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Abstract

Motivated by the potential tension between coordination, which may require discriminating between identical agents, and social comparisons, which may call for small pay differentials, we analyze the optimal reward scheme in an organization involving agents with social preferences whose tasks are complementary. Although a tension exists between the effects of inequality aversion and altruism, there is always more reward inequality when agents are inequality-averse and altruistic than when they are purely self-interested. We then highlight how our results differ when agents are not altruistic but rather inequality-averse *a la* Fehr and Schmidt (1999).

JEL classification: D91, D86, D62

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1 Introduction

While social comparisons do affect workers' performance, well-being and pay,[1] little is known about how organizations should account for these features. Should inequality aversion yield a decrease in reward inequalities within organizations? Should it be associated with lower monetary incentives? More generally, how should social comparisons affect the distribution of rewards within organizations?

In order to address these questions, we analyze the implications of social preferences for the optimal design of reward schemes. Specifically, when workers' decisions in an organization exhibit complementarity effects, there is a potentially important tension between social comparisons and incentives. Indeed, a given worker may fear the risk that the other workers shirk, making her own effort useless. One solution is to offer a sufficiently high reward to a worker, so that exerting effort is beneficial to her even when the other workers shirk. This in turn removes the risk (for others) that this worker does not perform her task. Using this argument in a contracting setting with externalities,[2] different contributions highlight that the need to ensure agents' coordination implies that the optimal reward scheme is discriminatory, meaning that workers obtain different rewards even if they are identical (Segal, 2003; Winter, 2004). When workers are averse to inequalities, unequal rewards are likely to negatively affect them and, as such, to weaken the power of incentives. Yet, when workers are altruistic this may not be a problem, since they are willing to work to increase production in order to make other workers better off. As such, a tension seems to exist between the likely effects of aversion to inequalities and altruism.

We explore the implications of such tension and introduce an organization setting with multiple agents who exhibit social preferences and whose tasks are complementary. We first consider the case where symmetric agents are inequality averse and altruistic, in that an agent's social component of utility always increases when the other agents' payoffs increase. We then consider the case where agents are inequality averse a la Fehr and Schmidt (1999), a prominent form of social preferences which is such that an agent's social component of utility may decrease when other agents' payoffs increase. Specifically, each agent then negatively values the difference between her material payoff and that of any other agent: her utility thus decreases as another agent's payoff increases provided she obtains a smaller payoff than this given agent.

Our first main result addresses how inequality aversion with altruism (IAwA) affects the distribution of rewards. The optimal reward scheme (the least cost scheme inducing all agents' effort provision as the unique equilibrium of the induced game) is such that the agents obtain lower rewards, and inequality in the reward distribution is higher, when the agents are inequal-

[1] See Bandiera et al. (2005); Card et al. (2012); Cohn et al. (2014); Breza et al. (2017); Dube et al. (2019)
[2] The seminal paper is Segal (1999).

ity averse and altruistic rather than purely selfish. This result is far from straightforward, and the intuition goes as follows. If the agents exhibit no social preferences, as more agents exert effort, smaller rewards are required to induce the remaining agents to exert effort. The optimal reward exhibits a divide and conquer property: agents are ranked, and each agent is indifferent between exerting effort and shirking when the higher ranked agents exert effort and the lower ranked agents shirk. Thus agents are discriminated as identical agents get different rewards. Social preferences may result in more or less inequality in the reward distribution. Indeed, social preferences have both an extensive margin and an intensive margin effect on inequalities.

At the extensive margin, each additional agent who decides to exert effort generates a positive externality for the agent just ranked above her. Since the agents are altruistic and thus value the positive externalities they generate for others, the principal can decrease each agent's reward. Therefore, the extensive margin effect tends to increase inequality in the reward distribution. However, along the intensive margin, each additional agent who decides to exert effort generates positive externalities for all the higher ranked agents, who already benefit from the positive externalities they generate for each other. Since the agents are averse to inequality, an additional agent's marginal contribution to the social component of utility is lower as more agents exert effort. As more agents exert effort, the principal must increase the rewards allocated to the remaining agents to induce them to also exert effort. Thus the intensive margin effect tends to decrease inequality in the reward distribution. Moreover, the extensive margin effect applies to the agent whose material payoff is the lowest, while the intensive margin effect applies to the other agents who exert effort and who get higher rewards. Since the agents are averse to inequality, the extensive margin effect is always stronger than the intensive margin effect. A final implication is then that inequality in the reward distribution is higher when the agents are inequality averse and altruistic rather than purely selfish.

Our second main result addresses how inequality aversion a la Fehr and Schmidt (1999) affects the distribution of rewards. We show that this type of inequality aversion (IneqA) should be associated with larger rewards: compared to the case where agents are selfish, agents exhibiting inequality aversion should be all provided with larger rewards. We then show that inequality is unambiguously lower compared to a situation where the agents are not inequality averse. But provided the agents are already inequality averse, an increase in inequality aversion may actually result in an increase in inequality.[3] We further highlight that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. Finally, the distribution of rewards may also be non monotonic, and the most inequality-averse agents may lie at both ends of the reward distribution.

The mechanism at work can here be described as follows. When designing the optimal

[3] Montero (2007) also shows (in a very different setting) that inequality aversion may increase inequality.

reward scheme, the principal takes into account that offering a reward to an agent has both direct and indirect effects on the other agents' decisions. The direct effect is due to social comparisons, while the indirect effect is due to the complementarity effect in the agents' efforts. While the indirect effect is clearly positive, the direct effect is ambiguous, as an agent's payoff may increase or decrease when her effort increases. This follows from the characterization of the optimal reward scheme, which again exhibits the divide and conquer property. This implies that, in this case, an agent's material payoff equals the sum of the opportunity cost and of the increase in disutility due to the inequality resulting from this agent's decision to exert effort. As in the case of IAwA social preferences, the logic for the connection between the unique implementation requirement and the divide and conquer property is fairly consistent with the existing literature (Segal, 2003; Winter, 2004; Halac et al., 2020).

Our third main result concerns the role of coordination. So far the principal is assumed to explicitly account for the existence of a coordination problem (due to the potential existence of multiple equilibria) when designing the reward scheme: the optimal reward scheme must induce her desired outcome as a *unique* equilibrium of the induced game. We come back to the same problem and instead assume that the principal can costlessly select her most preferred equilibrium outcome. As such, the principal now offers the least-cost reward scheme that implements her desired outcome as *one* (of possibly many) equilibrium outcome of the induced game. This has a notable effect on the analysis, as several qualitative conclusions are reversed. For IAwA-type of social preferences, while symmetric agents always obtain identical rewards unlike in the previous case, there is not much difference regarding the other main results: for instance, the level of rewards is lower compared to the case where the agents exhibit no social preferences. By contrast, for IneqA-type of social preferences, all important results are reversed: inequality aversion results in a decrease in the individual rewards, and advantageous inequality aversion is then of first order importance compared to disadvantageous inequality aversion. The fact that effort provision by all agents be only one of several equilibria allows the principal to reduce the overall cost of the contractual scheme by relaxing the agents' incentive constraints: this actually implies that advantageous inequality aversion parameters then become most relevant. Moreover, the reward distribution becomes very different from the one derived when the principal has to explicitly solve for the coordination problem.

Overall, our results highlight that agents' altruism is beneficial to the principal and results in lower agents' rewards, while agents' inequality aversion is detrimental to the principal. When the agents are altruistic, our results suggest that inequality aversion has less effect than altruism, a result that is consistent with empirical evidence (Gueye et al., 2020). We also find that optimal rewards are necessarily higher when agents are (IneqA) inequality averse rather than purely selfish. This is also an important result, which contradicts a conjecture made by Cohn et al. (2014) that inequality aversion should cause a reduction in pay inequality, and that it should be associated with smaller monetary incentives. Another important implication of our analysis is that in any case, social preferences may yield more inequality.

Literature. This paper is related to two different strands in the literature. The first one relates to an empirical literature on the potential effect of social comparisons on workers' well-being and performance in organizations.[4] Recent empirical studies highlight that social comparisons do affect workers' pay and job satisfaction (Card et al., 2012), workers' performance (Bandiera et al., 2005; Cohn et al., 2014), output and attendance (Breza et al., 2017), and even decisions to quit (Dube et al., 2019). While social comparisons seem to strongly affect workers well-being and performance, little is known about how organizations should account for these features. This is the main goal of this paper which, to our knowledge, is the first to analyze the interplay between the problem of coordinating agents' decisions and social preferences. We provide theoretical results, and as such general conclusions, about the specific characteristics of the optimal reward scheme when multiple agents exhibiting social preferences interact within an organization.

The present contribution also relates to the literature on behavioral contract theory.[5] Specifically, this study relates to the literature focusing on optimal contracting with multiple inequality-averse agents (Demougin et al., 2006; Rey-Biel, 2008).[6] Very few papers consider a principal - multiple agent relationship when agents are inequality averse.[7] A part of this literature provides results related to team-based incentives, as for instance Bartling and Von Siemens (2010, 2011), Rey-Biel (2008), or Itoh (2004). The general focus of these studies is on how the principal tailors agents' incentives to account for agents' preferences by offering more equitable contracts or team-based incentive schemes. All these papers differ notably from the present contribution in terms of the setting considered and the research questions. Moreover, all these contributions focus on two-agent settings and none of them is suited to analyze the (possibly non monotonic) distribution of rewards. Finally, these contributions do not tackle the problem where a principal uses rewards to explicitly induce coordination among agents.[8]

Structure of the paper. The remainder of the contribution is organized as follows. Section 2 introduces the model. In Section 3 we characterize the optimal reward scheme that induces effort provision and coordination when the agents exhibit IAwA-type of social preferences. In Section 4 we study the case where they exhibit IneqA-type of social preferences. In Section 5

[4]Recent laboratory experimental evidence clearly rejects the assumption that individuals care only about their material payoffs (Camerer, 2003).

[5]See Koszegi (2014), DellaVigna (2009), and Rabin (1998) for extended reviews.

[6]Englmaier and Wambach (2010) mainly consider the effect of inequality aversion on contract design in a single principal - single agent setting when the agent cares for the principal's material payoffs.

[7]Goel and Thakor (2006) analyze the case where agents envy each other: The focus is on contracts inducing surplus sharing in the case of homogeneous agents. Gürtler and Gürtler (2012) analyze the effect of inequality aversion on individuals' behavior in a quite general setting. Yet, the focus is on the externalities resulting from such preferences in an homogeneous population setting.

[8]Dhillon and Herzog-Stein (2009) analyze the coordination problem in a very different setting where agents are status-seeking.

we analyze the differences that emerge when the principal can costlessly solve the coordination problem.

2 The Model

A principal offers individual bilateral contracts to several agents in an environment characterized by positive externalities between the agents. First, the principal offers a publicly observable reward scheme to a set of agents. Second, the agents observe the principal's proposition and simultaneously decide whether to exert effort or shirk.

The vector of agents' decisions is $e = (e_1, ..., e_n) \in \{0, 1\}^n$, where $e_i = 1$ means that agent i chooses to exert effort and $e_i = 0$ means that that agent decides to shirk. In the absence of monetary incentives, agent i's payoff is denoted $b_i(\boldsymbol{e})$. Any agents' effort e_i generates positive externalities for the other agents and they are complementary (e.g. effort cost-reducing externalities): b is strictly increasing in each of its arguments and supermodular. When an agent (say, agent i) is the only one to exert effort, her payoff is normalized to zero, that is $b_i(\boldsymbol{e}) = 0$ if $\sum_{j \neq i} e_j = 0$ and $e_i = 1$. An agent who decides to shirk receives an outside option c, that is $b_i(\boldsymbol{e}) = c$ if $e_i = 0$.

The principal aims at inducing all agents to exert effort at least cost. To reach this goal, the principal offers a reward scheme $v = (v_1, v_2, ..., v_n) \in R^n$ to agents $i = 1, 2, ..., n$, to induce them to exert effort. An agent's reward is conditional on this agent exerting effort: agent i obtains reward v_i from the principal if he exerts efforts and 0 otherwise. The reward scheme v is designed such agent $i \in N$ receives a unique offer v_i, meaning the principal can use individualized rewards. We will denote $\boldsymbol{y}_{y_i=a}$ the vector $\boldsymbol{y} = (y_1, ..., y_n) \in R^n$ which ith component equals a. We will denote $\boldsymbol{y}_{-i}$ the vector $\boldsymbol{y}$ which ith component is removed. For instance, if $n = 3$ we have $\boldsymbol{\pi}_{-1} = (\pi_2, \pi_3)$. Then $\boldsymbol{y}^k = (y, ..., y)$ denotes the k-dimensional vector such that each component equals $y \in R$. Finally, $\boldsymbol{t} = (\boldsymbol{z}, \boldsymbol{y})$ denotes the vector which first components correspond to those of vector $\boldsymbol{z}$ and last components correspond to those of vector $\boldsymbol{y}$.

Agent i's material payoff is:

$$\pi_i(\boldsymbol{e}, v_i) = e_i v_i + b_i(\boldsymbol{e}). \quad (1)$$

The agents exhibit social preferences and they award relative weight to their own payoff (a "self-interested" motive) and to the distribution of all agents' payoffs (a social motive). Formally, the social utility of agent i is:

$$U_i(\boldsymbol{e}, \boldsymbol{v}) = u_i(\pi_i(\boldsymbol{e}, v_i)) + \theta W_i(\boldsymbol{\pi}(\boldsymbol{e}, \boldsymbol{v})), \quad (2)$$

where $\boldsymbol{\pi}(\boldsymbol{e}, v)$ denotes the vector of the agents' payoffs, u_i is the selfish component of utility and W_i denotes the agent's social component of utility. Parameter $\theta \geq 0$ measures the magnitude of the social component of an agent's utility. When $\theta = 0$, the agents are purely self-interested and when $\theta > 0$ the agents exhibit social preferences. The individual utility function u_i is strictly increasing.

The principal seeks to implement full effort (the outcome where all agents exert effort) in the following way. A reward system $\boldsymbol{v}$ *implements full effort as a unique Nash equilibrium* if $\boldsymbol{e} = \mathbf{1}^n$ is the unique Nash equilibrium of the game induced by $\boldsymbol{v}$.[9] The set of schemes that implement full effort as a unique Nash equilibrium is open because $\boldsymbol{v}$ takes on continuous values. Thus, we define the *optimal reward scheme* as the least cost scheme such that, for any $\epsilon > 0$, increasing v_i by ϵ for any agent $i \in N$ implements full effort as a unique Nash equilibrium.[10] More precisely, we characterize the reward vector v^* that solves the following optimization problem:

$$Min_{\boldsymbol{v} \in \Re^n} \sum_{j \in N} v_j \tag{3}$$

s.t. for all $\epsilon > 0$, full effort ($\boldsymbol{e} = \mathbf{1}^n$) is a Nash equilibrium when the rewards are increased by ϵ:

$$U_i(\mathbf{1}^n, \boldsymbol{v} + \boldsymbol{\epsilon^n}) \geq U_i(\mathbf{1}^n_{e_i=0}, \boldsymbol{v} + \boldsymbol{\epsilon^n}), \tag{NE}$$

for all $i \in N$, and there is no other Nash equilibrium, that is, for all $\boldsymbol{e} \neq \mathbf{1}^n$, $\exists i \in N$ such that $e_i = a$, $a \in \{0, 1\}$ and:

$$U_i(\boldsymbol{e}_{e_i=1-a}, \boldsymbol{v} + \boldsymbol{\epsilon^n}) > U_i(\boldsymbol{e}, \boldsymbol{v} + \boldsymbol{\epsilon^n}). \tag{UC}$$

The set of constraints (NE) ensures that each agent has incentives to exert effort when all other agents also exert efforts, and the set of constraints (UC) ensures that, for each of the other outcomes, at least one agent has incentives to deviate.

3 Inequality Aversion with Altruism

This section concerns the implications of IAwA-type of social preferences on the distribution of rewards. We first set general assumptions on the social component of the utility function, then we study specific reward schemes (which exhibit the divide and conquer property), and finally provide our main result on optimal rewards and inequality.

Les us introduce our assumptions on the social component of the utility function.

[9] In Section 5 we investigate the case where the principal can costlessly select her most preferred equilibrium outcome. Thus, we characterize the least-cost contract that ensures that the outcome where all agents exert effort be one (of possibly many) equilibrium (partial implementation), as is done in Segal (1999) for instance.

[10] This solution concept corresponds to that of Winter (2004) and Halac et al. (2020). Halac et al. (2021) use a similar concept in a Bayesian game.

Assumption C-SupM (Inequality aversion): For any $i \in N$ the social component W_i is concave and supermodular.

Assumption A (altruism): For any $i \in N$ the social component W_i is strictly increasing in π_j for any $j \in N$.

Assumption NDC (no double counting): For any $i \in N$ the social component W_i does not depend on π_i. Formally $W_i = W_i(\boldsymbol{\pi_{-i}}(\boldsymbol{e}, \boldsymbol{v_{-i}}))$ where $\boldsymbol{\pi_{-i}}(\boldsymbol{e}, \boldsymbol{v_{-i}})$ denotes the vector of the agents' payoffs except agent i and v_{-i} denotes the vector of rewards of all agents except agent i.

Assumption S (symmetry): The agents are symmetric, that is $u_i \equiv u$, $b_i \equiv b$ and $W_i \equiv W$ and then $U_i \equiv U$ for all i.

Notice that Assumption S implies that the level of social component W_i is the same for any permutation of the material payoff of the other agents: in this sense, others are anonymous. Assumption C-SupM models inequality aversion in the following natural way. Concavity guarantees that the social component of the utility function increases when transfering (part of) a reward from one agent to another poorer agent (Atkinson, 1970). Supermodularity enables to consider general inequality aversion preferences where the social component of utility is non-separable across the agents' rewards (Meyer and Mookherjee, 1987) and it implies that the social component of utility increases more with an agent's reward if the rewards of other agents are high.

Assumptions C-SupM and A are satisfied by well known social welfare functions, such as the the constant elasticity of substitution (CES) social welfare function, or the quasi-maximin social welfare function, and assumptions NDC and S can be applied to these functions.[11]

To illustrate our results and provide intuition, we will use the following example assuming a utilitarian social welfare function and linear externalities in the agents' efforts:

Example [Utilitarianism]: *Let u be a concave function, W_i the utilitarian social welfare function $W_i(\boldsymbol{\pi_{-i}}) = \sum_{k \neq i} u(\pi_k)$, and the externalities as modeled by function $b_i(\boldsymbol{e}) = e_i \sum_{j \neq i} e_j w + (1 - e_i)c$ where $w > 0$. Let $n = 3$ so $W_1(\pi_2, \pi_3) = u(\pi_2) + u(\pi_3)$, $W_2(\pi_1, \pi_3) = u(\pi_1) + u(\pi_3)$, $W_3(\pi_1, \pi_2) = u(\pi_1) + u(\pi_3)$, while $b_1(\boldsymbol{e}) = e_1(e_2 + e_3)w + (1 - e_1)c$, $b_2(\boldsymbol{e}) = e_2(e_1 + e_3)w + (1 - e_2)c$, and $b_3(\boldsymbol{e}) = e_3(e_1 + e_2)w + (1 - e_3)c$.*

[11] The CES social welfare function is such that $W_i(\boldsymbol{\pi_{-i}}) = \left[\sum_{k \neq i} u(\pi_k)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$, where $s \in]0, +\infty[$ and $s \neq 1$. The quasi-maximin social welfare function, introduced in Charness and Rabin (2002), is such that $W_i(\boldsymbol{\pi_{-i}}) = \eta min\{\pi_1, ..., \pi_{i-1}, \pi_{i+1}, ..., \pi_n\} + (1 - \eta) \sum_{j \in N} \pi_j$ where $\eta \in [0, 1]$.

The main results of this section highlight that IAwA-type of social preferences result in lower agents' rewards and in lower inequality in the distribution of rewards compared to purely selfish preferences.

3.1 Ranking and Reward Scheme

In this section, we analyze the principal's problem.

3.1.1 Ranking Agents and Reward Distribution

The following class of reward schemes will prove to be useful in the analysis:

Definition 1: *A reward scheme* $\boldsymbol{v}$ *is a* ***ranking scheme for agent*** i *if the agents are ranked in a given order, and agent* i *is indifferent between exerting effort or shirking when the higher ranked agents also exert efforts and the lower ranked agents shirk.*

To save on notations, the payoff that agent i obtains in this situation will be denoted $\tilde{\pi}_i$. It is such that:

$$\tilde{\pi}_i \equiv \pi_i(\mathbf{1}^i, \mathbf{0}^{n-i}) \tag{4}$$

Coming back to our example and assuming that the agents are ranked from 1 to 3, we have $\tilde{\pi}_1 = v_1$, $\tilde{\pi}_2 = v_2 + w$ and $\tilde{\pi}_3 = v_3 + 2w$.

Formally, Definition 1 means that, if the agents are ranked from 1 to n (without loss of generality), and $\boldsymbol{v}$ is a ranking scheme for agent i, we must have:

$$u_i(c) - u_i(\tilde{\pi}_i) = \theta \left[W_i(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) - W_i(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}})) \right] \tag{5}$$

This condition corresponds to the case where agent i exerts effort, while the higher ranked agents also exert efforts and the other agents shirk. It states that the marginal benefit from shirking (related to the selfish component of utility) is equal to the marginal benefit from exerting effort (related to the social component of utility).

In our example, we have the following condition for the highest ranked agent:

$$u_1(c) - u_1(\tilde{\pi}_1) = 0 \tag{6}$$

The highest ranked agent must be indifferent between exerting effort or shirking, when no other agent exerts effort. Hence, his reward has to be equal to his opportunity cost, $\tilde{\pi}_1 = v_1 = c$.

Now consider the indifference condition for agent 2:

$$u_2(c) - u_2(\tilde{\pi}_2) = \theta \left[\underbrace{u_1(\tilde{\pi}_1 + w) - u_1(\tilde{\pi}_1)}_{B_{21}} \right] \tag{7}$$

This condition states that this agent's marginal benefit from shirking (related to the selfish component of utility) must equal the social marginal benefits that he provides to higher ranked agent, namely agent 1.

Finally, consider the third agent's indifference condition:

$$u_3(c) - u_3(\tilde{\pi}_3) = \theta \left[\underbrace{u_1(\tilde{\pi}_1 + 2w) - u_1(\tilde{\pi}_1 + w)}_{B_{31}} + \underbrace{u_2(\tilde{\pi}_2 + w) - u_2(\tilde{\pi}_2)}_{B_{32}} \right] \tag{8}$$

This condition states that this agent's marginal benefit from shirking (related to the selfish component of utility) must equal the sum of the social marginal benefits that this agent provides to the higher ranked agents (agents 1 and 2).

Using these conditions, we can show that the agents' payoffs are lower than their opportunity cost:

Proposition 1 [Payoffs under IAwA]: *If $\boldsymbol{v}$ is a ranking scheme for agent i and all higher ranked agents exert efforts while the lower ranked agents shirk, then agent i's material payoff is lower than the opportunity cost, $\tilde{\pi}_i \leq c$.*

The intuition of this result is as follows. When an agent exerts effort, he generates positive externalities to the other agents who also exert efforts, which in turn increases the level of his social component of utility. It is thus not necessary to compensate for the full opportunity cost to induce this agent to exert effort. Inspecting conditions (6), (7), (8) highlights that this holds in the case of our example.

We now show a much less intuitive result:

Proposition 2 [Distribution of Payoffs under IAwA]: *If $\boldsymbol{v}$ is a ranking scheme for the first i agents according to a common ranking (1 to n without loss of generality), then $\tilde{\pi}_{k-1} \geq \tilde{\pi}_k$ for all $2 \leq k \leq i$.*

In this Proposition, we compare the payoffs of two subsequent agents in two different situations. For each agent, we consider the situation where they exert effort together with the higher ranked agents while the other agents shirk. We show that the highest ranked agent among the two subsequent agents obtains a higher material payoff than the other one. The two payoffs are equal when the agents exhibit no social preferences ($\theta = 0$) and they are different when they do exhibit such preferences ($\theta > 0$). We explain the intuition in both cases below.

If the agents exhibit no social preferences ($\theta = 0$), agent 1 must be indifferent between exerting effort and shirking when no other agent exerts effort. In this case, the agents do not benefit from positive externalities. Agent 2 must be indifferent between exerting effort and shirking when agent 1 also exerts effort. Agent 2 thus benefits from a positive externality and has greater incentives to exert effort. Agent 2 thus obtains a reward that is equal to the difference between agent 1's reward and the positive externality generated by this agent. This reasoning holds for any two subsequent agents.

Next consider the case where the agents exhibit social preferences ($\theta > 0$). In our example one can notice that $\tilde{\pi}_1 = c > \tilde{\pi}_2$ (by Proposition 1). Looking at the difference between the indifference conditions for agents 3 and 2 (conditions (8) and (7)) we have:

$$u_2(\tilde{\pi}_2) - u_3(\tilde{\pi}_3) = \theta \times \{\underbrace{B_{31} - B_{21}}_{(-)} + \underbrace{B_{32}}_{(+)}\} \tag{9}$$

The sign of this difference is a priori ambiguous. The first term between brackets on the right hand side is the difference between the social marginal benefit agent 1 provides to agent 3 and to agent 2. This intensive margin effect is negative because of concavity of u. The second term on the right hand side is the marginal social benefit agent 3 provides agent 2 with. This extensive margin effect is positive.

We can rewrite the difference as follows:

$$u(\tilde{\pi}_2) - u(\tilde{\pi}_3) = \theta \left[\underbrace{B_{31}}_{(+)} - \underbrace{\left[\underbrace{u_1(\tilde{\pi}_1 + w) - u_1(\tilde{\pi}_1)}_{B_{21}} \right] + \underbrace{u_2(\tilde{\pi}_2 + w) - u_2(\tilde{\pi}_2)}_{B_{32}}}_{(+)} \right] \tag{10}$$

The first term (B_{31}) denotes the marginal social benefit that agent 3 provides agent 1 with: it is positive. The second term ($B_{32} - B_{21}$) denotes the difference between the marginal social benefit that agent 3 provides to agent 2 and the marginal social benefit that agent 2 provides to agent 1. Since $\tilde{\pi}_1 > \tilde{\pi}_2$ while the agents are symmetric and u is concave, this difference is positive, and we can conclude.

The condition above proves that Proposition 2 holds in the utilitarian case with three agents, concave utility u and linear externalities. The extensive margin effect applies to the agent with the lowest material payoff, while the intensive margin effect applies to the agents who also exert effort and whose material payoffs are higher. The concavity of W implies that the extensive margin effect is larger than the intensive margin effect.

Proposition 2 also holds for more general settings than the one of our utilitarianism example: the symmetry assumption together with the concavity of W and supermodularity of W and b allow to prove the result in the general case.

3.2 Optimal Reward Scheme and Inequality

We first prove a preliminary result that will be used to analyze the optimal scheme:

Lemma 1: *If $\tilde{\boldsymbol{v}}$ is such that each agent $k \leq i$ prefers to exert effort when the higher ranked agents (according to a common ranking, 1 to n, without loss of generality) also exert effort and the remaining agents shirk, then each agent k also prefers to exert effort when j agents ($k \leq j \leq i$) exert effort.*

The intuition of this result is simple if the agents exhibit no social preferences ($\theta = 0$). As more agents exert effort, the agents who exert effort benefit from higher positive externalities and thus they are more likely to exert effort.

If the agents exhibit social preferences ($\theta > 0$), the result is less straightforward. The monotonicity result of Proposition 2 is needed to prove the result. To provide some intuition, consider the case where externalities are linear and homogeneous and $w > 0$ denotes the externality generated by each agent exerting effort for any other agents who also exert effort. If $\tilde{\boldsymbol{v}}$ is a ranking scheme for the first i agents according to a common ranking, then their rewards are characterized by condition (5). Consider the situation where the first i agents exert effort. If either agent i or agent k chooses to shirk, the material payoff of each agent ranked higher than i is reduced by w. Moreover, if agent i shirks, agent k's payoff is reduced by w and if agent k shirks, agent i's payoff is reduced by w. Given that $v_k > v_i$ and W is concave, the social component of agent k's utility decreases more (when agent k decides to shirk) than the social component of agent i's utility (when agent i decides to shirk). Thus, agent k has lower incentives to deviate than agent i. Since agent i is indifferent between exerting effort and shirking, agent k strictly prefers to exert effort.

The following class of reward schemes will be useful to characterize the optimal reward scheme:

Definition 2: *A reward scheme $\boldsymbol{v}$ is a **global ranking scheme** if it is a ranking scheme for all the agents according to a common ranking.*

Notice that this scheme is unique for a given ranking. Each agent is indifferent between exerting effort and shirking when the agents ranked before him also exert effort, and the remaining agents shirk. This definition is used to provide necessary conditions for a reward scheme to be optimal:

Theorem 1 [Inequality under IAwA]: *Any optimal reward scheme is a global ranking scheme. Moreover, individual rewards are lower and the degree of inequality in the reward distribution is higher when the agents exhibit IAwA-type of social preferences rather than purely self-interested ones.*

The first part of the Theorem builds on the strategic complementarity property embedded in the model due to the increasing externalities (in the sense of Segal 2003) and the supermodularity property of the social component of agents' utility. It states that the optimal scheme is a divide-and-conquer scheme. The first part of the Theorem thus shows the robustness of the divide-and-conquer property when agents exhibit IAwA-type of social preferences. Notice that the global ranking scheme is unique, up to a reordering of the (ex-ante identical) agents. This result together with Propositions 1 and 2 yield the second part of the Theorem. It connects the behavioral features of the model with the structure of the optimal scheme. If the agents exhibit no social preferences, when more agents exert efforts, an agent has higher incentives to exert effort since he benefits from higher positive externalities. If the agents exhibit social preferences, they derive utility from generating positive externalities to other agents: as such the principal can provide them with lower rewards, and she can even more reduce the rewards for the lower ranked agents. What is most surprising here, as discussed in Section 3.1.1, is that this result holds even if the agents are both inequality averse and altruistic.

To prove sufficiency, we need an additional assumption:

Assumption SOSC (second order social concerns): *For any two reward vectors* $\boldsymbol{v}$ *and* $\boldsymbol{v'}$*, we have* $|u(\pi_i(\boldsymbol{e}, v_i)) - u(\pi_i(\boldsymbol{e}, v_i'))| > \theta\, |W(\boldsymbol{\pi}(\boldsymbol{e}, \boldsymbol{v})) - W(\boldsymbol{\pi}(\boldsymbol{e}, \boldsymbol{v'}))|$.

This assumption implies that social concerns are of second-order importance compared to material payoffs (it has been also used in Cabrales and Calvo-Armengol 2008). Using this assumption, we obtain the following result:

Theorem 2 [Sufficiency]: *If assumption SOSC holds, any global ranking scheme is an optimal reward scheme.*

Theorem 2 provides sufficiency conditions. Namely, if the social component is of second-order importance compared to the private component of the utility function, then full effort is a unique Nash equilibrium under any global ranking scheme. Suppose assumption SOSC does not hold, and the agents are very inequality averse. An agent may prefer to shirk when the higher ranked agents exert effort and the remaining agents shirk if the agents' rewards are increased by a positive amount. Indeed, an increase in the agents' rewards simultaneously increases the marginal effect of exerting effort on the private component of utility and decreases the marginal effect of exerting effort on the social component of utility. Thus, full effort is the unique Nash Equilibrium under a global ranking scheme if the latter effect is larger than the former for each agent when the higher ranked agents exert effort while the lower ranked agents shirk.

3.3 Heterogeneity

Up to now, we have focused on the case of identical agents and homogenous externalities. One may wonder whether the main results of the paper hold in a situation where the agents exhibit heterogeneous preferences and under heterogeneous externalities.

Let us analyze the role of heterogeneity using a simple form of altruistic preferences. Assume that the agents exhibit heterogeneous preferences for the other agents' payoffs and that the social component of an agent's utility is a weighted form of the other agents' payoffs:

$$W_i(\boldsymbol{e}) = \sum_{j \neq i} \gamma_{ij} \pi_j, \tag{11}$$

where $\gamma_{ij} \geq 0$ denotes the intensity of the preference of agent i for agent j's payoff.

Also assume that externalities are heterogeneous: when agents i and j exert effort, agent j generates positive externalities for agent i whose related benefits are denoted $w_{ij} \geq 0$:

$$b_i(\boldsymbol{e}) = \sum_{j \neq i} w_{ij} e_j e_i + (1 - e_i)c. \tag{12}$$

To keep things simple, also assume that the private component of utility is such that $u_i(\pi_i) \equiv \pi_i$. Using all the assumptions above, agent i's utility can be written as:

$$U_i(\boldsymbol{e}, \boldsymbol{v}) = \pi_i + \theta \sum_{j \neq i} \gamma_{ij} \pi_j, \tag{13}$$

where

$$\pi_i = e_i \left(v_i + \sum_{j \neq i} e_j w_{ij} \right) + (1 - e_i)c. \tag{14}$$

We can derive the optimal reward scheme using similar proofs as in Bernstein and Winter (2012). We use their notations and denote i_j the agent characterized by rank j. If each agent i_j with $j < k$ exerts effort, then agent i_k also prefers to exert effort if:

$$v_{i_k} + \sum_{i_j < i_k} w_{i_k i_j} + \theta \sum_{i_j < i_k} \gamma_{i_k i_j} w_{i_j i_k} > c. \tag{15}$$

Thus, the optimal reward scheme is such that the agents are ranked from i_1 to i_n and the optimal reward of agent i_k is:

$$v^*_{i_k} = c - \sum_{i_j < i_k} (w_{i_k i_j} + \theta\gamma_{i_k i_j} w_{i_j i_k}), \tag{16}$$

Hence, for a given ranking, the agents still obtain lower rewards when they exhibit heterogeneous altruistic preferences ($\theta > 0$) than when they do not ($\theta = 0$). The optimal ranking depends on the virtual popularity tournament described in section A in Bernstein and Winter (2012). In the present model, agent j beats agent k if:

$$(1 - \theta\gamma_{jk})w_{kj} < (1 - \theta\gamma_{kj})w_{jk} \tag{17}$$

This result is interesting since it implies that altruistic preferences affect the agents' optimal ranking. Indeed, one may conclude that i_j beats i_k if altruism is not accounted for ($w_{jk} > w_{kj}$) while i_k actually beats i_j when altruism is accounted for ($(1-\theta\gamma_{kj})w_{jk} < (1-\theta\gamma_{jk})w_{kj}$). This occurs for instance when agent i_j derives benefits from high externalities generated by agent i_k and agent i_k strongly values agent i_j's payoff.

How the introduction of altruistic preferences ($\theta > 0$) affects inequality is not straightforward. Let us consider the spread of the rewards distribution:

$$\max_{k \in N} v_{i_k} - \min_{k \in N} v_{i_k} = \sum_{i_j < i_l} (w_{i_l i_j} + \theta\gamma_{i_l i_j} w_{i_j i_l}), \tag{18}$$

where i_l is characterized by $\min_{k \in N} v_{i_k} = v_{i_l}$ (notice that we have $l > 1$). We have the following conclusion:

Proposition 3 [Reward Distribution under Heterogeneity]: *The spread of the reward distribution is higher when the agents exhibit altruistic heterogeneous preferences than when they exhibit no social preferences.*

Thus, in this sense, altruistic preferences result in more inequality when one introduces heterogeneity in the model. It is however important to notice that the results of this section are limited to the case where utility is linear in the agents' actions and externalities are additive. They cannot be easily extended to the more general case of non linear preferences (that encompasses inequality aversion) or of non linear externalities considered in the remainder of the section.

4 Inequality Aversion without Altruism

Up to now, we have assumed that the agents' social component of utility is always increasing when the other agents' payoffs increase. In this section, we depart from this assumption and study a situation where the agents are assumed to be inequality averse *a la* Fehr and Schmidt (1999). Specifically, the social component of agent i's utility function is:

$$W_i(\boldsymbol{\pi}) = -\frac{\alpha_i}{n-1}\sum_{k\neq i}\max\{\pi_k(e,v_k)-\pi_i(e,v_i),0\}-\frac{\beta_i}{n-1}\sum_{k\neq i}\max\{\pi_i(e,v_i)-\pi_k(e,v_k),0\}, \quad (19)$$

where $0 \leq \beta_i < 1$ and $\beta_i \leq \alpha_i$. The first term on the right hand side of this equality corresponds to disadvantageous inequality, while the second term on the right hand of the equality corresponds to advantageous inequality. This specification of preferences to model aversion to inequalities has been widely used in the behavioral economics literature due to its simplicity and to the consistency of theoretical findings with experimental evidence in different game situations.

This function departs from the assumptions considered in Section 3. It does not satisfy Assumption A (altruism), since the social component of agent i's utility can decrease when the payoff of another agent (say, j) increases. Indeed, assume there are only two agents (1 and 2) and that agent 1 is disadvantaged ($\pi_1 < \pi_2$), then the social component of utility becomes $W_1(\pi_1,\pi_2) = -\alpha_1(\pi_2-\pi_1)$ and thus it decreases when the second agent's payoff increases. It does not satisfy Assumption NDC (no double counting) since it depends on the differences between the agent's payoffs and those of the other agents. However, it does satisfy Assumption C-SupM since it is both concave and supermodular.

For simplicity, the externalities are assumed to be linear and homogenous:

$$b_i(\boldsymbol{e}) = \sum_{j\neq i} we_je_i + (1-e_i)c, \quad (20)$$

where $w > 0$.

We also assume that the private component of utility is linear and that $\theta = 1$, so agent i's utility writes as follows:

$$U_i(\boldsymbol{e},\boldsymbol{v}) = e_i\left(v_i + \sum_{j\neq i} we_j\right) + (1-e_i)c - \frac{\alpha_i}{n-1}\sum_{k\neq i}\max\{\pi_k(\boldsymbol{e},v_k)-\pi_i(\boldsymbol{e},v_i),0\}$$
$$-\frac{\beta_i}{n-1}\sum_{k\neq i}\max\{\pi_i(\boldsymbol{e},v_i)-\pi_k(\boldsymbol{e},v_k),0\}. \quad (21)$$

As in the previous section, we will characterize the least-cost contract that implements effort provision from all agents as a unique Nash equilibrium of the induced game.[12]

[12]In Section 5 we analyze the case where the principal can costlessly select her most preferred equilibrium outcome. Thus, we characterize the least-cost contract that ensures that the outcome where all agents exert efforts be one (of possibly many) equilibrium (partial implementation), as in Segal 1999 for instance.

4.1 A simple example

Before developing the generic results and characterizations, we first provide the intuition underlying the characterization of the optimal reward scheme implementing full effort as a unique Nash equilibrium of the induced game (which will be provided in Proposition 5). We do this by using a simple example. Suppose there are two agents, 1 and 2, and that the principal's reward scheme is a global ranking scheme, that is, agent 1 is indifferent between exerting effort and shirking when agent 2 shirks and agent 2 is indifferent between exerting effort and shirking when agent 1 exerts effort. A restatement of the global ranking scheme property given the agents' inequality-averse preferences is that the following properties hold (assuming $v_1 \geq v_2$):

1. The *advantageous* inequality generated by agent 1 exerting effort *when agent* 2 *shirks* equals agent 1's selfish payoff difference between exerting efforts and shirking. Formally:

   $$\beta_1 (v_1 - c) = v_1 - c \Longleftrightarrow v_1 = c$$

   where the equivalence statement follows from $\beta_1 < 1$.

2. The net *disadvantageous* inequality generated by agent 2 exerting efforts *when agent* 1 *exerts efforts* equals agent 2's selfish payoff difference between exerting efforts and shirking. Formally,

   $$\underbrace{\alpha_2 (v_1 - v_2)}_{\text{disadvantageous inequality when making efforts}} - \underbrace{\alpha_2 (v_1 - c)}_{\text{disadvantageous inequality under shirk}} = \underbrace{v_2 + w}_{\text{selfish payoff when exerting efforts}} - \underbrace{c}_{\text{selfish payoff under shirk}}$$

   Using $v_1 = c$ we notice that $v_2 = c - \frac{w}{1+\alpha_2}$.

From these simple calculations, we can understand why the agents must obtain a higher reward when they are inequality averse. The counter-intuitive nature of this result lies in the following reasoning: when agent 2 exerts efforts, she actually *increases* disadvantageous inequality, whereas commonplace intuition suggests that exerting efforts should reduce it (inequality of effort is reduced). This property emerges because agent 2 is getting a lower reward than agent 1 *in spite of generating positive externalities that increases agent* 1*'s selfish payoffs*, that is, in spite of her effort contributions. Hence, each agent must receive higher monetary incentives to exert efforts, rather than downward reward compression.

4.2 The optimal reward scheme

The first step to characterize the optimal reward scheme is to show that it is characterized by the global ranking scheme property. The set of reward schemes exhibiting this property is obtained by ranking agents in an arbitrary fashion, and by providing each agent with a reward that would induce her to exert effort assuming that all the higher ranked agents also exert effort and all lower ranked agents shirk. Intuitively, lower ranked agents are induced to exert effort by the others' choice to do so and can be offered smaller rewards.

So we first consider an arbitrary ranking of the set of agents, and we provide a first result:

Proposition 4 *A reward scheme is a global ranking scheme if and only if it is an optimal reward scheme.*

This characterization result provides an important feature of the overall problem. Indeed, the principal's optimization problem then boils down to finding the optimal ranking and the optimal scheme for that ranking. We now proceed in two steps. First, fixing the ranking, we characterize the optimal scheme. Let π_i^j denote agent i's material payoff when the first j agents exert effort while the remaining $n-j$ agents shirk. Let $D_i^j = \frac{\alpha_i}{n-1}\sum_{k\neq i} max\{\pi_k^j - \pi_i^j, 0\} + \frac{\beta_i}{n-1}\sum_{k\neq i} max\{\pi_k^j - \pi_i^j, 0\}$ denote agent i's total disutility resulting from inequity. Finally, introducing $\Delta D_i^i = D_i^i - D_i^{i-1}$ we obtain the following result:

Proposition 5 [Reward scheme under IneqA]: *If the agents are averse to inequality, the optimal reward scheme that implements full effort as a unique Nash equilibrium of the induced game is such that:*[13]

$$v_i^* = c - (i-1)w + \Delta D_i^i, \tag{22}$$

where

$$\Delta D_i^i = \frac{(i-1)\alpha_i}{n-1+(i-1)\alpha_i-(n-i)\beta_i}w, \tag{23}$$

for all $1 \leq i \leq n$.

This result characterizes the additional material payoff that the agents obtain because they exhibit aversion to inequality. It highlights that the stronger the aversion to inequality (i.e. the larger α_i or β_i), the larger the agent's reward. This static comparative result follows from straightforward computations using the fact that disadvantageous inequality and advantageous inequality increase when agent i exert effort rather than when she shirks (see the discussion based on a simple example in Section 4.1). When an agent is more averse to inequality, these increases in inequality have to be compensated by an increase in the agent's reward. The Proposition also yields an interesting implication, which can be summarized as follows:

[13]The unique Nash equilibrium induced by these schemes is strict.

Lemma 2: *For any agent i, the reward scheme satisfies $\pi_i^i - c = \Delta D_i^i \geq 0$ and this implies that any agent obtains a larger payoff if she is averse to inequality rather than selfish.*

This conclusion follows directly from Proposition 5, yet an initial intuition would suggest that inequality aversion may cause a decrease in pay inequalities within organizations, which in turn may be associated with smaller monetary rewards (see Cohn et al. 2014). In our model, an intuition suggests that the principal, in order to minimize the rewards, might have incentives to make the material payoffs smaller than the opportunity cost. However, this is not possible due to the uniqueness constraint.

To understand the mechanism further, the main arguments are as follows. In order to induce the highest ranked agent to exert effort while all the other agents shirk, the principal has to provide him with a reward that is at least equal to the opportunity cost, that is $\pi_1^1 \geq c$. For the other agents, it is actually not possible that the result holds for the first $i-1$ agents and not for agent i. Indeed, if the first $i-1$ agents obtain a reward that is larger than c when the higher ranked agents exert effort, their payoff increases when more agents exert effort. Hence, when the agents who are ranked higher than agent i exert effort, if agent i receives a reward that is lower than the opportunity cost, she obtains the lowest payoff, whatever her decision. In this case, only disadvantageous inequality aversion plays a role and agent i is more disadvantaged when she exerts effort rather than when she shirks. We conclude that agent i's payoff must satisfy $\pi_i^i \geq c$. Hence, agent i's reward is $v_i^* = c-(i-1)w+\Delta D_i^i$, where $\Delta D_i^i \geq 0$.

It is also interesting to notice that disadvantageous inequality aversion is of first-order importance compared to advantageous inequality aversion. Indeed, using a first order approximation of agent i's reward around $(\alpha_i, \beta_i) = (0, 0)$, we have $v_i \sim c-(i-1)w+(i-1)\alpha_i w$ which depends on α_i but not on β_i. This is due to the fact that, when the higher ranked agents exert effort, an agent is only advantaged compared to the lower ranked agents who do not exert effort and who get the opportunity cost c. The disutility that is due to advantageous inequality aversion is thus proportional to the increase in the agent's material payoff when she decides to exert effort instead of shirking (still when only the higher ranked agents do exert effort). The divide and conquer nature of the optimal reward schemes awards a lot of importance to higher ranked agents, and as such to disadvantageous inequality-aversion parameters.

The optimal scheme is a global ranking scheme with the optimal ranking, that is, the ranking that minimizes the principal's aggregate cost of providing incentives to exert effort. As such, it is important to notice that Proposition 5 does not provide insights on the agents' optimal ranking.

4.3 Payoffs and inequality

The characterization provided in Proposition 5 can be used to provide some insights about the effects driven by inequality aversion compared to the case where the agents do not exhibit social preferences. We obtain the following result:

Proposition 6 [Effect of inequality aversion under IneqA]: *Under the optimal reward scheme, we have the following conclusions:*
(i) An agent's material payoff is larger when he is inequality averse rather than purely selfish (i.e. when $\beta_i \geq 0$ and $\alpha_i \geq 0$ instead of $\alpha_i = \beta_i = 0$).
Assuming that the agents are symmetric ($\alpha_k = \alpha$ and $\beta_k = \beta$ for all k), we also have that:
(ii) The magnitude of the difference between any two subsequent agents' material payoffs, $|\pi_i - \pi_{i+1}|$, is lower when the agents are averse to inequality rather than purely selfish (i.e. when $\beta \geq 0$ and $\alpha \geq 0$ instead of $\alpha = \beta = 0$)

These properties mostly follow from Proposition 5.[14] For instance, regarding point (i), the optimal reward scheme is such that disadvantageous inequality and advantageous inequality increase when agent i exerts effort rather than when she shirks. These increases in inequality levels have to be compensated by an increase in the agent's reward when she is averse to inequality.

We can also use the characterization of the optimal reward scheme to highlight some non-intuitive effects of social comparisons on inequality levels. Here we just assume the simplest case where $\alpha_k = \alpha$ and $\beta_k = \beta$ for any $k \in N$, and we obtain:

Proposition 7 [Effect on inequality under IneqA]: *Under the optimal reward scheme, we have the following conclusions:*

(i) A marginal increase in disadvantageous inequality aversion may lead to an increase in inequality at the bottom of the reward distribution. Formally, $\frac{\partial|\pi_i - \pi_{i+1}|}{\partial \alpha} \geq 0$ if and only if $i \geq 2$ and $\alpha \geq \sqrt{\frac{n-1-(n-i)\beta}{i(i-1)}}$.
(ii) A marginal increase in advantageous inequality aversion may lead to an increase in inequality at the bottom of the reward distribution. Formally, $\frac{\partial|\pi_i - \pi_{i+1}|}{\partial \beta} \geq 0$ if and only if $i = n-1$ or $\beta \in \left[1 - \sqrt{\frac{i(i-1)}{(n-i)(n-i-1)}}(1+\alpha), 1\right[$.

These effects directly result from the characterization of optimal rewards provided in Proposition 5. The two cases highlight that an increase in the intensity of aversion to inequalities may actually result in higher inequality through the impact of both types of inequality aversion

[14] The optimal ranking is generically unique: there might be knife-edge cases where two rankings are optimal (for instance, when agents are fully symmetric).

on the agents' rewards. Yet, these results also highlight that such effect differs depending on whether the focus is on disadvantageous inequalities or on advantageous inequalities.

4.4 Optimal ranking

As a final step, notice that we cannot provide a full characterization of the optimal ranking as heterogeneity is bi-dimensional, each agent i being characterized by a pair of inequality-aversion parameters, (α_i, β_i). However, we can characterize the optimal ranking when the agents are identical with respect to one dimension and heterogeneous with respect to the other one.

Let us first consider the case where the agents have different disadvantageous inequality-aversion parameters:

Proposition 8 [Disadvantageous inequality under IneqA]: *Assume $\beta_j = \beta$ for all j. (i) If the agents are weakly averse to disadvantageous inequality ($\alpha_j < 1$ for all j), then the optimal ranking is such that an agent's rank decreases as her disadvantageous inequality parameter is lower ($\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n$). (ii) If the agents are strongly averse to disadvantageous inequality ($\alpha_j > 1$ for all j), then the optimal ranking is such that the agents' rank is a U-shaped function of the disadvantageous inequality parameters ($\exists 1 < k < n$ such that $\alpha_1 \geq ... \geq \alpha_k$ and $\alpha_k \leq ... \leq \alpha_n$).*

If the agents value disadvantageous inequality more than their own monetary payoff (case (i)), the most averse agents lie at the top of the reward distribution. However, if the agents value their own payoff more than disadvantageous inequality (case (ii)), then the optimal ranking is non monotonic and the most inequality-averse agents lie at both ends of the reward distribution.

To get some intuition on this result, it is sufficient to focus on the disutility resulting from disadvantageous inequality. Assume that agent i is ranked at position j. Her disutility resulting from disadvantageous inequality is $\alpha_i \Delta A_j^j = \frac{\alpha_i}{n-1}\left[w - (\pi_j^j - c)\right] j$. A marginal increase in α_i leads to a marginal disutility level $\Delta A_j^j + \alpha_i \frac{\partial A_j^j}{\partial \alpha_i}$. The first term increases, while the second term decreases, when the rank j increases. When the degree of aversion to disadvantageous inequality is sufficiently low ($\alpha_i < 1$), the first term dominates, and then the principal has incentives to first rank the most inequality-averse agents. When the degree of aversion to disadvantageous inequality is sufficiently large ($\alpha_i > 1$), the second term is sufficiently strong and the optimal ranking is non monotonic. In other words, the agents' ranking is characterized by the magnitude of the disutility resulting from disadvantageous inequalities. An agent's disadvantageous inequality-aversion parameter has both a direct effect on this disutility and an indirect effect resulting from its impact on this agent's optimal reward. The direct effect is positive, while the indirect effect is negative, and the net effect depends on the fundamentals.

Proposition 8 only provides a partial characterization of the case where agents are homogeneous with respect to the advantageous inequality parameters. The following example highlights that, when $\alpha_j < 1$ is satisfied only for some agents $j \in N$, the optimal ranking will depend on the relative values of agents' disadvantageous inequality parameters together with the absolute value of the advantageous inequality parameter. Thus, no fairly generic conclusion may be expected in this case.

Let us consider the three-agent case. It is easily checked that the highest ranked agent's disadvantageous inequality parameter must satisfy $\alpha_1 \geq \alpha_2$. Indeed, the optimal ranking must be such that the principal saves cost by relying on this ranking rather than switching the position of the first two highest ranked agents. Using Proposition 5 this implies that $\frac{\alpha_1}{2+\alpha_1-\beta} \geq \frac{\alpha_2}{2+\alpha_2-\beta}$ must be satisfied, which in turn is equivalent to $\alpha_1 \geq \alpha_2$. Now, moving on to the comparison between the second highest ranked agent and the last agent in the ranking, the optimal ranking must be such that the principal saves cost by relying on this ranking rather than switching the position of these two agents. Using Proposition 5 this is equivalent to

$$\frac{\alpha_2}{2+\alpha_2-\beta} + \frac{2\alpha_3}{2+2\alpha_3} \leq \frac{\alpha_3}{2+\alpha_3-\beta} + \frac{2\alpha_2}{2+2\alpha_2}$$

or

$$(1-\beta)\frac{\alpha_3}{(1+\alpha_3)(2+\alpha_3-\beta)} \leq (1-\beta)\frac{\alpha_2}{(1+\alpha_2)(2+\alpha_2-\beta)} \tag{24}$$

Since $\beta < 1$ is satisfied we only need to focus on function $\Phi(\alpha) = \frac{\alpha}{(1+\alpha)(2+\alpha-\beta)}$ and we obtain $\Phi'(\alpha) = \frac{2-\beta-\alpha^2}{(1+\alpha)^2(2+\alpha-\beta)^2}$ so the sign of $\Phi'(\alpha)$ is that of function $g(\alpha) = 2-\beta-\alpha^2$, which is concave in α, increases up to $\alpha = 1-\frac{\beta}{2}$ then decreases thereafter. This implies that, as $g(\alpha) = 0$ if and only if $\alpha = \sqrt{2-\beta} > 1$, that Φ increases up to $\alpha = \sqrt{2-\beta}$ and decreases thereafter. Thus inequality (24) holds when $\alpha_3 \leq \alpha_2 \leq \sqrt{2-\beta}$ or $\sqrt{2-\beta} < \alpha_2 < \alpha_3$ holds. Yet, these are not the only cases where this inequality holds. When $\alpha_2 < 1 < \sqrt{2-\beta} \leq \alpha_3$ holds then the conclusion depends on the relative values of α_2 and α_3.

We now characterize the optimal ranking when the agents only differ in terms of their aversion to advantageous inequality:

Proposition 9 [Advantageous inequality under IneqA]: *Assume $\alpha_j = \alpha$ for all j. The optimal ranking is such that the rank is a U-shaped function of the advantageous inequality parameters ($\exists 1 < k < n$ such that $\beta_1 \geq ... \geq \beta_k$ and $\beta_k \leq ... \leq \beta_n$).*

The optimal ranking of heterogeneous agents who are averse to advantageous inequality is non monotonic: The most inequality-averse agents lie at both ends of the reward distribution. When agents are heterogeneous in their advantageous inequality-aversion parameters only, the agents' ranking is characterized by the magnitude of the disutility resulting from advantageous inequalities. An agent's disadvantageous inequality-aversion parameter has both a direct effect on this disutility and an indirect effect resulting from its impact on this agent's optimal reward. As in the previous case, there is a trade-off between direct and indirect effects, but this time the trade-off has always bite: there is a U-shape relationship.

5 (How) does coordination matter?

In the previous sections, we have highlighted several qualitative properties of the optimal reward scheme that solves any potential coordination problem. This raises the question about whether the coordination problem does actually matter. Does the existence of such a problem drastically affect the characterization of the optimal reward scheme, or does it have little effect on it? To answer this question, we now solve for the optimal partial implementation reward scheme, that is, the least-cost scheme inducing all agents to exert effort as one of the Nash equilibria of the induced game. We look for the solution to the principal problem that can now be stated as follows:

$$Min_{\boldsymbol{v}\in\Re^n} \sum_{j\in N} v_j \tag{25}$$

s.t. full effort ($\boldsymbol{x} = \mathbf{1}^n$) is a Nash equilibrium:

$$U_i(\mathbf{1}^n, \boldsymbol{v}) \geq U_i(\mathbf{1}^n_{x_i=0}, \boldsymbol{v}), \tag{26}$$

for all $i \in N$.

The set of constraints (26) ensures that each agent has an incentive to exert efforts when all other agents also exert efforts. In order to minimize the cost of the scheme, the principal will necessarily choose a reward scheme such that the constraints (NE) are binding, or:

$$u_i(c) - u_i(\pi_i(\mathbf{1}^n)) = \theta \left[W_i(\boldsymbol{\pi_{-i}}(\mathbf{1}^n, \boldsymbol{v_{-i}})) - W_i(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, 0, \mathbf{1}^{n-i}), \boldsymbol{v_{-i}})) \right], \tag{27}$$

for all $i \in N$. This condition states that for each agent the marginal benefit from shirking (related to the private component of utility) must equal the marginal benefit of exerting effort (related to the social component of utility) when all the other agents exert efforts.

In each case (IAwA and IneqA) the assumptions made in the previous sections are maintained. Thus, for IAwA-type of preferences we keep the general functional form and assume symmetric agents, while we keep the more specific form of utility *a la* Fehr and Schmidt (1999) for IneqA-type of preferences and consider asymmetric agents.

5.1 The role of coordination under IAwA-type of social preferences

The properties of the optimal scheme under assumptions A, NDC, C-SupM and S can be deduced from inspecting condition (27). Indeed, using symmetry, agent i's optimal reward is characterized by:

$$v_i^* = u^{-1}\left(u(c) - b(\mathbf{1}^n) - \theta\left[W(\boldsymbol{\pi}_{-i}(\mathbf{1}^n, \boldsymbol{v}_{-i}^*)) - W(\boldsymbol{\pi}_{-i}((\mathbf{1}^{i-1}, 0, \mathbf{1}^{n-i}), \boldsymbol{v}_{-i}^*))\right]\right). \tag{28}$$

This leads us to the following conclusion:

Proposition 10 [Partial implementation under IAwA]: *The optimal partial implementation reward scheme is such that the agents obtain a lower reward when they exhibit IAwA-type of social preferences ($\theta > 0$) than when they exhibit no social preferences ($\theta = 0$). Moreover, all the (symmetric) agents obtain the same reward.*

Hence, as in the case of unique implementation analyzed in Section 3, the agents obtain lower rewards when they have IAwA-type social preferences rather than selfish ones. However, unlike the case of unique implementation, there is no inequality here whenever the (symmetric) agents exhibit social preferences or are purely self-interested. Notice that here, there is no need to rank the agents as in the case of unique implementation.

An interesting implication of IAwA-type of social preferences is that it affects the difference between the rewards agents obtain in the unique and in the partial implementation cases. When agents are purely self-interested ($\theta = 0$), the lowest ranked agent gets the same reward in the unique implementation case and in the partial implementation case. On the other hand, when agents exhibit IAwA-type of social preferences ($\theta > 0$), the lowest ranked agent gets a lower reward in the partial implementation case compared to the unique implementation case. Indeed, the highest ranked agents get a lower reward in the partial implementation case (compared to the unique implementation case). Thus, the marginal benefit (related to the social component of utility) that the lowest ranked agent obtains when he decides to exert effort instead of shirking is higher in the partial implementation case, because he generates positive externalities to the other agents who obtain lower rewards than in the unique implementation case.

5.2 The role of coordination under IneqA-type of social preferences

We now turn to analyze the role of coordination when the agents exhibit IneqA-type of social preferences. We obtain the following first result:

Proposition 11 [Partial Implementation under IneqA]: *If the agents exhibit IneqA-type of social preferences, the optimal partial implementation reward scheme is such that:*

$$v_i^* = c - (n-1)w - J_i w, \tag{29}$$

where

$$J_1 = \frac{\beta_1}{1-\beta_1} \tag{30}$$

and

$$J_i = \frac{\beta_i + \frac{\alpha_i+\beta_i}{n-1}\left(\sum_{j=1}^{i-1} J_j\right)}{1-\beta_i + \frac{i-1}{n-1}(\alpha_i+\beta_i)}, \tag{31}$$

for all $2 \leq i \leq n$ and given the ranking where higher ranked agents obtain higher rewards.

The main arguments of the proof of this result go as follows. First, the outcome where all agents exert effort is a Nash equilibrium of the induced effort choice game if and only if, for any $i \in N$, the following incentive constraint is satisfied:

$$(1-\beta_i)\left[v_i + (n-1)w - c\right] + \beta_i w - \frac{\alpha_i+\beta_i}{n-1}\sum_{k\neq i}\left[max\{v_k - v_i, 0\} - max\{v_k + (n-2)w - c, 0\}\right] \geq 0$$

Second, it must necessarily hold that $v_k + (n-2)w - c \leq 0$ for all $k \in N$: otherwise, the reward scheme considered would not be least cost. This property enables us to simplify the incentive constraint: the highest ranked agent's reward has to satisfy the following incentive constraint

$$(1-\beta_1)\left[v_1 + (n-1)w - c\right] + \beta_1 w \geq 0$$

To induce the lowest feasible payment, this condition must be satisfied as an equality. Then, for the agent whose reward is the ith largest one, the following constraint must be satisfied:

$$(1-\beta_i)\left[v_i + (n-1)w - c\right] + \beta_i w - \frac{\alpha_i+\beta_i}{n-1}\sum_{k<i}\left[v_k - v_i\right] \geq 0$$

Again, to induce the lowest payment it must be satisfied as an equality. Solving the resulting system of $(n-1)$ equalities as functions of $(v_2, ..., v_n)$, we obtain the desired expressions.

Proposition 11 allows to conclude that the coordination problem does matter as it deeply affects the characterization of the optimal reward scheme. Indeed, using Lemma 2 and Proposition 11, we conclude that optimal unique and partial implementation reward schemes drastically differ. Under partial implementation, inequality aversion negatively affect monetary incentives: compared to the case of self-interested agents, the reward scheme provides all agents with lower rewards. This conclusion is entirely reversed when dealing with unique implementation: all agents are provided with higher rewards (compared to the case where they do not exhibit social preferences). One must be cautious about the rankings corresponding to the unique and partial implementation cases: in general, these rankings will differ.

It is also interesting to notice that aversion to advantageous inequality is of first-order importance compared to disadvantageous inequality aversion. Indeed, using a first-order approximation of agent i's reward around $(\alpha_i, \beta_i) = (0, 0)$, we have $v_i \sim c - (n-1)w - \beta_i w$ which depends on β_i but not on α_i. This qualitative result is also entirely reversed when dealing with unique implementation: disadvantageous inequality aversion is of first-order importance in this case.
Intuitively, the fact that full effort be only one of several Nash equilibria allows the principal to reduce the overall cost of the reward scheme by relaxing the incentive constraints for all agents: this actually implies that advantageous inequality-aversion parameters then become the most relevant parameters. This has to be contrasted with the case of optimal unique implementation, where the divide and conquer structure of the scheme tends to award more importance to higher ranked agents, and thus to aversion to disadvantageous inequalities.

We now rely on this characterization to highlight another notable effect of the coordination problem: namely, the induced differences in terms of the agents' optimal ranking. Since there is a fundamental difference with the case of optimal unique implementation in terms of the characterization of the optimal reward scheme, the same type of qualitative differences is expected for the next results. In order to understand the effect of each fundamental, we proceed with the analysis in several steps. First, we assume that agents share the same degree of aversion to advantageous inequalities. We then consider the case where they share the same degree of aversion to disadvantageous inequalities. Finally, we provide some partial insights about the general case. We have the first following result:

Proposition 12 [Disadvantageous inequality under IneqA]: *Assume $\beta_j = \beta$ for all j. Then aversion to disadvantageous inequality has no effect on the agents' ranking: $v_i = v$ $\forall i \in N$.*

This important feature is perfectly illustrated by Proposition 12: Aversion to advantageous inequalities has a first-order effect on the characterization of the optimal ranking induced by the partial implementation reward scheme. We now analyze the polar case where the agents' degree of aversion of disadvantageous inequalities is the same. We obtain the following results:

Proposition 13 [Advantageous inequality under IneqA]: *Assume $\alpha_j = \alpha$ for all j. The optimal ranking is increasing in the magnitude of aversion to advantageous inequality: $\beta_1 \leq ... \leq \beta_n$.*

This case highlights the asymmetric effect of inequality aversion. Aversion to advantageous inequality has a first-order effect on the optimal ranking, as it flattens the distribution of payments when agents are homogeneous with respect to this fundamental. Aversion to disadvantageous inequality does not have a similar type of effect: when agents are homogeneous with respect to this fundamental, the optimal ranking is characterized by increasing degrees of aversion to advantageous inequality for lower-ranked agents.

We now move on to the general case, and we specifically highlight the potential non-monotonicity of the optimal ranking. We obtain:

Proposition 14 [General case]: *The optimal ranking is characterized by the following conditions:* $\beta_1 \leq \beta_2$ *and, for any* $i \geq 3$*:*

$$(\beta_i - \beta_{i-1}) + \frac{\sum_{l=1}^{i-2} I_l}{n-1}\left[(\alpha_i + \beta_i) - (\alpha_{i-1} + \beta_{i-1}) - \beta_{i-1}\alpha_i + \beta_i\alpha_{i-1}\right] + \frac{i-2}{n-1}\left[\beta_i\alpha_{i-1} - \beta_{i-1}\alpha_i\right] \geq 0 \quad (32)$$

As such, a sufficient condition for a ranking to be optimal is that it satisfies $\frac{\beta_i}{\beta_{i-1}} > \frac{\alpha_i}{\alpha_{i-1}} \geq 1$ *for any* $i \geq 3$*, which also implies that this ranking satisfies* $\alpha_i \geq \alpha_{i-1}$ *and* $\beta_i > \beta_{i-1}$ *for any* $i \geq 3$*. When these conditions are not satisfied, there are cases where the optimal ranking satisfies* $\alpha_i \leq \alpha_{i-1}$ *for some* $i \in$*.*

The fact that it is not possible to obtain a closed-form characterization was fairly expected as heterogeneity is again bi-dimensional. Nonetheless, Proposition 14 highlights a sufficient condition ensuring that the optimal ranking satisfies some particular form of monotonicity. When this condition is not satisfied, it is not possible to obtain a clear-cut conclusion and the optimal ranking may be non-monotonic.

Appendix

Proof of Proposition 2:

Assume that $\boldsymbol{v}$ is a ranking scheme for the first i agents. Using (5) for $i = 1$, we have

$$u(c) - u(v_1) = \theta \left[W(\boldsymbol{c}^{n-1}) - W(\boldsymbol{c}^{n-1}) \right] = 0, \tag{33}$$

and then $v_1 = c$.

Let $\hat{b}_i \equiv b_i(\mathbf{1}^i, \mathbf{0}^{n-i})$ for all i. Now we use induction to show that $\pi_k(\mathbf{1}^k, \mathbf{0}^{n-k}) = v_k + \hat{b}_{k+1} \leq \pi_{k-1}(\mathbf{1}^{k-1}, \mathbf{0}^{n-k+1}) = v_{k-1} + \hat{b}_k$ for all $k \leq i$. Assume that this inequality holds for all $k \leq j+1$ with $j+2 \leq i$. Using condition (5) for agents $j+1$ and $j+2$, we have:

$$\begin{aligned} & u(v_{j+1} + \hat{b}_{j+1}) - u(v_{j+2} + \hat{b}_{j+2}) \\ & \quad = \theta \left[W(\boldsymbol{\pi}_{\mathbf{-(j+2)}}((\mathbf{1}^{j+2}, \mathbf{0}^{n-j-2}), \boldsymbol{v}_{\mathbf{-(j+2)}})) - W(\boldsymbol{\pi}_{\mathbf{-(j+2)}}((\mathbf{1}^{j+1}, \mathbf{0}^{n-j-1}), \boldsymbol{v}_{\mathbf{-(j+2)}})) \right] \\ & \qquad - \theta \left[W(\boldsymbol{\pi}_{\mathbf{-(j+1)}}((\mathbf{1}^{j+1}, \mathbf{0}^{n-j-1}), \boldsymbol{v}_{\mathbf{-(j+1)}})) - W(\boldsymbol{\pi}_{\mathbf{-(j+1)}}((\mathbf{1}^{j}, \mathbf{0}^{n-j}), \boldsymbol{v}_{\mathbf{-(j+1)}})) \right]. \end{aligned} \tag{34}$$

It is sufficient to show that the right hand side term in condition (34) is positive. Let $\boldsymbol{y}, \boldsymbol{\Delta}, \boldsymbol{\delta} \in R^{n-1}$. If $\delta_k \leq \Delta_k$ for $1 \leq k \leq n-1$, concavity of W implies $W(\boldsymbol{y}+\boldsymbol{\delta}) + W(\boldsymbol{y}-\boldsymbol{\delta}) \geq W(\boldsymbol{y}+\boldsymbol{\Delta}) + W(\boldsymbol{y}-\boldsymbol{\Delta})$ (Rothschild and Stiglitz, 1970). Letting $y_k = \frac{v_k + v_{k+1}}{2} + \frac{\hat{b}_j + \hat{b}_{j+2}}{2}$ for $1 \leq k \leq j$ and $y_k = c$ for $j+1 \leq k \leq n-1$, $\delta_k = \frac{v_k - v_{k+1}}{2} - \frac{\hat{b}_{j+2} - \hat{b}_j}{2}$ for $1 \leq k \leq j$ and $\delta_k = 0$ for $j+1 \leq k \leq n-1$, and $\Delta_k = \frac{v_k - v_{k+1}}{2}$ for $1 \leq k \leq j$ and $\Delta_k = 0$ for $j+1 \leq k \leq n-1$, we must have:

$$\begin{aligned} & W(v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-1}) + W(v_1 + \hat{b}_j, ..., v_j + \hat{b}_j, \boldsymbol{c}^{n-j-1}) \\ & \geq W(v_1 + \frac{\hat{b}_j + \hat{b}_{j+2}}{2}, ..., v_j + \frac{\hat{b}_j + \hat{b}_{j+2}}{2}, \boldsymbol{c}^{n-j-1}) + W(v_2 + \frac{\hat{b}_j + \hat{b}_{j+2}}{2}, ..., v_{j+1} + \frac{\hat{b}_j + \hat{b}_{j+2}}{2}, \boldsymbol{c}^{n-j-1}). \end{aligned} \tag{35}$$

Since b is supermodular, we must have $\frac{\hat{b}_j + \hat{b}_{j+2}}{2} \geq \hat{b}_{j+1}$. Hence condition (36) implies:

$$\begin{aligned} & W(v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-1}) + W(v_1 + \hat{b}_j, ..., v_j + \hat{b}_j, \boldsymbol{c}^{n-j-1}) \\ & \qquad \geq W(v_1 + \hat{b}_{j+1}, ..., v_j + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-1}) + W(v_2 + \hat{b}_{j+1}, ..., v_{j+1} + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-1}). \end{aligned} \tag{36}$$

Since the agents are symmetric, we can permute the material payoffs without affecting the level of W. Hence, we obtain:

$$W(c, v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-2}) - W(c, v_2 + \hat{b}_{j+1}, ..., v_{j+1} + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-2}) \geq W(v_1 + \hat{b}_{j+1}, ..., v_j + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-1}) - W(v_1 + \hat{b}_j, ..., v_j + \hat{b}_j, \boldsymbol{c}^{n-j-1}). \quad (37)$$

Moreover, using supermodularity of W and $v_1 = c$, we have

$$W(v_1 + \hat{b}_{j+1}, v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-2}) + W(c, v_2 + \hat{b}_{j+1}, ..., v_{j+1} + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-2}) \geq W(c, v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-2}) + W(v_1 + \hat{b}_{j+1}, v_2 + \hat{b}_{j+1}, ..., v_{j+1} + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-2}). \quad (38)$$

Combining (37) and (38), we find:

$$W(v_1 + \hat{b}_{j+1}, v_2 + \hat{b}_{j+2}, ..., v_{j+1} + \hat{b}_{j+2}, \boldsymbol{c}^{n-j-2}) - W(v_1 + \hat{b}_{j+1}, v_2 + \hat{b}_{j+1}, ..., v_{j+1} + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-2}) \geq W(v_1 + \hat{b}_{j+1}, ..., v_j + \hat{b}_{j+1}, \boldsymbol{c}^{n-j-1}) - W(v_1 + \hat{b}_j, ..., v_j + \hat{b}_j, \boldsymbol{c}^{n-j-1}). \quad (39)$$

Since W and b are non decreasing functions, condition (39) implies that the right hand side in (34) is indeed positive.□

Proof of Lemma 1: Assume that agent i prefers to exert effort when the higher ranked agents also exert effort while the remaining agents shirk (assuming they are ranked from 1 to n, without loss of generality). Hence:

$$u(v_i + \hat{b}_i) + \theta W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) \geq u(c) + \theta W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}})), \quad (40)$$

where $\hat{b}_i = b_i(\mathbf{1}^i, \mathbf{0}^{n-i})$. Assume that agent $k < i$ weakly prefers to shirk when each agent $j \leq i, j \neq k$ exerts effort while the remaining agents do not. Hence, we must have

$$u(c) + \theta W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{e_k=0}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-k}})) \geq u(v_k + \hat{b}_i) + \theta W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-k}})) \quad (41)$$

Using (40), we obtain:

$$u(v_i + \hat{b}_i) + \theta \left[W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-i}})) - W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-i}}))\right] \geq u(v_k + \hat{b}_i) + \theta \left[W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i, \mathbf{0}^{n-i}), \boldsymbol{v_{-k}})) - W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{e_k=0}, \mathbf{0}^{n-i+1}), \boldsymbol{v_{-k}}))\right] \quad (42)$$

which is equivalent to

$$\theta\left[W(\boldsymbol{\pi_{-i}}((\mathbf{1}^i,\mathbf{0}^{n-i}),\boldsymbol{v_{-i}}))-W(\boldsymbol{\pi_{-i}}((\mathbf{1}^{i-1},\mathbf{0}^{n-i+1}),\boldsymbol{v_{-i}}))\right]$$
$$-\theta\left[W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i,\mathbf{0}^{n-i}),\boldsymbol{v_{-k}}))-W(\boldsymbol{\pi_{-k}}((\mathbf{1}^i_{e_k=0},\mathbf{0}^{n-i+1}),\boldsymbol{v_{-k}}))\right]$$
$$\geq u(v_k+\hat{b}_i)-u(v_i+\hat{b}_i). \quad (43)$$

Using symmetry and writing the arguments more explicitly, we obtain:

$$\theta\left[W(v_1+\hat{b}_i,...,v_{i-1}+\hat{b}_i,\boldsymbol{c}^{n-i})-W(v_1+\hat{b}_{i-1},...,v_{i-1}+\hat{b}_{i-1},\boldsymbol{c}^{n-i})\right]$$
$$-\theta\left[W(v_1+\hat{b}_i,...,v_{k-1}+\hat{b}_i,v_i+\hat{b}_i,v_{k+1}+\hat{b}_i,...,v_{i-1}+\hat{b}_i,\boldsymbol{c}^{n-i})\right.$$
$$\left.-W(v_1+\hat{b}_{i-1},...,v_{k-1}+\hat{b}_{i-1},v_i+\hat{b}_{i-1},v_{k+1}+\hat{b}_{i-1},...,v_{i-1}+\hat{b}_{i-1},\boldsymbol{c}^{n-i})\right]$$
$$\geq u(v_k+\hat{b}_i)-u(v_i+\hat{b}_i). \quad (44)$$

The second term in brackets in the left hand side term in (44) is identical to the first term in brackets except that v_k is replaced by v_i, which is smaller. Thus, concavity of W implies that the left hand side term in (43) is negative. Hence:

$$u(v_k+\hat{b}_i)-u(v_i+\hat{b}_i)\leq 0 \quad (45)$$

Thus, we must have $v_k \leq v_i$, where $k < i$. This contradicts Proposition 2.□

Proof of Theorem 1: Assume that $\boldsymbol{v}$ is an optimal reward scheme. Hence, if no agent exerts effort, one agent, say 1, must prefer to exert effort (otherwise $\mathbf{0}^n$ is a Nash equilibrium). Assume that each agent $k \leq i$ prefers to exert effort when the agents ranked before this agent (according to a common ranking, 1 to n without loss of generality) exert effort while the remaining agents do not. When the first i agents exert effort, none of these agents has an incentive to deviate. Hence, another agent, say $i+1$, has an incentive to exert effort, otherwise $(\mathbf{1}^i,\mathbf{0}^{n-i})$ is a Nash equilibrium. Hence, the set of constraints (UC) can be reduced to:

$$u(v_{i+1}+\hat{b}_{i+1}+\epsilon)+\theta W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i+1},\mathbf{0}^{n-i-1}),\boldsymbol{v}_{-(i+1)}+\boldsymbol{\epsilon^{n-1}}))$$
$$> u(c)+\theta W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i},\mathbf{0}^{n-i}),\boldsymbol{v}_{-(i+1)}+\boldsymbol{\epsilon^{n-1}})), \quad (46)$$

Given that the principal's objective function is linear, the optimal reward v_{i+1} is characterized either by the following condition:

$$u(v_{i+1}+\hat{b}_{i+1})-u(c)=\theta\left[W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i},\mathbf{0}^{n-i}),\boldsymbol{v}_{-(i+1)}))-W(\boldsymbol{\pi}_{-(i+1)}((\mathbf{1}^{i+1},\mathbf{0}^{n-i-1}),\boldsymbol{v}_{-(i+1)}))\right], \quad (47)$$

or by $U(\mathbf{1}^n, \boldsymbol{v}) = U(\mathbf{1}^n_{e_{i+1}=0}, \boldsymbol{v})$, which is equivalent to

$$u(v_{i+1} + \hat{b}_n) - u(c) = \theta \left[W(\boldsymbol{\pi}_{-(i+1)}(\mathbf{1}^n_{e_{i+1}=0}, \boldsymbol{v}_{-(i+1)})) - W(\boldsymbol{\pi}_{-(i+1)}(\mathbf{1}^n, \boldsymbol{v}_{-(i+1)})) \right]. \quad (48)$$

Let us denote v^*_{i+1} the solution to (47) and v'_{i+1} the solution to (48). We must have $v'_{i+1} \leq v^*_{i+1}$, which means that (47) is the binding constraint. Hence $\boldsymbol{v}$ is a global ranking scheme.□

Proof of Theorem 2: Let $\boldsymbol{v}$ be a global ranking scheme. Theorem 1 ensures that the solution has to be a global ranking scheme. Lemma 1 (see the proof) ensures that constraints (NE) are not satisfied for $i = 1, ..., n-1$. It is thus sufficient to check that each agent strictly prefers to exert effort under scheme $\boldsymbol{v} + \boldsymbol{\epsilon}$, where $\epsilon > 0$ takes on infinitesimal values, when the higher ranked agents (according to the ranking 1,...,n without loss of generality) also exert effort while the remaining agents shirk. This follows immediately from Assumption SOSC.□

Proof of Proposition 3: To show the result, it is sufficient to show that any permutation induced in the optimal ranking by the introduction of social preferences leads to a decrease in the minimum rewards within the permuted pair of agents. Assume that j beats k when altruism is not taken into account ($w_{jk} > w_{kj}$) while k beats j when altruism is accounted for ($(1 - \theta\gamma_{kj})w_{jk} < (1 - \theta\gamma_{jk})w_{kj}$). This implies that the minimum reward within the paired agents is v_k in the former situation while it is v_j in the latter. Thus, moving from a situation with no social preferences to a situation with altruism leads to a marginal change $w_{jk} - (w_{kj} + \theta\gamma_{kj}w_{jk}) < 0$ in the minimum reward.□

Proof of Proposition 4:
Sufficient condition: We show that the least cost global ranking scheme is such that the outcome in which all the agents exert effort is the unique Nash equilibrium of the effort choice game. In the proof of Proposition 5 we characterize the least cost global ranking scheme and we show that it implements full effort as a Nash equilibrium. This scheme is such that any outcome where the first i agents ($0 \leq i < n$) exert effort is not a Nash equilibrium. It remains to show that the remaining possible outcomes are not Nash equilibria.

Let us show that when the least cost global ranking scheme is implemented, agent i has an incentive to exert effort as long as $i-1$ other agents exert effort and the remaining agents shirk. Let P_{i-1} denote the set composed by the $i-1$ other agents who exert effort. The material payoff of an agent k who exerts effort when i agents exert effort is $\pi^i_k = c + (i-k)w + I_k w$. Agent i's material payoff is $\pi^i_i = c + \Delta D^i_i \geq c$.

When all agents $k \in P_{i-1}$ exert effort and all the remaining agents shirk, agent i has no incentive to deviate if and only if:

$$(1-\beta_i)\left(\pi_i^i - c\right) + \frac{i-1}{n-1}\beta_i \geq \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k\in P_{i-1}}\left(\max\{\pi_k^i - \pi_i^i, 0\} - \max\{\pi_k^i - w - c, 0\}\right)\right], \tag{49}$$

or,

$$(1-\beta_i)I_i w + \frac{i-1}{n-1}\beta_i w \geq \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k\in P_{i-1}}(\max\{(i-k+I_k-I_i)w, 0\} - \max\{(i-k+I_k-1)w, 0\})\right]. \tag{50}$$

Let $\underline{P}_i$ be the set of agents $k < i$. Notice that $-1 \leq I_k - I_i \leq 1$ and $-1 \leq I_k - 1 \leq 0$. Hence, condition (50) can be rewritten as follows:

$$(1-\beta_i)I_i w + \frac{i-1}{n-1}\beta_i \geq \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k\in P_{i-1}\cap \underline{P}_i} \max\{(i-k+I_k-I_i)w, 0\}\right]. \tag{51}$$

The term in brackets in the right hand side term takes on its maximum value when the set P_{i-1} is only composed of agents k with $k < i$. We know that condition (51) holds in this case (by definition of the global ranking scheme). This is sufficient to conclude the proof of this step.

Necessary condition: We show that an optimal reward scheme is a global ranking scheme.
An optimal reward scheme is such that, for any outcome $e \neq 1^n$, at least one agent has an incentive to deviate. Consider outcome $(0, ..., 0)$, that is, no agent does exert effort. In this case, at least one agent has an incentive to deviate and to change her choice from shirking to exerting effort (i.e. the ranking scheme property holds for this agent). Let us rank this agent first. Consider the outcome in which only the first agent exerts effort, $(1, 0, ..., 0)$. In this case, at least one agent who shirks has an incentive to deviate (i.e. the ranking scheme property holds for this agent). Let us rank this agent second. Consider outcome $(1, 1, 0..., 0)$, that is, only the first two agents exert effort. There are two possibilities here: (i) one agent who shirks has an incentive to deviate (i.e. the ranking scheme property holds for this agent) or the first agent has an incentive to deviate. Assume that the latter holds, that is, agent 1 prefers $(0, 1, 0..., 0)$ over $(1, 1, 0, ..., 0)$, then:

$$(1-\beta_1)(c - v_1 - w) - \frac{\beta_1}{n-1}w - \frac{\alpha_1+\beta_1}{n-1}(\max\{v_2 - c, 0\} - \max\{v_2 - v_1, 0\}) \geq 0. \tag{52}$$

We know that the ranking scheme property holds for agent 1 and that it is equivalent to $v_1 \geq c$. Thus, the left hand side term in condition (52) is negative, which is a contradiction.

Hence, at least one agent who shirks has an incentive to deviate. Let us rank this agent in third position.

Let us show that this result holds for any outcome such that the first j agents exert effort and the ranking scheme property holds for these agents. Using the same argument as in Step 1 of the proof of Proposition 5, we can show that the rewards of these agents are the rewards they receive under the optimal reward scheme. The payoff of these agents, when the first j agents exert effort, is denoted π_k^j with $k = 1, ..., j$ and it is thus such that $\pi_k^j \geq c + w$ for all $k < j$. Consider the outcome where the first j agents exert effort and the other agents shirk. Agent $i < j$ has an incentive to deviate if and only if:

$$(1-\beta_i)\left(c - \pi_i^j\right) - \frac{\beta_i}{n-1}(j-1)w - \frac{\alpha_1 + \beta_1}{n-1}\left(\sum_{k \leq j, k \neq i}\left[\max\{\pi_k^j - w - c, 0\} - \max\{\pi_k^j - \pi_i^j, 0\}\right]\right) \geq 0. \tag{53}$$

We have $\pi_i^j \geq c + w$ and then the left hand side term in condition (53) is negative, which is a contradiction. We consider a unique implementation scheme here, hence at least one agent who shirks has an incentive to deviate. Let us rank this agent in position $j+1$. This concludes the proof of our claim and of the result.□

Proof of Proposition 5:

Notice that agent i's utility function can be rewritten as follows:

$$U_i(e, v) = (1-\beta_i)\pi_i(e, v) + \frac{\beta_i}{n-1}\sum_{k \neq i}\pi_k(e, v) - \frac{\alpha_i + \beta_i}{n-1}\sum_{k \neq i}\max\{\pi_k(e, v) - \pi_i(e, v), 0\}. \tag{54}$$

The divide and conquer property holds if, for a given ordering of the agents $1, ..., n$, each agent prefers to exert effort when all the preceding agents exert effort and the remaining agents shirk. Using (54), we can show that this condition holds for agent $i = 1$ if and only if $v_1 \geq c$ and for $i \geq 2$ if and only if:

$$\begin{aligned}(1-\beta_i)\left[v_i + (i-1)w - c\right] + (i-1)\frac{\beta_i}{n-1}w \geq \\ \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k - v_i, 0\} - \max\{v_k + (i-2)w - c, 0\}\right)\right] \\ + \frac{\alpha_i + \beta_i}{n-1}(n-i)\max\{c - v_i - (i-1)w, 0\}\end{aligned} \tag{55}$$

Step 1: We show that, the least cost scheme that is characterized by the global ranking scheme property is such that (5) is binding for all i.

Assume that there exists an agent i such that (55) is not binding. This implies that we have to solve a problem with $n-1$ inequalities and n unknowns. Thus, there exists an agent l such that $v_l = -\infty$. We can thus rewrite condition (55) for this agent as follows:

$$\left(1-\beta_l+\frac{n-l}{n-1}\left(\alpha_l+\beta_l\right)\right)\left(v_l+(l-1)w-c\right)+\frac{l-1}{n-1}\left(\alpha_l+\beta_l\right)v_l \geq \frac{\alpha_l+\beta_l}{n-1}\sum_{k<i}\left[\left(v_k-\max\{v_k+(l-2)w-c,0\}\right)-\frac{\beta_l}{n-1}w\right] \quad (56)$$

This implies that there exists an agent $l' < l$ such that $v_{l'} = -\infty$. Reiterating this reasoning, we can conclude that $v_1 = -\infty$, which is a contradiction since $v_1 \geq c$. We conclude that the least cost global ranking scheme is characterized by $v_1 = c$ and:

$$\left(1-\beta_i\right)\left[v_i+(i-1)w-c\right]+(i-1)\frac{\beta_i}{n-1}w = \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k-v_i,0\}-\max\{v_k+(i-2)w-c,0\}\right)\right] + \frac{\alpha_i+\beta_i}{n-1}(n-i)\max\{c-v_i-(i-1)w,0\}, \quad (57)$$

for all $i \geq 2$.

Step 2: We show that, the optimal reward scheme that is characterized by the divide and conquer property is such that $v_i = c-(i-1)w+\Delta D_i^i$ for all i.

After some computations, condition (57) can be rewritten as follows:

$$\left[1-\beta_i+\left(\alpha_i+\beta_i\right)\frac{i-1}{n-1}\right]\left(v_i+(i-1)w-c\right) = \frac{\alpha_i+\beta_i}{n-1}\left[\sum_{k<i}\max\{v_i-v_k,0\}\right]+\alpha_i\frac{i-1}{n-1}w \quad (58)$$

Thus, in order to minimize the cost of the scheme, the principal chooses to rank the agents such that $v_i \leq v_k$ for all $k < i$ and $v_i = c-(i-1)w+\Delta D_i^i$ for all i. Notice that $v_i - v_{i-1} = \left(I_i - I_{i-1} - 1\right)w$, which is non positive if and only if $I_i - I_{i-1} \leq 1$, this is always true since $I_k \leq 1$ for all k.

Step 3: We show that the reward scheme described in the Proposition is such that the situation where all the agents exert effort is a Nash equilibrium of the effort choice game.

The reward scheme in the Proposition exhibits the global ranking scheme property. Hence, agent n has no incentive to deviate from the situation where all the agents exert effort. Let us consider agent $i < n$. Let us denote π_k^n agent k's material payoff when all agents exert effort. We have $\pi_k^n = c+(n-k)w+I_k w$. The difference between two successive terms is $\pi_{k+1}^n - \pi_k^n = \left(I_{k+1}-I_k-1\right)w \leq 0$. When agent i deviates from the situation where all the agents exert effort, agent k's material payoff ($k \neq i$) is $\pi_k^n - w$. This payoff is larger than c. Indeed, we have $\pi_k^n - w - c = (n-k-1)w + I_k w \geq 0$ if and only if $k < n$.

Using these remarks, agent $i < n$ has no incentive to deviate from the situation where all the agents exert effort if and only if:

$$(1-\beta_i)\left(\pi_i^n - c\right) + \beta_i w + \frac{\alpha_i + \beta_i}{n-1}(i-1)\left(\pi_i^n - w - c\right) + \frac{\alpha_i + \beta_i}{n-1}\sum_{i<k<n}\left(\pi_k^n - w - c\right) \geq 0 \quad (59)$$

This concludes the proof of Step 3.

Step 4: It remains to show that increasing each reward by an arbitrarily small amount makes the reward scheme a unique implementation scheme.

One can show that the scheme characterized in Proposition 5 satisfies this definition. Consider that agent i's reward is $v_i^* + \epsilon$, where $\epsilon > 0$ is arbitrarily small. First, let us show that each agent (strictly) prefers to exert effort when all the preceding agents exert effort and the remaining agents shirk. For agent $i = 1$, we have $v_1^* + \epsilon = c + \epsilon > c$. For $i \geq 2$, the needed condition is:

$$\begin{aligned}(1-\beta_i)\left[v_i^* + \epsilon + (i-1)w - c\right] + (i-1)\frac{\beta_i}{n-1}w > \\ \frac{\alpha_i + \beta_i}{n-1}\left[\sum_{k<i}\left(\max\{v_k^* - v_i^*, 0\} - \max\{v_k^* + \epsilon + (i-2)w - c, 0\}\right)\right] \\ + \frac{\alpha_i + \beta_i}{n-1}(n-i)\max\{c - v_i^* - \epsilon - (i-1)w, 0\} \quad (60)\end{aligned}$$

First, notice that $v_k^* + \epsilon + (i-2)w - c = (I_k - 1)w + \epsilon < 0$ because $I_k - 1 < 0$ and ϵ is arbitrarily small. Second, notice that $c - v_i^* - \epsilon - (i-1)w = -I_i - \epsilon < 0$. Hence, condition (60) is equivalent to $(1-\beta_i)\epsilon > 0$, which is satisfied. Hence, for each outcome except the outcome where all the agents exert effort, at least one agent strictly prefers to deviate.

Moreover, the outcome in which all agents exert effort is a (strict) Nash equilibrium. It is sufficient to notice that condition (59) holds with a strict inequality when adding ϵ to each reward.□

Proof of Lemma 2:

If $i = 1$, we have

$$\pi_1^1 - c = D_1^1 - D_1^0 = D_1^1 = \alpha_1 \max\{c - \pi_1^1, 0\} + \beta_1 \max\{\pi_1^1 - c, 0\}. \quad (61)$$

Hence, we must have $\pi_1^1 - c \geq 0$.

Now, assume that $\pi_k^k \geq c$ for all $k < i$ while $\pi_i^i < c$. Hence, we have $\pi_i^i < c = \pi_j^i \leq \pi_k^i$ for $k < i$ and $j > i$, and $\pi_i^{i-1} = c = \pi_j^{i-1} \leq \pi_k^k \leq \pi_k^{i-1}$. Hence, in this case $B_i^i = B_i^{i-1} = 0$ and:

$$A_i^i - A_i^{i-1} = \frac{1}{n-1} \sum_{k<i} \left[(\pi_k^i - \pi_i^i) - \max\{\pi_k^i - w - c, 0\}\right] + \frac{1}{n-1} \sum_{k>i} \left[(c - \pi_i^i) - 0\right] > 0. \quad (62)$$

This is sufficient to prove the result.□

Proof of Proposition 6: Proof of part (i) results from the fact that $I_i \geq 0$. To prove part (ii), notice that we have $v_1 \geq v_2 \geq ... \geq v_n$ and $\pi_i - \pi_{i+1} = (1 + I_i - I_{i+1})\, w$. Moreover, when $\alpha_k = \beta_k = 0$ for $k = i, i+1$, we have $\pi_i - \pi_{i+1} = w$. Hence it is sufficient to show that $I_i - I_{i+1} \leq 0$. One can easily show that this is equivalent to $-(n-1) - (n-i)\beta + (i-1)\beta \leq 0$, which is always true because $\beta < 1$.□

Proof of Proposition 7: Let us prove point (i). We have $\alpha_k = \alpha$ and $\beta_k = \beta$ for all k. We can show that:

$$\frac{\partial\, |\pi_i - \pi_{i+1}|}{\partial \alpha} = \frac{(n-1)(1-\beta)\left[i(i-1)\alpha^2 - (n-1-(n-i)\beta)\right]}{\left(n-1+(i-1)\alpha-(n-i)\beta\right)^2 \left(n-1+i\alpha-(n-i-1)\beta\right)^2} w, \quad (63)$$

which is positive if and only if $\alpha \geq \sqrt{\frac{n-1-(n-i)\beta}{i(i-1)}}$ and $i \geq 2$.

Now we prove point (ii). We have $\alpha_k = \alpha$ and $\beta_k = \beta$ for all k. We can show that:

$$\frac{\partial\, |\pi_i - \pi_{i+1}|}{\partial \beta} = \frac{-(n-i)(n-i-1)(n-1)(\beta^2+2\beta) + (i-1)(n-i)(n-1+i\alpha)^2 - i(n-i-1)(n-1+(i-1)\alpha)^2}{\left(n-1+(i-1)\alpha-(n-i)\beta\right)^2 \left(n-1+i\alpha-(n-i-1)\beta\right)^2} \alpha w, \quad (64)$$

which is positive if and only if $i = n-1$ or $\beta \in \left[1 - \sqrt{\frac{i(i-1)}{(n-i)(n-i-1)}}(1+\alpha), 1\right[$.□

Proof of Proposition 8: To prove the result, we can focus on the function $I(i, \alpha) \equiv \frac{(i-1)\alpha}{n-1+(i-1)\alpha-(n-i)\beta}$ and its cross derivative with respect to i and α. It is easily checked that the cross derivative has the same sign as $n - 1 + \alpha - n\beta - i(\alpha - \beta)$. This expression is a linear function that decreases when i increases and takes on value $(n-1)(1-\beta) > 0$ when $i = 1$ and $(n-1)(1-\alpha)$ when $i = n$. Hence, if $\alpha < 1$, the cross derivative is always positive. If $\alpha \geq 1$, the cross derivative is positive when i lies below a threshold $1 < k < n$ and negative when i lies above this threshold.□

Proof of Proposition 9: To prove the result, we can focus on the function $I(i, \beta) \equiv \frac{(i-1)\alpha}{n-1+(i-1)\alpha-(n-i)\beta}$ and its cross derivative with respect to i and β. It is easily checked that the cross derivative has the same sign as $1+\alpha+(1-\beta)n-i(\alpha-\beta+2)$. This expression is a linear function that decreases when i increases and takes on value $(n-1)(1-\beta)>0$ when $i=1$ and $-(1+\alpha)(n-1)<0$ when $i=n$. Hence, the cross derivative is positive when i lies below a threshold $1<k<n$ and negative when i lies above this threshold.□

Proof of Proposition 11:
Notice that agent i's utility function can be rewritten as follows:

$$U_i(e,v)=(1-\beta_i)\pi_i(e,v)+\frac{\beta_i}{n-1}\sum_{k\neq i}\pi_k(e,v)-\frac{\alpha_i+\beta_i}{n-1}\sum_{k\neq i}\max\{\pi_k(e,v)-\pi_i(e,v),0\}, \quad (65)$$

The outcome in which all agents exert effort is a Nash equilibrium of the induced effort choice game if and only if we have, for any $i\in N$:

$$(1-\beta_i)\left[v_i+(n-1)w-c\right]+\beta_i w-\frac{\alpha_i+\beta_i}{n-1}\sum_{k\neq i}\left[max\{v_k-v_i,0\}-max\{v_k+(n-2)w-c,0\}\right]\geq 0 \quad (66)$$

We claim that the optimal reward scheme must satisfy $v_k+(n-2)w-c\leq 0$ for all $k\in N$. Assume that there is (at least) one agent for which this does not hold. Let us denote $\bar{N}\subset N$ such that $\forall k\in\bar{N}$ we have:

$$v_k+(n-2)w-c>0$$

while any $j\notin\bar{N}$ is such that $v_j+(n-2)w-c\leq 0$ is satisfied. This implies that $\max_{j\notin\bar{N}}v_j<\min_{k\in\bar{N}}v_k$ is satisfied. Thus, ranking all agents in decreasing order with respect to their rewards, if $|\bar{N}|=m$ then subset $\bar{N}$ includes exactly the first m agents in this ranking $v_1\geq ...\geq v_n$. Moreover, by definition: $\forall j\geq m+1$ we know that $v_j+(n-2)w-c\leq 0$ is satisfied.

Let us consider the first agent in the ranking. This agent's related participation constraint is:

$$(1-\beta_1)\left[v_1+(n-1)w-c\right]+\beta_1 w+\frac{\alpha_1+\beta_1}{n-1}\sum_{k=2}^{m}\left[v_k+(n-2)w-c\right]\geq 0 \quad (67)$$

This constraint is actually vacuous as v_1 satisfies $v_1>c-(n-2)w$ by definition of $\bar{N}$, while the above inequality can be rewritten as

$$v_1\geq c-(n-1)w-\frac{1}{1-\beta_1}\left[\beta_1 w+\frac{\alpha_1+\beta_1}{n-1}\sum_{k=2}^{m}\left[v_k+(n-2)w-c\right]\right]$$

and it is easily checked that the following inequality is satisfied:

$$c-(n-2)w>c-(n-1)w-\frac{1}{1-\beta 1}\left[\beta_1 w+\frac{\alpha_1+\beta_1}{n-1}\sum_{k=2}^{m}\left[v_k+(n-2)w-c\right]\right]$$

In other words, $v_1+(n-2)w-c>0$ is the relevant constraint: compared to the benchmark situation where $v_1 + (n-2)w - c \leq 0$ would hold, this agent's reward has increased. Now, looking at the second agent in the ranking, his participation constraint is:

$$(1-\beta_2)\left[v_2 + (n-1)w - c\right] + \beta_2 w - \frac{\alpha_2+\beta_2}{n-1}\left[v_1 - v_2\right] + \frac{\alpha_2+\beta_2}{n-1}\sum_{k\in\bar{N},k\neq 2}\left[v_k + (n-2)w - c\right] \geq 0 \tag{68}$$

or

$$(1-\beta_2)\left[v_2 + (n-1)w - c\right] + \beta_2 w + \frac{\alpha_2+\beta_2}{n-1}\sum_{k\in\bar{N},k\neq 1}\left[v_k + (n-2)w - c\right] \geq 0 \tag{69}$$

Finally, we can rewrite this inequality as:

$$v_2 \geq c - (n-1)w - \frac{1}{1-\beta_2}\left[\beta_2 w + \frac{\alpha_2+\beta_2}{n-1}\sum_{k\in\bar{N},k\neq 1}\left[v_k + (n-2)w - c\right]\right]$$

and it is easily checked that the following inequality is satisfied:

$$c - (n-2)w > c - (n-1)w - \frac{1}{1-\beta_2}\left[\beta_2 w + \frac{\alpha_2+\beta_2}{n-1}\sum_{k\in\bar{N},k\neq 1}\left[v_k + (n-2)w - c\right]\right]$$

In other words, $v_2 + (n-2)w - c > 0$ is the relevant constraint: compared to the benchmark situation where $v_2 + (n-2)w - c \leq 0$ would hold, this agent's reward has also increased. A similar reasoning allows one to quickly conclude that the rewards of all agents in $\bar{N}$ have increased compared to the benchmark situation.

Now, for agent $m+1$ the participation constraint is:

$$(1-\beta_{m+1})\left[v_{m+1} + (n-1)w - c\right] + \beta_{m+1} w - \frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k - v_{m+1}\right]$$

$$+ \frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k + (n-2)w - c\right] \geq 0 \tag{70}$$

or

$$(1-\beta_{m+1})\left[v_{m+1} + (n-1)w - c\right] + \beta_{m+1} w + \frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_{m+1} + (n-2)w - c\right] \geq 0 \tag{71}$$

Finally, we can rewrite this inequality as:

$$(1-\beta_{m+1})\left[v_{m+1} + (n-1)w - c\right] + \beta_{m+1} w \geq \frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[c - (n-2)w - v_{m+1}\right] \tag{72}$$

while we would have, in the benchmark situation, the following inequality:

$$(1-\beta_{m+1})\left[v_{m+1}+(n-1)w-c\right]+\beta_{m+1}w \geq \frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+1}\right] \tag{73}$$

Keep in mind that, in the benchmark situation, we have $v_k+(n-2)w-c \leq 0$ for any $k \in \bar{N}$, and we then easily check that:

$$\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+1}\right]-\frac{\alpha_{m+1}+\beta_{m+1}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+1}\right]=\sum_{k\in\bar{N}}\left[v_k+(n-2)w-c\right]$$

Since the final term is non-positive in the benchmark situation, this inequality implies that, compared to the benchmark situation, agent $m+1$'s reward has at best remained the same.

Finally, for agent $m+2$, we obtain the following participation constraint:

$$(1-\beta_{m+2})\left[v_{m+2}+(n-1)w-c\right]+\beta_{m+2}w \geq \frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+2}\right]$$
$$+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{74}$$

while we would have, in the benchmark situation, the following inequality:

$$(1-\beta_{m+2})\left[v_{m+2}+(n-1)w-c\right]+\beta_{m+2}w \geq \frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{75}$$

Again, in the benchmark situation, we have $v_k+(n-2)w-c \leq 0$ for any $k \in \bar{N}$, and we then easily check that:

$$\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[v_k-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right]$$
$$\leq \frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\sum_{k\in\bar{N}}\left[c-(n-2)w-v_{m+2}\right]+\frac{\alpha_{m+2}+\beta_{m+2}}{n-1}\left[v_{m+1}-v_{m+2}\right] \tag{76}$$

This inequality implies that, compared to the benchmark situation, agent $m+2$'s reward has at best remained the same. A similar reasoning applies to any agents who do not belong to $\bar{N}$. To summarize, compared to the benchmark situation, the rewards of all agents in $\bar{N}$ have increased and those of non-members of $\bar{N}$ have at best remained the same. This contradicts the fact that this scheme is least cost, and we conclude by contradiction that $v_k+(n-2)w-c \leq 0$ for all $k \in N$.

Now, the agent (say, agent 1) whose reward is the highest has to satisfy the following constraint:

$$(1-\beta_1)\left[v_1+(n-1)w-c\right]+\beta_1 w \geq 0 \tag{77}$$

To induce the lowest reward, this condition must be satisfied as an equality. Then, for the agent whose reward is the ith highest one, the following constraint must be satisfied:

$$(1-\beta_i)\left[v_i+(n-1)w-c\right]+\beta_i w-\frac{\alpha_i+\beta_i}{n-1}\sum_{k<i}\left[v_k-v_i\right] \geq 0 \tag{78}$$

To induce the lowest reward, this condition must be satisfied as an equality. Solving the resulting system of $(n-1)$ equalities as functions of $(v_2,...,v_n)$, we obtain the desired expressions.□

Proof of Proposition 12: We have:

$$v_1-v_2=\frac{\beta-(1-\beta)J_1}{1-\beta+\frac{\alpha_2+\beta}{N-1}}=0 \tag{79}$$

Then, using the expression of v_i for $i \geq 2$ it is easily checked that $v_i = v_{i-1}$ as $J_i = J_{i-1} = J_1$ is satisfied.□

Proof of Proposition 13: We obtain after straightforward computations:

$$v_{i-1}-v_i=\frac{(\beta_i-\beta_{i-1})+\frac{i-2}{n-1}(\beta_i-\beta_{i-1})\alpha+\frac{(\beta_i-\beta_{i-1})(1+\alpha)}{n-1}\left[\sum_{j=1}^{i-2}J_j\right]}{\left[1-\beta_i+\frac{i-1}{n-1}(\alpha+\beta_i)\right]\left[1-\beta_{i-1}+\frac{i-2}{n-1}(\alpha+\beta_{i-1})\right]} \tag{80}$$

We conclude that $v_{i-1} \geq v_i$ if and only if $\beta_i \geq \beta_{i-1}$ is satisfied.□

Proof of Proposition 14: Looking at the optimal ranking satisfying $v_{i-1} \geq v_i$ for any $i \in N$, we first obtain:

$$v_1 \geq v_2 \Longleftrightarrow \frac{\beta_2+\frac{\alpha_2+\beta_2}{n-1}J_1}{1-\beta_2+\frac{\alpha_2+\beta_2}{n-1}} \geq J_1 \tag{81}$$

which, after simplification, is equivalent to:

$$\beta_2 \geq (1-\beta_2)J_1=(1-\beta_2)\frac{\beta_1}{1-\beta_1} \tag{82}$$

or

$$\frac{\beta_2}{1-\beta_2} \geq \frac{\beta_1}{1-\beta_1} \tag{83}$$

which is equivalent to $\beta_2 \geq \beta_1$. We now move on to the case where $i \geq 3$ holds, and we have:

$$v_{i-1} \geq v_i \Longleftrightarrow \frac{\beta_i+\frac{\alpha_i+\beta_i}{n-1}\left[\sum_{l=1}^{i-1}J_l\right]}{1-\beta_i+(i-1)\frac{\alpha_i+\beta_i}{n-1}} \geq J_{i-1} \tag{84}$$

Rewriting and simplifying the right hand side term of this equivalence, then using the expression of J_{i-1} as a function of $\sum_{l=1}^{i-2}$, we conclude that $v_{i-1} \geq v_i$ is equivalent to:

$$(\beta_i - \beta_{i-1}) + \frac{\sum_{l=1}^{i-2} J_l}{n-1} \left[(\alpha_i + \beta_i) - (\alpha_{i-1} + \beta_{i-1}) - \beta_{i-1}\alpha_i + \beta_i\alpha_{i-1}\right] + \frac{i-2}{n-1} \left[\beta_i\alpha_{i-1} - \beta_{i-1}\alpha_i\right] \geq 0. \tag{85}$$

Now, if agents' aversion to inequality parameters are such that there exists a ranking satisfying $\frac{\beta_i}{\beta_{i-1}} > \frac{\alpha_i}{\alpha_{i-1}} \geq 1$ for any $i \geq 3$ then it satisfies $\alpha_i \geq \alpha_{i-1}$ and $\beta_i > \beta_{i-1}$ for any $i \geq 3$, and it satisfies condition (32).

Finally, assuming that this type of ranking does not exist, we provide an example where the optimal ranking necessarily satisfies $\alpha_i < \alpha_{i-1}$ for some $i \in N$. Indeed, let us consider $n = 3$: we know that the agent ranked first satisfies $\beta_1 \leq \beta_2$, and we here assume that this inequality is strict. Then we consider a situation where $\beta_2 = \beta_3 = \beta > \beta_1$. Using condition (32) for $i = 3$ yields that $v_2 \geq v_3$ is equivalent to:

$$\beta + (1-\beta)\frac{\alpha_3 + \beta}{n-1} J_1 + \frac{\beta}{n-1}\alpha_2 \geq \beta + (1-\beta)\frac{\alpha_2 + \beta}{n-1} J_1 + \frac{\beta}{n-1}\alpha_3 \tag{86}$$

Rewriting, we obtain:

$$\frac{(1-\beta)\beta_1}{(n-1)(1-\beta_1)} (\alpha_3 - \alpha_2) \geq \frac{\beta}{n-1} (\alpha_3 - \alpha_2) \tag{87}$$

If $\alpha_3 \geq \alpha_2$ this inequality is easily checked to be equivalent to $\beta_1 \geq \beta$, which contradicts our initial finding. As such, we conclude that $\alpha_3 < \alpha_2$ must necessarily hold.□

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