Environmental Productivity Assessment: an Illustration with the Ecuadorian Oil Industry

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Abstract

In this paper, environmental productivity change is analysed through the production theoretic approach to index numbers. Specifically, pollution-adjusted Malmquist and Hicks-Moorsteen productivity indices are considered. These productivity indices are defined as combination of multiplicative distance functions. Non convex pollution-generating technology is assumed to estimate the pollution-adjusted Malmquist and Hicks-Moorsteen productivity measures. Moreover, the main sources of the environmental productivity change are displayed. An empirical illustration is provided by considering a sample of 20 Ecuadorian oil companies over the period 2014-2018. The results are estimated through a non parametric analytic framework.

Keywords: Data Envelopment Analysis (DEA), Ecuadorian Oil Industry, Environmental Efficiency, Productivity Indices, Non Convexity, Pollution-generating Technology.

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1 Introduction

Oil represents 32 percent of the global energy consumption sources [20]. The World Energy Outlook [20], claims that energy generated from fossil fuels will remain the major source and is still expected to meet about 84 percent of energy demand in 2030. According to British Petroleum [10], South and Central America have 18.7 percent (324.1 thousand of millions of barrels) of the world's proven reserves. In terms of production, Ecuador is the fifth oil producer in South America with an average production of 27.94 million of tons from 2009 to 2019. There is research into other reliable energy resources to replace fossil fuel, considering its depletion and the environmental impacts generated by this industry. However, it is expected that the energy market will continue to depend on fossil fuels for at least the next few decades.

Among all industry sectors, the petroleum industry is of particular interest to Ecuador because of its economic and environmental significance. Oil is the second most important sector for the Ecuadorian economy. The contribution of the oil sector was 11 percent of the Gross Domestic Product (GDP) for the period 2011-2018 approximately. Oil is also important for the Ecuadorian energy sector; in 2018, there was a primary energy production of 216 million Barrels of Oil Equivalent (BOE). Of the total produced, 86.9 percent was made up of oil. According to the Third National Communication on Climate Change and First Biennial Update Report [31], the energy sector produced 37 594 Gg of carbon dioxide equivalent (CO_2e) which represents 47 percent of total GreenHouse Gas (GHG) emissions in 2012. Energy industry is a significant contributor of GHG emissions in the country, especially for the burning of fossil fuels. In 2012 this activity accounted for 36 822.54 Gg (CO_2e) which represents the 97.95 percent of emissions of the energy sector. Thus, oil companies need to be more efficient and make a balance between pollution mitigation and economic success.

The performance change assessment of Ecuadorian petroleum companies needs to consider a methodology integrating the companies' environmental indicators with their operational measures. Following the production theoretic approach to index numbers, the performance variation is commonly assessed through productivity indices [25]. In this paper, environmental productivity is appraised through the non convex Pollution-adjusted Malmquist (PM) and Hicks-Moorsteen (PHM) productivity indices [1, 2]. The PM productivity measure takes the form of the Malmquist index [15, 12] whilst the PHM productivity measure inherits the structure of the Hicks-Moorsteen index [7]. Specifically, the PM and the PHM productivity indices display the change of economic and polluting outputs induced by inputs variation by relaxing the convexity property of pollution-generating technology. It is worth noting that pollution-generating processes encompass many human and ecological interactions that can induce non linearities [32, 14][1]. Although relaxing the convexity property of production technologies has been investigated in the literature [8], few studies consider non convex pollution-generating technologies. The only theoretical model that considers non convex pollution-generating processes has been introduced in Abad and Briec [3]. This model provides axiomatic

[1]Dasgupta and Mäler [14] mention that: "The word convexity is ubiquitous in economics, but absent from ecology".

foundation of the non convex version of the Murty *et al.*'s by-production model [36, 23]. Knowing the prominent drivers of productivity change is a major concern in applied economics literature [5, 22, 26]. As a result, this paper displays the main components of the pollution-adjusted productivity variation by considering the case of the Ecuadorian oil industry. Moreover, the main sources of environmental productivity change are highlighted by separating polluting and non polluting dimensions.

Recently, numerous papers investigate environmental efficiency and productivity variation of the oil sector [34, 35, 28, 29]. In this area, non parametric mathematical programming methods for production analysis are widely applied to appraise technical productivity variation[2] [6, 13, 18, 30, 37]. In this paper, non parametric production model is considered to highlight the practicability of the approach provided. Specifically, non convex Free Disposal Hull (FDH) by-production model is defined to estimate the PM and the PHM indices [3, 33]. Interestingly, this modelling does not need to explicitly specify the mathematical form of the production function which characterised the pollution-generating technologies. Moreover, it allows to assess the environmental productivity of multi-input and multi-output production units by relaxing the convexity property of the pollution-generating technologies. To the best of our knowledge, there has been no research performed in the field of oil industry that analyses pollution-adjusted productivity change based upon non convex FDH approximation of pollution-generating production model.

An empirical illustration is provided by considering a sample of 20 Ecuadorian private oil companies over the period 2014-2018. Almost all private companies are engaged in exploration and production, while only a relatively small fraction of firms participate in other activities such as transport and distribution. The outputs of the oil companies are separated into economic (*i.e.*, desirable) and polluting (*i.e.*, undesirable) components; number of oil barrels and CO_2 emissions, respectively. In the segment, CO_2 emissions are generated directly through drilling processes and fossil fuel combustion and indirectly through well leaks and venting.

The remainder of this paper is structured as follows. Section 2 displays the production theoretic framework allowing to define the PM and the PHM indices. The non parametric estimation of the pollution-generating technology and the environmental productivity indices are proposed in Section 3. The empirical illustration is provided in Section 4. Finally, Section 5 discusses and concludes.

2 Methodology

This section presents the theoretical basis that is considered in this paper. Based upon this theoretical framework, pollution-adjusted efficiency and productivity indices are introduced.

[2]Note that allocative productivity change can also be identified essentially focusing on cost and profit functions [17, 19, 21].

2.1 Pollution-generating process: definition and properties

Assume that the outputs are separated into economic (*i.e.*, desirable) and polluting (*i.e.*, undesirable) components[3]. Let $\mathtt{I}$ and $\mathtt{O}$ be the input and output sets such that $\mathtt{I} := \{x_t \in \mathbb{R}^n_+ : n \in \mathbb{N}^*\}$ and $\mathtt{O} := \{y_t = (y^{\mathtt{d}}_t, y^{\mathtt{u}}_t) \in \mathbb{R}^{m^{\mathtt{d}}+m^{\mathtt{u}}}_+ : m^{\mathtt{d}}, m^{\mathtt{u}} \in \mathbb{N}^*\}$. The input and output vectors for the period (t) are defined as $(x_t, y_t) \in \mathbb{R}^{m+n}_+$, where $m = m^{\mathtt{d}} + m^{\mathtt{u}}$.

The pollution-generating technology is defined as follows,

$$T_t := \left\{(x_t, y_t) \in \mathbb{R}^{n+m}_+ : x_t \text{ can produce } (y^{\mathtt{d}}_t, y^{\mathtt{u}}_t)\right\}. \tag{2.1}$$

Usual characterisations of T_t are the output set, $P : \mathbb{R}^n_+ \mapsto 2^{\mathbb{R}^m_+}$, and the input correspondence, $L : \mathbb{R}^m_+ \mapsto 2^{\mathbb{R}^n_+}$,

$$P(x_t) := \left\{(y^{\mathtt{d}}_t, y^{\mathtt{u}}_t) \in \mathbb{R}^m_+ : (x_t, y_t) \in T_t\right\} \tag{2.2}$$

and

$$L(y^{\mathtt{d}}_t, y^{\mathtt{u}}_t) := \left\{x_t \in \mathbb{R}^n_+ : (x_t, y_t) \in T_t\right\}. \tag{2.3}$$

In this paper, alternative characterisations of the pollution-generating process are considered through the undesirable set, $\mathcal{Q} : \mathbb{R}^{m^{\mathtt{d}}}_+ \mapsto 2^{\mathbb{R}^{n+m^{\mathtt{u}}}_+}$, and the desirable correspondence, $\mathcal{Z} : \mathbb{R}^{m^{\mathtt{u}}}_+ \mapsto 2^{\mathbb{R}^{n+m^{\mathtt{d}}}_+}$,

$$\mathcal{Q}(y^{\mathtt{d}}_t) := \left\{(x_t, y^{\mathtt{u}}_t) \in \mathbb{R}^m_+ : (x_t, y_t) \in T_t\right\} \tag{2.4}$$

and

$$\mathcal{Z}(y^{\mathtt{u}}_t) := \left\{(x_t, y^{\mathtt{d}}_t) \in \mathbb{R}^m_+ : (x_t, y_t) \in T_t\right\}. \tag{2.5}$$

The sets (2.4)and (2.5) restrict T_t to the subspace of inputs and economic outputs and, to the subspace of inputs and polluting outputs, respectively [4].

In such case,

$$\left.\begin{array}{l} x_t \in L(y^{\mathtt{d}}_t, y^{\mathtt{u}}_t) \\ (y^{\mathtt{d}}_t, y^{\mathtt{u}}_t) \in P(x_t) \\ (x_t, y^{\mathtt{d}}_t) \in \mathcal{Z}(y^{\mathtt{u}}_t) \\ (x_t, y^{\mathtt{u}}_t) \in \mathcal{Q}(y^{\mathtt{d}}_t) \end{array}\right\} \Leftrightarrow (x_t, y_t) \in T_t \tag{2.6}$$

[3]Throughout the paper, the superscripts d and u denote the desirable and undesirable outputs, respectively.

Assume that the pollution-generating technology satisfies the following usual properties [16]:

$\mathcal{A}1$: *No free lunch and Inaction*; $(0,0) \in T_t$, $(0, y_t) \in T_t \Rightarrow y_t = 0$.
$\mathcal{A}2$: *Boundedness*; $T(y_t) = \{(x_t, v_t) \in T_t : v_t \leq y_t\}$ is bounded for all $y_t \in \mathbb{R}^m_+$.
$\mathcal{A}3$: *Closedness*; T_t is closed.

Let C be the convex cone such that: $\mathrm{C} := \{y_t \in \mathbb{R}^m : y^u_t \leq 0 \text{ and } y^d_t \geq 0\}$. In addition of the traditional axioms $\mathcal{A}1 - \mathcal{A}3$, suppose that the pollution-generating process satisfies the B-disposal assumption [3]:

$\mathcal{A}4$: *B-disposability*; $T_t := \Big(\big(T_t + (\mathbb{R}^n_+ \times -\mathbb{R}^m_+)\big) \cap \big(T_t + (\mathbb{R}^n_+ \times -\mathrm{C})\big) \Big) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+)$.

The theoretical model based upon the properties $\mathcal{A}1 - \mathcal{A}4$ permits to define the pollution-generating process as an intersection of sub-technologies: $T_t + (\mathbb{R}^n_+ \times -\mathbb{R}^m_+) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+)$ and $T_t + (\mathbb{R}^n_+ \times -\mathrm{C}) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+)$ [23]. The intended production activities of firms satisfy the usual strong disposability assumption; *ie.*, $T_t + (\mathbb{R}^n_+ \times -\mathbb{R}^m_+) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+)$. Moreover, partially reversed free disposal axiom applies for the polluting residuals generation; *ie.*, $T_t + (\mathbb{R}^n_+ \times -\mathrm{C}) \cap (\mathbb{R}^m_+ \times \mathbb{R}^n_+)$. It is worth noting that axioms $\mathcal{A}1 - \mathcal{A}4$ define a fairly weak axiomatic framework such that the convexity assumption is not required to define pollution-generating processes.

2.2 Pollution-adjusted efficiency and productivity indices

This section lays out the pollution-adjusted efficiency and productivity indices. Moreover, the main sources of the pollution-adjusted productivity change are highlighted by separating the polluting and non polluting dimensions.

2.2.1 Pollution-adjusted multiplicative distance function

The following definition presents the pollution-adjusted multiplicative distance function.

Definition 2.1 *Let T_t be a pollution-generating process that satisfies properties $\mathcal{A}1 - \mathcal{A}4$. For any $(x_t, y_t) \in \mathbb{R}^{n+m}_+$, where $y_t = (y^{\mathrm{d}}_t, y^{\mathrm{u}}_t) \in \mathbb{R}^m_+$, the multiplicative pollution-adjusted distance function, $\mathfrak{D}^\phi : \mathbb{R}^{n+m}_+ \longrightarrow \mathbb{R} \cup \infty$, is defined as follows :*

$$\mathfrak{D}^\phi_t(x_t, y_t) := \begin{cases} \inf\limits_\beta \left\{ \beta \in]0,1] : \ \left(\beta^\alpha x_t, \beta^{\gamma^{\mathrm{d}}} y^{\mathrm{d}}_t, \beta^{\gamma^{\mathrm{u}}} y^{\mathrm{u}}_t\right) \in T_t \right\} \\ \qquad\qquad \textit{if } \left(\beta^\alpha x_t, \beta^{\gamma^{\mathrm{d}}} y^{\mathrm{d}}_t, \beta^{\gamma^{\mathrm{u}}} y^{\mathrm{u}}_t\right) \in T_t, \beta > 0 \\ \infty \qquad\qquad \textit{else} \end{cases} \tag{2.7}$$

where $\phi = (\alpha, \gamma^{\mathrm{d}}, \gamma^{\mathrm{u}}) \in \{0,1\} \times \{-1,0\} \times \{0,1\}$.

Let us consider the following orientations for the pollution-adjusted distance function.

Proposition 2.2 *For any* $(x_t, y_t) \in \mathbb{R}_+^{n+m}$ *where* $y_t = (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in \mathbb{R}_+^m$*,*

i. $\mathfrak{D}_t^{1,-1,0}(x_t, y_t) \equiv \mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}(x_t, y_t)$.

ii. $\mathfrak{D}_t^{1,0,1}(x_t, y_t) \equiv \mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}(x_t, y_t)$.

iii. $\mathfrak{D}_t^{1,0,0}(x_t, y_t) \equiv \mathfrak{D}^{\mathtt{I}}(x_t, y_t)$.

iv. $\mathfrak{D}_t^{0,-1,0}(x_t, y_t) \equiv \mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}(x_t, y_t)$.

v. $\mathfrak{D}_t^{0,0,1}(x_t, y_t) \equiv \mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t)$.

The pollution-adjusted efficiency indices presented in the aforementioned results **i.**-**v.** fully identify pollution-generating processes [1, 2]. Moreover, remark that efficiency measures **i.** and **ii.** take the form of hyperbolic distance functions [16] whereas **iii.**, **iv.** and **v.** are the standard Shephard distance functions [27].

In such case,

$$\left.\begin{array}{l} \mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}(x_t, y_t) \in]0;1] \Leftrightarrow (x_t, y_t^{\mathtt{d}}) \in \mathcal{Z}(y_t^{\mathtt{u}}) \\ \mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}(x_t, y_t) \in]0;1] \Leftrightarrow (x_t, y_t^{\mathtt{d}}) \in \mathcal{Q}(y_t^{\mathtt{d}}) \\ \mathfrak{D}^{\mathtt{I}}(x_t, y_t) \in]0;1] \Leftrightarrow x_t \in L(y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \\ \mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}(x_t, y_t) \in]0;1] \Leftrightarrow (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in P(x_t) \\ \mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t) \in]0;1] \Leftrightarrow (y_t^{\mathtt{d}}, y_t^{\mathtt{u}}) \in P(x_t) \end{array}\right\} \Leftrightarrow (x_t, y_t) \in T_t \tag{2.8}$$

2.2.2 Malmquist pollution-adjusted productivity index

The next result displays the Malmquist pollution-adjusted productivity index [2].

Definition 2.3 *Suppose that* T_t *is a pollution-generating process satisfying properties* $\mathcal{A}1 - \mathcal{A}4$*. For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$*, where* $y_{t,t+1} = (y_{t,t+1}^{\mathtt{d}}, y_{t,t+1}^{\mathtt{u}}) \in \mathbb{R}_+^m$*, the Malmquist pollution-adjusted productivity index is defined as follows :*

$$PM_{t,t+1}^{\phi}(x_{t,t+1}, y_{t,t+1}) = \Big[PM_t^{\phi}(x_{t,t+1}, y_{t,t+1}) \times PM_{t+1}^{\phi}(x_{t,t+1}, y_{t,t+1})\Big]^{\frac{1}{2}} \tag{2.9}$$

such that $\phi = (\alpha, \gamma^{\mathtt{d}}, \gamma^{\mathtt{u}}) \in \{0,1\} \times \{-1,0\} \times \{0,1\}$.

The Malmquist pollution-adjusted productivity indices for the periods (t) and $(t+1)$ are presented in the next results.

$$PM_t^{\phi}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}(x_t, y_t)} \times \frac{\mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}(x_{t+1}, y_t^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}(x_t, y_t)} \tag{2.10}$$

and

$$PM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}. \tag{2.11}$$

If the multiplicative productivity index $PM^{\phi}_{t,t+1}$ is larger than 1 then, pollution adjusted productivity gains arise over periods (t) and $(t+1)$. In such case, the firms produce more non polluting outputs and operate managerial efforts to reduce their inputs and polluting outputs. Remark that the pollution-adjusted Malmquist productivity index defined in this paper, is a hyperbolic-based Malmquist productivity measure[4].

The following statement defines a decomposition of the Malmquist pollution-adjusted productivity variation by separating polluting and non polluting dimensions.

Proposition 2.4 *For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$ *where* $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}^m_+$,

$$PM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = PM^{\mathtt{d}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \times PM^{\mathtt{u}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}). \tag{2.12}$$

Where,

$$PM^{\mathtt{d}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t}(x_t, y_t)} \times \frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{d}}}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}\right]^{\frac{1}{2}}$$

and

$$PM^{\mathtt{u}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \left[\frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t}(x_t, y_t)}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{\mathtt{I},0^{\mathtt{u}}}_{t+1}(x_{t+1}, y_{t+1})}\right]^{-\frac{1}{2}}$$

Assume that the no polluting Malmquist productivity index $PM^{\mathtt{d}}_{t,t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than 1. In such case, more desirable outputs are produced and less inputs are used between the periods (t) and (t+1), for a given level of undesirable outputs. In the same vein, if the polluting Malmquist productivity index $PM^{\mathtt{u}}_{t,t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, less undesirable outputs are produced and less inputs are used between the periods (t) and (t+1), for a given level of desirable outputs. Obviously, reciprocal reasoning holds when $PM^{\mathtt{d}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \leq 1$ and $PM^{\mathtt{u}}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) \leq 1$.

The prominent components of the Malmquist pollution-adjusted productivity variation are presented in the next result.

[4]In contrary with the traditional input- and output-oriented Malmquist index, the hyperbolic version of the Malmquist productivity measure is defined as combination of hyperbolic distance functions. Although this version of the Malmquist index has been rarely empirically applied [38], this paper provides an empirical estimation of the PM productivity measure through a non parametric enumerative approach.

Definition 2.5 *Let T_t be a pollution-generating technology satisfying assumptions $\mathcal{A}1-\mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$, the decomposition of the Malmquist pollution-adjusted productivity change is defined as follows:*

$$PM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = EC^{\phi}_{t,t+1} \times TC^{\phi}_{t,t+1} \tag{2.13}$$

where $EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$ correspond to the efficiency variation and technological change components, respectively.

$EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$ are laid out in the next results.

$$\begin{aligned} EC^{\phi}_{t,t+1} &= EC^{\mathtt{d}}_{t,t+1} \times EC^{\mathtt{u}}_{t,t+1} \\ &= \frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t}(x_t, y_t)} \times \frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t+1}(x_{t+1}, y_{t+1})}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t}(x_t, y_t)} \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} TC^{\phi}_{t,t+1} &= \left(TC^{\mathtt{d}}_{t,t+1} \times TC^{\mathtt{u}}_{t,t+1}\right)^{\frac{1}{2}} \\ &= \Bigg[\left(\frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t}(x_t, y_t)}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t+1}(x_t, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})} \times \frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}_{t+1}(x_{t+1}, y_{t+1})}\right) \times \\ &\qquad \left(\frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t}(x_t, y_t)}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t+1}(x_t, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_t)} \times \frac{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t}(x_{t+1}, y^{\mathtt{d}}_t, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}_{t+1}(x_{t+1}, y_{t+1})}\right)\Bigg]^{\frac{1}{2}}. \end{aligned} \tag{2.15}$$

If the efficiency change $EC^{\phi}_{t,t+1}$ is greater than 1 then, efficiency progress arises over the periods (t) and $(t+1)$. Moreover, technological improvement occurs between the periods (t) and $(t+1)$ when $TC^{\phi}_{t,t+1} \geq 1$. Note that the main sources of the pollution-adjusted productivity variation, namely $EC^{\phi}_{t,t+1}$ and $TC^{\phi}_{t,t+1}$, are separated into polluting and non polluting components.

2.2.3 Hicks-Moorsteen pollution-adjusted productivity index

The Hicks-Moorsteen pollution-adjusted productivity index is presented in the next statement [1].

Definition 2.6 *Let T_t be a pollution-generating technology that satisfies assumptions $\mathcal{A}1-\mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$, with $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}_+^m$, the Hicks-Moorsteen pollution-adjusted productivity index is defined as follows,*

$$PHM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \Big[PHM^{\phi}_{t}(x_{t,t+1}, y_{t,t+1}) \times PHM^{\phi}_{t+1}(x_{t,t+1}, y_{t,t+1})\Big]^{\frac{1}{2}} \tag{2.16}$$

where $\phi = (\alpha, \gamma^{\mathtt{d}}, \gamma^{\mathtt{u}}) \in \{0,1\} \times \{-1,0\} \times \{0,1\}$.

$PHM_t^{\phi}(x_{t,t+1}, y_{t,t+1})$ and $PHM_{t+1}^{\phi}(x_{t,t+1}, y_{t,t+1})$ display the Hicks-Moorsteen pollution-adjusted productivity indices for the periods (t) and $(t+1)$, respectively. These productivity indices are defined as follows:

$$PHM_t^{\phi}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{d}}}(x_t, y_{t+1}^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{d}}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_t^{\mathtt{I}}(x_{t+1}, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{I}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \tag{2.17}$$

and

$$PHM_{t+1}^{\phi}(x_{t,t+1}, y_{t,t+1}) = \frac{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{d}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{d}}}(x_{t+1}, y_t^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{u}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{u}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_t, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}. \tag{2.18}$$

With regard to the aforementioned results, if $PHM_{t,t+1}^{\phi}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, pollution-adjusted productivity growth occurs.

The polluting and non polluting parts of the PHM productivity variation are defined in the following result.

Proposition 2.7 *For any* $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}_+^{n+m}$*, such that* $y_{t,t+1} = (y_{t,t+1}^{\mathtt{d}}, y_{t,t+1}^{\mathtt{u}}) \in \mathbb{R}_+^m$*,*

$$PHM_{t+1}^{\phi}(x_{t,t+1}, y_{t,t+1}) = PHM_{t+1}^{\mathtt{d}}(x_{t,t+1}, y_{t,t+1}) \times PHM_{t+1}^{\mathtt{u}}(x_{t,t+1}, y_{t,t+1}). \tag{2.19}$$

Where $PHM_{t+1}^{\mathtt{d}}(x_{t,t+1}, y_{t,t+1})$ *and* $PHM_{t+1}^{\mathtt{u}}(x_{t,t+1}, y_{t,t+1})$ *display the non polluting PHM index and the polluting PHM index, respectively.*

The productivity indices $PHM_{t+1}^{\mathtt{d}}(x_{t,t+1}, y_{t,t+1})$ and $PHM_{t+1}^{\mathtt{u}}(x_{t,t+1}, y_{t,t+1})$ are defined as follows,

$$\begin{aligned} PHM_{t+1}^{\mathtt{d}}(x_{t,t+1}, y_{t,t+1}) = {} & \left[\frac{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{d}}}(x_t, y_{t+1}^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{d}}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{d}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{d}}}(x_{t+1}, y_t^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}\right]^{1/2} \times \\ & \left[\frac{\mathfrak{D}_t^{\mathtt{I}}(x_{t+1}, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{I}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_t, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}\right]^{1/4} \end{aligned} \tag{2.20}$$

and

$$\begin{aligned} PHM_{t+1}^{\mathtt{u}}(x_{t,t+1}, y_{t,t+1}) = {} & \left[\frac{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{O}^{\mathtt{u}}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{u}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{O}^{\mathtt{u}}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_t^{\mathtt{u}})}\right]^{1/2} \times \\ & \left[\frac{\mathfrak{D}_t^{\mathtt{I}}(x_{t+1}, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})}{\mathfrak{D}_t^{\mathtt{I}}(x_t, y_t^{\mathtt{d}}, y_t^{\mathtt{u}})} \times \frac{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_{t+1}, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}{\mathfrak{D}_{t+1}^{\mathtt{I}}(x_t, y_{t+1}^{\mathtt{d}}, y_{t+1}^{\mathtt{u}})}\right]^{1/4} \end{aligned} \tag{2.21}$$

If $PHM^{\mathtt{d}}_{t+1}(x_{t,t+1}, y_{t,t+1})$ is greater than unity then, productivity improvement arises between the period (t) and $(t+1)$ with respect to the non polluting components. Correspondingly, $PHM^{\mathtt{u}}_{t+1}(x_{t,t+1}, y_{t,t+1}) > 1$ shows productivity growth in the polluting dimension.

The next statement allows to go a bit further in detail by highlighting the main components of the Hicks-Moorsteen pollution-adjusted productivity index.

Proposition 2.8 *Assume that T_t is a pollution-generating process satisfying properties $\mathcal{A}1 - \mathcal{A}4$. For any $(x_{t,t+1}, y_{t,t+1}) \in \mathbb{R}^{n+m}_+$, where $y_{t,t+1} = (y^{\mathtt{d}}_{t,t+1}, y^{\mathtt{u}}_{t,t+1}) \in \mathbb{R}^m_+$, the Hicks-Moorsteen pollution-adjusted productivity index is decomposed as follows:*

$$PHM^{\phi}_{t,t+1}(x_{t,t+1}, y_{t,t+1}) = \mathcal{EC}^{\phi}_{t,t+1} \times \mathcal{TC}^{\phi}_{t,t+1} \times \Sigma^{\phi}_{t,t+1} \tag{2.22}$$

such that $\mathcal{EC}^{\phi}_{t,t+1}$, $\mathcal{TC}^{\phi}_{t,t+1}$ and $\Sigma^{\phi}_{t,t+1}$ respectively correspond to the efficiency change, the technological variation and the residual components over the periods (t) and $(t+1)$.

Remark that the residual component $\Sigma^{\phi}_{t,t+1}$ is a wide component highlighting the part of the pollution-adjusted productivity which does not come from efficiency and technological changes. As a result, the traditional scale efficiency change component is incorporated within.

$\mathcal{EC}^{\phi}_{t,t+1}$, $\mathcal{TC}^{\phi}_{t,t+1}$ and $\Sigma^{\phi}_{t,t+1}$ are defined as follows:

$$\begin{aligned}\mathcal{EC}^{\phi}_{t,t+1} &= \mathcal{EC}^{\mathtt{d}}_{t,t+1} \times \mathcal{EC}^{\mathtt{u}}_{t,t+1} \\ &= \frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})},\end{aligned} \tag{2.23}$$

$$\begin{aligned}\mathcal{TC}^{\phi}_{t,t+1} &= \mathcal{TC}^{\mathtt{d}}_{t,t+1} \times \mathcal{TC}^{\mathtt{u}}_{t,t+1} \\ &= \left[\frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t+1}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{d}}}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}\right]^{\frac{1}{2}} \times \\ &\quad \left[\frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t+1}(x_{t}, y^{\mathtt{d}}_{t}, y^{\mathtt{u}}_{t})} \times \frac{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}{\mathfrak{D}^{\mathtt{O}^{\mathtt{u}}}_{t+1}(x_{t+1}, y^{\mathtt{d}}_{t+1}, y^{\mathtt{u}}_{t+1})}\right]^{\frac{1}{2}}\end{aligned} \tag{2.24}$$

and

$$\Sigma^{\phi}_{t,t+1} = \Sigma^{\mathtt{d}}_{t,t+1} \times \Sigma^{\mathtt{u}}_{t,t+1} \tag{2.25}$$

such that,

$$\Sigma^{\mathtt{d}}_{t,t+1} = \frac{PHM^{\mathtt{d}}_{t,t+1}}{\mathcal{EC}^{\mathtt{d}}_{t,t+1} \times \mathcal{TC}^{\mathtt{d}}_{t,t+1}} \tag{2.26}$$

and

$$\Sigma^{\mathtt{u}}_{t,t+1} = \frac{PHM^{\mathtt{u}}_{t,t+1}}{\mathcal{EC}^{\mathtt{u}}_{t,t+1} \times \mathcal{TC}^{\mathtt{u}}_{t,t+1}} \tag{2.27}$$

When $\mathcal{EC}^{\phi}_{t,t+1} > 1$ and $\mathcal{TC}^{\phi}_{t,t+1} > 1$ then, pollution-adjusted efficiency and technological improvements arise between the periods $(t, t+1)$[5]. Remark that the prominent drivers of the Hicks-Moorsteen pollution-adjusted productivity index (2.23)-(2.25) are separated into polluting and non polluting components.

3 Pollution-adjusted productivity change: non parametric specification

In this section, non parametric pollution-generating production technology is considered. Specifically, the pollution-adjusted productivity measures are defined through the FDH version of the by-production model [3, 23, 33].

3.1 Non convex pollution-generating production process

Let us introduce the following notation: $(x_t, y_t) \equiv (x, y)$. Assume that $\mathcal{U} := \{(x_s, y_s) : s \in \mathcal{S}\}$ is a set of production units, where $\mathcal{S}$ is an index set of integers. Moreover, for any $(x_s, y_s) \in \mathcal{U}$, consider the following individual production possibility sets:

$$\mathcal{I}_t(x_s, y_s) := \Big\{(x, y) \in \mathbb{R}^{n+m}_+ : x_i \geq x_{s,i},\ y_j \leq y_{s,r},\ i \in [n],\ r \in [m^{\mathtt{d}}]\Big\} \tag{3.1}$$

and

$$\mathcal{J}_t(x_s, y_s) := \Big\{(x, y) \in \mathbb{R}^{n+m}_+ : x_i \geq x_{s,i},\ y_l \geq y_{s,l},\ i \in [n],\ l \in [m^{\mathtt{u}}]\Big\}. \tag{3.2}$$

The next result presents the definition of the FDH non convex pollution-generating technology.

Definition 3.1 *Let T_t be a pollution-generating process that satisfies the axioms $\mathcal{A}1-\mathcal{A}4$. For any $(x_s, y_s) \in \mathcal{U}$, the FDH non convex pollution-generating technology is*

[5]Notice that the Hicks-Moorsteen pollution-adjusted productivity index is decomposed through an output orientation. As this productivity index is a total factor productivity measure, it can be decomposed following either an output orientation or an input orientation as well [1].

defined as follows,

$$T_t^{FDH} := \Big\{(x,y) \in \mathbb{R}_+^{n+m} : (x,y) \in \big(\cup_{s\in\mathcal{S}} \mathcal{I}_t(x_s,y_s)\big) \cap \big(\cup_{s\in\mathcal{S}} \mathcal{J}_t(x_s,y_s)\big)\Big\}. \quad (3.3)$$

The FDH by-production set (3.3) is defined as an intersection of non convex sub-technologies. $\cup_{s\in\mathcal{S}}\mathcal{I}_t(x_s,y_s)$ displays the usual strong disposal FDH sub-technology. The partially reversed free disposal FDH sub-technology corresponds to the non convex set $\cup_{s\in\mathcal{S}}\mathcal{J}_t(x_s,y_s)$.

Non parametric approximation of the non convex FDH pollution-generating technology is laid out in the following result.

Proposition 3.2 *Assuming that the pollution-generating process T_t satisfies the properties $\mathcal{A}1-\mathcal{A}4$ and Variable Returns to Scale (VRS), non parametric approximation of the FDH non convex pollution-generating technology is defined as follows,*

$$\begin{aligned} T_t^{FDH} := \Big\{(x,y) \in \mathbb{R}_+^{n+m} : x_i \geq \sum_{s\in\mathcal{S}} \lambda_s x_{s,i},\ y_r \leq \sum_{s\in\mathcal{S}} \lambda_s y_{s,r},\ x_i \geq \sum_{s\in\mathcal{S}} \nu_s x_{s,i}, \\ y_l \geq \sum_{s\in\mathcal{S}} \nu_s y_{s,l},\ \sum_{s\in\mathcal{S}} \nu_s = \sum_{s\in\mathcal{S}} \lambda_s = 1,\ \nu,\lambda \in \{0,1\},\ i \in [n] \\ r \in [m^{\mathrm{d}}],\ l \in [m^{\mathrm{u}}]\Big\}. \end{aligned} \quad (3.4)$$

3.2 Productivity index on non parametric pollution-generating technology

The next statement defines the pollution-adjusted multiplicative distance function with respect to the non parametric specification of the FDH non-convex pollution-generating process.

Definition 3.3 *For any $(x,y) \in \mathbb{R}_+^{n+m}$ and any $\phi = (\alpha,\gamma^d,\gamma^u) \in \{0,1\} \times \{-1,0\} \times \{0,1\}$, the pollution-adjusted multiplicative distance function is defined within T_t^{FDH} as follows,*

$$\begin{array}{lll} \mathfrak{D}_t^{\phi,FDH}(x_0,y_0) & = \min & \beta \\ s.t. & & \beta^{\alpha} x_{0,i} \geq \sum_{s\in\mathcal{S}} \lambda_s x_{s,i},\ i \in [n] \\ & & \beta^{\gamma^d} y_{0,r} \leq \sum_{s\in\mathcal{S}} \lambda_s y_{s,r},\ r \in [m^d] \\ & & \beta^{\alpha} x_{0,i} \geq \sum_{s\in\mathcal{S}} \nu_s x_{s,i},\ i \in [n] \\ & & \beta^{\gamma^{\mathrm{u}}} y_{0,l} \geq \sum_{s\in\mathcal{S}} \nu_s y_{s,l},\ l \in [m^u] \\ & & \sum_{s\in\mathcal{S}} \nu_s = \sum_{s\in\mathcal{S}} \lambda_s = 1 \\ & & \nu,\lambda \in \{0,1\}. \end{array} \quad (3.5)$$

The next proposition provides an enumeration process to evaluate the pollution-adjusted multiplicative distance function through the above (3.5) mathematical program, see [9]. Remark that the pollution-adjusted multiplicative efficiency index is non-linear and hence, would be related to a non-linear optimisation. To overcome this issue, the FDH non-parametric approach is used allowing to assess hyperbolic-based efficiency measure [16] through an enumeration process avoiding approximation results [38].

Proposition 3.4 *Assume that T_t is a pollution-generating process that satisfies properties $\mathcal{A}1 - \mathcal{A}4$. For any $(x, y) \in \mathbb{R}_+^{n+m}$, where $y = (y^{\mathtt{d}}, y^{\mathtt{u}}) \in \mathbb{R}_+^m$, the FDH pollution-adjusted multiplicative efficiency index is defined below:*

$$\mathfrak{D}_t^{\phi,FDH}(x_0, y_0) \equiv \begin{cases} \mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{d}}}(x_0, y_0) = \max\limits_{s \in \mathcal{S}} \left(\min \left\{ \max\limits_{i \in [n]} \left(\dfrac{x_{s,i}}{x_{0,i}} \right); \max\limits_{r \in [m^{\mathtt{d}}]} \left(\dfrac{y_{0,r}}{y_{s,r}} \right) \right\} \right) \\ \mathfrak{D}_t^{\mathtt{I},\mathtt{O}^{\mathtt{u}}}(x_0, y_0) = \max\limits_{s \in \mathcal{S}} \left(\min \left\{ \max\limits_{i \in [n]} \left(\dfrac{x_{s,i}}{x_{0,i}} \right); \max\limits_{l \in [m^{\mathtt{u}}]} \left(\dfrac{y_{s,l}}{y_{0,l}} \right) \right\} \right) \\ \mathfrak{D}_t^{\mathtt{I}}(x_0, y_0) = \max\limits_{s \in \mathcal{S}} \left(\min \left\{ \max\limits_{i \in [n]} \left(\dfrac{x_{s,i}}{x_{0,i}} \right) \right\} \right) \\ \mathfrak{D}_t^{\mathtt{O}^{\mathtt{d}}}(x_0, y_0) = \max\limits_{s \in \mathcal{S}} \left(\min \left\{ \max\limits_{r \in [m^{\mathtt{d}}]} \left(\dfrac{y_{0,r}}{y_{s,r}} \right) \right\} \right) \\ \mathfrak{D}_t^{\mathtt{O}^{\mathtt{u}}}(x_0, y_0) = \max\limits_{s \in \mathcal{S}} \left(\min \left\{ \max\limits_{l \in [m^{\mathtt{u}}]} \left(\dfrac{y_{s,l}}{y_{0,l}} \right) \right\} \right) \end{cases} \tag{3.6}$$

See proof in Appendix I.

4 Empirical illustration

This empirical illustration focuses on the oil companies having production and extraction activities in Ecuador over the period 2014-2018. The results are provided through a FDH non convex DEA model.

4.1 Data in brief

A sample of 20 private oil companies in Ecuador is considered over the period 2012–2018. The data set used in this research is built with the population of registered oil Ecuadorian formal firms, constructed from the balance sheets and financial statements registered on the official website of the Superintendencia de Compañias, Valores y Seguros (SCVS). This information is reported annually directly by firms to the SCVS.

Two inputs are selected: **(i)** number of formal employees of each company and **(ii)** net tangible assets (capital stock). Information about the number of legally registered employees **(i)** is declared by each company. The capital stock **(ii)** is set as the sum of the real dollar value of buildings, machinery and vehicles by assuming a depreciation of 5, 10, and 20 percent. Precisely, the methodology of Camino-Mogro and Bermudez-Barrezueta [11] is employed. Hence, the capital stock is valued considering the gross investment in equipment in year (t), net fixed assets in real value (physical capital in year $(t-1)$), a depreciation rate and the price index for equipment at the industry level obtained from the Ecuadorian National Institute of Statistics. These inputs permit to produce different outputs. Thus, we consider one desirable output, **(iii)** number of oil barrels and one undesirable output represented by **(iv)** CO_2 emissions. The number of extracted barrels of oil **(iii)** is defined based on the variable "sales" (American dollars) reported in the balance sheets and financial statements registered on the official website of the SCVS. Obviously, we divide it by the price (American dollars/barrel) to obtain the variable "number of extracted barrels of oil". The reference price (WTI) is considered allowing comparisons with other international research in the same field. The CO_2 emissions (tons of CO_2 equivalents) **(iv)** is measured by using the methodology of the 2006 IPCC Guidelines for National Greenhouse Gas Inventories.

Table 1 presents the descriptive statistics of the variables used in this study.

Variables	**Min**	**Max**	**Median**	**S.D.**	**Mean**
Labor	1	706	11.5	176.79	102.09
Capital stock	1760.86	173359632.8	629671.59	27682885.45	9293764.69
Oil production	725.15	9718986.9	315659.3	1411965.6	791729.5
CO_2 **emissions**	7.03	94270.37	3061.77	13695.51	7679.47

Table 1: Characteristics of inputs and outputs.[6]

4.2 Results and discussions

Malmquist pollution-adjusted productivity index

The results outlined in Table 2 display the number of observations which face either increasing, decreasing or constant global productivity variation (PM - 3rd column from the right) as well as non polluting ($PM^{\mathtt{d}}$ - 9th column from the right) and polluting ($PM^{\mathtt{u}}$ - 6th column from the right) oriented productivity change, over the period 2014-2018. Moreover, each productivity measure has been decomposed to reveal the main drivers of the productivity variation namely the technological change (global TC, polluting $TC^{\mathtt{u}}$ and no polluting $TC^{\mathtt{d}}$ one) and the efficiency change (global EC, polluting $EC^{\mathtt{u}}$ and no polluting $EC^{\mathtt{d}}$), respectively. These results are summarised in Figure 1.

[6] The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Periods		PM^{d}	EC^{d}	TC^{d}	PM^{u}	EC^{u}	TC^{u}	PM^{ϕ}	EC^{ϕ}	TC^{ϕ}
2017-2018	<1	9	6	5	10	6	7	9	7	5
	$=1$	1	13	1	2	13	1	3	13	2
	>1	10	1	14	8	1	12	8	0	13
2016-2017	<1	10	2	11	12	1	16	11	1	14
	$=1$	1	14	0	2	14	1	2	14	1
	>1	9	4	9	6	5	3	7	5	5
2015-2016	<1	13	4	14	8	3	6	13	3	13
	$=1$	0	13	0	0	13	1	0	13	0
	>1	7	3	6	12	4	13	7	4	7
2014-2015	<1	15	2	16	4	2	3	8	2	11
	$=1$	0	11	0	0	11	0	0	11	0
	>1	5	7	4	16	7	17	12	7	9

Table 2: Number of observations per component and per value of the PM productivity index.

Results in Table 2 are illustrated by Figure 1 which shows that observations facing productivity loss are higher than those facing productivity gains except during the period 2014-2015. Indeed, although the observations may present non polluting (respectively polluting) oriented productivity growth, a substantial degradation of the polluting (respectively non polluting) productivity results into a high number of observations facing a global productivity decrease over most of the considered periods. The productivity scores are displayed in Appendix II.

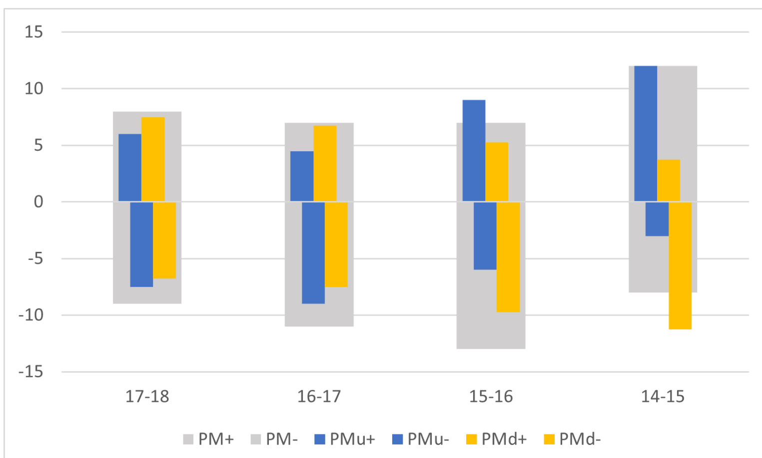

Figure 1: PM productivity index: observations per component and value.[7]

Table 3 reports the average annual PM productivity indices over the analysed period. In most of the period 2014-2018, the PM productivity indices indicate that there are pollution-adjusted productivity improvements (*ie.*, $PM^{\phi} > 1$), except during the period 2016-2017. Indeed, this period shows productivity losses in both desirable and polluting components which result in pollution-adjusted productivity decline (*ie.*, $PM^{\phi} < 1$),

[7]Remark that '+' and '-' indicate respectively productivity loss and gain. Thus, a value higher than zero indicates the number of observations presenting productivity gain whereas a value lesser than zero indicates the number of observations facing productivity loss.

Periods	PM^{d}	EC^{d}	TC^{d}	PM^{u}	EC^{u}	TC^{u}	PM^{ϕ}	EC^{ϕ}	TC^{ϕ}
2017-2018	1.044	0.934	1.117	0.962	0.864	1.114	1.004	0.807	1.245
2016-2017	0.797	0.984	0.810	0.897	1.217	0.737	0.715	1.198	0.597
2015-2016	0.932	1.052	0.886	1.189	1.328	0.895	1.108	1.397	0.793
2014-2015	0.813	1.136	0.715	1.328	1.270	1.046	1.079	1.443	0.748
Overall	0.891	1.024	0.870	1.081	1.154	0.936	0.963	1.182	0.815

Table 3: Average annual PM productivity change

during the period 2016-2017. The combination of polluting and non polluting productivity variations permits to show the main drivers of the pollution-adjusted productivity change. The gains in pollution-adjusted productivity over the period 2017-2018 come from increasing productivity in non polluting components (*ie.*, $PM^{\text{d}} > 1$). Indeed, during this period, a loss in polluting productivity arose (*ie.*, $PM^{\text{u}} < 1$). However, the non polluting productivity improvement compensates the polluting productivity decline for this period (*ie.*, $PM^{\text{d}} \times PM^{\text{u}} > 1$). Regarding the remaining periods, these latter present productivity gains in polluting (*ie.*, $PM^{\text{u}} > 1$) dimension whereas they face productivity decline (*ie.*, $PM^{\text{d}} < 1$) in non polluting components. This case results in global pollution-adjusted productivity growth (*ie.*, $PM^{\text{d}} \times PM^{\text{u}} > 1$). Globally, the average values of the Malmquist productivity indices over the period 2014-2018 indicate pollution-adjusted productivity loss which essentially comes from a substantial decreasing productivity in non polluting components. These outcomes are outlined by the Figure 2 which shows that the period 2015-2016 presents the higher pollution-adjusted Malmquist productivity growth over the considered periods.

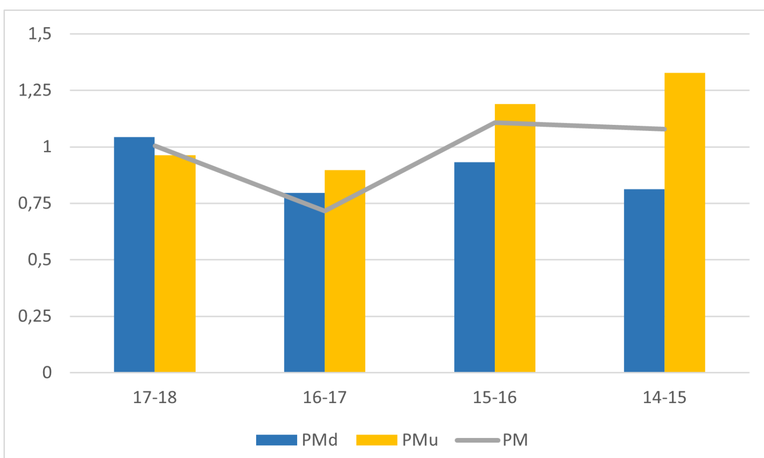

Figure 2: Trend of PM productivity change over periods

The decomposition of the PM pollution-adjusted productivity indices presents the main sources of productivity change in both polluting and non polluting dimensions. Table 3 displays the two main components of the average annual pollution-adjusted productivity variation, namely the efficiency variation and the technological change. It is worth noting that pollution-adjusted productivity variation especially comes from the efficiency change component over the analysed period.

Hicks-Moorsteen pollution-adjusted productivity index

Table 4 outlines the number of observations that presents increasing, decreasing or constant Hicks-Moorsteen pollution-adjusted productivity change (PHM^{ϕ} - 1st column from the right). This table also presents the non polluting (PHM^{d} - 3rd column form the right) and polluting (PHM^{u}- 2nd column form the right) productivity variations over the period 2014-2018. Moreover, the main components of the PHM productivity index has been provided, namely the efficiency change $\mathcal{EC}^{\phi}$, the technological variation $\mathcal{TC}^{\phi}$ and a residual component Σ^{ϕ}, as well as in both polluting ($\mathcal{TC}^{\mathrm{u}}$, $\mathcal{EC}^{\mathrm{u}}$, Σ^{u}) and non polluting ($\mathcal{TC}^{\mathrm{d}}$, $\mathcal{EC}^{\mathrm{d}}$, Σ^{d}) orientations.

		$\mathcal{EC}^{\phi}$	$\mathcal{EC}^{\mathrm{d}}$	$\mathcal{EC}^{\mathrm{u}}$	$\mathcal{TC}^{\phi}$	$\mathcal{TC}^{\mathrm{d}}$	$\mathcal{TC}^{\mathrm{u}}$	Σ^{ϕ}	Σ^{d}	Σ^{u}	PHM^{d}	PHM^{u}	PHM^{ϕ}
2017-2018	<1	5	5	5	6	5	12	8	8	6	8	12	10
	$=1$	4	12	5	1	0	0	2	2	4	0	0	4
	>1	11	3	10	8	10	3	5	5	5	12	8	6
	∞	0	0	0	5	5	5	5	5	5	0	0	0
2016-2017	<1	7	6	6	12	8	13	3	4	4	6	13	10
	$=1$	3	12	4	0	0	0	2	2	3	0	0	3
	>1	10	2	10	2	6	1	9	8	7	14	7	7
	∞	0	0	0	6	6	6	6	6	6	0	0	0
2015-2016	<1	8	4	9	10	13	2	7	9	7	14	5	10
	$=1$	3	8	5	0	0	0	0	0	0	0	0	0
	>1	9	8	6	6	3	14	9	7	9	6	15	10
	∞	0	0	0	4	4	4	4	4	4	0	0	0
2014-2015	<1	6	5	6	8	14	1	9	7	7	18	1	6
	$=1$	3	7	5	0	0	0	0	0	0	0	0	0
	>1	11	8	9	7	1	14	6	8	8	2	19	14
	∞	0	0	0	5	5	5	5	5	5	0	0	0

Table 4: Observations per component and per value for the PHM productivity index.

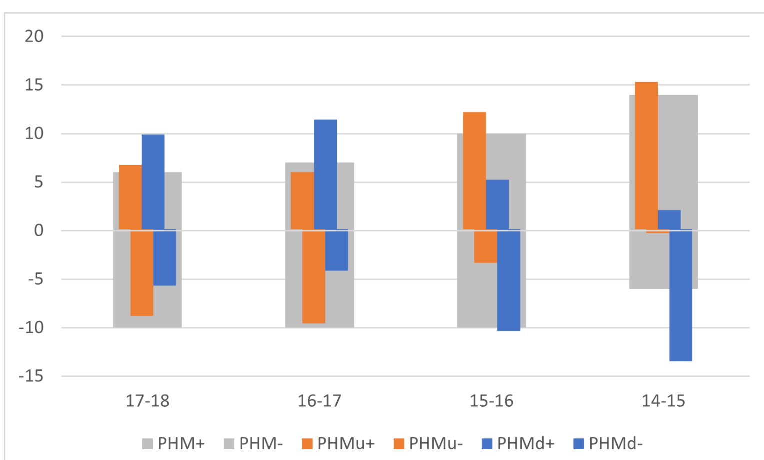

Figure 3: Observation per components and per variation for the PHM productivity index

The results in Table 4 are summarised in Figure 3. This figure highlights that most of observations face Hicks-Moorsteen productivity loss during the period 2014-2018, except during the period 2014-2015. Indeed, the non polluting productivity gain does not offset the productivity loss in the polluting dimension for most of the considered observations. The detailed Hick-Moorsteen productivity scores are presented in Appendix III.

(t)	$\mathcal{EC}^{\phi}$	$\mathcal{EC}^{\mathrm{d}}$	$\mathcal{EC}^{\mathrm{u}}$	$\mathcal{TC}^{\phi}$	$\mathcal{TC}^{\mathrm{d}}$	$\mathcal{TC}^{\mathrm{u}}$	Σ^{ϕ}	Σ^{d}	Σ^{u}	PHM^{d}	PHM^{u}	PHM^{ϕ}
2017-2018	0.804	0.923	0.871	0.933 [5]	1.093	0.853	1.360	1.097	1.240	1.133	0.867	0.982
2016-2017	1.246	0.952	1.310	0.439 [6]	0.867	0.507	1.410	1.277	1.104	1.012	0.860	0.870
2015-2016	0.895	1.129	0.793	0.928 [4]	0.659	1.410	1.630	1.031	1.581	0.728	1.581	1.150
2014-2015	1.412	1.054	1.339	1.028 [5]	0.459	2.241	0.676	1.019	0.664	0.519	2.215	1.150
Overall	1.061	1.011	1.049	0.791	0.732	1.081	1.206	1.101	1.095	0.811	1.271	1.031

[X] indicates the number of observations subjected to infeasibility in the technological change component.

Table 5: Average annual PHM productivity change

Table 5 shows the average annual PHM productivity index and its prominent drivers over the period 2014-2018. The average annual PHM productivity index scores indicate pollution-adjusted productivity growth (*ie.*, $PHM^{\phi} > 1$) over the analysed period. The polluting and non polluting parts of the PHM productivity variation permit to go a bit further in details. Specifically, two specific schemes of the productivity change arise over the period 2014-2018:

i. The pollution-adjusted productivity loss is driven by polluting productivity decrease during the periods 2016-2017 and 2017-2018. In such case, the non polluting productivity growth does not compensate the polluting productivity loss (*ie.*, $PHM^{\mathrm{d}} > 1$, $PHM^{\mathrm{u}} < 1$ and $PHM^{\mathrm{d}} \times PHM^{\mathrm{u}} < 1$).

ii. The improvement in pollution-adjusted productivity is driven by the polluting productivity gains during the periods 2015-2016 and 2015-2014 (*ie.*, $PHM^{\mathrm{d}} < 1$, $PHM^{\mathrm{u}} > 1$ and $PHM^{\mathrm{d}} \times PHM^{\mathrm{u}} > 1$). And the increasing productivity in polluting components offsets the productivity loss in non polluting dimension.

Globally, a similar design as **ii.** arises regarding the PHM pollution-adjusted productivity change for the overall analysed periods. These different schemes are illustrated in Figure 4.

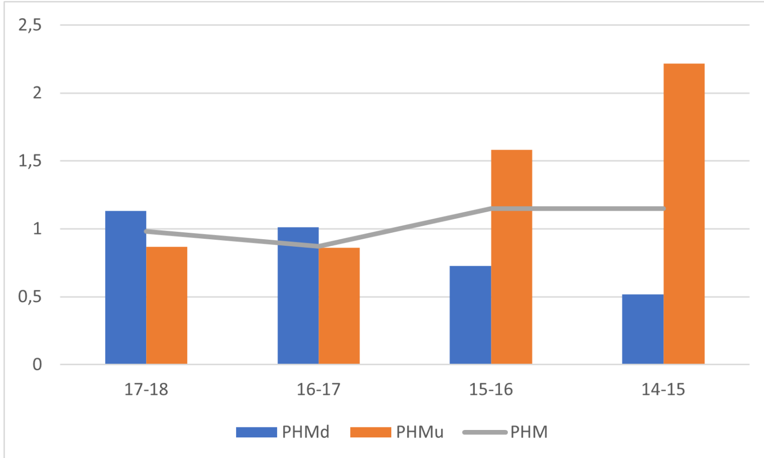

Figure 4: Trend of average annual PHM productivity variation over periods.

Table 5 also displays the three main drivers of the average annual PHM productivity variation, namely the efficiency change, the technological variation and a residual component. The decomposition of PHM indices (*ie.*, PHM^{d}, PHM^{u} and PHM^{ϕ}) shows that the pollution-adjusted productivity change is induced by the positive efficiency variations in both polluting and non polluting dimensions. The 1$^{\mathrm{st}}$ column from the right in Table 5 elicits the number of observations that are subjected to indeterminateness in the technological change variation[8] and in the residual component. Thus, the average of these components is only based upon the observations that do not present indeterminateness and the results should be interpreted with caution. However, even though the technological variation may present infeasibility, the PHM productivity indices scores always have finite values providing pollution-adjusted productivity insights for each production unit (see Appendix III). Hence, it is always possible to assess the productivity variation over periods in global, polluting and no polluting dimensions.

To summarise, the results provided by the PM productivity index coincide with those provided by the PHM productivity index, globally. However, the PM is a local technical variation measure whereas the PHM is a total factor productivity change measure which is defined as a ratio of input and output indices [24]. Moreover, the disaggregation of each productivity index, into polluting and non polluting components highlights the contribution of each dimension into the productivity variation. And results show that the productivity growth in the polluting dimension contributes the most in the global environmental productivity improvement.

5 Concluding comments

This paper aims to analyse environmental productivity change through the pollution-adjusted Malmquist and Hicks-Moorsteen productivity indices. Moreover, the promi-

[8]The technological change component is defined based upon complete cross-period distance functions; see (2.24). These complete cross-period distance functions induce infeasibility when they do not encounter the production frontier as mentioned in [4].

nent sources of the pollution-adjusted productivity variation are provided by considering polluting and no polluting parts of the productivity variation. To drive this investigation, non convex pollution-generating technology is considered through the free disposal hull production model.

The empirical illustration provided in this paper focuses on the Ecuadorian oil industry. Precisely, a sample of 20 Ecuadorian oil companies over the period 2014-2018 is selected. The results are provided through a FDH non convex DEA model. The proposed theoretical framework permits to characterise the pollution-adjusted productivity variation. Specifically, the approach provided in this paper permits to show that pollution-adjusted productivity change may be driven by either polluting or no polluting components. Moreover, the pollution-adjusted productivity decomposition allows to go a bit further in details by displaying the prominent drivers of both polluting and no polluting productivity variations. In this line, the empirical illustration shows that environmental productivity improvement comes mainly from an increasing productivity in polluting components. A further research could investigate this productivity analysis through additive-based productivity measures and could provide a comparison of the results with those of multiplicative-based measures.

Disclosure of interest

The authors report there are no competing interests to declare.

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Appendix I

Proof of Proposition 3.4: Assume that $\phi = (1,0,0)$. Following properties $\mathcal{A}1 - \mathcal{A}4$, the FDH pollution-generating process is defined as an intersection of non convex sub-technologies; see (3.3). Thus, for any $i \in [n],\ r \in [m^{\mathrm{d}}],\ l \in [m^{\mathrm{u}}]$, it follows that:

$$\begin{aligned}
\mathfrak{D}_t^{\phi,FDH}(x,y) &= \max_{s\in\mathcal{S}} \Bigg\{ \quad \min_{\beta} \left\{\beta \in]0,1] : \beta x_{0,i} \geq x_{s,i}, y_{0,r} \leq y_{s,r}\right\}; \min_{\beta} \left\{\beta \in]0,1] : \right. \\
&\qquad\qquad \left. \beta x_{0,i} \geq x_{s,i}, y_{0,l} \geq y_{s,l}\right\} \Bigg\}, \\
&= \max_{s\in\mathcal{S}} \Bigg\{ \quad \min_{\beta} \left\{\beta \in]0,1] : \beta \geq \max_{i\in[n]} \left(\frac{x_{s,i}}{x_{0,i}}\right)\right\} \Bigg\}.
\end{aligned}$$

The proof for $\phi = (1,0,1)$, $\phi = (1,-1,0)$, $\phi = (0,-1,0)$ and $\phi = (0,0,1)$ can be immediately deduced from the aforementioned results. $\square$

Appendix II: Malmquist Pollution-adjusted productivity index

DMUs	$PM^{\mathrm{d}}_{t,t+1}$	$EC^{\mathrm{d}}_{t,t+1}$	$TC^{\mathrm{d}}_{t,t+1}$	$PM^{\mathrm{u}}_{t,t+1}$	$EC^{\mathrm{u}}_{t,t+1}$	$TC^{\mathrm{u}}_{t,t+1}$	$PM^{\phi}_{t,t+1}$	$EC^{\phi}_{t,t+1}$	$TC^{\phi}_{t,t+1}$
				2017-2018					
1	0.995	0.852	1.169	1.000	0.457	2.190	0.995	0.389	2.560
2	0.960	0.672	1.429	0.960	0.672	1.429	0.922	0.452	2.041
3	1.079	1.000	1.079	0.915	1.000	0.915	0.988	1.000	0.988
4	1.011	1.000	1.011	0.938	1.000	0.938	0.948	1.000	0.948
5	0.753	0.874	0.862	0.753	0.874	0.862	0.567	0.764	0.743
6	1.115	1.000	1.115	1.115	1.000	1.115	1.243	1.000	1.243
7	1.597	1.000	1.597	1.600	1.000	1.600	2.555	1.000	2.555
8	0.981	1.000	0.981	1.070	1.000	1.070	1.049	1.000	1.049
9	0.714	0.500	1.429	0.714	0.500	1.429	0.510	0.250	2.041
10	0.883	0.857	1.030	0.883	0.857	1.030	0.779	0.734	1.061
11	1.725	1.741	0.991	0.612	0.428	1.429	1.056	0.746	1.415
12	1.361	1.000	1.361	1.050	1.000	1.050	1.429	1.000	1.429
13	0.995	1.000	0.995	1.005	1.000	1.005	1.000	1.000	1.000
14	0.995	0.683	1.458	1.005	1.091	0.921	1.000	0.745	1.343
15	1.138	1.000	1.138	0.997	1.000	0.997	1.135	1.000	1.135
16	1.024	1.000	1.024	0.915	1.000	0.915	0.937	1.000	0.937
17	1.008	1.000	1.008	1.008	1.000	1.008	1.016	1.000	1.016
18	1.008	1.000	1.008	0.977	1.000	0.977	0.985	1.000	0.985
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	0.985	1.000	0.985	1.069	1.000	1.069	1.053	1.000	1.053
				2016-2017					
1	1.020	1.000	1.020	1.000	1.000	1.000	1.020	1.000	1.020
2	0.204	0.417	0.490	0.204	0.417	0.490	0.042	0.174	0.240
3	0.817	1.000	0.817	0.610	1.000	0.610	0.499	1.000	0.499
4	1.053	1.000	1.053	1.053	1.000	1.053	1.110	1.000	1.110
5	0.930	1.000	0.930	0.729	1.000	0.729	0.678	1.000	0.678
6	1.072	1.000	1.072	0.957	1.000	0.957	1.026	1.000	1.026
7	1.021	1.000	1.021	0.822	1.000	0.822	0.839	1.000	0.839
8	1.024	1.000	1.024	1.115	1.000	1.115	1.142	1.000	1.142
9	1.000	1.667	0.600	1.000	1.667	0.600	1.000	2.778	0.360
10	0.857	1.000	0.857	0.857	1.000	0.857	0.735	1.000	0.735
11	0.528	0.507	1.040	1.025	2.091	0.490	0.541	1.061	0.509
12	0.231	1.000	0.231	0.214	1.000	0.214	0.050	1.000	0.050
13	1.020	1.000	1.020	0.981	1.000	0.981	1.000	1.000	1.000
14	1.054	1.303	0.808	1.149	1.925	0.597	1.210	2.509	0.482
15	0.830	1.090	0.762	0.929	1.169	0.795	0.771	1.274	0.605
16	0.599	1.000	0.599	0.810	1.000	0.810	0.485	1.000	0.485
17	0.658	1.000	0.658	0.841	1.000	0.841	0.553	1.000	0.553
18	0.902	1.000	0.902	0.714	1.000	0.714	0.644	1.000	0.644
19	1.479	1.450	1.020	12.071	15.583	0.775	17.851	22.603	0.790
20	1.175	1.000	1.175	1.180	1.000	1.180	1.386	1.000	1.386
				2015-2016					
1	0.9512	1.0000	0.9512	0.9966	1.0000	0.9966	0.9480	1.0000	0.9480
2	0.9437	1.0000	0.9437	1.0607	1.0000	1.0607	1.0010	1.0000	1.0010
3	0.7027	1.0000	0.7027	1.4142	1.0000	1.4142	0.9938	1.0000	0.9938
4	1.1878	1.0000	1.1878	1.2258	1.0000	1.2258	1.4560	1.0000	1.4560
5	1.1575	1.0000	1.1575	0.8043	1.0000	0.8043	0.9310	1.0000	0.9310
6	0.9215	1.0000	0.9215	0.9215	1.0000	0.9215	0.8492	1.0000	0.8492
7	0.9700	1.0016	0.9684	0.9436	42.5365	0.0222	0.9152	42.6051	0.0215
8	0.7508	1.0000	0.7508	0.7508	1.0000	0.7508	0.5637	1.0000	0.5637
9	0.7668	0.9200	0.8335	0.9200	0.9200	1.0000	0.7055	0.8464	0.8335
10	0.9437	1.0000	0.9437	0.9437	1.0000	0.9437	0.8906	1.0000	0.8906
11	0.5270	0.8358	0.6305	1.9255	1.7656	1.0906	1.0148	1.4758	0.6877
12	1.0453	1.0000	1.0453	1.2166	1.0000	1.2166	1.2717	1.0000	1.2717
13	0.7941	1.0000	0.7941	1.0797	1.0000	1.0797	0.8573	1.0000	0.8573

14	0.6633	0.9250	0.7171	1.3839	1.2889	1.0737	0.9179	1.1922	0.7699
15	0.8438	0.9173	0.9199	1.1186	0.8558	1.3072	0.9439	0.7850	1.2024
16	1.0705	1.0000	1.0705	1.0705	1.0000	1.0705	1.1459	1.0000	1.1459
17	1.1317	1.0000	1.1317	1.4608	1.0000	1.4608	1.6531	1.0000	1.6531
18	0.7271	1.0000	0.7271	1.0275	1.0000	1.0275	0.7471	1.0000	0.7471
19	1.3375	2.1212	0.6305	0.5567	0.4545	1.2247	0.7446	0.9642	0.7723
20	2.0118	1.9871	1.0125	10.9135	8.4512	1.2914	21.9559	16.7930	1.3074
2014-2015									
1	0.5223	1.0269	0.5086	0.6592	32.9026	0.0200	0.3443	33.7887	0.0102
2	3.8165	3.8022	1.0038	9.4916	7.5835	1.2516	36.2250	28.8342	1.2563
3	0.9978	1.0000	0.9978	1.8533	1.0000	1.8533	1.8493	1.0000	1.8493
4	0.7761	1.0000	0.7761	1.0909	1.0000	1.0909	0.8466	1.0000	0.8466
5	1.0756	1.0000	1.0756	1.1151	1.0000	1.1151	1.1993	1.0000	1.1993
6	0.8053	1.0000	0.8053	1.2516	1.0000	1.2516	1.0079	1.0000	1.0079
7	0.5661	1.8190	0.3112	0.8715	0.5642	1.5446	0.4934	1.0263	0.4807
8	1.0633	1.0000	1.0633	1.4917	1.0000	1.4917	1.5862	1.0000	1.5862
9	0.8696	1.0435	0.8333	0.8696	1.0435	0.8333	0.7561	1.0888	0.6944
10	1.1379	1.2000	0.9482	1.1379	1.2000	0.9482	1.2948	1.4400	0.8991
11	0.4853	1.2664	0.3832	2.2692	1.7968	1.2629	1.1011	2.2755	0.4839
12	1.9630	1.0000	1.9630	2.5619	1.0000	2.5619	5.0289	1.0000	5.0289
13	0.7098	1.0000	0.7098	1.4382	1.0000	1.4382	1.0209	1.0000	1.0209
14	0.2793	0.6122	0.4563	1.4607	1.1601	1.2592	0.4080	0.7102	0.5745
15	0.8986	1.0000	0.8986	1.4272	1.0000	1.4272	1.2825	1.0000	1.2825
16	0.8268	1.0000	0.8268	1.4142	1.0000	1.4142	1.1693	1.0000	1.1693
17	0.8032	1.0000	0.8032	1.1493	1.0000	1.1493	0.9232	1.0000	0.9232
18	0.6207	1.0000	0.6207	1.0517	1.0000	1.0517	0.6528	1.0000	0.6528
19	0.2889	0.6500	0.4445	0.2662	0.2259	1.1785	0.0769	0.1468	0.5238
20	0.9788	2.8735	0.3406	1.6138	1.4348	1.1248	1.5796	4.1229	0.3831
2013-2014									
1	0.8160	1.0392	0.7852	0.9162	0.1935	4.7344	0.7476	0.2011	3.7176
2	0.8864	1.0488	0.8452	1.0000	0.8883	1.1258	0.8864	0.9317	0.9514
3	1.4231	1.0000	1.4231	2.3312	1.0000	2.3312	3.3174	1.0000	3.3174
4	0.8592	1.0000	0.8592	1.0299	1.0000	1.0299	0.8849	1.0000	0.8849
5	0.7746	1.0000	0.7746	0.9757	1.0000	0.9757	0.7558	1.0000	0.7558
6	0.5305	1.0000	0.5305	0.5353	1.0000	0.5353	0.2840	1.0000	0.2840
7	0.8807	1.1320	0.7781	0.4329	0.2079	2.0820	0.3813	0.2354	1.6199
8	1.4269	1.0000	1.4269	1.4269	1.0000	1.4269	2.0360	1.0000	2.0360
9	0.7500	0.9375	0.8000	0.7500	0.9375	0.8000	0.5625	0.8789	0.6400
10	1.1250	1.0083	1.1157	1.1250	1.0083	1.1157	1.2655	1.0166	1.2448
11	1.0248	1.3172	0.7781	1.1579	1.1319	1.0230	1.1867	1.4908	0.7960
12	0.9317	1.0000	0.9317	0.9317	1.0000	0.9317	0.8681	1.0000	0.8681
13	1.2613	1.0000	1.2613	2.1759	1.0000	2.1759	2.7443	1.0000	2.7443
14	0.9704	0.7607	1.2757	0.9876	0.4830	2.0445	0.9583	0.3674	2.6082
15	0.7268	1.0000	0.7268	0.7558	1.0000	0.7558	0.5493	1.0000	0.5493
16	0.8646	1.0000	0.8646	0.9331	1.0000	0.9331	0.8067	1.0000	0.8067
17	0.7544	1.0000	0.7544	0.7544	1.0000	0.7544	0.5691	1.0000	0.5691
18	1.3679	1.0090	1.3557	2.4135	1.0090	2.3919	3.3014	1.0181	3.2426
19	1.2304	1.1345	1.0845	7.2042	6.9775	1.0325	8.8641	7.9161	1.1198
20	0.3388	0.2160	1.5688	0.1532	0.1150	1.3325	0.0519	0.0248	2.0904
2012-2013									
1	1.0538	1.0129	1.0405	1.1923	5.6640	0.2105	1.2565	5.7369	0.2190
2	1.5282	1.1695	1.3068	1.0883	1.1830	0.9200	1.6632	1.3835	1.2022
3	0.5191	1.0000	0.5191	0.2777	1.0000	0.2777	0.1442	1.0000	0.1442
4	1.1535	1.0000	1.1535	0.9217	1.0000	0.9217	1.0632	1.0000	1.0632
5	1.0777	1.0000	1.0777	1.0777	1.0000	1.0777	1.1615	1.0000	1.1615
6	1.3564	1.0000	1.3564	1.2371	1.0000	1.2371	1.6781	1.0000	1.6781
7	1.1136	1.0703	1.0405	1.7956	5.0097	0.3584	1.9995	5.3619	0.3729
8	0.8774	1.0000	0.8774	0.7071	1.0000	0.7071	0.6204	1.0000	0.6204
9	0.8667	0.8667	1.0000	0.8667	0.8667	1.0000	0.7511	0.7511	1.0000
10	1.6512	1.7948	0.9200	1.6512	1.7948	0.9200	2.7264	3.2215	0.8463
11	1.4913	1.4333	1.0405	0.6669	0.7843	0.8504	0.9946	1.1241	0.8848
12	1.0988	1.0000	1.0988	0.6489	1.0000	0.6489	0.7129	1.0000	0.7129
13	0.7122	1.0000	0.7122	0.2649	1.0000	0.2649	0.1886	1.0000	0.1886

14	1.7671	3.0312	0.5830	1.0311	3.1440	0.3280	1.8222	9.5303	0.1912
15	1.0000	1.0000	1.0000	0.9678	1.0000	0.9678	0.9678	1.0000	0.9678
16	0.9843	1.0000	0.9843	0.9843	1.0000	0.9843	0.9688	1.0000	0.9688
17	0.9122	1.0000	0.9122	0.8963	1.0000	0.8963	0.8176	1.0000	0.8176
18	0.5420	0.9911	0.5468	0.1585	0.9911	0.1599	0.0859	0.9822	0.0874
19	1.2836	2.3919	0.5366	0.7996	1.1474	0.6969	1.0263	2.7446	0.3739
20	2.8980	5.0876	0.5696	10.3248	13.7989	0.7482	29.9213	70.2038	0.4262

Table 6: PM productivity index under FDH production technology

Appendix III: Hicks-Moorsteen Pollution-adjusted productivity index

DMUs	$\mathcal{EC}^{\phi}_{t,t+1}$	$\mathcal{EC}^{\mathrm{d}}_{t,t+1}$	$\mathcal{EC}^{\mathrm{u}}_{t,t+1}$	$\mathcal{TC}^{\phi}_{t,t+1}$	$\mathcal{TC}^{\mathrm{d}}_{t,t+1}$	$\mathcal{TC}^{\mathrm{u}}_{t,t+1}$	$\Sigma^{\phi}_{t,t+1}$	$\Sigma^{\mathrm{d}}_{t,t+1}$	$\Sigma^{\mathrm{u}}_{t,t+1}$	$PHM^{\mathrm{d}}_{t,t+1}$	$PHM^{\mathrm{u}}_{t,t+1}$	$PHM^{\phi}_{t,t+1}$
						2017-2018						
1	1.114	0.852	1.308	1.226	1.589	0.771	0.732	0.732	1.000	0.991	1.009	1.000
2	1.018	0.663	1.536	0.975	1.264	0.771	0.967	0.987	0.980	0.827	1.161	0.960
3	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.186	0.833	0.988
4	1.115	1.000	1.115	0.853	1.106	0.771	0.986	1.018	0.969	1.126	0.833	0.938
5	0.338	0.251	1.349	0.093	0.121	0.771	23.828	27.454	0.868	0.834	0.903	0.753
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.344	0.829	1.115
7	1.052	1.000	1.052	1.350	1.750	0.771	1.126	0.890	1.265	1.558	1.027	1.600
8	0.848	0.848	1.000	0.206	0.912	0.226	5.603	1.076	5.210	0.832	1.179	0.981
9	1.308	1.507	0.868	1.327	1.720	0.771	0.411	0.487	0.845	1.262	0.566	0.714
10	0.020	1.064	0.019	7.740	1.419	5.456	5.602	0.843	6.644	1.273	0.694	0.883
11	0.521	0.694	0.751	1.518	1.967	0.771	0.805	1.009	0.798	1.377	0.462	0.637
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.517	0.903	1.369
13	1.000	1.000	1.000	1.000	0.991	1.009	1.000	1.000	1.000	0.991	1.009	1.000
14	1.308	1.000	1.308	0.936	1.213	0.771	0.816	0.816	1.000	0.991	1.009	1.000
15	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.271	0.784	0.997
16	1.205	1.502	0.802	1.017	1.214	0.838	0.835	0.826	1.012	1.506	0.680	1.024
17	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.278	0.789	1.008
18	1.232	1.000	1.232	0.669	0.868	0.771	1.185	1.199	0.988	1.040	0.939	0.977
19	0.848	1.000	0.848	1.180	0.991	1.191	1.000	1.000	1.000	0.991	1.009	1.000
20	1.459	1.000	1.459	1.139	1.476	0.771	0.644	0.623	1.034	0.919	1.163	1.069
						2016-2017						
1	1.039	1.000	1.039	0.962	1.040	0.926	1.000	1.000	1.000	1.040	0.962	1.000
2	0.512	4.974	0.103	0.237	4.503	0.053	1.682	0.127	13.261	2.841	0.072	0.204
3	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.005	0.557	0.560
4	1.204	1.000	1.204	0.968	1.046	0.926	0.904	0.881	1.026	0.921	1.144	1.053
5	0.882	1.000	0.882	∞	∞	∞	∞	∞	∞	1.046	0.697	0.729
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.396	0.735	1.026
7	0.819	1.000	0.819	0.859	0.928	0.926	1.168	1.288	0.906	1.195	0.688	0.822
8	0.774	0.774	1.000	3.326	0.914	3.637	0.398	1.151	0.346	0.814	1.258	1.024
9	1.039	1.000	1.039	0.745	0.805	0.926	1.291	1.291	1.000	1.040	0.962	1.000
10	39.199	0.728	53.837	0.016	1.176	0.013	1.400	1.085	1.291	0.929	0.922	0.857
11	1.039	0.507	2.048	0.359	0.388	0.926	2.764	2.722	1.015	0.536	1.925	1.031
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.449	0.384	0.172
13	2.083	1.000	2.083	0.480	1.040	0.462	1.000	1.000	1.000	1.040	0.962	1.000
14	1.039	0.890	1.168	0.677	0.731	0.926	1.707	1.558	1.095	1.013	1.184	1.200
15	0.273	1.090	0.250	∞	∞	∞	∞	∞	∞	1.229	0.647	0.795
16	1.292	0.678	1.906	0.430	0.856	0.502	1.079	0.940	1.147	0.545	1.099	0.599

17	0.250	1.000	0.250	∞	∞	∞	∞	∞	∞	1.118	0.588	0.657
18	0.886	1.000	0.886	0.733	0.792	0.926	1.099	1.301	0.845	1.030	0.693	0.714
19	27.555	0.396	69.564	0.035	0.151	0.232	17.636	28.388	0.621	1.701	10.034	17.071
20	1.159	1.000	1.159	1.049	1.134	0.926	0.971	0.893	1.086	1.013	1.165	1.180
						2015-2016						
1	0.830	1.000	0.830	1.014	0.749	1.354	1.184	1.186	0.998	0.888	1.123	0.997
2	0.225	0.225	1.000	0.585	0.242	2.417	7.165	1.216	5.892	0.066	14.243	0.944
3	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.415	2.443	1.013
4	0.770	1.000	0.770	2.452	1.811	1.354	0.649	0.586	1.107	1.061	1.155	1.226
5	0.356	1.000	0.356	0.525	0.388	1.354	4.294	4.788	0.897	1.858	0.433	0.804
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.602	1.530	0.922
7	0.735	1.002	0.734	0.098	0.073	1.354	12.935	13.366	0.968	0.973	0.962	0.937
8	1.833	1.833	1.000	0.960	0.854	1.125	0.427	0.640	0.667	1.001	0.750	0.751
9	1.171	0.950	1.232	0.868	0.641	1.354	0.905	0.944	0.959	0.575	1.601	0.920
10	0.035	1.887	0.019	7.337	8.160	0.899	3.670	0.075	49.139	1.150	0.821	0.944
11	1.171	0.836	1.401	0.673	0.497	1.354	2.220	1.678	1.323	0.697	2.510	1.750
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.814	1.494	1.217
13	0.653	1.000	0.653	1.786	0.735	2.428	0.902	0.879	1.026	0.647	1.627	1.052
14	1.171	0.925	1.266	0.560	0.414	1.354	2.196	1.829	1.200	0.700	2.058	1.441
15	1.218	1.523	0.800	0.490	0.549	0.891	1.550	0.721	2.151	0.603	1.533	0.925
16	0.785	1.497	0.525	1.708	0.771	2.215	0.798	0.846	0.944	0.976	1.096	1.070
17	8.302	2.073	4.005	∞	∞	∞	∞	∞	∞	0.757	1.929	1.461
18	1.415	1.000	1.415	0.985	0.727	1.354	0.737	0.727	1.014	0.529	1.943	1.027
19	1.171	2.121	0.552	1.429	1.055	1.354	0.356	0.462	0.772	1.033	0.577	0.596
20	3.259	1.987	1.640	0.834	0.616	1.354	4.014	1.215	3.304	1.487	7.340	10.914
						2014-2015						
1	1.037	1.027	1.010	0.138	0.068	2.044	4.594	5.658	0.812	0.393	1.676	0.659
2	28.384	1.196	23.724	0.968	0.106	9.137	0.345	9.047	0.038	1.147	8.274	9.492
3	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.619	2.992	1.853
4	0.939	1.000	0.939	1.841	0.901	2.044	0.631	0.604	1.044	0.544	2.004	1.091
5	0.526	1.000	0.526	4.395	2.151	2.044	0.482	0.457	1.056	0.982	1.135	1.115
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.515	2.176	1.119
7	1.572	1.819	0.864	1.486	0.727	2.044	0.225	0.310	0.725	0.410	1.280	0.525
8	0.661	0.661	1.000	6.672	0.553	12.067	0.241	1.269	0.190	0.463	2.295	1.063
9	1.572	3.263	0.482	1.125	0.550	2.044	0.492	0.527	0.933	0.947	0.918	0.870
10	33.783	1.136	29.728	0.167	0.473	0.353	0.202	1.096	0.184	0.590	1.930	1.138
11	1.572	1.266	1.241	0.615	0.301	2.044	1.934	1.414	1.368	0.539	3.471	1.871
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.783	3.271	2.562
13	3.243	1.000	3.243	∞	∞	∞	∞	∞	∞	0.373	3.853	1.438
14	1.572	0.612	2.568	0.713	0.349	2.044	0.959	0.925	1.037	0.198	5.441	1.075
15	0.753	0.602	1.250	0.800	0.627	1.277	1.613	1.270	1.270	0.479	2.027	0.972
16	0.612	0.612	1.000	∞	∞	∞	∞	∞	∞	0.417	2.406	1.002
17	0.120	0.482	0.250	1.344	0.471	2.854	4.963	1.922	2.582	0.436	1.840	0.803

18	1.122	1.000	1.122	0.910	0.445	2.044	1.030	1.004	1.026	0.447	2.351	1.052
19	1.572	1.326	1.186	1.949	0.954	2.044	0.061	0.142	0.434	0.179	1.051	0.188
20	1.572	2.874	0.547	0.900	0.440	2.044	1.161	0.906	1.282	1.146	1.433	1.643
						2013-2014						
1	1.313	1.039	1.264	1.765	1.803	0.979	0.395	0.413	0.957	0.774	1.184	0.916
2	0.342	0.297	1.153	1.689	1.725	0.979	1.729	1.729	1.000	0.886	1.128	1.000
3	1057.527	1.000	1057.527	0.129	3.397	0.038	0.017	0.348	0.049	1.181	1.974	2.331
4	1.460	1.000	1.460	1.259	1.287	0.979	0.560	0.552	1.015	0.710	1.451	1.030
5	1.945	1.000	1.945	0.717	0.733	0.979	0.700	0.708	0.988	0.519	1.881	0.976
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.616	0.862	0.531
7	1.313	1.132	1.160	2.294	2.344	0.979	0.111	0.192	0.579	0.510	0.657	0.335
8	1.260	1.260	1.000	1.795	0.839	2.139	0.631	1.064	0.593	1.126	1.268	1.427
9	0.625	0.758	0.825	1.438	1.469	0.979	0.834	0.963	0.866	1.073	0.699	0.750
10	1.209	1.003	1.205	0.825	0.843	0.979	1.128	1.063	1.061	0.899	1.251	1.125
11	1.313	1.317	0.997	1.070	1.094	0.979	0.824	0.766	1.076	1.103	1.050	1.158
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	0.986	0.945	0.932
13	4.345	1.000	4.345	0.504	1.703	0.296	0.994	0.674	1.475	1.148	1.896	2.176
14	1.313	1.226	1.071	1.224	1.251	0.979	0.614	0.618	0.994	0.948	1.042	0.988
15	4.000	1.000	4.000	∞	∞	∞	∞	∞	∞	0.735	1.029	0.756
16	9.129	1.000	9.129	∞	∞	∞	∞	∞	∞	0.722	1.292	0.933
17	4.000	1.000	4.000	∞	∞	∞	∞	∞	∞	0.734	1.028	0.754
18	1.289	1.019	1.266	3.412	3.486	0.979	0.549	0.353	1.554	1.254	1.925	2.413
19	1.313	1.058	1.242	0.331	0.339	0.979	7.402	4.124	1.795	1.477	2.181	3.222
20	0.342	0.270	1.269	2.285	2.335	0.979	0.196	0.500	0.391	0.315	0.486	0.153
						2012-2013						
1	2.283	1.013	2.254	0.185	0.440	0.421	2.819	2.582	1.092	1.151	1.036	1.192
2	8.753	5.632	1.554	0.249	0.592	0.421	0.499	0.478	1.043	1.594	0.683	1.088
3	2.392	1.000	2.392	0.098	0.232	0.421	1.188	2.254	0.527	0.523	0.531	0.278
4	1.620	1.000	1.620	0.336	0.797	0.421	1.695	1.765	0.960	1.408	0.655	0.922
5	1.365	1.000	1.365	0.856	2.033	0.421	0.923	0.889	1.038	1.807	0.597	1.078
6	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	1.522	0.891	1.356
7	2.283	1.070	2.133	0.157	0.373	0.421	6.469	4.250	1.522	1.695	1.367	2.317
8	5.651	5.651	1.000	0.209	1.178	0.177	0.744	0.394	1.888	2.623	0.335	0.877
9	2.541	2.540	1.000	0.162	0.386	0.421	2.101	2.257	0.931	2.210	0.392	0.867
10	1.818	1.179	1.542	0.446	1.059	0.421	2.037	1.586	1.285	1.980	0.834	1.651
11	2.283	1.433	1.593	0.365	0.868	0.421	1.203	1.201	1.002	1.494	0.672	1.004
12	1.000	1.000	1.000	∞	∞	∞	∞	∞	∞	2.006	0.355	0.713
13	0.315	1.000	0.315	0.726	0.475	1.528	1.158	1.127	1.028	0.535	0.495	0.265
14	2.283	1.881	1.213	0.339	0.805	0.421	1.333	1.313	1.015	1.988	0.519	1.031
15	1.207	4.824	0.250	1.627	0.533	3.052	0.493	0.501	0.984	1.288	0.752	0.968
16	0.661	2.372	0.279	1.724	0.565	3.052	0.864	0.870	0.992	1.166	0.844	0.984
17	0.250	1.000	0.250	2.000	0.655	3.052	1.793	1.893	0.947	1.241	0.722	0.896
18	2.135	0.982	2.175	0.103	0.245	0.421	0.721	1.811	0.398	0.435	0.364	0.158

19	2.283	1.258	1.815	0.459	1.089	0.421	0.764	0.854	0.894	1.170	0.683	0.800
20	8.753	4.070	2.151	0.121	0.289	0.421	9.709	3.021	3.213	3.549	2.909	10.325

Table 7: PHM productivity index under FDH production process