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Abstract

We study optimal harvesting of a renewable resource with stochastic dynamics. To focus on the effect of risk aversion, we consider a resource user who is indifferent with respect to intertemporal variability. We find that a constant escapement strategy is optimal, i.e. the stock after harvesting is constant. Under common specifications of risk aversion, increasing risk and risk aversion increase current resource use, the reason being a substitution effect, i.e. the resource user substitutes assets away from the risky resource stock. We apply the model to the case of the Eastern Baltic cod fishery and, in contrast to the previous literature, find a strong effect of risk and risk aversion on optimal harvesting.

Keywords : Resource Economics | Investment under Uncertainty | Risk Aversion | Prudence | Precautionary Savings
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1 Introduction

Renewable natural resources are inherently volatile assets, exemplified by forests affected by fires and pest outbreaks (Patto and Rosa, 2022), freshwater reservoirs subject to fluctuating rainfall , and the growth of fish stocks depending on environmental conditions (Möllmann et al., 2021). In many cases, climate change amplifies volatility, and thus the riskiness of incomes derived from natural resource use is of high and increasing concern both for risk-averse individual resource users and for policies regulating resource use. A key problem thus is to characterize optimal management of a dynamic renewable natural resource if the resource users are risk averse.

Analysing the optimal use of natural resources inherently requires a dynamic analysis: not only risk aversion, but also time preferences matter – in particular the preference for intertemporal smoothing. Various ways for disentangling risk and time preferences have been proposed in the literature (Kreps and Porteus, 1978; Epstein and Zin, 1989; Traeger, 2009; Bommier and Grand, 2018), and empirical studies show that these aspects of preferences are distinct (Epstein and Zin, 1991; Knapp and Olson, 1996; Epaulard and Pommeret, 2003).

In this paper, we focus on the effect of risk aversion. To analytically isolate the effect of risk aversion, we adopt the strong assumption that the decision-maker, albeit risk-averse, is indifferent with respect to intertemporal variability. In a market setting, this assumption is valid if the resource user has access to perfect capital markets, but insurance is not available. While reality is not black and white, this assumption tends to go in the right direction. Capital markets are typically accessible to resource users just as they are for other actors in the economy. Financial insurance of incomes from resource use remains very difficult, though. Even for farming, financial insurance is available in few countries only, and where it is available, this is only possible due to high governmental subsidies (Smith and Glauber, 2012).

The previous literature on the topic has mostly considered a setting where the resource users are neutral with respect to both, intertemporal variability and risk. Reed (1979) has shown that optimal resource use then is characterized by a 'constant escapement' strategy, i.e. it is optimal to harvest all of the resource stock that goes beyond some constant quantity that should optimally be left in stock (i.e. 'escape' harvesting). This is because the statistical distribution of the future stock can be fully controlled by the user by adjusting current escapement. Costello and Polasky (2008) and Costello et al. (2015) apply this type of model in a spatial setting; Costello et al. (2019) consider both volatility of resource growth and the possibility of a regime shift.

Another strand of literature considers risk aversion, but without disentangling risk aversion from the preference for intertemporal consumption smoothing. McGough et al. (2009) linearized the problem around the optimal steady state to characterize how optimal harvesting depends on risk and the decision maker's preferences. Roughgarden and Smith (1996), Weitzman (2002), and Sethi et al. (2005) consider, on top of volatility of resource growth, additional uncertainties in form of measurement error and policy implementation error.

Recent studies considered both risk aversion and a preference for intertemporal consumption smoothing, using an Epstein and Zin (1989) setting to disentangle these two aspects of risk- and time preferences. Quaas et al. (2019) use this framework to characterize the insurance value of natural capital. Augeraud-Véron et al. (2019) and Augeraud-Véron et al. (2021) consider a similar model to study the insurance value of biodiversity. However, these studies rely on very specific assumptions on the biological model that allow analytical insights.

The model set up in this paper allows for a general resource growth function, and risk aversion, while assuming neutrality with respect to intertemporal variability of income. We show that under these assumptions optimal harvesting is still characterized by a constant escapement strategy. This reveals that the essential assumption to obtain this result is neutrality with respect to intertemporal variability, but not the assumption of risk neutrality. On this basis, we characterize how optimal resource use depends on risk and risk aversion. We decompose the optimality condition for the constant escapement size into three different effects, namely wealth, substitution, and gambling effect. We show how the wealth effect is linked to prudence. If the resource may be diminished in the future there is a propensity to invest in the stock (by increasing escapement) to guarantee future returns. The substitution effect acts in the opposite direction and represents the user's dislike of risk. If the resource may be diminished in the future the user has a propensity to harvest more now (by decreasing escapement) and invest in a risk-free asset instead. The gambling effect is concerned with the curvature of the cost function, as discussed by Kapaun and Quaas (2013). The user has a propensity to increase escapement and bet on good environmental conditions as this may reduce the cost of harvesting in the future. These effects have close parallels to the optimal savings decision seen in asset-management literature Sandmo (1969). Specifically this framework, like Sandmo (1969), relates the impact of risk aversion to the curvature of the function representing risk preferences. Finally, we apply the model to the Baltic Sea Cod fishery. This case study shows a strong effect of risk and risk aversion on optimal management, in contrast to previous contributions that have not disentangled risk and time preferences (Kapaun and Quaas, 2013; Tahvonen et al., 2018).

The paper is structured as follows: Section 2 outlines the model. Section 3 characterizes the optimal escapement policy for a user facing constant unit costs and Section 4 derives the optimal escapement policy for a user facing convex costs. Both section 3 and section 4 discuss comparative statics and use an example with constant absolute risk aversion (CARA) preferences to illustrate the theoretical findings. Section 5 then applies the model to the Baltic Sea Cod fishery. Section 6 provides a discussion of the results and concludes.

2 The Model

We consider the stock x_t of a renewable resource in discrete time (index t). Resource harvest h_t diminishes the stock down to the 'escapement' $s_t := x_t - h_t$. The dynamics of the resource stock are described by the following stochastic Markovian transition equation

$$x_{t+1} = z_{t+1}g(x_t - h_t) = z_{t+1}g(s_t), \tag{1}$$

where z_{t+1} is a series of independently and identically distributed random shocks with unit expectation. Shocks are distributed on a finite interval $[\underline{z}, \bar{z}]$, where $0 < \underline{z} \leq 1 \leq \bar{z} < \infty$, with some given probability density function.

The expected resource stock at the beginning of the next period, $g(s_t)$, is an increasing, strictly concave, and twice differentiable function of escapement, $g'(s_t) > 0$ and $g''(s_t) < 0$. We also assume that $g'(s_t) > 1$ for sufficiently small s_t, whereas $g'(s_t) < 1$ for sufficiently large s_t. Thus, the expected growth of the resource, which is $g(s_t) - s_t$, is positive for sufficiently small s_t. There is, however, some value of $s_{\max}$ such that $s_{\max} = g(s_{\max})$, i.e., $s_{\max}$ is the expected carrying capacity of the resource. Given stochasticity, the next period's stock is a random variable, and so is the carrying capacity. As the support of the shock z_{t+1} is finite, there is an interval of sufficiently small escapement levels where the stock will grow for sure, i.e., where $z_{t+1}\, g(s_t) \geq \underline{z}\, g(s_t) > s_t$ with certainty.

There is a single resource user with risk averse preferences. The resource we consider is small relative to the overall size of the market, and thus for the resource user the market price p of resource harvest is given. To focus on the effect of stochastic resource dynamics we keep p constant over time.

Unit harvesting costs $c(y)$ are (weakly) decreasing and (weakly) convex in the current stock size y, $c'(y) \leq 0$ and $c''(y) \geq 0$. For some resources (e.g., fish) search costs decrease with the resource abundance, $c'(y) < 0$. For other resources, such as a forest, such a stock dependence of cost may be negligible, $c'(y) = 0$. Net revenues of resource harvesting in period t are obtained by integrating the flow of profit over the harvesting season,

$$\pi(x_t, s_t) = \int_{s_t}^{x_t} (p - c(y))dy \tag{2}$$

In order to simplify notation, the stem function $C(y)$ is used to denote total costs such that $C'(y) = c(y)$.

In the analysis, we consider two cases. First, we consider a resource where unit harvesting costs are independent of stock size, i.e. $C(y) = cy$, and then we consider the case where unit costs are strictly decreasing and strictly convex in current stock size, $c'(y) < 0$ and $c''(y) > 0$.

The risk-averse user aims to maximise the present value of current and expected future utility over time subject to the growth constraints of the resource stock. Risk aversion is characterized using the recursive preference structure of Traeger (2009) represented by the following generalized utility relation

$$U_t = W[\pi(x_t, s_t), \mu(U_{t+1})] \tag{3}$$

where $W(\cdot,\cdot)$ is an aggregator function representing the present value of utility, U_t, given the resource user's preferences for intertemporal consumption smoothing. Furthermore, $\pi(x_t, s_t)$ represents the current period net revenues, whilst $\mu(\cdot) := f^{-1}\mathbb{E}[f(\cdot)]$ is the certainty equivalent operator, where the curvature of the function $f(\cdot)$ represents risk preferences in the Arrow-Pratt sense (Traeger, 2009), and $\mathbb{E}[\cdot]$ is the expectation over the stochastic shock z_{t+1}.

Whereas we focus on general risk preferences for the most part of the analysis, we will also consider the two examples of constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). The assumption of a constant absolute risk aversion $\rho = -f''(v)/f'(v) > 0$ implies the following functional forms for $f(\cdot)$ and certainty equivalence operator:

$$CARA: \quad f(v) = -e^{-\rho v}; \quad \mu(v) = -\frac{1}{\rho}\ln\left(\mathbb{E}\left[e^{-\rho v}\right]\right). \tag{4}$$

The assumption of a constant relative risk aversion $\eta = -v f''(v)/f'(v) > 0$ implies

$$CRRA: \quad f(v) = \frac{1}{1-\eta}v^{1-\eta}; \quad \mu(v) = \left(\mathbb{E}\left[v^{1-\eta}\right]\right)^{\frac{1}{1-\eta}}. \tag{5}$$

In the stochastic dynamic setting, it is usually not possible to simultaneously satisfy the common rationality requirements for decision-making over time (such as time consistency, Koopmans 1960) and for decision-making under risk (such as independence of irrelevant alternatives, von Neumann and Morgenstern 1944). Thus, while recursive preferences are time consistent, ordinal dominance (or preference monotonicity) is usually not satisfied (Bommier and Grand, 2018). This means that in order to derive unambiguous results for comparative static analysis, further assumptions must be made. These are discussed in detail in Section 3 and Section 4.

To focus on the effect of risk aversion, the user is assumed to exhibit no preference for intertemporal consumption smoothing. All that counts is the expected present value of income, no matter how variable income is over time. These preferences are represented by the affine aggregator function

$$U_t = \pi(x_t, s_t) + \beta\mu(U_{t+1}), \tag{6}$$

where $\beta = 1/(1+r)$ is the discount factor and r the corresponding discount rate.

Using $V(x_t)$ to represent the value function, the user's stochastic optimisation problem can be represented by the following Bellman equation that describes how the user maximises the expected present value of income, i.e. net revenues of resource harvesting, over an infinite horizon by choosing escapement in every period

$$V(x_t) = \max_{s_t}\left\{\pi(x_t, s_t) + \beta\,\mu\left(V(z_{t+1}\,g(s_t))\right]\right\}. \tag{7}$$

With zero risk, $z_{t+1} \equiv \mathbb{E}[z] = 1$, the second term on the right-hand side of the Bellman equation (7) simplifies to $\beta V(g(s_t))$. Furthermore, for a risk neutral user, the certainty equivalent operator $\mu(\cdot)$ is the identity, and the second term on the right-hand side of the Bellman equation (7) becomes $\beta\,\mathbb{E}[V(z_{t+1}(s_t))]$.

As stock growth and net revenues are strictly concave functions of x_t and s_t, and as the certainty equivalent operator $\mu(\cdot)$ is sub-additive, the Bellman equation (7) has a unique solution (Stokey et al., 1989, Theorem 9.6).

The solution of (7) is interior in the first period of time, $0 < s_0 < x_0$, if the initial resource stock x_0 is large enough. This is always the case if the initial stock is at the expected carrying capacity, $x_0 = s_{\max}$, which we assume to be the case.

The next section solves the model for the case of constant unit costs, $c(y) = c$, while Section 4 considers the case of convex costs, $c'(y) < 0$ and $c''(y) > 0$.

3 The Case of Constant Marginal Costs

In the case of constant marginal costs, $c(y) = c < p$, marginal net benefit is constant. Without loss of generality we normalize the marginal net benefit to one, $p - c = 1$. The Bellman equation (7) simplifies to

$$V(x_t) = \max_{s_t} \{x_t - s_t + \beta\,\mu\,(V(z_{t+1}\,g(s_t))]\}\,. \tag{8}$$

We show in Appendix A that the value function is linear in the stock size, and can be written as

$$V(x) = x + b, \tag{9}$$

with some constant $b \in \mathbb{R}$. Optimal resource harvesting is characterized by a 'constant escapement' strategy, i.e. it is optimal to always let a constant stock size s^* remain. The following proposition, proven in Appendix A, characterizes this optimal escapement strategy.

Proposition 1. *For $x_0 > s^* > 0$, the solution is interior and a constant escapement policy is optimal such that optimal escapement is $s_t = s^*$, with s^* implicitly determined by the two conditions*

$$1 = \beta\,g'(s^*)\,\frac{\mathbb{E}[z_{t+1}\,f'(z_{t+1}\,g(s^*)+b)]}{f'(\mu\,(z_{t+1}\,g(s^*)+b))} \tag{10}$$

$$b = -s^* + \beta\mu\,(z_{t+1}\,g(s^*)+b)\,. \tag{11}$$

The optimality condition (10) states that at the optimum, the user chooses escapement to balance their current returns with discounted future returns. This means current period marginal profits of the last unit harvested (the left-hand side of the condition) must equal the discounted expected marginal returns resulting from leaving an additional unit of stock unharvested for the next period. Condition (11) specifies the constant b as the expected present value of the net benefit of conserving a quantity s^* of the resource for future use.

Note that both the optimal escapement level s^* and the constant term b in the value function (9) appear in both conditions, (10) and (11). If the user is risk averse, i.e. if $f(\cdot)$ is concave, neither for s^* nor for b an explicit expression can be derived.

To study the optimality condition in more depth we consider, independently, how an increase in the degree of risk aversion and magnitude of risk impact the optimal escapement decision. Defining an increase in risk as a mean preserving spread (Rothschild and Stiglitz, 1970), we find conditions for the optimal escapement to be decreasing in both the degree of risk aversion and the magnitude of risk. In order to do this, we first look at how a risk neutral user would optimally use the resource.

3.1 The Case of a Risk Neutral User

For a risk neutral user, the function $f(\cdot)$ characterizing risk preferences is the identity function. The Bellman equation (8) simplifies to

$$V(x_t) = \max_{s_t} \left\{ x_t - s_t + \beta \, \mathbb{E}[z_{t+1} \, g(s_t)] \right\} = \max_{s_t} \left\{ x_t - s_t + \beta \, g(s_t) \right\} \tag{12}$$

Using s^*_{RN} to denote the optimal escapement policy for a risk neutral user, the constant optimal escapement policy is characterized by

$$1 = \beta g'(s^*_{RN}) \quad \Leftrightarrow \quad r = g'(s^*_{RN}) - 1 \tag{13}$$

Equation (13) is independent of the random variable z_{t+1} meaning an increase in the magnitude of risk has no impact on the optimal escapement decision. This is the same optimality condition for a user facing zero risk, s^*_{DET} such that $s^*_{DET} = s^*_{RN}$, where DET stands for the deterministic case $z_{t+1} \equiv \mathbb{E}[z_{t+1}] = 1$ for all t. The return resulting from a marginal reduction in current escapement, r, must equal the discounted return from keeping that marginal unit of current escapement in the sea for the next period $g'(s^*_{RN}) - 1$.

3.2 The Case of a Risk-Averse User

For a risk-averse user with constant marginal costs, the optimality condition in (10) shows that the optimal escapement policy depends on the random variable, and hence the magnitude of risk.

Given that the left-hand sides of equations (10) and (13) are the same, risk aversion decreases the optimal escapement relative to the risk-neutral case if and only if the right-hand side of (10) is less than the right-hand side of (13). The same holds if we consider a risk-averse user and compare the optimal escapement levels under stochastic resource dynamics with random shocks to the deterministic problem where $z_{t+1} \equiv \mathbb{E}[z_{t+1}] = 1$ for all t.

Thus, the optimal escapement under risk aversion is greater (less) than the optimal escapement under risk neutrality (and the deterministic case) if and only if the following condition holds:

$$s^* \gtreqless s^*_{RN} = s^*_{DET} \quad \Leftrightarrow \quad \frac{\mathbb{E}[z_{t+1} f'(z_{t+1} g(s^*) + b)]}{f'(\mu(z_{t+1} g(s^*) + b))} \gtreqless 1 \tag{14}$$

In order to explain the user's escapement decision in more detail, we rewrite (10) using the joint expectation formula.

$$1 = \beta g'(s^*) \left\{ \frac{\mathbb{E}[f'(z_{t+1} g(s^*) + b)]}{f'(\mu(z_{t+1} g(s^*) + b))} + \frac{\mathrm{cov}(z_{t+1}, f'(z_{t+1} g(s^*) + b))}{f'(\mu(z_{t+1} g(s^*) + b))} \right\} \tag{15}$$

This allows the optimal escapement decision to be decomposed into two different effects influencing escapement, a wealth effect and a substitution effect. The wealth effect corresponds to the first term in the brackets of equation (15). If this term is greater than one, the wealth effect works to increase escapement under uncertainty. The substitution effect corresponds to the second term in the brackets. If this term is positive, the substitution effect works to increase escapement under uncertainty. The following propositions (2) and (3) state conditions for the wealth effect to increase or decrease escapement under uncertainty and state the result that the substitution effect is always negative for a risk-averse user.

The following proposition is proven in Appendix B.

Proposition 2. *Assume that a decreasing function $\varphi(\cdot)$ exists such that $f'(\cdot) = \varphi(f(\cdot))$. The wealth effect is positive, i.e. it tends to increase optimal escapement under risk, if and only if $\varphi(\cdot)$ is convex.*

Proposition 2 provides conditions for the wealth effect to increase optimal escapement relative to the deterministic and risk neutral cases. We call this the *wealth effect*, because a risk-averse user increases escapement under uncertainty to protect against low levels of future wealth. The wealth effect here resembles the wealth (or income) effect known from the asset management literature (Sandmo, 1970).

The condition $\phi(\cdot)$ is convex means that $f'(\cdot)$ is convex relative to $f(\cdot)$. Given that $f''(\cdot) < 0$, if $f'(\cdot)$ is convex, then we can say that $f(\cdot)$ exhibits *prudence* in the sense of Kimball (1990). Therefore, one can also understand Proposition 2 in the way that positive absolute prudence $-f'''(\cdot)/f''(\cdot)$ is a necessary condition for a positive wealth effect, i.e. that it tends to increase optimal escapement under risk.

However, even a positive wealth effect is not sufficient for the result that optimal escapement is higher under risk than in the deterministic case. The reason is that optimality condition (15) also includes the substitution effect, captured by the covariance term. The substitution effect is always negative:

Proposition 3. *The substitution effect tends to decrease optimal escapement under risk,*

$$\frac{\mathrm{cov}(z_{t+1}, f'(z_{t+1} g(s^*) + b))}{f'(\mu(z_{t+1} g(s^*) + b))} < 0 \tag{16}$$

Proof. See Appendix C. □

This effect decreases optimal escapement relative to the deterministic and risk neutral case. Intuitively, this represents the user's dislike of risk and desire to divest away from the risky stock. We call this the *substitution effect*, as the user has a propensity to substitute the risky asset with risk free alternatives by decreasing escapement under uncertainty. This substitution effect here plays the same role as the substitution effect seen in Sandmo (1970).

Proposition 3 shows that prudence in the sense of Kimball (1990) is not sufficient for the result that optimal escapement would increase with risk. The reason is that the substitution effect counteracts the wealth effect that comes about due to prudence. The following proposition states that if prudence is low enough, risk will always decrease optimal escapement:

Proposition 4. *Optimal escapement is lower in the risky setting than in the deterministic case if the following condition on (relative) prudence holds*

$$-\frac{z_{t+1}\, g(s^*)\, f'''(z_{t+1}\, g(s^*) + b)}{f''(z_{t+1}\, g(s^*) + b)} \leq 2. \tag{17}$$

Proof. See Appendix D. □

The case of preferences with constant absolute risk aversion (CARA, equation 4) provides a benchmark of particular interest. With CARA preferences, the first-order condition for optimal escapement, condition (10), becomes independent of the constant b from equation (11). It reads

$$1 = \beta\, g'(s^*)\, \frac{\mathbb{E}\left[z_{t+1}\, e^{-\rho\, z_{t+1}\, g(s^*)}\right]}{\mathbb{E}\left[e^{-\rho (z_{t+1}\, g(s^*))}\right]} \tag{18}$$

Moreover, due to the fact that risk aversion is independent of wealth with CARA preferences, the wealth effect does not change optimal escapement compared to the deterministic case. That is, as risk increases there is no propensity to increase escapement to ensure a certain level of future wealth. On the other hand, the substitution effect (the negative co-variance term) still remains (Proposition 3). Overall, we have

$$1 = \beta g'(s^*) \left(1 + \frac{\operatorname{cov}\left(z_{t+1}, e^{-\rho\, z_{t+1}\, g(s^*)}\right)}{\mathbb{E}\left[e^{-\rho\, z_{t+1}\, g(s^*))}\right]}\right) < \beta\, g'(s^*). \tag{19}$$

In conclusion, optimal escapement decreases under both risk, and risk aversion with CARA preferences, but not necessarily if absolute risk aversion is decreasing with wealth.

4 Optimal Escapement Policy With Convex Costs

In the case of convex marginal costs, marginal benefit $p - c(y)$ is increasing in the current stock size y. We show in Appendix E that the value function is linear in the profit,

$$V(x_t) = p x_t - C(x_t) + b. \tag{20}$$

with a constant $b \in \mathbb{R}$.

Also in the case of variable marginal costs, optimal resource harvesting is characterized by a constant escapement strategy. The following proposition, proven in Appendix E, characterizes this strategy.

Proposition 5. *For $x_0 > s^* > 0$, the solution is interior and a constant escapement policy is optimal such that the optimal escapement is $s_t = s^*$, with s^* implicitly determined by*

$$p - c(s^*) = \beta g'(s^*) \frac{\mathbb{E}[z_{t+1} f'(p z_{t+1} g(s^*) - C(z_{t+1} g(s^*)) + b)(p - c(z_{t+1} g(s^*)))]}{f'(\mu(p z_{t+1} g(s^*) - C(z_{t+1} g(s^*)) + b))} \tag{21}$$

$$b = -(p s^* - C(s^*)) + \beta \mu (p z_{t+1} g(s^*) - C(z_{t+1} g(s^*)) + b). \tag{22}$$

This optimality condition includes a couple of extra terms compared to the case of constant marginal costs. The convexity of costs creates an additional stock effect which interacts with the user's risk preference. In a similar manner to Section 3, we first look at the deterministic case, and the case of a risk neutral user. These cases are then compared with that of a risk-averse user to provide a discussion on how an increase in the magnitude of risk and degree of risk aversion impact s^*.

4.1 Baseline Cases of Zero Risk and Risk Neutral User

With convex costs, the optimality condition for a user facing zero risk becomes

$$p - c(s^*_{DET}) = \beta g'(s^*_{DET})(p - c(g(s^*_{DET}))) \tag{23}$$

such that at the optimum, the escapement decision of the user depends on the curvature of their marginal cost function. In case of zero risk, $\mu(\cdot)$ is the identity. We thus can directly solve (22) to obtain the constant b. This, in turn, gives the next period's value function as

$$V_{DET}(g(s^*)) = \frac{p g(s^*) - C(g(s^*)) - (p s^* - C(s^*))}{1 - \beta}, \tag{24}$$

which is simply the present value of the constant stream of net benefits from resource harvesting.

With convex costs, the optimality condition for a risk neutral user is

$$p - c(s^*_{RN}) = \beta g'(s^*_{RN})\mathbb{E}\left[z_{t+1}\left(p - c(z_{t+1}(s^*_{RN}))\right)\right] \tag{25}$$

such that at the optimum, the user's current marginal profits (LHS) must equal their expected discounted future marginal profits. Kapaun and Quaas (2013) show that by assuming $yc(y)$ is concave in the random variable y (coinciding with empirical estimates), an increase in the magnitude of risk increases the optimal escapement policy. They refer to this effect (as reflected in the right-hand side of equation (25)) as the 'gambling effect'. This is because there is a positive skew (fat tail at high rents) in the distribution of future returns meaning the user has a propensity to bet on environmental conditions and increase escapement under uncertainty. We consider this case in the subsequent analysis, i.e. we assume that $yc(y)$ is concave in y.

Due to the convexity of marginal costs, the optimal escapement for a risk neutral user is no longer independent of the random variable z_{t+1}, and hence comparative static analysis of an increase in the magnitude of risk is no longer the same as the analysis for an increase in the degree of risk aversion.

4.2 The Case of a Risk-averse User

In the convex marginal cost case, the optimality condition now depends on the risk preferences of the user, as well as on the curvature of the cost function. Using the joint expectation formula we can rewrite equation (21) as

$$\begin{aligned} p - c(s^*) = \beta g'(s^*)\Bigg(& \mathbb{E}\left[z_{t+1}\left(p - c\left(z_{t+1}\, g(s^*)\right)\right)\right] \frac{\mathbb{E}[f'(p z_{t+1}\, g(s^*) - C(z_{t+1}\, g(s^*)) + b)]}{f'\left(\mu\left(p z_{t+1}\, g(s^*) - C(z_{t+1}\, g(s^*)) + b\right)\right)} \\ & + \frac{\operatorname{cov}\Big(z_{t+1}\left(p - c\left(z_{t+1}\, g(s^*)\right)\right), f'\left(p z_{t+1}\, g(s^*) - C(z_{t+1}\, g(s^*)) + b\right)\Big)}{f'\left(\mu\left(p z_{t+1}\, g(s^*) - C(z_{t+1}\, g(s^*)) + b\right)\right)}\Bigg). \end{aligned} \tag{26}$$

Comparing the optimality condition with the constant marginal cost case, the two terms in brackets in (26) can again be interpreted as wealth and substitution effects. Both effects are now modulated by the 'gambling effect'. The wealth effect term is multiplied by the additional term $\mathbb{E}[z_{t+1}(p - c(z_{t+1}\, g(s^*)))]$, which is exactly similar to the positive gambling effect highlighted in equation (25).

Similarly, the substitution effect term contains the covariance of $f'(\cdot)$ not only with the shock z_{t+1}, as in the constant marginal cost case, but with the term $z_{t+1}\,(p - c\,(z_{t+1}\, g(s^*)))$, which again captures a 'gambling effect' as the expression $z_{t+1}\, c(z_{t+1}\, g(s^*))$ is concave in z_{t+1}. Thus, unlike the constant marginal cost case, risk aversion is no longer sufficient for the substitution effect to be negative. The gambling effect counteracts the risk averse user's propensity to divest away from the risky stock. The sign of the substitution effect now depends on the trade off between the effect driven by risk aversion and that driven by the curvature of harvesting costs.

The curvature of marginal costs also implies that the comparative statics results on an increase in the degree of risk aversion and magnitude of risk may no longer be the same. The impact on escapement of an increase in the degree of risk aversion only depends on the net wealth and substitution effect, while the impact on escapement of an increase in the magnitude of risk is more complex and also depends on the gambling effect. We quantify these effects in the numerical example in the next section.

For the case of constant absolute risk aversion (CARA, equation 4), the constant b drops out of condition (21), which then determines the constant optimal escapement level s^*. Also condition (22) simplifies, as with CARA preferences we have $\mu(v+b) = \mu(v)+b$, for any constant b. With this, expected next period's wealth can be written as

$$\mathbb{E}[V(z_{t+1}\,g(s^*))] = \frac{1}{1-\beta}\Bigg[(1-\beta)\,\mathbb{E}[p\,z_{t+1}\,g(s^*) - C(z_{t+1}\,g(s^*))] \\ + \beta\,\mu\,(p\,z_{t+1}\,g(s^*) - C(z_{t+1}\,g(s^*))) - (p\,s^* - C(s^*))\Bigg] \quad (27)$$

As the certainty equivalent of future income is smaller than the expected future income, expected next period's wealth is smaller than the next period's wealth in the deterministic case, as given by (24).

5 Numerical Example: Eastern Baltic Sea Cod Fishery

An empirical example of the Baltic Sea Cod fishery is used to quantitatively illustrate the theoretical findings. The cod fishery is one of the economically most important fisheries in the Baltic Sea. The stock reproduction strongly depends on environmental conditions (temperature, oxygen content, salinity), which are highly variable. This is due to the brackish nature of the Baltic Sea, where especially oxygen content and salinity change depending on irregular inflow events from the North Sea (Roeckmann et al., 2005; Kapaun and Quaas, 2013; Tahvonen et al., 2018).

The fishery has been extensively studied in fish biology and fisheries economics, such that biological and economic parameters are readily available from the literature. For the biological growth function we use estimates from Froese and Proelß (2010). Reproduction is modeled using the logistic function

$$x_{t+1} = z_t\left(s_t + r\,s_t\left(1-\frac{s_t}{K}\right)\right), \quad (28)$$

with an intrinsic growth rate $r = 0.48/\text{year}$, and a carrying capacity $K = 2{,}283$ thousand tons. As Kapaun and Quaas (2013), we assume the stochastic shocks to be log-normally distributed with unit mean, such that the magnitude of risk is fully determined by the variance σ^2 of the shocks. We vary this variance parameter to study how optimal escapement depends on environmental risk.

Tahvonen et al. (2018) specify a marginal harvesting cost function

$$c(y) = c\,y^{-\chi}, \tag{29}$$

with parameters $c = 6.604$ and $\chi = 0.426$. Thus, the function $y\,c(y)$ is concave. In line with the discussion in Section 4.1, we thus expect a gambling effect that would tend to increase optimal escapement under uncertainty. For the price we use the average of the years 2015–2020, which was 2 Euros / kg of fish (BLE, 2016–2021). We use a discount rate of 5% per year, i.e. we set $\beta = 0.952$.

We consider risk preferences with constant absolute risk aversion (CARA, equation 4) as well as with constant relative risk aversion (CRRA, equation 5). For any given level of wealth, v_0, the relationship between relative risk aversion η and absolute risk aversion ρ is $\eta = \rho\,v_0$. To compare results for changing the degree of risk aversion with CARA and CRRA preferences, we translate absolute risk aversion ρ into a corresponding coefficient of relative risk aversion using the expected next period's wealth derived from the fishery in the CARA optimum, obtained by solving (21) – which is independent of the constant b in case of CARA preferences – and using it in (27), which gives expected next period's wealth.

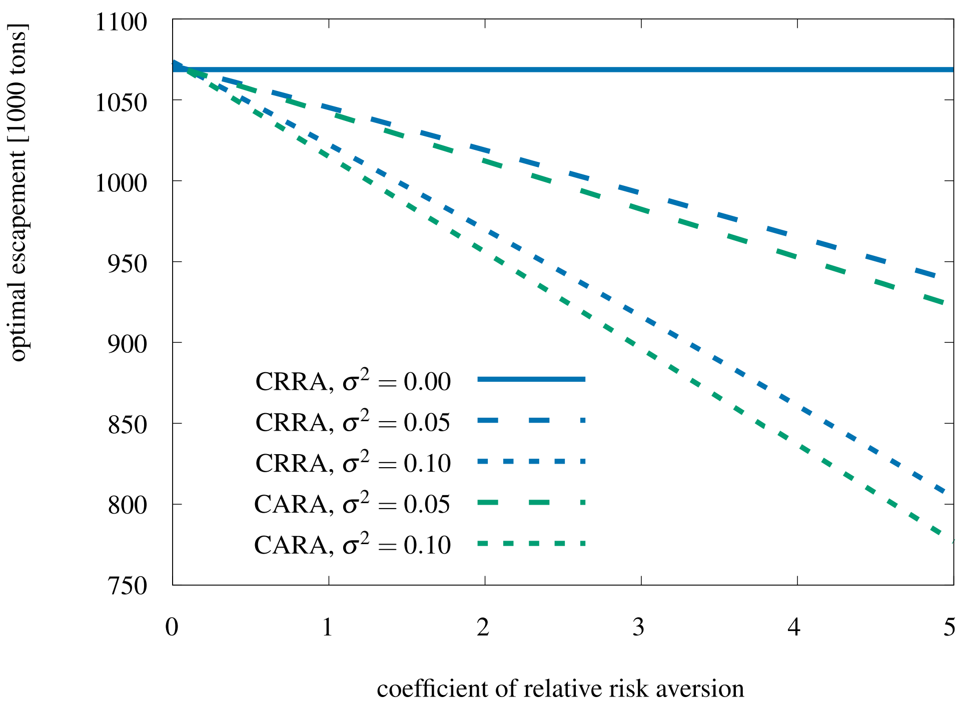

Figure 1 – Optimal escapement for risk preferences with constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) for three different levels of environmental risk.

Figure 1 shows the results for the optimal escapement for three different levels of risk and for varying the coefficients of risk aversion over the range that is generally deemed sensible in the literature (Gollier, 2001).

In case of zero risk, $\sigma^2 = 0$, the optimal escapement level is the same for both types of risk preferences and independent of the degree of risk aversion. If there is environmental risk, optimal escapement is decreasing in both the degree of risk aversion and the amount of risk. The substitution effect dominates both the wealth effect and the gambling effect.

Moreover, risk and risk aversion have a large effect on optimal escapement. This is in clear contrast to the findings in the previous literature that find only very small effects of environmental risk on optimal harvesting (Kapaun and Quaas, 2013; Tahvonen et al., 2018). Indeed, for a risk neutral resource user, Figure 1 does show an effect of uncertainty on optimal escapement, which is very small, though. The rather large effect of risk on optimal escapement is due to risk aversion.

The figure also shows the comparison of different types of risk preferences (for CARA preferences, the coefficient of relative risk aversion is computed for expected wealth). Whereas the difference in optimal escapement for different types of risk preferences is not very large, there is a consistent difference: For any given degree of risk aversion, optimal escapement is larger for CRRA preferences than for CARA preferences. The reason is that with CARA preferences there is no wealth effect that would tend to increase optimal escapement under risk. With CRRA preference, there is such an effect, thus leading to a relatively higher optimal escapement level.

6 Discussion and Conclusion

In the course of climate change, environmental risks are expected to increase (IPCC, 2021). This is an important and increasing issue for renewable resource use, where formal insurance is typically not available. Here, we have addressed the question how optimal resource harvesting depends on risk and the resource user's risk-aversion.

We have presented a theoretical model of harvesting a renewable resource with stochastic dynamics, focusing on the effect of risk aversion on optimal harvesting decisions. We have shown that optimal harvesting is characterized by a constant escapement strategy even for a risk-averse user, provided that the per-period payoff is linear in harvest and that the user is indifferent with respect to intertemporal variability.

Three effects determine how risk affects the optimal escapement level: The wealth, substitution, and gambling effects. The gambling effect depends on the curvature of harvesting costs only and tends to increase optimal escapement under risk. It vanishes if marginal harvesting costs are linear; otherwise is present independent of risk aversion.

The two other effects depend on the type of risk preferences. With constant marginal harvesting costs (a common assumption for natural resources such as forests) and with risk preferences characterized by constant absolute risk aversion, only the substitution effect is present. It always tends to decrease optimal escapement with risk, due to the propensity to divest away from the risky stock and avoid risk.

The wealth effect is present if absolute risk aversion is non-constant, and then typically tends to increase optimal escapement with risk. This effect depends on the degree of prudence. We show that if prudence is low enough, the net result of the substitution effect and the wealth effect, is that optimal escapement decreases with risk.

To quantify our theoretical results we apply our model to the Eastern Baltic Sea cod fishery and highlight that risk aversion can have a sizeable impact on the user's management decision. The more risk-averse the user, the lower the optimal escapement level, supporting our theoretical results. This result differs from that of a risk-neutral user, who increases escapement under increased risk. Previous empirical work focusing on risk neutral agents (Sethi et al. 2005, Kapaun and Quaas 2013, Tahvonen et al. 2018) find that biological uncertainty only has a small effect on the escapement level, suggesting that using models with deterministic recruitment may not be detrimental to policy-making. Our paper shows this may not be the case when the user is risk-averse. Depending on the degree of risk aversion, and magnitude of risk, the optimal escapement policy may differ considerably and this in turn may have significant implications for management.

Whilst these results are insightful and provide an important contribution to the literature on renewable resource management, the stylised nature of tractable models and the assumptions on which they are based must be acknowledged. As said initially, to focus on the effect of risk aversion, we considered a user who is indifferent with respect to intertemporal variability. In practice also credit markets will be imperfect, such that decision makers may actually exhibit a preference for intertemporal smoothing for income from resource use. Furthermore, we focus on a resource stock described by a single stock variable. For some interesting cases it seems appropriate to consider a more differentiated description of resource dynamics, taking into account the size structure of populations (Patto and Rosa, 2022; Tahvonen et al., 2018), or interactions with other resource stocks (Costello and Polasky, 2008). Developing this analysis in such settings constitutes an interesting avenue for further research.

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Appendices

Appendix A Proof of Proposition 1

We conjecture the value function is of the form $V(x) = ax + b$ with some constants $a > 0$ and b, and show that this solves the Bellman equation (8) when $a = 1$ and b is determined by (10) and (11). Substituting the guess $V(x) = ax + b$ into equation (8) gives

$$ax + b = x - s + \beta f^{-1}(E[f(a(z_{t+1}g(s)) + b)]) \tag{30}$$

As this equality must hold for any stock size x, it follows that $a = 1$. With $a = 1$, equation (30) becomes (11), where $s = s^*$. The first-order condition for optimal escapement, using the guess $V(x) = x + b$, is

$$1 = \beta g'(s)(f^{-1})'(E[f(z_{t+1}g(s) + b)])E[z_{t+1}f'(z_{t+1}g(s) + b)]. \tag{31}$$

As the optimality condition is independent of the current stock, the condition characterizes a stock-independent, constant optimal escapement level $s = s^*$. Applying the inverse function theorem, condition (31) becomes (10).

Appendix B Proof of Proposition 2

Recall that $f'(\cdot)$ is positive and decreasing, capturing risk aversion. Thus, its inverse function $f'^{-1}(\cdot)$ exists and must be a decreasing function as well. Hence,

$$\frac{\mathbb{E}[f'(z_{t+1}g(s^*) + b)]}{f'(\mu(z_{t+1}g(s^*) + b))} \gtreqless 1 \tag{32}$$

$$\Leftrightarrow \quad f'^{-1}(\mathbb{E}[f'(z_{t+1}g(s^*) + b)]) \lesseqgtr \mu(z_{t+1}g(s^*) + b) \tag{33}$$

$$\Leftrightarrow \quad \varphi\left(f\left(f'^{-1}(\mathbb{E}[f'(z_{t+1}g(s^*) + b)])\right)\right) \gtreqless \varphi(\mathbb{E}[f(z_{t+1}g(s^*) + b)]) \tag{34}$$

$$\Leftrightarrow \quad \mathbb{E}[\varphi(f(z_{t+1}g(s^*) + b))] \gtreqless \varphi(\mathbb{E}[f(z_{t+1}g(s^*) + b)]) \tag{35}$$

Applying Jensen's inequality, the last inequality holds with $>$ if $\varphi(\cdot)$ is convex, with equality if $\varphi(\cdot)$ is linear, and with $<$ if $\varphi(\cdot)$ is concave.

Appendix C Proof of Proposition 3

As $f''(\cdot) < 0$, we have $(z_{t+1} - 1)\,(f'(z_{t+1}\,g(s^*) + b) - f'(g(s^*) + b)) \leq 0$ for all z_{t+1} with strict inequality for all $z_{t+1} \neq 1$. Taking the expectation, we thus get

$$
\begin{aligned}
0 &> \mathbb{E}\left[(z_{t+1} - 1)\,\left(f'(z_{t+1}\,g(s^*) + b) - f'(g(s^*) + b)\right)\right] && (36)\\
&= \mathbb{E}\left[z_{t+1}\,f'(z_{t+1}\,g(s^*) + b)\right] - \mathbb{E}\left[f'(z_{t+1}\,g(s^*) + b)\right] && (37)
\end{aligned}
$$

Rearranging, we find

$$
\begin{aligned}
\mathbb{E}\left[f'(z_{t+1}\,g(s^*) + b)\right] &> \mathbb{E}\left[z_{t+1}\,f'(z_{t+1}\,g(s^*) + b)\right] && (38)\\
&= \mathbb{E}\left[f'(z_{t+1}\,g(s^*) + b)\right] + \operatorname{cov}\left[z_{t+1}, f'(z_{t+1}\,g(s^*) + b)\right], && (39)
\end{aligned}
$$

thus $\operatorname{cov}\left[z_{t+1}, f'(z_{t+1}\,g(s^*) + b)\right] < 0$.

Appendix D Proof of Proposition 4

As $\mu(z_{t+1}\,g(s^*) + b) < g(s^*) + b$, and $f'(\cdot)$ is decreasing, it follows that $f'(\mu(z_{t+1}\,g(s^*) + b)) > f'(g(s^*) + b)$. Thus, the numerator on the left-hand side of (14) is smaller than the denominator, if the expression in the expectation operator is weakly concave in z_{t+1}, as then

$$
\mathbb{E}[z_{t+1} f'(z_{t+1}\,g(s^*) + b)] \leq f'(g(s^*) + b) < f'(\mu(z_{t+1}\,g(s^*) + b)). \tag{40}
$$

The curvature of this expression in the expectation operator is

$$
\begin{aligned}
&\frac{d^2}{dz_{t+1}^2}\left(z_{t+1}\,f'(z_{t+1}\,g(s^*) + b)\right)\\
&\qquad = \frac{d}{dz_{t+1}}\left(f'(z_{t+1}\,g(s^*) + b) + z_{t+1}\,g(s^*)\,f''(z_{t+1}\,g(s^*) + b)\right)\\
&\qquad = z_{t+1}\,(g(s^*))^2\,f'''(z_{t+1}\,g(s^*) + b) + 2\,g(s^*)\,f''(z_{t+1}\,g(s^*) + b), \qquad (41)
\end{aligned}
$$

which is weakly negative if and only if (17) holds.

Appendix E Proof of Proposition 5

We conjecture the value function is of the form $V(x) = a(px - C(x)) + b$. with some constants $a > 0$ and b, and show that this solves the Bellman equation (7) when $a = 1$ and b is determined by (21) and (22). Substituting the guess into equation (7) gives

$$a\,(p\,x - C(x)) + b = p\,(x-s) - C(x) + C(s) + \beta\, f^{-1}(E[f(a\,(p\,z_{t+1}\,g(s) - C(z_{t+1}\,g(s)) + b)]) \tag{42}$$

As this equality must hold for any stock size x, it follows that $a = 1$. With $a = 1$, equation (42) becomes (22), where $s = s^*$. The first-order condition for optimal escapement, using the guess for the value function, is

$$p - c(s) = \beta\, g'(s)\,(f^{-1})'(E[f(z_{t+1}\,g(s) + b)])\,E[(p\,z_{t+1} - z_{t+1}\,c(z_{t+1}\,g(s)))\,f'(z_{t+1}\,g(s) + b)]. \tag{43}$$

As the optimality condition is independent of the current stock, the condition characterizes a stock-independent, constant optimal escapement level $s = s^*$. Applying the inverse function theorem, condition (43) becomes (21).

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