Applying Entropy Criterion to Cost Allocation: an Empirical Analysis of Fertilizer Cost Estimates for European countries

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Fertilizer drill (19th century), *Life Museum*, AgroParisTech (photo library)

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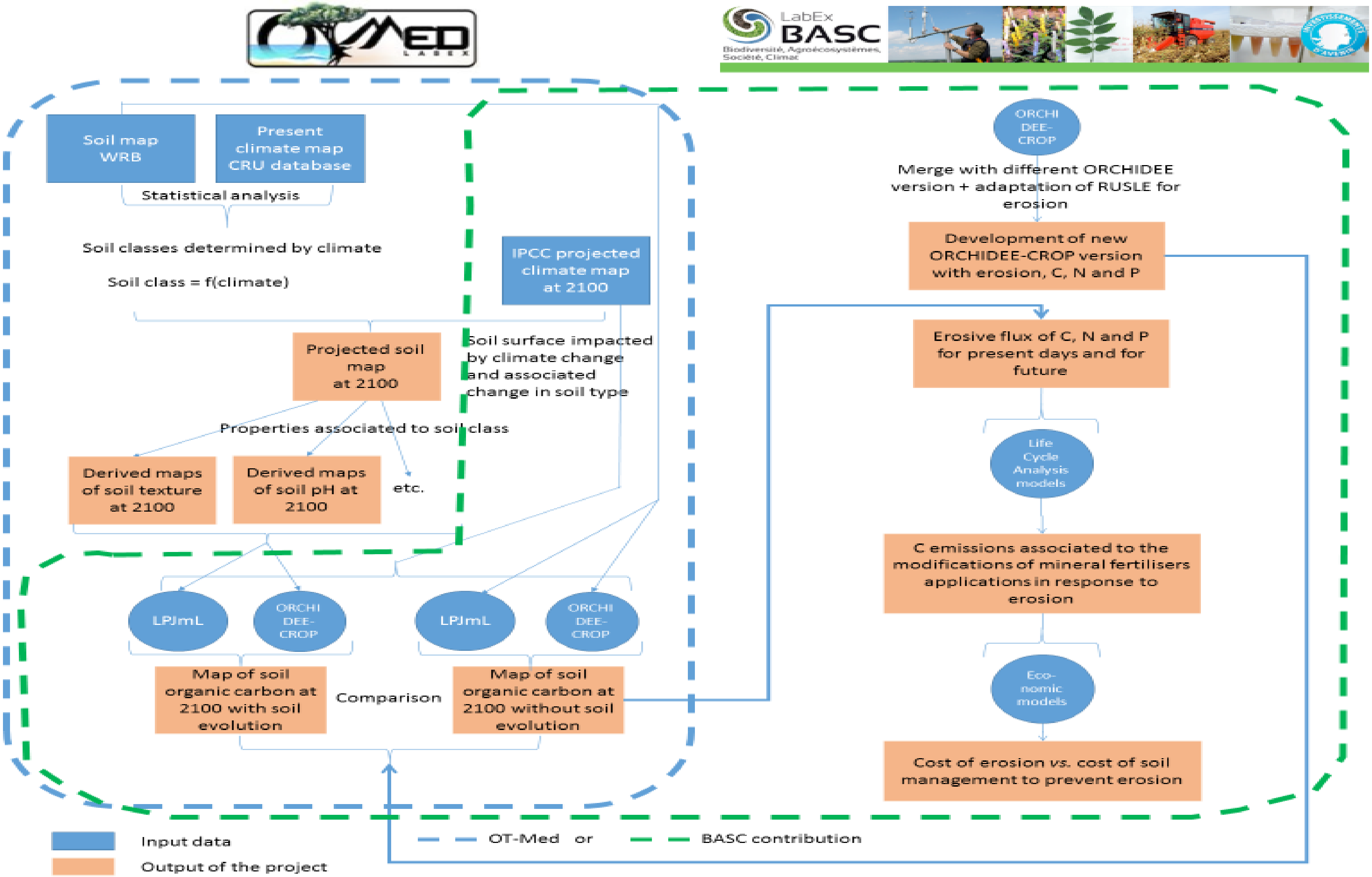
ASSESS : OTE-Med/BASC project



A debated conjecture: as a proxy, could the fertilisation cost be used to estimate the recovering costs of the soil fertility as a land service for agriculture ?

The economic assesment of erosion cost(on site/off site)

I) on-site costs:

- Loss of soil;
- **Loss of nutrients;**
- Loss of organic matter;
- Decreased chemical, physical and biological fertility;
- Damage to plantations and amendments;
- Falling yields;
- Production losses;
- Decrease of the cultivated area;

II) off site costs:

- Sedimentation
- Sedimentation of lakes and rivers;
- Decreased water retention capacity;
- floods;
- Flash floods;
- Landslides;
- Destruction of transport infrastructure;
- Obstruction of waterways;
- eutrophication;
- Loss of biodiversity;
- Unsafe water quality;
- Negative effects on water treatment;
- Negative effects on the production of electrical energy;
- Decrease in food production;
- Restrictions on recreational uses of water;

Estimates of Fertilizer Costs: an Input-Output Methodology

Econometric modeling of agricultural production costs :

$$x_l^i = \sum_{k=1}^{K} \beta_k^i y_l^k + \varepsilon_l^i$$

for input *i* and countries *l= 1, ... ,* L

Where

- x_l^i denotes country *l*'s expenditure on input *i*
- y_l^k denotes the value of good *k* produced by country *l*,
- the regression coefficient β_k^i denotes the intermediate consumption of input *i* in order to produce one unit of output value of good *k*,

with

- the term ε_l^i being an input-*i* and country-*l* specific hazard.

Estimation of Fertilizer Costs: The Generalized Maximum Entropy (GME) Method

GME estimation allows the introduction of **restrictions** :

- $\sum_{i=1}^{I} \beta_k^i = \quad 1$ *(2)*

  which derives from the accounting identity balancing expenditure and income for each good *k* produced in a country *l*

- $\beta_k^i \geq 0$ *(3)*

  non-negativity of the regression coefficients regardless of the positive or zero input expenditures ($x_l^i \geq 0$) as conditioning for the estimation.

Estimation of Fertilizer Costs:
The reparametrization of coefficients and hazards

Reparametrization:

i) for technical coefficients

- $\beta_k^i = \sum_{m=1}^{M} z_m p_{ik}^m$, *for i = 1, … , I and k = 1, … , K* *(3)*

  where z_m denotes the points on the M-dimensional support for

  β_k^i the regression coefficients

  and

  p_{ik}^m the associated probabilities.

(ii) for hazards

- $\varepsilon_l^i = \sum_{n=1}^{N} v_n w_{il}^n$, *for i = 1, … , I and l= 1, … , L* *(4)*

  where

  v_n denotes the *N*-dimensional grid points for

  the random variable ε_l^i ,

  and

  w_{il}^n the associated probabilities.

Estimation of Fertilizer Costs:
The reparametrization of coefficients and hazards

Estimation:

The coefficients β_k^i and the hazards ε_l^i are estimated as the optimal solution of the equation:

- $\max_{(p,w)} \left\{ H = -\sum_{m=1}^{M} p_{ik}^{m} \ln p_{ik}^{m} - \sum_{n=1}^{N} w_{il}^{n} \ln w_{il}^{n} \right\}$,

  for any triplet (*i*, *k*, *l*) (5)
- under the following constraints :
- $x_l^i = \sum_{k=1}^{K} \beta_k^i y_l^k + \varepsilon_l^i = \sum_{k=1}^{K} \left(\sum_{m=1}^{M} z_m p_{ik}^{m} + \sum_{n=1}^{N} v_n w_{il}^{n} \right)$

  for all i and l *(5.1)*
- $\sum_{i=1}^{I} \beta_k^i = \sum_{i=1}^{I} z_m p_{ik}^{m} = 1$

  for all k and m *(5.2)*
- $\sum_{m=1}^{M} p_{ik}^{m} = 1$

  for all l and k *(5.3)*
- $\sum_{n=1}^{N} w_{il}^{n} = 1$

  for all i and l *(5.4)*

Estimation of Fertilizer Costs: Normalized Entropy Estimators

i) For the coefficients $\beta_{\mathrm{k}}^{\mathrm{i}}$,

the normalized entropy indicator $\mathrm{S}(\hat{\mathrm{p}})$ is defined as follows:

- $\mathrm{S}(\hat{\mathrm{p}}) = -\sum_{\mathrm{m}=1}^{\mathrm{M}} (\mathrm{p}_{\mathrm{ik}}^{\mathrm{m}} \ln \mathrm{p}_{\mathrm{ik}}^{\mathrm{m}}) / (\mathrm{KI} \ln \mathrm{M})$

  whatever i and k *(6)*

  where $\mathrm{S}(\hat{\mathrm{p}}) \in [0,1]$.

ii) For hazards $\varepsilon_{\mathrm{l}}^{\mathrm{i}}$,

the normalized entropy indicator $\mathrm{S}(\widehat{\mathrm{w}})$) is defined as follows:

- $\mathrm{S}(\widehat{\mathrm{w}}) = -\sum_{\mathrm{n}=1}^{\mathrm{N}} (\mathrm{w}_{\mathrm{il}}^{\mathrm{n}} \ln \mathrm{w}_{\mathrm{il}}^{\mathrm{n}}) / (\mathrm{I}L \ln N)$

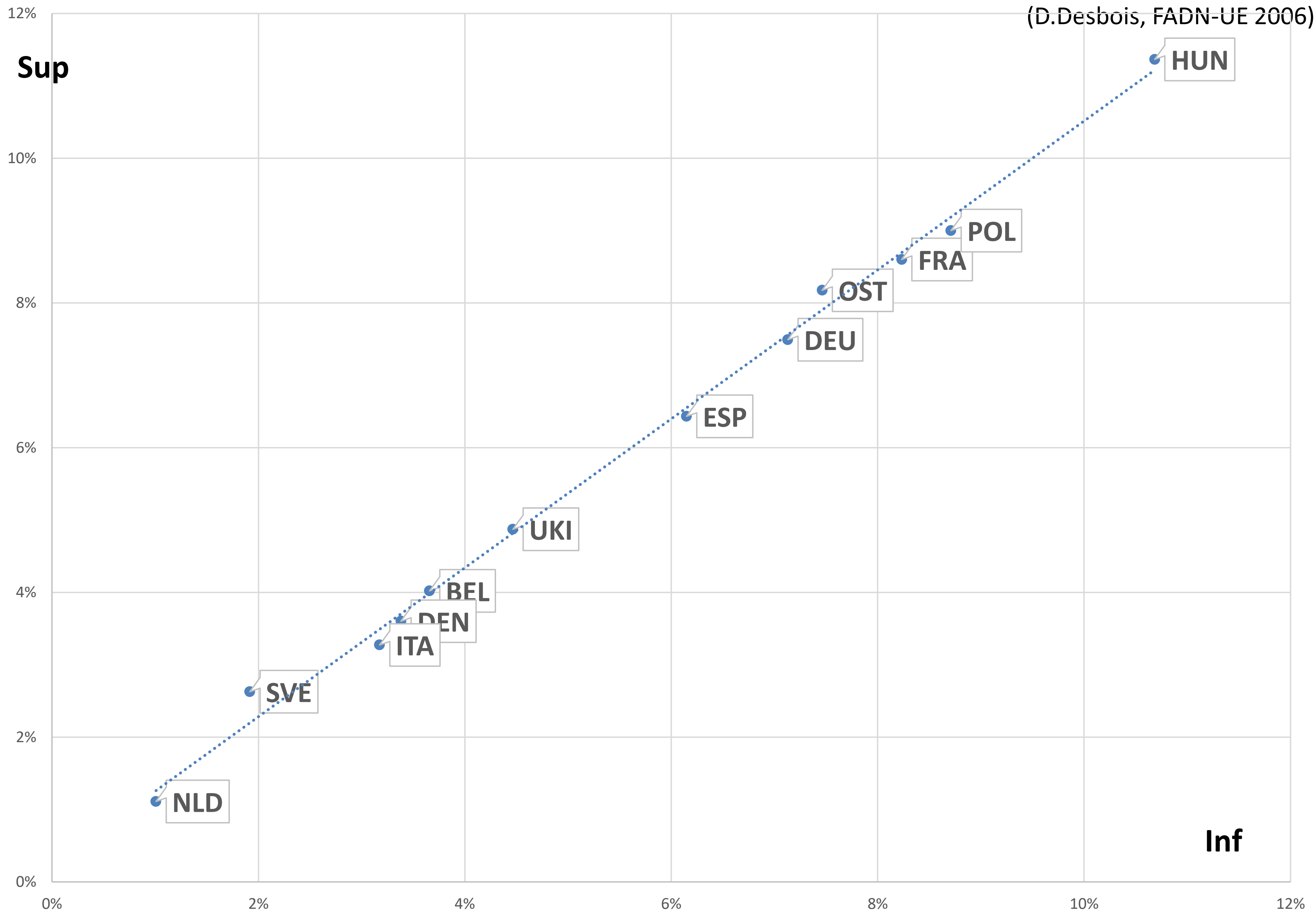
  whatever i and l *(7)*

  where $\mathrm{S}(\widehat{\mathrm{w}}) \in [0,1]$.

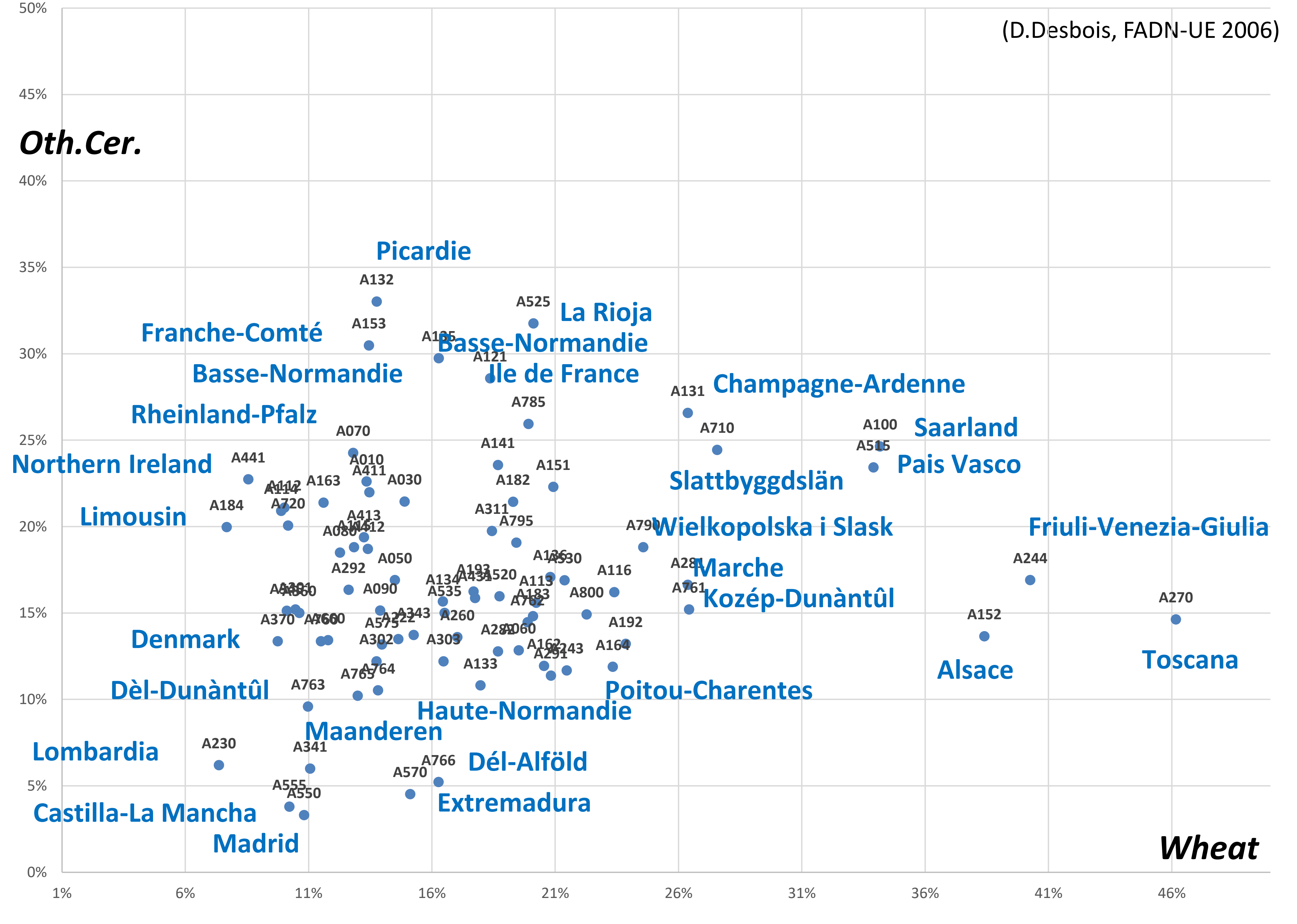
Estimation of Fertilizer Costs: Normalized Entropy Estimates (12 Countries)

Runs	Number of Support points	Distribution	Distribution of Selected Support Points												S(p)	S(w)
A	12	Symmetric	0 ;	0.05 ;	0.10 ;	0.20 ;	0.30 ;	0.40 ;	0.50;	0.60 ;	0.70;	0.80;	0.90 ;	1	**0.526**	**0.980**
B (Beta(i,k)<0.5)	12	Symmetric	0 ;	0.04 ;	0.08 ;	0.018 ;	0.26 ;	0.36 ;	0.44 ;	0.54 ;	0.62 ;	0.72 ;	0.74 ;	1	**0.546**	**0.966**

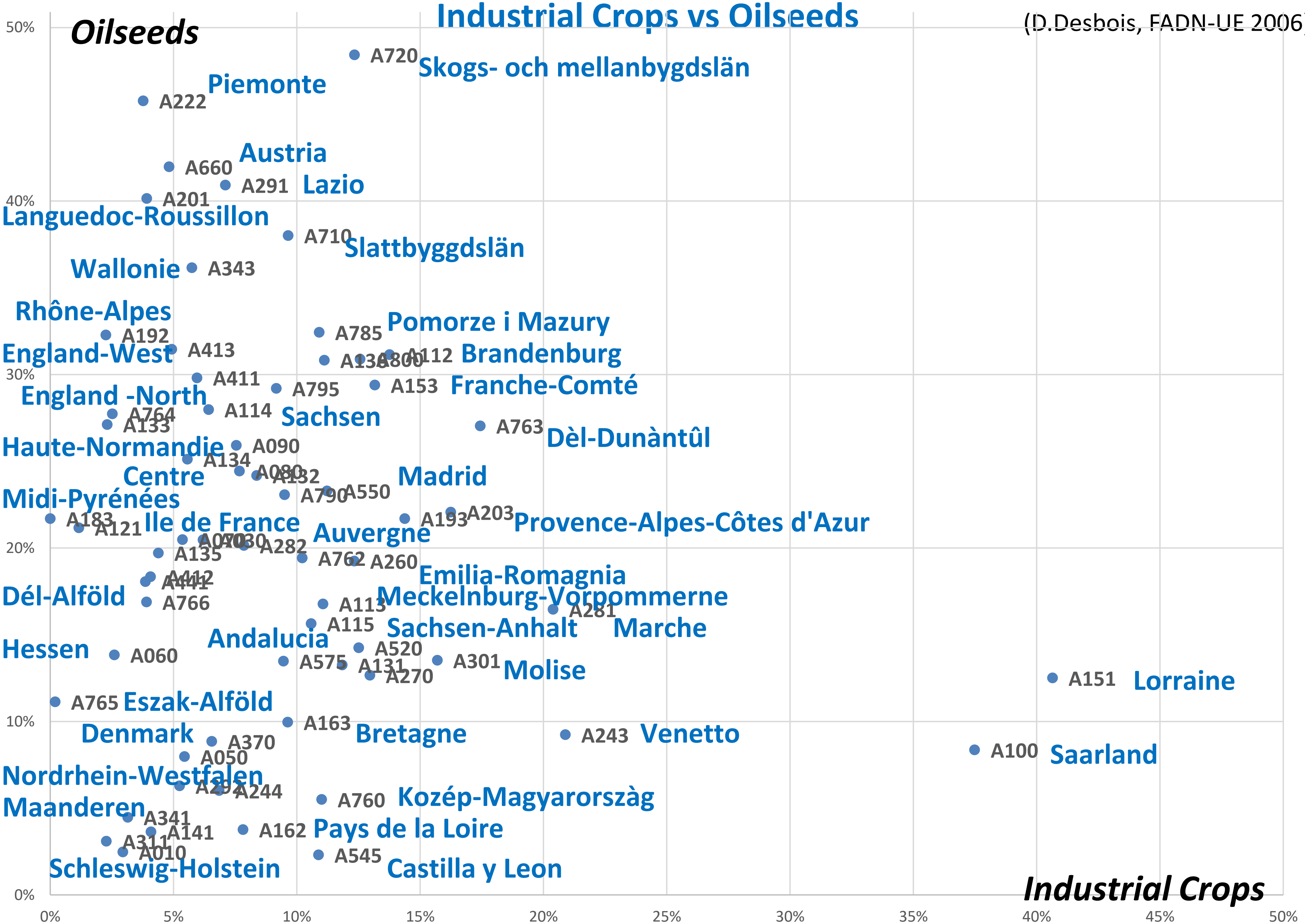
12 European Countries, Fertilisation Costs (% of Gross Product) : Annual Crops



European Regions, Fertilisation Costs in % of Gross Product: Wheat vs Other Cereals



European Regions, Fertilisation Costs in % of Gross Product: Industrial Crops vs Oilseeds



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This work is dedicated to the memory of

Prof. Yves Surry (Swedish Academy of Agriculture, SLU, Uppsala)

who was my thesis supervisor
and introduced me to the use of entropy in econometric methods.