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Microscopic mechanism of particle detachment in granular materials subjected to suffusion in anisotropic stress states

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Abstract: Suffusion refers to a special form of internal erosion characterized by the selective erosion of the finest particles of a soil under the action of an internal fluid flow. In this work, the microscopic mechanism of particle detachment in binary mixtures subjected to suffusion under different flow directions is analyzed. We use the coupled lattice Boltzmann method (LBM) and discrete element method (DEM) to simulate the suffusion process in a granular sample subjected to an anisotropic stress state. When the macro flow direction is aligned with the principal direction of compression, it is found that the fluid flow is more intense, which increases erosion. The stress anisotropy also influences the detachment direction that is not necessarily correlated with the macroscopic flow direction. The sample's anisotropic stress state is responsible for directional variations in microstructural properties during the suffusion under different flow directions. From a micro scale point of view, a contact sliding index P and a particle detachment index $\mathcal{I}$ are defined to demonstrate that fluid-induced sliding dominates for particles about to detach.

Keywords: Suffusion, LBM-DEM, Detachment mechanism, Anisotropic stress,
Particle migration

1. Introduction

Suffusion refers to one of the typical phenomena of internal erosion, which consists of interaction between particles and seepage flow. This phenomenon can be described as the process of detachment and transport of the finest particles within the porous medium formed by coarse particles [1]. In geotechnical engineering, this complex phenomenon leads to loss of soil mass and non-negligible modifications in the constitutive behavior of soils. Such modification of the soil is one of the most common reasons for failure in dams, roads, and railways [2][3][4]. In the field of embankment dams and levees, about 50% of total failures originated from different forms of internal erosion [5][6]. However, the development process of internal erosion phenomena such as suffusion, which occurs inside the soil structure, lacks of direct observations and is poorly documented and understood. At the scale of hydraulic structures, visible signs of internal erosion are often detected long after it initiates and frequently too late to prevent a subsequent collapse of the structure [7]. Therefore, the serious consequences of internal erosion of hydraulic structures and its unobservable characteristics at the macro-scale emphasize the importance of a comprehensive understanding of suffusion phenomena including particle separation and transportation.

The initial research on internal erosion dates back from Terzaghi's criterion for heave, which relates erosion damage to the critical hydraulic gradient corresponding to the vanishing of the effective mean stress [7]. Since then, many studies have been

conducted to understand the mechanism of internal erosion, especially the relationship between the critical hydraulic gradient and soil properties [8][9]. However, the onset and continuation of suffusion are influenced by a series of factors including material susceptibility [10][11][12], stress state [13][14], and hydraulic conditions [15][16]. Extensive experimental tests have been carried out to study internal stability (i.e. material susceptibility¹) and critical hydraulic gradient in internal erosion [2][17][18][19]. Moffat et al.[20] conducted laboratory tests to study the spatial and temporal progression of internal erosion in cohesionless soils. The influence of confining pressure on internal erosion has also been examined via stress-controlled testing apparatus [21]. The reduction of the soil strength with suffusion was observed when the soil sample experience a monotonic compression test [22]. But there are also several studies showing that the eroded soil samples may demonstrate stronger resistance against liquefaction and stronger undrained shear strength [23][24]. With advanced techniques such as micro-CT and plane laser-induced fluorescence (PLIF), the visualization of suffusion phenomenon can be obtained during experimental tests [25][26]. Such experimental investigations of suffusion enriched the understanding of the phenomenon, but the lack of high resolution in devices and migration characteristics of suffusion, the micromechanical particle detachment and transportation process during suffusion is hard to detect in laboratory tests. In addition, some scholars have carried out a series of studies on internal erosion from the perspective of poromechanics,

¹ Internal stability is defined as the sensitivity of a material to suffusion. No systematic link has been established between internal instability and mechanical instability.

resulting in the proposal of several extend constitutive models reflecting internal erosion directly at macro continuum scale [27][28][29].

With the development of particle scale simulation and high-performance parallel computing technology, numerical modeling provides a feasible method to simulate the suffusion process at a much elementary scale and gain knowledge related to the basic mechanism of suffusion. The discrete element method (DEM) has significant advantages in tracing the information of every single particle [30]. When applied to suffusion simulation, the fine particle removal methods were first used to mimic the process in pure DEM simulation without accounting for the fluid phase [31][32][33]. This selective removal approach is however not able to reproduce realistic microstructure modifications during seepage induced suffusion, and in particular the modifications of the pore space resulting from the displacements and clogging of free particles. In addition, the soil fabric generated by this procedure may be sensitive to the frequency and rate of the removal algorithm that does not necessarily reflect the rate imposed by an internal fluid flow. Artificial removal of grains can artificially affect the stress-strain response as well as the final gradation distribution of eroded samples [32]. Therefore, the fluid phase was introducing into the simulations by solid-fluid coupling methods in recent years [34][35]. The coupled computational fluid dynamics and discrete element method (CFD-DEM) was widely used to study internal erosion [36][37][38][39][40][41][42], but the pore space complex geometry is frequently averaged by using unresolved scheme for the fluid. Kawano et al. [43] analyzed the initiation of suffusion with varying fines content and relative density in gap-graded

91 samples. Suffusion-induced deformation and mechanical change in granular soils were
92 investigated by Hu et al. [44][45]. Coupled Pore scale Finite Volumes and DEM (PFV-
93 DEM) is another numerical way to solve the interaction between solid and fluid [46]
94 through a resolved but approximated approach. Wautier et al. [47][48] adopted this
95 method to explored particle transport, flow impact on granular force chains and induced
96 mechanical instability during suffusion. Different from other traditional fluid phase
97 solvers, the lattice Boltzmann method (LBM) was revealed to be equivalent to a finite
98 difference approximation to Navier-Stokes equations for incompressible fluids [49][50],
99 which can be coupled with DEM through the immersed moving boundaries [51][52].
100 The three-dimensional LBM-DEM has been adopted in the soil erosion in granular
101 filters [53] and suffusion in binary mixtures [54]. This approach is spatially resolved
102 and is able to take into account the full complexity of the pore space geometry.

103 So far, most of the studies devoted much attention to the mechanical results of the
104 post-eroded sample rather than microscale variation and stress state during the suffusion
105 process. The researches on the mechanism of particle detachment and its corresponding
106 evolution process are limited. Therefore, the particle detachment mechanism
107 considering different flow directions in the anisotropic stress state during suffusion is
108 investigated in this paper for a bi-disperse granular material thanks to three-dimensional
109 LBM-DEM numerical simulations. A brief introduction of the LBM-DEM scheme and
110 detailed modeling setup for suffusion simulation are given in Section 2. Section 3 gives
111 the results and analysis in the microscopic directional analysis as well as detachment
112 mechanism of suffusion. Finally, Section 4 summarizes the results and presents a

conclusion of the particle detachment mechanism in anisotropic stress states.

2. Simulation of suffusion

2.1 LBM-DEM method

A three-dimensional and spatially resolved fluid-solid coupling method based on LBM-DEM is adopted to simulate the suffusion process. Among the coupling methods mentioned in the introduction, LBM-DEM has inherent advantages in terms of explicit formulation and large-scale parallel computing. The open source software LIGGGHTS is selected to solve DEM part [36], while fluid motion is controlled with the open source software Palabos [55]. The fluid solid coupling calculation is achieved by establishing ports in the two open source software. We coded the coupling in C++ based on the work of Seil & Pirker [56]. The immersed moving boundary (IMB) is adopted because of its superiority over other methods in terms of accuracy and stability [57][58].

We use the D3Q19 model in the LBM, because it possesses a good balance between accuracy and efficiency among other 3D LBM models [57][59]. According to the Bhatnagar-Gross-Krook approximation, the lattice Boltzmann equation is written as [60]:

$$f_i(\mathbf{x} + \mathbf{c}_i \delta_t, t + \delta_t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)] \quad (1)$$

where $f_i(\mathbf{x}, t)$ denotes the density distribution function whose coordinate is $\mathbf{x}$ directing in the i^{th} path at time t ; $\mathbf{c}_i$ represents 19 discrete velocity vectors; and τ is a relaxation coefficient, which controls the stability of LBM simulations.

The equilibrium distribution functions $f_i^{eq}(\mathbf{x}, t)$ in the right side of Eq. (1) is defined as

$$f_i^{\text{eq}} = w_i \rho_{\text{fluid}} \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}_{\text{fluid}}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u}_{\text{fluid}})^2}{2c_s^4} - \frac{|\mathbf{u}_{\text{fluid}}|^2}{2c_s^2} \right] \quad (i=0, \dots, 18) \quad (2)$$

where w_i ($i=0, \dots, 18$) are the weighting factors, $w_0=1/3$, $w_{1-6}=1/18$, $w_{7-18}=1/36$; ρ_{fluid} and $\mathbf{u}_{\text{fluid}}$ are the density and velocity of fluid, respectively. The sound speed is taken as $c_s = 1/\sqrt{3}$ in lattice units in the 3D LBM model [61].

According to the transfer method, the fluid velocity, fluid density, and fluid pressure can be calculated at each discretization point from LBM quantities as follows [62][63]:

$$\rho_{\text{fluid}} = \sum_{i=0}^{18} f_i \quad (3)$$

$$\mathbf{u}_{\text{fluid}} = \frac{1}{\rho_{\text{fluid}}} \sum_{i=0}^{18} \mathbf{c}_i f_i \quad (4)$$

$$p_{\text{fluid}} = c_s^2 \rho_{\text{fluid}} \quad (5)$$

Equation 5 reveals that the incompressible flow is satisfied approximately as slightly compressible flow in LBM simulation. To achieve the simulation of the slightly compressible flow, the computational Mach number must fulfill $Ma \ll 1$, which is defined as maximum simulated velocity in the flow divided by sound speed in the fluid [64]. The limitation requires the setting of parameters in LBM, and the relationship between parameters can be described as follows,

$$\nu_{\text{fluid}} = c_s^2 \left(\tau - \frac{1}{2} \right) \frac{(\delta x)^2}{\delta t} \quad (6)$$

where ν_{fluid} is the kinematic viscosity of fluid, δx is grid spacing, and δt is the timestep of LBM. From the perspective of numerical simulation, the commonly used method is to set δx and τ first, then compute δt based on the Eq. (6) [64][65].

The value of δx affects the accuracy of LB calculation and the cost of computational resources. It is evident that the smaller δx leads to better simulation accuracy, but it also increases the computational cost. And the relaxation parameter τ greatly determines the stability of LB calculation, with the requirement $\tau > 0.5$ in equation (6). The LB formulation will be more prone to numerical instability when τ is approaching 0.5 [64][66]. Feng et al. [64][65] discussed in details the choice of these parameters to achieve a satisfying tradeoff between the desired accuracy, reasonable computational cost and numerical stability of the calculation.

In DEM, the spherical particles' displacement is controlled by the Newton's second law of motion [30]:

$$m_i \mathbf{a} = \mathbf{F}_c + \mathbf{F}_{\text{fluid}} \quad (7)$$

$$I_i \dot{\boldsymbol{\omega}} = \mathbf{T}_c + \mathbf{T}_{\text{fluid}} \quad (8)$$

where I_i is the moment of inertia of particle, m_i is the mass of the particle; $\mathbf{a}$ is the motion acceleration; $\boldsymbol{\omega}$ is the angular velocity; $\mathbf{F}_c$ are the contact forces between particles and $\mathbf{T}_c$ is corresponding torques provided by particle collision; $\mathbf{F}_{\text{fluid}}$ and $\mathbf{T}_{\text{fluid}}$ are the hydrodynamic force and the corresponding torque provided by the fluid part, respectively. The normal force $\mathbf{F}_n$ and tangential force $\mathbf{F}_t$ between two particles are calculated by the Hertz-Mindlin contact model [67][68]. A rolling resistance is used all along the simulation according to the torque model proposed by Zhou et al [69]. The aim of introducing rolling resistance is to account for non-spherical particle shapes to some extent, the value of rolling friction parameter in this study is the same as those in other DEM studies of suffusion [44][45]. In relation with suffusion,

the rolling resistance is known to increase the number of small particles involved in force transmission.

Fluid-solid interaction calculations are achieved by adding a collision term Ω_i^s to Eq. (1), the govern equation becomes

$$f_i(\mathbf{x} + \mathbf{c}_i \delta_t, t + \delta_t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} \left[f_i(\mathbf{x}, t) - f_i^{\text{eq}}(\mathbf{x}, t) \right] (1 - B) + B \Omega_i^s \quad (9)$$

$$\Omega_i^s = f_{-i}(\mathbf{x}, t) - f_{-i}^{\text{eq}}(\rho_{\text{fluid}}, \mathbf{u}_{\text{fluid}}) + f_i^{\text{eq}}(\rho_{\text{fluid}}, \mathbf{u}_s) - f_i(\mathbf{x}, t) \quad (10)$$

where $B = \frac{\varepsilon(\tau - 0.5)}{(1 - \varepsilon) + (\tau - 0.5)}$ is a weighting function of the relaxation coefficient τ

and solid ratio ε of the LBM cell. $\varepsilon = V_{\text{solid}} / V_{\text{cell}}$, and thus, $\varepsilon = 0(1)$ yields $B = 0(1)$.

The subscript $-i$ in the Eq. (10) represents the opposite direction to i .

Taking the effect of particle rotation into consideration, the velocity of the solid part $\mathbf{u}_s$ is calculated as [51]

$$\mathbf{u}_s = \mathbf{u}_p + \boldsymbol{\omega}_p \times [(\mathbf{x} + 0.5\mathbf{c}_i \delta_t) - \mathbf{x}_p] \quad (11)$$

where $\mathbf{u}_p$ and $\boldsymbol{\omega}_p$ are translational and angular velocities of solid particle, and $\mathbf{x}_p$ is the position of solid particle.

Lastly, the hydrodynamic force $\mathbf{F}_{\text{fluid}}$ and the corresponding hydraulic torque $\mathbf{T}_{\text{fluid}}$ applied by fluid motion can be calculated as:

$$\mathbf{F}_{\text{fluid}} = \sum_{j=1}^n B_j \sum_{i=0}^{18} \Omega_i^s \mathbf{c}_i \quad (12)$$

$$\mathbf{T}_{\text{fluid}} = \sum_{j=1}^n \left[B_j (\mathbf{x}_j - \mathbf{x}_p) \times \sum_{i=0}^{18} \Omega_i^s \mathbf{c}_i \right] \quad (13)$$

where n is the total number of lattice cells covered by the solid part; B_j refer to the weight coverage function in j -th lattice cell among n lattice cells.

To achieve a stable DEM simulation, the DEM time step, Δt_{DEM} , needs to be

smaller than a critical value proportional to $\sqrt{m/K}$, where m and K are the mass and stiffness of the particles [70]. For problems in the geotechnical field due to the large stiffness of soils and rocks, the calculation time step length of LBM is larger than DEM [51][71]. Therefore, a sub-cycle method is used to solve the communication between LBM and DEM computation parts. The implementation of a series of repeated DEM time steps is allowed within a single LBM time step. Particle information in the DEM is transported to the LBM to identify the node occupied by solid in the domain every $N_{\text{sub}} = \delta t / \Delta t_{\text{DEM}}$ DEM cycles [51]. In this study, considering accuracy and computational cost, N_{sub} is selected as 10, which has been proposed by Owen et al. from an algorithmic perspective [51] and has also been adopted in previous simulations of suffusion [53][54].

2.2 Sample generation and pre-stress procedure

The numerical simulation takes macroscopic fluid direction, hydraulic gradient, and vertical stress levels as factors to conduct research. The schematic figure of the simulation is shown in Fig. 1.

Fig.1 Numerical procedure from sample preparation (radius expansion technique and dry oedometer compression) to suffusion simulations (fully coupled DEM-LBM simulations) for different flow directions.

The binary packing used in this study consists of two types of monodisperse particles, with a particle size ratio of 10. This size ratio is large enough to create space for fine particles to move in the pore structure formed by coarse particles. The fine content (FC) of binary mixture is known to be an important parameter that drives the sample's mechanical behavior depending whether the force transmission skeleton is composed of coarse or fine particles. In this paper, FC is set to 30% which is in the transition region between “underfilled” and “overfilled” configurations [72][2]. In this case, fine and coarse particles are both involved in stress transmission.

Gravity is not considered during the entire sample preparation and simulation process. The specific steps are shown in Figure 1. First, a binary packing is randomly inserted in a cubic box. Then, the radius expansion method is used to isotropically

consolidate the sample to a small stress value and obtain particles with the targeted radius value. After that, the boundaries of the sample are moved to consolidate the sample under an isotropic stress state with mean pressure of 100 kPa. This preparation procedure ends with a prepared binary mixture with dimension $0.11 \times 0.11 \times 0.11$ m. For the sake of disregarding any boundary effects, the subsequent statistical analyses are only based on the central volume ($0.10 \times 0.10 \times 0.10$ m). We emphasize that the sample in our simulations is probably too small to be considered as a representative elementary volume with respect to material constitutive properties (the sample contains only large particles), however, the present study focuses on the micromechanical detachment mechanism, which can still be analyzed in such small samples.

Eventually, the binary packing is compressed vertically to prescribed stress levels with a servo-controlled method in the top boundary, while the lateral walls and the bottom wall are kept fixed (oedometer compression). This results in an anisotropic stress state, the principal compression direction being aligned with the z-axis, referred to as the “vertical” direction (even in the absence of gravity forces). When the equilibrium of the sample is reached under prescribed compressive stress, it will be considered as the end of sample preparation and the beginning of suffusion simulation.

2.3 Suffusion simulation step

After generating an anisotropic stress state in the sample, an internal fluid flow is simulated by applying a fluid pressure difference on the sample boundaries in two selected directions (x-axis for a horizontal flow, z-axis for a vertical flow). The fluid pressure drop Δp applied on the upstream and downstream relates to the hydraulic

gradient I as $\Delta p = \rho g h I$ ². Permeable walls are used in the flow direction while zero flux conditions are imposed in other directions. As shown in Figure 1, the downstream wall is permeable to both the fluid and the small particles to allow for suffusion to occur. Only small particles are able to escape from the sample through the permeable wall which ensure that the stress state in the sample is kept constant. For the boundary conditions in LBM, pressure boundary conditions are used in the direction of fluid flow and the half-way bounce-back scheme is applied in other wall boundaries (zero flux boundary conditions) [73]. The values of the hydraulic gradient considered in this study are 0.1 and 1.0, and the values of the vertical stress levels are 0.1 MPa and 0.5 MPa. They are summarized in Table 1.

As is shown in Eq. (6), it is difficult to ensure that δx is small enough to spatially resolve the pore space geometry. To comply with this constraint, τ has to be close to 0.5 when model in the fluid flows with a small viscosity like water [64]. In this simulation, the kinematic viscosity of fluid $\nu_{\text{fluid}} = 1.01 \times 10^{-6} \text{ m}^2/\text{s}$. The smallest value of grid spacing $\delta x = 2.75 \times 10^{-4} \text{ m}$ is adopted to achieve highest solution accuracy under the condition of maximum computing capacity. In order to carry out detailed simulation of particle motion in DEM simulation, the time step Δt_{DEM} is chosen ten times smaller than the LBM timestep δt . Therefore, after considering the computational cost and the influence of the analytical accuracy, we set $\delta t = 4.45 \times 10^{-7} \text{ s}$, which leads to an extremely small value of $(\tau - 0.5) = 2 \times 10^{-5}$. However, this is in accordance with the

² Note that, since gravity is not taken into account, the hydraulic gradient is simply a dimensionless scaling of the fluid pressure drop.

values in a series of numerical simulation by LBM-DEM [53][65][66][74], especially in the similar simulations of soil erosion in granular filter using LBM-DEM, in which an even smaller value $(\tau - 0.5) = 5 \times 10^{-7}$ was adopted [53][74].

The other detailed parameters used in DEM and LBM simulations are summarized in Table 2. They are similar to those used in [75] and [54].

The three-dimensional simulation in this research is carried out using one node (Intel Xeon(R) CPU E5-2620 v4@2.10 GHz) in the supercomputer center of Wuhan University. For the sake of numerical stability, single thread simulations are adopted in this study. Consequently, it takes about 160 h 42 min for each simulation. It will be challenging to apply this method to larger size problems without using massive parallelization techniques.

Table 1 The simulations carried out based on various conditions

Conditions	Vertical stress, σ_v (MPa)	Stress ratio, $\eta = q/p$	Hydraulic gradient, I	Flow direction
Case 1	0.1	0.7	1.0	Horizontal
Case 2	0.5	1.1	0.1	Horizontal
Case 3	0.5	1.1	1.0	Horizontal
Case 4	0.5	1.1	1.0	Vertical

Table 2 Parameters of the LBM-DEM simulations

	Parameters	Units	Values
DEM	Particle density, ρ	kg/m ³	2600
	Particle number, N_{coarse}/N_{fine}		60/26728
	Contact model		Hertz-Mindlin
	Young's module, Y	GPa	25
	Poisson ratio, ν		0.3
	Maximum diameter, d_{max}	mm	30
	Minimum diameter, d_{min}	mm	3

	Particle size ratio		10
	Sliding friction, μ		0.5
	Rolling friction, μ_r		0.1
	DEM time step, Δt_{DEM}	s	4.45×10^{-8}
LBM	Fluid density, ρ_{fluid}	kg/m ³	1000
	Kinematic viscosity, ν_{fluid}	m ² /s	1.01×10^{-6}
	LBM time step, δt	s	4.45×10^{-7}

3 Results and discussions

3.1 Effects of the flow conditions on suffusion intensity

The suffusion phenomenon under different flow directions is simulated and the evolution of erosion rate and void ratio e in the granular assembly in horizontal and vertical flow directions are compared. The results of e are displayed in the condition $I = 1.0$, vertical stress $\sigma_v = 0.5$ MPa (Case 3 and Case 4) in Figure 2. Obviously, the void ratio e increases, due to the seepage-induced loss of fines. Subsequently, due to the settling and reorganization of coarse particles after the erosion of fine particles (FC), the inter-granular void ratio [76], $e_{eq} = \frac{e + FC}{1 - FC}$ is significantly reduced in the vertical flow³. This also corresponds to the higher erosion rate in Figure 3 under vertical flow when the hydraulic gradient is the same in both flow directions. A larger erosion ratio compared with $I = 0.1$ condition is also achieved when $I = 1.0$ in the horizontal flow as shown in Figure 3. The anisotropic stress state is the main reason for the variations in the suffusion process in the vertical and horizontal flow and will be the subject of detailed analysis in the following. In order to quantify the microstructure anisotropy,

³ Note that the present intergranular void ratio definition assumes that no fine particles participate to stress transmission, which is not exact for transitional materials. The definition should be updated by considering the non-active fine content only.

the contact based and contact force based fabric tensor in the sample before the fluid erosion are computed. The fabric tensor of contact normal are quantified by the definition proposed by Oda [77]:

$$\phi_{ij} = \frac{1}{N_c} \sum_{c \in N_c} n_i n_j \quad (14)$$

where n is the unit vector of the contact normal direction, N_c is the number of contacts. The fabric tensor of normal contact force can be defined as follows [78]:

$$\chi_{ij}^n = \frac{1}{N_c} \sum_{c \in N_c} \frac{f^n n_i n_j}{1 + a_{kl}^c n_k n_l} \quad (15)$$

where contact anisotropy tensor $a_{ij}^c = 15/2 \phi'_{ij}$. According to the above formulas, we calculate the fabric tensor of contact normal (ϕ_{ij}) and normal contact force (χ_{ij}^n) in the sample: $\phi_{xx} = 0.3232$, $\phi_{yy} = 0.3258$, $\phi_{zz} = 0.3510$ and $\chi_{xx}^n = 0.1046$, $\chi_{yy}^n = 0.1055$, $\chi_{zz}^n = 0.1349$ (off diagonal terms are negligible).

Fig.2 The variation of void ratio, e and inter-granular void ratio e_{eq} ($I = 1.0$, $\sigma_v =$

0.5MPa)

Fig.3 The variation of erosion ratio $m_{\text{eroded}}/m_{\text{total}}$ ($\sigma_v = 0.5\text{MPa}$)

3.2. Fluid impact on contact populations

Based on the two types of particles that come into contact, we can distinguish coarse-coarse contacts (C-C), coarse-fine contacts (C-F), and fine-fine contacts (F-F).

The distributions of the number of different contact pairs are summarized in Table 3.

Table 3 Distribution of the number of contact pairs under different conditions before ($I = 0$) and after ($I \neq 0$) applying a fluid flow.

Pair type	Vertical stress, $\sigma_v = 0.5\text{MPa}$				Vertical stress, $\sigma_v = 0.1\text{MPa}$	
	$I = 0$	$I = 0.1(\text{horizontal})$	$I = 1.0(\text{horizontal})$	$I = 1.0(\text{vertical})$	$I = 0$	$I = 1.0(\text{horizontal})$
Coarse-Coarse	95	95	95	95	94	94
Coarse-Fine	9975	9615	9228	9122	9186	7765
Fine-Fine	57495	55376	53432	53306	54438	47385

The flow effect is visible through the destruction of contact pairs C-F and F-F, which leads to the loss of fine particles. The degree of influence on the contact pair intensifies with an increase in hydraulic gradient and when the macro flow direction is

aligned with the principal direction of compression (vertical flow case). A similar finding was also observed in [48].

To better analyze this mechanism, the stress tensor computed from contact forces can be split in three terms as in [75],

$$\sigma_{ij} = \frac{1}{V} \left(\sum_{c \in c-c} f_i^{c-c} d_j^{c-c} + \sum_{c \in c-f} f_i^{c-f} d_j^{c-f} + \sum_{c \in f-f} f_i^{f-f} d_j^{f-f} \right) \quad (16)$$

where i and j denote either x , y , or z to generate the 6 components of the symmetric tensor, f_i and d_j represent the contact force and branch vector connecting the centers of two contacting particles, respectively. By defining the mean pressure as $p = \frac{1}{3} \sum_i \sigma_{ii}$ and the deviatoric stress as $q = \sigma_{zz} - \sigma_{xx}$, p and q can also be seen as the sum of mean pressure and deviatoric stress computed from the coarse-coarse (C-C), the coarse-fine (C-F) and the fine-fine (F-F) contacts. Figure 4 shows the relative contribution of the three contact populations to the mean stress and deviatoric stress under different loading conditions. As exposed in Figure 4ab, C-C contact pairs represent the smallest fraction of the contacts but they transmit most of the stress, especially for deviatoric stress. The contribution of F-F contacts to the deviatoric stress (in Figure 4b) is very small without significant difference whatever the loading conditions. More fine particles participate in stress transmission before fluid flow is applied when the vertical stress is increased from 0.1 MPa to 0.5 MPa. As fine particles are eroded by the flow, the stress contribution of C-F and F-F contacts decreases while the contribution of C-C increases. This increase is even larger when the sample is subjected to vertical fluid flow. Despite of the great number in F-F contact pairs shown in Table 3, such contacts do not participate much in stress and have always been weak, corresponding to a great

reduction in pair number with hydraulic loading, which is also one of the reasons for fine particles to be eroded.

Fig.4 Stress repartition according to the three different types of contact: (a) the distribution of mean stress; (b) the distribution of deviatoric stress. Conceptual schemes (on the left) recall the different loading conditions.

3.3. Directional porosity and suffusion sensitivity

To clarify why erosion is stronger in the vertical direction (i.e. when the flow aligns with the stress principal compression direction), we propose a method to compute directional pipe porosity of the initial sample before fluid pressure drop is applied. The

initial sample selected is in the equilibrium under the loading of the vertical stress ($\sigma_v = 0.5\text{MPa}$) before applying the hydraulic gradient. Then, rattlers (contact number less than one) are removed to define the effective pore structure. Figure 5 shows a schematic diagram of the pore structure and the definition of pores and throats based on network models [79]. The throat size corresponds to the minimum size of diameter that can pass through the narrow channels. Then, the directional pipe porosity matrix is estimated over all constriction pipes by calculating,

$$\boldsymbol{\varphi} = \frac{1}{V_{\text{total}}} \sum_{i \in \text{constriction}} V_i \mathbf{m}_i \otimes \mathbf{m}_i \quad (17)$$

where $\mathbf{m}_i$ is the direction of pipe i , and V_i is the cylinder volume of pipe i ⁴.

Fig. 5 Schematic image of the constriction pipe in the pore structure

Finally, the computation result of directional pipe porosity when $\sigma_v = 0.5\text{MPa}$ is given below in xyz coordinate system, z being the principal compression direction, as shown in Figure 1.

⁴ The length of pipe i is defined as the distance between the contact points of pores as shown in the Figure 5. Such length controls the hydraulic conductivity better than the length including also the pore radius.

$$\boldsymbol{\varphi} = \begin{pmatrix} 0.5025 & 0.0052 & 0.0062 \\ 0.0052 & 0.5031 & 0.0061 \\ 0.0062 & 0.0061 & 0.5121 \end{pmatrix} * 10^{-3}$$

Note that $\text{Tr}(\boldsymbol{\varphi})$ is corresponding to the total volume of the pipes divided by the sample volume. It represents an effective porosity. Because the volume of constriction pipes is just based on the length of the connection channels, disregarding the volume of the pores (see Figure 5), the results of directional pipe porosity is much smaller compare to the sample porosity. It can be seen from the diagonal coefficients that the directional pipe porosity is almost the same in the x and y directions, but the directional porosity is larger along the z -axis (the principal compression direction). For eroded particles, large channels are more frequently observed in the z -direction. The results of directional porosity are consistent with the sample's directional permeability ($K_x = 0.1499 \mu\text{m}^2$, $K_y = 0.1501 \mu\text{m}^2$, $K_z = 0.1661 \mu\text{m}^2$) computed by Finite-difference method stokes solver (FDMSS) [80].

This difference of permeability between horizontal and vertical directions affects the distribution of fluid forces as displayed in Figure 6 for $t=0.6\text{s}$ (half of the simulation time) and a hydraulic gradient of 1. The mean fluid force value is 0.105 N for the horizontal flow while it is 0.136 N for the vertical flow. Although faced with same hydraulic gradient in both flow direction, the directional porosity and permeability lead to a different mean fluid force intensity. In the vertical flow case, the larger permeability results in larger fluid forces, which explains why particle detachment is more likely to occur. In contrast, the maximum proportion of fluid forces in the case of horizontal flow is in the interval $[0-0.01]$ N which explain why erosion is less pronounced for the

horizontal flow case.

Fig. 6 Distribution of fluid force under different flow direction ($I = 1.0$, $\sigma_v = 0.5\text{MPa}$)

3.4. Detachment mechanism of suffusion

To describe the particle sensitivity to fluid forces, the ratio between the mean contact force and fluid force of the eroded particles is judged by the following index that compare the mean contact force magnitude to the mean fluid force magnitude:

$$CR = \frac{\frac{1}{N_{\text{contact}}} \sum_{i=1}^{N_{\text{contact}}} \|F_{\text{contact}}\|}{\|F_{\text{fluid}}\|} \quad (18)$$

Particle information in DEM is recorded every 10,000 DEM steps in this simulation. Therefore, the calculation of CR is done every 4.45×10^{-4} s. The time evolution of CR for three typical particles are displayed in Figure 7. The particles detach when $CR = 0$. An increase in CR corresponds to a relative increase in contact force, and the movability of the particles is expected to decrease accordingly. The process of monotonous descent in Fig. 7a corresponds to a particle moving continuously without encountering local blockage. Figure 7(b)(c) corresponds to the typical behavior

395 of particles trapped in a local pore structure, as the particles lose and gain contacts
 396 several times. Overall, the general decrease in CR index in Figure 7 illustrates that the
 397 fluid force gradually dominates the competition with contact.

Fig. 7 Three typical CR evolution curves of eroded particles

398 The statistics for eroded particles below are given at a critical time corresponding
 399 to the moment when the eroded particles are about to lose contact. Hence, the critical
 400 time step is not exactly to $CR=0$, but a little earlier. As shown in Figure 8, the sharp
 401 drop period of CR is considered as Δt_c , then the red point which is ahead of the point
 402 $CR=0$ in time interval Δt_c is taken as the critical point for subsequent analysis ($t_c=t_{CR=0}-$
 403 Δt_c). Considering the CR evolution of typical particles in Figure 8, Δt_c is estimated once
 404 for all and set to $\Delta t_c=0.02s$. Since then, the successive contents involving critical time
 405 in this article is based on this determination method to obtain the corresponding critical
 406 point.

Fig. 8 The determination of critical time t_c

Fig.9 Distribution characteristics of CR

The distributions of CR are also computed in the stable particles as well as eroded particles in Figure 9. The results are obtained by averaged methods in all time steps for stable particles. While the distributions of eroded particles are calculated in the critical time as defined in Figure 8. As shown in Figure 9, most stable particles have a CR magnitude larger than 1.0; whereas most of eroded particles have a CR in the interval [0-0.2]. The dominant position of contact force for stable particles resist the fluid

erosion, and the influence of the fluid prevails when the eroded particles are going to become free.

In order to get a deeper understanding of the process leading to the vanishing of CR for detached particles, a contact-sliding index P is introduced. This index is design to estimate the potential for sliding of a given particle contact when an additional force (e.g. a fluid force) is applied to the particle in a given direction $\mathbf{m}$ (vertical or horizontal in the present case). Figure 10 illustrate the idealized situation used to derive the P index.

Fig. 10 Idealized configuration to estimate the sliding index P of a given particle contact

We assume that the particle p is at equilibrium and we want to assess the impact of a perturbation force that generate a displacement in direction $\mathbf{m}$. The idea is then to estimate the impact on the normal to tangential force ratio at contact. If this ratio increases, the contact is approaching its sliding limit and the perturbation is deemed to have a destabilization effect on the contact. Based on this analysis, a contact-sliding index P is defined such to indicates whether the contact is going to slide ($P < 0$) or strengthen ($P > 0$) under the action of the fluid force. For the sake of brevity, the detailed derivation of the sliding index P is given in the Appendix I. All computations done, this index is expressed for a contact i belonging to particle p as,

$$P^{(i)}(\mathbf{m}) = \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[\frac{k_m^{(i)} \mathbf{m} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} - \frac{k_t^{(i)} \mathbf{t}_i \cdot \mathbf{m}}{\|\mathbf{F}_t^{(i)}\|} \right] \quad (19)$$

where $\mathbf{m}$ is the direction of the fluid force acting on particle p , $\mathbf{m}_i$ and $\mathbf{t}_i$ are the normal and tangential vector of the i -th contact of the particle, $\mathbf{F}_t^{(i)}$ and $\mathbf{F}_m^{(i)}$ are the tangential force and contact force of i -th contact pair acting on particle p , $k_m^{(i)}$ and $k_t^{(i)}$ are the stiffness in the normal and tangential direction, $\varphi^{(i)}$ is the friction angle of i -th contact pair. According to the definition, $P < 0$ reflects that the contact pair is going to slide and open if a fluid force in direction $\mathbf{m}$ is applied to the considered particle p . $P^{(i)}$ index is attached to a given contact i . In equation (19), two kinds of perturbations contribute to a negative value for $P^{(i)}$: i) normal unloading of the contact ($\mathbf{m} \cdot \mathbf{m}_i < 0$), ii) increase in the tangential force ($\mathbf{t}_i \cdot \mathbf{m} > 0$).

In order to construct a detachment index for the particle we define $\Delta = \text{Number}(P^{(i)}(\mathbf{m}) > 0) - \text{Number}(P^{(i)}(\mathbf{m}) < 0)$ which correspond to the difference between the number of contacts reinforced and weakened under the action of a fluid force in direction $\mathbf{m}$. As a result, a negative value for Δ is proposed to characterize the overall detachment sensitivity of a given particle subjected to a fluid force in direction $\mathbf{m}$.

Figure 11 shows the Δ distribution for the eroded particles with different contact number (CN) at critical time (i.e. when the particles are about to lose contact and become free particles). At this time, the calculated value of Δ is based on the direction of the local fluid force of the individual particles. The distribution of Δ for the contacts of eroded particles just prior to their detachment well reflects the sliding tendency of contact pairs, because of the large proportion of negative value. Especially for the

condition when CN=5 in horizontal flow, all contact pairs of the eroded particles show a sliding tendency. Detached particles are thus characterized by a set of contact forces unable to balance the additional force applied by the fluid without sliding or opening.

Fig.11 Distribution of Δ considering different CN and flow direction

A negative value for $P(\mathbf{m})$ is obtained in equation (19) either by increasing the tangential force ($\mathbf{t}_i \cdot \mathbf{m} > 0$) at contact or by decreasing the normal force ($\mathbf{m}_i \cdot \mathbf{m}_i < 0$). To assess which part of the contact sliding potential is simply coming from geometric considerations (decrease in normal force), we also consider the angle between the normal contact vector of eroded particle and macroscopic flow direction at the critical time in Figure 12. The x-axis in Figure 12 represents the classification according to the coordination number of eroded particles. The results in Figure 12(a) illustrate that particle detachment often occurs when most of the contacts are perpendicular to the fluid force direction in the horizontal case, which is no longer the case when the fluid align with the principal compression direction (vertical flow case in Figure 12(b)). In addition, for large coordination numbers in horizontal flow in Figure 12(a), the

proportions of angle larger than 90 degrees from the flow direction become more obvious. This means that, with the increase in the contact number, the direction of the contact normal corresponding to the critical time tends to be closer to the opposite direction of the fluid direction. In vertical flow, the angle distribution is much wider than in the horizontal flow case. In a horizontal flow, the contact state of a typical eroded particle can be described as shown in Figure 13a. The normal vectors of the particle contact are all almost perpendicular to the direction of the fluid; on the contrary, a relatively broad distribution is obtained in vertical flow, as shown in Figure 13b.

The absence of correlation between detachment and contact orientation in the vertical flow case could be the signature that particle detachment relies widely on local fluid force fluctuations with respect to the macroscopic direction. As a result, this observation could be interpreted as a smaller sensitivity to particle detachment when the macroscopic flow aligns with the principal stress direction. This is obviously not the case as seen in Figures 2 and 3. As a result, we can conclude that the larger directional pipe porosity, permeability, and fluid force in the vertical direction overwhelm such disadvantage caused by vertical distributed force chains.

Fig.12 Distribution of angle between flow direction and normal contact vector at critical time for eroded particles: (a) horizontal flow; (b) vertical flow

Fig.13 Typical detachment configurations with contact normal directions for (a) horizontal flow and (b) vertical flow directions.

To support the conjecture on the dependence of particle detachment to fluid force fluctuations in the vertical flow case, we also explore the relationship between the displacement direction of eroded particles and the macroscopic flow direction at the critical time. The results of this correlational analysis are summarized in Figure 14. The angle β is defined in Figure 14 as the angle between the particle displacement and flow direction.

Fig.14 Distribution of angle between flow direction and particle displacement at

critical time: (a) horizontal flow; (b) vertical flow

An angle of zero means that the displacement is in the same direction of the macroscopic flow (positive x -direction in the horizontal case and negative z -direction in the vertical case) while an angle of 90° correspond to a perpendicular direction to the macroscopic flow. Figure 14(a) demonstrates that most eroded particles detached in the direction of macroscopic flow in the horizontal case. A relatively flat distribution is observed in the vertical flow case for angles less than 90 degrees, which support our conjecture. In the vertical flow case, a significant proportion of detachment events occur in the perpendicular direction to the macroscopic flow (in the x - y plane). This feature is linked to the anisotropic contact forces induced by the imposed stress boundary conditions. The greater anisotropy of normal contact force and contact normal in the z -direction (see section 3.1) illustrates the difference of the particle detachment mechanism in different flow direction. This effect also corresponds to the angular distribution when the particles are detached as shown in Figure 12.

4 Conclusions and outlook

This paper adopted a three-dimensional coupled LBM-DEM method to simulate the phenomenon of suffusion considering anisotropic stress state. The microscale detachment mechanism is investigated by considering a sliding index and the contact force ratio when the fluid flow is parallel (horizontal) or perpendicular (vertical) to the principal compression direction. The main conclusions can be summarized as follows:

- Regardless of the initial state, the ratio between the contact force and the fluid force of the eroded particles displays a downward trend over time, reflecting the gradual dominance of the fluid forces, which eventually leads to particle detachment and erosion.
- Compared with horizontal flow, the erosion rate and mean fluid force are greater in vertical flow. This is a consequence of the stress anisotropy, which generates a larger permeability in the direction aligned with the principal stress direction (vertical) and larger fluid forces. The larger directional porosity in the vertical direction also facilitates particle transport and subsequent erosion.
- Based on the comparison between the direction of the fluid force and the direction of the contact force, a contact sliding index P has been proposed to determine whether a given particle contact is going to slide or strengthen under the action of a fluid force. The distribution of P for the contacts of eroded particles just prior to their detachment reflects that fluid-induced sliding dominates.
- The majority of particle migration is directed by the fluid. It follows the flow direction in horizontal flow, but in the vertical flow case, a significant proportion of detachment events occur in the perpendicular direction to the macroscopic flow (in the x - y plane). In this case, where most of the contacts are aligned with the macroscopic direction, particle detachment relies largely on local fluid force fluctuations. This dependence on fluid force fluctuations is expected to reduce the occurrence of particle detachment, but overall, the increase in the fluid forces resulting from larger directional porosity is dominant. To isolate this effect, it

should be possible to simulate a vertical fluid flow with a reduced hydraulic gradient so that the fluid force magnitude corresponds to the horizontal flow case. Such a simulation is expected to have less erosion than the present horizontal flow case.

The conclusion obtained in this study will prove useful for suffusion mitigation in practical applications by assessing the relative orientation between the internal fluid flow and the principal stress direction. On the simulation side, further exploration is needed to be able to simulate larger volumes corresponding to representative elementary volume (REV) for suffusion. A better correlation between fluid force (magnitude and fluctuations) and permeability needs to be established as well in subsequent researches. A fine understanding of such features (or possibly the use of machine learning techniques on a large data set of detachment events) will probably make it possible to evaluate the a priori sensitivity of detachment without performing coupled simulations.

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Appendix A

Definition of a sliding index for a particle contact

Let us consider a given contact i of a particle p with normal direction $\mathbf{m}_i$. At contact i , the neighboring particle applies a contact force on particle p that can be decomposed into normal and tangential components $\mathbf{F}_m^{(i)}$ and $\mathbf{F}_t^{(i)}$ respectively. We assume that the particle p is at equilibrium and we want to assess the impact of a perturbation force that generate a displacement $\delta\mathbf{u}$ in direction $\mathbf{m}$. While this force is applied on particle p , we also assume that the neighboring particles do not move. This configuration is illustrated in the following Figure.

Fig. A.1 Idealized configuration to estimate the sliding index P of a given particle contact

The normal and tangential displacement in the direction of one exist contact pair $\mathbf{u}_m^{(i)}$, $\mathbf{u}_t^{(i)}$ can be calculated by

$$\delta\mathbf{u}_m^{(i)} = (\delta\mathbf{u} \cdot \mathbf{m}_i) \mathbf{m}_i \quad (\text{A.1})$$

$$\delta\mathbf{u}_t^{(i)} = (\mathbf{m}_i \wedge \delta\mathbf{u}) \wedge \mathbf{m}_i \quad (\text{A.2})$$

$\delta\mathbf{u}$ is the displacement in the direction of unbalanced force. An elastic contact force increment is then computed as

$$\delta\mathbf{F}_m^{(i)} = -k_m^{(i)} \delta\mathbf{u}_m^{(i)} \quad (\text{A.3})$$

$$\delta \mathbf{F}_t^{(i)} = -k_t^{(i)} \delta \mathbf{u}_t^{(i)} \quad (\text{A.4})$$

We define the normalized distance to sliding of a contact according to the friction

angle φ :

$$d_p^{(i)} = \frac{\tan \varphi^{(i)} - \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}}{\tan \varphi^{(i)}} \quad d_p^{(i)} \in [0,1] \quad (\text{A.5})$$

When tangential force $\mathbf{F}_t^{(i)}$ become zero, $d_p^{(i)}$ is 1 corresponding to no sliding;

while $\tan \varphi^{(i)} = \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}$, $d_p^{(i)}$ is 0 which can be considered as critical state of sliding.

We perform differential calculations on $d_p^{(i)}$,

$$\begin{aligned} \delta d_p^{(i)} &= \frac{\tan \varphi^{(i)} - \frac{\|\mathbf{F}_t^{(i)} + \delta \mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)} + \delta \mathbf{F}_m^{(i)}\|} - \tan \varphi^{(i)} + \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}}{\tan \varphi^{(i)}} \\ &= \frac{1}{\tan \varphi^{(i)} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}} \times \left[1 - \frac{\frac{\|\mathbf{F}_t^{(i)} + \delta \mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_t^{(i)}\|} + \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_t^{(i)}\|}}{\frac{\|\mathbf{F}_m^{(i)} + \delta \mathbf{F}_m^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} + \frac{\|\mathbf{F}_m^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}} \right] \end{aligned} \quad (\text{A.6})$$

Then, the last fraction in brackets can be further simplified. Because when $\delta \mathbf{u}$

is small enough, the denominator part can be simplified as shown below,

$$\frac{1}{\frac{\|\mathbf{F}_m^{(i)} + \delta \mathbf{F}_m^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} + \frac{\|\mathbf{F}_m^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|}} = \frac{1}{\left\| -\mathbf{m}_i - k_m^{(i)} \frac{\delta \mathbf{u} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} \mathbf{m}_i \right\|} = 1 - k_m^{(i)} \frac{\delta \mathbf{u} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} = 1 + \frac{\delta \mathbf{F}_m^{(i)}}{\|\mathbf{F}_m^{(i)}\|} \quad (\text{A.7})$$

Also, we can easily derive that

$$\|\mathbf{x} + d\mathbf{x}\|^2 = \|\mathbf{x}\|^2 + 2\mathbf{x} \cdot d\mathbf{x} + o(\|d\mathbf{x}\|^2) \quad (\text{A.8})$$

$$\|\mathbf{x} + d\mathbf{x}\| = \|\mathbf{x}\| \sqrt{1 + \frac{2\mathbf{x} \cdot d\mathbf{x}}{\|\mathbf{x}\|^2}} = \|\mathbf{x}\| \left(1 + \frac{\mathbf{x} \cdot d\mathbf{x}}{\|\mathbf{x}\|^2} \right) \quad (\text{A.9})$$

Then the numerator part can also be simplified,

$$\left\| \frac{\mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|} + \frac{\delta \mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|} \right\| = 1 + \frac{\mathbf{F}_t^{(i)} \cdot \delta \mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|^2} \quad (\text{A.10})$$

Finally, we can obtain the following formula,

$$\begin{aligned} \delta d_p^{(i)} &= \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[1 - \left(1 + \frac{\delta \mathbf{F}_m^{(i)}}{\|\mathbf{F}_m^{(i)}\|} \right) \left(1 + \frac{\mathbf{F}_t^{(i)} \cdot \delta \mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|^2} \right) \right] \\ &= \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[-\frac{\delta \mathbf{F}_m^{(i)}}{\|\mathbf{F}_m^{(i)}\|} - \frac{\mathbf{F}_t^{(i)} \cdot \delta \mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|^2} \right] \end{aligned} \quad (\text{A.11})$$

We build the partial differential equation between $d_p^{(i)}$ and displacement in the unbalanced force direction,

$$\frac{\delta d_p^{(i)}}{\delta u}(\mathbf{m}) = \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[\frac{k_m^{(i)} \mathbf{m} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} - \frac{\mathbf{t}_i \cdot (k_t^{(i)} (\mathbf{m}_i \wedge \mathbf{m}) \wedge \mathbf{m}_i)}{\|\mathbf{F}_t^{(i)}\|} \right] \quad (\text{A.12})$$

Owing to that

$$\mathbf{t}_i = -\frac{\mathbf{F}_t^{(i)}}{\|\mathbf{F}_t^{(i)}\|} \quad (\text{A.13})$$

$$(\mathbf{m}_i \wedge \mathbf{m}) \wedge \mathbf{m}_i = (\mathbf{m}_i \cdot \mathbf{m}_i) \cdot \mathbf{m} - (\mathbf{m} \cdot \mathbf{m}_i) \cdot \mathbf{m}_i = \mathbf{m} - (\mathbf{m} \cdot \mathbf{m}_i) \cdot \mathbf{m}_i \quad (\text{A.14})$$

$$\mathbf{t}_i \cdot \mathbf{m}_i = 0 \quad (\text{A.15})$$

Finally, it comes to the conclusion that,

$$\frac{\delta d_p^{(i)}}{\delta u}(\mathbf{m}) = \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[\frac{k_m^{(i)} \mathbf{m} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} - \frac{k_t^{(i)} \mathbf{t}_i \cdot \mathbf{m}}{\|\mathbf{F}_t^{(i)}\|} \right] \quad (\text{A.16})$$

We ended up writing it as follows:

$$P^{(i)}(\mathbf{m}) = \frac{1}{\tan \varphi^{(i)}} \frac{\|\mathbf{F}_t^{(i)}\|}{\|\mathbf{F}_m^{(i)}\|} \left[\frac{k_m^{(i)} \mathbf{m} \cdot \mathbf{m}_i}{\|\mathbf{F}_m^{(i)}\|} - \frac{k_t^{(i)} \mathbf{t}_i \cdot \mathbf{m}}{\|\mathbf{F}_t^{(i)}\|} \right] \quad (\text{A.17})$$

If the incremental displacement $\mathbf{m}$ is positively collinear with $\mathbf{m}_i$, then it generates an increase in the normal force (particle interpenetration increases). Therefore, the considered contact is getting away from the sliding condition which corresponds to $P > 0$.

For the opposite direction $-\mathbf{m}_i$, particle interpenetration decreases which increases the risk of sliding ($P < 0$).

If the incremental displacement $\mathbf{m}$ is positively collinear with $\mathbf{t}_i$ (the opposite direction to the tangential force $\mathbf{F}_t^{(i)}$), the tangential displacement between the two particles will increase, so as the tangential force magnitude. Therefore, the considered contact is approaching the sliding condition which corresponds to $P < 0$. For the opposite direction $-\mathbf{t}_i$, the tangential force decreases which decreases the risk of sliding ($P > 0$).

For arbitrary directions, the sign of $P(\mathbf{m})$ tells whether contact i is getting closer or not to sliding. Note that this index does not tell which perturbation magnitude is necessary to make the contact slide. Also, note that the sign of the index is attached to a given particle. From the action reaction law, the sliding index P will be of opposite sign for the neighboring particle and the same perturbation displacement. This is not an issue since P index is design to estimate in a second step the detachment potential of particle P by integrating the sliding information over all contacts of the particle. This is done in the present paper by defining the Δ index:

$$\Delta = \text{Number}(P^{(i)}(\mathbf{m}) > 0) - \text{Number}(P^{(i)}(\mathbf{m}) < 0) \quad (\text{A.18})$$