Environmental Productivity Analysis : an Illustration with the Ecuadorian Oil Industry

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Abstract

In this paper, environmental productivity variation is analysed through the pollution-adjusted Malmquist and Hicks-Moorsteen productivity indices. These productivity indices are defined as combination of multiplicative pollution-adjusted distance functions. Non convex pollution-generating technology is assumed to estimate the pollution-adjusted Malmquist and Hicks-Moorsteen productivity measures. The main sources of the pollution-adjusted productivity change are displayed. An empirical illustration is provided by considering a sample of 20 Ecuadorian oil companies over the period 2012-2018. The results are estimated through a non parametric analytic framework.

Keywords: Non Convexity, Ecuadorian Oil industry, Environmental Efficiency and Productivity Indices, Pollution-generating Technology.

JEL: C61, D24, Q50

1 Introduction

Oil represents 32 percent of the global energy consumption sources (IEA, 2019). The World Energy Outlook (IEA, 2019), claims that energy generated from fossil fuels will remain the major source and is still expected to meet about 84 percent of energy demand in 2030. According to British Petroleum (2020), South and Central America have 18.7 percent (324.1 thousand of millions of barrels) of the world's proven reserves. In terms of production, Ecuador is the fifth oil producer in South America with an average production of 27.94 million of tons from 2009 to 2019. There is research into other reliable energy resources to replace fossil fuel, considering its depletion and the environmental impacts generated by this industry. However, it is expected that the energy market will continue to depend on fossil fuels for at least the next few decades.

Among all industry sectors, the petroleum industry is of particular interest to Ecuador because of its economic and environmental significance. Oil is the second most important sector for the Ecuadorian economy. The contribution of the oil sector was 11 percent of the GDP for the period 2011-2018 approximately. Oil is also important for the Ecuadorian energy sector; in 2018, there was a primary energy production of 216 million BEP. Of the total produced, 86.9 percent was made up of oil. According to the Third National Communication on Climate Change and First Biennial Update Report (2017), the energy sector produced 37 594 Gg of carbon dioxide equivalent (CO_2e) which represents 47 percent of total GHG emissions in 2012. Energy industry is a significant contributor of GHG emissions in the country, especially for the burning of fossil fuels. In 2012 this activity accounted for 36 822.54 Gg (CO_2e) which represents the 97.95 percent of emissions of the energy sector. Thus, oil companies need to be more efficient and make a balance between pollution mitigation and economic success.

The environmental productivity change assessment of Ecuadorian petroleum companies needs to consider a methodology integrating the companies' environmental indicators with their operational measures. In this paper, environmental productivity is appraised through the non convex Pollution-adjusted Malmquist (PM) and Hicks-Moorsteen (PHM) productivity indices (Abad and Ravelojaona, 2021, 2022). The PM productivity measure takes the form of the Malmquist index (Färe et al., 1995; Caves et al., 1982) while the PHM productivity measure inherits the structure of the Hicks-Moorsteen index (Bjurek, 1996). Specifically, the PM and the PHM productivity indices display the change of economic and polluting outputs induced by inputs variation by relaxing the convexity property of pollution-generating technology. It is worth noting that pollution-generating processes encompass many human and ecological interactions that can induce non linearities (Tschirhart, 2012; Dasgupta and Mäler, 2003)[1]. Although relaxing the convexity property of production technologies has been investigated in the literature (Briec et al., 2020), few studies consider non convex pollution-generating technologies. The only theoretical model that considers non convex pollution-generating processes has been introduced in Abad and Briec (2019). This model extends the work of Briec et al. (2016, 2018) on congested production technologies. The approach of

[1]Dasgupta and Mäler (2003) mention that: "The word convexity is ubiquitous in economics, but absent from ecology".

Abad and Briec (2019) follows a by-production framework such that it provides axiomatic foundation of the non convex version of the Murty et al. (2012) by-production model (Yuan et al., 2021). In this contribution, the non convex free disposal hull pollution-generating production technology (Abad and Briec, 2019) is considered to estimate the PM and the PHM indices.

Knowing the prominent drivers of environmental productivity change is a major concern in applied economics literature (Becerra-Peña and Santin, 2021; Miao et al., 2019; Aparicio et al., 2018; Shen et al., 2017; Valadkhani et al., 2016). This paper displays the main components of the pollution-adjusted productivity variation considering Ecuadorian oil companies. The identification of the main sources of pollution-adjusted productivity change allow to display internal (technological processes, management skills, etc.) or external (environmental policies, economic context, etc.) constraints that influence productivity variation.

Recently, numerous papers investigate environmental efficiency and productivity variation of the oil sector; see, *e.g.*, Tavana et al. (2020), Wegener and Amin (2019), Sueyoshi and Wang (2018, 2014), De Alencar Bezerra et al. (2017), Azedeh et al. (2015), Song et al. (2015), Sueyoshi and Goto (2015), among others. Moreover, non parametric mathematical programming methods for production analysis are widely applied to set environmental efficiency and productivity studies (Sueyoshi et al., 2017; Zhou et al. 2008). In this paper, non parametric free disposal hull production model (Tulkens, 1993) is applied as a practical approach to evaluate the pollution-adjusted productivity change of Ecuadorian petroleum companies. This modelling does not need to explicitly specify a mathematical form for the production function. Moreover, it allows to assess the environmental efficiency of multi-inputs and multi-outputs production units by relaxing the convexity property of the pollution-generating technologies. To the best of our knowledge, there has been no research performed in the field of oil industry that analyses environmental productivity change considering non convex free disposal hull pollution-generating production model.

An empirical illustration is provided by considering a sample of 20 Ecuadorian private oil companies over the period 2012-2018 to estimate the PM and the PHM productivity indices. Almost all private companies are engaged in exploration and production, while only a relatively small fraction of firms participate in other activities such as transport and distribution. The outputs of the oil companies are separated into economic (desirable) and polluting (undesirable) components; number of oil barrels and CO_2 emissions, respectively. In the segment, CO_2 emissions are generated directly through drilling processes and fossil fuel combustion and indirectly through well leaks and venting.

The remainder of this paper is structured as follows. Section 2 displays the methodological framework. Non convex modelling for the pollution-generating technology and the environmental efficiency indices are defined in Section 3. The empirical illustration is provided in Section 4. Finally, Section 5 discusses and concludes.

2 Methodology

This section presents the theoretical framework adopted throughout the paper. Following this background, pollution-adjusted efficiency and productivity indices are introduced.

2.1 Pollution-generating process: definition and properties

Assume that the outputs are separated into economic (desirable) and polluting (undesirable) components[2]. Let $\mathtt{I}$ and $\mathtt{O}$ be the input and output sets such that $\mathtt{I} = [n]$ and $\mathtt{O} = (\mathtt{O}^{\mathtt{d}}, \mathtt{O}^{\mathtt{u}}) = [m]$, where $[n] = \#\mathtt{I}$ and $[m] = \#\mathtt{O}^{\mathtt{d}} + \#\mathtt{O}^{\mathtt{u}}$. The input and output vectors for the period (t) are defined as $(x_t, y_t) \in \mathbb{R}_+^{m+n}$.

The pollution-generating technology is defined as follows,

$$T_t = \left\{(x_t, y_t) \in \mathbb{R}_+^{n+m} : x_t \text{ can produce } (y_t^{\mathtt{d}}, y_t^{\mathtt{u}})\right\}. \tag{2.1}$$

Usual characterisations of T_t are the output set, $P : \mathbb{R}_+^n \mapsto 2^{\mathbb{R}_+^m}$, and the input correspondence, $L : \mathbb{R}_+^m \mapsto 2^{\mathbb{R}_+^n}$,

$$P(x_t) = \left\{(y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in \mathbb{R}_+^m : (x_t, y_t) \in T_t\right\} \tag{2.2}$$

and

$$L(y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) = \left\{x_t \in \mathbb{R}_+^n : (x_t, y_t) \in T_t\right\}. \tag{2.3}$$

In this paper, alternative characterisations of the pollution-generating processes are considered through the undesirable set, $\mathcal{Q} : \mathbb{R}_+^{m^{\mathtt{d}}} \mapsto 2^{\mathbb{R}_+^{n+m^{\mathtt{u}}}}$, and the desirable correspondence, $\mathcal{Z} : \mathbb{R}_+^{m^{\mathtt{u}}} \mapsto 2^{\mathbb{R}_+^{n+m^{\mathtt{d}}}}$,

$$\mathcal{Q}(y_t^{\mathtt{d}}) = \left\{(x_t, y_t^{\mathtt{u}}) \in \mathbb{R}_+^m : (x_t, y_t) \in T_t\right\} \tag{2.4}$$

and

$$\mathcal{Z}(y_t^{\mathtt{u}}) = \left\{(x_t, y_t^{\mathtt{d}}) \in \mathbb{R}_+^m : (x_t, y_t) \in T_t\right\}. \tag{2.5}$$

The sets (2.5) and (2.4) restrict T_t to the subspace of inputs and economic outputs and, to the subspace of inputs and polluting outputs, respectively (Abad, 2015).

Hence, the next statement holds:

[2]The superscripts $\mathtt{d}$ and $\mathtt{u}$ respectively denote desirable and undesirable outputs throughout the paper.

$$\left.\begin{array}{l} x_t \in L(y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \\ (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in P(x_t) \\ (x_t, y_t^{\mathtt{d}}) \in \mathcal{Z}(y_t^{\mathtt{u}}) \\ (x_t, y_t^{\mathtt{u}}) \in \mathcal{Q}(y_t^{\mathtt{d}}) \end{array}\right\} \Leftrightarrow (x_t, y_t) \in T_t \tag{2.6}$$

Assume that the pollution-generating technology satisfies the following usual properties (Färe et al., 1985):

$\mathcal{A}1$: *No free lunch and Inaction*; $(0,0) \in T_t$, $(0, y_t) \in T_t \Rightarrow y_t = 0$.
$\mathcal{A}2$: *Boundedness*; $T(x_t, y_t) = \{(x_t, v_t) \in T_t : v_t \leq y_t\}$ is bounded for all $y_t \in \mathbb{R}_+^m$.
$\mathcal{A}3$: *Closedness*; T_t is closed.

Let C be the convex cone such that: $\mathrm{C} = \{y_t \in \mathbb{R}^m : y_t^u \leq 0 \text{ and } y_t^d \geq 0\}$. In addition of the traditional axioms $\mathcal{A}1 - \mathcal{A}3$, suppose that the pollution-generating process satisfies the B-disposal assumption (Abad and Briec, 2019):

$\mathcal{A}4$: *B-disposability*; $T_t = \Big(\big(T_t + (\mathbb{R}_+^n \times -\mathbb{R}_+^m)\big) \cap \big(T_t + (\mathbb{R}_+^n \times -\mathrm{C})\big)\Big) \cap (\mathbb{R}_+^m \times \mathbb{R}_+^n)$.

The theoretical model based upon the properties $\mathcal{A}1 - \mathcal{A}4$ permits to define the pollution-generating process as an intersection of sub-technologies (Abad and Briec, 2019): $T_t + (\mathbb{R}_+^n \times -\mathbb{R}_+^m) \cap (\mathbb{R}_+^m \times \mathbb{R}_+^n)$ and $T_t + (\mathbb{R}_+^n \times -\mathrm{C}) \cap (\mathbb{R}_+^m \times \mathbb{R}_+^n)$. As in the by-production framework (Murty and Russell, 2020; Murty et al., 2012), the intended production activities of firms satisfy the usual strong disposability assumption; *ie.* $T_t + (\mathbb{R}_+^n \times -\mathbb{R}_+^m) \cap (\mathbb{R}_+^m \times \mathbb{R}_+^n)$. Moreover, a partially reversed free disposal axiom applies for the polluting residuals generation; *ie.* $T_t + (\mathbb{R}_+^n \times -\mathrm{C}) \cap (\mathbb{R}_+^m \times \mathbb{R}_+^n)$. It is worth noting that axioms $\mathcal{A}1 - \mathcal{A}4$ define a fairly weak axiomatic framework such that the convexity assumption is not required to define pollution-generating processes.

2.2 Pollution-adjusted efficiency and productivity indices

This section displays the pollution-adjusted efficiency and productivity indices. In addition, the core components of the pollution-adjusted productivity change are presented.

2.2.1 Pollution-adjusted multiplicative distance function

Multiplicative distance functions have become standard tools to define technical efficiency measures (Debreu, 1951; Farrell, 1957; Shephard, 1953). These distance functions fully characterise multiple inputs-outputs production processes.

The following definition presents the pollution-adjusted multiplicative distance function (Abad, 2018).

Definition 2.1 *Let T_t be a pollution-generating process that satisfies properties $\mathcal{A}1-\mathcal{A}4$. For any $(x_t, y_t) \in \mathbb{R}_+^{n+m}$, where $y_t = (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in \mathbb{R}_+^m$, the multiplicative pollution-adjusted distance function, $\mathfrak{D}^{\phi} : \mathbb{R}_+^{n+m} \longrightarrow \mathbb{R} \cup \infty$, is defined below :*

$$\mathfrak{D}_t^{\phi}(x_t, y_t) = \begin{cases} \inf\limits_{\beta} \left\{ \beta \in]0,1] : \quad \left(\beta^{\alpha} x_t, \beta^{\gamma^{\mathtt{d}}} y_t^{\mathtt{d}}, \beta^{\gamma^{\mathtt{u}}} y_t^{\mathtt{u}}\right) \in T_t \right\} \\ \qquad\qquad \text{if } \left(\beta^{\alpha} x_t, \beta^{\gamma^{\mathtt{d}}} y_t^{\mathtt{d}}, \beta^{\gamma^{\mathtt{u}}} y_t^{\mathtt{u}}\right) \in T_t, \beta > 0 \\ \infty \qquad\qquad \text{else} \end{cases} \tag{2.7}$$

where $\phi = (\alpha, \gamma^{\mathtt{d}}, \gamma^{\mathtt{u}}) \in \{0,1\}^n \times \{-1,0\}^{m^{\mathtt{d}}} \times \{0,1\}^{m^{\mathtt{u}}}$.

Let us consider the following orientations for the pollution-adjusted distance function.

Proposition 2.2 *For any $(x_t, y_t) \in \mathbb{R}_+^{n+m}$ where $y_t = (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in \mathbb{R}_+^m$,*

i. $\mathfrak{D}_t^{1,-1,0}(x_t, y_t) \equiv \mathfrak{D}^{\mathcal{Z}}(x_t, y_t)$.

ii. $\mathfrak{D}_t^{1,0,1}(x_t, y_t) \equiv \mathfrak{D}^{\mathcal{Q}}(x_t, y_t)$.

iii. $\mathfrak{D}_t^{1,0,0}(x_t, y_t) \equiv \mathfrak{D}^{L}(x_t, y_t)$.

iv. $\mathfrak{D}_t^{0,-1,0}(x_t, y_t) \equiv \mathfrak{D}^{\hat{P}}(x_t, y_t)$.

v. $\mathfrak{D}_t^{0,0,1}(x_t, y_t) \equiv \mathfrak{D}^{\tilde{P}}(x_t, y_t)$.

The pollution-adjusted efficiency indices presented in the aforementioned results **i.** and **v.** fully identify pollution-generating processes (Abad and Ravelojaona, 2021, 2022).

In such case, the next statement holds:

$$\left.\begin{array}{l} \mathfrak{D}^{\mathcal{Z}}(x_t, y_t) \in]0;1] \Leftrightarrow (x_t, y_t^{\mathtt{d}}) \in \mathcal{Z}(y_t^{\mathtt{u}}) \\ \mathfrak{D}^{\mathcal{Q}}(x_t, y_t) \in]0;1] \Leftrightarrow (x_t, y_t^{\mathtt{u}}) \in \mathcal{Q}(y_t^{\mathtt{d}}) \\ \mathfrak{D}^{L}(x_t, y_t) \in]0;1] \Leftrightarrow x_t \in L(y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \\ \mathfrak{D}^{\hat{P}}(x_t, y_t) \in]0;1] \Leftrightarrow (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in P(x_t) \\ \mathfrak{D}^{\tilde{P}}(x_t, y_t) \in]0;1] \Leftrightarrow (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in P(x_t) \end{array}\right\} \Leftrightarrow (x_t, y_t) \in T_t \tag{2.8}$$

2.2.2 Malmquist pollution-adjusted productivity index

The next result displays the Malmquist pollution-adjusted productivity index (Abad and Ravelojaona, 2021).

Definition 2.3 *Suppose that T_t is a pollution-generating process satisfying properties $\mathcal{A}1-\mathcal{A}4$ over the periods (t) and $(t+1)$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$, the Malmquist pollution-adjusted productivity index is defined as follows :*

$$PM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \Big[PM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1}) \times PM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1})\Big]^{\frac{1}{2}} \tag{2.9}$$

such that $\phi = (\alpha, \gamma^{\mathtt{d}}, \gamma^{\mathtt{u}}) \in \{0,1\}^n \times \{-1,0\}^{m^{\mathtt{d}}} \times \{0,1\}^{m^{\mathtt{u}}}$.

The Malmquist pollution-adjusted productivity indices for the periods (t) and $(t+1)$ are presented in the next results.

$$PM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}^{1,-1,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{1,-1,0}_{t}(x_t, y_t)} \times \frac{\mathfrak{D}^{1,0,1}_{t}(x_{t+1}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,1}_{t}(x_t, y_t)} \tag{2.10}$$

and

$$PM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}^{1,-1,0}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{1,-1,0}_{t+1}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{1,0,1}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{1,0,1}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})}. \tag{2.11}$$

If the multiplicative productivity index $PM^{\phi}_{t,t+1}$ is larger than 1 then, pollution adjusted productivity gains arise over periods (t) and $(t+1)$. In such case, the firms produce more non polluting outputs and operate managerial efforts to reduce their inputs and polluting outputs.

The following statement defines a decomposition of the Malmquist pollution-adjusted productivity variation by separating polluting and non polluting dimensions.

Proposition 2.4 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$ where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$,*

$$PM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = PM^{1,-1,0}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \times PM^{1,0,1}_{t,t+1}(x_{t,t+1}, y_{t,t+1}). \tag{2.12}$$

Where,

$$PM^{1,-1,0}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{1,-1,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{1,-1,0}_{t}(x_t, y_t)} \times \frac{\mathfrak{D}^{1,-1,0}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{1,-1,0}_{t+1}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})}\right]^{\dfrac{1}{2}}$$

and

$$PM^{1,0,1}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{1,0,1}_{t}(x_t, y_t)}{\mathfrak{D}^{1,0,1}_{t}(x_{t+1}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{1,0,1}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{1,0,1}_{t+1}(x_{t+1}, y_{t+1})}\right]^{-\dfrac{1}{2}}$$

Assume that the no polluting Malmquist productivity index $PM^{1,-1,0}_{t,t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than 1. In such case, more desirable outputs are produced and less inputs are used between the periods (t) and (t+1), for a given level of undesirable outputs. In the same vein, if the polluting Malmquist productivity index $PM^{1,0,1}_{t,t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, less undesirable outputs are produced and less inputs are used between the periods (t) and (t+1), for a given level of desirable outputs. Obviously, reciprocal reasoning holds when $PM^{1,-1,0}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \leq 1$ and $PM^{1,0,1}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \leq 1$.

The prominent components of the Malmquist pollution-adjusted productivity variation are presented in the next result.

Definition 2.5 *Let T_t be a pollution-generating technology satisfying assumptions $\mathcal{A}1$–$\mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}^m_+$, the decomposition of the Malmquist pollution-adjusted productivity change is defined below:*

$$PM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = EC^{\phi}_{t,t+1} \times TC^{\phi}_{t,t+1} \tag{2.13}$$

where $EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$ respectively correspond to the efficiency variation and technological change components.

$EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$ are defined as follows,

$$\begin{aligned} EC^{\phi}_{t,t+1} &= EC^{1,-1,0}_{t,t+1} \times EC^{1,0,1}_{t,t+1} \\ &= \frac{\mathfrak{D}^{1,-1,0}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{1,-1,0}_{t}(x_t, y_t)} \times \frac{\mathfrak{D}^{1,0,1}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{1,0,1}_{t}(x_t, y_t)} \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} TC^{\phi}_{t,t+1} &= \left(TC^{1,-1,0}_{t,t+1} \times TC^{1,0,1}_{t,t+1}\right)^{\frac{1}{2}} \\ &= \left[\left(\frac{\mathfrak{D}^{1,-1,0}_{t}(x_t, y_t)}{\mathfrak{D}^{1,-1,0}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{1,-1,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{1,-1,0}_{t+1}(x_{t+1}, y_{t+1})}\right) \times \right. \\ &\qquad \left. \left(\frac{\mathfrak{D}^{1,0,1}_{t}(x_t, y_t)}{\mathfrak{D}^{1,0,1}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{1,0,1}_{t}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,1}_{t+1}(x_{t+1}, y_{t+1})}\right)\right]^{\frac{1}{2}}. \end{aligned} \tag{2.15}$$

If the efficiency change $EC^{\phi}_{t,t+1}$ is greater than 1 then, efficiency progress arises over the periods (t) and $(t+1)$. Moreover, technological improvement occurs between the periods (t) and $(t+1)$ when $TC^{\phi}_{t,t+1} \geq 1$. Note that the main sources of the pollution-adjusted productivity variation, namely $EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$, are separated into polluting and non polluting components.

2.2.3 Hicks-Moorsteen pollution-adjusted productivity index

The Hicks-Moorsteen pollution-adjusted productivity index (Abad and Ravelojaona, 2022) is presented in the statement below.

Definition 2.6 *Let T_t be a pollution-generating technology that satisfies assumptions $\mathcal{A}1 - \mathcal{A}4$ over the periods (t) and $(t+1)$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, with $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$, the Hicks-Moorsteen pollution-adjusted productivity index is defined as follows,*

$$PHM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \Big[PHM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1}) \times PHM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1})\Big]^{\frac{1}{2}} \quad (2.16)$$

where $\phi = (\alpha, \gamma^{\mathtt{d}}, \gamma^{\mathtt{u}}) \in \{0,1\}^n \times \{-1,0\}^{m^{\mathtt{d}}} \times \{0,1\}^{m^{\mathtt{u}}}$.

$PHM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1})$ and $PHM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1})$ respectively display the Hicks-Moorsteen pollution-adjusted productivity indices for the periods (t) and $(t+1)$. These productivity indices are defined as follows:

$$PHM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}^{0,-1,0}_{t}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{0,-1,0}_{t}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{0,0,1}_{t}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_{t}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{1,0,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{1,0,0}_{t}(x_t, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \quad (2.17)$$

and

$$PHM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{1,0,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,0}_{t+1}(x_{t}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}. \quad (2.18)$$

With regard to the aforementioned results, if $PHM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, pollution-adjusted productivity growth occurs.

The polluting and no polluting parts of the PHM productivity variation are defined in the following result.

Proposition 2.7 *For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, such that $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$,*

$$PHM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1}) = PHM^{0,-1,0}_{t+1}(x_{t,t+1}, y_{t,t+1}) \times PHM^{0,0,1}_{t+1}(x_{t,t+1}, y_{t,t+1}). \quad (2.19)$$

Where $PHM^{0,-1,0}_{t+1}(x_{t,t+1}, y_{t,t+1})$ and $PHM^{0,0,1}_{t+1}(x_{t,t+1}, y_{t,t+1})$ display the no polluting PHM index and the polluting PHM index, respectively.

The productivity indices $PHM^{0,-1,0}_{t+1}(x_{t,t+1}, y_{t,t+1})$ and $PHM^{0,0,1}_{t+1}(x_{t,t+1}, y_{t,t+1})$ are defined as follows,

$$PHM^{0,-1,0}_{t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{0,-1,0}_t(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,-1,0}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}\right]^{1/2} \times \left[\frac{\mathfrak{D}^{1,0,0}_t(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{1,0,0}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{1,0,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,0}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}\right]^{1/4} \tag{2.20}$$

and

$$PHM^{0,0,1}_{t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{0,0,1}_t(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,0,1}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}\right]^{1/2} \times \left[\frac{\mathfrak{D}^{1,0,0}_t(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{1,0,0}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{1,0,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,0}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}\right]^{1/4} \tag{2.21}$$

If $PHM^{0,-1,0}_{t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, no polluting productivity improvement arises between the period (t) and $(t+1)$. Correspondingly, $PHM^{0,0,1}_{t+1}(x_{t,t+1}, y_{t,t+1}) > 1$ shows polluting productivity growth.

The next statement allows to go a bit further in detail by displaying the main components of the Hicks-Moorsteen pollution-adjusted productivity index.

Proposition 2.8 *Assume that T_t is a pollution-generating process satisfying properties $\mathcal{A}1 - \mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}^m_+$, the Hicks-Moorsteen pollution-adjusted productivity index is separated as follows:*

$$PHM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \mathcal{EC}^{\phi}_{t,t+1} \times \mathcal{TC}^{\phi}_{t,t+1} \times \mathcal{SC}^{\phi}_{t,t+1} \tag{2.22}$$

such that $\mathcal{EC}^{\phi}_{t,t+1}$, $\mathcal{TC}^{\phi}_{t,t+1}$ and $\mathcal{SC}^{\phi}_{t,t+1}$ respectively correspond to the efficiency change, the technological variation and the scale efficiency change components over the periods (t) and $(t+1)$.

$\mathcal{EC}^{\phi}_{t,t+1}$, $\mathcal{TC}^{\phi}_{t,t+1}$ and $\mathcal{SC}^{\phi}_{t,t+1}$ are defined as follows:

$$\begin{aligned}\mathcal{EC}^{\phi}_{t,t+1} &= \mathcal{EC}^{0,-1,0}_{t,t+1} \times \mathcal{EC}^{0,0,1}_{t,t+1} \\ &= \frac{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,-1,0}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_t(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)},\end{aligned} \tag{2.23}$$

$$
\begin{aligned}
\mathcal{TC}^{\phi}_{t,t+1} &= \mathcal{TC}^{0,-1,0}_{t,t+1} \times \mathcal{TC}^{0,0,1}_{t,t+1} \\
&= \left[\frac{\mathfrak{D}^{0,-1,0}_{t}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,-1,0}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{0,-1,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \right]^{\frac{1}{2}} \times \\
&\qquad \left[\frac{\mathfrak{D}^{0,0,1}_{t}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,0,1}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{0,0,1}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \right]^{\frac{1}{2}}
\end{aligned}
\tag{2.24}
$$

and

$$
\mathcal{SC}^{\phi}_{t,t+1} = \mathcal{SC}^{0,-1,0}_{t,t+1} \times \mathcal{SC}^{0,0,1}_{t,t+1} \tag{2.25}
$$

such that,

$$
\begin{aligned}
\mathcal{SC}^{0,-1,0}_{t,t+1} &= \left[\frac{\mathfrak{D}^{0,-1,0}_{t}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,-1,0}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{0,-1,0}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,-1,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})} \right]^{\frac{1}{2}} \times \\
&\qquad \left[\frac{\mathfrak{D}^{1,0,0}_{t}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{1,0,0}_{t}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{1,0,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,0}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \right]^{\frac{1}{4}}
\end{aligned}
\tag{2.26}
$$

and

$$
\begin{aligned}
\mathcal{SC}^{0,0,1}_{t,t+1} &= \left[\frac{\mathfrak{D}^{0,0,1}_{t}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{0,0,1}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{0,0,1}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{0,0,1}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)} \right]^{\frac{1}{2}} \times \\
&\qquad \left[\frac{\mathfrak{D}^{1,0,0}_{t}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)}{\mathfrak{D}^{1,0,0}_{t}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{1,0,0}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{1,0,0}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})} \right]^{\frac{1}{4}}.
\end{aligned}
\tag{2.27}
$$

Remark that the prominent drivers of the Hicks-Moorsteen pollution-adjusted productivity index (2.23)-(2.25) are separated into polluting and no polluting components. Obviously, pollution-adjusted efficiency and technological improvements arise between the periods $(t, t+1)$ when $\mathcal{EC}^{\phi}_{t,t+1} > 1$ and $\mathcal{TC}^{\phi}_{t,t+1} > 1$, respectively. Besides, if the scale efficiency component $\mathcal{SC}^{\phi}_{t,t+1}$ is equal to unity then, the observation performs at the optimal scale over the analysed periods.

3 Pollution-adjusted productivity change: non parametric specification

In this section, non parametric pollution-generating production technology is considered. Specifically, the pollution-adjusted productivity measures are defined through the Free Disposal Hull (FDH) non convex production model (Abad and Briec, 2019; Tulkens, 1993).

3.1 Non convex pollution-generating production process

Assume that $\mathcal{U} = \{(x_s, y_s) : s \in \mathcal{S}\}$ is a set of production units, where $\mathcal{S}$ is an index set of integers.

FDH non convex pollution-generating technology is defined as follows:

$$T^{FDH}\Big\{(x,y) : (x,y) \in \big(\cup_{s\in\mathcal{S}} \mathcal{I}(x_s, y_s)\big) \cap \big(\cup_{s\in\mathcal{S}} \mathcal{J}(x_s, y_s)\big)\Big\}. \tag{3.1}$$

Where, for any $(x_s, y_s) \in \mathcal{U}$,

$$\mathcal{I}(x_s, y_s) = \Big\{(x,y) \in \mathbb{R}_+^{n+m} : x_i \geq x_{s,i},\ y_j \leq y_{s,j},\ i \in [n],\ j \in [m]\Big\} \tag{3.2}$$

and

$$\mathcal{J}(x_s, y_s) = \Big\{(x,y) \in \mathbb{R}_+^{n+m} : x_i \geq x_{s,i},\ y_r \leq y_{s,r},\ y_l \geq y_{s,l},\ i \in [n],\ r \in [m^{\mathtt{d}}],\ l \in [m^{\mathtt{u}}]\Big\}. \tag{3.3}$$

The above FDH pollution-generating process (3.1) is defined as an intersection of non convex sub-technologies. $\cup_{s\in\mathcal{S}}\mathcal{I}(x_s, y_s)$ displays the usual strong disposal FDH sub-technology. The partially reversed free disposal FDH sub-technology corresponds to the non convex set $\cup_{s\in\mathcal{S}}\mathcal{J}(x_s, y_s)$.

3.2 Productivity index on non parametric pollution-generating technology

The following statement defines the pollution-adjusted multiplicative distance function with respect to a FDH non-convex pollution-generating process.

Proposition 3.1 *Assume that T_t is a pollution-generating process that satisfies properties $\mathcal{A}1 - \mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$, the FDH pollution-adjusted multiplicative efficiency index for the period (t) is defined* *below:*

$$\mathfrak{D}_t^{\phi,FDH}(x_0,y_0)=\begin{cases}\max\limits_{s\in\mathcal{S}}\left(\min\left\{\max\limits_{i\in[n]}\left(\dfrac{x_{s,i}}{x_{0,i}}\right);\max\limits_{r\in[m^{\mathrm{d}}]}\left(\dfrac{y_{0,r}}{y_{s,r}}\right)\right\}\right) & if\ \phi=(1,-1,0).\\ \max\limits_{s\in\mathcal{S}}\left(\min\left\{\max\limits_{i\in[n]}\left(\dfrac{x_{s,i}}{x_{0,i}}\right);\max\limits_{l\in[m^{\mathrm{u}}]}\left(\dfrac{y_{s,l}}{y_{0,l}}\right)\right\}\right) & if\ \phi=(1,0,1).\\ \max\limits_{s\in\mathcal{S}}\left(\min\left\{\max\limits_{i\in[n]}\left(\dfrac{x_{s,i}}{x_{0,i}}\right)\right\}\right) & if\ \phi=(1,0,0).\\ \max\limits_{s\in\mathcal{S}}\left(\min\left\{\max\limits_{r\in[m^{\mathrm{d}}]}\left(\dfrac{y_{0,r}}{y_{s,r}}\right)\right\}\right) & if\ \phi=(0,-1,0).\\ \max\limits_{s\in\mathcal{S}}\left(\min\left\{\max\limits_{l\in[m^{\mathrm{u}}]}\left(\dfrac{y_{s,l}}{y_{0,l}}\right)\right\}\right) & if\ \phi=(0,0,1).\end{cases}\tag{3.4}$$

Proof of Proposition 3.1: Assume that $\phi=(1,-1,0)$. Following properties $\mathcal{A}1-\mathcal{A}4$, the FDH pollution-generating process is defined as an intersection of non convex sub-technologies; see (3.1). Thus, for any $i\in[n],\ r\in[m^{\mathrm{d}}],\ l\in[m^{\mathrm{u}}]$, it follows that:

$$\begin{aligned}\mathfrak{D}_t^{\phi,FDH}(x,y) &= \max_{s\in\mathcal{S}}\Bigg\{\ \min_{\beta}\left\{\beta\in]0,1]:\beta x_{0,i}\geq x_{s,i},\frac{y_{0,r}}{\beta}\leq y_{s,r},y_{0,l}\leq y_{s,l}\right\};\\ &\qquad\min_{\beta}\left\{\beta\in]0,1]:\beta x_{0,i}\geq x_{s,i},\frac{y_{0,r}}{\beta}\leq y_{s,r},y_{0,l}\geq y_{s,l}\right\}\Bigg\},\\ &= \max_{s\in\mathcal{S}}\left\{\ \min_{\beta}\left\{\beta\in]0,1]:\beta\geq\max_{i\in[n]}\left(\frac{x_{s,i}}{x_{0,i}}\right),\beta\geq\max_{r\in[m^{\mathrm{d}}]}\left(\frac{y_{0,r}}{y_{s,r}}\right)\right\}\right\}.\end{aligned}$$

The proof for $\phi=(1,0,1)$, $\phi=(1,0,0)$, $\phi=(0,-1,0)$ and $\phi=(0,0,1)$ can be directly deduced from the aforementioned results. □

4 Empirical illustration

This empirical illustration focuses on the oil companies having production and extraction activities in Ecuador over the period 2012-2018. The results are provided through a FDH non convex DEA model.

4.1 Data in brief

A sample of 20 private oil companies in Ecuador is considered over the period 2012–2018. The data set used in this research is built with the population of registered oil Ecuadorian formal firms, constructed from the balance sheets and financial statements registered on the official website of the Superintendencia de Compañias, Valores y Seguros (SCVS). This information is reported annually directly by firms to the SCVS.

Two inputs are selected: **(i)** number of formal employees of each company and **(ii)** net tangible assets (capital stock). Information about the number of legally registered employees **(i)** is declared by each company. The capital stock **(ii)** is set as the sum of the real dollar value of buildings, machinery and vehicles by assuming a depreciation of 5, 10, and 20 percent. Precisely, the methodology of Camino-Mogro and Bermudez-Barrezueta (2021) is employed. Hence, the capital stock is valued considering the gross investment in equipment in year (t), net fixed assets in real value (physical capital in year ($t-1$)), a depreciation rate and the price index for equipment at the industry level obtained from the Ecuadorian National Institute of Statistics. These inputs permit to produce different outputs. Thus, we consider one desirable output, **(iii)** number of oil barrels and one undesirable output represented by **(iv)** CO_2 emissions. The number of extracted barrels of oil **(iii)** is defined based on the variable "sales" (American dollars) reported in the balance sheets and financial statements registered on the official website of the SCVS. Obviously, we divide it by the price (American dollars/barrel) to obtain the variable "number of extracted barrels of oil". The reference price (WTI) is considered allowing comparisons with other international research in the same field. The CO_2 emissions (tons of CO_2 equivalents) **(iv)** is measured by using the methodology of the 2006 IPCC Guidelines for National Greenhouse Gas Inventories.

Table 1 presents the descriptive statistics of the variables used in this study.

Variables	**Min**	**Max**	**Median**	**S.D.**	**Mean**
Labor	1	706	11.5	181.43	112.525
Capital stock	1.76	399629.97	637.23	38819.16	10434.72
Oil production	0.38	11000.14	439.28	1676.60	985.68
CO_2 **emissions**	3.73	106697.08	4260.89	16262.35	9560.72

Table 1: Characteristics of inputs and outputs

4.2 Results

Malmquist pollution-adjusted productivity index

The results outlined in Table 2 (Appendix I) reveal the PM productivity indices scores and its decompositions over the period 2012-2018. Specifically, the third column from the right displays the Malmquist pollution-adjusted productivity index scores. The first two columns from the right show the main drivers of the pollution-adjusted productivity change, namely the technological change and the efficiency variation components, respectively. Columns 4, 5 and 6 from the right focus on polluting sources of productivity variation displaying $TC^{1,0,1}$, $EC^{1,0,1}$ and $PM^{1,0,1}$, respectively. Regarding to the no polluting part of productivity change, columns 7, 8 and 9 from the right indicate $TC^{1,-1,0}$, $EC^{1,-1,0}$ and $PM^{1,-1,0}$, respectively.

DMUs	$PM^{1,-1,0}$	$EC^{1,-1,0}$	$TC^{1,-1,0}$	$PM^{1,0,1}$	$EC^{1,0,1}$	$TC^{1,0,1}$	PM^{ϕ}	EC^{ϕ}	TC^{ϕ}
				2017-2018					
1	0.995	1.000	0.995	1.005	1.000	1.005	1.000	1.000	1.000
2	0.997	1.000	0.997	1.067	1.000	1.067	1.063	1.000	1.063

3	1.079	1.000	1.079	1.045	1.000	1.045	1.127	1.000	1.127
4	1.011	1.000	1.011	infty	1.000	infty	infty	1.000	infty
5	0.982	1.000	0.982	0.982	1.000	0.982	0.965	1.000	0.965
6	1.115	1.000	1.115	infty	1.000	infty	infty	1.000	infty
7	1.600	1.000	1.600	infty	1.000	infty	infty	1.000	infty
8	0.916	1.000	0.916	1.070	1.000	1.070	0.981	1.000	0.981
9	1.033	1.000	1.033	0.745	1.000	0.745	0.769	1.000	0.769
10	0.928	1.000	0.928	0.858	1.000	0.858	0.797	1.000	0.797
11	1.304	1.000	1.304	0.610	1.000	0.610	0.796	1.000	0.796
12	1.361	1.000	1.361	1.187	1.000	1.187	1.615	1.000	1.615
13	0.995	1.000	0.995	infty	1.000	infty	infty	1.000	infty
14	0.995	1.000	0.995	1.005	1.000	1.005	1.000	1.000	1.000
15	1.027	1.000	1.027	infty	1.000	infty	infty	1.000	infty
16	1.220	1.000	1.220	0.820	1.000	0.820	1.000	1.000	1.000
17	1.038	1.000	1.038	infty	1.000	infty	infty	1.000	infty
18	1.008	1.000	1.008	1.843	1.000	1.843	1.858	1.000	1.858
19	0.995	1.000	0.995	1.005	1.000	1.005	1.000	1.000	1.000
20	0.985	1.000	0.985	0.220	1.000	0.220	0.217	1.000	0.217
2016-2017									
1	1.020	1.000	1.020	1.314	1.000	1.314	1.340	1.000	1.340
2	1.586	1.000	1.586	0.486	1.000	0.486	0.771	1.000	0.771
3	0.820	1.000	0.820	0.591	1.000	0.591	0.484	1.000	0.484
4	1.053	1.000	1.053	infty	1.000	infty	infty	1.000	infty
5	0.930	1.000	0.930	0.728	1.000	0.728	0.677	1.000	0.677
6	1.072	1.000	1.072	infty	1.000	infty	infty	1.000	infty
7	1.021	1.000	1.021	infty	1.000	infty	infty	1.000	infty
8	0.919	1.000	0.919	1.115	1.000	1.115	1.024	1.000	1.024
9	1.020	1.000	1.020	0.981	1.000	0.981	1.000	1.000	1.000
10	0.938	1.000	0.938	0.938	1.000	0.938	0.879	1.000	0.879
11	0.576	1.000	0.576	1.557	1.000	1.557	0.897	1.000	0.897
12	0.231	1.000	0.231	0.196	1.000	0.196	0.045	1.000	0.045
13	1.020	1.000	1.020	infty	1.000	infty	infty	1.000	infty
14	1.054	1.000	1.054	1.139	1.000	1.139	1.200	1.000	1.200
15	0.830	1.000	0.830	infty	1.000	infty	infty	1.000	infty
16	0.748	1.000	0.748	1.118	1.000	1.118	0.836	1.000	0.836
17	0.678	1.000	0.678	infty	1.000	infty	infty	1.000	infty
18	0.860	1.000	0.860	infty	1.000	infty	infty	1.000	infty
19	0.815	1.000	0.815	1.736	1.000	1.736	1.414	1.000	1.414
20	1.175	1.000	1.175	infty	1.000	infty	infty	1.000	infty
2015-2016									
1	0.951	1.000	0.951	0.798	1.000	0.798	0.759	1.000	0.759
2	0.285	1.000	0.285	3.316	1.000	3.316	0.944	1.000	0.944
3	0.650	1.000	0.650	infty	1.000	infty	infty	1.000	infty
4	1.188	1.000	1.188	infty	1.000	infty	infty	1.000	infty
5	1.158	1.000	1.158	1.153	1.000	1.153	1.335	1.000	1.335
6	0.922	1.000	0.922	0.726	1.000	0.726	0.669	1.000	0.669
7	0.970	1.000	0.970	infty	1.000	infty	infty	1.000	infty
8	0.760	1.000	0.760	0.751	1.000	0.751	0.571	1.000	0.571
9	0.649	1.000	0.649	1.297	1.000	1.297	0.842	1.000	0.842
10	0.993	1.000	0.993	0.974	1.000	0.974	0.967	1.000	0.967
11	0.558	1.000	0.558	1.750	1.000	1.750	0.976	1.000	0.976
12	1.045	1.000	1.045	0.979	1.000	0.979	1.023	1.000	1.023
13	0.835	1.000	0.835	infty	1.000	infty	infty	1.000	infty
14	0.969	1.000	0.969	1.600	1.000	1.600	1.551	1.000	1.551
15	0.873	1.000	0.873	1.527	1.000	1.527	1.333	1.000	1.333
16	1.040	1.000	1.040	1.070	1.000	1.070	1.113	1.000	1.113
17	0.969	1.000	0.969	1.506	1.000	1.506	1.460	1.000	1.460
18	0.734	1.000	0.734	infty	1.000	infty	infty	1.000	infty
19	1.262	1.000	1.262	0.865	1.000	0.865	1.091	1.000	1.091
20	1.007	1.000	1.007	2.425	1.000	2.425	2.441	1.000	2.441
2014-2015									
1	0.674	1.000	0.674	infty	1.000	infty	infty	1.000	infty
2	1.220	1.000	1.220	2.204	1.000	2.204	2.690	1.000	2.690
3	1.168	1.000	1.168	infty	1.000	infty	infty	1.000	infty

4	0.776	1.000	0.776	infty	1.000	infty	infty	1.000	infty
5	1.076	1.000	1.076	1.081	1.000	1.081	1.163	1.000	1.163
6	0.805	1.000	0.805	0.789	1.000	0.789	0.636	1.000	0.636
7	0.507	1.000	0.507	1.765	1.000	1.765	0.896	1.000	0.896
8	0.713	1.000	0.713	1.492	1.000	1.492	1.063	1.000	1.063
9	1.160	1.000	1.160	1.160	1.000	1.160	1.346	1.000	1.346
10	0.814	1.000	0.814	1.397	1.000	1.397	1.138	1.000	1.138
11	0.409	1.000	0.409	2.449	1.000	2.449	1.001	1.000	1.001
12	1.963	1.000	1.963	2.031	1.000	2.031	3.987	1.000	3.987
13	0.966	1.000	0.966	infty	1.000	infty	infty	1.000	infty
14	0.198	1.000	0.198	6.052	1.000	6.052	1.197	1.000	1.197
15	0.697	1.000	0.697	1.596	1.000	1.596	1.113	1.000	1.113
16	0.647	1.000	0.647	infty	1.000	infty	infty	1.000	infty
17	0.686	1.000	0.686	1.426	1.000	1.426	0.979	1.000	0.979
18	0.696	1.000	0.696	infty	1.000	infty	infty	1.000	infty
19	0.428	1.000	0.428	1.641	1.000	1.641	0.703	1.000	0.703
20	1.212	1.000	1.212	1.194	1.000	1.194	1.447	1.000	1.447
2013-2014									
1	0.941	1.000	0.941	1.220	1.000	1.220	1.149	1.000	1.149
2	0.941	1.000	0.941	1.062	1.000	1.062	1.000	1.000	1.000
3	1.776	1.000	1.776	infty	1.000	infty	infty	1.000	infty
4	0.859	1.000	0.859	infty	1.000	infty	infty	1.000	infty
5	0.775	1.000	0.775	0.822	1.000	0.822	0.637	1.000	0.637
6	0.531	1.000	0.531	0.549	1.000	0.549	0.291	1.000	0.291
7	0.851	1.000	0.851	0.841	1.000	0.841	0.716	1.000	0.716
8	1.373	1.000	1.373	1.427	1.000	1.427	1.959	1.000	1.959
9	0.964	1.000	0.964	0.783	1.000	0.783	0.754	1.000	0.754
10	1.128	1.000	1.128	1.293	1.000	1.293	1.458	1.000	1.458
11	1.133	1.000	1.133	1.051	1.000	1.051	1.191	1.000	1.191
12	0.932	1.000	0.932	0.816	1.000	0.816	0.761	1.000	0.761
13	1.919	1.000	1.919	infty	1.000	infty	infty	1.000	infty
14	1.029	1.000	1.029	1.048	1.000	1.048	1.079	1.000	1.079
15	0.756	1.000	0.756	infty	1.000	infty	infty	1.000	infty
16	0.865	1.000	0.865	infty	1.000	infty	infty	1.000	infty
17	0.754	1.000	0.754	0.628	1.000	0.628	0.474	1.000	0.474
18	2.072	1.000	2.072	infty	1.000	infty	infty	1.000	infty
19	1.227	1.000	1.227	1.405	1.000	1.405	1.723	1.000	1.723
20	0.565	1.000	0.565	1.091	1.000	1.091	0.616	1.000	0.616
2012-2013									
1	1.045	1.000	1.045	0.969	1.000	0.969	1.013	1.000	1.013
2	1.038	1.000	1.038	0.746	1.000	0.746	0.775	1.000	0.775
3	0.533	1.000	0.533	infty	1.000	infty	infty	1.000	infty
4	1.153	1.000	1.153	infty	1.000	infty	infty	1.000	infty
5	0.980	1.000	0.980	1.208	1.000	1.208	1.183	1.000	1.183
6	1.356	1.000	1.356	1.882	1.000	1.882	2.553	1.000	2.553
7	1.144	1.000	1.144	0.951	1.000	0.951	1.088	1.000	1.088
8	1.468	1.000	1.468	0.598	1.000	0.598	0.877	1.000	0.877
9	0.949	1.000	0.949	0.519	1.000	0.519	0.492	1.000	0.492
10	1.285	1.000	1.285	1.035	1.000	1.035	1.330	1.000	1.330
11	1.189	1.000	1.189	0.760	1.000	0.760	0.903	1.000	0.903
12	1.099	1.000	1.099	1.034	1.000	1.034	1.136	1.000	1.136
13	0.468	1.000	0.468	infty	1.000	infty	infty	1.000	infty
14	1.503	1.000	1.503	0.684	1.000	0.684	1.028	1.000	1.028
15	1.107	1.000	1.107	0.874	1.000	0.874	0.968	1.000	0.968
16	1.067	1.000	1.067	0.922	1.000	0.922	0.984	1.000	0.984
17	1.026	1.000	1.026	0.889	1.000	0.889	0.912	1.000	0.912
18	0.364	1.000	0.364	infty	1.000	infty	infty	1.000	infty
19	1.023	1.000	1.023	0.877	1.000	0.877	0.898	1.000	0.898
20	2.108	1.000	2.108	0.923	1.000	0.923	1.946	1.000	1.946

Table 2: PM productivity index under FDH production technology

Table 3 reports average annual PM productivity indices over the analysed period. In most of the period 2012-2018, the PM productivity indices indicate that there are pollution-adjusted productivity improvements (*ie.*, $PM^{\phi} > 1$), especially in the period 2014-2015. Meanwhile in the periods 2013-2014 and 2016-2017 there are pollution-adjusted productivity decline (*ie.*, $PM^{\phi} < 1$). The combination of polluting and no polluting productivity variations permits to show the main drivers of the pollution-adjusted productivity change. The gains in pollution-adjusted productivity over the periods 2017-2018 and 2012-2013 come from increasing productivity in no polluting components (*ie.*, $PM^{1,-1,0} > 1$). Indeed, for these periods a loss in polluting productivity arise (*ie.*, $PM^{1,0,1} < 1$). However, the no polluting productivity growth compensates the polluting productivity decline for the periods 2017-2018 and 2012-2013 (*ie.*, $PM^{1,-1,0} \times PM^{1,0,1} > 1$). Regarding the periods 2014-2015 and 2015-2016, the reverse results occur. In such case, the pollution-adjusted productivity progress come from increasing productivity in polluting components (ie., $PM^{1,-1,0} < 1$, $PM^{1,0,1} > 1$ and $PM^{1,-1,0} \times PM^{1,0,1} > 1$). Remark that over the period 2013-2014, the no polluting productivity decrease does not compensate polluting productivity improvement such that pollution-adjusted productivity decline arises (*ie.*, $PM^{1,-1,0} < 1$, $PM^{1,0,1} > 1$ and $PM^{1,-1,0} \times PM^{1,0,1} < 1$). In the same way pollution-adjusted productivity loss occurs over the period 2016-2017 however, this productivity decrease results from both polluting and no polluting productivity decline (*ie.*, $PM^{1,-1,0} < 1$, $PM^{1,0,1} < 1$ and $PM^{1,-1,0} \times PM^{1,0,1} < 1$). Globally, the average values of the Malmquist productivity indices over the period 2012-2018 indicate pollution-adjusted productivity growth which essentially comes from increasing productivity in polluting components.

T	$PM^{1,-1,0}$	$EC^{1,-1,0}$	$TC^{1,-1,0}$	$PM^{1,0,1}$	$EC^{1,0,1}$	$TC^{1,0,1}$	PM^{ϕ}	EC^{ϕ}	TC^{ϕ}	**Infty**
17-18	1.057	1.000	1.057	0.962	1.000	0.962	1.013	1.000	1.013	6
16-17	0.888	1.000	0.888	0.992	1.000	0.992	0.881	1.000	0.881	8
15-16	0.896	1.000	0.896	1.382	1.000	1.382	1.138	1.000	1.138	5
14-15	0.849	1.000	0.849	1.877	1.000	1.877	1.383	1.000	1.383	6
13-14	0.939	1.000	0.939	1.003	1.000	1.003	0.986	1.000	0.986	6
12-13	1.212	1.000	1.212	0.929	1.000	0.929	1.130	1.000	1.130	4
Mean	0.973	1.000	0.973	1.191	1.000	1.191	1.089	1.000	1.089	6

Table 3: Average annual PM productivity change

The decomposition of the PM pollution-adjusted productivity indices presents the main sources of productivity change in both polluting and no polluting dimensions. Table 3 displays the two main components of the average annual pollution-adjusted productivity variation, namely the efficiency variation and the technological change. It is worth noting that pollution-adjusted productivity variation especially comes from the technological change component over the analysed period. This outcome is not surprising. Indeed, the PM productivity indices are estimated based on FDH pollution-generating process which maps the smallest non convex production possibility set satisfying the B-disposal property. Hence, the considered DMUs belong to the production frontiers such that they are efficient for each year. Obviously, this result may be biaised by slacks. In addition, the proposed pollution-adjusted productivity decomposition does not permit to determine each part of the productivity change inducing infeasibility; see the first column from the right in Table 3. Table 2 allows to go further in details by describing the main driver of pollution-adjusted productivity infeasibility; see *eg.*, DMU 4 over the period 2017-2018. Remark that the issue of infeasibility comes from the technological change component[3]. Besides, regarding polluting and no polluting productivity decomposition, the infeasibility issue specifically comes from the polluting technological change over the period 2012-2018.

Hicks-Moorsteen pollution-adjusted productivity index

The results displayed in Table 4 show the PHM index scores and its main components, namely the efficiency change $\mathcal{EC}^{\phi}$, the technological variation $\mathcal{TC}^{\phi}$ and the scale efficiency change $\mathcal{SEC}^{\phi}$. Precisely, the three first columns from the right present the PHM index outcomes and, its polluting and no polluting components; *ie.*, PHM^{ϕ}, $PHM^{0,0,1}$ and $PHM^{0,-1,0}$, respectively. Columns 6, 9 and 12 from the right reveal the main drivers of the PHM productivity change, namely the scale efficiency change $\mathcal{SEC}^{\phi}$, the technological variation $\mathcal{TC}^{\phi}$ and the efficiency change $\mathcal{EC}^{\phi}$, respectively. Besides, columns 4-5, 7-8 and 10-11 focus on the polluting and no polluting parts of the PHM productivity variation displaying $\mathcal{SEC}^{0,0,1}$–$\mathcal{SEC}^{0,-1,0}$, $\mathcal{TC}^{0,0,1}$–$\mathcal{TC}^{0,-1,0}$ and $\mathcal{EC}^{0,0,1}$–$\mathcal{EC}^{0,-1,0}$, respectively.

DMUs	$\mathcal{EC}^{\phi}$	$\mathcal{EC}^{0,-1,0}$	$\mathcal{EC}^{0,0,1}$	$\mathcal{TC}^{\phi}$	$\mathcal{TC}^{0,-1,0}$	$\mathcal{TC}^{0,0,1}$	$\mathcal{SC}^{\phi}$	$\mathcal{SC}^{0,-1,0}$	$\mathcal{SC}^{0,0,1}$	$PHM^{0,-1,0}$	$PHM^{0,0,1}$	PHM^{ϕ}
2017-2018												
1	1.000	1.000	1.000	1.349	1.449	0.931	0.741	0.684	1.084	0.991	1.009	1.000
2	1.000	1.000	1.000	1.162	1.046	1.111	0.915	0.832	1.100	0.870	1.222	1.063
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.186	0.833	0.988
4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.126	0.833	0.938
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.952	1.032	0.982
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.344	0.829	1.115
7	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	2.605	1.716	4.471
8	1.000	1.000	1.000	infty	infty	0.226	infty	infty	5.210	0.832	1.179	0.981
9	1.000	1.000	1.000	0.718	0.860	0.835	1.072	1.524	0.703	1.310	0.587	0.769
10	1.000	1.000	1.000	1.009	1.325	0.761	0.851	0.947	0.898	1.255	0.684	0.858
11	1.000	1.000	1.000	0.776	1.028	0.754	1.296	1.682	0.770	1.730	0.581	1.005
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.517	0.903	1.369
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.991	1.009	1.000
14	1.000	1.000	1.000	1.031	1.515	0.681	0.970	0.654	1.483	0.991	1.009	1.000
15	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.271	0.784	0.997
16	1.000	1.000	1.000	infty	infty	0.953	infty	infty	0.714	1.506	0.680	1.024
17	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.278	0.789	1.008
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.040	0.939	0.977
19	1.000	1.000	1.000	1.975	2.051	0.963	0.506	0.483	1.048	0.991	1.009	1.000
20	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.919	1.163	1.069
2016-2017												
1	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.040	0.962	1.000
2	1.000	1.000	1.000	infty	infty	0.090	infty	infty	1.245	4.437	0.112	0.498
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.005	0.557	0.560
4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.921	1.144	1.053
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.046	0.697	0.729
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.396	0.735	1.026
7	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.195	0.688	0.822
8	1.000	1.000	1.000	infty	infty	0.824	infty	infty	1.527	0.814	1.258	1.024
9	1.000	1.000	1.000	0.985	0.796	1.238	1.015	1.306	0.777	1.040	0.962	1.000
10	1.000	1.000	1.000	infty	infty	0.850	infty	infty	1.135	0.972	0.965	0.938
11	1.000	1.000	1.000	1.670	1.705	0.979	0.618	0.314	1.966	0.536	1.925	1.031
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.449	0.384	0.172
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.040	0.962	1.000
14	1.000	1.000	1.000	1.529	1.726	0.886	0.752	0.575	1.309	0.992	1.160	1.150
15	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.229	0.647	0.795
16	1.000	1.000	1.000	infty	infty	1.168	infty	infty	1.050	0.609	1.227	0.748
17	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.118	0.588	0.657
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.030	0.693	0.714
19	1.000	1.000	1.000	2.429	1.830	1.327	2.877	0.595	4.837	1.089	6.420	6.989
20	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.013	1.165	1.180
2015-2016												
1	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.888	1.123	0.997
2	1.000	1.000	1.000	infty	infty	2.417	infty	infty	5.892	0.066	14.243	0.944
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.415	2.443	1.013

[3] The technological change component is defined based on complete cross-period distance functions; see (2.24). These complete cross-period distance functions induce infeasibility when they do not encounter production frontiers (Abad, 2015).

4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.061	1.155	1.226
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.858	0.433	0.804
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.602	1.530	0.922
7	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.973	0.962	0.937
8	1.000	1.000	1.000	infty	infty	1.125	infty	infty	0.667	1.001	0.750	0.751
9	1.000	1.000	1.000	0.970	1.114	0.871	0.791	0.471	1.679	0.525	1.462	0.767
10	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.168	0.834	0.974
11	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.697	2.510	1.750
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.814	1.494	1.217
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.647	1.627	1.052
14	1.000	1.000	1.000	1.449	1.412	1.026	0.922	0.478	1.932	0.674	1.983	1.337
15	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.618	1.571	0.971
16	1.000	1.000	1.000	infty	infty	1.111	infty	infty	0.986	0.976	1.096	1.070
17	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.757	1.929	1.461
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.529	1.943	1.027
19	1.000	1.000	1.000	1.037	0.916	1.132	0.657	1.205	0.545	1.104	0.617	0.681
20	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.880	4.343	3.821
2014-2015												
1	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.438	1.866	0.818
2	1.000	1.000	1.000	infty	infty	4.700	infty	infty	1.348	0.878	6.333	5.561
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.619	2.992	1.853
4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.544	2.004	1.091
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.982	1.135	1.115
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.515	2.176	1.119
7	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.352	1.100	0.388
8	1.000	1.000	1.000	infty	infty	2.313	infty	infty	0.992	0.463	2.295	1.063
9	1.000	1.000	1.000	1.730	1.296	1.335	0.671	0.844	0.795	1.094	1.061	1.160
10	1.000	1.000	1.000	5.812	3.013	1.929	0.196	0.196	1.001	0.590	1.930	1.138
11	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.487	3.135	1.527
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.783	3.271	2.562
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.373	3.853	1.438
14	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.366	10.080	3.688
15	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.479	2.027	0.972
16	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.417	2.406	1.002
17	1.000	1.000	1.000	infty	infty	1.426	infty	infty	1.290	0.436	1.840	0.803
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.447	2.351	1.052
19	1.000	1.000	1.000	1.559	1.562	0.998	0.251	0.165	1.520	0.258	1.518	0.392
20	1.000	1.000	1.000	1.702	1.475	1.154	0.748	0.684	1.093	1.009	1.261	1.272
2013-2014												
1	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.809	1.238	1.002
2	1.000	1.000	1.000	1.449	1.438	1.008	0.690	0.617	1.119	0.886	1.128	1.000
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.181	1.974	2.331
4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.710	1.451	1.030
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.519	1.881	0.976
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.616	0.862	0.531
7	1.000	1.000	1.000	0.958	1.149	0.834	0.570	0.566	1.007	0.651	0.839	0.546
8	1.000	1.000	1.000	infty	infty	0.560	infty	infty	2.263	1.126	1.268	1.427
9	1.000	1.000	1.000	0.787	0.807	0.976	0.994	1.359	0.732	1.096	0.714	0.783
10	1.000	1.000	1.000	5.995	5.074	1.181	0.216	0.190	1.136	0.964	1.342	1.293
11	1.000	1.000	1.000	1.303	1.240	1.051	0.889	0.889	0.999	1.103	1.050	1.158
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.986	0.945	0.932
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.148	1.896	2.176
14	1.000	1.000	1.000	0.958	1.196	0.801	1.093	0.816	1.340	0.976	1.073	1.048
15	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.735	1.029	0.756
16	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.722	1.292	0.933
17	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.734	1.028	0.754
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.254	1.925	2.413
19	1.000	1.000	1.000	1.138	0.834	1.364	3.372	1.933	1.745	1.612	2.380	3.836
20	1.000	1.000	1.000	1.000	0.825	1.212	0.433	0.642	0.674	0.530	0.817	0.433
2012-2013												
1	1.000	1.000	1.000	0.977	1.023	0.955	1.070	1.053	1.016	1.077	0.970	1.045
2	1.000	1.000	1.000	1.020	0.826	1.234	0.859	1.731	0.496	1.430	0.612	0.876
3	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.725	0.736	0.534
4	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.408	0.655	0.922
5	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.722	0.569	0.980
6	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.522	0.891	1.356
7	1.000	1.000	1.000	0.830	0.847	0.980	1.872	1.640	1.142	1.388	1.119	1.554
8	1.000	1.000	1.000	infty	infty	0.523	infty	infty	0.640	2.623	0.335	0.877
9	1.000	1.000	1.000	0.612	0.631	0.971	0.937	2.852	0.329	1.798	0.319	0.574
10	1.000	1.000	1.000	0.624	0.508	1.230	2.058	3.438	0.598	1.746	0.736	1.285
11	1.000	1.000	1.000	0.767	0.980	0.783	0.983	1.322	0.744	1.295	0.582	0.754
12	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	2.006	0.355	0.713
13	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.535	0.495	0.265
14	1.000	1.000	1.000	1.003	1.004	0.999	0.897	1.850	0.485	1.857	0.485	0.900
15	1.000	1.000	1.000	1.098	0.585	1.876	0.881	2.200	0.401	1.288	0.752	0.968
16	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.166	0.844	0.984
17	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	1.241	0.722	0.896
18	1.000	1.000	1.000	infty	infty	infty	infty	infty	infty	0.647	0.542	0.350
19	1.000	1.000	1.000	1.088	1.308	0.832	0.825	0.948	0.870	1.240	0.724	0.898
20	1.000	1.000	1.000	0.621	0.602	1.033	5.490	3.390	1.619	2.039	1.672	3.410

Table 4: PHM productivity index under FDH production technology

Table 5 shows the average annual PHM productivity index and its prominent drivers over the period 2012-2018. The average annual PHM productivity index scores indicate pollution-adjusted productivity growth (*ie.*, $PHM^{\phi} > 1$) over the analysed period. Regarding the polluting and no polluting parts of the PHM productivity variation permits to go a bit further in details. Specifically, three specific schemes of the pollution-adjusted productivity change arise over the period 2012-2018:

i. The pollution-adjusted productivity variation is driven by no polluting productivity improvement over the periods 2012-2013 and 2017-2018. In such case, the raise of the no polluting PHM productivity index compensates the polluting productivity loss (*ie.*, $PHM^{0,-1,0} > 1$, $PHM^{0,0,1} < 1$ and $PHM^{0,-1,0} \times PHM^{0,-1,0} > 1$).

ii. A reciprocal result occurs over the periods 2013-2014, 2014-2015 and 2015-2016. Although no polluting productivity decline arises over these periods, the growth in pollution-adjusted productivity comes from polluting productivity gains (*ie.*, $PHM^{0,-1,0} < 1$, $PHM^{0,0,1} > 1$ and $PHM^{0,-1,0} \times PHM^{0,-1,0} > 1$).

iii. The improvement in pollution-adjusted productivity is driven by both polluting and no polluting productivity gains over the period 2016-2017 (*ie.*, $PHM^{0,-1,0} > 1$, $PHM^{0,0,1} > 1$ and $PHM^{0,-1,0} \times PHM^{0,-1,0} > 1$). Besides, pollution-adjusted productivity growth essentially comes from increasing productivity in polluting dimension over the period 2016-2017 (*ie.*, $PHM^{0,-1,0} > 1$, $PHM^{0,0,1} > 1$ and $PHM^{0,-1,0} < PHM^{0,0,1}$).

Globally, a similar design as **iii.** arises regarding the PHM pollution-adjusted productivity change for the overall analysed periods.

T	$\mathcal{EC}^{\phi}$	$\mathcal{EC}^{0,-1,0}$	$\mathcal{EC}^{0,0,1}$	$\mathcal{TC}^{\phi}$	$\mathcal{TC}^{0,-1,0}$	$\mathcal{TC}^{0,0,1}$	$\mathcal{SC}^{\phi}$	$\mathcal{SC}^{0,-1,0}$	$\mathcal{SC}^{0,0,1}$	$PHM^{0,-1,0}$	$PHM^{0,0,1}$	PHM^{ϕ}
17-18	1.000	1.000	1.000	1.146 [13]	1.325 [13]	0.802 [11]	0.907 [13]	0.972 [13]	1.446 [11]	1.235	0.940	1.181
16-17	1.000	1.000	1.000	1.653 [16]	1.514 [16]	0.920 [12]	1.316 [16]	0.697 [16]	1.731 [12]	1.148	1.163	1.154
15-16	1.000	1.000	1.000	1.152 [17]	1.147 [17]	1.280 [14]	0.790 [17]	0.718 [17]	1.950 [14]	0.813	2.202	1.186
14-15	1.000	1.000	1.000	2.701[16]	1.836 [16]	1.979 [13]	0.466 [16]	0.472 [16]	1.148 [13]	0.577	2.732	1.501
13-14	1.000	1.000	1.000	1.699 [12]	1.570 [12]	0.999 [11]	1.032 [12]	0.876 [12]	1.224 [11]	0.918	1.306	1.268
12-13	1.000	1.000	1.000	0.864 [10]	0.831 [10]	1.038 [9]	1.587 [10]	2.042 [10]	0.758 [9]	1.438	0.706	1.007
Mean	1.000	1.000	1.000	1.536	1.371	1.170	1.016	0.963	1.376	1.021	1.508	1.216

[X] indicates the number of infeasibility

Table 5: Average annual PHM productivity change

Table 5 displays the three main drivers of the average annual PHM productivity variation, namely the efficiency change, the technological variation and the scale efficiency change. The PHM indices (*ie.*, $PHM^{0,-1,0}$, $PHM^{0,0,1}$ and PHM^{ϕ}) decomposition indicates that the pollution-adjusted productivity change is neutral with respect to the efficiency variation. Precisely, the PHM productivity change especially comes from the technological variation and the scale efficiency change components. The FDH specification of the technology results in this outcome since it sets the smallest non convex production technology. Interestingly, the scale efficiency change components permits to appraise the change in productivity along the production boundaries. Hence, a remarkable result arises since slacks are ubiquitous in the FDH model. Even though the technological variation and the scale efficiency change may present infeasibility, the PHM productivity indices scores always have finite values providing pollution-adjusted productivity insights for each production unit (*ie.*, DMUs: 1-20) at each period (*ie.*, T: 2012-2018). Table 4 shows that the infeasibility issue arises simultaneously in the technological variation and the scale efficiency change dimensions over the analysed period[4]. Regarding the polluting and no polluting parts of the technological variation and the scale efficiency change, Table 4 indicates that the infeasibility issue essentially arises from the no polluting components; *eg.*, see the DMUs 8 and 16 over the period 2017-2018.

To summarize regarding the PM and the PHM pollution-adjusted productivity outcomes, it is worth noting that the PHM productivity index permits to display a comprehensive pollution-adjusted productivity analysis by avoiding infeasibility issue. However, it might be impossible to decompose the PM and the PHM pollution-adjusted productivity change. Specifically, infeasibility issue happens from the technological variation and the scale efficiency change components which are defined through cross-period distance functions. Remarkably, the PM and the PHM pollution-adjusted productivity indices are neutral with respect to the efficiency change over the analysed period. The neutrality with respect to the efficiency change component especially comes from the selected non convex FDH analytical framework.

5 Concluding comments

This paper aims to analyse environmental productivity change through the pollution-adjusted Malmquist and Hicks-Moorsteen productivity indices. Moreover, the prominent sources of the pollution-adjusted productivity variation are provided by considering polluting and no polluting parts of the productivity variation. To drive this investigation, non convex pollution-generating technology is considered through the free disposal hull production model.

The empirical illustration provided in this paper focuses on the Ecuadorian oil industry. Precisely, a sample of 20 Ecuadorian oil companies over the period 2012-2018 is selected. The results are provided through a FDH non convex DEA model. The proposed theoretical framework permits to characterize the pollution-adjusted productivity variation. Specifically, the empirical outcomes show that pollution-adjusted productivity change may be driven by either polluting or no polluting components. The pollution-adjusted productivity decomposition allows to go a bit further in details by displaying the prominent drivers of both polluting and no polluting productivity variations.

In the analysed period, pollution-adjusted productivity change especially comes from the technological variation and the scale efficiency change components which might induce infeasibility. Besides, the pollution-adjusted productivity variation appears as neutral with regard to the efficiency change. A further work could consider the non

[4]Note that, the infeasibility issue comes from the technological variation and the scale efficiency change components since they are defined by means of cross-period distance functions (Abad, 2015); see (2.24) and (2.25).

convex multiplicative production process (Banker and Mandiratta, 1986) to set the analysis.

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