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Can Askan Mavi, Benteng Zou

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Can Askan Mavi[1] and Benteng Zou[2]

[1] *University of Paris-Saclay, INRAE, AgroParisTech, Paris-Saclay Applied Economics, 91120, Palaiseau*

[2] *DEM, University of Luxembourg, Luxembourg*

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Abstract

In this study, we present a modified Hartwick rule encompassing the dynamics of pandemic, such as COVID-19. In our setting, the labor productivity gradually improves after the pandemic shock and may even go beyond its pre-pandemic level due to the remote work and digitalization as also suggested by the empirical evidence. We demonstrate that a gradual labor productivity increase helps to conserve natural resources. We provide a theoretical foundation for a"sooner-the-better" strategy to control a pandemic, and we show that policy maker should implement a "whatever it costs" response to ensure that the transmission rate of the virus is below the recovery rate from the very beginning of the pandemic. Otherwise, the economy cannot have a sustained utility. We also analyze the implications of an "uncertain" pandemic on the intertemporal dynamics of natural resource and capital accumulation under the maximin criterion. Another important finding is that there exists a new economic and public health trade-off since a strong prevention policy is shown to decrease capital accumulation.

Keywords: Uncertainty, Maximin principle, Intergenerational equity, Pandemic, Hartwick's rule.
JEL classification: Q01, Q3, D8, Q56.

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1 Introduction

Many countries have accumulated significant amounts of debt because of the COVID-19 crisis (Kose et al., 2021). Countries such as Norway that invest natural resource rents into man-made capital, however, have remained resilient against the COVID-19 crisis thanks to their sovereign wealth funds (SWFs), which have allowed them to avoid increasing their public debt rates (Bortolotti and Fotak, 2020). In this sense, the pandemic has intensified the debate around the use of wealth funds for disasters.[1] [2] The question that motivates this study is the following: How should an economy build a wealth fund (i.e., capital accumulation) that helps mitigate economic losses due to harmful events such as pandemics?

A sovereign wealth fund (SWF) can be efficiently created using Hartwick's rule so as to sustainably manage national wealth (van der Ploeg, 2017). According to this famous rule pioneered by Hartwick (1977), if an economy invests the rents stemming from the extraction of natural resources into the net accumulation of physical capital, it follows an equitable and sustainable growth path by maximizing the utility of the most deprived one (maximin criterion) (Solow, 1974). Evidence also shows that countries applying Hartwick's rule are wealthier than those not following it (Hamilton et al., 2005).

In the context of a pandemic, the question of why Hartwick's rule is still important is crucial. It is evident that the pandemic has caused a reallocation of production factors such as labor, natural resources, and capital (Gromling, 2021). Thus, with the pandemic shock being accompanied by productivity changes, it is crucial to understand how we should extract natural resources and invest the associated rents in man-made capital so as to sustain utility and build an efficient wealth fund. Therefore, it is essential to outline a pandemic-modified Hartwick rule that encompasses the dynamics of the pandemic outbreak. A pandemic-modified Hartwick rule will provide new prescriptions for how to build a sovereign wealth fund in the context of a pandemic.

The creation of wealth funds is also motivated by the possibility of uncertain harmful events that can potentially occur in the future (see p. 48 in World Bank, 2014). Hence, it is important to have an idea of how the economy should build a wealth fund prior to the occurrence of an uncertain harmful event, with the aim of increasing the resilience of an economy against uncertain future shocks. In this line, there is a recent literature on precautionary savings in the context of uncertain harmful events such as droughts, floods, and hurricanes (van der Ploeg and de Zeeuw, 2016, 2017). However, these studies do not take into account the dynamics of a pandemic and do not focus on sustainability.

A large strand of the literature, starting with Hartwick (1977), strives to understand the connection between Hartwick's rule and sustainable development by ensuring a constant utility level over time (Hartwick, 1978; Dixit et al., 1980; Buckholtz and Hartwick, 1989; Hamilton, 1995; Asheim et al., 2007; Martinet, 2007; D'Autume and Schubert, 2008; D'Autume et al., 2010; Hartwick and Long, 2018). However, there are only a few papers studying uncertainty in the maximin context. Cairns and Long (2006) investigate the maximin case under uncertainty, with events that stochastically affect the evolution of stock variables. They show that a constant utility level cannot be guaranteed due to shocks that occur. In the same vein, Butterfield (2003) concentrates on the uncertainty regarding the future prices of extracted natural resources, presenting a version of Hartwick's rule with uncertainty. Van Long and Tian (2003) investigate Hartwick's rule where the uncertainty comes from international trade.

[1] See https://www.ft.com/content/46a6bdf4-c965-48ff-be58-820067b04e81

[2] see https://bit.ly/3sVkizv

To provide answers to the questions posed above, we recall the Dasgupta–Heal–Solow–Stiglitz (DHSS) model under the uncertainty of a pandemic occurring and in which we embed the simple epidemiological susceptible–infected–removed (SIR) model. Prior to the pandemic, the economy faces the risk of a pandemic occurring. Once the pandemic takes place, the active population is supposed to be constant but labor productivity changes over time. Labor productivity depends on the qualitative features of the disease, such as the transmission rate, the recovery rate (or removal), and prevention policy, which consists of measures such as lockdowns, social distancing, vaccination, etc. We adopt a similar approach to Bosi et al. (2021) for the modeling of prevention policy, but the economic literature also includes many recent contributions with policy instruments to mitigate the impacts of COVID-19 (Nævdal, 2020; d'Albis and Augeraud-Véron, 2021; Barbier, 2021).

We suppose that labor productivity decreases abruptly due to the pandemic shock but increases gradually afterwards and may go beyond the pre-pandemic productivity level. At first glance, this may seem unrealistic. However, the current evidence regarding COVID-19 shows that labor productivity, after decreasing sharply at the very beginning of lockdowns,[3] increases significantly due to the accelerated digitalization and automation that allowed for remote work (Bloom et al., 2020; Chernoff and Warman, 2020; IMF, 2021; Petropoulos, 2021). To ensure a robust analysis, in Section 4.1 we also consider the cases where labor productivity decreases, by relaxing the assumption that labor productivity increases after the pandemic.

The contribution of the current study is threefold. First, to the best of our knowledge this is the first rigorous framework seeking to understand the connection between sustainability and a pandemic. One of the main results of the study is that the policymaker should implement a "whatever it costs" policy response to decrease the ratio of infected/susceptible individuals (i.e., the prevalence rate). Otherwise, Hartwick's rule cannot be implemented and utility is not sustained over time. In this sense, our study offers an analytical basis for a "sooner the better" strategy to control a pandemic. An important result is that the policymaker should implement a "whatever it costs" policy if the natural resource is an essential and exhaustible input for production. We show that if there is an available substitute such as solar energy or wind instead of exhaustible resources, then the utility can be sustained even though the prevalence rate is increasing over time. In this sense, this result also shows the importance of energy substitutes regarding the sustainability of an economy.

The second contribution is that we present a new trade-off between public health policies (such as prevention) and capital accumulation in the context of Hartwick's rule. The mechanism is as follows: When the policymaker implements a prevention policy, labor productivity increases. This productivity increase naturally leads to lower natural resource extraction. It follows that there is a lower amount of rents stemming from the extraction of natural resources invested in physical capital accumulation. This result shows that sovereign wealth funds composed of natural resources rents may be at risk (Bortolotti and Fotak, 2020).[4] In other words, a policymaker faces an important trade-off between constructing a wealth fund and protecting the public health by a prevention policy. Thus, to get rid of this tradeoff, policymakers should redesign wealth funds to be built from other sources of revenue such as trade surpluses.

Third, this study is the first to examine the impacts of the probability of a pandemic occurring on man-made capital and natural resource depletion dynamics under the maximin criterion. To build a resilient economy

[3] The sharp decrease in labor productivity is mainly due to the lack of adaptation of many firms to remote work.

[4] Another risk mentioned by Bortolotti and Fotak (2020) is the sharp decrease in oil prices. Our study abstracts from the price dynamics of oil.

before a harmful event, it is important to know how an economy under uncertainty should build wealth funds (i.e., capital accumulation) prior to the occurrence of a pandemic. The probability of a pandemic is shown to generate precautionary behavior for natural resource conservation. Of course, this leads to lower capital accumulation since natural resource rents decrease. Again, policymakers should think about how to design a wealth fund in a world with uncertain harmful events. Last but not the least, we show that the probability of a harmful event decreases the maximum achievable utility. At this point, a prevention policy that increases labor productivity may help an economy compensate the decrease in utility due to the pandemic risk if the cost of the prevention policy is not too high.

The remainder of the paper is organized as follows. Section 2 introduces the economy with the expected pandemic. Section 3 illustrates the outbreak of a pandemic, where a SIR model is solved analytically, and the pre-pandemic and post-pandemic regimes are presented. The main analysis and results are given in Section 4. Section 5 offers some concluding remarks. All proofs are relegated to the end of the paper, in the Appendix.

2 The economy with an expected pandemic

Denote as $T(>)$ the uncertain future time when a pandemic indeed occurs. Denote as $F(t) = \Pr\{T \leq t\}$ the cumulative distribution of epidemics occurring up to time t and as $f(t)$ the corresponding density function. The instantaneous conditional probability of a pandemic occurring, given its non-appearance in time, is assumed constant:

$$\theta = \frac{F'(t)}{1 - F(t)} = \frac{f(t)}{1 - F(t)}.$$

In other words, the probability of a pandemic and its density function is

$$F(t) = 1 - e^{-\theta t}, \tag{1}$$

$$f(t) = \theta [1 - F(t)]. \tag{2}$$

For simplicity, we suppose that the pandemic occurs just once,[5] entailing a penalty that decreases with the natural resource stock (Mavi, 2019, 2020; Tsur and Zemel, 1996, 1998, 2015, 2016). The evidence shows that the incidence of a zoonotic disease is related to many factors such as habitat conversion and biodiversity loss (see Barbier (2021); Augeraud-Véron et al. (2021)). However, there is no evidence showing that zoonotic disease becomes a pandemic causing significant economic losses because of the biodiversity loss or the habitat conversion. Hence, the probability of a pandemic occurring is supposed to be constant.

3 The outbreak of a pandemic

When the pandemic breaks out, the total population (L) becomes the sum of three different types of people in society: the susceptible x, infected y, and recovered j, thus forming the SIR model (Kermack and McK-

[5]Obviously, in reality an epidemic or pandemic may happen multiple times, like the seasonal flu, bird flu, plague, and so on, and different epidemics and pandemics appear all the time, such as SARS 2002–2003, COVID-19, etc. Nonetheless, including multiple epidemics and pandemics significantly increases the calculation difficulty without commensurate gains in insight. Here, we use the situation of just one pandemic to illustrate the importance of taking into account a potential pandemic in the study of Hartwick's rule.

endrick, 1927; Gersovitz and Hammer, 2004; Acemoglu et al., 2021). The total population $x+y+j=\bar{L}$ is assumed to be constant. Denote as b the infection rate and as c the recovery rate if no prevention policy (lockdown, for example) is imposed. A regulator can impose a lockdown policy that contains a portion $\lambda \in [0,1]$ of society. Suppose each individual meets a number of other individuals k per unit of time. Then, on average, an infective individual meets $k\frac{x}{x+y}(1-\lambda)$ susceptible individuals. Of course, if there is a total lockdown $\lambda = 1$, an infective individual meets no one. The total number of meetings between infective and susceptible people is $k\frac{x}{x+y}(1-\lambda)\,y$. The new number of infective people is $pk\frac{x}{x+y}(1-\lambda)\,y$, where p is the probability of transmission of the diseases during a meeting between a susceptible and an infective person. Indeed, any action that decreases the probability of contracting the disease (e.g., lockdown, vaccination, social distancing), $p(1-\lambda)$, can be considered a prevention policy.

The evolution of the number of susceptible and infected individuals is given by

$$\dot{x} = -\tilde{b}\frac{xy}{x+y}, \tag{3}$$

where $\tilde{b} = b(1-\lambda)$ and $b = pk$. This dynamic shows that $\frac{\tilde{b}xy}{x+y}$ susceptible individuals become infected at each moment in time. Naturally, the evolution of the infected population is given by

$$\dot{y} = \frac{\tilde{b}xy}{x+y} - cy, \tag{4}$$

which includes the newly infected minus the recovered and in which c is the recovery rate of the disease. Consequently, the dynamics of the removed (recovery) individuals is

$$\dot{j} = cy. \tag{5}$$

Following Bohner et al. (2019), the analytical solution for the differential system (3), (4), and (5) (see Appendix *A*.1 for calculation details) can be given by the following: the number of susceptible individuals is

$$x(t) = x_0 (1+\kappa)^{\frac{\tilde{b}}{\tilde{b}-c}} \left(1+\kappa e^{\left(\tilde{b}-c\right)t}\right)^{-\frac{\tilde{b}}{\tilde{b}-c}}, \tag{6}$$

and the number of infected individuals is

$$y(t) = y_0 (1+\kappa)^{\frac{\tilde{b}}{\tilde{b}-c}} \left(1+\kappa e^{\left(\tilde{b}-c\right)t}\right)^{-\frac{\tilde{b}}{\tilde{b}-c}} e^{\left(\tilde{b}-c\right)t}. \tag{7}$$

The evolution of the number of susceptible, infected, and recovered (removed) individuals can be represented as in Figure 1.

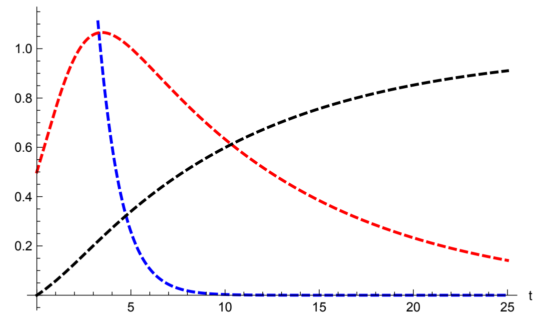

Figure 1: The evolution of the number of susceptible x (blue), infected (red) y, and recovered r (black) individuals over time t, with $\kappa = 7$, $c = 0.3$, $b = 0.8$, and $\lambda = 0.1$.

Denote the infected/susceptible ratio as

$$z(t) = \frac{y}{x}.$$

It is straightforward to have

$$z(t) = \kappa e^{(\tilde{b}-c)t}, \tag{8}$$

where $\kappa = \frac{y_0}{x_0}$ is the initial value.

In particular, $x_0 > 0$ and $y_0 > 0$ are the initial numbers of susceptible and infected individuals. This infected/susceptible ratio can be considered as the prevalence rate. Given that the share of infected individuals has a negative impact on labor productivity, the infected/susceptible ratio is an important indicator of the damaging cost of a pandemic. Hence, controlling the disease through a prevention policy λ is necessary to increase labor productivity. The choice of an SIR model is also related to its analytical tractability. Note that in the remainder of the paper, we show that this choice does not have a qualitative impact on our results (see Appendix A.5.)

3.1 The economy after the pandemic

After the occurrence of the pandemic, economic activities continue. Once the only expected pandemic occurs, the optimization problem becomes a deterministic one. However, we assume that labor supply, $\bar{L}$, is a fixed constant and it is normalized to 1 in the analysis.[6] Note that the number of workers does not change but productivity decreases with the prevalence rate of the pandemic $z(t) = \frac{y(t)}{x(t)}$, where, as above, $x(t)$ and $y(t)$ are the number of susceptible and infected people, respectively. Labor productivity $\tilde{f}(z(t))$ is a function of the ratio of susceptible people $z(t)$. Similar to d'Albis and Augeraud-Véron (2021), we reformulate the labor supply as

$$L(t) = \bar{L}\tilde{f}(z(t))$$

and impose the following assumption: The number of workers in production does not change. Thus, we can interpret the function $\tilde{f}(z(t))$ as a productivity factor.

Assumption 1 *The number of workers $\bar{L}$ is constant, but labor productivity $\tilde{f}(z(t))$ is a decreasing and concave function of the prevalence rate $z(t)$ if the transmission rate of the disease $\tilde{b}$ is higher than the recovery*

[6]Obviously, during some pandemics, such as COVID-19, lives are lost and labor supply decreases. Nonetheless, a constant labor supply with lower average productivity is a reasonable approximation.

rate c: $\tilde{f}(z) > 0$, $\tilde{f}'(z) < 0$, *and* $\tilde{f}''(z) < 0$.

Assumption 1 indicates that the pandemic has a negative impact on production through a decrease in the labor supply if the ratio of infected/susceptible people $z(t)$ is increasing. If the economy implements an efficient prevention policy λ such that the recovery rate c is higher than the transmission rate of the disease $b(1-\lambda)$, implying $c > b(1-\lambda)$, the productivity of labor increases. It is known that European countries experienced a labor productivity slowdown prior to the pandemic,[7] but this has changed after the pandemic. At first glance, the increase in labor productivity might be considered unrealistic since the pandemic has caused large economic losses. However, the evidence shows that the pandemic has increased labor productivity per hour due to greater digitalization and automation in many sectors (Bloom et al., 2020; IMF, 2021).

For these reasons, we can also interpret the parameter λ as a measure of the level of remote working, which helps decrease the transmission of the disease b. In addition, the evidence shows that remote work[8] is accompanied by digitalization, such as the use of technological devices during lockdown, which increased labor productivity (Lopez-Garcia and Szorfi, 2021). In other words, λ may be considered as technical progress that increases labor productivity.

In a world with a pandemic, the social planner still aims to maximize the utility of the most "deprived" $\min u_t$ with the condition $u_t \geq \bar{u}_2$. Denote consumption as $C(t)$, and capital and non-renewable resources employed in production as $K(t)$ and $R(t)$, respectively. Consider output function $F(K(t), R(t), L(t))$ following a Cobb–Douglas form (Solow, 1974):

$$Y = F(K, R, L) = K^{\alpha} R^{\beta} L^{\gamma},$$

where parameters $\alpha, \beta, \gamma \in (0, 1)$ are the share of capital, natural resources, and labor, respectively, in the output.

The pandemic has led to significant economic losses stemming from the destruction of capital by the exit of vulnerable firms from the market.[9] (OECD, 2020) In our specification, we note the economic losses from capital destruction in the post-pandemic regime, which we explain in more detail later.

The policymaker's optimal control problem is similar to Van Long (1992), where the Bellman value function after the occurrence of the harmful pandemic is defined as

$$\max_{C} V_2(K, S) = \int_{T}^{\infty} \rho \bar{u}_2 e^{-\rho(t-T)} dt = \bar{u}_2, \tag{9}$$

[7] See https://ec.europa.eu/eurostat/databrowser/view/tesem160/default/line?lang=en

[8] Remote work started just after governments imposed lockdowns in many countries. Higher levels of lockdown (higher λ) mean a higher level of remote work.

[9] See also https://voxeu.org/article/lasting-scars-covid-19-crisis

subject to

$$\dot{K}(t) = F(K(t), R(t), L(t)) - C(t), \tag{10}$$

$$\dot{S}(t) = -R(t), \tag{11}$$

$$U_2(C(t)) \geq \bar{u}_2, \tag{12}$$

$$\lim_{t\to\infty} S(t) = 0,$$

$$S(T) = S_T > 0, \tag{13}$$

$$K(T) = K_T > 0, \tag{14}$$

where positive parameter ρ is the pure rate of time preference. In the post-pandemic regime, there is capital destruction and the initial condition K_T may even fall below K_0, which is the initial capital level in the pre-pandemic world.

3.2 The economy before the pandemic

Before the pandemic, the economy takes into account the probability of a pandemic occurring and the disutility associated with the pandemic possibly happening at a future unknown time T (see Barbier (2021) for similar modeling). We denote the disutility as $\psi(S)$, which depends on the natural capital S which is supposed to have an amenity value (see D'Autume and Schubert (2008); D'Autume et al. (2010)). The preservation of natural capital helps to decrease the disutility (penalty) since it has an amenity value. For example, the preserved natural environments are disrupted by the extraction of productive resource inputs such as oil (Krautkraemer, 1985). Furthermore, more recent studies, focusing on the link between the natural resource protection, biodiversity and the epidemic diseases (Augeraud-Véron et al., 2021), shows that the biodiversity conservation may help to dampen the negative impacts of pandemic. Then, preserving the natural environment as a source of well-being for individuals helps to decrease the disutility due to a future harmful event such as a pandemic.

Assumption 2 *The penalty rate due to the occurrence of a future pandemic decreases if the natural capital level is high:* $\psi^{'}(S) < 0$.

Following Clarke and Reed (1994) and Tsur and Zemel (2016), the post-pandemic value function takes the following form:

$$\varphi(K, S) = \underbrace{V_2}_{=\bar{u}_2} - \psi(S). \tag{15}$$

Since the economy takes into account the disutility of an event that may possibly happen in the future, the term $\psi(S)$ presents in this post-value function. Contrary to our specification, the existing literature normally considers u_2 as a minimum utility level $u(c_{min})$ (Clarke and Reed, 1994; Bommier et al., 2015; Tsur and Zemel, 2016; Mavi, 2019) and specifies a post-value function very similar to $\varphi(K, S) = u(c_{min}) - \psi(S)$, with an inflicted penalty $\psi(S)$ due to the occurrence of the harmful event. This formulation states that economic activity stops and the consumption level is reduced to a minimum level. An important difference in our study is that economic activity continues after the occurrence of the harmful event, and we calculate the value of u_2 instead of just assuming that it is an arbitrary constant minimum utility $u(c_{min})$.

To sum up, the pandemic damage is twofold. On the one hand, the damage goes indirectly through the production process, and on the other hand, after the occurrence of the pandemic there is a future disutility, $\psi(S)$. Since the arrival time of the pandemic T is uncertain, the expected utility of the economy before the pandemic is thus

$$E_T\left\{\int_0^T U(C(t))e^{-\rho t}dt + \varphi(K(T), S(T))e^{-\rho T}\right\}. \tag{16}$$

Taking expectations of the expression (16) with respect to the distribution of T and having (1) and (2) yield the initial value function as (see Appendix *A*.6 for the proof):

$$V_1(K_0, S_0) = \max_{C(t)} \int_0^\infty \left[U(C(t)) + \theta(V_2(K(t), S(t)) - \psi(S))\right] e^{-(\rho+\theta)t}dt. \tag{17}$$

More precisely, the social planner aims to maximize the utility of the poorest generation, $\min u_t$, under technical constraints and the damage due to pandemic in the future. In other words, we can define the optimal control problem before the pandemic as

$$\int_0^\infty (\rho+\theta)\,\bar{u}_1 e^{-(\rho+\theta)t}dt = \bar{u}_1, \tag{18}$$

subject to

$$\dot{K} = I = F(K, R) - C, \tag{19}$$

$$\dot{S} = -R, \tag{20}$$

$$U(C) + \theta(V_2(K, S) - \psi(S)) \geq \bar{u}_1, \tag{21}$$

$$\lim_{t\to\infty} S(t) = 0, \tag{22}$$

$$S(0) = S_0 > 0, \tag{23}$$

$$K(0) = K_0 > 0. \tag{24}$$

In the following, we analyze the impact of the risk of a pandemic occurring on the optimal paths of capital accumulation and resource extraction when the social planner's objective is to maximize the "lowest" utility in society by taking into account the cost of a potential pandemic. The role of the pandemic enters into the utility through the post-event value function, $\varphi(K, S)$, which we discuss in more detail throughout the remainder of the paper.

After describing the pandemic in the next section, we present the maximin problem, the constant utility level, and the optimal paths of physical capital accumulation and natural resource exploitation that sustain the utility in an economy facing the risk of a pandemic.

4 Analysis and results

The analysis follows backward induction by first investigating the economy after the pandemic shock and then studying the economy prior to this shock, looking at how the optimal choice of consumption can be made while taking into account potential pandemic damage.

4.1 The economy after the pandemic

In this section, we treat three different cases:

- *Knife's edge*, where the social planner should immediately control the pandemic outbreak by ensuring $\tilde{b} = b(1-\lambda) < c$ to ensure a sustained utility level.
- *Not a knife's edge*, where functional specifications of labor productivity and the evolution of the disease are relaxed.
- *Free of natural resource constraints*, where the economy is assumed to have a constant stream of natural resources, similar to Dasgupta and Heal (1974a).

The "knife's edge" case contains a full analytical resolution of the model, which shows that the social planner should ensure $\tilde{b} = b(1-\lambda) < c$. Otherwise, the economy cannot evolve along an optimal maximin path.

The "not a knife's edge" case solves the knife's edge problem without functional specifications for labor productivity and pandemic dynamics and without presenting analytical results. It is shown that the results in the knife's edge case are robust and that the social planner should still control the pandemic to sustain utility.

The "free of natural resource constraints" case focuses on an economy where the natural resource constraint does not exist, due to a constant stream of substitutes (such as solar and wind for energy supply). We show that the economy can sustain a constant utility level even though the ratio of infected to susceptible individuals increase over time. Of course, this case is far from the Dasgupta–Heal–Solow–Stiglitz framework and from Hartwick's rule. The aim is to show that if there is a substitute for the exhaustible natural resource stock, then the utility can be sustained even though there is an increasing prevalence rate, implying $\tilde{b} = b(1-\lambda) > c$.

4.1.1 A knife's edge case

After experiencing the expected one-time pandemic, the economy returns to a deterministic optimal control problem with the given production function, initial capital K_T, and natural resource reserve S_T, and labor expressed in efficiency units is $L = \bar{L}\tilde{f}(z(t)) = \frac{\bar{L}}{z(t)} = \frac{e^{-(\tilde{b}-c)t}}{\kappa}$, with $\bar{L} = 1$ and $\kappa = \frac{y_T}{x_T}$. Once the pandemic occurs, the labor supply immediately becomes $L = \frac{\bar{L}}{\kappa}$ with $\kappa > 1$ at time T. Taking COVID-19 as an example, productivity at the beginning of the outbreak is below the pre-pandemic level, as most firms and employees were trying to adapt to remote working during the first weeks of lockdown (Al-Habaibeh et al., 2021). However, if the pandemic is under control, i.e., $\tilde{b} < c$), labor productivity increases over time and after some time goes beyond pre-pandemic levels due to digitalization and automation, which increase after the occurrence of the pandemic, as mentioned above in reference to COVID-19 (see (Bloom et al., 2020; Chernoff and Warman, 2020; IMF, 2021; Petropoulos, 2021)).

Indeed, when the pandemic is under control through a prevention policy, Increasing labor productivity over time can be considered technological progress that augments the stock of natural resources and diminishes the capital stock (D'Autume and Schubert, 2008). The post-pandemic production function is given by

$$F\left(K, R, \tilde{f}(z)\right) = K^{\alpha} R^{\beta} L^{\gamma} = K^{\alpha} R^{\beta} \kappa^{-\gamma} e^{-\gamma\left(\tilde{b}-c\right)(t-T)}, \ \ \forall t \geq T.$$

The functional specification for labor productivity $\tilde{f}(z(t))$ and the use of an SIR model is for complete analytical tractability. In next sections, we show that the results are robust even without functional specifications.

In order to solve the post-pandemic optimization problem presented in the previous section, we employ the same argument as Cairns and Long (2006). First, we translate the non-autonomous system into an autonomous one. To do so, we take time as an extra state variable by defining variable $W(t)$ as

$$W(t) = t - T, \ \ \forall t \geq T.$$

Thus,

$$\dot{W} = 1$$

with initial condition $W(T) = 0$. Then, the Hamiltonian of the planner is defined as

$$\mathcal{H} = -p_1 R + p_2 \left(K^{\alpha} R^{\beta} (\kappa)^{-\gamma} e^{-\gamma(\bar{b}-c)W(t)} - C \right) + p_3,$$

where p_i $(i = 1, 2, 3)$ are costate variables of natural resources, capital, and time, respectively.

Given the inequality constraint $U(C) \geq \bar{u}_2$, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{H} + \Phi \cdot (U_2(C) - \bar{u}_2),$$

with Φ being the Kuhn–Tucker multiplier.

The first-order necessary condition for the choice variables (where the second-order condition holds as well) yields that

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial R} = -p_1 + p_2 F_R = 0, \\ \frac{\partial \mathcal{L}}{\partial C} = -p_2 + \Phi U_2^{'}(C) = 0, \\ \Phi \geq 0, \ \ \Phi \cdot (U_2(C) - \overline{u}_2) = 0. \end{cases} \tag{25}$$

Obviously, the first equation

$$p_1 = p_2 F_R$$

states that the in situ price of a natural resource, p_1, is determined by its rental value employing $p_2 F_R$, which is the product of the rental price of capital p_2 and the marginal output of resource F_R. While the rental price of capital p_2 is given by the second equation

$$p_2 = \Phi U_2^{'}(C),$$

which is indeed the optimal value of the objective function coming from employing an extra unit of capital. Furthermore, this value is given by the product of the marginal utility and the shadow value, Φ. Thus, the rental value p_2 takes into account the trade-off between the minimum level of utility, $\overline{u}_2$, and its consequences on the marginal utility $U_2'(C)$.

Implicitly, as long as the rental price $p_2 > 0$, which should hold for all times $t \geq T$, we must have

$$\Phi > 0,$$

given $U_2'(C) > 0$. In other words, for any $t \geq T$ along the maximin optimal trajectory, it is necessary that

$$U_2(C) = \bar{u}_2.$$

Furthermore, the first-order condition with respect to the three state variables S, K, and W, yields the following dynamics of the co-state variables:

$$\begin{cases} \dot{p}_1 = 0, & (a) \\ \dot{p}_2 = -p_2 F_K, & (b) \\ \dot{p}_3 = p_2 \gamma \left(\tilde{b} - c\right) F, & (c) \\ -p_1 R + p_2 \dot{K} + p_3 = \mathcal{H} = 0, & (d) \end{cases} \tag{26}$$

with transversality conditions $\lim_{t \to +\infty} p_3 = 0$ and $\lim_{t \to +\infty} e^{-\rho t} p_2(t) K(t) = 0$.

The first transversality condition simply indicates that the shadow value of time p_3 vanishes when $t \to +\infty$. The second transversality condition is a standard one stating that the discounted far future value of capital is zero.

Importantly, equation (a) states that the in situ price p_1 is constant over time for any $t \geq T$. Equation (b) indicates that the growth rate of the shadow value of capital, $\frac{\dot{p}_2}{p_2}$, is negatively related to the marginal product of capital. Thus, given $p_1 = p_2 F_R$, the growth of marginal product, F_R, from employing natural resources must exactly compensate the growth of price p_2 in order for the product, $p_2 F_R$, to be constant over time.

The dynamic value of time, p_3, is determined not only by the production and rental price of natural resources but also by the transmission and infection of the pandemic disease, given the time explicitly entering the production process through the impact on labor efficiency that is damaged partially due to the pandemic. By using $p_1 = p_2 F_R$, the value of time, p_3, can be reformulated via equation (c) as the following:

$$\dot{p}_3 = p_2 \gamma \left(\tilde{b} - c\right) F = p_2 \gamma \left(\tilde{b} - c\right) \frac{F_R R}{\beta} = -p_1 \frac{\gamma \left(\tilde{b} - c\right)}{\beta} \dot{S}. \tag{27}$$

Combining this with transversality condition $\lim_{t \to +\infty} p_3 = 0$, integrating (27) over $[t, +\infty)$, $\forall t \geq T$, yields

$$p_3(t) = -p_1 S(t) \frac{\gamma \left(\tilde{b} - c\right)}{\beta}. \tag{28}$$

In other words, the value of time after the pandemic shock is determined by the value of natural resources, which is the product of the in situ price p_1 and the reserve of the resource itself, S, also taking into account impacts from the pandemic, $\frac{\gamma(\tilde{b}-c)}{\beta}$. Given the resource reserve $S(t) \to 0$ as $t \to +\infty$ with p_1 being constant, the transversality condition, $\lim_{t \to \infty} p_3(t) = 0$, is then straightforward.

With the above preparation, we are ready to find the optimal consumption, $\overline{C}$, and the minimum utility level, $\overline{u}_2$, after the pandemic under the maximin principle. To do so, we employ the arguments proposed by Cairns and Long (2006), guessing a special candidate solution and then checking the necessary conditions for optimality.

The special candidate is defined as follows: (a) take $K(t) = K(T)$ along the optimal path, $\forall t > T$, thus $\dot{K} = 0$; (b) take $R(t) = -\frac{\gamma(\tilde{b}-c)}{\beta} S(T) e^{\left(\frac{\gamma(\tilde{b}-c)}{\beta}\right)(t-T)}$. In order to ensure that the resource constraint $\int_T^\infty R(t)\,dt = S_T$ holds if and only if $\widetilde{b} = b(1-\lambda) < c$, (c) the candidate maximum achievable consumption is given by

$$\bar{C} = Y(T) = Y(t) = \frac{K_T^\alpha S_T^\beta}{\kappa^\gamma} \left(\frac{\gamma\left(c - \tilde{b}\right)}{\beta} \right)^\beta. \tag{29}$$

As a by-product of the above candidate, from $\widetilde{b} = b(1-\lambda) < c$, labor supply $L = \kappa e^{-(\widetilde{b}-c)(t-T)}$ must increase over time after $t > T$. In other words, the shock from the pandemic diminishes over time and labor productivity returns to the pre-pandemic level in the long run. The condition $\widetilde{b} = b(1-\lambda) < c$ is essential for the existence of maximin sustainable consumption. The rest is to show that, indeed, this candidate fulfills the above first-order necessary conditions, which we present in Appendix A.3.

Proposition 1 *For a given infection and recovery rate b and c, and lockdown policy λ, the social planner should ensure $\widetilde{b} = b(1-\lambda) < c$ in order to guarantee the constant maximum achievable utility that ensures equity across generations.*

Proof. See Appendix A.2

As shown in Appendix A.2, a social planner cannot implement sustained utility if she is not able (or willing) to control the pandemic from the very beginning. Thus, this proposition offers a theoretical basis on which to defend the "sooner the better" strategy, even though it has a cost (see d'Albis and Augeraud-Véron, 2021 for a similar discussion). Note that this proposition is not sensitive to the functional form we use. In sections (4.1.2) and (4.1.3), we show that the social planner should ensure a decreasing prevalence rate $z(t)$ without any functional specifications. We also show that the choice of an SIR model does not have a qualitative effect on Proposition 1. It is shown that even if we use an SIS model, the social planner still needs to ensure $\widetilde{b} = b(1-\lambda) < c$ (see Appendix (A.5)).

Proposition 2 *Assume that the social planner implements $\widetilde{b} = b(1-\lambda) < c$. Then, if the pandemic occurs at date T, following the maximin principle the maximum achievable post-pandemic consumption is*

$$C(t) = \bar{C} = Y(T) = Y(t) = \frac{K_T^\alpha S_T^\beta}{\kappa^\gamma} \left(\frac{\gamma\left(c - \tilde{b}\right)}{\beta} \right)^\beta, \quad \forall t \geq T \tag{30}$$

and the corresponding sustainable utility is

$$V_2(K, S) = \bar{u}_2 = U_2\left(\bar{C}\right). \tag{31}$$

Furthermore, at T,

$$S_T = K_T^{\frac{1-\alpha}{\beta}} \left(\frac{\gamma(c - \tilde{b})}{\beta} \right)^{\frac{1-\beta}{\beta}} \kappa^{\frac{\gamma}{\beta}}.$$

The fundamental mechanism for the above proposed sustainable consumption relies on the fact that labor productivity increases over time after the pandemic shock, which offers the possibility of a decreasing exploitation rate of natural resources while keeping capital at its pre-pandemic level. In other words, the exploitation rate must decrease at the same rate as the growth of labor productivity.

Arguably, this mechanism works by essentially relying on the prevention policy λ. With $\lambda = 0$, i.e., no prevention at all, the pandemic situation reads $b > c$, meaning the infection rate is higher—even much higher—than the recovery rate. Thus, the damage from the pandemic will be much greater, such that sustainable consumption cannot be guaranteed.

From the above proposition, it is straightforward that $\frac{\partial \overline{C}}{\partial \lambda} > 0$, meaning that the prevention policy, λ, increases the sustained consumption level. However, we must be cautious when interpreting this result. The above findings are based on the assumption that the prevention policy does not damage essential economic activities such as production, services, and main consumption, etc. In other words, there is no cost of implementing a prevention policy in terms of economic activity loss (see Section 6 for an extension).

Obviously, with $\lambda = 1$, a complete lockdown that shuts down all economic activities, a different story will unfold. Systematic studies of the impacts of lockdown, in the context of COVID-19, can be found in Bosi et al. (2020) and Acemoglu et al. (2021).

In the rest of this paper, we focus on the pandemic's impact on Hartwick's rule with a relatively low λ, such that Proposition 1 holds. Thus, we can investigate the impacts of the prevention policy and ignore the negative effects of prevention policy.

Before further investigation of Hartwick's rule, we must mention that the proposed candidate for a sustainable path in Proposition 2 is not the only one. We could easily imagine different combinations of capital and resources such that the gain from the improvement in labor productivity is shared, by taking into account the elasticity of substitution between capital and resources. In the next section, we take this into account, instead of only working on constant capital, by means of numerical simulations.

The pandemic-modified Hartwick rule

Hartwick's rule states that if the economy invests the rents obtained from the use of natural resources into man-made capital, the economy stays on a sustainable, constant path of utility, which implies equity across generations. In other words, if the economy is on a sustainable path, the genuine savings are always zero and the utility and consumption levels are maintained as constants.

Under the current setting, a new pandemic-modified Hartwick rule can be obtained. To do so, we rely on the shadow values from the above analysis. We rename the shadow values p_1 and p_3 in terms of the shadow price of capital, p_2, as

$$q = \frac{p_1}{p_2} \text{ and } q_z = \frac{p_3}{p_2}.$$

By (28), the relative value of time in terms of shadow price, q_z, is

$$q_z(t) = -q\frac{\gamma\left(\tilde{b} - c\right)}{\beta}S(t). \tag{32}$$

Furthermore, from (d) in (26), the pandemic-modified Hartwick rule, $\mathcal{H} = 0$, can be rewritten as

$$\dot{K}(t) = \frac{p_1}{p_2}R(t) - \frac{p_3}{p_2} = qR(t) - q_z = qR + q\frac{\gamma\left(\tilde{b} - c\right)}{\beta}S(t). \tag{33}$$

Obviously, without a pandemic, $b = c = 0$, it follows that $\dot{K} = qR > 0$. When there is a pandemic and the social planner controls it, i.e., $\tilde{b} - c < 0$, capital accumulation slows down or stabilizes if $\dot{K} = 0$ for all $t \geq T$, as a special solution. Arguably, the slowdown process depends on the prevention policy λ. In the special sustainable constant consumption rule proposed in Proposition 2, the last equation (33) yields

$$-R(t) = \frac{\gamma(\widetilde{b} - c)}{\beta} S(t) = \dot{S}, \quad \forall t \geq T.$$

In other words,

$$\frac{\dot{S}}{S} = \frac{\gamma(\widetilde{b} - c)}{\beta} < 0,$$

and thus,

$$S(t) = S(T) e^{\frac{\gamma(\widetilde{b-c})(t-T)}{\beta}} \quad \forall t \geq T.$$

Obviously, the natural resource exploitation rate is lower with a stronger prevention policy. Figure 2 graphically illustrates this analytical expression of $S(t)$.

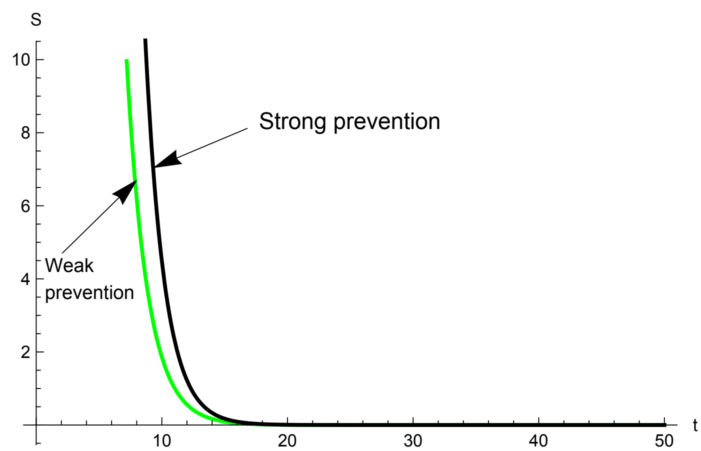

Figure 2: Strong prevention ($\lambda = 0.15$) (red) vs. weak prevention ($\lambda = 0.1$) (blue) - Candidate solution in Proposition 2

Figure 2 shows the optimal path of natural resource stocks over time for the outcome in Proposition 2[10]. Note that we do not present a graphical illustration for K for the candidate solution since we assume $\dot{K} = 0$.

It is clear that a stronger prevention policy lowers natural resource extraction, thus leaving a larger natural reserve $S(t)$. The reason for this result comes from the fact that the labor productivity is higher with a stronger prevention policy. In other words, the prevention policy plays the role of an increase in productivity and substitutes for natural resource extraction.

We can now present the dynamics of the system under maximin in the general case, instead of the constant capital scenario. Denote $X(t) = S(t)L(t)^{\frac{\gamma}{\beta}}$ as a deflated variable (see Appendix A.4 for details) in order to

[10]The parameters for the numerical simulation are $\beta = 0.1$, $\alpha = 0.7$, $c = 0.15$, $b = 0.1$, $\kappa = 1$. For strong prevention; $\lambda = 0.15$ and for weak prevention; $\lambda = 0.1$

have an autonomous system. Then, the dynamic system becomes the following:

$$\begin{cases} \dot{K} = \beta Y - q_z, \\ \dot{X} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \kappa^{-\frac{\gamma}{\beta}} - \frac{\gamma(\tilde{b}-c)}{\beta} X, \\ \dot{q}_z = \gamma \left(\tilde{b} - c\right) Y + r q_z. \end{cases} \tag{34}$$

where $r = \alpha \frac{Y}{K}$. Unfortunately, an analytical solution becomes impossible for this general case. Thus, we rely on numerical analysis to illustrate the main ideas.

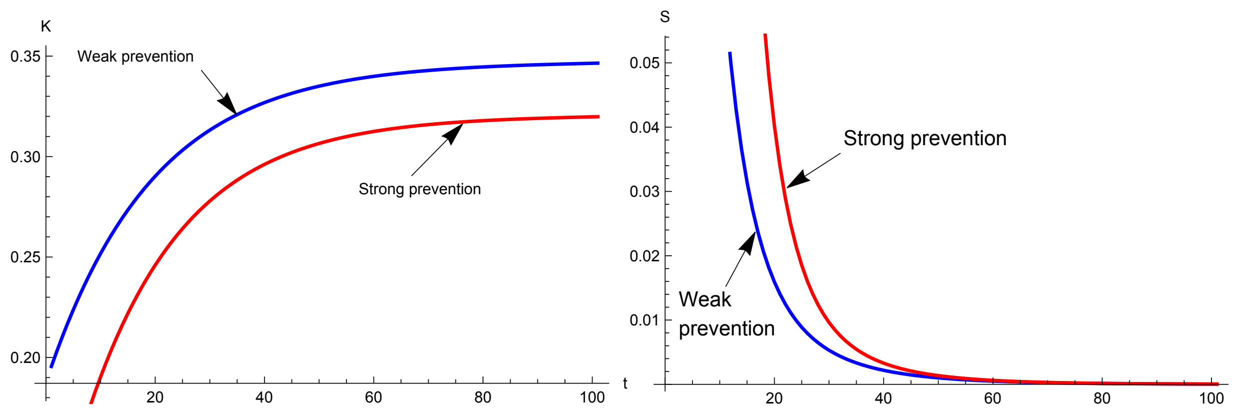

Figure 3: Strong prevention ($\lambda = 0.15$) (red) vs. weak prevention ($\lambda = 0.1$) (blue)

Figure 3 describes the trajectory dynamics of capital stock K and the reserve of resources S[11]. It is consistent with the prediction in Proposition 1 that a strong prevention policy helps prevent the exploitation of natural resources (with constant capital or not). But the cost is paid by decreasing capital accumulation, i.e., a strong prevention policy yields a lower capital stock. Furthermore, as previously mentioned, keeping consumption—and thus utility—constant over time is possible via the combination of variable capital and exploiting, instead of only constant capital as in Proposition 1.

In the next subsection, the analysis is conducted without functional specifications on labor productivity $\tilde{f}(z(t))$.

4.1.2 Not a knife's edge case

In this section, we take the same economy but with general functional forms for labor productivity $\tilde{f}(z(t))$ and for the prevalence rate $z(t)$. Assumption (1) always holds. The labor expressed in efficiency units takes the form $L = \bar{L}\tilde{f}(z)$. The production function takes the following form, as in the previous section:

$$F\left(K, R, \tilde{f}(z)\right) = K^{\alpha} R^{\beta} L^{\gamma} = K^{\alpha} R^{\beta} \left(\bar{L}\tilde{f}(z)\right)^{\gamma}, \ \ \forall t \geq T.$$

[11]The parameters for the numerical simulation are the same as for Figure 2. $\beta = 0.1$, $\alpha = 0.7$, $c = 0.15$, $b = 0.1$, $\kappa = 1$. For strong prevention; $\lambda = 0.15$ and for weak prevention; $\lambda = 0.1$

Since labor is replaced by productivity $\tilde{f}(z)$, it is convenient to track the evolution of z as a state variable.

$$\dot{z} = g(z)$$

We do not specify a functional form for the evolution of the pandemic, nor do we impose any condition on $g(z)$. The Hamiltonian is defined in a different form by taking the evolution of the prevalence rate $z(t)$ into account, which affects labor productivity:

$$\mathcal{H} = -\hat{p}_1 R + \hat{p}_2 \left(K^{\alpha} R^{\beta} \left(\bar{L} \tilde{f}(z) \right)^{\gamma} - C \right) + \hat{p}_3 \left(g(z) \right). \tag{35}$$

Given the inequality constraint $U(C) \geq \bar{u}_2$, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{H} + \varPhi \cdot \left(U_2(C) - \bar{u}_2 \right).$$

The optimality conditions yield the following canonical system (see Appendix A.5 ii for details):

$$\begin{cases} \dot{K} = \beta Y - \hat{q}_z g(z), \\ \dot{S} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \left(\bar{L} \tilde{f}(z) \right)^{-\frac{\gamma}{\beta}}, \\ \dot{z} = g(z), \\ \dot{\hat{q}}_z = -\gamma Y \frac{\tilde{f}'(z)}{\tilde{f}(z)} - \left(g'(z) - r \right) \hat{q}_z, \end{cases} \tag{36}$$

where $Y = \frac{\bar{C} + \hat{q}_z g(z)}{1-\beta}$ and $\hat{q}_z = \frac{\hat{p}_3}{\hat{p}_2}$.

With the above system, Appendix A.5 proves the following equivalency.

Corollary 1 *The dynamic system* (36) *is equivalent to* (34) *if precise functional forms and an SIR model are used as the knife's edge case.*

Nevertheless, the following non-existence result is straightforward.

Proposition 3 *The policymaker cannot ensure a sustained utility level over time if the prevalence rate* $z(t)$ *is increasing.*

To see the reason behind this result, we can express $\dot{K} = 0$ at the steady state, implying $\beta Y^* = \hat{q}_z^* g(z^*)$ where variables with an asterisk (*) stand for the steady-state values. Then, we have $Y^* = 0$ if the prevalence rate $\dot{z} = 0$ at the steady state, which implies $g(z^*) = 0$. Thus, $Y^* = \frac{\bar{C} - \hat{q}_z^* g(z^*)}{1-\beta} = 0$ implies $\bar{C} = 0$.

To give an example, consider the previous SIS model. Then, $\dot{z} = g(z(t)) = \left(\tilde{b} - c(1+z) \right) z$. At the steady state, we have $\dot{z} = 0$. Inequality $\tilde{b} > c$ yields $z^* = \frac{\tilde{b}-c}{c}$. Note that if the policymaker can control the pandemic by guaranteeing $\tilde{b} - c < 0$, we show that the model becomes a standard DHSS model. Then, sustained utility over time can be achieved (see the SIS model example in Appendix A.5).

This result can be understood by the fact that the natural resources are exhaustible. If there is a productivity decrease of labor due to an increasing prevalence rate $z(t)$, the economy cannot compensate this decrease by increasing natural resource extraction R since the natural resource stocks S is exhaustible. In the next subsection, we show that when the economy is not constrained by natural resource stocks, the presence of an increasing prevalence rate does not jeopardize the presence of sustained consumption.

4.1.3 Free of natural resource constraints case

In this section, our aim is to show that the economy can ensure a sustained utility level even when the number of infected people is increasing over time. We show that this is possible if the natural resource constraint S is relaxed thanks to the discovery of a substitute. It is evident that we are far from Hartwick's rule. However, this section provides important insights regarding the importance of the limited natural resource stocks when there is a pandemic.

For this, we employ the modeling proposed by Dasgupta and Heal (1974a). Suppose that there is a substitute (for example, solar or wind for energy supply) entering steadily into the economy and that it relaxes the resource constraint on S. Instead of having $\dot{S} = -R$, we suppose

$$\dot{S} = m - R,$$

where m is a constant service provided by the substitute. Integrating $\dot{S}$ over $[T;\infty]$ yields $\int_T^\infty \frac{dS}{dt} dt = [mt]_T^\infty - \int_T^\infty R(t)\, dt$, giving an unstable solution that diverges to infinity. Then, similar to Dasgupta and Heal (1974a), the solution is to jump to the steady state by stating $m = R$ for $\forall t$. Of course, in this context the aim is not to have a Hartwick rule but to show that the natural resource constraint impedes sustained utility.

Labor supply is $L = \bar{L}\tilde{f}(z(t))$, and productivity decreases with the ratio of infected to susceptible individuals (i.e., $\tilde{f}'(z(t)) < 0$). Then, physical capital accumulation is

$$\dot{K} = K^\alpha m^\beta \tilde{f}(z(t))^\gamma - C, \tag{37}$$

where $\bar{L} = 1$. The evolution of labor productivity is

$$\dot{z}(t) = g(z(t)), \tag{38}$$

where $g(z(t)) > 0$. The Hamiltonian is

$$\mathcal{H} = \tilde{p}_2 \left(K^\alpha m^\beta \tilde{f}(z(t))^\gamma - C\right) + \tilde{p}_3 (g(z(t))), \tag{39}$$

and the Lagrangian with the constraint $U_2(C) \geq \bar{u}_2$ is

$$\mathcal{L} = \mathcal{H} + \varPhi \cdot (U_2(C) - \bar{u}_2).$$

The first-order condition $\Phi U_2' = \tilde{p}_2$ holds as before, and the dynamics of the co-state variables are

$$\begin{cases} \dot{\tilde{p}}_3 = -\tilde{p}_2 K^\alpha m^\beta \tilde{f}'(z) - \tilde{p}_3 g'(z), \\ \dot{\tilde{p}}_2 = -F_K \tilde{p}_2. \end{cases} \tag{40}$$

Denote $\tilde{q}_z = \frac{\tilde{p}_3}{\tilde{p}_2}$. Then the dynamics of the economy can be expressed as follows:

$$\begin{cases} \dot{\tilde{q}}_z = -K^{\alpha} m^{\beta} \tilde{f}'(z) - \tilde{q}_z g'(z) + \tilde{q}_z F_K, \\ \dot{K} = K^{\alpha} m^{\beta} \tilde{f}(z(t))^{\gamma} - C, \\ \dot{z}(t) = g(z(t)). \end{cases} \tag{41}$$

From the above system, the following results can be concluded:

Proposition 4 *With $\mathcal{H} = 0$, the dynamics of capital accumulation are*

$$\dot{K} = -\tilde{q}_z g(z) \tag{42}$$

and follow a path proportional to the dynamics of the prevalence rate.

Note that the shadow value $\tilde{q}_z$ can be considered as a shadow cost and has a negative value. Capital accumulation is proportional to the dynamics of the pandemic $g(z)$ weighted by the relative shadow value of the prevalence rate $\tilde{q}_z$. The economic intuition is that the economy may compensate the disutility stemming from an increasing prevalence rate affecting labor productivity by accumulating physical capital. Hence, the utility can be sustained in this manner.

Proposition 5 *Even though there is a pandemic with an increasing prevalence rate $z(t)$ over time, the social planner can sustain the utility level if the natural resource stocks are not limited thanks to the presence of a substitute.*

To understand this result, we rewrite the steady state of this economy as a function of z^* and $\dot{z}(t) = g(z(t)) = 0$ as follows:

$$\begin{cases} \tilde{q}_z^* = -\frac{K^*(z^*)^{\alpha} m^{\beta} \tilde{f}'(z^*)}{g'(z^*) - \alpha \frac{\bar{C}}{K^*(z^*)}}, \\ K^* = \left(\frac{\bar{C}}{m^{\beta} \tilde{f}(z^*)^{\gamma}} \right), \end{cases} \tag{43}$$

where $F_K = \alpha \frac{Y}{K}$. As our aim is not to solve a complete model, the above steady state is sufficient to show that a meaningful steady state exists with a Y^* different from zero and a positive $\bar{C}$.

To see clearly why $\tilde{q}_z^*$ is negative, assume that under an SIS model, as in Appendix A.5, $z(t)$ is increasing over time when $\tilde{b} > c$ and tends to $z^* = \frac{b-c}{c}$. Since $g(z(t)) = \left(\tilde{b} - c(1+z)\right) z$ and $g'(z^*) < 0$, it follows that $\tilde{q}_z^* < 0$, as mentioned above.

5 The economy before the pandemic

The economy faces the possibility of a pandemic occurring and the date is unknown. In order to solve the maximization problem for the pre-pandemic situation, we follow the method proposed by Van Long (1992) and D'Autume and Schubert (2008). The social planner seeks to maximize the following problem presented in Section 3.2:

$$V = \int_0^{\infty} (\rho + \theta) \bar{u}_1 e^{-(\rho+\theta)t} dt = \bar{u}_1, \tag{44}$$

subject to constraints (19) and (20). The production function before the pandemic takes the Cobb–Douglas form $Y = K^{\alpha}R^{\beta}$, where labor is supposed to be unity.

The current-value Hamiltonian is defined as

$$\mathcal{H} = (\rho + \theta)\,\bar{u}_1 e^{-(\rho+\theta)t} - \pi_1 R + \pi_2\left(K^{\alpha}R^{\beta} - C\right). \tag{45}$$

Taking into account the utility constraint $U(C) + \theta(\bar{u}_2 - \psi(S)) \geq \bar{u}_1$, the Lagrangian can then be defined as

$$\mathcal{L} = \mathcal{H} + \Lambda[U(C) + \theta(\bar{u}_2 - \psi(S)) - \bar{u}_1],$$

with Λ being the Kuhn–Tucker multiplier.

The first-order necessary conditions with respect to the choice variables (and the second-order sufficient conditions being guaranteed) are

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial R} = -\pi_1 + \pi_2\left(\beta K^{\alpha}R^{\beta-1}\right) = 0, \\ \frac{\partial \mathcal{L}}{\partial C} = -\pi_2 + \Lambda U^{'}(C) = 0, \\ \frac{\partial \mathcal{L}}{\partial \Lambda} = [U(C(t)) + \theta(\bar{u}_2 - \psi(S))] - \bar{u}_1, \quad \Lambda \geq 0, \ \ \Lambda\frac{\partial \mathcal{L}}{\partial \Lambda} = 0. \end{cases} \tag{46}$$

In the above first-order condition, we did not impose the non-negative constraint $R \geq 0$. The reason is twofold: On the one hand, given the Cobb–Douglas form of the production function, natural resources are an essential input à la Dasgupta and Heal (1974b) and it is not optimal to exhaust them in finite time; on the other hand, as assumed in the last section, after the outbreak of the pandemic there is still a reserve of natural resources to be exploited. Thus, imposing the non-negative constraint is redundant.

As in the previous section, the in situ price of natural resources, π_1, is equal to the rental value of natural resources, $\pi_2 F_R$, which is composed of the shadow price of the physical capital and the marginal output of natural resources. The second equation states that the rental price of capital, π_2, is given by

$$\pi_2 = \Lambda U^{'}(C),$$

which is the optimal value of the pre-disease objective function stemming from using an extra unit of capital. This value is composed of the shadow value Λ of the constraint $U(C(t)) + \theta(\bar{u}_2 - \psi(S)) - \bar{u}_1 \geq 0$ and the marginal utility of consumption $U^{'}(C)$. The rental price of capital, π_2, accounts for the trade-off between the minimum utility $\bar{u}_1$ and its consequences on the marginal utility $U^{'}(C)$. As long as the rental price of physical capital is $\pi_2 > 0$, we have $\Lambda > 0$, which implies

$$\bar{u}_1 = U(C(t)) + \theta(\bar{u}_2 - \psi(S)).$$

The dynamics of the co-state are

$$\begin{cases} \dot{\pi}_1 = \Lambda\theta\psi^{'}(S), & (a) \\ \dot{\pi}_2 = -\pi_2\left(\alpha K^{\alpha-1}R^{\beta}\right). & (b) \end{cases} \tag{47}$$

Equation (a) in (47) shows that the dynamics of the in situ price of natural resources depends on the marginal damage due to the pandemic with respect to the natural resource stocks $\psi^{'}(S)$ weighted by the probability of a pandemic occurring θ and the shadow value Λ of the constraint $U(C(t)) + \theta(\bar{u}_2 - \psi(S)) - \bar{u}_1 \geq 0$. In contrast to the post-pandemic regime, the evolution of the rental price of natural resources, $\dot{\pi}_1$, depends also on the probability of a pandemic occurring, θ, and on the marginal damage due to the pandemic. Hence, the trade-off between the minimum utility $\bar{u}_1$ and the marginal utility $U'(C)$ is also affected by the uncertainty regarding the occurrence of a pandemic, through the term $\pi_1 = \pi_2 F_R$. As in the previous section, the growth rate of the shadow value $\frac{\dot{\pi}_1}{\pi_1}$ is negatively related to the marginal productivity of capital.

Furthermore, from the above optimality conditions (46), Appendix (A.7) shows that the last two terms in the current-value Hamiltonian, (45) verify that $\pi_1 R = \pi_2 I$. More precisely, the relative shadow prices, $\frac{\pi_1}{\pi_2}$, are the same as the investment/extraction ratio $\frac{I}{R}$, that is,

$$\frac{\pi_1}{\pi_2} = \frac{I}{R} = \beta K^{\alpha} R^{\beta - 1}. \tag{48}$$

Thus, capital accumulation (19) becomes

$$\dot{K} = I = \beta K^{\alpha} R^{\beta} = \beta Y, \tag{49}$$

and natural resource extraction (see Appendix (A.7) for details) is

$$\dot{S} = -R = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}}, \tag{50}$$

where output and consumption varify $(1-\beta)Y = C(\bar{u}_1, \bar{u}_2, S)$.

Additionally, if taking a linear pandemic cost function $\psi(S) = a - \bar{\psi} S$ with $a > \bar{\psi} S_0 > 0$ and combining it with CRRA utility $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$, it follows that the optimal consumption is given by

$$C(t) = C(S(t)) = \left[(1-\sigma)\left[\bar{u}_1 - \theta\left(\bar{u}_2 - (a - \bar{\psi} S(t))\right)\right]\right]^{\frac{1}{1-\sigma}}.$$

Intuitively, before time t there was no pandemic and pandemic outbreaks had a probability of θ at time t, where the reserves of resource are $S(t)$. Then, the above equation provides the optimal consumption at time t and after the occurrence of the pandemic and the optimization enters the second period, which we studied in the previous subsection. Obviously, this pre-pandemic optimal consumption is a precautionary optimal policy in that it takes into account the outbreak of a pandemic and its related cost. If $\theta = 0$, the probability of a pandemic outbreak is almost nil and consumption would reach its maximum level, given $0 < \theta < 1$. In addition, it is easy to see that $\frac{dC}{dS} < 0$, that is, the latter the pandemic happens the lower the resource reserve will be, and the higher consumption it will yield.

Now we can solve the two-dimensional dynamic system (S, K). To do so, we eliminate the time variable by dividing equation (50) by equation (49) and rearranging terms. It follows that the resources change in terms of capital accumulation according to:

$$\begin{aligned} \frac{dS}{dK} &= -\frac{R}{\beta Y} = -\frac{1}{\beta} K^{-\frac{\alpha}{\beta}} Y^{\frac{1-\beta}{\beta}} = -\frac{1}{\beta} K^{-\frac{\alpha}{\beta}} \left(\frac{C(S)}{1-\beta}\right)^{\frac{1-\beta}{\beta}} \\ &= -\frac{1}{\beta} K^{-\frac{\alpha}{\beta}} \frac{\left[(1-\sigma)\left[\bar{u}_1 - \theta\left(\bar{u}_2 - \bar{\psi}\left(a - \bar{\psi} S\right)\right)\right]\right]^{\frac{1-\beta}{\beta(1-\sigma)}}}{(1-\beta)^{\frac{1-\beta}{\beta}}}, \end{aligned} \tag{51}$$

which is a non-linear but separable differential equation of S and K. Rearranging terms and taking integrals on both sides gives the maximum achievable utility $\bar{u}_1$ (see Appendix A.7). With the above analysis, we conclude the following:

Proposition 6 *For the above given CRRA utility function, the Cobb–Douglas output function and the linear pandemic function, let $0 < \beta < \max\{\alpha, 1/2\}$ and $\sigma \in (0,1)$. Then,*

1) Hartwick's rule before the pandemic is given by

$$\dot{K} = K^{\alpha} R^{\beta} - C = \beta Y;$$

2) the maximin consumption before the pandemic is given by

$$C(t) = C(S(t)) = \left[(1-\sigma)\left[\bar{u}_1 - \theta(\bar{u}_2 - (a - \bar{\psi} S(t)))\right]\right]^{\frac{1}{1-\sigma}}; \tag{52}$$

3) the maximum achievable utility before the pandemic $\bar{u}_1$ encompasses the utility level after the pandemic, $\bar{u}_2$, and is given implicitly by the following relationship:

$$\frac{\beta(1-\beta)^{\frac{1-\beta}{\beta}}}{\bar{\psi}\theta} \; \frac{C(S_0)^{\beta(2-\sigma)-1} - C(S_\infty)^{\beta(2-\sigma)-1}}{-(\beta(2-\sigma)-1)} = \frac{K_0^{1-\frac{\alpha}{\beta}}}{\alpha - \beta}, \tag{53}$$

where $S_\infty = 0$, $C(S_0)$ and $C(S_\infty)$ are given by (52).

The condition $\beta < 1/2$ is in line with Solow (1974) and Van Long (1992) in that although the exhaustible resources is essential in term of Dasgupta and Heal (1974b), Usually, the share of exhaustible resources in production is much lower than the share of capital input.

Obviously, the maximum consumption in the above (52) is different from the classical one via the maximin principle, where consumption is constant over time; see, for example Solow (1974), or Example 9.6.1 in Van Long (1992), which is the same as our study except that there is no pandemic shock in Example 9.6.1. Our modified maximin consumption before the pandemic is no longer constant over time. More precisely, pre-pandemic consumption depends on the reserve of exhaustible resources and it increases over time. This result essentially comes from the maximin principle that the instantaneous utility should be constant over time to ensure equity across generations: $\frac{C^{1-\sigma}}{1-\sigma} + \theta\left(\bar{u}_2 - \psi\left(S\right)\right) = \bar{u}_1$. Naturally, the pre-pandemic Hartwick rule is different from the classic idea as well—the genuine savings are no longer zero, and are not even fixed constant, rather, they increase over time $\dot{K} > 0$ without limit, which is in line with the literature, such as Example 9.6.1 in Van Long (1992). The main reason rests on the precautionary savings by taking into account the potential uncertain pandemic damage. Arguably, after the pandemic happens constant consumption is reestablished. Of course, this result also rests on the assumption that the pandemic happens only once. In a more complete setting where pandemics or other types of shocks happen repeatedly, then similar but more complicated results would be observed.

Nonetheless, despite the fact that pre-pandemic consumption is not constant, the maximin principle still implies that the utility, $\overline{u}_1$, is constant. The maximum instantaneous utility $\overline{u}_1$ is implicitly given by equation (53), where the left-hand side is the aggregate consumption over the period until exhaustion of the initial natural reserves in the long run, assuming no pandemic happens yet, while the right-hand side is essentially the initial extraction rate depending on the initial capital level and technology of production. To see this last point, recall that in (51), output is given by $Y = K^{\alpha} R^{\beta}$, thus $R_0 = \left(\frac{Y_0}{K_0^{\alpha}}\right)^{1/\beta}$ and $Y_0 = \frac{C(S_0)}{1-\beta}$. Hence, the maximum achievable utility must take into account the consumption of all generations as well as the initial stock of capital, natural resources, and production technology. The combination of all yields the optimal maximin choice.

Through direct calculation, given in Appendix A.8, the following properties are proven.

Proposition 7 *Under the conditions of Proposition 6,*

a) the maximum sustainable utility $\bar{u}_1$ increases with the post-pandemic utility: $\frac{\partial \bar{u}_1}{\partial \bar{u}_2} > 0$;

b) a higher probability of a pandemic θ decreases the maximum achievable utility $\bar{u}_1$: $\frac{\partial \overline{u_1}}{\partial \theta} < 0$;

c) a higher probability of a pandemic θ decreases capital accumulation K and increases the conservation of natural resource stock S prior to the occurrence of the pandemic: $\frac{\partial K}{\partial \theta} < 0$ and $\frac{\partial S}{\partial \theta} > 0$;

d) a stronger announced lockdown policy yields a higher maximum achievable utility $\bar{u}_1$: $\frac{\partial \bar{u}_1}{\partial \lambda} > 0$;

e) lockdown policy has no impact on capital accumulation and natural resource extraction during the pre-pandemic period: $\frac{\partial K}{\partial \lambda} = 0$ and $\frac{\partial S}{\partial \lambda} = 0$.

According to the precautionary policy, the maximum achievable utility $\bar{u}_1$ relies on the consequences of post-pandemic policies. Hence, a strong prevention policy λ, which increases the maximum achievable utility $\bar{u}_2$ (as in the previous subsection), will naturally also increase the pre-pandemic maximum achievable utility $\bar{u}_1$. In other words, prevention policy has a positive impact on the maximum achievable utility prior to the occurrence of the pandemic as well as after the pandemic.

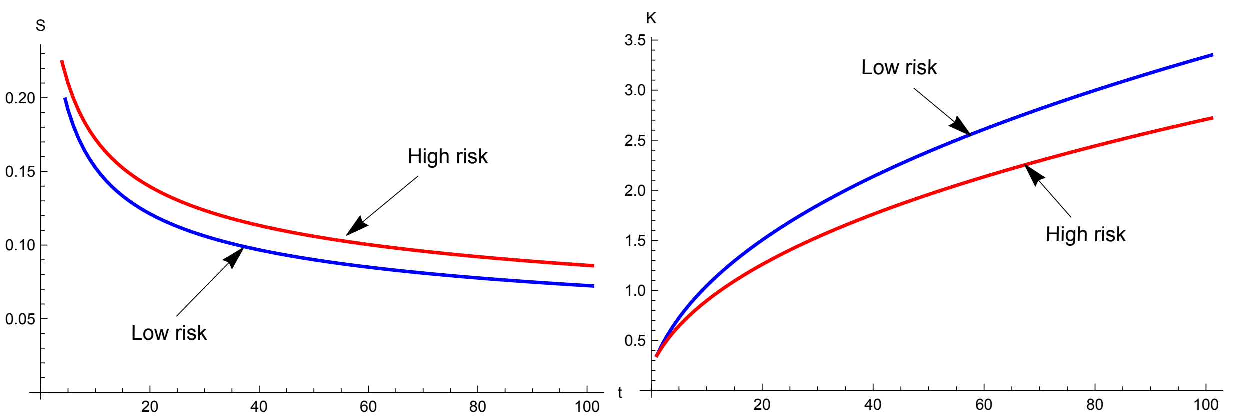

Figure 4: High probability ($\theta = 0.15$) (red) vs. low probability of a pandemic occurring $\theta = 0.1$ (blue)

Figure (4) clearly shows that natural resource extraction decreases[12] with a higher probability of a pandemic occurring. Recall that the natural resources has an amenity value that helps to alleviate the negative impact of the penalty ψ. Since a higher probability increases the marginal damage from the pandemic,

[12]This implies that the natural resource stock S is more conserved with a higher probability θ as shown in Figure (4). Of course, the natural resource stock S always decreases over time.

there is a greater incentive to conserve the natural resource stocks. Consequently, this implies a decrease in the consumption level (and consequently in production) in order to keep the utility constant. Due to the decrease in production, the economy needs less capital.

The prevention policy increases the maximum achievable utility $\bar{u}_1$. This is plausible since the maximum achievable utility after the pandemic increases due to higher labor productivity. However, the prevention policy has no effect on capital accumulation and natural resource extraction given the prevention policy has no effect on the aggregate consumption level prior to the occurrence of the pandemic.

To understand this result, note that the prevention policy increases the utility before and after the pandemic. In addition, $C(S_\infty)$ increases with the utility before the pandemic but decreases with the utility after the pandemic. Therefore, the effect of the prevention policy on the aggregate consumption level vanishes due to the opposite effects of utility $\bar{u}_1$ and $\bar{u}_2$ on pre-pandemic consumption. (See Appendix A.8 for the analytical proof of this result.) Hence, the prevention policy does not affect physical capital accumulation before the pandemic.

6 Extension: Prevention with cost

In this section, we assume that the prevention policy (vaccination, Prevention of social distancing, mask production) has a cost in terms of production loss. Denote as ζ the unit cost of the prevention policy. Then, the post-pandemic production function encompassing the cost of prevention is given by

$$F\left(K,R,D\left(z\right)\right)=\underbrace{\left(1-\zeta\lambda\right)}_{\text{Cost of the prevention}}K^{\alpha}R^{\beta}\kappa^{-\gamma}e^{\overbrace{-\gamma\left(b\left(1-\lambda\right)-c\right)\left(t-T\right)}^{\text{Benefit of the prevention}}},\ \ \forall t\geq T.$$

After doing the same calculations as in previous sections, and supposing a pandemic outbreak at T, then following the maximin principle the maximum achievable post-pandemic consumption, bearing the cost of prevention, is

$$C(t)=\bar{C}=Y(T)=Y(t)=\left(1-\zeta\lambda\right)K_T^{\alpha}S_T^{\beta}\kappa^{-\gamma}\left(\frac{\gamma\left(c-\tilde{b}\right)}{\beta}\right)^{\beta}, \tag{54}$$

and the corresponding sustainable utility is

$$V_2\left(K,S\right)=\bar{u}_2=U_2\left(\bar{C}\right). \tag{55}$$

One may remark that there should be an optimal level of prevention policy.

Proposition 8 *The optimal level of prevention policy that maximizes the sustained consumption level is*

$$\lambda^*=\frac{b\left(1+\zeta\beta^2\right)-c}{\left(1+\zeta\beta^2\right)}. \tag{56}$$

Obviously, prevention becomes optimal if $b\left(1+\zeta\beta^2\right)-c>0$, meaning that the transmission rate of the virus is already high. If the transmission rate is sufficiently low to be $b\left(1+\zeta\beta^2\right)-c<0$, then the prevention policy is not optimal and only decreases the sustained consumption level.

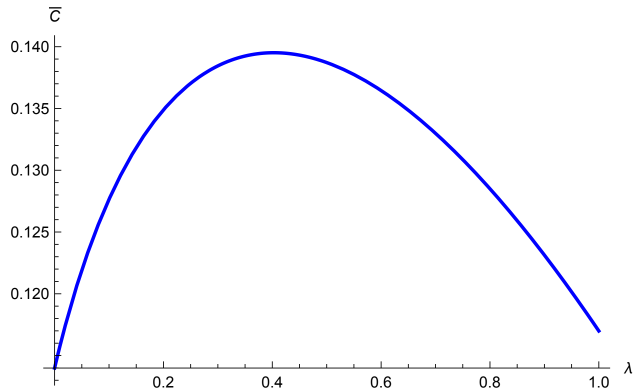

Figure 5: The effect of prevention policy, λ, on the post-pandemic sustained consumption level

Figure (5) shows that a prevention policy has a threshold beyond which it becomes suboptimal to impose a stronger prevention policy since it decreases production.

7 Conclusion

This paper illustrates a sustainable economy that follows an equitable growth path under the uncertainty of a pandemic. We analytically present the intertemporal paths of man-made capital and natural resource depletion under the maximin criterion. We also present the impact of the probability of a pandemic on the these optimal dynamics. Moreover, a prevention policy is shown to be beneficial in ensuring a higher constant sustainable utility and can compensate the decrease in utility due to the probability of a pandemic occurring.

This paper offers a theoretical foundation for the "whatever it costs" strategy to control an epidemic disease from the very beginning, in order to guarantee a sustained utility over time. We also show that if there is an available substitute for the exhaustible resource stock, then the utility can be sustained even though the prevalence rate increases and there is no control over the disease.

Another important finding is that there exists a trade-off between the public health policy (i.e, prevention) and the construction of a wealth fund (i.e, capital accumulation) since it is shown that a strong prevention policy increases the labor productivity and decreases the exploitation of natural resources. Then, the capital accumulation decreases due to a lower exploitation of natural resources (rents from the natural capital).

One of the limitations of this study is that the prevention policy is an exogenous variable that provides us analytical results. In future work, endogenous prevention policies are needed to capture the different phases of the pandemic's development and its impacts on Hartwick's rule. Moreover, the prevention policies also shape production activities, such that a threshold for prevention can be reached. Thus, more complex dynamics as well policy recommendations can be delivered without analytical results.

A Appendix

A.1 Dynamics of SIR

Consider equations (3) and (4)

$$\frac{\dot{x}}{x} = \frac{\tilde{b}y}{x+y} \tag{57}$$

and

$$\frac{\dot{y}}{y} = \frac{\tilde{b}y}{x+y} - c. \tag{58}$$

Subtracting the last two equations, it follows that

$$\frac{\dot{y}}{y} = \frac{\dot{x}}{x} + \left(\tilde{b} - c\right),$$

which is equivalent to

$$\frac{d}{dt}\left(\ln y\right) = \frac{d}{dt}\left(\ln x\right) + \left(\tilde{b} - c\right). \tag{59}$$

Integrating and taking the exponential of equation (59) gives

$$y\left(t\right) = x\left(t\right)\kappa e^{\left(\tilde{b}-c\right)t}, \tag{60}$$

with $\kappa = \frac{y_0}{x_0}$. Replacing (60) in (57) gives

$$\frac{\dot{x}}{x} = -\tilde{b}\left(\frac{\kappa e^{\left(\tilde{b}-c\right)t}}{1+\kappa e^{\left(\tilde{b}-c\right)t}}\right) = -\frac{\tilde{b}}{\tilde{b}-c}\left(\frac{du}{u}\right), \tag{61}$$

where $u = 1 + \kappa e^{\left(\tilde{b}-c\right)t}$. Integrating (61) gives

$$\ln x = -\frac{\tilde{b}}{\tilde{b}-c}\left(\ln u + K_1\right). \tag{62}$$

Taking the exponential of (62) and rearranging terms, we have

$$x\left(t\right) = x_0\left(1+\kappa\right)^{\frac{\tilde{b}}{\tilde{b}-c}}\left(1+\kappa e^{\left(\tilde{b}-c\right)t}\right)^{-\frac{\tilde{b}}{\tilde{b}-c}}, \tag{63}$$

$$y\left(t\right) = y_0\left(1+\kappa\right)^{\frac{\tilde{b}}{\tilde{b}-c}}\left(1+\kappa e^{\left(\tilde{b}-c\right)t}\right)^{-\frac{\tilde{b}}{\tilde{b}-c}} e^{\left(\tilde{b}-c\right)t}. \tag{64}$$

A.2 Proof of Proposition 1

The proof is quite trivial. To ensure the condition $\lim_{t\to\infty}\mathcal{H} = 0$, we should have $\lim_{t\to\infty}p_3 = -p_1\frac{\gamma\left(\tilde{b}-c\right)}{\beta}S\left(t\right) = 0$. If we have an increasing number of infected persons, which requires $c < \tilde{b}$, this (i.e, $\lambda < 1 - \frac{c}{b}$). Natural resource extraction becomes $\frac{\dot{S}}{S} = \frac{\gamma\left(\tilde{b}-c\right)}{\beta} > 0$ in the long run. This implies that $\lim_{t\to\infty}p_3 \neq 0$ and the transversality condition $\lim_{t\to\infty}\mathcal{H} = 0$ are not respected. Note also that the increase in natural resource extraction is in contradiction with the constraint $\int_0^\infty R\left(t\right)dt = S_0$. Hence, we should have $c > \tilde{b}$ (i.e, $\lambda > 1 - \frac{c}{b}$)

for an optimal solution under maximin optimization that implies the existence of a sustainable utility path.

A.3 Proof of Proposition 2

To check that the proposal candidate indeed establishess the first-order condition, differentiating two first-order conditions in (25) with respect to time, it follows that

$$\frac{\dot{F}_q}{F_q} = \frac{\dot{\Phi}}{\Phi} = -F_K = -\alpha \frac{Y(t)}{K(t)} = -\alpha \frac{Y_T}{K_T}. \tag{65}$$

Thus,

$$\Phi(t) = \alpha \frac{Y_T}{K_T} e^{-\left(\alpha \frac{Y_T}{K_T}\right)(t-T)}. \tag{66}$$

The second first-order condition in (25) gives

$$p_2(t) = U_2^{'}\left(\bar{C}\right) \alpha \frac{Y_T}{K_T} e^{-\left(\alpha \frac{Y_T}{K_T}\right)(t-T)}. \tag{67}$$

Since $\dot{K} = 0$, from the last equation of (26) we can write

$$p_3(t) = p_1 R = p_1 \frac{\gamma\left(c - \tilde{b}\right)}{\beta} S(T) e^{-\left(\frac{\gamma(c-\tilde{b})}{\beta}\right)(t-T)}. \tag{68}$$

Differentiating (68) yields

$$\dot{p}_3(t) = -p_1 \left(\frac{\gamma\left(c - \tilde{b}\right)}{\beta}\right)^2 S(T) e^{-\left(\frac{\gamma(c-\tilde{b})}{\beta}\right)(t-T)}. \tag{69}$$

On the other hand, recall equation (c) of (26):

$$\dot{p}_3 = -p_2 \gamma\left(c - \tilde{b}\right) Y = -p_2 \gamma\left(c - \tilde{b}\right) Y_T = -U_2^{'}\left(\bar{C}\right) \alpha \frac{\left(Y_T\right)^2}{K_T} e^{-\left(\alpha \frac{Y_T}{K_T}\right)(t-T)} \gamma\left(c - \tilde{b}\right), \tag{70}$$

where the second equality comes from the fact that along the candidate path production is constant and the last equality comes from (67).

The last two equations are consistent if and only if

$$\alpha \frac{Y_T}{K_T} = \frac{\gamma\left(c - \tilde{b}\right)}{\beta} \tag{71}$$

and

$$p_1 \left(\frac{\gamma\left(c - \tilde{b}\right)}{\beta}\right)^2 = U_2^{'}\left(\bar{C}\right) \alpha \frac{\left(Y_T\right)^2}{K_T}. \tag{72}$$

Combining equations (29) and (71), we have the following relationship between K_T and S_T:

$$S_T = K_T^{\frac{1-\alpha}{\beta}} \left(\frac{\gamma \left(c - \tilde{b} \right)}{\beta} \right)^{\frac{1-\beta}{\beta}} (\kappa)^{\gamma} .$$

Substituting this S_T into $\overline{C}$, it follows that

$$\overline{C} = K_T \left(\frac{\gamma(c - \widetilde{b})}{\beta} \right)^{1+\beta-\alpha} .$$

By plugging equation (29) into the aggregate utility $U_2 \left(\bar{C} \right)$, we can express the maximum sustainable utility level as

$$\bar{u}_2 = U_2 \left(\frac{K_T^{\alpha} S_T^{\beta}}{\kappa^{\gamma}} \left(\frac{\gamma \left(c - \tilde{b} \right)}{\beta} \right)^{\beta} \right) . \tag{73}$$

That completes the proof.

A.4 Long-term behavior

In order to characterize the long-run behavior, consider labor productivity (when $z(t)$ decreases) as augmenting the natural resource stocks and diminishing physical capital. Then, the production function becomes

$$Y = \left(K L^{-m_K} \right)^{\alpha} \left(R L^{-m_X} \right)^{\beta} L^{m_L},$$

with $J = KL^{-m_K}$ and $X = SL^{m_X}$. Thus, $-\alpha m_K + \beta m_X + m_L = 1 - \alpha - \beta = \gamma$. To guarantee that K and Y grow at the same rate, it must be that $m_K = m_L$. In the long run, $\dot{J} = 0$ and $\dot{X} = 0$ yield $g_K = m_K g_L$. From equation $Y = \frac{C - q_z}{1-\beta}$, it is easy to derive $g_Y = g_{q_z} = 0$. Furthermore, $g_K = g_Y$ implies $m_L = m_K = 0$. Then, it is easy to see that $m_X = \frac{1-\alpha-\beta}{\beta} = \frac{\gamma}{\beta}$ and $J = K$. So the dynamic system can be expressed as

$$\begin{cases} \dot{K} = \beta Y - q_z, \\ \dot{X} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \kappa^{-\frac{\gamma}{\beta}} - \frac{\gamma \left(\tilde{b} - c \right)}{\beta} X, \\ \dot{q}_z = \gamma \left(\tilde{b} - c \right) Y + r q_z, \end{cases}$$

where $q_z = \frac{p_3}{p_2}$. Thus, in the long run the above state and co-state variables can be expressed in terms of the constant consumption $\bar{C}$:

$$\begin{cases} K^* = \frac{\alpha \beta \bar{C}}{\gamma \left(c - \tilde{b} \right)}, \\ X^* = \left(\frac{\beta}{\gamma \left(c - \tilde{b} \right)} \right)^{\left(1 - \frac{\alpha}{\beta} \right)} \alpha^{-\frac{\alpha}{\beta}} \kappa^{-\frac{\gamma}{\beta}} \left(\bar{C} \right)^{\frac{(1-\alpha)}{\beta}} , \\ q_z^* = \beta \bar{C}. \end{cases} \tag{74}$$

$X = SL^{\frac{\gamma}{\beta}}$ yields that $\frac{\dot{X}}{X} = \frac{\dot{S}}{S} - \frac{\gamma \left(\tilde{b} - c \right)}{\beta}$. Thus, in the long run, from $\dot{X} = 0$ it follows that $\frac{\dot{S}}{S} = \frac{\gamma \left(\tilde{b} - c \right)}{\beta}$. Since exhaustible natural resource stocks cannot increase in the long run, we should have $\tilde{b} - c < 0$, which is possible with prevention policy $\lambda > 1 - \frac{c}{b}$. In other words, the natural resource stocks decrease at rate $-\frac{\gamma(\tilde{b}-c)}{\beta}$.

A.5 Not a knife's edge case

In order to see that the system is equivalent to the knife's edge case with the SIR model, denote $\hat{q}_z = \frac{q_z z}{\tilde{b}-c}$. Recall that the knife's edge case implies the following functional forms: $g(z) = \left(\tilde{b} - c\right) z$, $\tilde{f}(z) = \frac{1}{z}$, and $X = SL^{\frac{\gamma}{\beta}}$. Then, it is easy to see that we can reduce the dimension of the system by keeping z out of the dynamic system:

$$\begin{cases} \dot{K} = \beta Y - q_z, \\ \dot{X} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \kappa^{-\frac{\gamma}{\beta}} - \frac{\gamma\left(\tilde{b}-c\right)}{\beta} X, \\ \dot{q}_z = \gamma\left(\tilde{b} - c\right) Y + r q_z. \end{cases}$$

ii) For the sake of clarity, we rewrite the Lagrangian as

$$\mathcal{L} = -\hat{p}_1 R + \hat{p}_2 \left(K^\alpha R^\beta \left(\bar{L}\tilde{f}(z)\right)^\gamma - C\right) + \hat{p}_3 \left(g(z)\right) + \Phi \cdot \left(U_2(C) - \bar{u}_2\right).$$

The first-order conditions are

$$\begin{cases} \hat{p}_1 = \hat{p}_2 F_R, \\ \Phi U_2^{'}(c) = \hat{p}_2. \end{cases}$$

We can write the dynamics of the economy in the following form:

$$\begin{cases} \dot{K} = \beta Y - \hat{q}_z g(z), \\ \dot{S} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} \left(\bar{L}\tilde{f}(z)\right)^{-\frac{\gamma}{\beta}}, \\ \dot{z} = g(z), \\ \dot{\hat{p}}_2 = -\hat{p}_2 F_K. \\ \dot{\hat{p}}_3 = -\hat{p}_2 K^\alpha R^\beta \gamma \bar{L} \left(\tilde{f}(z)\right)^{\gamma-1} \tilde{f}^{'}(z) - \hat{p}_3 g^{'}(z). \end{cases} \tag{75}$$

With the variable $\hat{q}_z = \frac{\hat{p}_3}{\hat{p}_2}$ and $X = SL^{\frac{\gamma}{\beta}}$, it follows that

$$\begin{cases} \dot{K} = \beta Y - \hat{q}_z g(z), \\ \dot{X} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}} + \frac{\gamma}{\beta} \frac{\tilde{f}^{'}(z)}{\tilde{f}(z)} \dot{z} X, \\ \dot{z} = g(z), \\ \dot{\hat{q}}_z = -\gamma Y \frac{\tilde{f}^{'}(z)}{\tilde{f}(z)} - \left(g^{'}(z) - r\right) \hat{q}_z. \end{cases} \tag{76}$$

That completes the proof.

A.5.1 Example: SIS model

To have a clear idea of why the social planner should ensure $\tilde{b} - c < 0$, we give an example using an SIS model. Following d'Albis and Augeraud-Véron (2021), denote as x susceptible individuals and as y infected individuals. Assume there is no immunity in this model, meaning that the infected population becomes susceptible and can be infected several times. Then, the dynamics are

$$\dot{x}(t) = cy(t) - \tilde{b}x(t)\frac{y(t)}{x(t)+y(t)},$$
$$\dot{y}(t) = -cy(t) + \tilde{b}x(t)\frac{y(t)}{x(t)+y(t)},$$

where $\tilde{b} = b(1-\lambda)$. Recall that $z(t) = \frac{y(t)}{x(t)}$. We can write the dynamics $\dot{z} = \left(\tilde{b} - c(1+z)\right)z$. The differential equation has the following general solution:

$$z(t) = \frac{\tilde{b} - c}{\frac{\tilde{b}-c+z(0)c}{z(0)}e^{-\left(\tilde{b}-c\right)t} + c}. \tag{77}$$

There are two steady-state solutions: $z^* = 0$, which is unstable if $\tilde{b} - c < 0$, or $z^* = \frac{\tilde{b}-c}{c}$, which is stable if $\tilde{b} - c > 0$. Then, if $z^* = \frac{\tilde{b}-c}{c}$, $\dot{z} = 0$ at the steady state, leading to $\bar{C} = 0$. This shows that with increasing prevalence $z(t)$ over time, the economy cannot sustain itsutility level.

However, if the policymaker can control the pandemic from the beginning by ensuring $\tilde{b} - c < 0$, then $z(t)$ jumps to zero from the beginning and the model becomes a standard DHSS model. Then, sustained utility over time can be achieved.

A.6 An economy facing an uncertain pandemic

Taking the expectations of (16) gives

$$E_T\left[\int_0^T U(C(t))e^{-\rho t}dt + e^{-\rho T}\varphi(K(T), S(T))\right]. \tag{78}$$

Note that the probability distribution and density function are

$$f(t) = \theta e^{-\theta t} \text{ and } F(t) = 1 - e^{-\theta t}. \tag{79}$$

We write the following expression:

$$\int_0^\infty f(T)\left[\int_0^T U(C(t))e^{-\rho t}dt + e^{-\rho T}\varphi(K(T), S(T))\right]dT =$$
$$\underbrace{\int_0^\infty f(T)\left[\int_0^T U(C(t))e^{-\rho t}dt\right]dT}_{\text{A}} + \underbrace{\int_0^\infty f(T)\left[e^{-\rho T}\varphi(K(T), S(T))\right]dT}_{\text{B}}. \tag{80}$$

Integrating by parts, A yields

$$dX = f(T) \implies X = \int_0^T f(s)\,ds Y = \int_0^T U(C(t))\,e^{-\rho t}dt \implies dY = U(C(T))\,e^{-\rho T}.$$

Using $\int Y dX = XY - \int X dY$ yields

$$A = \left[\left(\int_0^T f(s)\,ds\right)\left(\int_0^T U(C(t))\,e^{-\rho t}dt\right)\right]_{T=0}^{\infty} - \int_0^\infty F(T)\,U(C(T))\,e^{-\rho T}dT. \tag{81}$$

Recall that $\int_0^\infty f(s)\,ds = 1$. Part A leads to

$$\int_0^\infty U(C(t))\,e^{-\rho t}dt - \int_0^\infty F(t)\,U(C(t))\,e^{-\rho t}dt. \tag{82}$$

Taking the sum $A + B$, it follows that

$$\int_0^\infty \left[(1 - F(t))\,U(C(t)) + f(t)\,\varphi(K(T), S(T))\right]e^{-\rho t}dt.$$

Thus, inserting the probability distribution and density function gives

$$\int_0^\infty \left[U(C(t)) + \varphi(K(T), S(T))\right]e^{-(\rho+\theta)t}dt.$$

A.7 Proof of Proposition 6 - Pre-pandemic regime

The detailed proof follows the same process as Example 9.6.1 in Van Long (1992) (pages 300–304). First, by Theorem 7.11.1 of Van Long (1992) (page 255), it follows that

$$\int_0^\infty \frac{\partial \mathcal{L}}{\partial \bar{u}_1}dt = 0,$$

which yields

$$\int_0^\infty \left((\rho+\theta)\,\bar{u}_1 e^{-(\rho+\theta)t} - \Lambda(t)\right)dt = 0 \tag{83}$$

or

$$\int_0^\infty \Lambda(t)\,dt = 1. \tag{84}$$

In addition, Theorem 9.6.1 in Van Long (1992) implies that

$$e^{(\rho+\theta)t}\mathcal{H} = (\rho+\theta)\,\overline{u_1}. \tag{85}$$

Combing with equations (44) and (45) gives

$$-\,e^{(\rho+\theta)t}\pi_1 R + e^{(\rho+\theta)t}\pi_2 I = 0, \tag{86}$$

where

$$I = \dot{K} = K^{\beta} R^{\alpha} - C.$$

From (86), it follows that

$$\frac{\pi_1}{\pi_2} = \frac{I}{R}.$$

From the second equation in the first-order conditions (46), we have

$$\frac{\pi_1}{\pi_2} = \left(\beta K^{\alpha} R^{\beta-1}\right).$$

Then,

$$\dot{K} = I = \beta K^{\alpha} R^{\beta}. \tag{87}$$

We reformulate the dynamics of capital accumulation as follows:

$$C = Y - I = (1-\beta) K^{\alpha} R^{\beta}. \tag{88}$$

Thus, the dynamics of the natural resource stocks S and the dynamics of capital K can be rewritten as

$$\dot{S} = -Y^{\frac{1}{\beta}} K^{-\frac{\alpha}{\beta}},$$
$$\dot{K} = \beta Y \quad \text{where} \quad Y = \frac{C\left(\bar{u}_1, \bar{u}_2, S\right)}{1-\beta}.$$

Hence, Hartwick's rule in the first period can be expressed in the following way by using (88):

$$\dot{K} = K^{\beta} R^{\alpha} - C = \beta Y = \tilde{q} R,$$

with $\tilde{q} = \frac{\pi_1}{\pi_2} = \frac{I}{R}$.

Furthermore, from the CRRA utility function $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ with $0 < \sigma < 1$, it follows that $\frac{C^{1-\sigma}}{1-\sigma} + \theta\left(\bar{u}_2 - \psi\left(S\right)\right) = \bar{u}_1$. Suppose a linear pandemic cost function $\psi\left(S\right) = a - \bar{\psi} S$ where parameters fulfill $a > \bar{\psi} S_0 > 0$, then it is easy to check

$$C(t) = \left[(1-\sigma)\left[\bar{u}_1 - \theta\left(\bar{u}_2 - \left(a - \bar{\psi} S(t)\right)\right)\right]\right]^{\frac{1}{1-\sigma}}.$$

Eliminating the time variable from the two dynamics equations above yields

$$\frac{dS}{dK} = -\frac{1}{\beta} K^{-\frac{\alpha}{\beta}} \frac{\left[(1-\sigma)\left[\bar{u}_1 - \theta\left(\bar{u}_2 - \left(a - \bar{\psi} S\right)\right)\right]\right]^{\frac{1-\beta}{\beta(1-\sigma)}}}{(1-\beta)^{\frac{1-\beta}{\beta}}}. \tag{89}$$

Integrating on both sides of (89) gives

$$(1-\beta)^{\frac{1-\beta}{\beta}} (1-\sigma)^{-\frac{1-\beta}{\beta(1-\sigma)}} \int_{S}^{S_0} \left[\bar{u}_1 - \theta\left(\bar{u}_2 - \bar{\psi}\left(a - \bar{\psi}\xi\right)\right)\right]^{-\frac{1-\beta}{\beta(1-\sigma)}} d\xi = \frac{1}{\beta} \int_{K_0}^{K} \Upsilon^{-\frac{\alpha}{\beta}} d\Upsilon,$$

with the assumption that $\alpha > \beta$ (Solow, 1974; Van Long, 1992). Furthermore, noticing when $K \to \infty$ and $S \to 0$, the above integral becomes

$$\frac{(1-\beta)^{\frac{1-\beta}{\beta}}(1-\sigma)^{-\frac{1-\beta}{\beta(1-\sigma)}}}{-\left(1-\frac{1-\beta}{\beta(1-\sigma)}\right)\bar{\psi}\theta}\left[\left[\bar{u}_1-\theta\left(\bar{u}_2-\left(a-\bar{\psi}S_0\right)\right)\right]^{1-\frac{1-\beta}{\beta(1-\sigma)}}-\left[\bar{u}_1-\theta\left(\bar{u}_2-a\right)\right]^{1-\frac{1-\beta}{\beta(1-\sigma)}}\right] = \frac{K_0^{1-\frac{\alpha}{\beta}}}{\alpha-\beta}. \tag{90}$$

The last equation gives the implicit expression for $\bar{u}_1$:

$$\bar{u}_1 = \bar{u}_1\left(\bar{u}_2, K_0, S_0, \theta, \bar{\psi}, a, k, \alpha, \beta, \sigma\right).$$

A similar result to (90) can be obtained in terms of any $S(t), K(t)$ instead of $S(0), K(0)$. To do so, taking the integral of (89) over $[t, +\infty]$ with the initial condition $S(t), K(t)$, it follows that

$$-\frac{(1-\beta)^{\frac{1-\beta}{\beta}}(1-\sigma)^{-\frac{1-\beta}{\beta(1-\sigma)}}}{\left(1-\frac{1-\beta}{\beta(1-\sigma)}\right)\bar{\psi}\theta}\left[\left[\bar{u}_1-\theta\left(\bar{u}_2-\left(a-\bar{\psi}S\left(t\right)\right)\right)\right]^{1-\frac{1-\beta}{\beta(1-\sigma)}}-\left[\bar{u}_1-\theta\left(\bar{u}_2-a\right)\right]^{1-\frac{1-\beta}{\beta(1-\sigma)}}\right] = \frac{K\left(t\right)^{1-\frac{\alpha}{\beta}}}{\alpha-\beta}. \tag{91}$$

To get a precise $S(t)$ and $K(t)$, denote

$$\tilde{S}(t) = \bar{u}_1-\theta\left(\bar{u}_2-\left(a-\bar{\psi}S\left(t\right)\right)\right)$$

such that

$$\tilde{S}\left(t\right) = \left(\frac{K\left(t\right)^{1-\frac{\alpha}{\beta}}}{B_1\left(\alpha-\beta\right)}+B_2\right)^{\frac{1}{B_2}}, \tag{92}$$

where B_1, B_2, and B_3 are constant terms.

$$\begin{cases} B_1 = -\frac{(1-\beta)^{\frac{1-\beta}{\beta}}(1-\sigma)^{-\frac{1-\beta}{\beta(1-\sigma)}}}{\left(1-\frac{1-\beta}{\beta(1-\sigma)}\right)\bar{\psi}\theta} > 0 \\ B_2 = 1-\frac{1-\beta}{\beta(1-\sigma)} < 0 \\ B_3 = \left[\bar{u}_1-\theta\left(\bar{u}_2-a\right)\right]^{1-\frac{1-\beta}{\beta(1-\sigma)}} > 0 \end{cases} \tag{93}$$

Substituting (92) in for capital accumulation,

$$\dot{K} = \frac{\beta}{1-\beta}\left[(1-\sigma)\left(\bar{u}_1-\theta\left(\bar{u}_2-\left(a-\bar{\psi}S\right)\right)\right)\right]^{\frac{1}{1-\sigma}},$$

it follows that

$$\dot{K} = \frac{\beta}{1-\beta}(1-\sigma)^{\frac{1}{1-\sigma}}\left[\frac{K\left(t\right)^{1-\frac{\alpha}{\beta}}}{B_1\left(\alpha-\beta\right)}+B_3\right]^{\frac{1}{B_2(1-\sigma)}}. \tag{94}$$

It is possible to solve the last differential equation (94) analytically, even though it yields an implicit solution for $K(t)$:

$$\frac{\beta}{1-\beta}(1-\sigma)^{\frac{1}{1-\sigma}}t+\frac{K(0)}{(B_3)^{\frac{\beta}{\beta(1-\sigma)-(1-\beta)}}}\,{}_2F_1\left(\frac{\beta}{\beta(1-\sigma)-(1-\beta)},\frac{1}{1-\frac{\alpha}{\beta}};1+\frac{1}{1-\frac{\alpha}{\beta}};-\frac{K(0)}{B_1(\alpha-\beta)(B_3)}\right)$$

$$=\frac{K(t)}{(B_3)^{\frac{\beta}{\beta(1-\sigma)-(1-\beta)}}}\,{}_2F_1\left(\frac{\beta}{\beta(1-\sigma)-(1-\beta)},\frac{1}{1-\frac{\alpha}{\beta}};1+\frac{1}{1-\frac{\alpha}{\beta}};-\frac{K(t)}{B_1(\alpha-\beta)(B_3)}\right). \tag{95}$$

A.8 Proof of Proposition 7

(i) To assess the impact of the probability of a disease occurring, θ, on the maximum sustainable utility $\bar{u}_1$, we take the total differential with respect to equation (90) and rearrange the terms. It follows that

$$\frac{\partial\bar{u}_1}{\partial\theta}=$$

$$\frac{\overbrace{-\frac{1}{\theta}\left[\bar{A}^{1-\frac{1-\beta}{\beta(1-\sigma)}}-\bar{B}^{1-\frac{1-\beta}{\beta(1-\sigma)}}\right]}^{<0}+\overbrace{\frac{1-\beta}{\beta(1-\sigma)}\left[\left(\bar{u}_2-\bar{\psi}(a-kS_0)-\bar{u}_1\frac{\beta(1-\sigma)}{(1-\beta)\theta}\right)\bar{A}^{\frac{1-\beta}{\beta(1-\sigma)}}-\left(\bar{u}_2-\bar{\psi}a\right)\bar{B}^{\frac{1-\beta}{\beta(1-\sigma)}}\right]}^{<0}}{-\underbrace{\left(1-\frac{1-\beta}{\beta(1-\sigma)}\right)}_{<0}\underbrace{\left(\frac{\bar{A}^{1-\frac{1-\beta}{\beta(1-\sigma)}}-\bar{B}^{1-\frac{1-\beta}{\beta(1-\sigma)}}}{\theta}\right)}_{>0}}$$

$$<0,$$

where $\bar{A}=\bar{u}_1-\theta\left(\bar{u}_2-\bar{\psi}(a-kS_0)\right)$ and $\bar{B}=\bar{u}_1-\theta\left(\bar{u}_2-\bar{\psi}a\right)$.

ii) To assess the impact of the probability of a pandemic on physical capital accumulation, we take the total differential of equation 95 and rearrange terms, yielding

$$\frac{\partial K(t)}{\partial\theta}=-\frac{\overbrace{-\frac{1}{1-\sigma}\left(\bar{u}_1-\theta\left(\bar{u}_2-\bar{\psi}\right)\right)^{-\frac{1}{1-\sigma}-1}}^{<0}\overbrace{\left(\frac{\partial\bar{u}_1}{\partial\theta}-\left(\bar{u}_2-\bar{\psi}\right)\right)}^{<0}\overbrace{\tilde{K}}^{<0}+{}_{<0}\overbrace{\frac{\partial\tilde{K}}{\partial\theta}}}{{}_{<0}-\underbrace{\frac{\partial\tilde{J}}{\partial K}}}<0, \tag{96}$$

where

$$\tilde{J}=\frac{K(t)}{(B_3)^{\frac{\beta}{\beta(1-\sigma)-(1-\beta)}}}\,{}_2F_1\left(\frac{\beta}{\beta(1-\sigma)-(1-\beta)},\frac{1}{1-\frac{\alpha}{\beta}};1+\frac{1}{1-\frac{\alpha}{\beta}};-\frac{K(t)}{B_1(\alpha-\beta)(B_3)}\right),$$

$$\frac{\partial\tilde{J}}{\partial K}=\frac{1}{(B_3)^{\frac{\beta}{\beta(1-\sigma)-(1-\beta)}}}\,{}_2F_1\left(\frac{\beta}{\beta(1-\sigma)-(1-\beta)},\frac{1}{1-\frac{\alpha}{\beta}};1+\frac{1}{1-\frac{\alpha}{\beta}};-\frac{K(t)}{B_1(\alpha-\beta)(B_3)}\right)$$

$$+\frac{\frac{\beta}{\beta(1-\sigma)-(1-\beta)}\left(\frac{1}{1-\frac{\alpha}{\beta}}\right)^2}{\left(1+\frac{1}{1-\frac{\alpha}{\beta}}\right)B_1(\alpha-\beta)(B_3)}\,{}_2F_1\left(1+\frac{\beta}{\beta(1-\sigma)-(1-\beta)},1+\frac{1}{1-\frac{\alpha}{\beta}};2+\frac{1}{1-\frac{\alpha}{\beta}};-\frac{K(t)}{B_1(\alpha-\beta)(B_3)}\right).$$

(iii) By differentiating equation (90), it is easy to see the impact of the prevention policy λ on the maximum achievable utility $\bar{u}_1$:

$$\frac{\partial \bar{u}_1}{\partial \lambda} = \beta b \theta \left[\left(\frac{c - b(1-\lambda)}{\beta} \right)^{\beta} \frac{K_0^{\alpha} S_0^{\beta}}{\kappa} \right]^{-\sigma} \left(\frac{c - b(1-\lambda)}{\beta} \right)^{\beta - 1} > 0, \tag{97}$$

$$\frac{\partial \bar{u}_2}{\partial \lambda} = \beta b \left[\left(\frac{c - b(1-\lambda)}{\beta} \right)^{\beta} \frac{K_0^{\alpha} S_0^{\beta}}{\kappa} \right]^{-\sigma} \left(\frac{c - b(1-\lambda)}{\beta} \right)^{\beta - 1} > 0. \tag{98}$$

It is also obvious that the analytical solution of $K(t)$ is not affected by the prevention policy λ. The only term that implies λ is B_3. The variation of B_3 with respect to λ is

$$\frac{\partial B_3}{\partial \lambda} = \frac{\partial \bar{u}_1}{\partial \lambda} - \theta \frac{\partial \bar{u}_2}{\partial \lambda} = 0.$$

That completes the proof.

A.9 Proof of Proposition 8

In order to see the impact of the prevention policy, λ, we differentiate the consumption with respect to λ, which yields

$$\frac{\partial \bar{C}}{\partial \lambda} = \left(\frac{\gamma (c - b(1-\lambda))}{\beta} \right)^{\beta} \frac{K_T^{\alpha} S_T^{\beta}}{\kappa^{\gamma}} \left[-\zeta + (1 - \zeta\lambda) \gamma b \beta \left(\frac{\gamma (c - b(1-\lambda))}{\beta} \right)^{-1} \right] \lesseqgtr 0.$$

Thus, the first-order condition for optimality yields

$$\lambda^* = \frac{b\left(1 + \zeta\beta^2\right) - c}{\left(1 + \zeta\beta^2\right)}.$$

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