Confronting the Carbon Pricing Gap: Second Best Climate Policy

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Abstract

Confronted with political opposition to the implementation of efficient direct carbon pricing, climate policy relies on alternative policy interventions, such as subsidies to renewables. This paper uses a dynamic macroeconomic model under a carbon budget to study climate policies constrained to keeping a constant level of the carbon tax. We find that it is possible to implement the optimal trajectory by combing an increasing tax on electricity consumption with a feed-in-premium paid to electricity produced from renewable sources. Otherwise, when the climate policy relies on the second instrument only, the subsidy to renewables should be so large to foster rapid build up of specialized capital, that it would imply large investment costs and financial burden on the public budget, unless the carbon tax level could be initially set at a high level. Unfortunately, the two solutions with no or low welfare losses raise concerns on their political acceptability too.

JEL codes: Q54, H2, Q4

Key words: Energy transition; Carbon tax; Renewable energy; Policy acceptability.

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1 Introduction

There is a wide consensus among economists to support the carbon tax and other market-based policy instruments forcing firms and consumers to take into account the externalities from their emissions of greenhouse gases.[1] As illustrated in Figure **??**, an overview of the direct pricing of carbon emissions around the globe makes clear that there is a substantial gap between the suggested and the observed price of carbon (OCDE, 2021). Carbon pricing is getting more and more adopted in the world (World Bank, 2021), and emission trading efficiency has improved, as can be seen with the ETS price reaching nearly €90 per ton of CO_2 in December 2021. However, in several countries, the announcement of an increasing carbon tax has received strong political opposition (e.g. Douenne and Fabre, 2022). Hence, in most cases mitigation policies do not rely exclusively or even mainly on pricing carbon emissions. They typically include implicit or explicit subsidies (Hoel, 2020 or Meckling, Sterner and Wagner, 2017). Governments may combine these tools to target specific objectives, such as limiting emissions within a carbon budget, or ensuring fossil phase out by some target date.

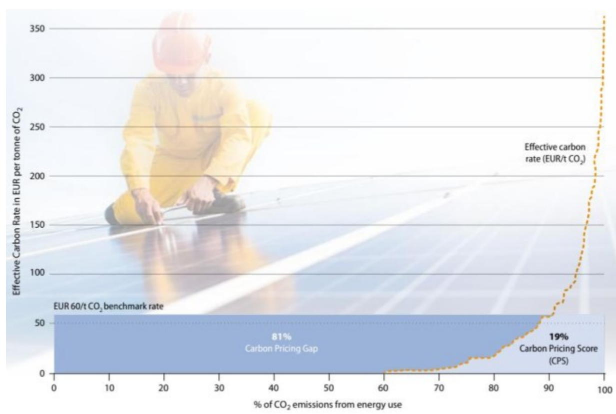

Figure 1: The carbon pricing score 2018 (OECD, 2021).

In this paper, we account for this political economy constraint and consider the case where the regulator is only able to charge a constant carbon tax, insufficiently high to ensure, on its own, that the carbon budget is respected. In a second best framework, we derive the additional instrument that fills the policy gap while maximizing welfare. Therefore, this paper contributes to the literature on the issue of climate policy instrument design (reviews are provided by Goulder and Parry, 2008 and Phaneuf and Requate, 2017) by adding the acceptability dimension embedded in this choice. Our analytical approach also links this article to the vast literature that considers the

[1] See for instance the 2021 statement on carbon pricing by the European association of environmental and resource economists at `https://www.eaere.org/statement/`.

problem of intertemporal allocation of scarce natural resources (Hotelling, 1931), and its extension to climate change issue with Chakravorty et al. (2006) that considers as policy objective a ceiling on carbon concentration: in order to avoid climate-related catastrophic outcomes, atmospheric carbon concentration shall stay below a ceiling. Finally, this paper makes a contribution to the current "equivalent" climate policy debate by providing a ranking of the policy instruments that can fill the pricing gap.

Tackling the climate change issue requires an energy transition, for which producing energy by renewable sources plays an important role. The production of renewable electricity has grown rapidly in recent years and at the end of 2020, the estimated renewable energy share in global electricity production is 27% (see IEA, 2020). However, renewables cannot be deployed at the required pace as they are still more costly on average than fossil fuels, and, on the other hand, they are non-dispatchable and are not continuously available. The last issue can be solved using storage solutions (for instance batteries for the short run and hydrogen electrolysis and methanation for the longer run) at the cost of even more expensive electricity systems. Hence, public policy is needed to incentivize investment in the renewable electricity infrastructures. We focus on the implementation of such a policy, by accounting for the political economy constraint that prevents reaching the first best solution.

We use a stylized dynamic model of the optimal choice of the electricity mix (fossil and renewable), where fossil energy is abundant and CO_2-emitting, while renewable energy is clean. Producing energy from renewable sources requires specialized capital: "green capital". The latter is composed of solar panels, wind turbines and the equipment necessary to make renewable energy dispatchable. Fossil, hydro, wind and solar energy are assumed to be available at zero variable cost. It is also assumed that, at the beginning of the planning horizon, fossil-fired plants already exist, so that there is no capacity constraint on carbon intensive electricity generation. On the contrary, existing solar capacity is small and requires costly investment in green capital.

The framework is a simple version of Pommeret and Schubert (2022) and Pommeret, Ricci, Schubert (2022). The former focuses on the intermittency problem and storage devices. The latter considers the scarcity of the raw materials for investing in green capital. There we also study a case where the policy design is constrained. In the present paper, we abstract from the material content of green capital, in order to be able to conduct an in-depth analysis of climate policy under a carbon pricing gap due to political constraints. We implement a rigorous approach to second-best policy design and use it for a larger scope of policy instruments and their combinations.

The optimal energy transition does not exceed a given carbon budget while maximizing the present value of the social value of electricity consumption net of investment cost. It solves the trade-off between, on the one hand, fossil energy which is expensive to use in terms of CO_2 emissions and, on the other hand, renewable energy which is expensive because it requires costly investment in green capital. The solution characterizes a path of fossil resource exhaustion, including the fossil phase out date, and the path of investment in green capital.

Turning to policy design, we consider the decentralized version of the economy, where the only market failure is climate change. A carbon tax increasing at the social discount rate would solve for this market failure and guarantee that the economy reaches the first best. However, we assume that the regulator cannot commit to levy such a tax as it is not accepted by the population. To account for this acceptability issue, we consider the case where the regulator is only able to charge a constant carbon tax, that is not sufficient to ensure, on its own, that the carbon budget is respected. To fill this carbon pricing gap, an additional public policy needs to be implemented that can take the form of a subsidy to renewable energy or a tax on electricity consumption. We consider two possible subsides: a feed-in-premium for electricity from renewable sources, or a subsidy to investment in renewable capacity. We study the required dynamic profile of each of the different instruments, and their combinations, in order to meet the carbon budget constraint. We analyze the consequences in terms of timing of the energy transition and we compute the welfare cost of each second best (suboptimal) policy in order to provide recommendations regarding the best way to fill the carbon pricing gap.

The policy instruments we consider as alternatives or complement to efficient carbon pricing, are representative of real-world policy interventions. In several European countries, electricity consumption is severely taxed, for various reasons including explicit goals to reduce it (e.g. the TEE market in Italy[2]) and to finance subsidies to renewables (e.g. the EEG Umlage in Germany[3]. Concerning subsidies to investment, investment in capacity for renewable electricity production is directly subsidized in the US through tax credits on the federal budget.[4] Several schemes have been adopted to subsidize the production of electricity from renewable sources. In Germany feed-in-tariffs (FiT) were introduced in 1991, requiring the distribution power companies to buy for 20 years power from non-utility scale new renewable producers at a fixed proportion of the wholesale electricity price, then –from 2001 and up to 2012– at a constant seller price. In the UK, public authorities signed contracts for differences (CfD) with renewable (and nuclear) power producers promising future payments to cover the difference between the price on the wholesale electricity market and the contractually predetermined tariff, in the event that the latter is above the former, and vice-versa, allowing the producer to be facing a fixed net seller price. In France, public authorities use the *complement de remuneration*, i.e. a feed-in-premium (FiP): a variant of the CfD whereby the transfer can become nil in the event of low demand for electricity (close to zero wholesale

[2]The Italian market of white certificates (Titoli di Efficienza Energetica) aims at subsidizing energy efficiency investment, but increases the marginal cost for power suppliers (`https://www.mise.gov.it/index.php/it/incentivi/energia/certificati-bianchi`

[3]According to the the renewable sources act of 2000 the cost of subsidizing investment in new renewable production capacity do not involve public finance, since it is temporarily paid by grid-operators, that in turn charge it –based on cost-pus regulation– on the purchaser price for electricity consumers (except for power intensive industries) (see `https://www.diw.de/de/diw_01.c.411881.de/presse/glossar/eeg_umlage.html` or `https://doi-org.ezpum.scdi-montpellier.fr/10.1016/S0038-092X(00)00144-4`)

[4]The Build Back Big package proposed by the Biden administration includes the extension of such tax credit schemes.

price). Alternative instruments require distributors to comply with a minimum volume or share of renewable energy in their sales, as for instance renewable portfolio standards or clean energy standards in the US.

In the literature, the design of second best climate policy relying on policy instruments other than carbon pricing, has been mostly analyzed through the lenses of industrial organization applied to energy and environmental economics (e.g. Requate, 2015, for an early review Bennear and Stavins, 2007). In most cases, these analyses encompass additional market failures on top of the externatilty for greenhouse gas emissions,[5] and consider that the policymaker cannot freely manage as many instruments as there are market failures to be corrected.[6] In this vein, Fischer et al. (2021) establish the optimal adjustment of the available free tools to compensate for the constrained ones, and evaluate the implied welfare loss with respect to the unfeasible first best reference point. Second best climate policy is also analyzed using quantitative prospective modeling of the energy transition scenarios. For instance, Stock and Stuart (2021) use a partial equilibrium dynamic model of the electricity sector in the US and consider different combinations of the three main instruments of the bill proposed by the administration: clean energy standards, tax credits to investment in low carbon technologies, and a carbon tax. Moreover, the second-best policy approach is at the heart of the double-dividend hypothesis, due to interactions between environmental regulation and preexisting distortions due to the fiscal system in static general equilibrium settings (e.g. Goulder et al., 1997). The general equilibrium approach has been extended to consider the endogenous dynamics resulting from accumulation or innovation, namely in integrated assessment models that quantitatively appraise the welfare cost of relying on subsidies to renewables rather than on carbon pricing. Kalhkul et at (2013) show that the welfare costs of renewable energy subsidies are multiple times higher than first-best mitigation costs under a carbon price policy. This is mainly due to their inability to crowd out fossil fuels in a timely manner. Results further worsen in case of –even small– deviations from the second best optimum. In addition, Rezai and van der Ploeg (2017) stress that the welfare costs also significantly increase in case of lack of credibility of second best policies.[7] In as much we also rely on a dynamic general equilibrium analysis, these latter two articles are closest to our. However, we focus on a unique market failure and propose an original approach to derive analytical results characterizing the trajectories.

The structure of the paper is the following. Section 2 presents the model of optimal energy transition. In Section 3, we consider the decentralized equilibrium and analyze the first best policy.

[5]Relevant for climate policies are those in the energy sector, aimed at coping with positive dynamic externalities from intentional investment to promote innovation or from learning-by-doing, or externalities from energy efficiency investment due to imperfect appropriation of the private returns or behavioral and information biases.

[6]Another typical issue is the one on overlapping regulations, as for instance for the interaction between national policies and the EU cap and trade CO_2 scheme (e.g. Böhringer and Rosendahl, 2010).

[7]Concerning renewable electricity quotas, Goulder et al (2016) compare the Clean Energy Standards to carbon cap and trade regulations using a static general equilibrium framework. Amigues et al (2020) consider the case of renewable portfolio standards in a dynamic general equilibrium setting with a carbon ceiling, comparing the floor to the share requirement regulatory approach.

The case of a carbon tax constrained to be constant is studied in section 4. Section 5 concludes.

2 Optimal energy transition

We begin by presenting the main assumptions on preferences, technology and resource constraints, then characterize the optimal paths for the economy when the production capacity of electricity from renewable sources is initially low.[8]

We consider a closed economy with a representative household, and study it in continuous time. The consumption of electricity, $e(t)$, results from two sources, fossil or renewables, considered as perfect substitutes by the consumer. Electricity is produced with a linear technology, either from the combustion of a flow of fossil resources, $x(t)$, or from the use of "green" capital, $Y(t)$, a stock of specialized equipment for renewable energy generation and storage:

$$e(t) = x(t) + \phi Y(t) \tag{1}$$

with $\phi > 0$ a measure of the efficiency of green electricity generation.

While the production of fossil resources entails no direct cost, their use implies carbon emissions, which accumulate in the atmosphere:

$$\dot{X}(t) = x(t) \tag{2}$$

where $X(t)$ denote cumulative carbon emissions and we normalize the ε is the emission coefficient to unity. Climate policy aims at limiting the temperature increase, compared to the pre-industrial equilibrium temperature. It is represented by a targeted carbon budget $\overline{X}$, i.e. the maximal amount of cumulative emissions compatible with the chosen temperature objective.

The stock of green capital depreciates at the constant rate $\delta \in (0, 1)$. It evolves with specific investment, $I(t)$, as follows:

$$\dot{Y}(t) = I(t) - \delta Y(t) \tag{3}$$

Investment inputs are available at no cost, but investment entails capital adjustment costs, $C(I(t))$, assumed increasing and strictly convex (i.e. C', $C'' > 0$).

Denoting by Y_0 the initial endowment in green capital, we can write the set of constraints that apply to the flow and stock variables as:

$$X(t) \leq \overline{X}, \quad x(t) \geq 0, \quad X(0) = 0 \text{ and } Y(0) = Y_0 \geq 0 \text{ given} \tag{4}$$

[8] In essence, the analysis in this section is a special case of Pommeret and Schubert (2022) and of Pommeret et al. (2022).

The representative household is characterized by an instantaneous utility function that is assumed to be concave in electricity consumption, according to $u(e(t))$, with $u' > 0$ and $u'' < 0$, and quasi-linear in the generic consumption good. He applies a constant discount rate, $\rho > 0$.

The benevolent social planner seeks to maximize society's net surplus, generated by power consumption net of the investment costs:

$$\max \int_0^\infty e^{-\rho t}[u(e(t)) - C(I(t))]dt \tag{5}$$

subject to the technology and resource constraints (1)-(3), as well as the non-negativity and carbon budget constraints for given initial conditions (4).

In order to be able to characterize the solution to this optimization problem, we define the costate variables associated to the stocks: $\mu(t)$ the value of the green capital, and $\lambda(t)$ the carbon value.

We can now characterize the optimal energy transition.

PROPOSITION 1. Optimal energy transition. *For a small green capital endowment, the optimal path of the economy, solving the program given by the objective function (5), under the technology and resource constraints (1)-(3), the non-negativity and carbon ceiling constraints and the initial conditions (4), is determined by the vector* $\{\mu^\circ(0), \lambda^\circ(0), T^\circ\}$, *such that the economy converges towards the steady state* (Y^*, μ^*), *defined by:*

$$\delta Y^* = C'^{-1}(\mu^*) \tag{6}$$

$$(\rho + \delta)\mu^* = \phi u'(\phi Y^*) \tag{7}$$

which is unique and saddle-path stable, and exists if the following condition holds

$$u'(0) > \frac{\rho + \delta}{\phi} C'(0) \tag{8}$$

The optimal path reaches asymptotically the steady state undergoing two phases, a carbon era, when $x(t) > 0$ $\forall t < T$, *followed by a clean era when* $x(t) = 0$ *for* $t \geq T$, *characterized by the system of differential equations:*

$$\dot{Y}^\circ(t) = C'^{-1}\left(\mu^\circ(t)\right) - \delta Y^\circ(t) \quad \forall t \geq 0 \tag{9}$$

$$\dot{\mu}^\circ(t) = \begin{cases} (\rho + \delta)\mu^\circ(t) - \phi\lambda^\circ(t) & \forall t < T^\circ \\ (\rho + \delta)\mu^\circ(t) - \phi u'(\phi Y^\circ(t)) & \forall t \geq T^\circ \end{cases} \tag{10}$$

$$\dot{\lambda}^\circ(t) = \rho\lambda^\circ(t) \ \forall t < T^\circ \tag{11}$$

The carbon budget $\overline{X}$ is exhausted at date T°, satisfying

$$\lambda^\circ(T^\circ) = u'\left(\phi Y^\circ\left(T^\circ\right)\right) \tag{12}$$

Proof. In order to simplify notation, we do not write the dependency of variables on time. The Hamiltonian and the Lagrangian of the planner's problem are

$$\mathcal{H} = u(x + \phi Y) - C(I) - \lambda x + \mu(I - \delta Y)$$

$$\mathcal{L} = \mathcal{H} + \omega_x x + \omega_X(\overline{X} - X)$$

The first order conditions are

$$u'(x + \phi Y) = \lambda - \omega_x \tag{13}$$

$$C'(I) = \mu \tag{14}$$

$$\dot{\lambda} = \rho\lambda - \omega_X \tag{15}$$

$$\dot{\mu} = (\rho + \delta)\mu - \phi u'(x + \phi Y) \tag{16}$$

We consider the case when Y_0 is sufficiently small for investment in green capital to take place at the beginning of the planning horizon ($C'(0) < \mu(0)$).

According to (14), (9) holds for any ω_x and ω_X.

Carbon era: $x > 0, \omega_x = 0, \omega_X = 0$. From (15), the carbon value evolves according to (11). Together with (13) this implies that

$$u'(x + \phi Y) = \lambda(0)e^{\rho t} \tag{17}$$

which gives (10) for $t < T$, and also the path of fossil resource use :

$$x^\circ(t) = u'^{-1}\left(\lambda^\circ(0)e^{\rho t}\right) - \phi Y^\circ(t) \tag{18}$$

The carbon era comes to an end at T such that $x(T) = 0$, $Y(T) = \frac{1}{\phi}u'^{-1}(\lambda(0)e^{\rho T})$ and the carbon budget is exhausted: $\int_0^T x^\circ(t)dt = \overline{X}$ with (18).

Clean era: $x = 0$, $\omega_x > 0$, $\omega_X > 0$. Substituting for $x = 0$ into (16) gives (10) for $t \geq T$. These two differential equations define the steady state (6)-(7), which is saddle path stable under (8). $\square$

Along the optimal trajectory, electricity consumption is non-monotonous. At first, it is decreasing to reflect the ever increasing stringency of the carbon budget. When the later is exhausted however, consumption increases along with production capacity from renewable energy sources.

3 Decentralized equilibrium

In the decentralized economy, the government handles a set of policy instruments, households demand electricity, power companies supply electricity, and to do so they invest in green capital and demand fossil resource inputs, and fossil resource managers supply the latter. Here we characterize the behaviors of these agents for given policy tools, and then present the main features of the general dynamic equilibrium.

We consider four possible policy tools available to the regulator: a carbon tax $\tau(t)$; a feed-in-premium (FIP) for electricity from renewable sources $\sigma(t)$; a subsidy to investment in green capital $s(t)$; a tax on electricity consumption $\theta(t)$.

The public budget is balanced through lump-sum transfers or taxes, $\mathcal{T}(t)$, to the household:

$$\tau(t)x(t) + \theta(t)e(t) = \sigma(t)\phi Y(t) + s(t)I(t) + \mathcal{T}(t) \tag{19}$$

3.1 Behaviors

The representative household maximizes his intertemporal utility. Utility is derived from the consumption of a generic good z and electricity services, e. The utility function is quasi-linear in good z, taken as numeraire. Hence, the program of the representative consumer is $\forall t \geq 0$:

$$\begin{aligned} \max_{e(t),z(t)} \quad & \int_0^\infty e^{-\rho t}\left[z(t) + u\left(e(t)\right)\right] dt \\ \text{s.t.} \quad & \dot{a}(t) = r(t)a(t) - z(t) - \left(p_e(t) + \theta(t)\right) e(t) + \Pi_x(t) + \Pi_e(t) + \mathcal{T}(t) \\ & a(0) = a_0 \text{ given} \end{aligned} \tag{20}$$

where a is the household's financial wealth and a_0 its endowment, r is the real rate of return on financial wealth as well as the interest rate at which the household can borrow, while Π_x and Π_e are the profits of the fossil resource producers and of the electricity producers respectively. The households' optimal saving behavior ensures that $\forall t \geq 0$ $r(t) = \rho$, while its behavior on the electricity market is characterized by the inverse demand schedule:

$$p_e(t) + \theta(t) = u'(e(t)) \tag{21}$$

Perfectly competitive utilities produce electricity from either fossil resources or from a specific green capital. The representative power utility acts under perfect competition on the electricity market and on the markets for natural resources, where it takes prices as given. The firm seeks to maximize the present value of its profits, taking into account that building up its green capital to produce renewable energy implies adjustment costs. It therefore solves an intertemporal program, based on the expected evolution of the electricity price p_e, the prices of the fossil resource p_x, as well as policy tools, namely, the carbon tax $\tau(t)$, the FIP $\sigma(t)$ and the subsidy $s(t)$. Defining $R(t) \equiv \int_0^t r(u)du$, the program of the power producer is:

$$\begin{aligned} \max_{x(t),I(t)} \quad & \int_0^\infty e^{-R(t)} \left[p_e(t)x(t) + (p_e(t)+\sigma(t))\,\phi Y(t) - (p_x(t)+\tau(t))\,x(t) - C(I(t)) + s(t)I(t)\right] dt \\ \text{s.t.} \quad & (3),\ x(t) \geq 0,\ I(t) \geq 0 \text{ and } Y(0) = Y_0 \ \text{ given.} \end{aligned} \tag{22}$$

Let us write the Hamiltonian of this program, denoting μ_d the shadow price of green capacity:

$$\mathcal{H}^u = p_e x + (p_e + \sigma)\phi Y - (p_x + \tau)x - C(I) + sI + \mu_d(I - \delta Y) \tag{23}$$

The Hamiltonian is linear in x. Therefore, when some of the electricity is optimally produced using fossil inputs, the seller price of electricity equals the cost of fossil resource use, comprehensive of the price of the resource and of the carbon tax:

$$x(t) > 0 \quad \Leftrightarrow \quad p_e(t) = p_x(t) + \tau(t) \tag{24}$$

On the other hand, when the price of electricity is smaller than the cost of the fossil input, the fossil input is not used.

The first order conditions characterizing the accumulation of green capital by electricity producers are

$$C'(I) = \mu_d + s \tag{25}$$

$$\dot{\mu}_d = (r+\delta)\mu_d - \phi(p_e + \sigma) \tag{26}$$

According to (25), investment in green capital takes place up to the point where the marginal investment cost is equal to the private shadow value of green capital, augmented by the subsidy on investment. The dynamics of green capital is therefore given by

$$\dot{Y}(t) = C'^{-1}\left(\mu_d(t) + s(t)\right) - \delta Y(t) \tag{27}$$

The power utility chooses the accumulation path of green capital knowing in advance the evolution of the electricity price and of policy tools. A necessary condition for its choice to be optimal is the continuity of the value of green capital:[9]

$$\mu_d(t) \quad \text{continuous} \tag{28}$$

[9] A formal proof is provided in Appendix A.1 for the case considered in Section 4.2.

Our focus being on an effective climate policy, which makes the environmental problem more stringent than resource scarcity, the scarcity of the natural resource stock is not binding at equilibrium on the market for fossil resources. Producers act as if their resources were abundant, so that, rather than solving an intertemporal optimization problem, they maximize their current profit at each date. Given that the production costs are assumed nil and that the market is perfectly competitive, the equilibrium seller's price for fossil resources is nil:

$$p_x(t) = 0 \tag{29}$$

3.2 Equilibrium

In equilibrium, when the fossil resource is used to produce power, (26) can be written using the equality of the interest and discount rates, and equations (24) and (29), to get

$$\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi(\tau(t) + \sigma(t)) \quad \text{if } x(t) > 0 \tag{30}$$

Moreover, the equilibrium on the electricity market is obtained by combining (21), (24) and (29), to get

$$x(t) = u'^{-1}\left(\tau(t) + \theta(t)\right) - \phi Y(t) \quad \text{if } x(t) > 0 \tag{31}$$

When instead power relies exclusively on renewable sources, i.e. $x(t) = 0$, (26) can be written using (21), to get

$$\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi[u'(\phi Y(t)) - \theta(t) + \sigma(t)] \quad \text{if } x(t) = 0 \tag{32}$$

In this case the supply on the electricity market is perfectly rigid, so that the full incidence of the tax on consumption is on the power utility. As a consequence, it is conceivable that the level of the consumption tax be so high that some capacity could remain idle

$$e(t) = u'^{-1}(\theta(t)) \leq \phi Y(t) \tag{33}$$

Of course, there is no reason to impose such a high level of taxation.

Let us notice that when the fossil resource is not exploited, the consumption tax and the FIP are equivalent policy instruments, since they only affect the evolution of the private value of green capital, and do so in the same way (see (32)).

Moreover, the level of green capital at the date T when the fossil input stops being used is, according to (31):

$$Y(T) = \frac{1}{\phi}u'^{-1}\left(\tau(T) + \theta(T)\right) \tag{34}$$

Therefore, the regulation determines this threshold level of green capital, implying a change in regime (the carbon tax is not relevant thereafter).

3.3 First best carbon pricing

Since there is one market imperfection related to the carbon budget, one policy instrument is sufficient to set the decentralized equilibrium of the economy on the optimal path.

PROPOSITION 2. Optimal carbon pricing. *The decentralized equilibrium coincides with the optimal path when the carbon tax equals the carbon value along the optimal energy transition:*

$$\tau^{fb}(t) = \lambda^{\circ}(0)e^{\rho t}, \;\; t \leq T^{\circ} \tag{35}$$

where λ° *and* T° *are defined in Proposition 1, and no other instrument is used:* $\sigma^{fb}(t) = s^{fb}(t) = \theta^{fb}(t) = 0$.

The optimal policy calls for an increasing carbon tax. As argued in the introduction, in practice such a policy seems to raise political opposition, making its implementation unlikely. We therefore turn to the question of what can be done when the regulator can only commit to a constant carbon tax, and what is the welfare implication of this constraint on climate policy design.

4 Second best policy with a constant carbon tax

For the remainder of the article, we assume that the carbon tax is set a constant level $\widetilde{\tau}$ for acceptability. We consider alternative configurations of policy tools. First we consider the use of a FIP with a consumption tax. Next we consider the case of a FIP only. Finally we consider the case of a subsidy to investment in green capacity.

The objective of the regulator is to design the climate policy tools in order to maximize social welfare, taking into account the climate constraint and the reaction functions of households.

Let us first derive the regulator's objective function. Using the equation of evolution of household's wealth, the profits and the public budget constraint, and the transversality condition, we obtain:

$$a_0 = \int_0^{\infty} e^{-\rho t} \left[z(t) + (p_e(t) + \theta(t))e(t) - \Pi_e(t) - \mathcal{T}(t) \right] dt = \int_0^{\infty} e^{-\rho t} \left[z(t) + C(I(t)) \right] dt$$

Therefore

$$\int_0^{\infty} e^{-\rho t} z(t) dt = a_0 - \int_0^{\infty} e^{-\rho t} C(I(t)) dt$$

and, since a_0 is given, the regulator's objective function reads:

$$W = \int_0^\infty e^{-\rho t} \left[u(x(t) + \phi Y(t)) - C(I(t))\right] dt \tag{36}$$

4.1 Combining the FIP with the tax on electricity consumption

Remember that during the carbon era the optimal consumption path is decreasing, although the production of electricity from renewable sources is increasing. This is possible only if the use of fossil resources declines. Relying on a constant carbon tax as the sole instrument implies however that the private marginal cost of using fossil inputs for the power utility is constant during this phase. As a result, there are no incentives to reduce production of electricity from fossil resources.

Nevertheless, there is one possibility to induce the power utility to use ever less fossil inputs. In fact, it uses them only to cope with demand above the base power $\phi Y(t)$ produced by the already installed capacity with nil marginal cost. In other words, the power utility employs fossil resources $x(t) = e(t) - \phi Y(t)$ if the demand $e(t)$ is above the base load $e(t) > \phi Y(t)$. Hence, the regulator can induce a decline in fossil resource use by appropriately influencing the demand for electricity, thanks to the consumption tax.

In fact, setting $\tau(t) = \widetilde{\tau}$ in (31), we see that at the decentralized equilibrium $u'(x(t) + \phi Y(t)) = \widetilde{\tau} + \theta(t)$ while at the optimum $u'(x(t) + \phi Y(t)) = \lambda^\circ(0)e^{\rho t}$. Hence, along a trajectory where green capital investment is optimal, the fossil input use could be reduced at the optimal pace by setting a consumption tax to compensate for the carbon pricing gap:

$$\widetilde{\theta}(t) = \lambda^\circ(t) - \widetilde{\tau} \tag{37}$$

which is increasing and becomes positive as soon as $\lambda^\circ(t)$ is larger than $\tilde{\tau}$.

However, given the carbon pricing gap, there is no reason for investment in green capital to follow the optimal path when only the constant carbon tax and the electricity consumption tax are implemented. Inspecting the evolution of the green capital value in (30) when $\tau(t) = \widetilde{\tau}$, and comparing with its optimal path in (10), one sees that the consumption tax does not influence the value of green capital, and cannot therefore cope with the inability of the constant carbon tax to signal the increasing social value of carbon. However, the FIP could be increased in order to solve the problem. Its optimal value would indeed be identical to the electricity consumption tax. Hence, subsidies would be entirely financed through the tax on electricity consumption.[10] To summarize:

Proposition 3. Optimal constrained policy. *The optimal trajectory can be implemented in a decentralized equilibrium with a constant carbon tax $\widetilde{\tau}$ by levying a tax on electricity consumption*

[10] Do note that this contrasts with the current reform of the EEG levy, which involves financing subsidies to renewables using revenues from the auctions of carbon emissions allowances from consumer instead of taxing electricity consumption.

$\widetilde{\theta}(t)$ defined by (37), while subsidizing electricity production from renewable sources through a FIP $\widetilde{\sigma}(t) = \widetilde{\theta}(t)$. This policy implies a surplus of the public budget $\mathcal{T}(t) = \lambda^{\circ}(t)x^{\circ}(t)$ during the carbon era and a balanced budget during the clean area.

Two remarks are worthwhile. First, if the constant carbon tax is set at a level above the initial level of the optimal carbon tax, i.e. if $\widetilde{\tau} > \lambda^{\circ}(0)$, then it would be optimal to initially subsidize electricity consumption, while taxing the production of electricity from renewables. Second, if potential political opposition to an increasing path of the carbon tax is the underlying constraint on policy design, one wonders if there would not be political opposition also to an increasing path of taxes on electricity consumption as in (37). Proposition 3 may therefore be of little practical guidance.

4.2 Relying only on the FIP

In our approach, consumers, electricity producers and the regulator play a Stackelberg policy game, where the leader is the regulator (see Dockner at al, 2000, chapter 5). At date 0, the constant carbon tax $\widetilde{\tau}$ is exogenously set at a level acceptable by society and the regulator announces the FIP path $\sigma(t)$, as well as the date $\widetilde{T}$ at which it will become nil, and commits to this plan. The other policy tools are not used: $s(t) = \theta(t) = 0$. Taking into account this information on climate policy, households optimally decide how much electricity to consume, according to (31) or (33), and electricity producers choose the electric mix and the amount of green capital to install according to (30) or (32). The game is solved by backward induction: the regulator, knowing the agents' best responses to his policy, proceeds to choose the FIP path maximizing his objective function while complying with the carbon budget, conditional to the exogenous constant carbon tax $\widetilde{\tau}$.

Lemma 1. The clean era with a constant carbon tax and FIP.

(i) The date $\widetilde{T}^{\diamond}$ at which the regulator lifts the FIP is necessarily equal to the date at which fossil phase out occurs, that is $\widetilde{T}^{\diamond} = T^{\diamond}$.

(ii) The clean era is described by the same system of differential equations (9)-(10) as at the first best. Green capital and its shadow value converge asymptotically along the saddle path to their steady state levels defined by (6)-(7). However, the clean era starts at the different date than in the first best, from a different level of green capital.

(iii) The initial level of green capital in the clean era is $Y^{\diamond}(\widetilde{T}^{\diamond}) = u'^{-1}(\widetilde{\tau})/\phi$, a decreasing function of the carbon tax. It is larger than at the first best iff $\widetilde{\tau} < \tau^{fb}(T^{\circ}) = \lambda^{\circ}(0)e^{\rho T^{\circ}}$.

(iv) For a small enough carbon tax, $\widetilde{\tau} < \underline{\tau} \equiv u'(\phi Y^{*})$, the green capital at the date of fossil phase out is larger than its long run value, $Y^{\diamond}(\widetilde{T}^{\diamond}) > Y^{*}$: second best climate policy entails overshooting the long term accumulation target, to compensate for the weakness of the carbon tax.

Proof. Suppose that the regulator provides a positive FIP to electricity producers during the carbon era and continues to do so after it ends. The regulator's program in the clean era then reads:

$$\begin{aligned}
&\max_{\sigma(.)} \int_{\widetilde{T}}^{\infty} e^{-\rho t}\left[u(\phi Y(t)) - C(C'^{-1}(\mu_d(t)))\right] dt \\
&\dot{Y}(t) = C'^{-1}(\mu_d(t)) - \delta Y(t) \\
&\dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi(u'(\phi Y(t)) + \sigma(t)) \\
&Y(\widetilde{T}) \text{ given}
\end{aligned}$$

The Hamiltonian is:

$$\mathcal{H}_1^r = u(\phi Y) - C(C'^{-1}(\mu_d)) + \zeta_2(C'^{-1}(\mu_d) - \delta Y) + \zeta_3\left[(\rho+\delta)\mu_d - \phi(u'(\phi Y) + \sigma(t))\right]$$

where ζ_2 is the second best value of green capital and ζ_3 the shadow price of μ_d, which is treated here as a state variable.

Concerning the first order condition on the FIP, notice that the Hamiltonian is linear in σ, and that $\partial\mathcal{H}/\partial\sigma = -\zeta_3\phi$. Therefore, assuming a positive FIP and an upper bound σ_{max} on its value, the condition yields:

$$\sigma(t)\begin{cases} = 0 & \text{if} \quad \zeta_3 > 0 \\ \in [0, \sigma_{max}] & \text{if} \quad \zeta_3 = 0 \quad \text{(switching manifold)} \\ = \sigma_{max} & \text{if} \quad \zeta_3 < 0 \end{cases} \tag{38}$$

The other first order conditions are:

$$\dot{\zeta}_2 = (\rho+\delta)\zeta_2 - \phi u'(\phi Y) + \zeta_3\phi^2 u''(\phi Y) \tag{39}$$

$$\dot{\zeta}_3 = -\delta\zeta_3 + \frac{\partial C'^{-1}(\mu_d)}{\partial \mu_d}(\mu_d - \zeta_2) \tag{40}$$

In the case where $\sigma > 0$ over a non-degenerate interval of time, $\zeta_3 = \dot{\zeta}_3 = 0$ over this interval and equation (40) yields $\mu_d = \zeta_2$; comparing the equations of evolution of μ_d and ζ_2 (32) and (39) shows that this is only possible if $\sigma = 0$, a contradiction. Therefore $\sigma = 0$ all along the clean era, which proves (i).

Direct inspection of the dynamic systems characterizing the evolution of green capital and its shadow value in the clean era at the first and second best shows that they are identical. However, at the first best the clean era starts at date T° with a green capital stock $Y^\circ(T^\circ) = u'^{-1}(\lambda^\circ(0)e^{\rho T^\circ})/\phi$ (equation (12)), whereas at the second best it starts at date $\widetilde{T}$ with a green capital stock $Y(\widetilde{T}) = u'^{-1}(\widetilde{\tau})/\phi$ (equation (34) with $\tau(t) = \widetilde{\tau}$ and $\theta = 0$). This proves (ii) and (iii).

(iv) is directly derived from the definition of $Y(\widetilde{T})$: $Y(\widetilde{T}) > Y^* \Leftrightarrow u'^{-1}(\widetilde{\tau})\phi > Y^* \Leftrightarrow \widetilde{\tau} <$

$u'(\phi Y^*)$. □

Notice that since $\widetilde{\tau}$ has been set to make carbon taxation acceptable, it will most plausibly be smaller than the optimal carbon value at the date of fossil phase out $\tau^{fb}(T^\circ) = \lambda^\circ(0)e^{\rho T^\circ}$. Hereafter we assume this to be the case. Therefore the green capital at the date of fossil phase out is larger at the second best than at the first best. However, at this stage, nothing can be said concerning the date of the switch to the clean area: a priori the energy transition can be delayed or accelerated at the second best, compared to what is optimal.

Let us consider the climate policy in the carbon era. First notice that under our assumption of zero production costs in the fossil resources, the carbon budget is attainable only if the carbon tax is a above a minimum exogenous threshold $\tau^{\min} \equiv u'\left(\phi(1-\delta)Y_0 + C^{-1}(a_0)\right)$.[11] We assume that it is socially acceptable to set the tax above this threshold, and that the constant carbon tax is indeed above it. We now characterize the second best climate policy in the carbon era, taking this condition as granted.

Proposition 4. The carbon area with a constant carbon tax and FIP.

(i) The optimal trajectory cannot be implemented in a decentralized equilibrium with only a constant carbon tax and a FIP, as the electricity consumption is then constant up to the date of fossil phase out.

(ii) At each level of the constant carbon tax corresponds a second best carbon value, denoted $\zeta_1^\diamond$, initially all the higher since the carbon tax is small, and increasing over time at the discount rate. The carbon pricing gap *is the difference between this second best carbon value and the effective constant carbon tax, $\zeta_1^\diamond(0)e^{\rho t} - \widetilde{\tau}$. It is therefore increasing over time.*

(iii) The optimal date of the switch to the clean era $\widetilde{\tau}$ is such that the marginal benefit of delaying the switch is nil, which implies the continuity of the social value of green capital $\zeta_2(t)$ at the date of the switch.

(iv) Given that $Y_0 < Y^*$,

a. *If the carbon tax is above $\overline{\tau}_2 = u'(\phi Y_0)$, the fossil resource is not used, and the optimal FIP is nil.*[12]

[11] This lower bound on carbon taxation is defined by setting the investment in green capital as high and as soon as possible, to be able to phase out fossils upon instantaneous exhaustion of the carbon budget. In fact, from (31) with $\theta = 0$ and taking (4) into account, fossil use at date 0 is $x(0) = u'^{-1}(\tau^{\min}) - \phi Y_0 = \overline{X}$ but $x(0+dt) = 0$. As stated in Lemma 1, for $\widetilde{T} = 0 + dt$ $u'^{-1}(\tau^{\min}) = \phi Y(0+dt)$, where $Y(0+dt) = (1-\delta)Y_0 + C^{-1}(a_0)$, from (3) and investing all available assets $I_0^{\max} \equiv C^{-1}(a_0)$.

[12] Actually taxing the production of electricity from renewables to induce consumption of some fossil resource until the carbon budget is met would be welfare improving (recall that there is no cost associated to fossil resource if the carbon budget is not exhausted).

b. If the carbon tax is $\widetilde{\tau} \in (\overline{\tau}_1, \overline{\tau}_2]$, the fossil resource is used, but the carbon pricing gap is always negative and therefore the FIP nil. $\overline{\tau}_1$ is defined by $\overline{\tau}_1 = \zeta_1^{\diamond}(0) e^{\rho \widetilde{T}^{\diamond}}|_{\overline{\tau}_1}$.[13]

c. If the carbon tax is $\widetilde{\tau} \in (\overline{\tau}, \overline{\tau}_1]$, the carbon pricing gap is initially negative and the FIP nil, up to the date T_0 when the carbon pricing gap becomes nil. After T_0 the FIP compensates for the carbon pricing gap:

$$\sigma^{\diamond}(t) = \zeta_1^{\diamond}(t) - \widetilde{\tau} \tag{41}$$

$\overline{\tau}$ is defined by $\overline{\tau} = \zeta_1^{\diamond}(0)|_{\overline{\tau}}$.

d. If $\widetilde{\tau} \in (\underline{\tau}, \overline{\tau}]$, with $\underline{\tau} = u'(\phi Y^*)$, the FIP is optimally provided from the start, according to (41). The accumulation of green capital is monotonically increasing toward Y^*.

e. If $\widetilde{\tau} \in (\underline{\tau}_1, \underline{\tau}]$, the FIP is optimally provided from the start, according to (41), and there is overshooting of green capital, i.e. $Y^{\diamond}(\widetilde{T}^{\diamond}) > Y^*$.

f. If $\widetilde{\tau} \leq \underline{\tau}_1$, the FIP is capped at a value σ_{max} to prevent an overshooting so large that it would entail disinvestment in the clean era. In the case of logarithmic utility and quadratic adjustment costs, $\underline{\tau}_1$ is defined by $\underline{\tau}_1 = u'\left(\frac{\omega}{\omega+\delta}\phi Y^*\right)$, with $\omega = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\right] < 0$.

Proof. (i) derives directly from equation (21); since the consumer price of electricity is $\widetilde{\tau}$, demand for electricity is at each date in the carbon era is equal to $u'^{-1}(\widetilde{\tau})$.

To prove the other parts of the proposition we must solve the regulator's program.

In the carbon era, the behavior of the power producer is described by (30)-(31) together with (25) and (27). Therefore, for a given $\widetilde{\tau}$ chosen by the regulator, his program reads:

$$\begin{aligned}
&\max_{\sigma(.)} \int_0^{\widetilde{T}} e^{-\rho t}\left[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d(t)))\right] dt \\
&\dot{X}(t) = u'^{-1}(\widetilde{\tau}) - \phi Y(t) \\
&\dot{Y}(t) = C'^{-1}(\mu_d(t)) - \delta Y(t) \\
&\dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi(\widetilde{\tau} + \sigma(t)) \\
&X(t) \leq \overline{X} \\
&\mu_d(\widetilde{T}_-) = \mu_d(\widetilde{T}_+) \\
&X(0) = 0\ ,\ Y(0) = Y_0\ ,\ Y(\widetilde{T}) = \frac{1}{\phi}u'^{-1}(\widetilde{\tau}),\ \mu_d(0)\ \text{ free}
\end{aligned}$$

[13] The notation means that $\overline{\tau}_1$ is the fixed point of the equation, with both terms on the right-hand-side depending on $\widetilde{\tau}$.

The associated Hamiltonian is:

$$\mathcal{H}_2^r = u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d)) \\ - \zeta_1(u'^{-1}(\widetilde{\tau}) - \phi Y) + \zeta_2(C'^{-1}(\mu_d) - \delta Y) + \zeta_3\left[(\rho+\delta)\mu_d - \phi(\widetilde{\tau}+\sigma)\right]$$

where ζ_1 is the second best carbon value.

As before, the Hamiltonian is linear in σ, and conditions (38) apply. The other first order conditions are:

$$\dot{\zeta}_1 = \rho\zeta_1 \tag{42}$$

$$\dot{\zeta}_2 = (\rho+\delta)\zeta_2 - \zeta_1\phi \tag{43}$$

$$\dot{\zeta}_3 = -\delta\zeta_3 + \frac{\partial C'^{-1}(\mu_d)}{\partial \mu_d}(\mu_d - \zeta_2) = -\delta\zeta_3 + \frac{1}{c_2}(\mu_d - \zeta_2) \tag{44}$$

where the last identity holds in the case of quadratic investment costs, $C(I) = c_1 I + \frac{1}{2}c_2 I^2$.

Equation (42) shows that ζ_1, the second best carbon value, obeys the Hotelling rule, as it is the case for the first best carbon value λ. This proves (ii).

The prove of (iii) is presented in detail in Annex A.2.

$$W \equiv \int_0^{\widetilde{T}} e^{-\rho t}\left[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d(t)))\right]dt + \int_{\widetilde{T}}^{\infty} e^{-\rho t}\left[u(\phi Y(t)) - C(C'^{-1}(\mu_d(t)))\right]dt$$

The proof of (iv)a. is trivial.

Suppose that we are in the case $\sigma \in [0, \sigma_{max}]$. Then $\zeta_3 = 0$ (see conditions (38)), and equation (44) yields $\mu_d = \zeta_2$, meaning that thanks to the FIP, the private and social shadow values of green capital are the same. Then equations (30) and (43) imply (41).

The lower bound of the second best carbon value is $\zeta_1(0)$. When $\widetilde{\tau} < \overline{\tau} = \zeta_1(0)|_{\overline{\tau}}$, the economy is on the switching manifold from the beginning of the horizon on, meaning that the FIP is positive all along the carbon area. When $\widetilde{\tau} > \overline{\tau}$, there is at the beginning of the horizon a period when the FIP should be negative, which is unfeasible according to our assumption $\sigma \in [0, \sigma_{max}]$. The phase where $\sigma(t) > 0$ is necessarily preceded by a phase where, because the carbon tax is higher than the initial second best carbon value, $\zeta_3(t) > 0$ and $\sigma(t) = 0$. This phase is all the longer since $\widetilde{\tau}$ is high. At the limit, this phase lasts until fossil phase out, which occurs for $\widetilde{\tau} = \overline{\tau}_1 = \zeta_1(0)e^{\rho\widetilde{T}}|_{\overline{\tau}_1}$. This proves (iv) b. and c.

Coming back to the case $\widetilde{\tau} < \overline{\tau}$, we have shown in Lemma 1 that the carbon tax may be so small ($\widetilde{\tau} < \underline{\tau}$) that it entails optimal overshooting of the green capital, compared to its long term value.

The limit case of this situation is when this overshooting is so large that disinvesting in the clean era would be optimal, which is not possible according to our assumptions. The limit is provided by the equality of the private shadow value of green capital for electricity producers and the marginal investment cost for $I = 0$ at the beginning at the clean era: $\mu_d(\widetilde{T}^+) = c_1$. This limit translates into an upper bound for $Y(\widetilde{T})$, which in turn translates into a lower bound $\underline{\tau}_2$ for the carbon tax, and an upper bound $\sigma_{max} = \sigma(\widetilde{T})|_{\widetilde{\tau}=\underline{\tau}_2}$. For $\widetilde{\tau} < \underline{\tau}_2$ the FIP is first positive, then, as the carbon pricing gap widens, it reaches at a date T_{max} in the future its upper bound σ_{max}, implying that $\zeta_3(t) < 0$ from this date on. This proves (iv) d–f.

We provide in Appendices A and B a thorough analysis of the dynamic systems characterizing the carbon era in the different cases, based on the specification of quadratic adjustment costs and logarithmic utility, as well as a proof of that the sign of ζ_3 is compatible with the optimality conditions in the different cases. $\square$

Notice that according to Eq. (41) $\sigma^\diamond(t)$ is increasing over time, as is $\zeta_1^\diamond(t)$. But just after $\widetilde{T}$ it becomes nil, which means that it jumps at this date from a positive value $\zeta_1^\diamond(\widetilde{T}^\diamond) - \widetilde{\tau}$ to 0.

For any $\widetilde{\tau}$ chosen by the planner the FIP path allowing the climate objective to be satisfied is unique. Computing numerically the value functions in each case and comparing them will allow us to evaluate the *welfare cost of acceptability*, at least for illustrative purposes. This is done in sub-section 4.5.

4.3 Imposing a balanced public budget

Studying second best policies is mainly driven by the objective to reduce the opposition to climate policies. According to the literature, this opposition is weaker when the revenue from the tax is used to finance subsidies to renewable energy production (Douenne and Fabre, 2020, Klenert et al., 2018). Therefore, we study the case where the regulator pays the FIP with the resulting tax revenue, hence ensuring a balanced budget without transfers ($\mathcal{T}(t) = 0$):

$$\widetilde{\tau} x(t) = \sigma(t) \phi Y(t) \tag{45}$$

Substituting for the decentralized use of fossils (31) for $\theta = 0$, in the budget balance above, we see that the the FIP is determined by the carbon tax and the stock of green capital by:

$$\sigma^c(t) = \widetilde{\tau} \left(\frac{u'^{-1}(\widetilde{\tau})}{\phi Y^c(t)} - 1 \right) > 0 \ \forall t \leq \widetilde{T}^c \tag{46}$$

It decreases and then is nil from $\widetilde{T}^c$ onward.

The carbon area is then characterized by:

$$\dot{Y}^c = C'^{-1}(\mu_d^c) - \delta Y^c$$
$$\dot{\mu}_d^c = (\rho+\delta)\mu_d^c - \frac{\widetilde{\tau}(u'^{-1}(\widetilde{\tau})}{Y^c} \tag{47}$$

Points (ii)-(iv) of lemma 1, and (i) and (ii) in Proposition 4 are still valid.

First, imposing a balanced budget further constrains the programme, hence necessarily generating an adverse effect on welfare. Second, it cannot ensure reaching both the climate and acceptability objectives. In case $\widetilde{\tau}$ is still set exogenously to ensure acceptability, the FIP being out of control as well since it is simply determined to ensure a balanced budget, nothing ensures that the carbon budget be met (see previous sub-section for the uniqueness of the FIP path to meet the carbon budget). If $\widetilde{\tau}$ is set for the economy to stay within the carbon budget, then it is shown in Appendix B that there is only one level of the tax that exactly meets the carbon budget. Therefore, nothing ensures that this carbon tax is lower (in particular at time $t = 0$) than the optimal tax, hence potentially missing the acceptability target (see Pommeret et al. (2022) that exhibits numerically this case). As a result, limiting the FIP to be exclusively financed out of carbon tax revenues does not seem to be a wise policy option.

4.4 Relying on a subsidy to investment in green capital

We now study the case where the regulator puts in place a subsidy to investment in green capital to complement the constant carbon tax. The policy game is similar to the one implemented to design the optimal FIP. Given a constant carbon tax $\widetilde{\tau}$, the regulator announces and commits to a date $\widetilde{T}^s$ and a subsidy path $s(t)$, while FIP and electricity consumption tax are set to zero. The investment subsidy is chosen by the regulator who takes the agents' best responses into account when maximizing its objective function under the carbon budget constraint.

Proposition 5. Constant small carbon tax and subsidy to green investment.

(i) When it is positive, the subsidy fills the gap between the social and the private shadow values of renewable capacity:

$$s^\diamond(t) = \zeta_2^\diamond(t) - \mu_d(t) \tag{48}$$

Equivalently, it is equal to the discounted present value of the future carbon policy gaps until the end of the carbon era:

$$s^\diamond(t) = \int_t^{\widetilde{T}^\diamond} e^{-(\rho+\delta)(u-t)}(\zeta_1^\diamond(u) - \widetilde{\tau})du \tag{49}$$

(ii) For a given constant carbon tax, there exists a unique path of subsidy to investment in green capital $s^\diamond$, leading to the same electricity consumption, the same timing of fossil phase out and the

same accumulation of green capital as a FIP $\sigma(t)$ provided to electricity producers as described in Proposition 4.

(iii) When the carbon tax is "small', $\sigma(t), s(t) > 0\ \forall t \in [O, \widetilde{T})$, and the relationship linking the two instruments is:

$$\dot{s}^{\diamond}(t) = (\rho + \delta)s^{\diamond}(t) - \phi\sigma^{\diamond}(t) \tag{50}$$

Choosing to fill the carbon pricing gap with a subsidy to investment in green capital rather than with a FIP induces a transfer from the public budget to the electricity producers, immaterial to households:

$$\int_0^{\widetilde{T}^{\diamond}} e^{-\rho t} s^{\diamond}(t) I^{\diamond}(t) dt = \int_0^{\widetilde{T}^{\diamond}} e^{-\rho t} \sigma^{\diamond}(t)\phi Y^{\diamond}(t) dt - s^{\diamond}(0)Y_0 \tag{51}$$

Proof. The proof is relegated to Appendix C. □

The main result in Proposition 5 hinges on the equivalence between using subsidies to investment or FIP's, which underlie one major distinguishing feature between the American and the European policy designs. The former relies relatively more on tax credits for investments to expand renewable capacity, while the latter relies more on feed-in-premiums and similar subsidies to purchases of electricity from renewable production units. In fact, the potential different weight on public finances outlined in (51) is not relevant in the case of these policies. This is because, in practice, FIP's only apply to electricity supplied from new production sites (i.e. not to preexisting green capital Y_0) and for limited time.[14]

4.5 Simulations

For illustration purposes we propose three simulations. We assume a log utility function $u(e) = \gamma \ln e$ and quadratic investment costs $C(I) = c_1 I + \frac{1}{2}c_2 I^2$. The parameters we use are given in Table 1.

ϕ	ρ	δ	γ	c_1	c_2	Y_0	$\overline{X}$
0.9	0.03	0.01	1.8	3	20	1	53.81

Table 1: Parameters

Table 2 and Figures 2–6 illustrate the optimal trajectory, as well as the second best trajectories with different levels of the constant carbon tax, "large", "small" without overshooting of its long run value by the green capital accumulated in the carbon era, and "small" with overshooting.

[14]Since our setting is deterministic, we do not explicitly consider the limited horizon over which the FIP to recently installed capacity. In this case, it is possible to equivalently reward investment in new capacity using a uniform time-variable FIP up to $\widetilde{T}$, or a vintage specific FIP held constant over a fixed interval of time. In the former case though, one avoids the complexity of the analysis araising from considering a vintage structure with differentiated seller net prices. In a stochastic setting however, the difference between the two approaches –tax credit vs FIP's– may be salient, as for instance in terms of the committed public spending.

Remember that "large" (resp. "small") refers to a carbon tax larger (resp. smaller) than the initial second best carbon value.

Figure 2 depicts the initial second best carbon value $\zeta_1(0)$ as a function of the carbon tax $\widetilde{\tau}$, as well as the initial optimal carbon value $\lambda(0)$. It shows that the relationship between $\widetilde{\tau}$ and $\zeta_1(0)$ is decreasing, convex for "small" carbon taxes, then concave for "large" carbon taxes. Moreover, it shows that a carbon tax that is considered "small" can actually be larger than the initial optimal carbon value.

Figure 3 depicts the date of fossil phase-out and the date of the introduction of the FIP as functions of the carbon tax, as well as the optimal initial carbon value and date of fossil phase out as benchmarks. It shows that $\widetilde{T}$ is an increasing function of $\widetilde{\tau}$, which may appear as counter-intuitive. It also shows that the date of fossil phase out may be postponed or brought forward, compared to what is optimal. When the constant carbon tax put in place by the regulator is so small that green capital has to be accumulated very fast to fill the carbon pricing gap, the carbon budget is used up early on (see Figure 4) and the switch to the clean era is accelerated.

Table 2 presents the main results for three values of the carbon tax. It displays in particular the *welfare cost of acceptability*, defined as the welfare loss at the second best compared to the optimum, measured by the equivalent constant additional electricity consumption that should be given to households to make them indifferent. It is given by:

$$w \equiv \exp\left(\frac{\rho}{\gamma}(W^{\circ} - W^{sb})\right) - 1 \tag{52}$$

It is small for "large" carbon taxes, but may become enormous for "small" carbon taxes with overshooting.

Table 2 also display the cost of the second best policy in terms of public budget balance, i.e. the present value of additional lump sum taxes or transfers that are levied on households:

$$b \equiv \int_0^{\widetilde{T}} e^{-\rho t}(\widetilde{\tau}x(t) - \sigma(t)Y(t))dt = \int_0^{\widetilde{T}} e^{-\rho t}\mathcal{T}(t)dt \tag{53}$$

	$\lambda(0)$	$\widetilde{\tau}$	$\zeta_1(0)$	T_0	$T_X, \widetilde{T}$	$w(\%)$	b
Optimum	0.25				18.5		13.6
Second best, "large" $\widetilde{\tau}$		0.33	0.26	7.8	26.3	1.2	9.0
Second best, "small" $\widetilde{\tau}$, no overshooting		0.23	0.76		20.6	31.5	-53.7
Second best, "small" $\widetilde{\tau}$, overshooting		0.18	1.57		16.8	148.6	-155.3

Table 2: Optimum and second best with constant carbon tax and FIP

With a "small" constant carbon tax, the FIP has to be very large to fill the carbon pricing gap. This policy fosters a very fast accumulation of green capital. Incidentally, electricity consumption is at each date higher than optimal. This result highlights the potential complementary role of consumption taxes. The policy based on very large subsidies to renewables is very costly in terms of public finance and in terms of welfare loss.

With a “large” constant carbon tax there is potentially a first phase where the FIP is zero. This recalls us that it might be desirable to tax the production of electricity from renewables. Thus, in the beginning, only a small FIP is required to fill the carbon pricing gap. The resulting accumulation of green capital is slightly faster than what it is at the optimum. The climate policy generates a public budget surplus, so that households receive lump sum transfers. The welfare loss is small.

Clearly, the situation where the regulator puts in place a “large” constant carbon tax is better. But it is difficult to believe that if he cannot commit to the optimal carbon tax for political economy reasons, he would be able to put in place a higher carbon tax early on. Hence, the case closer to the public concern seem to be the one of a “small” carbon tax. As we have seen, it entails strong distortions in terms of investment, implying large inefficiencies, and it may postpone or accelerate the energy transition.

5 Conclusion

Confronted with political opposition to the implementation of an efficient direct carbon pricing, policy makers have been relying on a set of alternative policy interventions, including in particular subsidies to the production of electricity from renewable sources (feed-in-tarifs, feed-in-premiums, renewable portfolio standards), subsidies to investment in specialized capital (tax credits, rebates on credit costs), as well as command and control regulation, or energy consumption taxes.

In this paper we explore the possibility of implementing a climate policy without relying on a carbon tax that increases at the optimal rate. We compare the performance in terms of welfare and related metrics the use of alternative patterns of policy tools. We do this in a dynamic macroeconomic model where the climate policy aims at using efficiently a given amount of cumulative carbon emissions.

We find that it could be possible to design two policy instruments, in such a way to replicate the same outcome as under optimal carbon pricing. This is the case when a tax is levied on electricity consumption, while a feed-in-premium is paid to electricity produced from renewable sources, or investment in production & storage capacity from renewables is directly subsidized. However, such a tax on electricity consumption would be increasing as well, making it implausible that it could be politically more feasible than an efficient carbon tax.

In the case that climate policy should rely on subsidizing renewable to complement a small constant carbon tax, we find that meeting the climate target would be much more costly. In fact the subsidies should be very large early on, in order to foster investment in renewables at a fast pace, resulting in important investment costs, weighing on households's budgets. Overall, larger investment costs and deformed consumption paths together reduce welfare. The key problem is that subsidies to renewables do not tackle directly the issue of limiting fossil resource use early on. They can crowd out fossils from the market, but the second best policy implies some strong and costly distortions such as boosting energy consumption, during the early phase when it should be falling.

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Appendices

A FIP, carbon era

After proving the continuity of the firm's value of green capital upon date $\widetilde{T}$, this appendix contains the detailed analysis of the cases considered in Proposition 4, developed for the case of a quadratic investment costs function, and of a logarithmic utility function.

A.1 The private value of green capital

Under the policy considered in Section 4.2, the FIP is lifted at a known future date $\widetilde{T}$. This discontinuity should however not alter abruptly the private value of the green capital stock, since its accumulation results of decentralized choices by the representative power utility. To see this, let us analyze the firm's problem as described in Section 3.1 from (22) to (27), in the case of this policy.

Given the discontinuity at date $\widetilde{T}$, the problem can be studied in two intervals of time. Denoting the value function for the firm as $V \equiv V_1 + V_2$, the continuation value depends on the stock at date $\widetilde{T}$: $V_2^*(\widetilde{T}, Y(\widetilde{T}))$. Since $x(t) = \sigma(t) = 0$ after $\widetilde{T}$, the firm's problem is

$$V_2 \equiv \max_{I(.)} \int_{\widetilde{T}}^{\infty} e^{-R(t)} \left[p_e \phi Y(t) - C\left(I(t)\right)\right] dt$$

with $\dot{Y}(t) = I(t) - \delta Y(t)$ and $\widetilde{T}$ given. The associated present value Hamiltonian is:

$$H_2 = \left[p_e \phi Y(t) - C\left(I(t)\right)\right] e^{-R(t)} + \mu_d(t) \left[I(t) - \delta Y(t)\right]$$

Denoting by * the value function when the first order conditions hold, and by $_+$ the time right after the date $\widetilde{T}$, the value of green capital at the beginning of the period is, by definition:

$$\mu_d(\widetilde{T}_+) = \frac{\partial V_2^*}{\partial Y(\widetilde{T})}$$

Up to date $\widetilde{T}$, the firm's problem is

$$V_1 \equiv \max_{x(t), I(t)} \int_0^{\widetilde{T}} e^{-R(t)} \left[p_e x(t) + (p_e + \sigma(t))\phi Y(t) - (p_x(t) + \tilde{\tau})x(t) - C(I(t))\right] dt$$

with $\dot{Y}(t) = I(t) - \delta Y(t)$. The associated present value Hamiltonian is

$$H_1 = \left[p_e x(t) + (p_e + \sigma(t))\phi Y(t) - (p_x(t) + \tilde{\tau}) - C\left(I(t)\right)\right] e^{-R(t)} + \mu_d(t) \left[I(t) - \delta Y(t)\right]$$

handing the first order conditions (24)-(25) with $s(t) = 0$. Using the definition of the Hamiltonian, and integrating by parts the second term of the integrand on the first line of the expression below, the value function can be written as follows:

$$\begin{aligned} V &= \int_0^{\widetilde{T}} \left(H_1(t) - \mu_d(t)\dot{Y}(t) \right) dt + V_2^*(\widetilde{T}, Y(\widetilde{T})) \\ &= \int_0^{\widetilde{T}} \left(H_1(t) + \dot{\mu}_d(t)Y(t) \right) dt + \mu_d(0)Y_0 - \mu_d(\widetilde{T})Y(\widetilde{T}) + V_2^*(\widetilde{T}, Y(\widetilde{T})) \end{aligned}$$

Since the firm chooses x, I and $Y(\widetilde{T})$, but not $\widetilde{T}$, the optimally conditions imply

$$dV = \int_0^{\widetilde{T}} \left(\frac{\partial H_1}{\partial x}dx + \frac{\partial H_1}{\partial I}dI + \frac{\partial H_1}{\partial Y}dY + \dot{\mu}_d(t)dY \right) dt + \frac{\partial V_2^*}{\partial Y(\widetilde{T})}dY(\widetilde{T}) - \mu_d(\widetilde{T}_-)dY(\widetilde{T}) = 0$$

where we denote by $\widetilde{T}_-$ the terminal date of the first part of the program. The condition is satisfied if (24), (25), (26) hold as well as if

$$\frac{\partial V_2^*}{\partial Y(\widetilde{T})} = \mu_d(\widetilde{T}_-)$$

We conclude that $\mu_d(t)$ is continuous at date $\widetilde{T}$ for the firm's choice to be optimal, since $Y(t)$ is continuous implying that $\mu_d(\widetilde{T}_-) = \mu_d(\widetilde{T}_+)$.

A.2 The social value of green capital and date $\widetilde{T}$

In this appendix the program of the regulator is analyzed to derive the optimally conditions and in particular the one concerning the date $\widetilde{T}$ and the continuity of ζ_2 and ζ_3.

The regulator's problem from date $\widetilde{T}$ onward is:

$$W_2 \equiv \max_{\sigma(.)} \int_{\widetilde{T}}^{\infty} e^{-\rho t} \left[u(\phi Y(t)) - C(C'^{-1}(\mu_d(t))) \right] dt$$

with $\dot{Y}(t) = C'^{-1}(\mu_d(t)) - \delta Y(t)$ and $\dot{\mu}_d(t) = (\rho + \delta)\mu_d(t) - \phi(u'(\phi Y(t)) + \sigma(t))$ and some initial conditions at T. In particular $Y(\widetilde{T}) = \tilde{Y}$ and $X(\widetilde{T}) = \overline{X}$ given. The associated present value Hamiltonian is:

$$\mathcal{H}_2^r = \left[u(\phi Y) - C(C'^{-1}(\mu_d)) \right] e^{-\rho t} + \zeta_2 \left[C'^{-1}(\mu_d) - \delta Y \right] + \zeta_3 \left[(\rho + \delta)\mu_d - \phi(u'(\phi Y) + \sigma(t)) \right]$$

Let us denote by the superscript * the paths identified by first order conditions. A crucial property of the corresponding value function is that it neither depends on $Y(\widetilde{T})$ nor on $X(\widetilde{T})$ as free variables, since they are fixed at $\tilde{Y}$ and $\bar{X}$ respectively. On the contrary, $\mu_d(\widetilde{T})$ is free. Hence the value-function W_2^* is a function of $\widetilde{T}$ and $\mu_d(\widetilde{T})$ and one can write: $W_2^*(\widetilde{T}, \mu(\widetilde{T}))$. Moreover the following envelop conditions apply:

$$\frac{\partial W_2^*}{\partial \widetilde{T}} = -H_2^*(\widetilde{T})$$
$$\frac{\partial W_2^*}{\partial \mu_d(\widetilde{T})} = \zeta_3^*(\widetilde{T})$$

The program up to date $\widetilde{T}$ is

$$\max_{\sigma} W = \int_0^{\widetilde{T}} e^{-\rho t}\left[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d(t)))\right] dt + W_2^*(\widetilde{T}, \mu_d(\widetilde{T}))$$

with $\dot{X}(t) = u'^{-1}(\widetilde{\tau}) - \phi Y(t)$, $\dot{Y}(t) = C'^{-1}(\mu_d(t)) - \delta Y(t)$ and $\dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi(\widetilde{\tau} + \sigma(t))$ and where $\widetilde{T}$ is free. The corresponding present value Hamiltonian is:

$$\begin{aligned}\mathcal{H}_1^r &= \left[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d))\right] e^{-\rho t} \\ &\quad - \zeta_1\left[u'^{-1}(\widetilde{\tau}) - \phi Y\right] + \zeta_2\left[C'^{-1}(\mu_d) - \delta Y\right] + \zeta_3\left[(\rho+\delta)\mu_d - \phi(\widetilde{\tau}+\sigma)\right]\end{aligned}$$

Writing the value-function in terms of the Hamiltonian, and integrating by parts, provides:

$$\begin{aligned}W &= \int_0^{\widetilde{T}} (\mathcal{H}_1^r - \zeta_1\dot{X} - \zeta_2\dot{Y} - \zeta_3\dot{\mu}_d)dt + W_2^*(\widetilde{T}, \mu_d(\widetilde{T})) \\ &= \int_0^{\widetilde{T}} (\mathcal{H}_1^r + \dot{\zeta}_1 X + \dot{\zeta}_2 Y + \dot{\zeta}_3\mu_d)dt \\ &\quad - \zeta_1(\widetilde{T})X(\widetilde{T}) + \zeta_1(0)X(0) - \zeta_2(\widetilde{T})Y(\widetilde{T}) + \zeta_2(0)Y(0) \\ &\quad - \zeta_3(\widetilde{T})\mu_d(\widetilde{T}) + \zeta_3(0)\mu_d(0) + W_2^*(\widetilde{T}, \mu_d(\widetilde{T}))\end{aligned}$$

First-order variation of W with respect to the state and control variables' paths, for free T yields:

$$\begin{aligned}dW &= \int_0^{T}\left[\frac{\partial \mathcal{H}_1^r}{\partial \sigma}d\sigma + \frac{\partial \mathcal{H}_1^r}{\partial X}dX + \frac{\partial \mathcal{H}_1^r}{\partial Y}dY + \frac{\partial \mathcal{H}_1^r}{\partial \mu_d}d\mu_d + \dot{\zeta}_1 dX + \dot{\zeta}_2 dY + \dot{\zeta}_3 d\mu_d\right] dt \\ &\quad + \left[\mathcal{H}_1^r(\widetilde{T}) + \dot{\zeta}_1(\widetilde{T})X(\widetilde{T}) + \dot{\zeta}_2(\widetilde{T})Y(\widetilde{T}) + \dot{\zeta}_3(\widetilde{T})\mu_d(\widetilde{T})\right] d\widetilde{T} + \frac{\partial W_2^*}{\partial \widetilde{T}}d\widetilde{T} + \frac{\partial W_2^*}{\partial \mu_d(\widetilde{T})}d\mu_d(\widetilde{T}) \\ &\quad + \left[-\dot{\zeta}_1(\widetilde{T})X(\widetilde{T}) - \dot{\zeta}_2(\widetilde{T})Y(\widetilde{T}) - \dot{\zeta_3(\widetilde{T})}\mu_d(\widetilde{T})\right] d\widetilde{T} - \zeta_3(\widetilde{T})d\mu_d(\widetilde{T}) \\ &= \int_0^{\widetilde{T}}\left[\frac{\partial \mathcal{H}_1^r}{\partial \sigma}d\sigma + \left(\frac{\partial \mathcal{H}_1^r}{\partial X} + \dot{\zeta}_1\right)dX + \left(\frac{\partial \mathcal{H}_1^r}{\partial Y} + \dot{\zeta}_2\right)dY + \left(\frac{\partial \mathcal{H}_1^r}{\partial \mu_d} + \dot{\zeta}_3\right)d\mu_d\right] dt \\ &\quad + \mathcal{H}_1^r(\widetilde{T})d\widetilde{T} + \frac{\partial W_2^*}{\partial \widetilde{T}}d\widetilde{T} + \frac{\partial W_2^*}{\partial \mu_d(\widetilde{T})}d\mu_d(\widetilde{T}) - \zeta_3(\widetilde{T})d\mu_d(\widetilde{T}))\end{aligned}$$

A trajectory is optimal if any departure from it decreases the value function, that is $\dot{W} \leq 0$ for any $dx(t)$, $dI(t)$, $dY(t)$ for $t \in [0, \tilde{T}]$ and $d\widetilde{T}$, which gives the following necessary conditions for an interior maximizer:

$$\begin{cases} \frac{\partial \mathcal{H}_1^r}{\partial \sigma} & = 0 \\ \frac{\partial \mathcal{H}_1^r}{\partial X} & = -\dot{\zeta}_1 \\ \frac{\partial \mathcal{H}_1^r}{\partial Y} & = -\dot{\zeta}_2 \\ \frac{\partial \mathcal{H}_1^r}{\partial \mu_d} & = -\dot{\zeta}_3 \\ \frac{\partial W_2^*}{\partial T} & = -\mathcal{H}_1(\widetilde{T}) \\ \frac{\partial W_2^*}{\partial \mu_d(\widetilde{T})} & = \zeta_3^*(\widetilde{T}) \end{cases}$$

Hence we have that the choice of $\widetilde{T}^\diamond$ is such that the marginal benefit of delay the transition is nil, since , which proves the first sentence of (iii) in Proposition 4.

Moreover, we have $\zeta_3^*(t)$ continuous at T (since it is equal to $\frac{\partial W_2^*}{\partial \mu_d(T)}$ in both programmes, and μ_d is continuous) and the continuity of the Hamiltonian $\mathcal{H}_1^{r*}(\widetilde{T}) = \mathcal{H}_2^{r*}(\widetilde{T})$. This implies:

$$\begin{aligned} &[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d(\widetilde{T}_-)))]e^{-\rho\widetilde{T}} \\ &- \zeta_1(\widetilde{T}_-)(u'^{-1}(\widetilde{\tau}) - \phi Y(\widetilde{T})) + \zeta_2(\widetilde{T}_-)(C'^{-1}(\mu_d(\widetilde{T}_-)) - \delta Y(\widetilde{T})) + \zeta_3(\widetilde{T}_-)\left[(\rho+\delta)\mu_d(\widetilde{T}_-) - \phi(\widetilde{\tau} + \sigma(\widetilde{T}_-))\right] \\ =&[u(\phi Y(\widetilde{T})) - C(C'^{-1}(\mu_d(\widetilde{T}_+)))]e^{-\rho\widetilde{T}} \\ &+ \zeta_2(\widetilde{T}_+)(C'^{-1}(\mu_d(\widetilde{T}_+)) - \delta Y(\widetilde{T})) + \zeta_3(\widetilde{T}_+)\left[(\rho+\delta)\mu_d(\widetilde{T}_+) - \phi(u'(\phi Y(\widetilde{T})) + \sigma(\widetilde{T}_+))\right] \end{aligned}$$

Taking into account (34) with (i) in Lemma 1 gives $u'^{-1}(\widetilde{\tau}) = \phi Y(\widetilde{T})$, and that in cases *c-f* $\sigma \in (0, \sigma_{max})$ we have $\zeta_3(\widetilde{T}) = \zeta_3(\widetilde{T}_-) = \zeta_3(\widetilde{T}_+) = 0$, the continuity of the Hamiltonians implies

$$\begin{aligned} &(-C(C'^{-1}(\mu_d(\widetilde{T}_-))))e^{-\rho\widetilde{T}} \\ &+ \zeta_2(\widetilde{T}_-)(C'^{-1}(\mu_d(\widetilde{T}_-)) - \delta Y(\widetilde{T})) \\ =&(-C(C'^{-1}(\mu_d(\widetilde{T}_+))))e^{-\rho\widetilde{T}} \\ &+ \zeta_2(\widetilde{T}_+)(C'^{-1}(\mu_d(\widetilde{T}_+)) - \delta Y(\widetilde{T})) \end{aligned}$$

Hence the continuity of the private value of green capital (28), i.e. $\mu_d(\widetilde{T}_-) = \mu_d(\widetilde{T}_+)$ which the regulator must respect, imply that the social value of green capital is continuous: $\zeta_2(\widetilde{T}_-) = \zeta_2(\widetilde{T}_+)$ as stated in the second sentence of (iii) in Proposition 4.

A.3 "Small" carbon tax

In this case, $\widetilde{\tau} < \zeta_1(0)$, $\zeta_3(t) = 0$ and $\sigma(t) \in [0, \sigma_{max}]$ $\forall t \in [0, \widetilde{T}]$. There may exist overshooting or not: $\widetilde{\tau} \in [\underline{\tau}_1, \overline{\tau}]$.

The dynamic system to be solved before $\widetilde{T}$ is:

$$\begin{aligned}
\dot{Y} &= \frac{1}{c_2}(\mu_d - c_1) - \delta Y \\
\dot{\mu}_d &= (\rho+\delta)\mu_d - \phi(\widetilde{\tau} + \sigma) \\
\sigma &= \zeta_1 - \widetilde{\tau} \\
\dot{\zeta}_2 &= (\rho+\delta)\zeta_2 - \phi\zeta_1 \\
\dot{\zeta}_1 &= \rho\zeta_1
\end{aligned}$$

Using the optimality conditions, it reduces to:

$$\dot{Y} = \frac{1}{c_2}(\mu_d - c_1) - \delta Y \tag{A.1}$$

$$\dot{\mu}_d = (\rho+\delta)\mu_d - \phi\zeta_1(0)e^{\rho t} \tag{A.2}$$

that is a differential system of two equations and two unknowns, Y and μ_d. This system yields the following second order linear differential equation in Y:

$$\ddot{Y} - \rho\dot{Y} - \delta(\rho+\delta)Y = (\rho+\delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\zeta_1(0)e^{\rho t}$$

The solution of this equation is the sum of the solution of the homogeneous equation and of a particular solution of the non-homogeneous equation. The homogeneous equation has two real roots of opposite sign, $\rho+\delta$ and $-\delta$. A particular solution of the non-homogeneous equation is found by the method of undetermined coefficients. We guess that the solution has the form $\alpha + \beta e^{\rho t}$. Then

$$-\delta(\rho+\delta)(\alpha + \beta e^{\rho t}) = (\rho+\delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\zeta_1(0)e^{\rho t}$$

which yields:

$$\alpha = -\frac{c_1}{c_2\delta} \quad \text{and} \quad \beta = \frac{\phi}{c_2\delta(\rho+\delta)}\zeta_1(0)$$

The solution is therefore

$$Y(t) = A_1 e^{(\rho+\delta)t} + A_2 e^{-\delta t} + \frac{\phi}{c_2\delta(\rho+\delta)}\zeta_1(0)e^{\rho t} - \frac{c_1}{c_2\delta} \tag{A.3}$$

where A_1 and A_2 are constants of integration.

Using (A.3) we can recover $\mu_d(t)$:

$$\mu_d(t) = c_2(\dot{Y}(t) + \delta Y(t)) + c_1 = A_1 c_2(\rho + 2\delta)e^{(\rho+\delta)t} + \frac{\phi}{\delta}\zeta_1(0)e^{\rho t} \tag{A.4}$$

The four unknowns are A_1, A_2, $\zeta_1(0)$ and $\widetilde{T}$. The four conditions allowing to compute them are the three boundary conditions (initial and terminal conditions on Y plus the continuity of μ_d at $\widetilde{T}$, proved below) and the carbon budget exhaustion equation:

$$\begin{aligned} &Y(0) = Y_0 \\ &Y(\widetilde{T}) = \frac{u'^{-1}(\widetilde{\tau})}{\phi} \\ &\mu_d(\widetilde{T}^-) = \mu_d(\widetilde{T}^+) \\ &\overline{X} = u'^{-1}(\widetilde{\tau})\widetilde{T} - \phi \int_0^{\widetilde{T}} Y(t)dt \end{aligned} \tag{A.5}$$

$\mu_d(\widetilde{T}^+)$ is the private value of green capital corresponding to an initial condition $Y(\widetilde{T})$ on the saddle path of the clean era. It depends on the steady state values Y^* and μ^*, on $Y(\widetilde{T})$ and on the parameters.

A.4 "Large" carbon tax

In this case, $\widetilde{\tau} > \zeta_1(0)$, $\zeta_3(t) > 0$ and $\sigma(t) = 0$ $\forall t \in [0, T_0]$. $\widetilde{\tau} \in [\overline{\tau}, \overline{\tau}_2]$.

The dynamic system to be solved for $t \in [0, T_0]$ is:

$$\dot{Y} = \frac{1}{c_2}(\mu_d - c_1) - \delta Y \tag{A.1}$$

$$\dot{\mu}_d = (\rho + \delta)\mu_d - \phi\widetilde{\tau} \tag{A.6}$$

It yields the following second order linear differential equation in Y:

$$\ddot{Y} - \rho\dot{Y} - \delta(\rho + \delta)Y = (\rho + \delta)\frac{c_1}{c_2} - \frac{\phi}{c_2}\widetilde{\tau}$$

which can be solved with the same method as in the previous case, to obtain:

$$Y(t) = B_1 e^{(\rho+\delta)t} + B_2 e^{-\delta t} + \frac{\phi}{c_2\delta(\rho + \delta)}\widetilde{\tau} - \frac{c_1}{c_2\delta} \tag{A.7}$$

where B_1 and B_2 are constants of integration.

Using (A.7) we can recover $\mu_d(t)$:

$$\mu_d(t) = B_1 c_2(\rho + 2\delta)e^{(\rho+\delta)t} + \frac{\phi}{\rho + \delta}\widetilde{\tau} \tag{A.8}$$

The solution of the dynamic system for $t \in [T_0, \widetilde{T}]$ is described by the same set of equations as in the case of a "small" carbon tax (equations (A.3) and (A.4)).

The seven unknowns are B_1, B_2, A_1, A_2, $\zeta_1(0)$, T_0 and $\widetilde{T}$. The seven conditions allowing to compute them are the four conditions (A.5), to which must be added the three following conditions of continuity at T_0:

$$\begin{aligned} &\zeta_1(0)e^{\rho T_0} = \widetilde{\tau} \\ &Y(T_0^-) = Y(T_0^+) \\ &\mu_d(T_0^-) = \mu_d(T_0^+) \end{aligned} \tag{A.9}$$

Using $\zeta_3(T_0) = 0$, equation (44) in the text integrates into:

$$\zeta_3(t) = \frac{1}{c_2}\int_t^{T_0} e^{\delta(s-t)}(\mu_d(s) - \zeta_2(s))ds$$

Before T_0 the social value of green capital ζ_2 is not equal to its private value μ_d, as it is the case after T_0. It is lower, which we are going to prove below. For them to be equal it would have been necessary to tax green capital instead of subsidizing it, in order to slow down its accumulation.

Equation (43) in the text integrates into:

$$\zeta_2(t) = \left(\zeta_2(0) - \frac{\phi}{\delta}\zeta_1(0)\right)e^{(\rho+\delta)t} + \frac{\phi}{\delta}\zeta_1(0)e^{\rho t}$$

Equation (A.8) may be written as:

$$\mu_d(t) = \left(\mu_d(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau}\right)e^{(\rho+\delta)t} + \frac{\phi}{\rho+\delta}\widetilde{\tau}$$

We have $\mu_d(T_0) = \zeta_2(T_0)$ and $\zeta_1(0)e^{\rho T_0} = \widetilde{\tau}$. Therefore, at T_0,

$$\left(\zeta_2(0) - \frac{\phi}{\delta}\widetilde{\tau}e^{-\rho T_0}\right)e^{(\rho+\delta)T_0} + \frac{\phi}{\delta}\widetilde{\tau} = \left(\mu_d(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau}\right)e^{(\rho+\delta)T_0} + \frac{\phi}{\rho+\delta}\widetilde{\tau}$$

i.e.

$$\mu_d(0) - \zeta_2(0) = \left(1 + \frac{\rho}{\delta}e^{-(\rho+\delta)T_0} - \left(1+\frac{\rho}{\delta}\right)e^{-\rho T_0}\right)\frac{\phi}{\rho+\delta}\widetilde{\tau}$$

Let $f(t) = 1 + \frac{\rho}{\delta}e^{-(\rho+\delta)t}$ and $g(t) = \left(1+\frac{\rho}{\delta}\right)e^{-\rho t}$. f and g are positive functions from $[0,+\infty)$ to $(0, 1+\frac{\rho}{\delta}]$, with $f(0) = g(0) = 1 + \frac{\rho}{\delta}$ and $\lim_{t\to\infty} f(t) = \lim_{t\to\infty} g(t) = 0$. Besides, $f'(t) = -\frac{\rho(\rho+\delta)}{\delta}e^{-(\rho+\delta)t}$ and $g'(t) = -\frac{\rho(\rho+\delta)}{\delta}e^{-\rho t} = f'(t)e^{\delta t}$. As f' and g' are negative, it implies that $g'(t) < f'(t)\ \forall t > 0$, meaning that the g function decreases faster than the f function, i.e. $g(t) < f(t)\ \forall t > 0$. We can then conclude that $\mu_d(0) - \zeta_2(0) > 0$.

We must finally prove that $\mu_d(t) - \zeta_2(t) > 0\ \forall t \in [0, T_0]$.

We have:

$$\left(\mu_d(t) - \zeta_2(t)\right) e^{-(\rho+\delta)t} = \mu_d(0) - \zeta_2(0) - \frac{\phi}{\rho+\delta}\widetilde{\tau} + \frac{\phi}{\delta}\zeta_1(0) + \frac{\phi}{\rho+\delta}\widetilde{\tau}e^{-(\rho+\delta)t} - \frac{\phi}{\delta}\zeta_1(0)e^{-\delta t}$$

Replacing $\mu_d(0) - \zeta_2(0)$ by its expression obtained above and $\widetilde{\tau}$ by $\zeta_1(0)e^{\rho T_0}$ we obtain:

$$\begin{aligned}
\frac{e^{-(\rho+\delta)t}}{\phi\zeta_1(0)}\left(\mu_d(t) - \zeta_2(t)\right) &= \frac{1}{\delta(\rho+\delta)}\left(\delta + \rho e^{-(\rho+\delta)T_0} - (\rho+\delta)e^{-\rho T_0}\right)e^{\rho T_0} \\
&\quad - \frac{1}{\rho+\delta}e^{\rho T_0} + \frac{1}{\delta} + \frac{1}{\rho+\delta}e^{\rho T_0}e^{-(\rho+\delta)t} - \frac{1}{\delta}e^{-\delta t} \\
&= \frac{e^{-\delta t}}{\delta(\rho+\delta)}\left(\delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)}\right)
\end{aligned}$$

Therefore

$$\mu_d(t) - \zeta_2(t) = \frac{\phi\zeta_1(0)e^{\rho t}}{\delta(\rho+\delta)}\left(\delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)}\right) \;\; \forall t \in [0, T_0]$$

Let $F(t) = \delta(e^{\rho(T_0-t)} - 1) - \rho(1 - e^{-\delta(T_0-t)})$. We have $F(0) = \delta(e^{\rho T_0} - 1) - \rho(1 - e^{-\delta T_0}) > 0$ since $\mu_d(0) - \zeta_2(0) > 0$, $F(T_0) = 0$, and $F'(t) = -\delta\rho(e^{\rho(T_0-t)} - e^{-\delta(T_0-t)}) < 0$. F is monotonously decreasing from a positive value to 0. Therefore $F(t) > 0 \; \forall t \in [0, T_0]$. We can then conclude that $\mu_d(t) - \zeta_2(t) > 0$.

A.5 "Very small" carbon tax

In this case, $\widetilde{\tau} < \zeta_1(0)$, $\zeta_3(t) = 0$ and $\sigma(t) \in]0, \sigma_{max}[\; \forall t \in [0, T_{max}[$ and $\sigma(t) = \sigma_{max} \; \forall t \in [T_{max}, \widetilde{T}]$. $\widetilde{\tau} \in]0, \underline{\tau}_2]$.

The dynamic system to be solved before T_{max} is (A.1)–(A.2). After T_{max} it is (A.1)–(A.6), where $\widetilde{\tau}$ is replaced by $\widetilde{\tau} + \sigma_{max}$ in the second equation. Boundary conditions are the four conditions (A.5), to which must be added the three following conditions of continuity at T_{max}:

$$\begin{aligned}
\zeta_1(0)e^{\rho T_{max}} &= \widetilde{\tau} + \sigma_{max} \\
Y(T_{max}^-) &= Y(T_{max}^+) \\
\mu_d(T_{max}^-) &= \mu_d(T_{max}^+)
\end{aligned} \tag{A.10}$$

In order to compute the value of $\underline{\tau}_1$ we linearize the dynamic sytem characterizing the evolution of the economy in the clean phase around the steady state. It yields:

$$\mu_d(t) = \mu^* + c_2(\omega + \delta)(Y(\widetilde{T}) - Y^*)e^{\omega(t - \widetilde{T})}$$

with

$$\omega = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\right] < 0$$

and

$$Y^* = \frac{\mu^* - c_1}{\delta c_2}$$

Then:

$$\begin{aligned}\mu_d(\widetilde{T}^+) - c_1 &= \mu^* - c_1 + c_2(\omega+\delta)(Y(\widetilde{T}) - Y^*)\\ &= c_2(\omega+\delta)Y(\widetilde{T}) - c_2\omega Y^*\end{aligned}$$

and

$$\mu_d(\widetilde{T}^+) \geq c_1 \Leftrightarrow (\omega+\delta)Y(\widetilde{T}) \geq \omega Y^* \Leftrightarrow (\omega+\delta)\frac{u'^{-1}(\widetilde{\tau})}{\phi} \geq \omega Y^*$$

We have to determine the sign of $\omega + \delta$.

$$\begin{aligned}&\omega = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\right] \Rightarrow \rho - 2\omega = \sqrt{(\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}}}\\ &\Rightarrow (\rho - 2\omega)^2 = (\rho+2\delta)^2 + \frac{4\gamma}{c_2 Y^{*2}} \Rightarrow -4(\rho+\delta-\omega)(\omega+\delta) = \frac{4\gamma}{c_2 Y^{*2}}\end{aligned}$$

which shows that $\omega + \delta < 0$.

We finally have:

$$\mu_d(\widetilde{T}^+) \geq c_1 \Leftrightarrow u'^{-1}(\widetilde{\tau}) \leq \frac{\omega}{\omega+\delta}\phi Y^* \Leftrightarrow \widetilde{\tau} \geq u'\left(\frac{\omega}{\omega+\delta}\phi Y^*\right) = \underline{\tau}_1$$

As $\omega < 0$, $\frac{\omega}{\omega+\delta} > 1$, implying that $u'\left(\frac{\omega}{\omega+\delta}\phi Y^*\right) < u'(\phi Y^*)$. Therefore $\underline{\tau}_1 < \underline{\tau}$.

Finally, $\sigma_{max} = \sigma(\widetilde{T}^-)|_{\underline{\tau}_1}$.

B Uniqueness of the tax ensuring a balanced budget and meeting the carbon budget

We show in this appendix that to each level of the instrument corresponds one and only one carbon budget, meaning that when the regulator wants to respect a given carbon budget he does not have any choice regarding the level of the carbon tax.

With a quadratic investment cost function, the dynamic system characterizing the evolution of the economy in the carbon era (equations (47) in the text) reads:

$$\begin{cases} \dot{Y}(t) = & \frac{1}{c_2}(\mu_d(t) - c_1) - \delta Y(t) \\ \dot{\mu}_d(t) = & (\rho+\delta)\mu_d(t) - \frac{\widetilde{\tau} u'^{-1}(\widetilde{\tau})}{Y(t)} \end{cases}$$

It is easy to show that the steady state equilibrium of this system is a saddle point, and to compute the saddle branch (parametrized by $\widetilde{\tau}$). Then, starting from an initial green capacity Y_0 of shadow value $\mu_d(0)$ on the stable branch, green capital increases, its shadow value decreases, until date $\widetilde{T}$ when $Y(\widetilde{T}) = \frac{1}{\phi} u'^{-1}(\widetilde{\tau})$.

Finally, fossil use in the first phase is $x(t) = u'^{-1}(\widetilde{\tau}) - \phi Y(t)$, parametrized by $\widetilde{\tau}$ as well, and the exhaustion condition gives the unique $\widetilde{\tau}$ that allows the economy to respect the carbon budget, i.e. to satisfy

$$\overline{X} = \int_0^{\widetilde{T}} x(t)dt = u'^{-1}(\widetilde{\tau})\widetilde{T} - \phi \int_0^{\widetilde{T}} Y(t)dt$$

C Comparing subsidies and FIP

For a given $\widetilde{\tau}$, the regulator's program in the carbon era reads:

$$\max_{s(.)} \int_0^{\widetilde{T}} e^{-\rho t} \left[u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d(t) + s(t))) \right] dt$$

$$\dot{X}(t) = u'^{-1}(\widetilde{\tau}) - \phi Y(t)$$

$$\dot{Y}(t) = C'^{-1}(\mu_d(t) + s(t)) - \delta Y(t)$$

$$\dot{\mu}_d(t) = (\rho+\delta)\mu_d(t) - \phi\widetilde{\tau}$$

$$X(t) \le \overline{X}$$

$$s(t) \ge 0$$

The Hamiltonian of the program is:

$$\begin{aligned} \mathcal{H} =& u(u'^{-1}(\widetilde{\tau})) - C(C'^{-1}(\mu_d + s)) \\ & - \zeta_1(u'^{-1}(\widetilde{\tau}) - \phi Y) + \zeta_2(C'^{-1}(\mu_d + s) - \delta Y) + \zeta_3 \left[(\rho+\delta)\mu_d - \phi\widetilde{\tau} \right] \end{aligned}$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \omega_s s$$

Considering the case of quadratic investment costs ($C(I) = c_1 I + \frac{1}{2}c_2 I^2$), the first order conditions are:

$$\mu_d + s = \zeta_2 + c_2\omega_s \tag{C.1}$$
$$\dot{\zeta}_1 = \rho\zeta_1 \tag{C.2}$$
$$\dot{\zeta}_2 = (\rho+\delta)\zeta_2 - \phi\zeta_1 \tag{C.3}$$
$$\dot{\zeta}_3 = -\delta\zeta_3 \tag{C.4}$$

Condition (C.1) shows that the subsidy during the carbon era is, when it is positive, equal to the difference between the private and social shadow values of renewable capacity (equation (48)). Besides, differentiating (C.1) with respect to time and using conditions (C.3) and (30) with $\tau(t) = \widetilde{\tau}$ and $\sigma(t) = 0$, yields:

$$\dot{s} = \dot{\zeta}_2 - \dot{\mu}_d = (\rho+\delta)s - \phi(\zeta_1 - \widetilde{\tau})$$

which can be integrated forward into (49). This proves (i).

Remember that the private green capital value μ_d is treated in the regulator problem as a state variable. However, this state variable is controllable (Dockner et al., 2000), in the sense that its initial value is free and can be controlled by the regulator through his choice of the subsidy. As a consequence, the initial value of the associated shadow price ζ_3 is nil. Then condition (C.4) implies that ζ_3 is nil all along the carbon era.

A straightforward comparison of the dynamic systems characterizing the evolution of the economy in the FIP case and in the subsidy to investment case shows that the two instruments are strictly equivalent. Indeed, the carbon era is characterized in the subsidy to investment case by;

$$\dot{Y} = \frac{1}{c_2}(\mu_d + s - c_1) - \delta Y$$
$$\dot{\mu}_d = (\rho+\delta)\mu_d - \phi\widetilde{\tau}$$
$$\mu_d + s = \zeta_2$$
$$\dot{\zeta}_2 = (\rho+\delta)\zeta_2 - \phi\zeta_1$$

and in the FIP case by:

$$\dot{Y} = \frac{1}{c_2}(\mu_d - c_1) - \delta Y$$
$$\dot{\mu}_d = (\rho+\delta)\mu_d - \phi(\widetilde{\tau} + \sigma)$$
$$\sigma = \zeta_1 - \widetilde{\tau}$$
$$\dot{\zeta}_2 = (\rho+\delta)\zeta_2 - \phi\zeta_1$$

It is easy to see that both systems coalesce into a dynamic system in Y and ζ_2, the social value of green capital, only:

$$\dot{Y} = \frac{1}{c_2}(\zeta_2 - c_1) - \delta Y$$
$$\dot{\zeta}_2 = (\rho + \delta)\zeta_2 - \phi\zeta_1(0)e^{\rho t}$$

Besides, the dynamic systems characterizing the evolution of the economy is the same in the clean era. Therefore, for a given level of the carbon tax $\widetilde{\tau}$, the solution is the same with the two instruments. ζ_2, the second best value of green capital, is the same in the two cases, and ζ_1, the second best carbon value, is the same as well. What differs are the private shadow value of green capital, μ_d, and the present value of subsidies or FIP provided to electricity producers during the carbon era. This proves (ii).

As Y and ζ_2 are the same with the two instruments, $\mu_d^s + s = \mu_d^\sigma$, where the superscripts s and σ denote respectively the case with a subsidy and the case with a FIP. It follows that:

$$\begin{aligned}
\dot{\mu}_d^s + \dot{s} = \dot{\mu}_d^\sigma &\Leftrightarrow (\rho + \delta)\mu_d^s - \phi\widetilde{\tau} + \dot{s} = (\rho + \delta)\mu_d^\sigma - \phi(\widetilde{\tau} + \sigma) \\
&\Leftrightarrow (\rho + \delta)\mu_d^s + \dot{s} = (\rho + \delta)\mu_d^\sigma - \phi\sigma \\
&\Leftrightarrow (\rho + \delta)\mu_d^s + \dot{s} = (\rho + \delta)(\mu_d^s + s) - \phi\sigma \\
&\Leftrightarrow \dot{s} = (\rho + \delta)s - \phi\sigma
\end{aligned}$$

that is equation (50) in the text.

We now turn to the comparison of the present values of subsidies and FIP. When the carbon tax is “small”, $\widetilde{\tau} \in [\underline{\tau}_1, \overline{\tau}]$, we are in the interior case ($s(t)$ and $\sigma(t) > 0\ \forall t \in [0, \widetilde{T}]$) and the discounted present value of the subsidies to green investment provided over the carbon era is:

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t) I(t) dt = \int_0^{\widetilde{T}} e^{-\rho t} s(t)(\dot{Y}(t) + \delta Y(t)) dt$$

By integration by parts,

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t)\dot{Y}(t) dt = e^{-\rho \widetilde{T}} s(\widetilde{T})Y(\widetilde{T}) - s(0)Y_0 - \int_0^{\widetilde{T}} e^{-\rho t}(\dot{s}(t) - \rho s(t))Y(t) dt$$

Then, with $s(\widetilde{T}) = 0$,

$$\int_0^{\widetilde{T}} e^{-\rho t} s(t) I(t) dt = -s(0)Y_0 - \int_0^{\widetilde{T}} e^{-\rho t}(\dot{s}(t) - (\rho + \delta)s(t))Y(t) dt$$

Finally, using equation (50) in the text we obtain (51. This proves (iii).

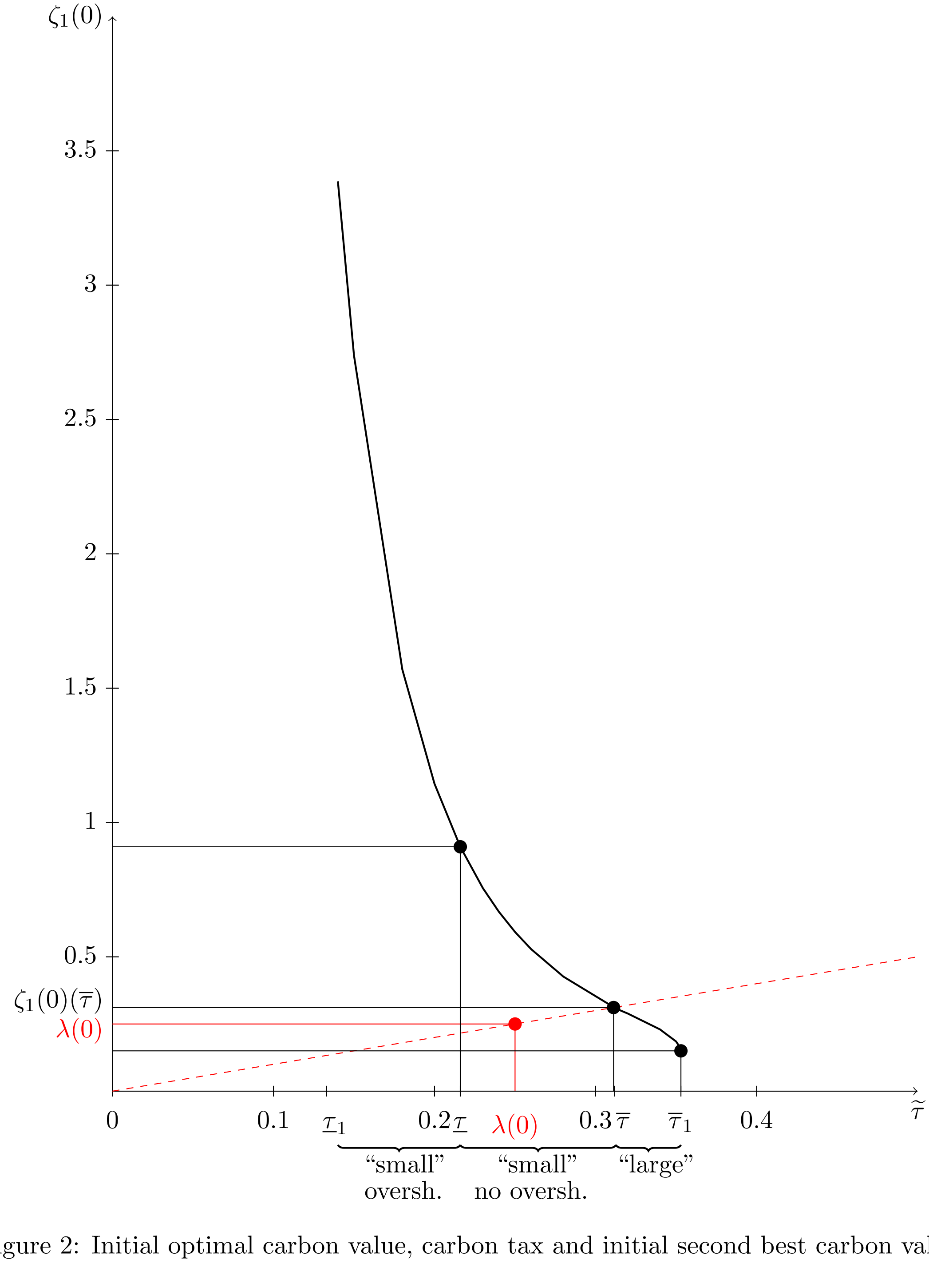

Figure 2: Initial optimal carbon value, carbon tax and initial second best carbon value

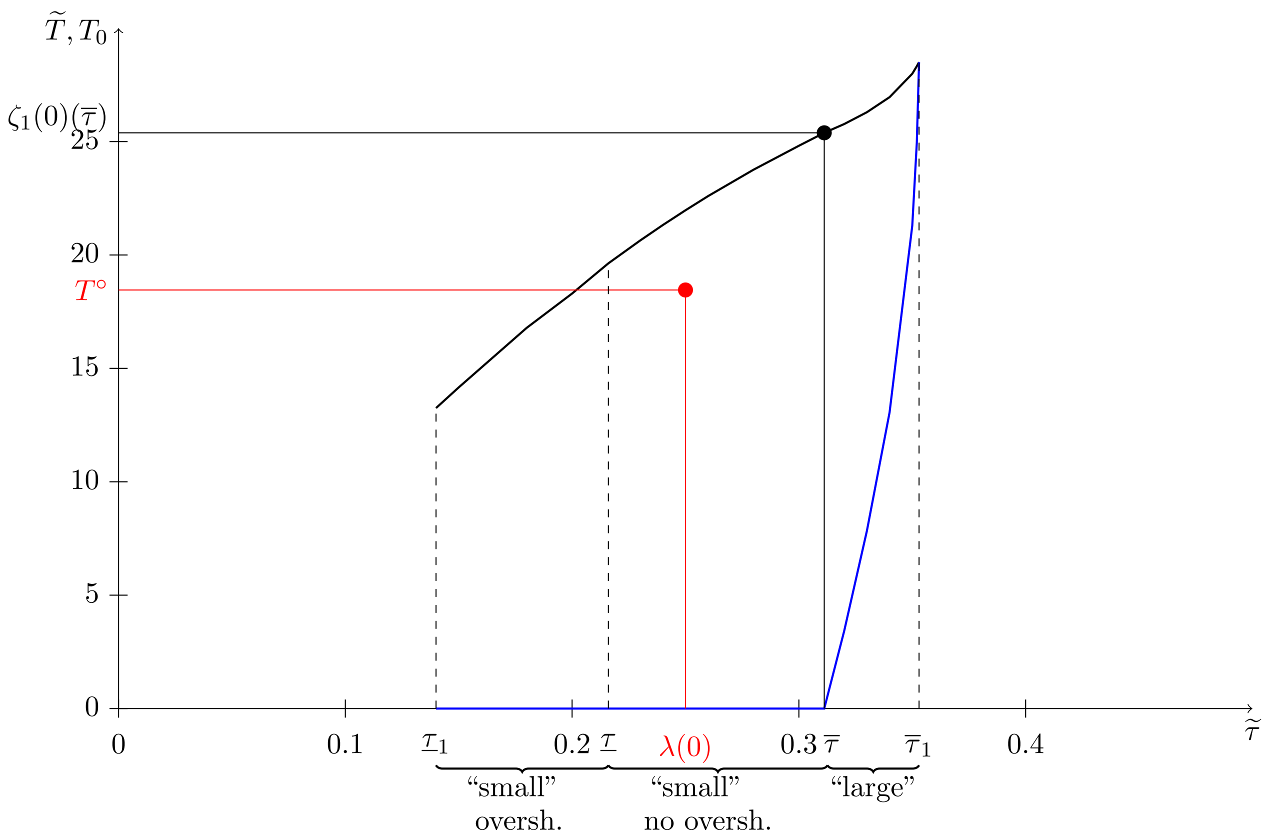

Figure 3: Dates of introduction of the FIP (T_0, blue) and of fossil phase out ($\widetilde{T}$, black) as functions of the carbon tax

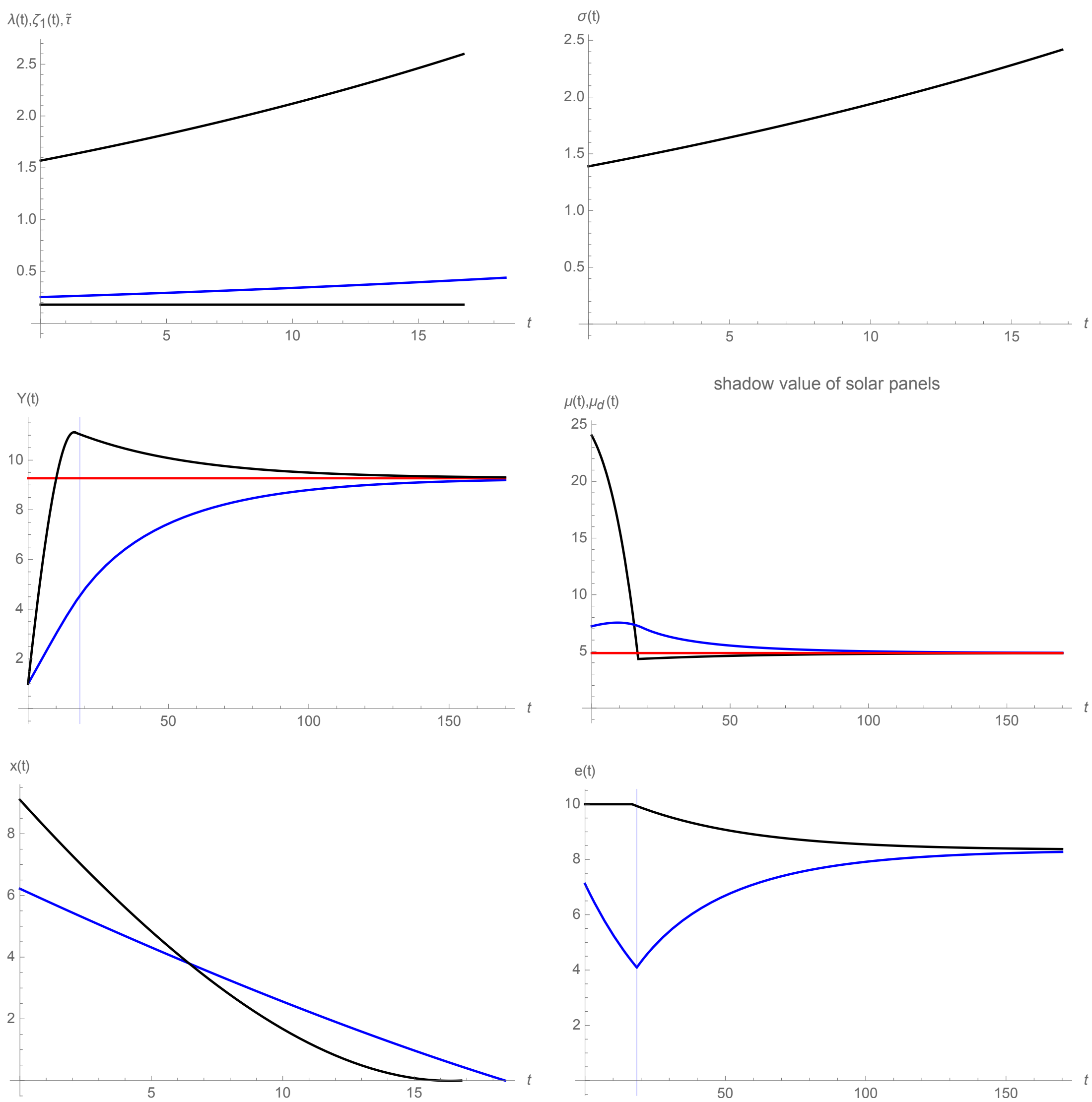

Figure 4: Optimal (blue) versus second best policy with very small constant carbon tax and FIP (black).

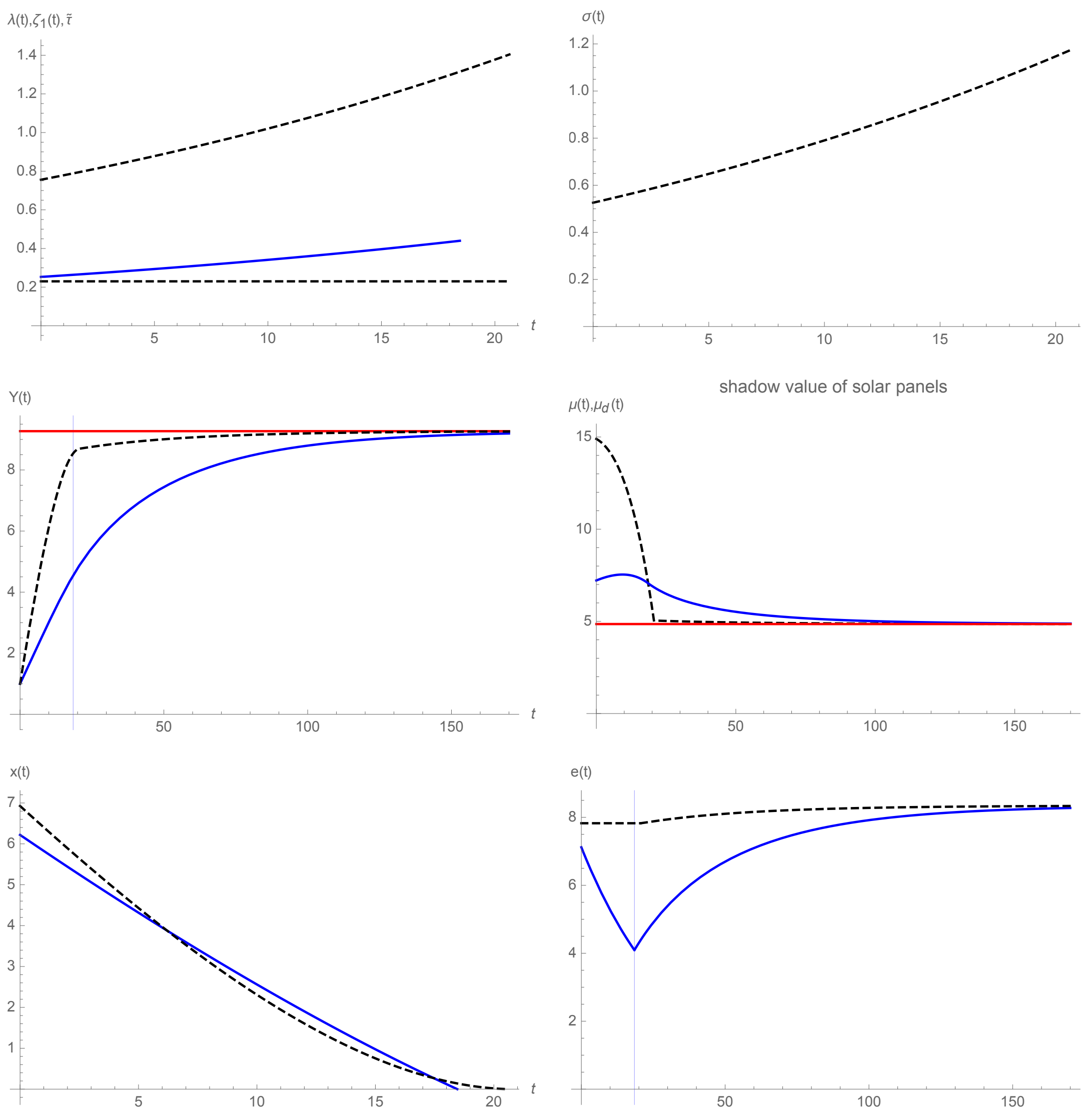

Figure 5: Optimal (blue) versus second best policy with small constant carbon tax and FIP (black, dashed).

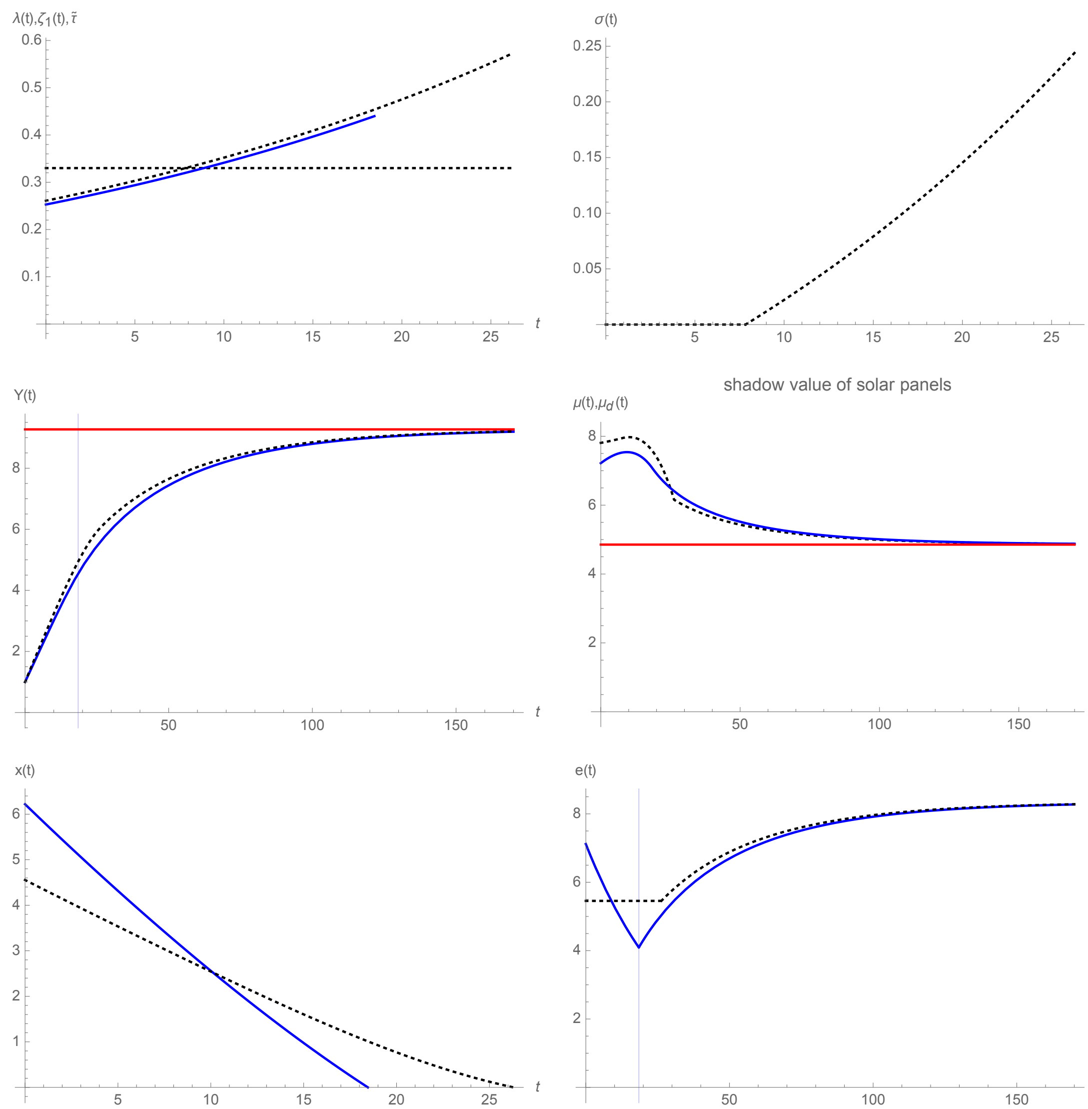

Figure 6: Optimal (blue) versus second best policy with large constant carbon tax and FIP (black, dashed).