An Infrastructure Perspective for Enhancing Multifunctionality of Forests: A Conceptual Modeling Approach

Mojtaba Houballah1, Jean-Denis Mathias1, and Thomas Cordonnier2

Key Points:
• A conceptual framework for a forest that is mathematically operationalized as a dynamical system
• Study of interactions between forest functions in a multifunctional management context
• Analyzing multifunctional forest management through the multifunctionality of road infrastructure networks

Supporting Information:
• Supporting Information S1

Abstract
Many forest resource systems depend heavily on shared and coupled infrastructures in applying their management strategies. Addressing a question of sustainability for relevant contemporary social-ecological systems (SES) can be tackled by understanding how these shared infrastructures mediate the interaction between human and ecological environment. Shared infrastructures, which are mainly composed of roads (accessibility utilities), highlight the relation between the performance of ecosystem services and the multifunctional use of the forest. However, dilemmas associated with road provision pose some problems when applied in a forest multifunctional management context, because roads potentially diminish or enhance forest functions in a complex way. In this context, maintaining, fostering, and improving multifunctional management where the development of an ecosystem function can affect the performance of others is challenging. We propose to develop a mathematical model based on a recent study that links multifunctional forest management to the multifunctionality of forest roads by using the SES and robustness frameworks. With this model, we analyze the evolution of the forest system and three key forest functions (wood production, tourism, and nature conservation) when impacted by decisions of road provision. We then examine how governance provision strategies can affect the performance of functions and how these strategies can potentially foster forest multifunctionality. This approach allows us to derive conditions of sustainability in which decisions of shared infrastructure provisions can play an important role in the functionalities and performance of the forest.

Plain Language Summary
To understand how the forest evolves in a multifunctional management context where shared infrastructures mediate the interaction of forest functions (wood production, tourism, and nature conservation), we develop a theoretical—but informed by a real case study—mathematical model based on the socio-ecological robustness framework that focuses on the infrastructure role in the performance of the forest’s functional systems. We define a concept of multifunctionality index as a way to quantify the performance of forest multifunctional management. This model integrates governance and highlights its ability to provide infrastructures. Analysis of the model results in an examination of the emergence of multifunctional forest management with a significant correlation with forest governance, and a study that deals with the sustainability of such ecosystems that are expressed as a clear relationship between biophysical and social structures.

1. Introduction
Services provided by forests are crucial to humans’ survival (Daily et al., 1997). They provide a wide variety of benefits that range between provisioning, regulating, supporting and cultural services, which stabilize climate, protect plants and animal species, provide food and shelter to local communities, protect critical human infrastructures such as settlements, roads, and railway lines from gravitational natural hazards, and isolate a large amount of carbon as a result of recycling of gases (Bonan, 2008; Gamfeldt et al., 2013; Millennium Ecosystem Assessment, 2005). These functions have also been claimed to be of great economic value (Costanza et al., 1997; Pearce et al., 2001). Unfortunately, in most cases, forests are unsustainably managed, resulting in the “mining” of the forest resource and widespread ecological degradation (Barnes et al., 1997). It is critical that in the future, forests are used in a way that sustains the resource.

In this context, sustainable forest management can be defined as the use of forest resources in a way and at a rate that maintains their biodiversity, productivity, regeneration capacity, and their ability to fulfill, now
and in the future, the relevant ecological, economic, and social functions (Martin-Garcia & Diez, 2012). However, while sustainable forest management, seen as a constant yield of wood supply, has been practiced in forestry for centuries, modern ideas of sustainability are broader in scope, embracing all the goods and services of the forest. And as a result, forests are increasingly being managed as multifunctional ecosystems (Farrell et al., 2000). Therefore, forests are viewed as complex social-ecological systems (SESs), requiring adaptive and multifunctional management (Messier et al., 2015). In this context, forest multifunctional management, which highlights the ecological and economic characters of forests, has become a fundamental objective for several European countries (e.g., France, Italy, and Germany; Slee, 2012).

Recent movements in sustainability science for forest SESs acknowledged the key role of infrastructures. For example, Anderies et al. (2019) argue for the importance of infrastructures in obtaining knowledge over how actions can manipulate and impact SESs, while Oberlack et al. (2015) attributed the regrowth of forests in the tropics to the presence of robust community institutions and co-management between communities and national government. Nonetheless, the capacity of societies to address forest sustainability hinges on their ability to deal with several social dilemmas associated with integrating their activity and cooperating concerning multiple uses of the forest as well as provisioning shared human-made infrastructure (Houballah et al., 2020; Muneeppeerkul & Anderies, 2017). Anderies et al. (2004) developed a framework (robustness framework) that combines the social and ecological facets around the concept of infrastructures. In this framework, infrastructures are broadly defined to include natural and human-made infrastructures that enable the operation of societies (Anderies et al., 2016). In the same vein, Clark et al. (1979) investigate the connection between sustainability of resource systems and management of infrastructures and present an example of how investments in fishing boats can affect the dynamics of fisheries SES. Investing in a shared and multifunctional road network can have positive (Houballah et al., 2020; Vilela et al., 2020) or negative (Fagua et al., 2019; Kleinschroth & Healey, 2017; Laurance & Useche, 2009) impacts on the multifunctional and sustainable use of forests.

Constructing and maintaining multifunctional forest roads are considered key elements for successful forest management. However, trade-offs between these two elements have negative and positive effects on different forest functions, which induces complexity in the decision-making process. For example, building a lot of forest roads can increase accessibility to the forest, which benefits wood extraction but can negatively affect the scenic beauty as well as the biodiversity of the forest (Li et al., 2013). In this context, Houballah et al. (2020) considered a new approach that combines the SES (Ostrom, 2009) and robustness frameworks to present a novel perceptive for understanding interactions in multifunctional forest management through an infrastructural point of view (see Figure 1a).

To fully integrate the role of shared infrastructures and their governance into ecosystem science, we propose a generic conceptual modeling approach, based on Houballah et al. (2020) study that links human and biophysical drivers, patterns, processes, and effects. Our main contributions are as follows: (1) development of a stylized—but informed by a real case study—mathematical model that operationalizes the modified robustness conceptual framework of Figure 1a to analyze the interactions in multifunctional forest management; (2) the study of the multifunctional forest management through analyzing the multifunctionality of road infrastructure. In particular, highlight the connection between the performance of forest functions and the provision of infrastructure, and finally (3) define an index of multifunctionality as a way to quantify the performance of forest multifunctional management, where we study the capacities and strategies for fostering forest multifunctionality. In particular, our study analyzes the three different forest functions (wood production, tourism, and nature conservation) and related governance strategies through the lens of the robustness framework and brings to clear focus, using mathematical expressions, the interactions between diverse forest functions, multifunctionality of road infrastructure, dynamics of the forest, and governance influence. Though our model construction is based on theoretical approximations, our insights discuss results proclaimed in different works of literature that do not consider such mixed and interwined assumptions.

2. Modeling Forest Multifunctionality

2.1. Introduction

The model construction is inspired by a real case study (Houballah et al., 2020). The Quatre-Montagne has about 17,000 ha of forest cover. Changing socioeconomic factors have led to a suite of land-use changes in for-
Earth’s Future

Three forest functions are considered as major economic and social drivers of exploitation: wood removal, tourism, and nature conservation. However, due to the mountainous terrain, these functions face particular difficulties linked to the accessibility of the resource (Avocat et al., 2012). In their approach, Houballah et al. (2020) introduced a systematic conceptualization of the multifunctional forest management in the Quatre-Montagne forest by connecting functions to relevant infrastructures. In particular, the authors established a connection between the variables found in the SES framework analysis and their related infrastructures as viewed by the robustness framework.

In this article, we base our model construction on the study reported in Houballah et al. (2020) to analyze the interaction of governance and the forest through infrastructures and the capacity of the system to withstand disturbances. In particular, we use the modified robustness framework (Figure 1a) to guide the development of the model and the analysis. We then explore the relationship between forest functions mentioned above (wood production, tourism, and nature conservation) and the ecosystem. Moreover, we examine the relation between functions’ performance and governance by delving into the role of governance in providing infrastructure (by which functions gain affordances to exploit). Figure 1b shows how we operationalize and adapt the robustness framework to help organize the presentation of the model and serve to answer our questions. While the model is based upon a real case study analysis, assumptions, analysis, and choice of parameters remain purely theoretical. All parameters of the model and their values are defined and outlined in Table S1 found in the supporting information (SI).

2.2. Forest Dynamics

The forest growth model has been developed and analyzed in Mathias et al. (2015) and has been modified to fit our analysis (the author analyzed the model according to different wood removal strategies). The innovation introduced in this model is the idea of linking the timber harvesting in the forest to the provisioning of roads. We consider monospecific silver fir stands and a 1 ha representative sample of each user’s forest stand. The stand is composed of two strata, the upper stratum \( x_1 \) (big trees) and the lower stratum \( x_2 \) (small trees) at time \( t \). We also consider that only trees in the upper stratum are removed for wood production. The dynamics of stratum 1 in the forest is assumed to be:

\[
\frac{dx_1}{dt} = \frac{\text{growth}}{\text{removal}} = \frac{h x_2(t)(1 - u g x_1(t))}{v_1} - m x_1(t) + \frac{m}{v_1}
\]
where $h$ is the intrinsic rate of the growth from stratum 2 to stratum 1, $u$ is the asymmetric competitive effect of stratum 1 on stratum 2, $g_i$ is the mean basal area of trees in stratum 1, $\rho$ is the intrinsic mortality in stratum 1, $v_1$ is the mean volume of trees in stratum 1, $m$ is the timber removal function which will be given later.

The dynamics of stratum 2 in the forest is assumed to be:

$$\frac{dx_2}{dt} = bg_2x_1(t)(1-s(g_1x_1(t)+g_2x_2(t)))-hx_2(t)(1-wg_2x_1(t)) - x_2(t)(zg_2x_1(t)+\rho)$$

where $b$ is the intrinsic recruitment rate, $s$ is the recruitment sensitivity to light interception by strata 1 and 2, $g_i$ is the mean basal area of trees in stratum 2, $z$ models the mortality process in stratum 2 due to asymmetric competition.

The volume of deadwood is considered a relevant indicator of the biodiversity (Bouget et al., 2012; Lassauce et al., 2011). Decaying deadwood provides habitats for small vertebrates, invertebrates, and other Saproxyllic species. Therefore, we introduce the deadwood volume dynamics as an indicator of biodiversity of the forest and therefore, the nature conservation function. The total deadwood dynamics can be expressed by the following equation:

$$\frac{dV_d}{dt} = v_2x_2(t)(zg_2x_1(t)+\rho) + m(1-p_e) - m_d + v_1x_1(t)p - \alpha v_d(t)$$

where $v_2$ is the mean volume of trees in stratum 2, $p_e$ is the ratio of tree volume that is effectively exported (in the case of whole tree extraction for wood energy, $p_e$ is 1), $m_d$ is the deadwood removal function and will be given later, $\alpha$ is the rate of decay of deadwood.

We consider that forest managers can partially control the wood harvest volume $m$, $m_d$ (since the harvest is controlled by managers and augmented by infrastructures). They generate decisions based on their economic objective, forest welfare, and biodiversity incentive (deadwood volume). The user harvest functions are considered to be enhanced by infrastructures in the forest and can be expressed as follows:

$$m = h_m \times H^F(S_1, C_1)$$

$$m_d = o \times (v_1x_1)p_e \times H^F(S_1, C_1)$$

where $h_m$ is the wood removal objective, $o$ is the ratio of deadwood removal per one road unit, $p_e$ is the ratio of timber volume that is effectively exported (in the case of whole tree extraction for wood energy, $p_e$ is 1), $p_e$ is the ratio of dead trees in stratum 1 that are removed for commercial purposes, $H^F$, $S_1(t)$, and $C_1(t)$ are the road enhancement function, road state dynamics, and road construction dynamics respectively that will be introduced later.

The financial aspect of forest managers can be expressed as a function of the yield from the harvest subtracted by the cost of the effort exerted by the manager. The revenue function of the users can be expressed by the following equation:

$$R = (p - c_m) \times m + (p_d - c_d) \times m_d \times (1 - T_{CF})$$

where $p$ is the price of 1 m$^3$ of timber (in euros), $c_m$ is the cost for extracting 1 m$^3$ of timber; $p_d$ is the price of 1 m$^3$ of deadwood, $c_d$ is the cost of extracting 1 m$^3$ of deadwood, $T_{CF}$ is the ratio of taxes taken from forestry users (both for timber and deadwood harvest).

### 2.3. Tourism Dynamics

The tourism industry has increased considerably in recent decades and has become one of the main sources of income in many countries (Nijkamp & Coccossis, 1995; Williams and Shaw 1988) and especially in the Vercors (ARANGE 2015; FORGECO 2014; Houballah et al., 2020). This development in the Vercors has been attribu-
ed to the scenic beauty of the mountainous terrain (FORGECO 2014; Tenerelli et al., 2016). For many tourist sites, the reward phase of development is characterized by long and intense growth in infrastructure and facilities. In fact, some destinations, after flourishing for a long time, have been abandoned by tourists in favor of more attractive sites newly available on the market (Butler, 1991). To compensate for this instability, local agents may seek increased investment and develop special facilities to attract tourists. Sometimes they are successful, but at the expense of the forest environment and its functionality where it may be severely degraded.

The dynamic model of tourism we propose here represents the “outside social demand” on the forest and we consider that tourism, as a forest function, is measured according to the number of tourists the forest can attract. This model is not thoroughly based on data but on very simple assumptions inspired by Casagrandi and Rinaldi (2002, to analyze the model solely, we refer the reader to this reference). These assumptions include interactions between three important components of the coupled system: the tourists, environment, and infrastructures that are based on so-called minimal models that are used to predict economic and environmental impact of any given policy (Anderies, 2005).

Imagine that tourists are asked to report on the attractiveness of the forest, $A$, and let us assume that these reports influence the decisions of potential new visitors (spread of information; Morley, 1998). Measuring $A$ in a suitable unit, we can then write the rate of change of tourists at a given site is equal to the product $TA$, that is,

$$\frac{dT(t)}{dt} = T(t) \times A(T, E, H^T),$$

where $E$ is a function describing the attractiveness of the forest’s environment, and $H^T$ that of infrastructures. $A$ refers here to relative attractiveness, namely the difference between the absolute attractiveness, $\hat{a}$, of the site (for which information on $T$, $E$, and $H^T$ is available) and a reference value, $a$, which can be thought of as the expected attractiveness of a generic site (i.e., the average value of the attractiveness of all potential tourist sites). Thus:

$$A(T, E, H^T) = \hat{a}(T, E, H^T) - a$$

where $a$ is influenced by several factors, including the price of alternative sites. In an abstract sense, $a$ is a measure of competition exerted by alternative tourist sites on the forest. The attractiveness of the site, being perceived by tourists, depends upon their sensitivity to the quality of the natural environment and their ability to detect it. It is the algebraic sum of three terms: (1) environmental quality, (2) availability and state of infrastructure, and (3) congestion of tourists. We consider here that the environmental attractiveness is affected by the forest structure (respective densities of the two different strata) where uneven-aged stands are considered most suitable for tourism in both winter and summer seasons (Clatterbuck et al., 2010; Meo et al., 2015; Paletto et al., 2017). This can be summarized by a minimum and a maximum number of trees in the forest (continuous cover) and a minimum ratio between trees of the two strata (structural complexity). Thus, to describe the quality of the forest environment, we consider the following two-dimensional Gaussian-like function (see Figure S1 in SI):

$$E(x_1, x_2) = \exp\left(-\left(\omega_1(x_1 - x_1^0)^2 + 2\omega_2(x_1 - x_1^0)(x_2 - x_2^0) + \omega_3(x_2 - x_2^0)^2\right)\right)$$

where $x_1^0$ and $x_2^0$ are the assumed forest most attractive structure for tourists; $\omega_1$, $\omega_2$, and $\omega_3$ are the rate of change of the forest attractiveness.

Finally, we assume that the congestion is proportional to $T$ and that attractiveness is linearly decreasing with congestion, we end up with the following dynamics for $T$:

$$\frac{dT}{dt} = T \left[\text{absolute attractiveness} \left(\hat{a}(T, E, H^T) - a\right)\right]
= T \left[\frac{\text{attractiveness of the environment}}{E} + \frac{\text{attractiveness of the infra}}{H^T(S, C)} - \frac{\text{congestion}}{\gamma T} - \frac{\text{a competition}}{a}\right]$$
where $\alpha_T$ the ratio of congestion of tourists, $a$ is the expected attractiveness of the forest, $H^F(S_t,C_I)$ will be given later as the attractiveness function that depends on the availability and state of roads.

We consider that the revenue function (economic indicator) for the tourism industry in the forest is scaled to the number of tourists in the area. Indeed, Stynes (1997) argues that one of the criteria to assess economic output for tourism is derived from the measure of the number of tourists at the site. For example, an increase in tourists staying overnight in hotels would directly yield increased sales in the hotel sector. The additional hotel sales and associated changes in hotel payments for wages and salaries, taxes, and supplies and services are direct effects of tourism spending. Therefore, we consider that a revenue function proportional to the number of tourism users can be expressed as follows:

$$R_T = \pi_T T(t) \times (1 - T_{cT})$$

where $\pi_T$ is the proportion of the money paid by the tourism users, $T_{cT}$ is the ratio of taxes taken from tourists to the government.

### 2.4. Road Infrastructure Enhancement Functions

$H^F(S_t,C_I)$ is the function that maps $S_t$ and $C_I$ to the productivity of users and is inspired by Muneepeerakul and Anderies (2017), where authors analyzed a resource system according to different infrastructure investment strategies. Many shared infrastructures exhibit nonlinear behavior in their productivity. For example, once the state of forest roads becomes so poor that it falls below a certain threshold, one that is related to major road blockage, the road’s employment in accessibility stops working. Moreover, the productivity of users is linked as well to the availability of infrastructure. Therefore, to capture such behavior, we assume the following piecewise function for $H^F(S_t(t),C_I(t))$:

$$H^F(S_t(t),C_I(t)) = \begin{cases} 0, & S_t(t) < S_{t0} \\ C_I(t) \frac{S_t(t) - S_{t0}}{S_{t1} - S_{t0}}, & S_{t0} \leq S_t(t) \leq S_{t1} \\ C_I(t), & S_t(t) > S_{t1} \end{cases}$$

where $S_{t0}$ is the threshold of $S_t$ below which $H^F$ is zero, $S_{t1}$ is the threshold of $S_t$ above which $H^F$ is maximum regarding the quality of available roads.

$H^F(S_t(t),C_I(t))$ is the function linking $S_t$ and $C_I$ to infrastructure attractiveness. It is considered that for a certain amount of roads the perception of tourists regarding the area’s attractiveness is considered to be the highest, after this value the perception starts declining due to the “congestion of infrastructure.” While infrastructures are of importance for the development of tourism, we also consider that its congestion negatively affects the natural scenic beauty of forests (Pastorella et al., 2016). For example, Thiel et al. (2008) concluded that infrastructures should be limited in certain forest areas to retain undisturbed forest patches within skiing areas. To capture the behavioral effect of tourists to infrastructure attractiveness, we assume the following Gaussian piecewise-like function:

$$H^F(S_t,C_I) = \begin{cases} 0, & S_t < S_{t0F} \\ \frac{(C_I - C_{0F})}{2\sigma_t^2} \frac{S_t - S_{t0F}}{S_{t1F} - S_{t0F}}, & S_{t0F} \leq S_t \leq S_{t1F} \\ \frac{(C_I - C_{0F})}{2\sigma_t^2}, & S_t > S_{t1F} \end{cases}$$

where $\sigma_t$ is the maximum attractiveness related to road availability, $C_{0F}$ is the number of roads in which the perception of tourists is considered the highest, $C_I$ is the rate of increase/decrease of roads attractiveness.
when the number of roads increases, \( S_{\text{int}} \) is the threshold of \( S_I \) below which, the attractiveness associated with the quality of infrastructure is zero, \( S_{\text{int}} \) is the threshold of \( S_I \) above which, the attractiveness associated with the quality of roads is maximum with respect to available roads.

### 2.5. Road Infrastructure Dynamics

For the sake of simplicity, we consider that all types of roads in the forest are used for all forest functions. On one hand, governance in the forest can decide to introduce new roads as a part of a strategy for increasing accessibility in the forest; this decision is based upon its measured effectiveness as well as the amount of money allocated for that purpose. To define a system of road network development, we first consider (1) the idea that existing roads trigger the development of more in and (2) the forest, being a finite space, can only withstand a maximum number of road units. Therefore, the dynamics of the number of road unit measured in km \( h^{-1} \) in the forest can be expressed by the following logistic growth equation:

\[
\frac{dC_I(t)}{dt} = \alpha_1 \times I_{BA} \times B_A(t) \times \mu \times C_I(t) \times \left(1 - \frac{C_I(t)}{C_I^{\text{max}}}ight),
\]

where \( \alpha_1 \) is the portion of the annual budget \( (I_{BA} \times B_A(t)) \) allocated for constructing roads, \( u_1 \) is the effectiveness of investment in constructing roads, \( C_I^{\text{max}} \) is the maximum carrying capacity for the number of road unit in the forest, \( \mu \) is the growth in CI(t) per unit of road.

On the other hand, governance is responsible for maintaining the road infrastructure in the forest, the behavior of such action is mediated by the amount of money allocated from the annual budget of the governance as well as the effectiveness of such action. Moreover, maintenance is reduced by the increasing number of road units as the effectiveness of the maintenance budget becomes less efficient. The function of maintenance can be expressed by the following equation:

\[
M(t) = \alpha_2 \times I_{BA} \times B_A(t) \times \left(u_2 \times \frac{1}{C_I(t) + k}\right)
\]

where \( \alpha_2 \) is the portion of the annual budget \( (I_{BA} \times B_A(t)) \) allocated for maintaining roads, \( u_2 \) is the effectiveness of investment in maintaining roads, \( k \) is the rate of decrease in road maintenance effectiveness.

Maintenance of the infrastructure is seen as a logistic growth of a road state dynamic. In particular, at a low state the growth is considered low due to the poor conditions of roads (using roads to maintain other roads), however, the growth increases with the increase of the state until it reaches very high quality and becomes costly to maintain. Moreover, introducing a new road has a positive effect on the state dynamics, where the newly built roads are considered to have the maximum quality, and consequently with time, impacted negatively by the depreciation effect. The dynamics of the state of roads \( S_I(t) \) is described as follows:

\[
\frac{dS_I(t)}{dt} = \frac{M(t) \times S_I(t) \times (1 - S_I(t))}{C_I(t)} + \frac{\epsilon \left(S_{\text{int}} - S_I(t)\right)}{C_I(t)} - \delta S_I(t)
\]

where \( \delta \) is the infrastructure’s depreciation rate, \( \epsilon \) is the number of roads introduced at time \( t \).

### 2.6. Governance of Infrastructures

Our analysis focuses on understanding the nature of the economic and political governance from an infrastructural point of view and within the dynamics of the robustness framework. In this context, governance (or public infrastructure providers in the robustness framework) in the forest is highlighted by the ability to provide public shared infrastructure.
The behavior of governance is manifested in the amount of resources collected from the forest functions that are appropriated by the governance for maintaining and constructing roads in the forest. The annual budget \((B_{A})\) of the governance is composed of taxes \((T_{cT}, T_{cT})\) paid by forest users (timber and tourism users), as well as subsidies \((\gamma)\), paid either by the French government or the European Union for forest management in the Western Alps, and is given by the following equation:

\[
B_{A}(t) = \frac{\text{forestry tax contribution ratio}}{\text{TcT}} \times \left( \frac{\text{revenue of timber harvest}}{m(t) \times (p - c_{m})} + \frac{\text{revenue of deadwood harvest}}{m_{d}(t) \times (p_{d} - c_{d})} \right) \]

\[
+ \frac{\text{revenue from tourism function}}{\text{tourism tax contribution ratio}} \times \frac{\text{subsidies}}{\text{TcT}} + \gamma
\]

### 2.7. Coupled Dynamics

Before proceeding with the results of the model, let us recall that we are analyzing the following system of six differential equations (to facilitate the comprehension of the model, we refer the reader to Figure S8 to visualize the interaction of different model components):

\[
\frac{dx_{1}}{dt} = \frac{\text{growth}}{v_{1}}(1 - u_{g_{1}}x_{1}) - \frac{\text{mortality}}{v_{1}}x_{1} - \frac{\text{removal}}{v_{1}}h_{m} \times H^{F}(S_{1}, C_{1})
\]

\[
\frac{dx_{2}}{dt} = \frac{\text{growth}}{v_{2}}(1 - u_{g_{2}}x_{2}) - \frac{\text{mortality}}{v_{2}}x_{2} - \frac{\text{removal}}{v_{2}}h_{m} \times H^{F}(S_{1}, C_{1})
\]

\[
\frac{dV_{d}}{dt} = v_{1}x_{1} + \frac{\text{debris from removal}}{\text{mortality}} - \frac{\text{lower stratum mortality}}{\text{decomposition}}
\]

\[
\frac{dT}{dt} = T \left[ \begin{array}{c}
\text{attractiveness of the environment} \\
\text{attractiveness of the infra} \\
\text{congestion}
\end{array} \right] + \left[ \begin{array}{c}
\text{depreciation} \\
\text{effect of introducing a new road}
\end{array} \right]
\]

\[
\frac{dS_{1}(t)}{dt} = \frac{\text{maintenance}}{M(t) \times S_{1}(t) \times (1 - S_{1}(t))} + \frac{\text{effect of introducing a new road}}{\text{depreciation}}
\]

\[
\frac{dC_{1}(t)}{dt} = \left( \frac{\text{Growth}}{\text{attractiveness of the infra} \times S_{1}(t) \times C_{1}(t) \times \left( 1 - \frac{C_{1}(t)}{C_{1,\text{max}}} \right)} \right)
\]

The model analysis reveals a rich set of results that highlights the interplay and trade-offs between forest functions mediated by the resource dynamics as well as infrastructure characteristics and focuses on an illustration on the emergence of sustainability and multifunctionality of forests in an infrastructure mediated context. In its core, the model refers to an area where, due to budgetary constraints, there is a trade-off between the decisions of investment for maintenance or construction of roads. The overall picture that guides our analysis is that forest functions are mediated by the availability and state of shared accessibility...
infrastructures. This offers governance control on the exploitation of the forest where it can severely impact the multifunctional management and consequently the resource’s sustainability.

3. Results

For the following scenario simulations, we use Euler implementation in MATLAB with a 0.1-time step to solve our coupled system using the parameter values given in Table S1 (see SI). In the subsequent discussions, we consider that users of different functions are maximizers of benefits in the sense that they do not care about damaging and degrading other functions. For example, timber harvest users only care about extracting wood regardless of the impact on the forest scenic beauty perceived by the tourists. In addition to the long-term equilibrium states of the simulations, we have chosen to represent the transient dynamics for a relatively short period (50 years, Mathias et al., 2015), to highlight the trade-offs that can occur between the different functions. Moreover, to quantify the wood removal function, we take into account the annual wood extracted from tree cuttings and the deadwood collected from the forest. In our analysis, we define a collapse of the forest system as the dysfunctionality of the forest ecosystem services. In our simulations, we specifically address the following questions: 1) Can the initial forest structure explain the preservation of forest functions whatever the investment strategies for infrastructures? 2) For what infrastructure strategies the different functions are maximized? 3) Are there any trade-offs between the three functions investigated? and finally, 4) What are the governance strategies that can foster multifunctionality?

3.1. Effect of Initial Forest Structure on Forest Functions

For the sake of clarity and comprehension of the model, we perform simulations first corresponding to initial forest structures ($x_1$, $x_2$). Figure 2 shows the final value of simulations of the functions (wood removal “WR,” tourism “T,” and nature conservation expressed by a biodiversity indicator measured as the dead-wood volume “DW” per ha) according to initial forest stand and a fixed investment from the governance for the construction and maintenance of roads (Figures 2a–2c). The figure shows also the evolution of the different system dynamics at three different points (A, B, and C) where functions have a change in behavior at their final values (Figures 2d–2f).

As shown in Figure 2a, on one hand when the number of trees at initial states in stratum 2 is not enough (insufficient number of small trees), the WR function undergoes a slow development and does not attain high values after 50 years (see point C). On the other hand, having a very high number of big trees incurs higher competition between the two strata which increases the small trees’ mortality and moves the forest toward a lower ability to generate big trees in the future. However, the maximum value for WR attained in 50 years is when there is a high number of small trees at initial states (see point A), in other words, where we have an unbalanced forest with high capability to produce big trees. Furthermore, point B shows that wood harvest function levels slightly decrease in the initial forest structure which maximizes T. This is due to the big amounts of money that can accompany a high attraction of tourists (Figure 2f), which leads to an overinvestment in infrastructures and then a high extraction of wood and finally a slight change in the structure of the forest, which disfavors T. This chain of effects can be seen as a closed-loop negative process, which leads to a peak early and settles for a lower value at sustainable state.

Moreover, Figure 2b indicates that the number of tourists reaches its high values at a low-aged forest structure (see point B with a high number of small trees). This is due to the effect of WR on the forest, in which it moves its structure to a state that slightly favors T (see Figures 2d and 2e). Such a behavior, with a relatively low $h_{\text{in}} = 10$ is a classic reaction of the compatibility of WR with T, where WR can help moving (through tree removal) the forest structure toward a favorable state.

Figure 2c shows that at points A and B, where the T and WR functions are fairly high, DW is low; this can be explained thanks to the high annual budget that can be obtained from the two functions, which allows for the development of road infrastructure, and eventually a high extraction capability for deadwood. At point C, where there are a lot of big trees and small trees, T and WR slowly develop due to the gradual development of tourism in the area (Figures 2e and 2f) which can be explained by the disadvantageous initial forest structure for the attractiveness function; this leads to a low annual budget, and therefore, low infrastructure.
investment. However, for the DW volume, and due to the low ability of extraction and the high ratio of tree mortality, the final value is maximized.

3.2. Influence of Governance

Accomplishing an objective of “harvesting more while preserving better” with achieving increases in WR, T, and biodiversity preservation (DW volume) requires improvements in the governance of forest infrastructures. As explained before, our model can address this issue, and we propose here to analyze the effect of different infrastructure governance scenarios on the forest system at the equilibrium state. For this purpose, we test different approaches of infrastructure governance including different strategies of investment in maintenance and construction of roads ($0 < \alpha_1 < 1$ such that $\alpha_1 + \alpha_2 = 1$); and corresponding to different actions of tax impositions ($0 < T_{CF} < 0.4$ and $0 < T_{CT} < 0.2$). Figure 3 presents the final value of forest functions relating to these strategies for a fixed initial forest stand. Moreover, the figure displays the evolution of the different model dynamics according to three different points (A, B, and C) where functions reach completely different final values (Figures 3d–3f).

Figure 3a shows that when the ratio of budget directed toward the construction of roads is very high (see point C), the functionality of the WR function decreases, this is because overinvesting in road construction
takes money from the investment in maintenance of these roads, and on top of that, as roads increase it becomes very costly to maintain them. In other words, with governance strategies of high road construction and low maintenance investments, the roads cannot preserve their state and will lead to the loss of their employment in WR. Point A shows that for the right amount of road construction investment and sufficient tax ratio imposition, WR function can be maximized (Figure 3d). However, as point B shows, not enough investment in road maintenance can lead to the slow development of WR function.

Moreover, Figure 3b represents T attraction in the forest and shows that the function is maximized with strategies that are directed toward maximizing wood extraction (with low investment in road construction and high tax imposition, see Figures 3d, 3e, and S3i). In the area where wood extraction is maximized (point A), the tourism function is also maximized (Figure 3e), this can be explained by the tree cutting effect on the structure of the forest which moves the forest to a more desired and attractive state. Furthermore, point C shows that high investment in road construction can cutback the infrastructure attractiveness and therefore gradually decrease the attraction of tourists (Figures 3e and S3i). Finally, Figure 3c shows that for a governance strategy that is directed toward high WR (point A), the deadwood volume is decreased (Figures 3d and 3f), while for a strategy directed at offering a low ability for wood extraction (points B and C), one can observe an increase in the values of DW.
3.3. Forest Multifunctionality With Extreme Cases

As shown previously in simulations, one can observe evidence of slight trade-offs between forest functions. Thus, we choose to highlight these trade-offs by taking extreme cases with functions’ objectives. For that, we consider the following two cases.

3.3.1. Case of High Wood Extraction

In this section, we focus on an important issue in multifunctional forest governance that can help in highlighting the trade-offs that occur in a relatively short-term period (50 years). We choose a case where we have intensive WR levels \( b_w = 30 \text{ m}^3 / \text{ha}, \sigma = 3 / \left[ C_f \right] \). Although this case refers to an unsustainable outcome for the forest (see SI), we are interested in the trade-offs that can occur in a relatively short-term period. Figure 4 presents the final value of forest functions according to these strategies for a fixed forest stand initial conditions. Moreover, the figure displays the evolution of the different model dynamics corresponding to three different points (A, B, and C) where functions reach completely different final values (Figures 4d–4f).

As the case in Section 3.2, Figure 4a shows that when the ratio of budget directed toward the construction of roads is very high (see point C), the functionality of the WR function decreases, while point A maximizes the function with the right strategies. However, with high WR objectives, the region of strategies with high WR expands, which indicates flexibility in the governance decision making.

Moreover, Figure 4b presents the tourism attraction in the forest and shows that the function is maximized with strategies that are not directed toward maximizing WR (with low investment in road construction and relatively low tax ratio, see Figures 4d, 4e, and S4i). In the area where WR is maximized (point A), T function gradually decreases (Figure 4e), this can be explained by the tree cutting effect on the structure of the forest which degrades its scenic beauty. Furthermore, point C shows that high investment in road construction can degrade the infrastructure attractiveness and therefore gradually decrease the attraction of tourists (Figures 4d and S4i).

Finally, Figure 4c shows that for a governance strategy that is directed toward high WR (point A), DW is decreased (Figures 4d and 4f), while for a strategy directed at offering a low ability for WR (points B and C), one can observe an increase for DW. The results of the simulations focus on and highlight the many trade-offs in the performance of each forest service. Maximizing one function can incur negative effects on others. The governance, being an infrastructure provider, or in other words, the offeror of capability for exploitation, plays an important role in maintaining and developing the different functions without affecting the overall economic and ecological performance of the forest. In conclusion, a highly intensified forest with high wood extraction levels incurs negative effects on the performance of other functions, specifically tourism. In particular, such high levels of tree removal change the structure of the forest toward an unfavorable place for tourism negatively affecting it.

3.3.2. Case of High Tolerance for Tourism

In some cases of tourism management, decision-makers can consolidate, through some management strategy, the negative effect of congestion of tourists on the overall perceived attractiveness of the forest (i.e., by building more resorts). In this section, we suppose that we have tolerance toward tourists’ congestion. We change the value of \( \alpha_T \) to be \( 5 \times 10^{-5} \), and consequently, we simulate our model corresponding to different infrastructures provision strategies. Figure 5 shows the evolution of the model in a high tourism tolerance environment.

The simulation suggests that tourism and wood extraction function development are compatible (point A, Figure 5a). However, as Figure 5d shows, even though we have a fast development of WR, this function is not sustainable in the long run, and the same goes for T (Figure 5e). This is because such high WR levels greatly affect the ability of the forest to sustain itself in the long term (Figure S5d). Consequently, as a result of the fast augmentation in infrastructures, DW is extracted at high levels, which can explain its low abundance in the forest (Figures 5c and 5f). This unsustainable behavior can be attributed to the peak in tourism function that increases the total annual budget, which enhances the provision of infrastructures augmenting wood removal, and finally affecting the sustainability of the forest;
Moreover, point B shows a slight decrease in T performance and a significant decrease in WR values. A slow development for road’s state slightly restraints T function due to its effect on infrastructure attractiveness, but adequately limits WR values (Figures 5a and 5d). This limitation on WR significantly accounted on one hand to a sustainable outcome for the forest by limiting its tree cutting (Figure S5d) and on the other hand allowed for the feasibility of its functions (Figures 5d–5f).

Finally, point C accounts for mono-oriented function management directed toward DW. This strategy leads to a sustainable outcome for the forest (Figure S5d) but drives its socioeconomic functions (T and WR) toward an eminent dysfunction (Figures 5d and 5e).

3.4. Multifunctionality Index as a Tool to Measure Governance Performance

With our presented model, we discuss the performance of multifunctional forest management from the perspective of each function, presenting the trade-offs and effects that interplays. However, the model allows us to present a global multifunctionality index that can quantify multifunctional management in forests. In this context, we define the multifunctionality index as follows:
MFI = \( k_1 WR_N + k_2 T_N + k_3 DW_N \)

where \( WR_N \), \( T_N \), and \( DW_N \) represents the standardized values for wood removal volume, tourism, and deadwood volume, respectively. \( k_1 \), \( k_2 \), and \( k_3 \) are the weight parameters corresponding to \( WR \), \( T \), and \( DW \) respectively, that can explain the importance of one function in some forest’s management context with:

\[ k_1 + k_2 + k_3 = 1 \]

Figure 6 represents simulations of MFI for different cases (referenced, high wood extraction, and a high tolerance for tourism) for different values of weights \( k_1 \), \( k_2 \), and \( k_3 \).

On one hand, in the cases where DW volume is not especially important (panels a1, a2, a3, b1, b2, b3, c1, c2, and c3), MFI is confined in a relatively big set [0.3, 0.8]; which indicates that decisions of governance in provisioning infrastructures are fairly important when it comes to fostering multifunctionality. For example, one decision can allow MFI to reach a high value of 0.8, while other decisions can drag its value to 0.3, which is not proper governance of multifunctionality. Nonetheless, multifunctionality is maximized in the area that is beneficial for \( T \) and \( WR \). On the other hand, in the case where DW is considered to be an important objective (panels d1, d2, and d3), MFI is restrained in the set [0.5, 0.66], which indicates a lower
The effect of governance on its outcomes. MFI is maximized with governance strategy decisions that boost T and slightly decrease in the area that boosts WR. This is a clear presentation of the trade-offs between WR and biodiversity conservation.

Moreover, in all cases with WR oriented management, we observe lower flexibility for the governance decision-making (with even lower flexibility in the high WR scenario). This behavior demonstrates the sensitivity of the forest system toward wood removal. This is also backed up with the high WR scenario (panels a2, b2, c2, and d2), where one can observe a lower performance of multifunctionality with strategies that maximize WR. Here, multifunctionality management performance is significantly lowered highlighting the effects and interplays that can arise with socioeconomic functions interactions.
4. Discussion and Conclusions

To address the problem of forest multifunctionality, we have mathematically operationalized the robustness framework conceptualization of forest multifunctionality based on Houballah et al. (2020) work. Here, we consider a particular relation between forest functions and governance highlighted through their ability to provision infrastructures. Namely, the idea proposed here is that infrastructures enable exploitation through accessibility needed either for tourism or wood removal. Naturally, such an assumption highlights the forest governance role in the development of ecosystem functions to meet the increasing demand of the market. We explored the extent of the model to represent the performance of the functions with simulations depending on different governance strategies for infrastructure provision in different extreme cases. Moreover, we have defined a multifunctionality index as a way of quantifying multifunctional forest management performance analyzing the different governance strategies in the present diverse extreme scenarios.

4.1. Trade-offs, Interplays, and Nonsymmetric Effects

Our findings highlighted the trade-offs and interplays that can occur between economic and social forest functions. In particular, our analysis gave a clear indication of the direct effect of wood removal on tourism and deadwood volume dynamics. This effect is backed up with the fact that wood removal, on one hand, can alter the structure of the forest and thus its scenic beauty, and ultimately affect the performance of tourism. On the other hand, through pursuing strategies that maximize wood removal, which falls in line with extracting more deadwood from the forest, it decreases the number of large trees that lead to reduce natural mortality in the forest. This ultimately affects biodiversity and nature conservation function. However, as shown in our analysis (see Sections 3.3.1 and 3.3.2) the effect of tourism on wood removal is positive in a direct manner. In our model’s context (Houballah et al., 2020), tourism permits the development of other functions by highly contributing to the annual budget directed toward infrastructure provision. As seen in the panels (a, b, and c) of Figures 2–5, the significant change observed for WR (and consequently for DW) at the final values of the simulation (yellow zone in panels a of Figures 2–5) is correlated in fact with the performance of T at the same initial forest structure (yellow zone in panels b of Figures 2–5) given the substantial effect tourism has on the infrastructure provision investments. This big change for T is controlled by the environmental attractiveness function, where at initial states, is a key element on how T will evolve. Yet, through its maximization, it excessively enables wood removal, which can backfire on tourism and have dramatic consequences on the forest in the long run. These insights have been confirmed in previous studies that discuss synergies and trade-offs of ecosystem services. In particular, Stevens (2003) discusses the direct impact of deforestation on the performance of tourism as well as the reversible indirect effect of tourism on the wood removal function. Moreover, Lafond et al. (2017) confirmed our hypothesis concerning the negative effect of wood removal on deadwood dynamics (because of the deadwood harvest), and as our model shows, this effect is limited with the fact that wood removal of standing trees yields deadwood ($pe = 0.9$, which refers to the ratio of the tree being removed). Furthermore, Ahtikoski et al. (2011) notice the negative effect of removing trees on the structure of the forest with implications on recreational activities in forests. Lexer and Bugmann (2017) also reported strong trade-offs occurring between wood removal on one hand and other forest functions on the other hand in mountain forests.

4.2. Fostering Multifunctional Forest Management

Many forest governance regimes have been, or are currently, shifting to multifunctional management mechanisms (La Notte, 2008), aimed at improving the applicability of one function-sided management strategies in the presence of other functions in the forestry sector. With our analysis, particular attention is given to the role of management of infrastructures in enabling the development of forest functions. One obvious result that has been highlighted by our model is the need for careful planning of road provisions due to its immense effect on the biodiversity indicator (deadwood volume, a finding that has also been concluded by other studies; Avon et al., 2010; Forman, 2000; Loucks et al., 2003; Selva et al., 2011). Through the enablement of functions, roads can have a detrimental effect on the dynamics of deadwood volume, thus affecting the biodiversity of the forest. Overall, our results confirm that roadless areas (Boston, 2016; Freudenberger et al., 2013; Strittholt & Dellasala, 2001) should be maintained to avoid negative effects on biodiversity and negative feedbacks on green tourism activities.
Nonetheless, fostering forest multifunctionality is a major problem in management where the simultaneous development of ecosystem functions is the focus (Smithusen, 2008). In a context where infrastructures play an important role in mediating the interactions between forest exploitation systems as well as its environment, we argue that on one hand, different infrastructure provision strategies can help reach a desirable outcome for forest multifunctionality. On the other hand, such strategies can reduce flexibility in decision-making for maximizing the performance of multifunctional forest management. Refining the optimal balance between these two processes should of paramount importance for future research.

As shown in Section 3.4, different infrastructure provision strategies may lead to different outcomes for multifunctionality index values. Figure 5 shows that in 50 years’ time horizon the area where wood removal is maximized one can notice a slight decrease in the multifunctionality index, which shows a negative effect on the overall performance of the forest functions. Negative effects appear within the forest ecosystem through empowering wood removal (also verified by Lafond et al., 2017). Moreover, analysis of the figure suggests that to maximize the performance of multifunctional forest management, in our model’s context, we have to minimize wood removal function as to the level that does not affect the perceived natural beauty of the forest (reported by several studies, Brown & Daniel, 1984; Klessig, 2011; Zhalnin et al., 2008).

Moreover, in all cases where we have \( k_3 = \frac{1}{2} \), the multifunctionality index is less sensitive to the governance strategies \((0.5 < \text{MFI} < 0.66)\). This indicates that infrastructure provision strategy is less efficient for multifunctionality in cases where biodiversity is given higher priorities. Moreover, governance has lower flexibility for fostering multifunctionality in the scenario where we have a high objective of wood removal (Lexer & Bugmann, 2017). In particular, the area which maximizes multifunctionality index in panels b1, b2, and b3 (Figure 6) is relatively smaller, which reveals rigidity in decision-making.

### 4.3. Long-Term and Short-Term Infrastructure Governance Strategies

The results of the simulation focus and highlight the many tradeoffs in the functionality of each forest service. Maximizing one function can incur negative effects on the functionality of others. The governance, being an infrastructure provider, can play an important role in maintaining and developing the functions without affecting the overall economic, social, and ecological performance of the forest. In addition to the long-term equilibrium states of the simulations, we have chosen to represent the transient dynamics for a relatively short period (50 years). However, trade-offs can occur between long-term and short-term governance strategies. On one hand, our analysis shows that a fast development of infrastructure, which accounts for fast development of functions (short-term investment), can have influential effects on the long-term sustainability (see Sections 3.3.1 and 3.3.2; has also been reported by Bebbington et al., 2018; Alamgir et al., 2019). Following strategies that do not allow wood removal to affect the forest structure may account for a sustainable outcome for the forest. On the other hand, following a long-term strategy in the governance decision making may not be able to satisfy the current needs of the market rendering the governance strategy not ideal. In other words, the government has to consolidate, through infrastructure provision (or offering affordances for exploitation), the current needs of the market with the objective of long-term sustainability of the forest.

### 4.4. Conclusion

Although our assumptions on the nature of infrastructures are fairly basic, the two functions \( H^T(S_t, C_t) \) and \( H^H(S_t, C_t) \), inspired by Muneepeerakul and Anderies (2017), defined a clear relationship between the ecosystem services and the biophysical environment of the forest. Moreover, such functions capture important aspects of infrastructures regarding the decision of exploitation (either functions use infrastructures for their benefit or do not). The idea that infrastructures can incur trade-offs among forest functions on one hand and between the forest functional system and its ecosystem, on the other hand, can pose problems of management when trying to maximize one forest functions through the provision of infrastructure. This model is assumed as a good approximation in cases where the government has already some control over the forest (a minimal set of soft and hard institutions is available). Our approach focuses on analyzing the forest multifunctional management through the provision of physical human-made infrastructure, which highlights the role of governance.
We hope that this work will contribute to the development of much-needed, systematic mathematical analysis of coupled infrastructure systems (Anderies et al., 2016), especially those focusing on multifunctionality concepts. Although our model is informed by a real case study, we believe that its analysis illustrates general dynamical features for forest functions and thus can be used in other contexts and for other systems; for instance, the derived results could serve as guidelines on how one might empirically measure multifunctionality in CISs. The nature of the model development adopted here was inspired by Muneepheerakul and Anderies (2017), in which we believe it holds systematic value in resource modeling science. There is still value in improving the model with a better indicator of biodiversity that can potentially better highlight ecological trade-offs in the forest. Moreover, much work is also needed with the introduction of the concept of nonphysical (or soft) infrastructures or “knowledge infrastructure” (Anderies et al., 2019) to the interplay between the forest functions and its ecosystem highlighting the adaptive management concept (Walters, 1987). From a general standpoint, viability theory (Aubin, 1991) can be useful in defining safe operating spaces (Carpenter et al., 2015, 2017; Mathias et al., 2018; Rockström et al., 2009) for governing functions as individuals, and common safe operating spaces for the forest multifunctionality. Such an approach can bring new insights to the management and development of SES encompassing a concept of multifunctionality.

**Data Availability Statement**

Data were not used, nor created for this research. The code used to produce figures is available at https://github.com/mojtaba-houballah-M/Earth'sFuture.git.

**Acknowledgments**

This research has been funded by the ANR (Agence Nationale de la Recherche) under the VIRGO project (ANR-16-CE03-0003) and the region of Auvergne. Moreover, this research was conducted on the Long-Term Socio-Ecosystem Research platform LTSER “Zone Atelier Alpes,” a member of the ILTER-Europe network.

**References**


Gamfeldt, L., Snüll, T., Bagchi, R., Jonsson, M., Gustafsson, L., Kjellander, P., et al. (2013). Higher levels of multiple ecosystem services are found in forests with more tree species. Nature Communications, 4, 1340. https://doi.org/10.1038/ncomms12328


