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Taking firms’ margin targets seriously in a model of competition in supply functions

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Taking firms’ margin targets seriously in a model of competition in supply functions *

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Abstract

We introduce price-markup objectives into a model of supply function competition. We characterize the corresponding supply-function equilibrium and study its qualitative properties. Adherence to price-markup targets is conducive to reduced market competition and increased firm profitability. While pursuing such goals reduces social welfare, welfare never drops below the level corresponding to a Cournot oligopoly. Finally, we establish conditions under which consumer preference for fair pricing inhibits the industry’s use of markups as a collusive device.

Keywords: supply function, markup pricing, price fixing, oligopoly.

1 Introduction

Cost-plus pricing is a lot like the romance novel genre, in that it’s widely ridiculed yet tremendously popular. (Dholakia, 2018)

Like it or not, markup (or cost-plus) pricing remains the most popular product pricing method. There is indeed ample survey evidence showing that most (price-making) firms set their product prices primarily by applying the following rule of thumb:

\[ p = (1 + m)AC, \]

where \( m \) is the gross profit margin and \( AC \) denotes the average production cost. In doing so, these firms set their prices at the average production cost plus some suitable markup.

The managerial literature has long recognized the empirical prevalence of this pricing routine. It provides many illustrative examples, among which the most notables are restaurants where ‘food is marked up three times direct costs, beer four times and liquor six times’ (Godin and Conley 1987, cited in Phillips 2011) and hotels where the ’Dollar per Thousands’ rule determines room rates (O’Neill, 2003). Dholakia (2018) claims he has known

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1In the remainder of this paper, we will use the expression ’markup pricing’ as a synonym of ’cost-plus-markup-pricing.’

2See, for example, Altomonte et al. (2015); Greenslade and Parker (2012, 2010); Rao and Kartono (2009); Hall and Hitch (1939).
of companies that apply markups ranging from 5% to 800%. These examples underline that the applied markup varies widely in practice.\(^3\)

By contrast, the economic profession welcomed initial survey evidence about markup pricing with skepticism. In its defense, one may note that Hall and Hitch (1939), in their seminal contribution on the topic, went as far as to suggest that the use of heuristic pricing methods is incompatible with profit maximization and, thus, provides a refutation of Marginalism. This challenging claim led to a heated debate known as the 'Marginalist Controversy' over the period ranging from 1939 to 1952.\(^4\) In this tense context, survey evidence supporting the markup pricing hypothesis was often erroneously interpreted as questioning mainstream economics. However, subsequent research quickly pointed out that average cost pricing is consistent with profit maximization provided certain conditions are satisfied. For example, markup pricing is rational when firms pursue long-run profits. Indeed, in this case, the profit-maximizing price \(p\) is such that the profit margin \(m\) decreases with the elasticity of demand \(\epsilon\); i.e.,

\[
p = (1 + m(\epsilon)) \times \text{AC}, \quad m'(\epsilon) < 0.
\]

Thus, markup pricing and profit maximization lead to the same market price whenever firms base their expected profit margin \(m\) on an accurate estimation of the long-run price elasticity of demand \(\epsilon\).\(^5\)

Such arguments combined with changes in economic methodology helped calm the marginalist controversy. Specifically, Herbert Simon's (1955) 'Behavioral Model of Rational Choice' offered a rationale for the reliance on rules-of-thumb in complex decision environments, emphasizing the virtues of pursuing satisfactory rather than optimal outcomes. Also, Milton Friedman's (1953) 'Methodology of Positive Economics' prompted a methodological shift from the realism of assumptions to their predictive power. Once the marginalist controversy subsided, empirical evidence of average cost pricing continued to pile up without raising economists' eyebrows. Yet, with the notable exception of Grant and Quiggin (1994), new advances in the understanding of firm pricing practices have hardly changed the modeling of firms' behavior in oligopolistic markets.\(^6\)

Recent studies confirm the prevalence of markup pricing. Altomonte et al. (2015) provide survey evidence on pricing behavior from a representative sample of 14000 European firms. Among the respondents, 60% stated they have market power. Of these price-making firms, 75% revealed that they rely on markup pricing. Likewise, the Bank of England (Greenslade and Parker, 2010, 2012) surveyed a representative sample of 2331 firms on the considerations guiding their pricing decisions for the main product they sell in the UK. Business respondents reported that their pricing decisions are primarily driven by cost considerations: 25% stated they rely on constant markup prices and 33% on variable markup prices. The study of pricing behaviors in other parts of the world leads to similar lessons. In this regard, one notable study is that of Rao and Kartono (2009), who surveyed a sample of firms operating in the USA, Singapore, and India. They asked firms to select up to five pricing approaches from a list of 19 options. Again, cost-plus pricing was the most frequently selected option.

There are several possible explanations for the enduring popularity of this pricing approach. First, companies often view it as financially prudent (Phillips 2011; Nagle et al. 2014). In particular, risk-averse managers might choose a price that covers the average cost as protection against the risk of loss (Shy 1996). Secondly, companies think this pricing approach is fair. As already argued by Hall and Hitch (1939), firms perceive this price as "the 'right' price, the one which ought to be charged." They believe they owe this price to their consumers as well as their competitors. Further contributions in economics and management provide support for this idea as applied to consumers. The selling price must be deemed acceptable by consumers. As Simon (2015) notes, consumers have a cost-plus mentality: they assume that the selling price reflects production costs and, ultimately, value. This mindset imposes constraints on companies' pricing strategies. If the price deviates excessively from the costs of production, consumers will feel resentful and businesses will suffer a loss of reputation. Equity considerations then translates into implicit price contracts between companies and consumers. Okun (1981)\(^1\)

\(^3\)Some pharmaceutical companies that distribute generic drugs are probably making even higher margins than this upper bound suggests. See, for example, Hill et al. (2018).

\(^4\)For a history and discussion of this controversy, see Lee (1984), Mongin (1992), and Nubbemeyer (2010)

\(^5\)See, for example, Koutsoyiannis (1979) and Langlois (1993; 1989).

\(^6\)In macroeconomics, Rotemberg (2005; 2011) and Eyster et al. (2020) have, however, proposed more sophisticated models of firm pricing behavior. These models aim to explain price rigidity by taking into account consumers’ sensitivity to fair pricing. See Section 4.2.
With this purpose in mind, we consider a simple model of supply function competition in which firms hope with the number of firms – a market parameter – to determine the market outcome. Market parameters alter firms’ strategy spaces and are argued by Grossman (1981) and popularized by Klemperer and Meyer (1989). They represent a natural avenue for modeling competition among firms that commit to fixed price-quantity schedules. First, they may resort to conventions and traditions in setting the margin rate (as is the case for the restaurant and hotel industries). Second, they may use the price set by a market leader as a guide in their pricing decisions. Finally, firms may coordinate their pricing decisions by relying on producer cost surveys published by trade associations. Following Phillips (2011), we may note that:

“[t]he cost-plus pricing approach appears to be objective and defensible. If all competitors in a market have similar cost structures, it would appear to be a reasonable way to ensure consistency with the competition”. (Phillips, 2011)

Here, the consistency requirement is to be interpreted as relating to price moderation, coordination, and stability. Despite its popularity, the markup pricing rule is inherently flawed. The error of cost-based pricing decisions become apparent when comparing Equations (1) with (2). The price markup $m$ should be linked to market demand, but cost-based pricing decisions ignore consumers’ willingness to pay. If it is perfectly correct to imply that cost information has a role to play in pricing, cost information alone cannot be the basis of sound pricing. Indeed, the average cost of a good depends on the quantity that is produced. This point is made clear by Nagle et al. (2014):

“The mistake that cost-plus pricers make is not that they consider costs in their pricing, but that they select the quantities they will sell and the buyers they will serve before identifying the prices they can charge. They then try to impose cost based prices that may be either more or less than what buyers will pay. In contrast, effective pricers make their decisions in exactly the opposite order. They first evaluate what buyers can be convinced to pay and only then choose quantities to produce and markets to serve.” (Nagle et al., 2014)

Markup pricing is a ‘pure’ pricing approach like value-based pricing -- where businesses price in line with perceived consumer value -- or market-based pricing -- where businesses price in keeping with competitive offerings (Phillips, 2011). All of these approaches are inadequate because they fail to recognize the multidimensional aspect of pricing decisions. Corporate pricing strategies account for both costs and market, demand, and competition imperatives. In the remainder of this paper, we develop a model where all these imperatives contribute jointly to firms’ pricing decisions. Specifically, our purpose is to analyze strategic interaction among firms when firms have expectations regarding the fixed margin (relative to production cost) they should obtain on a given market and commit to explicit (or implicit) price contracts that constrain their pricing decisions.

With this purpose in mind, we consider a simple model of supply function competition in which firms hope to achieve a certain margin on the average cost of production. Models of supply function competition were introduced by Grossman (1981) and popularized by Klemperer and Meyer (1989). They represent a natural avenue for modeling competition among firms that commit to fixed price-quantity schedules. A supply function describes the number of units of a good that a firm agrees to provide to consumers for any eligible price. As argued by Menezes and Quiggin (2012), this concept allows the construction of more elaborate strategy spaces than those usually encountered in oligopoly theory. It makes it possible to analyze how changes in the values of market parameters alter firms’ strategy spaces and vice-versa. Menezes and Quiggin (2012) illustrate this point by examining the trade-offs that competition authorities face when measures designed to increase competition in the marketplace might push some competitors out of business. They find that changes in the number of market participants may (or may not) alter competition in the market depending on whether firms’ supply schedules are aggressive (or not). In other words, the slope of firms’ supply functions – a strategy space parameter – interacts with the number of firms – a market parameter – to determine the market outcome.

Models of competition in supply functions have proven to be particularly suitable for modeling competition in electricity markets. See, for example, Baldick et al. (2004) and Willems et al. (2009).
In this paper, we follow the path set by Menezes and Quiggin (2012). Accordingly, we assume that firms compete in supply functions and condition their supply upon their ability to achieve a given profit margin on average production cost. The wider the gap between the actual market price and that which would result from the markup pricing rule, the greater the quantity supplied. Inversely, when this gap narrows, supply contracts. This modeling approach makes it possible to analyze how firms’ (profit) margin objectives alter market competition. We find that the higher the profit margin expectations, the more collusive the market outcomes. By committing to our augmented supply function, firms are able to maintain market power even when the market is highly competitive. Furthermore, they earn higher profits than when relying on standard linear supply function. Finally, since each firm profit is monotonically increasing in the profit margin \( m \), producer surplus is also monotonically increasing in \( m \). Consequently, firms have a collective incentive to increase without bound the profit margin they aspire to achieve. However, this increase would cause the supply curve to rotate around its \( y \)-intercept, so that the supply curve would become horizontal and Cournot’s solution would be obtained. Finally, we find that collusion over a positive but small profit margin only occurs when consumers are antagonized by pricing practices they perceive as unfair.

The paper that most closely resembles ours is that of Grant and Quiggin (1994). The authors assume that firms adhere strictly to the average cost pricing rule so that condition (1) always binds, and firms set their profit margins strategically. By contrast, we assume that the desired profit margin is constant, and firms adjust strategically the amount they produce to try to achieve it. Consequently, in our model, Condition 1 does not bind; i.e., the selling price is allowed to diverge from the markup price, and firms adjust quantities accordingly. The structure of the paper is as follows. Section 2 introduces the model, which is solved in Section 3. Equilibrium quantities and the main comparative statics results are presented in Subsection 3.1 and 3.2. These are compared with the results obtained by Menezes and Quiggin (2012) in Subsection 3.3. Section 4 analyzes firms’ incentives to agree on an industry price markup standard and explains how consumers’ preferences for fair pricing practices may limit applicable markup rates. Section 5 concludes the paper.

2 The model

Our purpose is to build a simple model of supply function competition that explicitly captures firms’ profit margin aspirations. We will work under the following customary assumptions. On the supply side, there are \( n \geq 2 \) identical firms (indexed by \( i = 1, 2, \ldots, n \)) producing a single commodity. The quantity supplied by firm \( i \) is \( q_i \) and the overall quantity supplied is \( Q = \sum_{i=1}^{n} q_i \). All firms share the same technology summarized by the cost function \( C(q_i) = cq_i + \frac{\gamma}{2}q_i^2 \), with \( c, v \geq 0 \). On the demand side, there is a representative consumer with a utility function given by \( U(Q, \xi) = aQ - \frac{b}{2}Q^2 + \xi \), where \( a \) and \( b \) are positive parameters, and \( \xi \) is a numeraire good. Accordingly, the inverse demand function is linear and given by \( p(Q) = a - bQ \). Firms’ profits are therefore given by \( \pi_i := p(Q)q_i - cq_i - \frac{\gamma}{2}q_i^2, i = 1, 2, \ldots, n \).

Now that we have specified the market structure, we move on to defining firms’ strategies. We assume that firms compete in supply functions. Grossman (1981), and Hart (1985), proposed models of supply function competition as alternatives to models of either price or quantity competition. In their approach, a supply function \( S_i(.) \) represents a commitment by firm \( i \) to choose price-quantity pairs \((p_i, q_i)\) satisfying \( q_i = S_i(p_i) \). Each firm selects its supply function \( S_i(.) \) to maximize its profit, taking as given competitors (equilibrium) supply functions. The set of Nash equilibria in supply functions then determines possible market outcomes.\(^8\)

The difficulty with this modeling approach is well known. If firms are allowed to select arbitrary supply functions, a profusion of Nash equilibria exists so that models of supply functions fail to predict the market outcome. Different avenues have been explored to address this multiplicity issue. Adding demand uncertainty (Klemperer and Meyer, 1989) or incomplete information (Vives, 2011) has been shown to limit the number of Nash equilibria. In this paper, to ensure the existence of a unique solution, we assume that firms choose among linear supply functions as Menezes and Quiggin (2012) and Delbono and Lambertini (2015) do. Namely, we restrict our attention to linear supply functions of the form:\(^9\)

\[ S_i(p) = a_i + \beta_i p, \tag{3} \]

---

\(^8\)For an introduction to models of competition in supply functions, see Vives (1999, chapter 7).

\(^9\)Delbono and Lambertini (2015) actually consider the slightly more general form
\[ S_i(p) = \alpha_i + \beta p, \]  

where \( \beta \) is a parameter describing how intense competition is and \( \alpha_i \) is the strategic variable which allows firm \( i \) to shift supply upward or downward. This assumption ensures the existence of a unique Nash equilibrium in supply schedules.

We will now specify further the form that firms’ supply functions take in our model. We assume that firms use
\[ \phi \]
where the intercept \( \beta \) is a parameter describing how intense competition is and \( \alpha_i \) is the strategic variable which allows firm \( i \) to shift supply upward or downward. This assumption ensures the existence of a unique Nash equilibrium in supply schedules.

We will now specify further the form that firms’ supply functions take in our model. We assume that firms use the markup price as a reference when making their supply decisions. More precisely, each firm conditions its supply to the difference between the market price and the price that would result from applying the markup pricing rule. The wider the divergence is, the greater the quantity supplied. Inversely, when this gap narrows, supply contracts. This supply behavior is captured by assuming that
\[ q_i = \phi_i + \psi (p - (1 + m)AC(q_i)), \]  

where the intercept \( \phi_i \) is not restricted in sign, the slope \( \psi \) and the gross margin \( m \) are non-negative, and \( AC(q_i) = C(q_i)/q_i \) denotes the average cost of production.

Note that the above behavioral assumption addresses the shortcomings of the standard average cost pricing routine. Firms do not behave on the false assumption that either i.) they can base their prices exclusively on costs considerations and ii.) they know their production costs before knowing the actual demand. In our model, firms set quantities, and the interaction of supply and demand determines the market price. Therefore, the market price depends on all the market parameters; the most relevant for our current purpose being the expected profit margin \( m \) and the degree of market competition \( \beta \).

Let us now reformulate Equation (6) in the general form \( q_i = S_i(p) \), with \( S_i(p) \) given by Equation (5). By substituting the average cost expression, we obtain the following equation:
\[ q_i = \phi_i + \psi \left( p - (1 + m) \left( c + \frac{vq_i}{2} \right) \right), \quad i = 1, 2, \ldots, n. \]  

Solving for \( q_i \) yields
\[ q_i := S_i(p) = \frac{2\phi_i - 2\psi c(m + 1)}{\psi(m + 1) + 2} + \frac{2\psi}{\psi(m + 1) + 2} p, \quad i = 1, 2, \ldots, n, \]  

which we rewrite as
\[ S_i(p) = \alpha_i(\phi_i, \psi; m, v) + \beta(\psi; m, v) p, \]  

by using the following change of variables:
\[ \beta(\psi; m, v) = \frac{2\psi}{\psi(m + 1)v + 2}, \quad \alpha_i(\phi_i, \psi; m, v) = \frac{2(\phi_i - \psi(m + 1)c)}{\psi(m + 1)v + 2}, \quad i = 1, 2, \ldots, n. \]  

Note that firms’ supply behaviors becomes more aggressive when the value of \( \beta(\psi; m, v) \) increases because, for the same market equilibrium price, their supply is higher as their supply function is steeper. Conversely, a decrease in \( \beta(\psi; m, v) \) leads to less aggressive market behaviors. It will prove helpful to understand how a change in the values of \( \psi \) and \( m \) affects the supply behavior of firms to interpret the model and its results. With this in mind note that

\[ S_i(p) = \beta_i p, \]  

10Replacing the average cost with the marginal cost in Equation (6) would not change the conclusion of the analysis. The parameter that plays the most significant role in our analysis is \( m \).

11As we will see below, firm \( i \)'s supply is always non-negative at the equilibrium, whatever the sign of the intercept \( \phi_i \). Hence, it is not necessary to introduce explicit positivity constraints on \( q_i \) and/or \( \phi_i \).
We solve the market game for a Nash equilibrium in linear supply functions, 
\[ S_i (\psi; m, v) = \alpha_i (\phi_i, \psi; 0, 0) + \beta (\psi; 0, 0) p, \]
\[ = \phi_i + \psi (p - c), \]

From now on we omit the arguments in the functions \( \alpha_i \) and \( \beta \) whenever this does not lead to any confusion. These arguments will only be specified when it is necessary to distinguish the solution of our general model from the particular case treated by Menezes and Quiggin (2012).

We assume that each Firm \( i \) sets strategically the intercept \( \alpha_i \) of its supply function. Consequently, we now express price and quantities as a function of the strategy chosen by Firm \( i \), \( \alpha_i \), and the profile of strategies chosen by all other firms \( \alpha_{-i} = (\alpha_1, \alpha_2, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n) \). Suppose that firm \( i \)'s supply function is given by Equation (5), \( i = 1, 2, \ldots, n \). Then, the overall industry’s supply function is

\[ \sum_{i=1}^{n} S_i (p) = \sum_{i=1}^{n} \alpha_i + n \beta p. \]

By replacing total output with overall industry supply in the inverse demand function, we obtain the equation
\[ p (\alpha_i, \alpha_{-i}) = \frac{a - b (\sum_{i=1}^{n} \alpha_i + n \beta p)}{1 + nb \beta}, \]

with partial derivative given by
\[ \frac{\partial p (\alpha_i, \alpha_{-i})}{\partial \alpha_i} = \frac{-b}{1 + nb \beta}. \]

Expressed in terms of supply functions, the profit of firm \( i \) can then be written as follows
\[ \pi_i (\alpha_i, \alpha_{-i}) = \left( p (\alpha_i, \alpha_{-i}) - c - \frac{\psi}{2} S_i (p) \right) S_i (p), \]
\[ = \frac{1}{2} (\alpha_i + \beta p (\alpha_i, \alpha_{-i})) \{ \alpha_i + \beta p (\alpha_i, \alpha_{-i}) - c \} - (\alpha_i + \beta p (\alpha_i, \alpha_{-i})) \}, \quad i = 1, 2, \ldots, n. \]

Having completed the model, we can turn to the analysis of firms’ equilibrium supply behavior.

### 3 Characterization of equilibrium supply functions

We solve the market game for a Nash equilibrium in linear supply functions, \( S_i (p) = \alpha_i + \beta p \). In doing so, recall that we assume that firms strategically choose the vertical intercept of their supply schedules. Selecting

\[ \lim_{\psi \to 0} \beta (\psi; m, v) = 0, \quad \frac{\partial \beta (\psi; m, v)}{\partial \psi} = \frac{4}{(2 + \psi (1 + m) v)^2} > 0, \quad \lim_{\psi \to -\infty} \beta (\psi; m, v) = \frac{2}{v (1 + m)}, \quad (11) \]

\[ \lim_{m \to 0} \beta (\psi; m, v) = \frac{2 \psi}{2 + v \psi}, \quad \frac{\partial \beta (\psi; m, v)}{\partial m} = -\frac{2 \psi^2}{(2 + \psi (1 + m) v)^2} < 0, \quad \lim_{m \to +\infty} \beta (\psi; m, v) = 0. \quad (12) \]

Firms’ supply functions are horizontal whenever \( \psi \) is zero, or the profit margin target \( m \) is arbitrarily large. In this case, the market will be uncompetitive, and Cournot’s solution will prevail. When \( \psi \) increases, or alternatively when \( m \) decreases, the supply behavior of firms becomes more aggressive. However, unless the marginal cost of production is constant (i.e., \( v = 0 \)), the slope of supply functions is finite so that they may be steep but never vertical. Finally, we have the following remark, which will be helpful for comparison purposes.

**Remark 1.** By assuming a constant marginal cost of production and no profit margin, we arrive at the same supply behavior as that posited by Menezes and Quiggin (2012). Indeed, if \( m = v = 0 \) then Equation (10) reduces to

\[ q_i := S_i (p) = \alpha_i (\phi_i, \psi; 0, 0) + \beta (\psi; 0, 0) p, \]
\[ = \phi_i + \psi (p - c), \quad (13) \]

\[ (14) \]

\[ \frac{\partial \beta (\psi; m, v)}{\partial \psi} = \frac{4}{(2 + \psi (1 + m) v)^2} > 0, \quad \lim_{\psi \to -\infty} \beta (\psi; m, v) = \frac{2}{v (1 + m)}, \quad (11) \]

\[ \lim_{\psi \to +\infty} \beta (\psi; m, v) = 0. \quad (12) \]

Actually, Menezes and Quiggin (2012) normalize the y-intercept. As noted by Delbono and Lambertini (2015) this normalization is useless.
a supply function \( S_i(.) \) for a firm \( i \) then reduces to setting the value of \( \alpha_i \). Formally, a profile of strategies \((\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*)\) is an equilibrium in supply functions if the following condition holds

\[
\alpha_i^* \in \arg\max_{\alpha_i} \pi_i \left( \alpha_i, \alpha_i^* \right),
\]  

(20)

for all \( i = 1, 2, \ldots, n \). Assuming an interior solution, the first-order conditions for profit maximization write as

\[
\frac{\partial \pi_i}{\partial \alpha_i} = 0, \quad \left( \frac{\partial p(\alpha_i^*, \alpha_i^*)}{\partial \alpha_i} + 1 \right) \left( p(\alpha_i^*, \alpha_i^*) (1 - \nu \beta) - \nu \alpha_i^* - c \right) + (\alpha_i^* + \beta p(\alpha_i^*, \alpha_i^*)) \frac{\partial p(\alpha_i^*, \alpha_i^*)}{\partial \alpha_i} = 0,
\]  

(21)

\( i = 1, 2, \ldots, n \). Turning to the second order optimality conditions, we have

\[
\frac{\partial^2 \pi_i}{\partial \alpha_i^2} = -((b \beta(n - 1) + 1)(b \beta(n - 1) \nu + \nu + 2b)) \left( \frac{\partial p(\alpha_i, \alpha_i^*)}{\partial \alpha_i} \right)^2 < 0, \quad i = 1, 2, \ldots, n,
\]  

(22)

where we used the fact that \( \partial^2 p/\partial \alpha_i^2 = 0 \). Hence, each firm \( i \)'s profit function is strictly concave in its strategic variable \( \alpha_i \) and the existence of a unique symmetric Nash equilibrium is ensured. This equilibrium is obtained by solving the system of first-order optimality conditions (21). We obtain the following proposition.

**Proposition 1.** There exists a unique Nash equilibrium in supply schedules. Equilibrium supply functions are identical and given by \( S_i^*(p) = \alpha^* + \beta p \), with

\[
\alpha^* = \frac{(a - c) - b \beta^2 (n - 1) (a v + b c n) + \beta (b (a (n - 2) - 2 c n + c) - a v)}{(b \beta (n - 1) (b n + v) + b n + b + v)},
\]  

(23)

and \( \beta := \beta(\psi; m, v) = \frac{2 \phi}{\phi(m+1)v+2}, \) for all \( i = 1, 2, \ldots, n \).

**Proof.** See Appendix A. \( \square \)

Now, by successively replacing \( \alpha_i^* = \alpha^* \) in Equations (16), (9), (15) and (18), we find the following equilibrium price-quantity pairs and profits:

\[
q^* = \frac{(a - c) (b (n - 1) \beta + 1)}{b (n - 1) \beta (b n + v) + b n + b + v} > 0, \quad p^* = \frac{b (n - 1) \beta (a v + b c n) + a (b + v) + b c n}{b (n - 1) \beta (b n + v) + b n + b + v} > 0,
\]  

(24)

\[
Q^* = \frac{n (a - c) (b (n - 1) \beta + 1)}{b (n - 1) \beta (b n + v) + b n + b + v} > 0, \quad \pi^* = \frac{(a - c)^2 (b (n - 1) \beta + 1) (b (n - 1) \nu \beta + 2b + v)}{2 (b (n - 1) \beta (b n + v) + b n + b + v)^2} > 0.
\]  

(25)

In subsection (3.1), we use Equation (10) to rewrite the above solution in terms of the initial parameters of the model \((\phi, \psi)\). Then, in subsection (3.2), we consider the two limit cases where supply functions are either horizontal \((\psi = 0)\) or relatively steep \((\psi \rightarrow +\infty)\). Finally, in subsection (3.3) we compare it to the solution obtained by Menezes and Quiggin (2012) which corresponds to the special case of our model for which \( m = v = 0 \). In the remainder of this paper, the superscript \( \star \), \( \ell \) and \( c \) are used to denote the general solution, the solution of Menezes and Quiggin (2012) and the Cournot solution.

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13From Equation (10), note that \( \beta(\psi; m, v) \) is monotonically increasing in \( \psi \) and has limit \( \beta(+\infty; m, v) = \frac{2}{\nu (m+1)} \). Consequently, the slope of firms' supply functions steepens as \( \psi \) increases. However, it remains finite unless the marginal cost of production is assumed to be constant (in which case firms' supply functions would be vertical).
3.1 Supply function equilibrium when firms have a price-markup target

Assume that firms’ supply behavior is given by Equation (6). Then, their respective equilibrium supply functions are given by \( S^*(p) = \alpha^* + \beta p \) and market equilibrium quantities are given by:

\[
q^* = \frac{(a - c)(2b\psi(n - 1) + \psi(m + 1)v + 2)}{2\psi b^2(n - 1)n + \psi b\psi((m + 3)n + m - 1) + 2b(n + 1) + v(\psi(m + 1)v + 2)},
\]

\[
p^* = \frac{ad\psi(b(m + 2n - 1) + mv + v) + 2a(b + v) + bcn(2b\psi(n - 1) + \psi(m + 1)v + 2)}{2\psi b^2(n - 1)n + \psi b\psi((m + 3)n + m - 1) + 2b(n + 1) + v(\psi(m + 1)v + 2)},
\]

\[
\pi^* = \frac{(a - c)^2(2b\psi(n - 1) + \psi(m + 1)v + 2)(\psi b(m + n) + mv + v + 4b + 2v)}{2(2\psi b^2(n - 1)n + \psi b\psi((m + 3)n + m - 1) + 2b(n + 1) + v(\psi(m + 1)v + 2))^2}.
\]

We obtain the following comparative statics results:

\[
\frac{\partial q^*}{\partial \psi} \geq 0, \quad \frac{\partial Q^*}{\partial \psi} \geq 0, \quad \frac{\partial p^*}{\partial \psi} \leq 0, \quad \frac{\partial \pi^*}{\partial \psi} \leq 0.
\]

Note that the above inequalities hold with equality whenever \( \psi = 0, v = 0 \) or \( m \) becomes arbitrarily large. An increase in the margin rate results in a reduction in sales volume. However, this quantity effect is more than offset by the increase in the selling price, so that the equilibrium profit increases. It is interesting to compare these comparative statics results with those which result from a change in the value of \( \psi \). Recall that the parameter \( \psi \) measures the degree of competitiveness of firms on the market. An increase in the degree of competitiveness of firms naturally leads to an increase in sales volumes and, in turn, a decrease in price. This time, however, the price effect outweighs the quantity effect, so that firms profits falls. Finally, it should be noted that comparative statics results in (29) are diametrically opposed to those in (30). In the face of aggressive competition \( \psi > 0 \), firms may increase their profits by increasing their benchmark margin rate. Hence, committing to pursue average-cost-plus pricing is a remedy for aggressive competition. Finally, one finds that individual sales volumes, market price and firms profits decrease monotonically with the number of competitors:

\[
\frac{\partial q^*}{\partial n} < 0, \quad \frac{\partial Q^*}{\partial n} < 0, \quad \frac{\partial p^*}{\partial n} < 0, \quad \frac{\partial \pi^*}{\partial n} < 0.
\]

Let us now consider the two limit cases of the model, which correspond respectively to a relatively uncompetitive and comparatively competitive market.

3.2 Limit cases: horizontal and steep supply functions

To begin with, let us consider the least competitive market situation. Recall that firms’ supply behavior coincides with Cournot’s behavior when \( \beta(\psi; m, v) = 0 \); i.e., when \( \psi \) is negligible or \( m \) becomes arbitrarily large. Hence, we have:

\[
\lim_{m \to +\infty} \pi^* = \pi^c = \lim_{\psi \to 0} \pi^*, \quad \lim_{m \to +\infty} p^* = p^c = \lim_{\psi \to 0} p^*, \quad \lim_{m \to +\infty} q^* = q^c = \lim_{\psi \to 0} q^*.
\]

We note that the Cournot equilibrium can be obtained by replacing \( \beta(\psi; m, v) = 0 \) in Equations (24–25). We obtain:

\[
q^c = \frac{a - c}{v + b(n + 1)}, \quad Q^c = \frac{n(a - c)}{v + b(n + 1)}, \quad p^c = \frac{a(b + v) + bcn}{v + b(n + 1)}, \quad \pi^c = \frac{(a - c)^2(2b + v)}{2(v + b(n + 1))^2}.
\]
Now we consider the opposite case where the market is relatively competitive. Firms behavior is most aggressive when $\psi$ becomes arbitrarily large. For this limiting case, the equilibrium price/quantity pair is given by:

$$\lim_{\psi \rightarrow +\infty} q^* = \frac{(a - c)(2b(n - 1) + (m + 1)v)}{2b^2(n - 1)n + bv((m + 3)n + m - 1) + (m + 1)v^2} > 0,$$  \hspace{1cm} (34)

$$\lim_{\psi \rightarrow +\infty} p^* = \frac{bv(a(m + 2n - 1) + c(m + 1)n + a(m + 1)v^2 + 2b^2c(n - 1)n)}{2b^2(n - 1)n + bv((m + 3)n + m - 1) + (m + 1)v^2} > 0.$$  \hspace{1cm} (35)

The price-markup over marginal cost is defined by

$$p^* - C'(q^*) = p^* - (c + \psi q^*) = \frac{b(a - c)}{b(n - 1)\beta(\psi; m, v)(bn + v) + bn + b + v},$$  \hspace{1cm} (36)

and

$$\lim_{\psi \rightarrow +\infty} (p^* - C'(q^*)) = \frac{b(m + 1)v(a - c)}{2b^2(n - 1)n + bv((m + 3)n + m - 1) + (m + 1)v^2} \geq 0.$$  \hspace{1cm} (37)

It is equal to zero if, and only if, $v = 0$; in which case,

$$\lim_{\psi \rightarrow +\infty} (p^* - C'(q^*)) |_{v=0} := \lim_{\psi \rightarrow +\infty} (p^* - c) = 0.$$  \hspace{1cm} (38)

and the Bertrand solution $p^* = c$ obtains. Finally, we find

$$\lim_{\psi \rightarrow +\infty} \pi^* = \frac{v(a - c)^2(2b(m + n) + mv + v)(2b(n - 1) + (m + 1)v)}{2(2b^2(n - 1)n + bv((m + 3)n + m - 1) + (m + 1)v^2)} \geq 0.$$  \hspace{1cm} (39)

Unless the marginal cost of production is constant (as in Menezes and Quiggin, 2012), the equilibrium price margin, and therefore profits, are strictly positive, whatever the degree of aggressiveness of firms on the market. We have the following proposition:

**Proposition 2.** Unless firms operate a constant marginal cost technology, the equilibrium price-markup $(p^* - C'(q^*))$ remains strictly positive as $\psi \rightarrow +\infty$.\(^4\)

Now that we have characterized the solution of our model, two questions arise. First, is it desirable for firms to adopt a profit margin target? Second, would this adoption harm social welfare? We consider these two questions successively below.

### 3.3 The desirability of a margin target

To know if setting a profit margin target is beneficial for firms, it is first necessary to calculate the equilibrium prices, quantities and profits in the absence of such a goal. This is equivalent to determining the equilibrium values of the standard model of competition in linear supply functions with quadratic production costs. In line with this, we replace Equations $\beta(\psi; m, v)$ in Equations (24) and (25) by

$$\beta|_{(\psi;m,v)=(\psi,0,0)} := \beta(\psi; 0, 0) = \psi.$$  \hspace{1cm} (40)

So doing, we obtain the following equilibrium values:

$$q^f = \frac{(a - c)(b(n - 1)\psi + 1)}{b(n - 1)\psi(bn + v) + bn + b + v} > 0, \quad p^f = \frac{b(n - 1)\psi(avn + bcn) + a(b + v) + bcn}{b(n - 1)\psi(bn + v) + bn + b + v} > 0,$$  \hspace{1cm} (41)

$$Q^f = \frac{n(a - c)(b(n - 1)\psi + 1)}{b(n - 1)\psi(bn + v) + bn + b + v} > 0, \quad \pi^f = \frac{(a - c)^2(b(n - 1)\psi + 1)(b(n - 1)\psi + 2b + v)}{2(b(n - 1)\psi(bn + v) + bn + b + v)} > 0,$$  \hspace{1cm} (42)

for all $i = 1, 2, \ldots, n$.

\(^4\)Obviously then, we also have $\lim_{\psi \rightarrow +\infty} (p^* - AC(q^*)) > 0$. 


In order to obtain more compact expressions, we define a function $\Phi(x) = b \left( (n - 1)(bn + v)x + (n + 1) \right) + v$. Also, the comparison of equilibrium quantities is facilitated if it is observed that:

$$\beta (\psi; 0, 0) - \beta (\psi; m, v) = \psi^2 (m + 1)v/(\psi (m + 1)v + 2) > 0.$$  \hspace{1cm} (43)

Building on this observation, we obtain:

$$p^\ell - p^* = -\frac{b^3(n-1)\left[\beta (\psi; 0, 0) - \beta (\psi; m, v)\right](a-c)}{\Phi (\beta(\psi,0)) - \Phi (\beta(\psi,m,v))} < 0, \quad q^\ell - q^* = \frac{b^3(n-1)(\beta(\psi,0,0) - \beta(\psi,m,v))(a-c)}{\Phi (\beta(\psi,0)) - \Phi (\beta(\psi,m,v))} > 0,$$

and

$$\frac{2(\pi^\ell - \pi^*)}{(a-c)^2} = -\frac{1}{\Phi (\beta(\psi,0)) - \Phi (\beta(\psi,m,v))} \left\{ b^3 (n - 1)^2 (\beta (\psi; 0, 0) - \beta (\psi; m, v)) (b\beta (\psi; m, v) (2n(bn + v) - v) + b\beta (\psi; 0, 0) (2b(n - 1)n\beta (\psi; m, v) (bn + v) + 2n(bn + v) - v) + 2(bn + b + v)) \right\} \leq 0.$$  \hspace{1cm} (45)

From Equation (44), note that each firm restricts its equilibrium output level when it targets a predetermined profit margin goal. The resulting reduction in total industry supply, in turn, leads to an increase in the market-clearing price. Equation (45) shows that this price increase more than compensates for the decrease in sales so that firms’ profits increase. We have the following proposition:

**Proposition 3.** If $\psi > 0$ then $\pi^* > \pi^\ell$. Firms achieve higher profits by conditioning their offer on a given margin target and market price rather than allowing their offer to vary only with the market price.

We conclude this section with a comparative study of levels of social well-being.

### 3.4 Price-markup targets and social welfare

Social-welfare is defined as the sum of consumer and producer surplus. Formally, we have:

$$SW := CS + PS = \frac{b}{2} Q^2 + \sum_{i=1}^{n} \pi_i,$$

$$= nq(\alpha - c) - \frac{1}{2} nq^2 (bn + v).$$  \hspace{1cm} (46)  \hspace{1cm} (47)

Let us now evaluate social-welfare at the supply function equilibrium. By plugging $q^*$ into Equation (47), we obtain

$$SW^* = \frac{n(a - c)^2 (b(n - 1)\beta + 1) (b(n - 1)\beta (bn + v) + b(n + 2) + v)}{2 (b(n - 1)\beta (bn + v) + bn + b + v)^2}. $$  \hspace{1cm} (48)

To compare social welfare levels, it is useful to note that

$$\beta (\psi; 0, v) \geq \beta (\psi; m, v) \geq \beta (0; m, v) := 0.$$  \hspace{1cm} (49)

The slope of standard linear supply functions is higher than the slope of markup-pricing supply functions, which is itself higher than the slope of Cournot supply functions (i.e. zero). In other words, competition is less intense when firms engage in supply function competition and have a margin target rather than no margin target. However, the intensity of competition will always be higher than in the case of a Cournot oligopoly.

By differentiating social welfare with respect to $\beta$, we obtain:

$$\frac{\partial SW^*}{\partial \beta} = \frac{b^3(n-1)n(a - c)^2}{(b(n - 1)\beta (bn + v) + bn + b + v)^2} > 0,$$

where $\beta := \beta (\psi; m, v) = \frac{2\psi}{\psi (m+1)v^2}$. Hence, social welfare increases as firms’ supply behavior becomes more aggressive. Given the ranking in Equation (49), we can state the following proposition:

**Proposition 4.** Social welfare levels rank as follows: $SW^\ell \geq SW^* \geq SW^c$.

Social welfare is always higher when firms engage in supply function competition than when they compete à la Cournot. However, social welfare will be maximum only if no margin target appears in firms’ supply functions.
4 Collusive price markup

So far, we have assumed a fixed and given price markup. This assumption may be justifiable for fragmented industries where considerations of profitability and equity have long crystallized into a conventional price markup around which market participants agree. However, this is less plausible for a concentrated industry where collusion opportunities arise since a limited number of firms supply the market. Consequently, we now investigate the possibility for firms to collude by setting the markup \( m \) strategically. In doing so, we consider two scenarios that differ in how consumers respond to an increase in the price markup. In the first, this increase leaves consumers indifferent, while in the second, it infuriates them so much that they reduce their demand.

4.1 Markup-insensitive Consumers

Consumers have a cost-plus mentality (Simon, 2015): they expect product prices to reflect production costs while agreeing that it is legitimate for business firms to make money. Most of the time, firms’ pricing practices agree with this mindset. For one thing, they imply a relatively small price markup that consumers welcome with leniency or a larger one that is sufficiently discreet to go unnoticed. For another, consumers may be uncertain as to what markup level is legitimate and thus acceptable. Then, as long as the price does not seem extravagant, they may forgive and forget quite noticeable price markups. The example of restaurants where hefty prices are charged for wine is well documented.\(^\text{15}\) Consumers may perceive those prices as necessary for restaurants to break even. High-tech companies enjoy a similar tolerance from consumers who recognize that the production of cutting-edge products deserves an extra reward.\(^\text{16}\) We start with the analysis of these situations where consumers are relatively insensitive to price markups.

Suppose that consumer demand is independent of the price markup \( m \). Then, the comparative statics results presented in the previous section hold. Unless firms compete à la Cournot \((\psi = 0)\) or operate a constant return to scale technology \((v = 0)\), from Equation (29), notice that \( \partial \pi_i^\ast / \partial m > 0 \) for \( m \) finite and \( \lim_{m \to +\infty} \partial \pi_i^\ast / \partial m = 0 \) for all \( i = 1, 2, \ldots, n \). Observe that the industry has incentives to increase \( m \) without bound. However, from Equation (10) note that the slope of the supply function is decreasing in \( m \) and eventually reaches zero as \( m \) becomes arbitrarily large – in which case the supply-function equilibrium replicates the standard Cournot equilibrium. We are now in a position to establish the following proposition.

**Proposition 5.** Suppose that firms are able to collude by setting strategically a common price-markup \( m \). If consumer demand is insensitive to \( m \), then price-markup expectations become so large that the Cournot solution obtains.

Hence, supply function competition in concentrated industries will likely lead to Cournot competition. This finding supports the thesis that markup pricing is popular because it favors collusion.\(^\text{17}\)

4.2 Markup-sensitive consumers

While consumers understand that businesses need to be profitable, they also demand to be treated fairly. Perceptions of unfair pricing practices can upset consumers, harm business goodwill, and reduce profits. In the past, one way for consumers to express their dissatisfaction was to participate in sporadic local protests and boycotts (Friedman, 1985). One early and well-documented example is the 1902 New York City kosher meat boycott which saw immigrant Jewish women protesting against the 'Meat Trust' decision to increase kosher meat price from 14 to 18 cents a pound.\(^\text{18}\) Instead, today when consumers feel wronged, they pour their anger

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15See, for example, Amspacher and Amspacher, 2011.
16In this regard, the example of Apple’s iPhone pricing is exemplary. According to Tilley (2018), gross profit margins for Apple smartphones are estimated at over 60%, well above industry standards. So although Apple’s sales accounted for less than 20% of all smartphone sales in 2017, they helped it capture 87% of the overall market profit.
17See the quotation of Phillips (2011) given in the introduction. Simon (2015) also notes that this pricing method

“[…] minimizes price competition for the product and encourages the sellers to compete on aspects other than price. Cost-plus pricing can create a de facto price cartel, with the resulting stability and predictability. All these factors explain why this method is so popular.”

18For a historical account, see Hyman, 1980.
out on social media, giving it a global audience. Even giant tech companies such as Apple Inc had to apologize to appease upset consumers. For example, the 2007 decision to cut the price of the iPhone by $200 just two months after it hit the market created such a backlash that the company had to offer early adopters a $100 credit to use on the Apple Store.

Consumers’ assertiveness is not always sufficient to mitigate price unfairness, especially when the price elasticity of demand is low. The pricing of pharmaceuticals demonstrates this. The pharmaceutical industry defends patented drugs prices increases by arguing they are justified by growing R&D costs or higher value for the patient. However, these arguments fail to explain why generic drugs have seen their price increase steadily over the last two decades (Furlow, 2015). For example, Hill et al. (2018) estimate that some generic drugs featured in the WHO essential drugs list sell in the UK at more than 100 times their production cost.

As previously noted, Hall and Hitch (1939) believed that the popularity of average cost pricing stems from its perception as a fair pricing practice to both consumers and firms. Also, Okun (1981) proposed to explain the relative stickiness of prices when demand suddenly increases by implicit price contracts among consumers and firms. Khaneman et al. (1986) summarized these ideas in a ‘dual entitlement principle’ according to which both firms and consumers are entitled to fair terms (i.e., price or profit) of trade. The introduction of consumer preference for fair prices in theoretical modeling is more recent. Rotemberg (2005; 2011) assumes that consumers expect some benevolence from firms and are upset whenever this expectation is not met. Since firms internalize such reactions into their decision-making process, they moderate their pricing practices. Rotemberg (2005) shows that firms will pass cost increases only partially onto their consumers. Also, Rotemberg (2011) highlights they will follow third-degree price discrimination based on income rather than demand elasticity. Finally, Eyster et al. (2020) embed consumer preference for a fair price in a New Keynesian model. The authors assume that consumers are upset by prices they regard as unfair. However, they cannot observe firms’ marginal costs of production. So the only option left for them is to infer these costs from prices. When doing so, they are assumed to overestimate the role price markups play in observed price increases. Eyster et al. (2020) shows that firms will cut back on their margins to moderate price increases.

For the limited purpose of this paper, we will assume here that consumers’ willingness to pay for the product is decreasing in the price-markup $m$. This assumption will suffice to show that the industry may choose a positive but small markup value.

Suppose that consumers are antagonized by an increase in the gross margin rate. Then, firm i’s profit may be written as

$$\pi_i^* = (a(m) - c)^2 \Gamma(\beta)$$

where

$$\Gamma(\beta) = \frac{(b(n - 1)\beta + 1)(b(n - 1)v\beta + 2b + v)}{2(b(n - 1)\beta(bn + v) + bn + b + v)^2} > 0,$$

and with

$$a'(m) < 0.$$  

We have

$$\frac{d\pi_i^*}{dm} = 2a'(m)(a(m) - c)\Gamma(\beta) + (a(m) - c)^2 \Gamma'(\beta)\beta'(m).$$

where

$$\Gamma'(\beta) = -\frac{b^2(n - 1)^3(b\beta n + 1)}{(\beta b(n - 1)(bn + v) + bn + b + v)^3} < 0,$$

$$\beta'(m) = -\frac{2v\psi^2}{((m + 1)v\psi + 2)^2} < 0.$$  

There are two cases in which it is straightforward to determine whether a change in the gross profit margin $m$ leads to higher profits or not. The first one is already familiar. As shown above, if $a'(m) = 0$, the first term on the right-hand side of Equation (53) vanishes so that firms’ equilibrium profit are monotonically increasing in $m$. 

12
The second one is more interesting. Observe that $\Gamma$ is independent of $m$ whenever $\beta$ is independent of $m$. This is the case if firms operate a constant return of scale technology ($v = 0$), or if the market is little competitive ($\psi = 0$). In both cases, an increase in the gross margin rate $m$ unambiguously leads to reduced profits. Indeed, Equation (53) reduces to:

$$\frac{\partial \pi_i^*}{\partial m} = 2a'(m) (a(m) - c) \Gamma'(\beta) < 0.$$  \hspace{1cm} (56)

We have the following two propositions:

**Proposition 6** (Collusive choice of profit margin with constant marginal costs). Suppose that the gross price margin antagonizes consumers and the technology of production is characterized by a constant marginal cost. If firms have the possibility to collude by choosing the industry gross margin strategically then they prefer to abandon any margin aspiration; i.e., they set $m^* = 0$.

**Proposition 7** (Collusive choice of profit margin with flat supply functions). Suppose that the gross price margin antagonizes consumers and supply functions are horizontal ($\psi = 0$) implying that firm supply is not aggressive. If firms have the possibility to collude by choosing the industry gross margin strategically then they prefer to abandon any margin aspiration; i.e., they set $m^* = 0$.

Except for these two special cases, it is impossible to sign Expression (53) without specifying further $a(m)$. Here, to avoid multiplicity, we assume that consumer demand decreases linearly with the industry gross margin rate $m$; i.e.,

$$a(m) = \bar{a} - \rho m, \text{ with } \rho \geq 0.$$ \hspace{1cm} (57)

The industry would never choose a gross margin rate $m$ causing so much resentment from consumers that they give up consuming its product. Plugging $a = \bar{a} - \rho m$ into Equation (26), we find that demand becomes zero if

$$a(m) - c = \bar{a} - \rho m - c < 0,$$ \hspace{1cm} (58)

or, equivalently, $m > (\bar{a} - c) / \rho := \bar{m}$. Accordingly, the industry never chooses a markup higher than $\bar{m}$.

We now show that the optimal margin rate for the industry can be positive or zero depending on the number of firms operating in the industry. Plugging $a = \bar{a} - \rho m$ into Equation (25), Firm $i$’s equilibrium profit becomes

$$\pi_i^* = \frac{(\bar{a} - \rho m - c)^2 (\beta(\psi;m,v) b(n-1) + 1) (\beta(\psi;m,v) b(n-1) v + 2b + v)}{2 (\beta(\psi;m,v) b(n-1) (bn+v) + bn + b + v)^2} > 0, \hspace{0.5cm} i = 1, 2, \ldots, n.$$ \hspace{1cm} (59)

For convenience, we define

$$\mu := \frac{2\rho}{2\rho + v (a - c + \rho) \psi}.$$ \hspace{1cm} (60)

In Appendix A.3, we prove that marginal profit is monotonically decreasing in $m$ if

$$n > n_3 = \frac{\sqrt{b^2 + \mu^2 v^2 (b + v) + 2b \mu v (b - v) + \mu v (b - v) + b} - \sqrt{b^2 v (b + v) - 2b \mu v (b - v) + \mu v (b - v) + b}}{2b \mu v (b - v)},$$ \hspace{1cm} (61)

in which case, the industry sets $m^* = 0$. Otherwise, there may exist a positive markup $m^*$ that maximizes profits. We are in a position to state the following proposition.

**Proposition 8.** If the number of firms is relatively large, $n > n_3$, then the industry prefers to abandon any expectation of margin; i.e., $m^* = 0$. Otherwise, the industry may collude over a positive markup; i.e., $m^* \in (0, \bar{m})$.

**Proof.** See Appendix A.3. 

$\square$
Figure 1 illustrates the case corresponding to the previous proposition where companies have the incentive to coordinate on a positive but small profit margin. For the chosen numerical values, the profit-maximizing markup is $m^* \approx 4.63/100$.

![Figure 1: The price markup as a collusion device. Parameter values are $a = 40$, $b = 1$, $\psi = 1$, $c = 0$, $n = 3$, $v = 1$, $\rho = 1$.](image)

5 Conclusion

Overall, businesses remain loyal to the markup pricing rule despite its many defects. Even when they do not rely on this pricing heuristic, they regard it as a convenient benchmark for judging whether the selling price is satisfactory and whether supply adjustments are desirable. Based on this observation, we have introduced a new supply function whereby the firm commits to increasing its supply as the selling price rises and deviates from the markup price. We have shown that this function is equivalent to a linear supply function whose y-intercept and slope are functions of the firms’ profit margin objectives and cost parameters. Therefore, it coincides with the supply function considered by Menezes and Quiggin (2012) whenever the firm has a zero profit margin target and a constant marginal cost technology.

We characterized the equilibrium in supply functions for an industry composed of $n$ firms. We performed a comparative statics analysis to investigate how the profit margin standard in the industry alters market supply, profitability, and social welfare. We found that the higher the industry margin target, the less aggressive firms’ equilibrium supply behavior becomes. The resulting reduction in supply leads to an increase in the equilibrium price, which results in higher equilibrium profits. However, the increase in profits fails to compensate for the loss in consumer surplus so that overall social welfare drops. Comparative statics analysis made it possible to elucidate the strategic role played by the profit margin targets of business firms. We showed that firms achieve higher profits when they condition their offers on the deviation of the selling price from the marked-up price rather than on the selling price itself (as is the case in Menezes and Quiggin, 2012).

We also analyzed how the equilibrium profit of firms varies with the standard margin level in the industry depending on whether or not unfair pricing practices antagonize consumers. When consumers exhibit a low sensitivity to fair pricing issues, we showed that firms’ profits increase monotonously with the industry’s profit margin standard and reach their Cournot levels when it becomes arbitrarily large. Therefore, firms have incentives to price-fixing by switching from supply function competition to quantity competition. The same conclusion may obtain in the more plausible context where consumers get upset when firms aim at excessive profit margins. In particular, this will be the case if firms use a constant return to scale technology or have a low capacity to adjust supply. However, there are many situations where consumers’ preference for fair treatment is enough to discipline the behavior of firms and cause them to compete in supply functions with relatively small markup over costs.

Future research could extend this work in several directions. First, we recall that our model assumed identical firms. Solving an $n$-firm game of supply function competition with asymmetric firms is difficult (Anderson and Xu, 2005). However, the duopoly case with asymmetric costs lends itself to analysis. Second, we assumed
a given and fixed industry profit margin standard. It would be interesting to study how this standard emerges in the first place. One may think of small businesses, endowed with limited computational skills and market experience, trying to learn from their rivals’ reactions what a suitable profit margin standard is for a given industry.

A Appendix

A.1 Proof of Proposition 1

Replacing $p(a_i, a_{-i})$ and $\partial p(a_i, a_{-i})/\partial a_i$ by their expressions (16) and (17) into Condition (21), and solving the resulting equation with respect to $a_i$, we get firm $i$’s inclusive\footnote{Here, we use the terminology of Wolfstetter (1999, page 91). For more details on the solution procedure, see Szidarovszky and Yakowitz (1977).} reaction function:

$$a_i^* = \Phi + \Psi \sum_{i=1}^{n} a_i^*, \quad i = 1, 2, \ldots, n. \tag{62}$$

where

$$\Phi = \frac{ab(\alpha - \beta) + a(v\beta - 1)(b(n-1)\beta + 1) + c(b(n-1)\beta + 1)(bn\beta + 1)}{(bn\beta + 1)(b(n-1)v\beta + b)}, \tag{63}$$

$$\Psi = \frac{b(n\beta + (v\beta - 1)(b(n-1)\beta + 1))}{(bn\beta + 1)(b(n-1)v\beta + b)}. \tag{64}$$

Summing over all $i$ yields the fixed-point equation

$$\sum_{i=1}^{n} a_i^* = n\Phi + n\Psi \sum_{i=1}^{n} a_i^*, \tag{65}$$

which can be solved to obtain

$$\sum_{i=1}^{n} a_i^* = \frac{n\Phi}{1 - n\Psi}. \tag{66}$$

Now, plugging Equation (66) back into the (inclusive-) reaction functions (62), we obtain firm $i$’s equilibrium intercept:

$$a_i^* = \frac{\Phi}{1 - n\Psi}, \quad i = 1, 2, \ldots, n. \tag{67}$$

Finally, substituting (63) and (64) into (67) and rearranging terms we obtain the $y$-intercept given by Equation (23).

A.2 Comparative statics

We have

$$\frac{\partial a^*}{\partial k} := \frac{\partial a^*}{\partial \beta} \frac{\partial \beta}{\partial k} = \frac{b^2(n-1)(a-c)}{(b(n-1)\beta)(bn+bv)+bn+b^2v} \frac{\partial \beta}{\partial k}, \quad \frac{\partial Q^*}{\partial k} := \frac{n^2 \partial a^*}{\partial \beta} \frac{\partial \beta}{\partial k} = \frac{nb^2(n-1)(a-c)}{(b(n-1)\beta)(bn+bv)+bn+b^2v} \frac{\partial \beta}{\partial k}, \tag{68}$$

$$\frac{\partial p^*}{\partial k} := \frac{\partial p^*}{\partial \beta} \frac{\partial \beta}{\partial k} = -\frac{b^2(n-1)n(a-c)}{(b(n-1)\beta)(bn+bv)+bn+b^2v} \frac{\partial \beta}{\partial k}, \quad \frac{\partial \pi^*}{\partial k} := \frac{\partial p^*}{\partial \beta} \frac{\partial \beta}{\partial k} = -\frac{b^2(n-1)^2(a-c)}{(b(n-1)\beta)(bn+bv)+bn+b^2v} \frac{\partial \beta}{\partial k}. \tag{69}$$

Now, since (from Eq. 10)

$$\frac{\partial \beta}{\partial m} = -\left(2\psi^2\nu\right)/\left(\psi(m+1)\nu+2\right)^2 \leq 0, \quad \text{and} \quad \frac{\partial \beta}{\partial \psi} = 4/\left(\psi(m+1)\nu+2\right)^2 > 0, \tag{70}$$

we obtain the comparative statics results (29)-(30).
A.3 Proof of Proposition 8

In order to simplify the proof of Proposition 8, it is convenient to define a new variable

$$
\mu := \frac{\beta (\psi; m, v)}{\psi} = \frac{2}{2 + \psi(m + 1)}.
$$

(71)

Because $m \in [0, +\infty[$, we have

$$
\mu \in [0, \bar{\mu}], \quad \bar{\mu} = \frac{2}{2 + \psi v}.
$$

(72)

Now, we rewrite the industry’s decision problem in terms of the new variable $\mu$. We begin with condition (58) whose satisfaction guarantees strictly positive profits. We solve Equation (71) for $m$ and plug its value into condition (58) to obtain

$$
\Omega := \bar{a} + \rho \left( 1 + \frac{2(\mu - 1)}{v\mu \psi} \right) - c > 0,
$$

(73)

This equation is satisfied provided that

$$
\mu > \frac{2\rho}{2\rho + v(a - c + \rho)\psi} := \mu^*.
$$

(74)

Here, we take note that $\mu \in [0, 1]$. We proceed by rewriting Firm $i$’s profit as a function of $\mu$. By plugging $\beta(\psi; m, v) = \mu \psi$ into Equation (25), we obtain

$$
\pi_i^*(\mu) = \frac{\Omega^2(b\mu(n - 1)\psi + 1)(b\mu(n - 1)v\psi + 2b + v)}{2(b\mu(n - 1)\psi(bn + v) + bn + b + v)^2}.
$$

(75)

Then the problem facing the industry can be rewritten as follows:

$$
\max_{\mu \in [\underbar{\mu}, \bar{\mu}]} \pi_i^*(\mu).
$$

(76)

Assuming an interior solution, the first-order condition for profit maximization is

$$
\frac{\partial \pi_i^*(\mu)}{\partial \mu} = 0,
$$

$$
\frac{\Omega}{v\mu^2 \psi(b + (bn + v)(1 + b(n - 1)\mu \psi))} \frac{\partial P(\mu)}{\partial \mu} = 0,
$$

(77)

where $P(\mu) = A_3\mu^3 + A_2\mu^2 + A_1\mu + A_0$, and

$$
\begin{align*}
A_0 &= 4\rho(2b + v)(bn + b + v) > 0, \\
A_1 &= 4b(n - 1)\rho \psi \left( b^2(5n + 1) + 3b(n + 2)v + 3v^2 \right) > 0, \\
A_2 &= 2b^2(n - 1)^2 \psi \left( bv\psi(-a + c + 6n\rho + 5\rho) + 6b^2n\rho \psi - 2b\rho + 6\rho v^2\psi \right), \\
A_3 &= 2b^3(n - 1)^2 \psi^2 \left( bn(v\psi(-a + c + 2n\rho - 3\rho) - 2\rho) + 2(n - 1)\rho v^2\psi \right).
\end{align*}
$$

(78-81)

We now study the properties of the marginal profit curve (77). To begin with, note that the signs of coefficients $A_2$ and $A_3$ depends on $n$. Indeed, we have $A_2 > 0$ iff

$$
n > \frac{bv\psi(a - c - 5\rho) + 2b\rho - 6\rho v^2\psi}{6b\rho \psi(b + v)} := n_2.
$$

(82)

Also, we have $A_3 > 0$ iff

$$
n > \frac{\sqrt{4b\rho v^2\psi (v\psi(-a + c + \rho) - 2\rho) + (bv\psi(a - c + 3\rho) + 2b\rho)^2 + 4\rho^2 v^4 \psi^2}}{4b\rho \psi} + \frac{bv\psi(a - c + 3\rho) + 2b\rho - 2\rho v^2\psi}{4b\rho \psi} := n_3.
$$

(83)
Using Equation (74), the above expression for \( n_3 \) may be written as a function of \( \mu \). This yields the more compact Equation (61). Finally, it can be easily checked that \( n_3 > n_2 \).

We now are in a position to establish Proposition (8). To this end, we must consider two cases separately. First, if \( n > n_3 \) then \( A_h > 0 \) for all \( h = 1, 2, 3, 4 \). We have \( \mathcal{P}(\mu) > 0 \) and thus \( \partial \pi^*_i / \partial \mu > 0 \). Since marginal profit is monotonically increasing in \( \mu \), maximum profit is reached for \( \mu^* = \hat{\mu} \). Using Equation (71) we obtain \( m^* = 0 \).

Second, if \( n < n_3 \) then we have \( \mathcal{P}(\mu) |_{\mu=0} > 0 \) and \( \lim_{\mu \to \infty} \mathcal{P}(\mu) = -\infty \). Because \( \mathcal{P}(\mu) \) is continuous, there exists a unique value of \( \mu \) that satisfies the equation \( \mathcal{P}(\hat{\mu}) = 0 \). Indeed, since \( A_3 < 0 \), the parabola \( \mathcal{P}'(\mu) = 3A_3\mu^2 + 2A_2\mu + A_1 \) opens downwards. This implies that \( \mathcal{P}(\mu) \) first increases reaching a maximum at \( \mu = \hat{\mu} \), and then decreases. Finally, since \( \hat{\mu} > \underline{\mu} \), it follows that the solution is \( \mu^* = \min\{\hat{\mu}, \bar{\mu}\} \).

References


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