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# Applying Entropy Criterion to Cost Allocation: an Empirical Analysis of Fertilizer Cost Estimates for European countries

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► **To cite this version:**

Dominique Desbois. Applying Entropy Criterion to Cost Allocation: an Empirical Analysis of Fertilizer Cost Estimates for European countries. 7th Stochastic Modeling Techniques and Data Analysis International Conference and Demographics 2022 Workshop, Jun 2022, Athènes, Greece. hal-03695425

**HAL Id: hal-03695425**

**<https://hal.inrae.fr/hal-03695425>**

Submitted on 14 Jun 2022

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# Applying Entropy Criterion to Cost Allocation: an Empirical Analysis of Fertilizer Cost Estimates for European countries

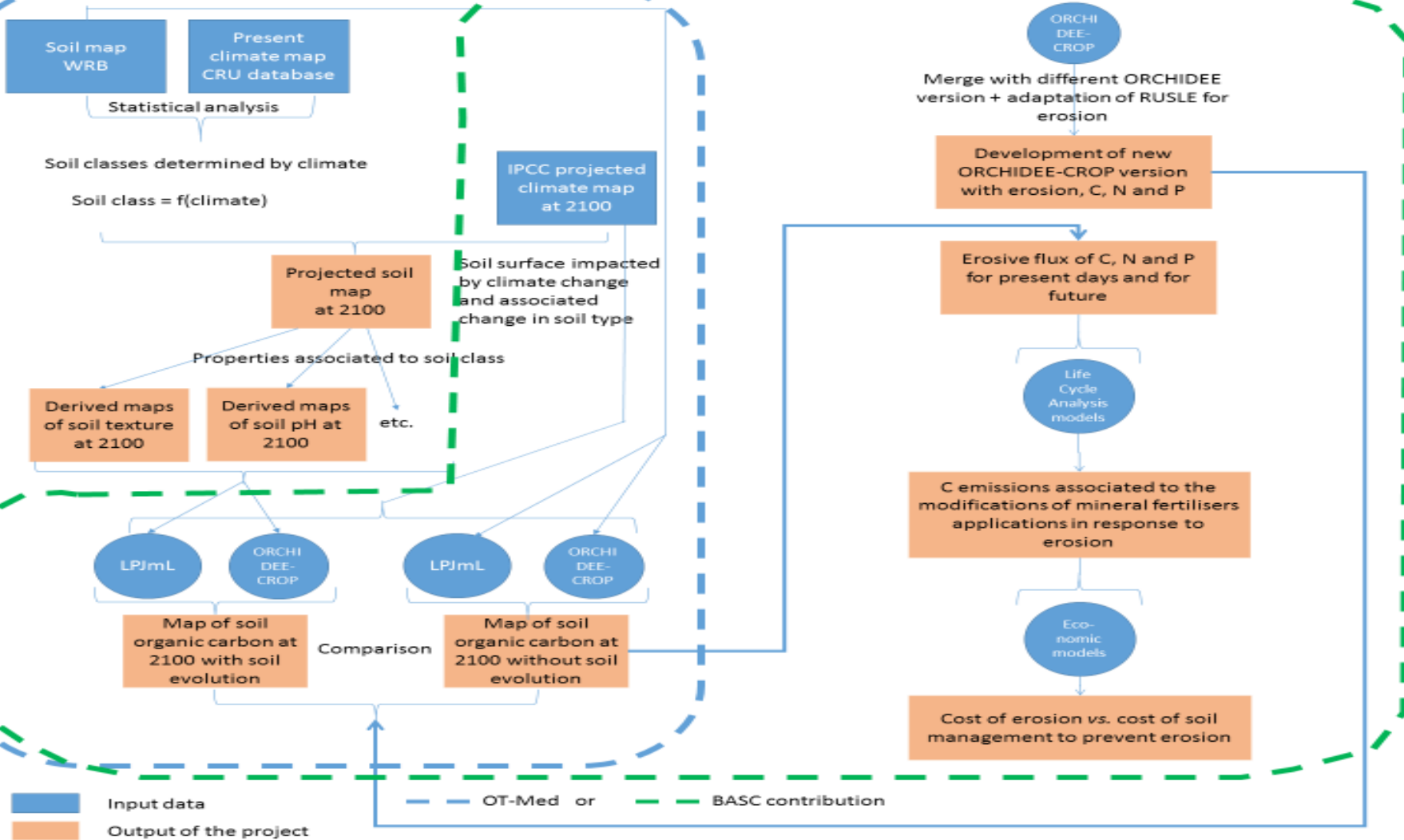


Fertilizer drill (19<sup>th</sup> century), *Life Museum*, AgroParisTech (photo library)

SMTDA\_2022, 7-10 June, Athens

Dominique DESBOIS (Paris Saclay Applied Economics, INRAE-AgroParisTech)

# ASSESS : OTE-Med/BASC project



**A debated conjecture: as a proxy, could the fertilisation cost be used to estimate the recovering costs of the soil fertility as a land service for agriculture ?**

# The economic assesment of erosion cost(on site/off site)

## I) on-site costs:

- Loss of soil;
- **Loss of nutrients;**
- Loss of organic matter;
- Decreased chemical, physical and biological fertility;
- Damage to plantations and amendments;
- Falling yields;
- Production losses;
- Decrease of the cultivated area;

## II) off site costs:

- Sedimentation
- Sedimentation of lakes and rivers;
- Decreased water retention capacity;
- floods;
- Flash floods;
- Landslides;
- Destruction of transport infrastructure;
- Obstruction of waterways;
- eutrophication;
- Loss of biodiversity;
- Unsafe water quality;
- Negative effects on water treatment;
- Negative effects on the production of electrical energy;
- Decrease in food production;
- Restrictions on recreational uses of water;

# Estimates of Fertilizer Costs: an Input-Output Methodology

**Econometric modeling of agricultural production costs :**

$$x_l^i = \sum_{k=1}^K \beta_k^i y_l^k + \varepsilon_l^i$$

for input  $i$  and countries  $l = 1, \dots, L$

Where

- $x_l^i$  denotes country  $l$ 's expenditure on input  $i$
- $y_l^k$  denotes the value of good  $k$  produced by country  $l$ ,
- the regression coefficient  $\beta_k^i$  denotes the intermediate consumption of input  $i$  in order to produce one unit of output value of good  $k$ ,

with

- the term  $\varepsilon_l^i$  being an input- $i$  and country- $l$  specific hazard.

# Estimation of Fertilizer Costs:

## The Generalized Maximum Entropy (GME) Method

**GME estimation** allows the introduction of **restrictions** :

- $$\sum_{i=1}^I \beta_k^i = 1 \quad (2)$$

which derives from the accounting identity balancing expenditure and income for each good  $k$  produced in a country  $l$

- $$\beta_k^i \geq 0 \quad (3)$$

non-negativity of the regression coefficients regardless of the positive or zero input expenditures ( $x_l^i \geq 0$ ) as conditioning for the estimation.

# Estimation of Fertilizer Costs:

## The reparametrization of coefficients and hazards

### Reparametrization:

i) for technical coefficients

- $\beta_k^i = \sum_{m=1}^M z_m p_{ik}^m$ , for  $i = 1, \dots, I$  and  $k = 1, \dots, K$  (3)

where  $z_m$  denotes the points on the M-dimensional support for

$\beta_k^i$  the regression coefficients

and

$p_{ik}^m$  the associated probabilities.

(ii) for hazards

- $\varepsilon_l^i = \sum_{n=1}^N v_n w_{il}^n$ , for  $i = 1, \dots, I$  and  $l = 1, \dots, L$  (4)

where

$v_n$  denotes the N-dimensional grid points for

the random variable  $\varepsilon_l^i$ ,

and

$w_{il}^n$  the associated probabilities.

# Estimation of Fertilizer Costs:

## The reparametrization of coefficients and hazards

### Estimation:

The coefficients  $\beta_k^i$  and the hazards  $\varepsilon_l^i$  are estimated as the optimal solution of the equation:

- $\max_{(p,w)} \{H = -\sum_{m=1}^M p_{ik}^m \ln p_{ik}^m - \sum_{n=1}^N w_{il}^n \ln w_{il}^n\}$  ,  
for any triplet  $(i, k, l)$  (5)

- under the following constraints :

- $x_l^i = \sum_{k=1}^K \beta_k^i y_l^k + \varepsilon_l^i = \sum_{k=1}^K (\sum_{m=1}^M z_m p_{ik}^m + \sum_{n=1}^N v_n w_{il}^n)$   
for all  $i$  and  $l$  (5.1)

- $\sum_{i=1}^I \beta_k^i = \sum_{i=1}^I z_m p_{ik}^m = 1$   
for all  $k$  and  $m$  (5.2)

- $\sum_{m=1}^M p_{ik}^m = 1$   
for all  $l$  and  $k$  (5.3)

- $\sum_{n=1}^N w_{il}^n = 1$   
for all  $i$  and  $l$  (5.4)



## Estimation of Fertilizer Costs: Normalized Entropy Estimators

i) For the coefficients  $\beta_k^i$ ,  
the normalized entropy indicator  $S(\hat{p})$  is defined as follows:

- $$S(\hat{p}) = - \sum_{m=1}^M (p_{ik}^m \ln p_{ik}^m) / (KI \ln M)$$

*whatever  $i$  and  $k$*  (6)

*where  $S(\hat{p}) \in [0,1]$ .*

ii) For hazards  $\varepsilon_l^i$ ,  
the normalized entropy indicator  $S(\hat{w})$  is defined as follows:

- $$S(\hat{w}) = - \sum_{n=1}^N (w_{il}^n \ln w_{il}^n) / (IL \ln N)$$

*whatever  $i$  and  $l$*  (7)

*where  $S(\hat{w}) \in [0,1]$ .*

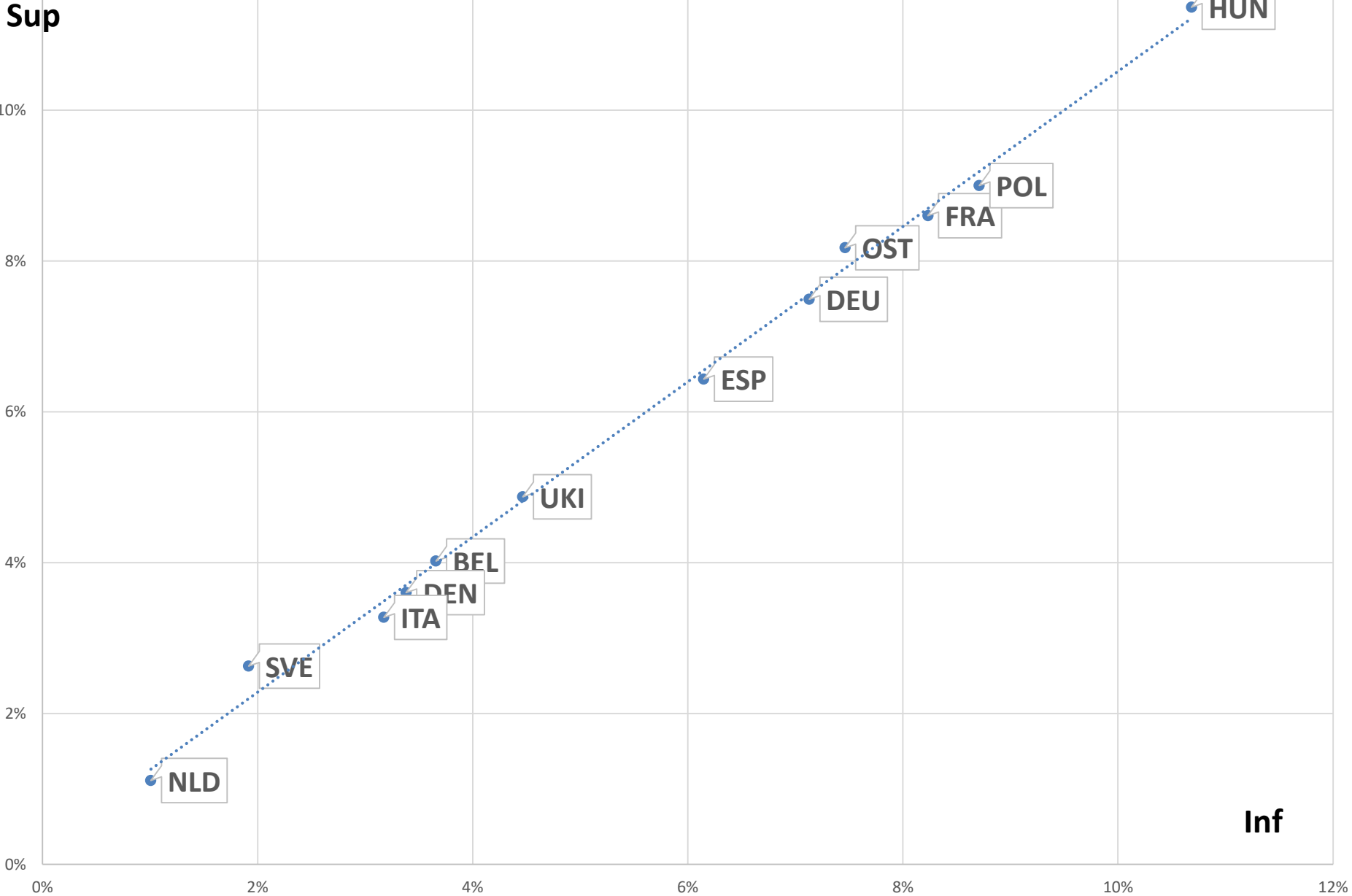
# Estimation of Fertilizer Costs: Normalized Entropy Estimates (12 Countries)

Runs	Number of	Distribution	Distribution																			
	Support points		of Selected Support Points																	S(p)	S(w)	
A	12	Symmetric	0 ; 0.05 ; 0.10 ; 0.20 ; 0.30 ; 0.40 ; 0.50 ; 0.60 ; 0.70 ; 0.80 ; 0.90 ; 1																		0.526	0.980
B (Beta(i,k)<0.5)	12	Symmetric	0 ; 0.04 ; 0.08 ; 0.018 ; 0.26 ; 0.36 ; 0.44 ; 0.54 ; 0.62 ; 0.72 ; 0.74 ; 1																		0.546	0.966

# 12 European Countries, Fertilisation Costs (% of Gross Product) : Annual Crops

(D.Desbois, FADN-UE 2006)

Sup



Inf

# European Regions, Fertilisation Costs in % of Gross Product: Wheat vs Other Cereals

(D.Desbois, FADN-UE 2006)

***Oth.Cer.***

**Northern Ireland**

**Limousin**

**Denmark**

**Dèl-Dunantùl**

**Lombardia**

**Castilla-La Mancha**

**Madrid**

**Franche-Comté**

**Basse-Normandie**

**Rheinland-Pfalz**

**Limousin**

**Denmark**

**Dèl-Dunantùl**

**Lombardia**

**Castilla-La Mancha**

**Madrid**

**Picardie**

**A132**

**A153**

**A165**

**A070**

**A010**

**A411**

**A012**

**A013**

**A090**

**A302**

**A763**

**A764**

**A570**

**A766**

**A570**

**Extremadura**

**Basse-Normandie**

**Île de France**

**Champagne-Ardenne**

**Saarland**

**Pais Vasco**

**Wielkopolska i Slask**

**Marche**

**Kozép-Dunantùl**

**Poitou-Charentes**

**Haute-Normandie**

**Dél-Alföld**

**Extremadura**

**La Rioja**

**Île de France**

**Champagne-Ardenne**

**Saarland**

**Pais Vasco**

**Wielkopolska i Slask**

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**Kozép-Dunantùl**

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**Wielkopolska i Slask**

**Marche**

**Kozép-Dunantùl**

**Poitou-Charentes**

**Haute-Normandie**

**Dél-Alföld**

**Extremadura**

**Alsace**

**Toscana**

**Wheat**

**Champagne-Ardenne**

**Île de France**

**Champagne-Ardenne**

**Saarland**

**Pais Vasco**

**Wielkopolska i Slask**

**Marche**

**Kozép-Dunantùl**

**Poitou-Charentes**

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**Haute-Normandie**

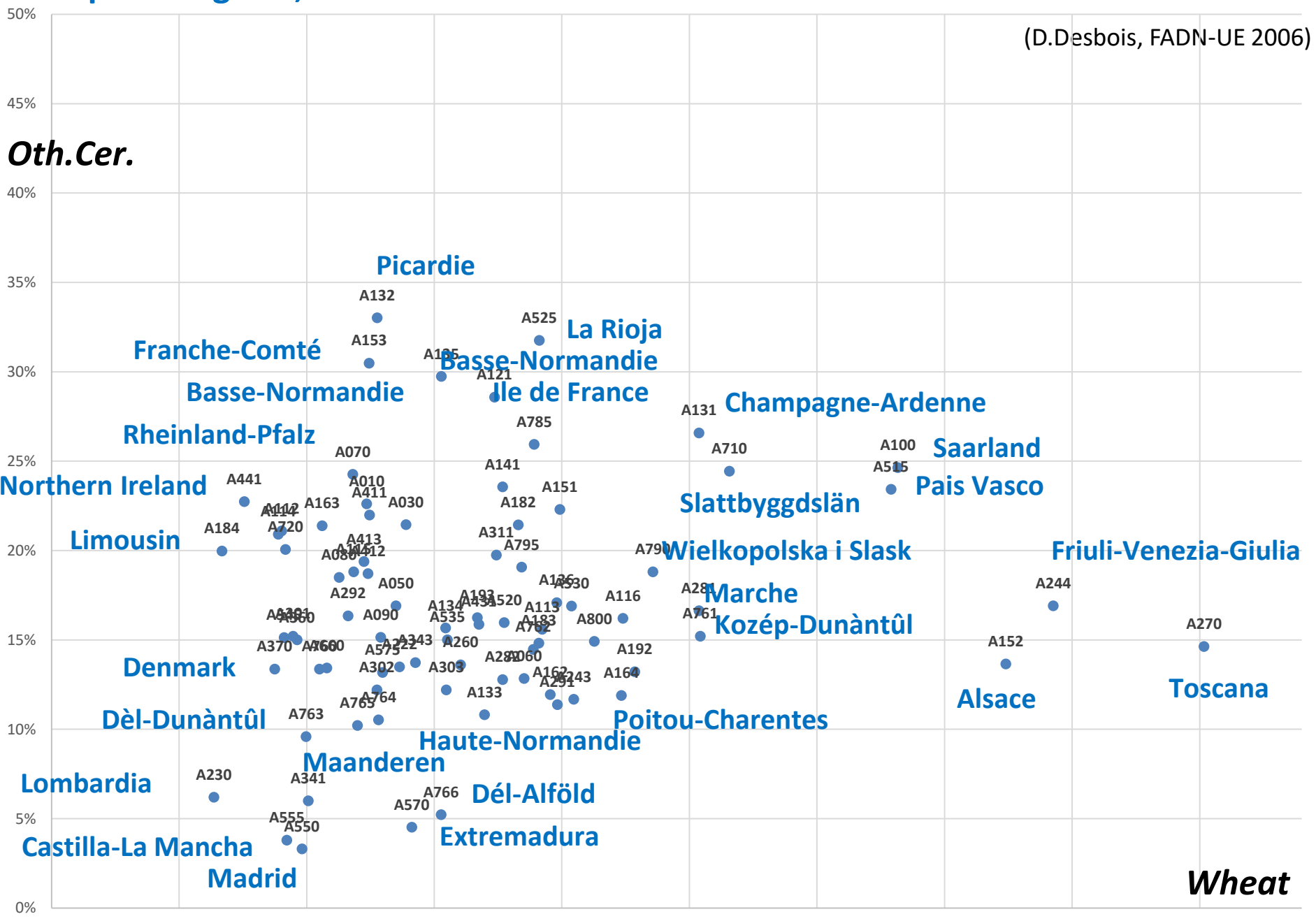
**Dél-Alföld**

**Extremadura**

**Alsace**

**Toscana**

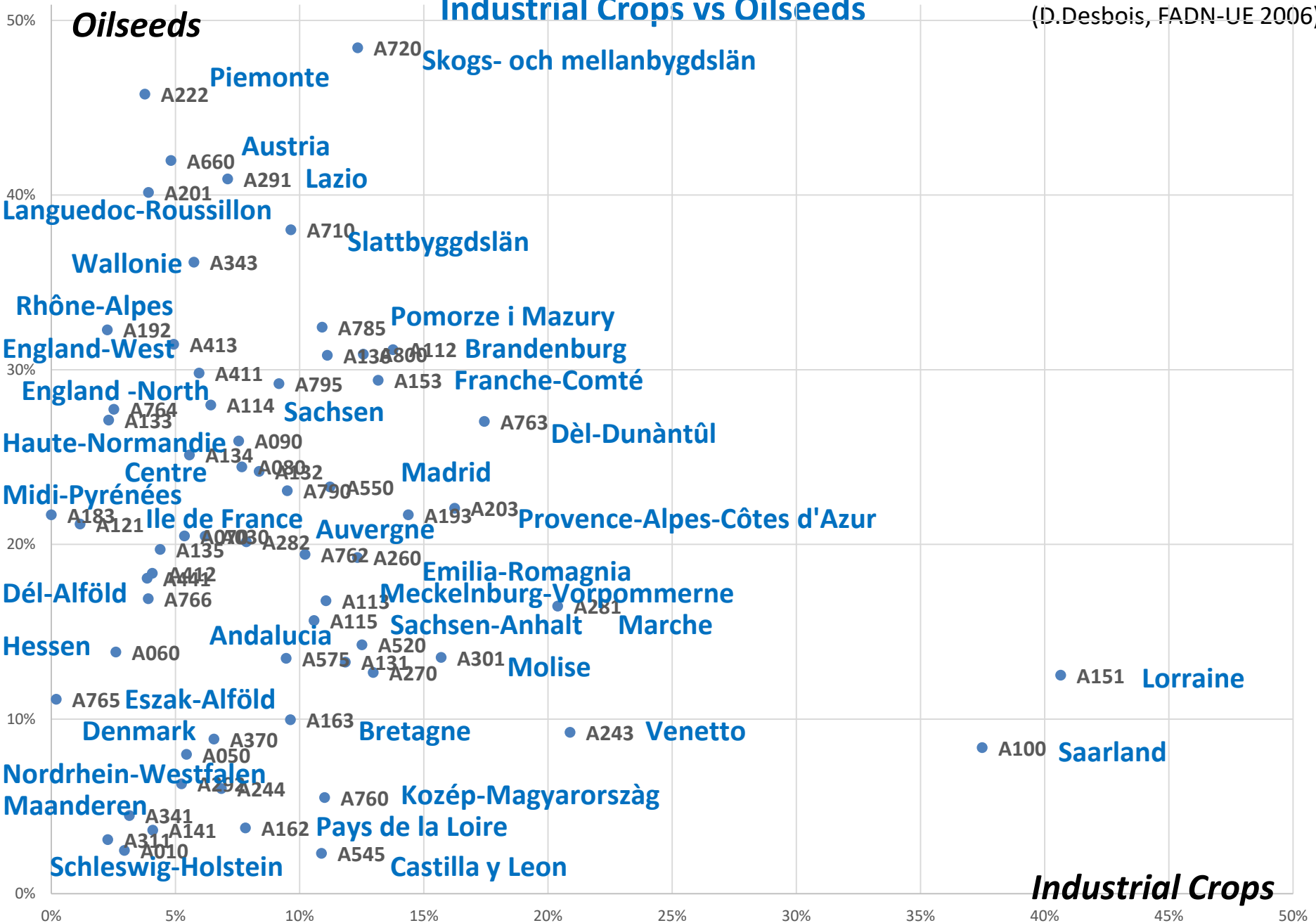
**Wheat**



# European Regions, Fertilisation Costs in % of Gross Product:

## Industrial Crops vs Oilseeds

(D.Desbois, FADN-UE 2006)



# References

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- Léon, Y., Peeters, L., Quinqu, M., Surry Y. (1999). The Use of the Maximum Entropy to Estimate Input-Output Coefficients from Regional Farms Accounting Data. *Journal of Agricultural Economics*, 50(3): 425-439.

This work is dedicated to the memory of

**Prof. Yves Surry (Swedish Academy of Agriculture, SLU, Uppsala)**

who was my thesis supervisor  
and introduced me to the use of entropy in econometric methods.