof of Lemma 3. According to Lemma 2, any remaining amount of minerals beyond date T_b is worthless. It is therefore optimal to exhaust all the stock of minerals M_0 by date T_b, thus $T_M = T_b$:

$$\int_0^{T_M} m(t)dt = M_0 \tag{46}$$

Moreover, during the interval of time $[T_m, T_M)$, when investment relies on minerals, their value is below the cost of the backstop, so that $\zeta(0) < \nu$. Concerning the exhaustion of fossils, notice first that, as long as green capital does not shrink indefinitely to zero, the level of consumption converges asymptotically toward a positive level. Therefore, the marginal utility of consumption cannot indefinitely grow exponentially at a constant rate, as required for positive fossil resource use, according to Lemma 1. At some finite date T_X, the use of fossil resource is nil for the first time $x(T_X) = 0$. It will then be nil forever, $x(t) = 0\ \forall t \geq T_X$. All the possible emissions of carbon will be exhausted by date T_X:

$$\int_0^{T_X} x(t)dt = \overline{X}/\varepsilon \tag{47}$$

□

Proof of Lemma 4. Last phase: $\forall t \geq \max\{T_X, T_M\}$ $X(t) = \overline{X}$ and $x(t) = 0$, $M(t) = 0$ and $m(t) = 0 \Rightarrow \omega_x > 0$, $\omega_m > 0$, while $b(t) > 0$ $\omega_b = 0$. Use these conditions with (4) and (9) to write (3) as $\dot{K}(t) = C'^{-1}\left(\mu(t) - \nu\right) - \delta K(t)$. Use the condition $x(t) = 0$ in (1) to set $e(t) = \phi K(t)$ into (44) as $\dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi u'(\phi K(t))$. This system of two differential equations for $t \geq \max\{T_X, T_M\}$ (i.e. the second lines in (13)-(14)) is characterized by the following two isoclines of the phase diagram in the (K, μ) plane

$$\begin{aligned} \mu_t|_{\dot{K}_t=0} &= \nu + C'\left(\delta K_t\right) \\ \mu_t|_{\dot{\mu}_t=0} &= \frac{\phi}{\rho+\delta} u'\left(\phi K_t\right) \end{aligned}$$

The $\dot{\mu}_t = 0$ locus is downward sloping, decreasing from $\frac{\phi}{\rho+\delta}u'(0)$ down to zero. The $\dot{K}_t = 0$ locus is upward sloping from $\nu + C'(0)$ to infinity. The steady state (12) exists if $\nu + C'(0) < \frac{\phi}{\rho+\delta}u'(0)$ and is unique. It implies $b^* = \delta K^* > 0$, thus $T_B = \infty$. The phase diagram is such that K declines in the region below the $\dot{K}_t = 0$ isocline, and vice versa, while μ declines in the region below the $\dot{\mu}_t = 0$ isocline. The optimal path is saddle point and stable, and is located below the $\dot{\mu}_t = 0$ isocline and above the $\dot{K}_t = 0$ isocline for $K_t < K^*$, and viceversa. Convergence to steady state is asymptotic. □

Proof of Proposition 1. In the case with early resource use $T_x = T_m = 0$, the transition toward the steady state goes through three phases.

First phase. $\forall t < \min\{T_X, T_M\}$: $X(t) < \overline{X}$ and $M(t) > 0$, $x(t) > 0$, $m(t) > 0 \Rightarrow \omega_x = \omega_m = 0$, while $b(t) = 0$ and $\omega_b > 0$. Use these conditions with (4) and (9) to write (3) as the first expression in (13). Use (40) with $\omega_x = 0$ into (44), to get the first expression in (14). The evolution of $\lambda(t)$ and $\zeta(t)$ over this phase, according to (15)-(16), is given in the proofs of Lemmas 1 and 2 respectively.

There are two distinct cases for the second phase, according to whether $T_X < T_M$ (Case 1), or $T_X > T_M$ (Case 2).

Second phase, case 1. $\forall t < [T_X, T_M)$: $X(t) = \overline{X}$ and $M(t) > 0$, $m(t) > 0 \Rightarrow \omega_m = 0$, while $x(t) = b(t) = 0$ $\Rightarrow \omega_x > 0$, $\omega_b > 0$. Use these conditions with (4) and (9) to write (3) as the first expression in (13). Use the condition $x(t) = 0$ in (1) to set $e(t) = \phi K(t)$ into (44), and get the second expression in (14). The evolution of $\zeta(t)$ over this phase, according to (15), is given in the proof of Lemma 2.
This phase starts at date T_X such that (47) holds for $x(t) = u'\left(\varepsilon\lambda(t)\right) - \phi K(t)$ from (1) and (40) with $\omega_x = 0$, reflecting the $\{K(t), \lambda(t)\}$ solution of the system given by the first expressions of (13)-(16). Moreover, the date T_X is such that for the first time $K(T_X)$, resulting of the solution of the system starting from K_0, is at a level such that (40) with $\omega_x = 0$ holds for $x(t) = 0$, i.e. the first part of (17).
This phase ends at date T_M such that (46) holds for $m(t)$ in (9) reflecting the $\{K(t), \mu(t), \zeta(t)\}$ solution of the system given by the first expressions of (13)-(16) up to date T_X, then of the first expressions in (13) and (15) but the second expressions in (14) and (16), and such that at date T_M for the first time $\zeta(t)$ equals the cost of the backstop technology ν, i.e. the second part of (17).

Second phase, case 2. $\forall t < [T_M, T_X)$: $X(t) < \overline{X}$ and $M(t) = 0$, $x(t) > 0$, $b(t) > 0 \Rightarrow \omega_x = \omega_b = 0$, while $m(t) = 0 \Rightarrow \omega_m > 0$. Use these conditions with (4) and (9) with $T_b = T_M$ to write (3) as the second expression in (13). Use (40) with $\omega_x = 0$ into (44), to get the first expression in (14). The evolution of $\lambda(t)$ over this phase is given in the proof of Lemma 1.
This phase starts at date T_M such that (46) holds for $m(t)$ in (9) reflecting the $\{K(t), \mu(t), \zeta(t)\}$ solution of the system given by the first expressions of (13)-(16). Moreover, the date T_M is such that for the first time $\zeta(t)$ equals the cost of the backstop technology ν, i.e. the second part of (17).
This phase ends at date T_X such that (47) holds for $x(t) = u'\left(\varepsilon\lambda(t)\right) - \phi K(t)$ from (1) and (40) with $\omega_x = 0$, reflecting the $\{K(t), \lambda(t)\}$ solution of the system given by the first expressions of (13)-(16) up to date T_M, then of the first expressions in (14) and (16) but the second expressions in (13) and (15), and such that at

date T_X for the first time $K(T_X)$ is at a level such that (40) with $\omega_x = 0$ holds for $x(t) = 0$, i.e. the first part of (17).

Third phase. $\forall t \geq \max\{T_X, T_M\}$ the dynamics are characterized in the proof of Lemma 4. $\square$

Sub-case with quadratic costs and logarithmic utility. For the simulations and part of the analysis, we consider the specific functional forms (18)-(19) and we focus on the case when the carbon budget is more stringent than minerals scarcity, i.e. $T_X < T_M$. According to Proposition 1, the optimal trajectory is given by (15)-(16) and

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases} \tag{48}$$

$$\forall t \in [T_X, T_M) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/K(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases} \tag{49}$$

$$\forall t \geq T_M \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/K(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \nu - c_1) - \delta K(t) \end{cases} \tag{50}$$

and, according to Lemma 4, approaches the steady state at which

$$\mu^* = \frac{\nu + c_1}{2}\left(1 + \sqrt{1 + 4\frac{c_2\delta\gamma}{(\rho+\delta)(\nu+c_1)^2}}\right) \quad , \quad K^* = \frac{\mu^* - \nu - c_1}{c_2\delta} \tag{51}$$

For this sequence to be optimal, the following restrictions must hold

$$\frac{\gamma}{\varepsilon\phi K^*} < \lambda(0) < \frac{\gamma}{\varepsilon\phi K_0} \quad \Leftrightarrow \quad x(0) > 0,\ T_x = 0,\ T_X > 0 \text{ finite} \tag{52}$$

$$\zeta(0) < \nu < \infty \quad \Leftrightarrow \quad b(0) = 0, \quad \zeta(0) < \mu(0) - c_1 \quad \Leftrightarrow \quad m(0) > 0,\ T_m = 0,\ T_M > 0 \text{ finite} \tag{53}$$

These restrictions involve endogenous variables, and one must check ex-post that the candidate solution satisfies them. They restrict the set of exogenous values of the stocks $\overline{X}, M_0, K_0$. They mean that the sequence under study is optimal for a relatively stringent climate policy, large minerals stock and small initial green capacity, and for a specific configuration of the costs.
For the case when minerals scarcity is binding first, the trajectory is described by (50)-(53), (48) with T_M replacing T_X, and (49) replaced by

$$\forall t \in [T_M, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases} \tag{54}$$

<u>*Sequence of the phases.*</u> Here we show, based on specifications (18)-(19), that the relative size of the stocks $\overline{X}$ and M_0 determines whether case 1 or case 2 for the second phase prevails.
We characterize the knife-edge case when $\overline{X}$ and M_0 are such that, for K_0 given, $T_X = T_M$, i.e. the carbon budget and the mineral stock are exhausted at the same date T, then we explore numerically the relationship between $\overline{X}$ and M_0 in this knife-edge case.
Phases 1 and 2 collapse in one single phase, ending at T, so that (15)-(17) give

$$x(T) = 0 \iff \frac{\gamma}{\phi K(T)} = \varepsilon\lambda(0)e^{\rho T} \tag{55}$$

$$\zeta(T) = \nu \iff e^{\rho T} = \frac{\nu}{\zeta(0)} \tag{56}$$

Taking into account the first expression in (16), the integration of the first expression in (14) yields

$$\mu(t) = e^{\rho t}\left[\left(\mu(0) - \frac{\varepsilon\phi}{\delta}\lambda(0)\right)e^{\delta t} + \frac{\varepsilon\phi}{\delta}\lambda(0)\right]$$

Moreover, using (15) in (4), we have

$$m(t) = \frac{\mu(t) - \zeta(0)e^{\rho t} - c_1}{c_2}$$

thus condition (46) can be expressed as follows

$$\begin{aligned} M(T) = 0 &\iff \int_0^T m(t)dt = M_0 \\ &\iff c_2 M_0 = \left(\mu(0) - \frac{\varepsilon\phi}{\delta}\lambda(0)\right)\frac{e^{(\rho+\delta)T} - 1}{\rho+\delta} + \left(\frac{\varepsilon\phi}{\delta}\lambda(0) - \zeta(0)\right)\frac{e^{\rho T} - 1}{\rho} - c_1 T \end{aligned} \tag{57}$$

Taking into account (4) with $b(t) = 0$, the integration of (3) yields

$$K(t) = K_0 e^{-\delta t} + e^{-\delta t}\int_0^t e^{\delta s} m(s) ds$$

Then, using the expressions for $m(t)$ and $\mu(t)$ above, prevailing up to date T, we have

$$\begin{aligned} K(T) = K_0 e^{-\delta T} &+ \frac{1}{c_2}\left[\left(\mu(0) - \frac{\varepsilon\phi}{\delta}\lambda(0)\right)\frac{e^{(\rho+\delta)T}}{\rho+2\delta} + \left(\frac{\varepsilon\phi}{\delta}\lambda(0) - \zeta(0)\right)\frac{e^{\rho T}}{\rho+\delta} - \frac{c_1}{\delta}\right] \\ &- e^{-\delta T}\frac{1}{c_2}\left[\left(\mu(0) - \frac{\varepsilon\phi}{\delta}\lambda(0)\right)\frac{1}{\rho+2\delta} + \left(\frac{\varepsilon\phi}{\delta}\lambda(0) - \zeta(0)\right)\frac{1}{\rho+\delta} - \frac{c_1}{\delta}\right] \end{aligned} \tag{58}$$

In order to pin down the trajectory of the value of green capital $\mu(t)$, we solve the dynamics for the last phase. The linearization around the steady state (51) of the last phase's dynamic system of the second expression in (50) yields

$$\mu(t) = \mu^* + c_2(\theta+\delta)(K(T) - K^*)e^{\omega(t-T)}$$

with

$$\theta = \frac{1}{2}\left[\rho - \sqrt{\rho^2 + 4\left((\rho+\delta)\delta + \frac{\gamma}{c_2 K^{*2}}\right)}\right]$$

which, using the expression of $\mu(t)$ computed for the first phase, yields

$$e^{\rho T}\left[\left(\mu(0) - \frac{\varepsilon\phi}{\delta}\lambda(0)\right)e^{\delta T} + \frac{\varepsilon\phi}{\delta}\lambda(0)\right] = \mu^* + c_2(\theta+\delta)(K(T) - K^*) \tag{59}$$

Condition (47) can be computed from (40), taking into account $\omega_x = 0$, (1) and the first expression in (16)

$$\overline{X} = \int_0^T x(t)dt = \int_0^T \left(\frac{\gamma}{\varepsilon\lambda(0)}e^{-\rho t} - \phi K(t)\right)dt = \frac{\gamma}{\rho\varepsilon\lambda(0)}(1 - e^{-\rho T}) - \phi\int_0^T K(t)dt$$

while (3), with (4) and $b(t) = 0$, and condition (46), together, imply

$$\dot{K}(t) = m(t) - \delta K(t) \Longrightarrow K(T) + \delta\int_0^T K(t)dt = K_0 + M_0$$

Eliminating the integral in the last two expressions, yields

$$\overline{X} = \frac{\gamma}{\rho\varepsilon\lambda(0)}(1 - e^{-\rho T}) - \frac{\phi}{\delta}(K_0 + M_0 - K(T)) \tag{60}$$

Equations (55) to (59) form a system of 5 equations with 5 unknowns $\{\mu(0), \zeta(0), \lambda(0), T, K(T)\}$, which can in principle be solved and allows to obtain the unknowns as functions of the cost parameters, preference

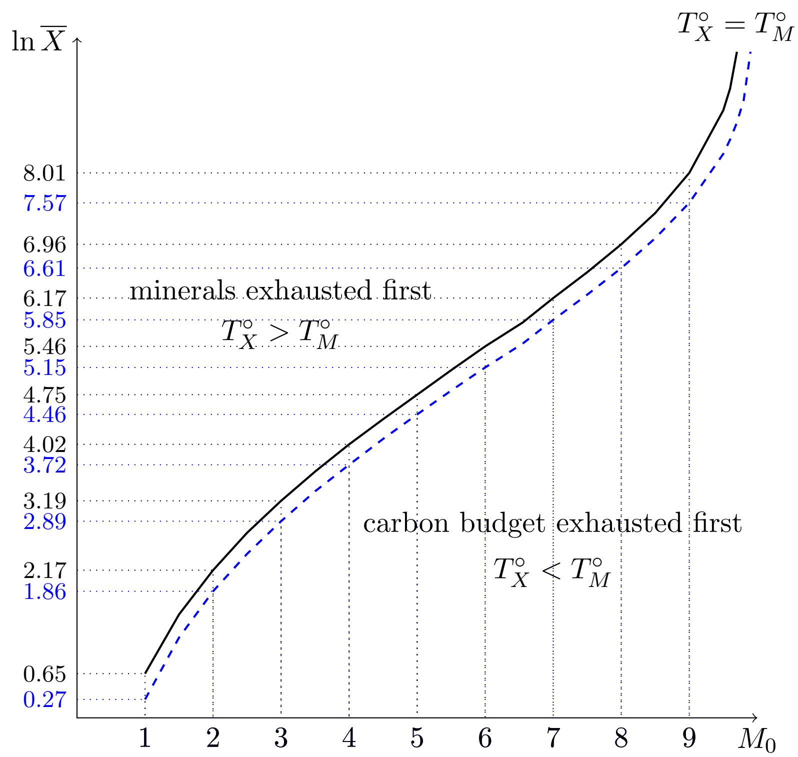

Figure 4: Frontier $(M_0, \ln \overline{X})$ for simultaneous exhaustion $T_X^\circ = T_M^\circ$. Continuous-black: $K_0 = 1$; dashed-blue: $K_0 = 0$.

parameter, the discount rate, the depreciation rate, and the initial stocks K_0 and M_0. Then equation (60) gives the level of the carbon budget for which exhaustion is simultaneous, and we can conclude that for a carbon budget smaller than this threshold the carbon budget will be exhausted first, and vice versa.

The system is quite complex and highly non-linear, and it cannot be solved in closed-form. Therefore we resort to numerical simulations, using the parameters reported in Table 3. Figure 4 shows the frontier $(M_0, \ln \overline{X})$ defined above, for $M_0 \in (1, 9.7)$ and $\overline{X} \in (1.91, 17930) \implies \ln \overline{X} \in (0.65, 9.79)$. Figure 4 also shows in dashed blue the frontier for the case with a lower endowment of green capital ($K_0 = 0$).

Comparative dynamics: We run simulations of the dynamic system with specifications (18)-(19). For illustrative purposes, we present three cases: the baseline case with parameters' values reported in Table 3; the case of a 20% lower carbon budget; and the case of a 10% lower stock of minerals. Under these configurations of parameters and stocks, case 1 prevails , i.e. the carbon budget binds first.

ϕ	ρ	δ	ε	γ	ν	c_1	c_2	K_0	M_0	$\overline{X}$
0.9	0.03	0.01	0.25	1.8	5	0.15	20	1	6	203.28

Table 3: Parameters for the baseline simulation.

The system of ordinary differential equations is solved using the backward shooting procedure from $\{\mu^*, K^*\}$ until $K(0) = K_0$ (Brunner and Strulik, 2002). The guess for the initial value of minerals, $\zeta(0)$, is adjusted by trial and error until cumulative mineral use equals M_0. For the baseline case, we normalize the initial value of carbon at $\lambda(0) = 1$ and set its constant growth rate equal to ρ. We then compute the cumulative use of fossil resources, and use this value to define the carbon budget $\overline{X}$ in Table 3. For the other cases, we adjust $\lambda(0)$ to ensure that cumulative emissions equal the predefined target $\overline{X}$ (or $0.8\overline{X}$).

A.2 Detailed derivation of results in Section 3

In this Appendix, we proceed in steps: we derive the necessary conditions for the solutions of (i) the household and (ii) power utility' problems (20) and (22); (iii) we prove Proposition 2; (iv) we obtain the social welfare function (28); (v) we prove Proposition 3; (vi) we prove Proposition 4; (vii) we present the case of isoelastic utility function; ($viii$) finally, we specify the solution in the case of quadratic costs and logarithmic utility.

(i) *The household's problem.* Let $w(t)$ be the value of wealth $a(t)$. The Hamiltonian of (20) is:

$$\mathcal{H} = z(t) + u\left(e(t)\right) + w(t)\left[r(t)a(t)e(t) + \pi_x(t) + \pi_m(t) + \pi_e(t) + \mathcal{T}(t) - z(t) - P_e(t)\right]$$

The first order conditions with respect to $z(t)$ and $e(t)$, and the Euler condition for $a(t)$ are:

$$\begin{aligned} 1 &= w(t) && (61)\\ u'(e(t)) &= w(t)P_e(t) && (62)\\ \dot{w}(t) &= (\rho - r(t))w(t) && (63) \end{aligned}$$

(61) in (62) gives (21), while (61) and (63) imply that $\forall t \geq 0$

$$r(t) = \rho \tag{64}$$

(ii) *The power utility's problem.* Given (3)-(4), the Lagrangian associated to program (22) is:

$$\begin{aligned}\mathcal{L} = P_e(t)x(t) + \left(P_e(t) + \sigma(t)\right)\phi K(t) - \left(P_x(t) + \varepsilon\tau(t)\right)x(t) - C(m(t) + b(t)) - P_m(t)m(t)\\ - \nu b(t) + \mu_d(t)\left(m(t) + b(t) - \delta K(t)\right) + \omega_x(t)x(t) + \omega_m(t)m(t) + \omega_b(t)b(t)\end{aligned}$$

μ_d denotes the value of green capital, and the $\omega_i(t)$ denote the Lagrange multipliers of non negativity constraints on the control variables $i \in \{x, m, b\}$. The first order conditions and Euler equation are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x(t)} &= P_e(t) - P_x(t) - \varepsilon\tau(t) + \omega_x(t) = 0 && (65)\\ \frac{\partial \mathcal{L}}{\partial m(t)} &= -P_m(t) - C'(I(t)) + \mu_d(t) + \omega_m(t) = 0 && (66)\\ \frac{\partial \mathcal{L}}{\partial b(t)} &= -\nu - C'(I(t)) + \mu_d(t) + \omega_b(t) = 0 && (67)\\ \dot{\mu_d}(t) &= \left(r(t) + \delta\right)\mu_d(t) - \left(P_e(t) + \sigma(t)\right)\phi && (68) \end{aligned}$$

Concerning investment, from (66)-(67) $m(t) > 0$ and $b(t) > 0$ (thus $\omega_m(t) = \omega_b(t) = 0$) if and only if $P_m(t) = \nu$. When $P_m(t) < \nu$, $m(t) > 0$ while $b(t) = 0$, hence $I(t) = m(t)$ and $\omega_m(t) = 0$, mineral use results of (66) and is:

$$m(t) = C'^{-1}\left(\mu_d(t) - P_m(t)\right) \tag{69}$$

When $P_m(t) \geq \nu$, $b(t) > 0$ while $m(t) = 0$, hence $I(t) = b(t)$ and $\omega_b(t) = 0$, so that (67) implies:

$$b(t) = C'^{-1}\left(\mu_d(t) - \nu\right) \tag{70}$$

Setting $x(t) > 0 \Leftrightarrow \omega_x(t) = 0$ in (65) gives (23).

(iii) *Proof of Proposition 2.* Consider first the behavior of mineral resource owners $j \in [0, J]$, each owning a resource stock $M_j(t)$, and producing $m_j(t)$ at no cost. The program is

$$\max_{m_j(t)} P_m(t)m_j(t) \text{ s.t. } \dot{M}_j(t) = -m_j(t) \text{ and } m_j(t) \geq 0 \text{ for } M_j(0) \text{ given}$$

Let ζ_j denote the value of the stock for the owner j, to write the Lagrangian as

$$\mathcal{L} = P_m(t)m_j(t) - \zeta_j(t)m_j(t) + \omega_m(t)m(t)$$

When $m_j(t) > 0$, $\omega_m(t) = 0$, the first order condition on m_j and the Euler condition on M_j imply that: $P_m(t) = \zeta_j(t)$ and $\dot{P}_m(t)/P_m(t) = r(t)$. Focusing on the representative firm of the sector, and taking into account (64), we get (24).

According to the analysis in (*ii*) above, the inverse demand function for minerals is truncated from above at the chock price ν. Hence the representative resource owner will exhaust the stock by date T_M such that $P_m(T_M) = \nu$ and (46) holds. Therefore, substituting in (3) for $I(t) = m(t)$ using (69) when $t < T_M$, and for $I(t) = b(t)$ using (70) when $t \geq T_M$, we get (25).

Let us now turn to the behavior of fossil resources owners. Given that the policy implies that fossil resources are not exhausted, and that the production costs are assumed nil, the supply schedule is perfectly elastic at:

$$P_x(t) = 0 \tag{71}$$

that, together with (23), implies that when $x(t) > 0$

$$P_e(t) = \varepsilon\tau(t) \tag{72}$$

Combining this with the demand function for electricity (21), we see that at equilibrium consumption is entirely driven by the carbon tax

$$e(t) = u'^{-1}\left(\varepsilon\tau(t)\right) \tag{73}$$

Using (72) and (64) into (68), gives the expression of (26) for $t < T_X$ prevailing when $x(t) > 0$. For $x(t) = 0$, consumption is $e(t) = \phi K(t)$. Using this and (21), as well as (64), into (68), gives the expression of (26) for $t \geq T_X$.

Finally we show that fossil resources are used from the start and over a continuous interval of time. The use of fossil resources is determined as the residual to ensure that the electricity supply covers demand (73)

$$x(t) = \max\{u'^{-1}\left(\varepsilon\tau(t)\right) - \phi K(t), 0\} \tag{74}$$

By definition, the user cost of fossil resources, $\varepsilon\tau(t)$, is non decreasing under a regular climate policy, hence in the case with early resource use the first term of the right-hand-side is non increasing in time. Moreover, under a regular climate policy, from (22) follows that the power producer should avoid any jump in the electricity price. Therefore, using (1) into (21), he gradually phases out fossil use, so that $x(t)$ reaches zero at date T_X, so that (27) applies. □

(*iv*) *The social welfare function.* Here we show that (28) holds. By definition, the household's initial wealth covers the difference between the present value of the flow of expenditure above income:

$$a_0 = \int_0^\infty e^{-\Omega(t)}\left[z(t) + P_e(t)e(t) - \pi_x(t) - \pi_m(t) - \pi_e(t) - \mathcal{T}(t)\right] dt$$

Taking into account (64), substituting for transfers $\mathcal{T}(t)$ from (29), the dividends from the power company:

$$\pi_e(t) = P_e(t)e(t) + \sigma(t)\phi K(t) - \left(P_x(t) + \varepsilon\tau(t)\right)x(t) - C(I(t)) - P_m(t)m(t) - \nu b(t)$$

together with $\pi_m(t) = P_m(t)m(t)$ and $\pi_x(t) = P_x(t)x(t)$, we have:

$$\int_0^\infty e^{-\rho t} z(t)dt = a_0 - \int_0^\infty e^{-\rho t}\left[C(I(t)) + \nu b(t)\right] dt$$

which links the left and the right-hand-sides of (28).

(*v*) *Proof of Proposition 3.* One needs to check that all the conditions defining the optimal path and those of the decentralized equilibrium coincide under this policy. For that compare equations (13), (14) and (15) at the optimum with the corresponding expression at the equilibrium (25), (26) and (24), respectively, and take into account (64). □

(*vi*) *Proof of Proposition 4.* The first part of (*c*) follows from $\tau(t) = \tau$ in (73). This together with (1) gives (*d*). Combining these results in (29) with $\mathcal{T} = 0$, one gets (*b*). The first line in (30) of (*e*) is obtained from (68) using $\tau(t) = \tau$ in (72), $\sigma(t)$ in (*b*), and (64).Continuity of $P_e(t)$, together with (1) and (*d*), imply the first part of (*f*). All other results are obtained as in the proof of Proposition 2. □

(*vii*) *Case with isoelastic utility function:* Consider the following utility function:

$$u(e_t) \equiv \gamma \frac{e(t)^{1-\omega}}{1-\omega} \qquad \text{with } \omega \geq 0 \ .$$

With this specification, (*b*) in Proposition 4 implies: $\sigma(t) = \frac{\gamma^{\frac{1}{\omega}}}{\phi K(t)} (\tau\varepsilon)^{\frac{\omega-1}{\omega}} - \tau\varepsilon$, which can be used in the first line of (30), to write the system of ordinary differential equations as follows (here only for case 2, i.e. $T_X < T_M$):

$$\forall t \in [0, T_X) \quad \begin{cases} \dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \left(\frac{\tau\varepsilon}{\gamma}\right)^{\frac{\omega-1}{\omega}} \frac{\gamma}{K(t)} \\ \dot{K}(t) = \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{\rho t} - c_1\right) - \delta K(t) \end{cases} \tag{75}$$

$$\forall t \in [T_X, T_M) \quad \begin{cases} \dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \frac{\gamma(\phi^{1-\omega})}{K(t)^{\omega}} \\ \dot{K}(t) = \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{\rho t} - c_1\right) - \delta K(t) \end{cases}$$

$$\forall t \geq \max\{T_M, T_X\} \quad \begin{cases} \dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \frac{\gamma(\phi^{1-\omega})}{K(t)^{\omega}} \\ \dot{K}(t) = \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) - \delta K(t) \end{cases}$$

The system converges to the steady state $\{K^*, \mu_d^*\}$, defined by

$$\mu_d^* \quad \text{s.t.} \quad \frac{\mu_d^*}{\delta c_2} - \left(\frac{\gamma}{\rho+\delta}\right)^{\frac{1}{\omega}} \phi^{\frac{1-\omega}{\omega}} (\mu_d^*)^{-\frac{1}{\omega}} - \frac{\nu + c_1}{\delta c_2} = 0 \quad \text{and} \quad K^* = \frac{\mu_d^* - \nu - c_1}{c_2 \delta}$$

There exists a unique μ_d^*, since the function defining it increases monotonically from $-\infty$ for $\mu_d^* = 0$, up to $+\infty$ for $\mu_d^* = \infty$.
According to (75), the policy affects the trajectory during the first phase $[0, T_X)$. The law of motion of μ_d is a decreasing (increasing) function of τ if the elasticity of intertemporal substitution ($1/\omega$) is smaller (greater) than unity. This affects the accumulation of green capital. However for unit elasticity of intertemporal substitution of electricity consumption, the FIP does not affect the value of green capital, nor the equilibrium path of green capital.

(*viii*) *Sub-case with quadratic costs and logarithmic utility.* For the numerical cases presented in Section 3, based on specifications (18)-(19), we have that (74) hands

$$x(t) = \begin{cases} \gamma/\left(\varepsilon\tau(t)\right) - \phi K(t) & t < T_X \\ 0 & t \geq T_X \end{cases}$$

(69) gives

$$m(t) = \begin{cases} \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) & t < T_M \\ 0 & t \geq T_M \end{cases}$$

while (70) implies

$$b(t) = \begin{cases} 0 & t < T_M \\ \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) & t \geq T_M \end{cases}$$

The dynamics of green capital is therefore

$$\dot{K}(t) = \begin{cases} \frac{1}{c_2}\left(\mu_d(t) - P_m(0)e^{rt} - c_1\right) - \delta K(t) & t < T_M \\ \frac{1}{c_2}\left(\mu_d(t) - \nu - c_1\right) - \delta K(t) & t \geq T_M \end{cases}$$

and the one of its value is

$$\dot{\mu}_d(t) = \begin{cases} (r+\delta)\mu_d(t) - \phi\left(\varepsilon\tau(t)/\sigma(t)\right) & t < T_X \\ (r+\delta)\mu_d(t) - \gamma/K(t) - \phi\sigma(t) & t \geq T_X \end{cases}$$

The steady state is given by (51), with $\mu(t)$ replaced by $\mu_d(t)$.

A.3 Detailed derivation of results in Section 4

Taking into account (34)-(35), the current value Hamiltonian associated to the social planner's program is:

$$\mathcal{H}(.) = u(x+\phi K) - C(m+b+\alpha\delta K) - \nu b - \eta(\alpha)\delta K - \lambda\varepsilon x + \mu(m+b-(1-\alpha)\delta K) + \zeta(-m)$$

The associated Lagrange function considers the additional constraints $\alpha \in [0,1]$ and writes:

$$\mathcal{L}(.) = \mathcal{H}(.) + \omega_x x + \omega_m m + \omega_b b + \omega_\alpha^0 \alpha + \omega_\alpha^1(1-\alpha) + \omega_X(\overline{X} - X) + \omega_M M$$

The first order conditions are now (40)-(43) and (45), while (44) is replaced by

$$-\frac{\partial \mathcal{L}}{\partial K} = -\phi u'(e) + C'(I)\alpha\delta + \eta(\alpha)\delta + (1-\alpha)\delta\mu = \dot{\mu} - \rho\mu \iff \dot{\mu} = (\rho + (1-\alpha)\delta)\mu - \phi u'(e) + \delta(C'(I)\alpha + \eta(\alpha)) \tag{76}$$

and the chosen recycling rate shall satisfy the condition

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \iff -C'(I)\delta K - \eta'(\alpha)\delta K + \mu\delta K + \omega_\alpha^0 - \omega_\alpha^1 = 0 \iff C'(I) + \eta'(\alpha) = \mu + \frac{\omega_\alpha^0 - \omega_\alpha^1}{\delta K} \tag{77}$$

According to Definition 2 and (33), $T_x = T_m = 0 < T_\alpha < T_M = T_b$ and $\omega_\alpha^1 = 0 \ \forall t \geq 0$.

First phase. There is a common initial phase with $x(t), m(t) > 0$ and $\alpha(t) = b(t) = 0 \ \forall t \in [0, \min\{T_X, T_\alpha\}]$. During this phase the first order conditions are the same as in the case without recycling, so that the dynamic system is given by (13)-(16) for $t < T_X$.

Cases. We then have three possible sequences of phases: (Case 1a), when $T_X < T_\alpha$; (Case 1b), when $T_X \in [T_\alpha, T_M]$; or (Case 2), when $T_M < T_X$. Here, case 1a is considered in detail, the other cases follow by applying a similar procedure.

Second phase, case 1a. $\forall t \in [T_X, T_\alpha)$: $X(t) = \overline{X}$, $M(t) > 0$ and $\zeta(t) < \eta'(0)$, $m(t) > 0 \Rightarrow \omega_m = 0$, while $x(t) = b(t) = \alpha(t) = 0 \Rightarrow \omega_x > 0$, $\omega_b > 0$, $\omega_\alpha^0 > 0$. Taking into account $\omega_\alpha^0 > 0$ and $b(t) = \alpha(t) = 0$ in (35), (77) implies that $\mu(t) < C'(m(t)) + \eta'(0)$. From this and (41) with $\omega_m = 0$, follows that $\zeta(t) < \eta'(0)$, meaning that the value of minerals in the ground is lower than the marginal cost of recycling a first unit of depreciated green capital. Moreover, from (32) $\alpha = 0$ implies that (76) boils down to (44). Therefore, the dynamic system is the one in (13)-(16) for $t \in [T_X, T_M)$.

This phase comes to an end at T_α, such that $\zeta(T_\alpha) = \eta'(0)$. At T_α, the backstop is not yet used $b(T_\alpha) = 0$, and investment relies on primary $m(T_\alpha) > 0$ and secondary resources $\alpha(T_\alpha)\delta K(T_\alpha)$, given that (33) holds $\eta'(0) < \nu$ for $T_\alpha < T_M = T_b$. This and (15) imply the definition of $T_\alpha^\star$ in (39).

Third phase, case 1a. $\forall t \in (T_\alpha, T_M)$: $X(t) = \overline{X}$, $M(t) > 0$ and $\zeta(t) \in (\eta'(0), \nu)$, $m(t) > 0$ and $\alpha(t) \in (0,1)$ $\Rightarrow \omega_m = \omega_\alpha^0 = 0$, while $x(t) = b(t) = 0 \Rightarrow \omega_x > 0$, $\omega_b > 0$. It follows that equations (41) and (77) read:

$$C'(I(t)) = \mu(t) - \zeta(t) \tag{78}$$

$$C'(I(t)) + \eta'(\alpha(t)) = \mu(t) \tag{79}$$

with $I(t)$ from (35) with $b(t) = 0$. Using (78) in (3), one gets $\dot{K}$ in (13). Moreover, $C'(I(t)) + \nu > \mu(t)$ since the backstop technology is still not used, so that $\omega_b > 0$ in (42). Together, (78)-(79) imply $\eta'(\alpha(t)) = \zeta(t)$, that is (38). Using this result and (79) in (76), we get (37) for $I_{t<T_X} = I_{t \geq T_M} = 0$ and $I_{t \geq T_X}$, $I_{t \in [T_\alpha, T_M)} > 0$:

$$\begin{aligned}
\dot{\mu}(t) &= (\rho+\delta)\mu(t) - \phi u'(\phi K(t)) - \delta\alpha(t)\left(\zeta(t) - \frac{\eta(\alpha(t))}{\alpha(t)}\right) \\
&= (\rho+\delta)\mu(t) - \phi u'(\phi K(t)) - \delta\left[\zeta(t)\eta'^{-1}(\zeta(t)) - \eta(\eta'^{-1}(\zeta(t)))\right]
\end{aligned}$$

The third phase comes to an end when the value of minerals reaches the marginal cost of the backstop technology. At date $T_M = T_b$, we have that $\zeta(T_M) = \eta'(\alpha^\star) = \nu$. The maximum recycling rate, $\alpha^\star$, prevailing from date T_M onward, is defined by (36) and bounded below unity by assumption (33). Moreover, date T_M is also characterized by the switch from minerals to the backstop input so that $I(T_M) = m(T_M^-) + \alpha(T_M)\delta K(T_M) = b(T_M^+) + \alpha(T_M)\delta K(T_M)$.

Last phase. $\forall t \geq T_M$: $X(t) = \overline{X}$, $M(t) = 0$ and $\eta'(\alpha^\star) = \nu$, $b(t) = 0$ and $\alpha(t) = \alpha^\star \Rightarrow \omega_b = \omega_\alpha^0 = 0$, while $x(t) = m(t) = 0 \Rightarrow \omega_x > 0$, $\omega_m > 0$. It follows from (35) and (76) that the motion of the stock of green capital and of its shadow value are given by (13) and (37). This system of differential equations admits a saddle path steady state, defined by (36).

Other sequences. The last phase is the same for any sequence of phases. Alternative sequences can be characterized, *mutatis mutandis*, with this same procedure. Overall, the explicit form of equation (37) is

$$\dot{\mu}^\diamond(t) = \begin{cases} (\rho+\delta)\mu^\diamond(t) - \phi\varepsilon\lambda^\diamond(t) & \forall t \in [0, \min\{T_X^\star, T_\alpha^\star\}) \\ (\rho+\delta)\mu^\diamond(t) - \phi u'\left(\phi K^\diamond(t)\right) & \forall t \in [T_X^\star, T_\alpha^\star) \text{ if } T_X^\star < T_\alpha^\star \\ (\rho+\delta)\mu^\diamond(t) - \phi u'\left(\phi K^\diamond(t)\right) - \delta\left[\eta'^{-1}(\zeta^\diamond(t))\zeta^\diamond(t) - \eta(\eta'^{-1}(\zeta^\diamond(t)))\right] & \forall t \in [T_\alpha^\star, T_M^\star) \text{ if } T_X^\star < T_\alpha^\star \\ (\rho+\delta)\mu^\diamond(t) - \phi\varepsilon\lambda^\diamond(t) - \delta\left[\eta'^{-1}(\zeta^\diamond(t))\zeta^\diamond(t) - \eta(\eta'^{-1}(\zeta^\diamond(t)))\right] & \forall t \in [T_\alpha^\star, T_X^\star) \text{ if } T_X^\star \in [T_\alpha^\star, T_M^\star) \\ (\rho+\delta)\mu^\diamond(t) - \phi u'\left(\phi K^\diamond(t)\right) - \delta\left[\eta'^{-1}(\zeta^\diamond(t))\zeta^\diamond(t) - \eta(\eta'^{-1}(\zeta^\diamond(t)))\right] & \forall t \in [T_X^\star, T_M^\star) \text{ if } T_X^\star \in [T_\alpha^\star, T_M^\star) \\ (\rho+\delta)\mu^\diamond(t) - \phi\varepsilon\lambda^\diamond(t) - \delta\left[\eta'^{-1}(\zeta^\diamond(t))\zeta^\diamond(t) - \eta(\eta'^{-1}(\zeta^\diamond(t)))\right] & \forall t \in [T_\alpha^\star, T_M^\star) \text{ if } T_X^\star > T_M^\star \\ (\rho+\delta)\mu^\diamond(t) - \phi\varepsilon\lambda^\diamond(t) + \alpha^\star\left(\nu - \eta\left(\alpha^\star\right)/\alpha^\star\right) & \forall t \in [T_M^\star, T_X^\star) \text{ if } T_X^\star > T_M^\star \\ (\rho+\delta)\mu^\diamond(t) - \phi u'\left(\phi K^\diamond(t)\right) + \alpha^\star\left(\nu - \eta\left(\alpha^\star\right)/\alpha^\star\right) & \forall t \geq \max\{T_X^\star, T_M^\star\} \end{cases}$$

Sub-case with quadratic costs and logarithmic utility: For the simulations in Section 4, we consider the specifications (18)-(19), and a quadratic cost function for recycling costs as follows:

$$\eta(\alpha(t)) = \eta_1\alpha(t) + \frac{\eta_2}{2}(\alpha(t))^2 \quad \Rightarrow \quad \eta'(\alpha(t)) = \eta_1 + \eta_2\alpha(t), \quad \eta'(0) = \eta_1, \quad \eta'(1) = \eta_1 + \eta_2 \tag{80}$$

with values in footnote 28, such that assumption (33) holds. The simulated sequence, corresponding to case $1a$, is characterized by (15)-(16) and

$$\forall t \in (0, T_X) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \phi\varepsilon\lambda(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases}$$

$$\forall t \in [T_X, T_\alpha) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/K(t) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases}$$

$$\forall t \in [T_\alpha, T_M) \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/K(t) - \delta(\zeta(t) - \eta_1)^2/(2\eta_2) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \zeta(t) - c_1) - \delta K(t) \end{cases}$$

$$\forall t \geq T_M \quad \begin{cases} \dot{\mu}(t) = (\rho+\delta)\mu(t) - \gamma/K(t) - \delta(\nu - \eta_1)^2/(2\eta_2) \\ \dot{K}(t) = \frac{1}{c_2}(\mu(t) - \nu - c_1) - \delta K(t) \end{cases}$$

Leading to the steady state, where

$$K^\star = \frac{\mu^\star - (\nu + c_1)}{\delta c_2} \quad , \quad \alpha^\star = \frac{\nu - \eta_1}{\eta_2} \quad , \quad b^\star = (1 - \alpha^\star)\delta K^\star$$

$$\mu^\star = \frac{1}{2}\left(\nu + c_1 + \frac{\delta\eta_2(\alpha^\star)^2}{2(\rho+\delta)}\right)\left[1 + \left(1 + 4\delta\frac{\gamma c_2 - (\nu + c_1)(\alpha^\star)^2\eta_2/2}{(\rho+\delta)\left(\nu + c_1 + \frac{\delta\eta_2(\alpha^\star)^2}{2(\rho+\delta)}\right)^2}\right)^{1/2}\right].$$

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