An indirect, Luenberger approach to price performance

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1. Introduction

For decision makers and accountants, business performance analyses are primarily based on changes in indicators expressed in monetary terms such as turnover, cost, profit, etc. Then, in a second step, the decomposition of these indicators variations, as a quantity effect and a price effect, makes it possible to refine the analysis of business performance. Regarding their business operations, such a decomposition provides crucial information to managers to identify what relates, on the one hand, to a better management of input and output quantities and, on the other hand, to a search for better market price opportunities by improving prices over time.

We introduce in this paper a new measure for evaluating price performance changes of a Decision Making Unit (DMU) considered in its activity environment and further decompose it. This complements the existing activity-monitoring toolkit available to decisions makers. We consider that DMUs under evaluation operate in non-fully competitive environments. This assumption is in line with a growing literature in production economics stating that real-life markets often involve imperfect competition: DMUs are not strictly price takers and the input and output prices can depend on many factors such as bargaining power (e.g., Cherchye et al., 2002; Camanho and Dyson, 2008; Sahoo and Tone, 2013). One can trace back imperfect competition to Cournot (1838) who proposed a theory of oligopoly based on quantity competition between a few firms selling a homogeneous product. Bertrand (1883) suggested that price choice might well be a more suitable strategy than quantity choice which ultimately opened the debate of quantity versus price strategies among economists leading to product differentiation frameworks (e.g., Launhardt, 1885; Hotelling, 1929; Chamberlin, 1933; Lancaster,1966). For Launhardt (1885) for example, the dispersion over space of firms gives them some market power over the customers situated in their vicinity, which allows them to manipulate their prices to their own advantage. Product differentiation comes nowadays in two distinct forms: vertical differentiation and horizontal differentiation. With vertical product differentiation, it is the intrinsic quality of the good which is modelled whereas, with horizontal product differentiation, the good quality is intrinsically the same across firms, but the services attached to it or the general context may differ. In our non-parametric approach, where the production technology is common to all firms, vertical differentiation is not considered as such. Therefore, differences in prices come from horizontal differentiation. Horizontal product differentiation can have various sources in real life settings. It can be “spatial” and modelled through the distance between each vendor and a consumer as in Hotelling (1929). It can also come from differences in the access to facilities and infrastructures, differences in channels of product valorisation (e.g., retail vs. wholesale), firm-specific advantages, such as advertising and/or reputation effects, etc.

Within the imperfect market framework, decision makers are also interested in the comparison of their prices policy to their peers. Or, traditional measures in the production economics literature, such as profit, revenue, cost, or allocative efficiencies are based only on the firm's own prices, ignoring peers' prices. In this article, we include such a comparison of prices across different firms by elaborating on Ayouba et al. (2019) who recently introduced the concept of price advantage based on a comparison of prices among peers. This price effect is calculated indirectly by the difference between two different inefficiency scores: the first based on data in quantity and the second on data in value. Consequently, Ayouba et al. (2019) interpreted the difference as a price advantage if a company's evaluation is better with values rather than quantities; in this case, the company benefits from a better price environment for inputs and outputs compared to its peers. The authors have clearly shown that this overall price advantage is different from the concept of allocative efficiency, which considers only the evaluated business unit's prices. Moreover, the price advantage measure can be broken down into price effects specific to each input and output. In their empirical application concerning French farms in the context of the successive common agricultural policy reforms leading to the liberalisation of agricultural prices (1992–2013), they have illustrated the operational aspect of this new concept of price advantage.

In the same spirit as in Ayouba et al. (2019), but in an inter-temporal perspective, we define an indirect measure of performance change due to prices, the 'total price performance indicator' by comparing total factor productivity changes based on quantities to variations in overall performance based on value data. In addition, we propose a decomposition of value overall performance changes that mirrors the traditional breakdown of productivity gains into a variation in efficiency (catching up to the frontier) and a technical progress component (shift of frontier). Simultaneously, we show that the time evolution of the price advantage measure introduced in Ayouba et al. (2019) measures the gap between the value overall inefficiency change and the technical inefficiency change. Finally, we propose a decomposition of the total price performance indicator into a price advantage change and a price environment change. The latter is obtained as the gap between the shift in the value-based benchmark and the shift of the quantity-base benchmark. While the price advantage change measures a decision-maker's efforts to improve their prices compared to its peers, the price environment change can be related to an exogenous price shock, which affects all DMUs in the same market, even though the capacity to absorb the shock may be different among them.

The original Luenberger indicator introduced by Chambers et al. (Chambers, Chung, & Färe, 1996; Chambers, Färe, & Grosskopf, 1996) and Chambers (2002) was constructed to measure total factor productivity, using quantity data for estimations. We extend this indicator to value-base data in order to estimate an overall performance indicator. Our indicator is well fit to respond to business unit managers and decision makers' quest for improved performance in the short- and mid-run and thus keeps the input/output mix almost unchanged. Note that analyses based on profit maximisation approach not only do not use peers' price information in the evaluation process but furthermore apply to long run decisions where the input/output mix is more flexible.

Juo et al. (2015) is one of the few analyses that used a Luenberger framework to disentangle quantities and prices effects from a profitability indicator change. They developed a profit-oriented productivity indicator and showed that it can be decomposed into changes in technical efficiency, changes in allocative efficiency, a shift in technology and a shift in relative inputs/outputs prices. Similarly, Lin et al. (2017) developed a cost-oriented productivity indicator based on the directional Russell measure that leads to a decomposition of productivity change into four components: technical efficiency change, allocative efficiency change, technical change and input price change. In the same vein, Zhao et al. (2019) used a value technology approach to decompose the ratio of revenue to expenses through Malmquist type indices, which account for the contribution of allocative efficiency. More specifically, their proposed decomposition is given by the product of a Malmquist input-oriented productivity index and an allocation Malmquist productivity index, both based on input and output values. However, our approach significantly differs from theirs by the fact that we do not consider allocative efficiency but compare what they call a value-based productivity to a traditional quantity-base productivity in order to deduce a price performance indicator and its economic drivers (efficiency and frontier change). Since the proposed indicator is built under the assumption that the evaluated DMU cannot significantly modify its input/output mix, we conjecture that it is thus more suitable than the profit maximisation approach for short- to mid-run decisions.

In the articles cited above, price effects are measured for allocative efficient DMU, implying a cost-free adjustment of the size and the input/output mix according to its own relative price structure. This makes sense for long-term decisions but not for short- or mid-term ones. Consequently, compared to their frameworks, our analysis is positioned in the short term, keeps the size and the mix almost unchanged and does not assume any restrictive profit or cost optimisation hypothesis related to the producer's behaviour. Finally, our approach differs from the previous ones in the sense that we compare DMUs' price systems. Indeed, in a traditional profit maximising framework, only prices of the evaluated DMU are considered for comparisons to peers. As a result, a profit efficiency analysis remains a pure quantity effect by reallocating physical resources and fails to include differences in absolute prices among DMUs. Our approach fills this gap by introducing comparisons among DMUs' price systems embedded in the value model we proposed. Therefore, the capacity of DMUs to take advantage of their favourable market price environments for their given input/output mix is introduced as a price effect.

Another strand of literature which has shown an interest in the decomposition of profitability between productivity and price recovery changes is that of traditional empirical indexes or indicators (Fisher, Törnqvist or Bennet). A complete review is developed in Grifell-Tatjé and Lovell (2015), who further proposed a direct theoretical price index called Könus price recovery index, which is also based on a comparison among peers' prices. However, the authors mention that, contrary to empirical indexes, a theoretical one such as Könus does not decompose the global price recovery change by output and/or input variables. Besides, a further decomposition into an individual efficiency effect and a price environment shock is not mentioned. Finally, our Färe-Lovell Luenberger (LFL hereafter) indicators tackle productivity analysis in a different way from the one developed in the aforementioned approaches. First, compared to empirical indexes, our LFL price performance indicator allows a decomposition by economic drivers (catching up to the frontier and frontier shift in time). Secondly, compared to theoretical Könus price index, a decomposition of the price performance indicator and its economic drivers by output and/or input variables can now be computed as the difference between their respective value and quantity effects.

The structure of the paper is as follows. Section 2 introduces the methodology used in this paper based on Färe-Lovell distance functions. Section 3 presents our main contributions related to the three Luenberger productivity indicators based on quantities, values and prices; their respective decompositions into economic drivers (efficiency and frontier change); and their further decompositions by specific input and output variables. Section 4 illustrates the model and the last section concludes.

2. Methodology

This section presents the general setting for our analysis with a specific focus on the Färe-Lovell directional distance function (Färe and Lovell, 1978; Briec, 2000).

2.1 Setup

We consider a set of *N* DMUs. For each DMU, denote the production vector of physical quantities by $\mathbf{Q} = (\mathbf{QI}, \mathbf{QO})$, where **QI** contains all *J* input quantities and **QO** all *K* output quantities. Let $\mathbf{P} = (\mathbf{PI}, \mathbf{PO})$ be the price vector where **PI** contains all *J* input prices and **PO** all *K* output prices.

Denote $\mathbf{V} = (\mathbf{VI}, \mathbf{VO})$ the value vector containing the input cost vector and the output revenue vector respectively. For an observed DMU *a* $(a = 1, ..., N)$, $VI_{j,a}$ denotes the cost incurred for the input *j* (*j=1,...,J*), obtained as the product between physical quantity and its price: $VI_{j,a} = QI_{j,a} \times PI_{j,a}$.

Furthermore, let $VI_a = \sum_{j=1}^{J} VI_{j,a}$ be the DMU a total cost. In the same way, $VO_{k,a}$ denotes the revenue from the output k ($k=1,...,K$), obtained as the product between physical quantity and its price: $VO_{k,a} = QO_{k,a} \times PO_{k,a}$. Moreover, let $VO_a = \sum_{k=1}^{K} VO_{k,a}$ be the DMU a total revenue.

2.2 A Färe-Lovell directional distance function and its economic interpretation

The production technology T_Q of DMUs represents the set of all feasible inputs-outputs combinations and is defined as follows:

$$T_Q = \left\{ \left(\mathbf{QI}, \mathbf{QO} \right) \in \mathrm{R}_+^{J+K} : \mathbf{QI} \text{ can produce } \mathbf{QO} \right\} \qquad (1).$$

Assumptions regarding technology are standard and are the followings: no free lunch, boundedness, closure, free disposability, and convexity (see Banker et al., 1984). In this frame, the Data Envelopment Analysis approximation of T_Q from a set of N observed DMUs, under constant returns to scale (CRS) is as follows:

$$\Psi_Q = \left\{ \left(\mathbf{QI}, \mathbf{QO} \right) : QI_j \geq \sum_{n=1}^{N} \lambda_n QI_{j,n}, \forall j = 1,...,J; QO_k \leq \sum_{n=1}^{N} \lambda_n QO_{k,n}, \forall k = 1,...,K; \lambda_n \geq 0, \forall n = 1...N \right\} \qquad (2).$$

The constant returns to scale (CRS) assumption regarding the production technology is by no means constraining and other assumptions could have been made, i.e., variable returns to scale or even non-increasing or non-decreasing returns to scale. Traditionally, all that one needs to do is to add a constraint on the sum of activity variables in equation (2) above $\left(\sum_n \lambda_n = 1, \sum_n \lambda_n \leq 1 \text{ and } \sum_n \lambda_n \geq 1, \text{respectively} \right)$.

Gaps between observed production plans and the estimated production technology's boundaries can be measured using a directional distance function (see Chambers, Chung, and Färe,1996), denoted $D_{Q,CCF} : (R_+^J \times R_+^K) \times (R_+^J \times R_+^K) \rightarrow R_+$ and defined as follows:

$$D_{Q,CCF}\left(\mathbf{Q}; \mathbf{G}^Q \right) = \sup_{\beta} \left\{ \beta \in \mathrm{R}_+ : \left(\mathbf{QI} - \beta \mathbf{GI}^Q, \mathbf{QO} + \beta \mathbf{GO}^Q \right) \in \Psi_Q \right\} \qquad (3),$$

with $\mathbf{G}^Q = \left(\mathbf{GI}^Q, \mathbf{GO}^Q \right)$ a strictly positive vector defining the direction of projection on the frontier, and *CCF* in $D_{Q,CCF}$ standing for Chambers, Chung, Färe. Properties of directional distance functions can be found in Chambers, Chung, et al. (1996). Note that $\left(\mathbf{QI}, \mathbf{QO} \right) \in \Psi_Q \Leftrightarrow D_{Q,CCF}(\mathbf{Q}; \mathbf{G}^Q) \geq 0$. β is a common scalar for all inputs and outputs defining a unique efficiency score for the DMU under evaluation.

In this paper, we go beyond this traditional setting to exhaust all input and output slacks, as proposed by Färe and Lovell (1978) and extended by Briec (2000). Thus, the measure obtained is fully Pareto efficient. Specifically, we make use of a Färe-Lovell directional distance function where $\boldsymbol{\beta}^{Q} = \left(\boldsymbol{\beta}\mathbf{I}^{Q}, \boldsymbol{\beta}\mathbf{O}^{Q}\right) \geq 0,$ a positive vector with specific components for each input and output. The resulting directional distance function (in the quantity space) is defined as:

$$D_{Q}\left(\mathbf{Q};\mathbf{G}^{Q}\right) = \sup_{\boldsymbol{\beta}^{Q}} \left\{ \begin{array}{l} \mathbf{PO}\ \boldsymbol{\beta}\mathbf{O}^{Q}\mathbf{GO}^{Q} + \mathbf{PI}\ \boldsymbol{\beta}\mathbf{I}^{Q}\mathbf{GI}^{Q} : \left(\mathbf{QI}\ - \boldsymbol{\beta}\mathbf{I}^{Q}\mathbf{GI}^{Q}, \mathbf{QO}\ + \boldsymbol{\beta}\mathbf{O}^{Q}\mathbf{GO}^{Q}\right) \in \Psi_{Q}, \\ \mathbf{GI}^{Q} \geq 0, \mathbf{GO}^{Q} \geq 0, \mathbf{G}^{Q} = \left(\mathbf{GI}^{Q}, \mathbf{GO}^{Q}\right) \neq 0, \boldsymbol{\beta}\mathbf{I}^{Q} \geq 0, \boldsymbol{\beta}\mathbf{O}^{Q} \geq 0 \end{array} \right\} \qquad (4).$$

Some interesting relations can be drawn between the distance function presented in equation *(4)* and the usual profit maximisation program (see *LPs 3-5*, in Appendix *A*). In terms of similarities, we notice that if the direction of the evaluation is the production plan of the evaluated DMU $\left(\mathbf{G}^{Q} = \left(\mathbf{GI}^{Q}, \mathbf{GO}^{Q}\right) = \left(\mathbf{QI}, \mathbf{QO}\right)\right)$, the objective function in equation (*4*) can be interpreted as a potential profit improvement resulting from the adoption of efficient input and output quantities for given prices. However, despite its interpretation as a means for profit improvement, $D_{Q}\left(\mathbf{Q};\mathbf{G}^{Q}\right)$ in equation *(4)* distinguishes itself from a classical profit maximisation measure in several regards.

Firstly, in a classical profit maximisation program, complete reallocation of inputs and outputs is possible: DMUs can always reduce the production of some output if its relative price is deemed disadvantageous and increase the use of some input if its relative cost is more advantageous. In our setting, DMUs maintain their size and input/output mix almost unchanged.[1] In equation (4), by imposing positive scores for all inputs and outputs, inputs and outputs reallocations within the optimal production plan are of limited impact. For any input *j,* a DMU cannot use more than its observed input quantity $QI_{j,a}$ and likewise, for every output *k*, a DMU cannot produce less than its observed output quantity $QO_{k,a}$. This assumption seems to fit better with short to mid-run decisions where decision makers do not have operational opportunities to modify their input/output mix through significant reallocations of their input and output quantities.[2]

Secondly, classical profit maximisation analysis is always performed under a variable returns to scale assumption. Indeed, with constant returns to scale, DMU's increase of outputs and inputs is

[1] Compared to a radial measure where the input/output mix is constant, the directional distance function approach may introduce a change in the DMU's mix. However, this alteration is limited in our approach where the direction is taken as the evaluated DMU production plan.

2 Recently, Färe et al. (2019) have proposed a Farrell-type measure for profit efficiency that synthesizes all previous approaches. However, in their framework too, it is assumed that producers can freely operate all necessary changes of input scale and input mix in addition to output changes.

unbounded and the solution to the maximisation problem is infinite. In our setting, we deal indirectly with this issue by constraining the input/output mix of the evaluated DMU. Indeed, by imposing positive input quantity contraction coefficients $\left(\boldsymbol{\beta}\mathbf{I}^{Q} \geq 0\right)$, DMUs cannot use a greater input quantity than the observed one. Therefore, they cannot infinitely increase their outputs. Consequently, the DMU size remains unchanged. This represents an advantage of our directional distance function, as it can be applied even to constant returns to scale analysis.

In our analysis, the direction is always defined by the production plan of the evaluated DMU $\left(\mathbf{G}^{Q} = \left(\mathbf{GI}^{Q},\, \mathbf{GO}^{Q}\right) = \left(\mathbf{QI}, \mathbf{QO}\right)\right)$. Moreover, we divide the directional distance function in equation (4) by a scalar, which is the total revenue observed for the evaluated DMU. Therefore, we obtain the directional distance function in equation (5):

$$D_{Q}\left(\mathbf{Q};\mathbf{G}^{Q}\right) = \sup_{\boldsymbol{\beta}^{Q}} \left\{ \begin{array}{l} \dfrac{1}{\mathbf{e}^{O}\mathbf{VO}}\left(\mathbf{PO}\ \mathbf{QO}\,\boldsymbol{\beta}\mathbf{O}^{Q} + \mathbf{PI}\ \mathbf{QI}\,\boldsymbol{\beta}\mathbf{I}^{Q}\right) : \left(\mathbf{QI}\ - \boldsymbol{\beta}\mathbf{I}^{Q}\mathbf{g}^{QI}, \mathbf{QO}\ + \boldsymbol{\beta}\mathbf{O}^{Q}\mathbf{GO}^{Q}\right) \in \Psi_{Q}, \\ \mathbf{GI}^{Q} \geq 0, \mathbf{GO}^{Q} \geq 0, \mathbf{G}^{Q} = \left(\mathbf{GI}^{Q}, \mathbf{GO}^{Q}\right) \neq 0, \boldsymbol{\beta}\mathbf{I}^{Q} \geq 0, \boldsymbol{\beta}\mathbf{O}^{Q} \geq 0 \end{array} \right\} \quad \textit{(5),}$$

where $\mathbf{e}^{0}$ is a unitary vector with the output quantity vector dimension.

Next, we define $\boldsymbol{\alpha}^{V} = \left(\boldsymbol{\alpha}^{VI}, \boldsymbol{\alpha}^{VO}\right)$ as a vector containing respectively all individual input cost and output revenue shares in the total revenue. For *DMU a*, two general terms in this vector would be $\alpha_{a,k}^{VO} = \dfrac{VO_{a,k}}{VO_{a}}$ and $\alpha_{a,j}^{VI} = \dfrac{VI_{a,j}}{VO_{a}}$. Note that while $\sum_{k} \alpha_{a,k}^{VO} = 1$, this is not the case for $\sum_{j} \alpha_{a,j}^{VI} = \dfrac{VI_{a}}{VO_{a}}$, which can be interpreted as a cost to revenue ratio for the *DMU a*. With this notation, the directional distance function becomes

$$D_{Q}\left(\mathbf{Q};\mathbf{G}^{Q};\boldsymbol{\alpha}^{V}\right) = \sup_{\boldsymbol{\beta}^{Q}} \left\{ \begin{array}{l} \boldsymbol{\beta}\mathbf{O}^{Q}\boldsymbol{\alpha}^{VO} + \boldsymbol{\beta}\mathbf{I}^{Q}\boldsymbol{\alpha}^{VI} : \left(\mathbf{QI}\ - \boldsymbol{\beta}\mathbf{I}^{Q}\mathbf{GI}^{Q}, \mathbf{QO}\ + \boldsymbol{\beta}\mathbf{O}^{Q}\mathbf{GO}^{Q}\right) \in \Psi_{Q}, \\ \mathbf{GI}^{Q} \geq 0, \mathbf{GO}^{Q} \geq 0, \mathbf{g}^{Q} = \left(\mathbf{GI}^{Q}, \mathbf{GO}^{Q}\right) \neq 0, \boldsymbol{\beta}\mathbf{I}^{Q} \geq 0, \boldsymbol{\beta}\mathbf{O}^{Q} \geq 0 \end{array} \right\} \quad \textit{(6)}$$

and measures the sum of output quantities increases and input quantity decreases weighed by their respective output and input shares. Thus, our directional distance function has a very convenient interpretation as a profit margin potential increase due to the adoption of optimal input and output quantities by the DMU.

Note that our directional distance function is different from the directional slacks-based measure proposed in Fukuyama and Weber (2009) and further used by Mahlberg and Sahoo (2011) to study Luenberger total factor productivity change in several regards. First, as shown above, because our directional distance function has a direct economic interpretation and, secondly, by imposing strictly positive scores, we ensure that the input/output mix and the DMU size stay relatively unchanged. However, both models share a common feature related to the estimation of individual inputs and outputs scores: the possibility to recover specific Luenberger indicators for each of the inputs and outputs.

The directional distance function in equation (6) is obtained using quantity-base data. However, a similar measure can be defined based on value data (revenues and costs). In line with Sahoo et al. (2014) and Zhao et al. (2019), we assume that the value-based technology satisfies the same standard assumptions as the quantity-base technology. Indeed, it is acceptable to consider that: (i) strictly positive revenues cannot be obtained from zero costs (no free lunch), (ii) infinite revenues are not allowed with a finite cost vector, in other words, there is always an upper bound for each output price (boundedness), (iii) the value-based technology is closed, (iv) less output revenues can always be obtained with more costs and inversely (free disposability), (v) if two cost vectors can be used to obtain two output revenue vectors, then any linear combination of these cost vectors can also be used to obtain some linear combination of these value vectors (convexity).

The resulting distance function is defined below:

$$D_V\left(\mathbf{V};\mathbf{G}^V;\boldsymbol{\alpha}^V\right)=\sup_{\boldsymbol{\beta}^V}\left\{\boldsymbol{\beta}\mathbf{O}^V\boldsymbol{\alpha}^{VO}+\boldsymbol{\beta}\mathbf{I}^V\boldsymbol{\alpha}^{VI}:\left(\mathbf{VI}-\boldsymbol{\beta}\mathbf{I}^V\mathbf{GI}^V,\mathbf{VO}+\boldsymbol{\beta}\mathbf{O}^V\mathbf{GO}^V\right)\in\Psi_V\right\} \qquad (7),$$

with $\mathbf{G}^{\mathrm{V}}=\left(\mathbf{GI}^{\mathrm{V}},\mathbf{GO}^{\mathrm{V}}\right)$ a strictly positive vector defining the direction of projection on the frontier and $\boldsymbol{\beta}^{\mathrm{V}}=\left(\boldsymbol{\beta}\mathbf{I}^{\mathrm{V}},\boldsymbol{\beta}\mathbf{O}^{\mathrm{V}}\right)\geq 0$ a strictly positive vector.

Compared to a classic profit optimisation program (see *LP 3-5* in Appendix *A*), this setting ensures that a DMU cannot reduce costs or increase revenue by a complete reallocation of the input/output mix. In equation (7), for any input j, a DMU cannot spend more than its observed input cost $VI_{j,a}$, and likewise, for every output k, a DMU cannot obtain less than its observed output revenue $VO_{k,a}$. This assumption seems more suitable for short- to mid-run decisions where decision makers do not have operational opportunities to modify their mix through significant reallocations of their input and output values.

The distance functions defined in (6) and (7) therefore give the profit margin rate if the DMU adopted optimal quantities/values, given the technical/value benchmark. It follows that these distance functions can be directly interpreted as technical/value inefficiency scores:

$$\begin{aligned} TE &= D_Q\left(\mathbf{Q};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) \\ VE &= D_V\left(\mathbf{V};\mathbf{G}^V;\boldsymbol{\alpha}^V\right) \end{aligned} \qquad (8).$$

The following section shows how Luenberger performance indicators can be constructed based on the Färe-Lovell directional distance functions presented here.

3. A Luenberger Färe-Lovell approach of productivity, overall performance and prices

In this section, we provide a method for computing Luenberger-inspired productivity, overall performance and price performance indicators based on the Färe-Lovell distance functions introduced above. Decompositions of each of these indicators into economic drivers (catching up to the frontier and frontier shift) is also provided. A further decomposition of all indicators and their respective economic drivers along the specific input/output variables is also possible.

3.1 A Luenberger Färe-Lovell approach of productivity change

Chambers (2002) has defined the Luenberger productivity indicator for a given period t as the difference between directional distance functions where technology is fixed in t and the evaluated DMU is considered alternatively in periods t and $t+1$. Using the directional distance function expressed in equation (6), we define the *LFL* productivity indicator for period t as

$$LQ^{t}\left(\mathbf{Q}^{t+1},\mathbf{Q}^{t};\mathbf{G}^{\mathbf{Q}};\boldsymbol{\alpha}^{\mathbf{V}}\right)=D_{Q}^{t}\left(\mathbf{Q}^{t};\mathbf{G}^{\mathbf{Q}};\boldsymbol{\alpha}^{\mathbf{V}}\right)-D_{Q}^{t}\left(\mathbf{Q}^{t+1};\mathbf{G}^{\mathbf{Q}};\boldsymbol{\alpha}^{\mathbf{V}}\right) \quad \textit{(9).}$$

Note that in Chamber's notations, while the observed DMU's production plan is considered at two different periods in time, the direction of evaluation ($\mathbf{G}^{\mathbf{Q}}$) is identical in the two terms in the equation (9).

In the same way, the Luenberger Färe-Lovell productivity indicator for period $t+1$ is defined as

$$LQ^{t+1}\left(\mathbf{Q}^{t+1},\mathbf{Q}^{t};\mathbf{G}^{\mathrm{Q}};\boldsymbol{\alpha}^{V}\right)=D_{Q}^{t+1}\left(\mathbf{Q}^{t};\mathbf{G}^{\mathrm{Q}};\boldsymbol{\alpha}^{V}\right)-D_{Q}^{t+1}\left(\mathbf{Q}^{t+1};\mathbf{G}^{\mathrm{Q}};\boldsymbol{\alpha}^{V}\right)$$

The *LFL* productivity indicator change between the two periods t and $t+1$ is defined as the arithmetic mean between the previous two indicators. Hereafter, we drop the arguments in the Luenberger productivity indicators for simplicity.

$$LQ^{t,t+1}=\frac{1}{2}\left[LQ^{t}+LQ^{t+1}\right]$$

A positive value for $LQ^{t,t+1}$ indicates that the DMU has observed an increase in its total factor productivity (*TFP*) between periods t and $t+1$.

As shown initially by Chambers, Färe, and Grosskopf (1996), and later for modified settings by Mahlberg and Sahoo (2011), this *LFL* productivity indicator change may be decomposed additively into quantity-efficiency change $(LQEC)$ and quantity-technology change $(LQTC)$ generally interpreted as the technical change in the spirit of the Malmquist productivity index decomposition in Färe et al. (1994):

$$LQ^{t,t+1} = LQEC + LQTC \quad (10),$$

with

$$LQEC = D_Q^t\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) - D_Q^{t+1}\left(\mathbf{Q}^{t+1};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) \quad (11),$$

$$LQTC = \frac{1}{2}\left\{\begin{array}{l}\left[D_Q^{t+1}\left(\mathbf{Q}^{t+1};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) - D_Q^t\left(\mathbf{Q}^{t+1};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right)\right] + \\ \left[D_Q^{t+1}\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) - D_Q^t\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right)\right]\end{array}\right\} \quad (12).$$

A positive value for the Luenberger quantity-efficiency change (*LQEC*) indicates that the DMU has increased its efficiency between periods *t* and *t+1*. Likewise, a positive sign for the Luenberger quantity-technology change (*LQTC*) can be interpreted as a sign that the DMU has benefitted from a positive upward shift of the production frontier.

3.2 A Luenberger Färe-Lovell indicator for overall performance change

While the previous Färe-Lovell Luenberger quantity productivity indicator (LQ) undeniably offers a precise image of the DMU performance based on its quantity information, its field of applicability is, nevertheless, restrained. Indeed, in everyday decisions, costs of production factors and output revenues are more likely to be considered by decision makers. In what follows, we incorporate output and input values to obtain a Färe-Lovell Luenberger indicator for overall performance change.

Following the quantity-base productivity indicators developed in the previous section, we introduce the equivalent inter-period Luenberger value-base indicators:

$$LV^t\left(\mathbf{V}^{t+1},\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) = D_V^t\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_V^t\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right)$$

$$LV^{t+1}\left(\mathbf{V}^t,\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right) = D_V^{t+1}\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_V^{t+1}\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right)$$

Finally, the change in the *LFL* overall performance indicator between the two periods *t* and *t+1* is defined as follows:

$$LV^{t,t+1} = \frac{1}{2}\left[LV^t + LV^{t+1}\right]$$

A positive value for $LV^{t,t+1}$ is interpreted as an increase of the evaluated DMU overall performance (in value) between the two periods. Improvements are due to a decrease in the inputs cost and/or an increase in the output revenue.[3]

As with its quantity-base counterpart, the *LFL* value-based indicator can be shown to decompose as the sum between a Luenberger value efficiency change indicator and a Luenberger value-technology change indicator.

$$LV^{t,t+1} = LVEC + LVTC \qquad (13).$$

The DMU's efforts to improve its value efficiency are revealed through the value efficiency change $(LVEC)$ which can arise either through an improved technical efficiency, or through a more advantageous input-output price system or a mix of both.

$$LVEC = D_V^t\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_V^{t+1}\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right) \qquad (14).$$

The value-technology change indicator $(LVTC)$ measures the shift of the frontier in the value space. It can be interpreted as the result of either technical progress and/or an exogenous price shock common to all DMUs.

$$LVTC = \frac{1}{2}\left\{\begin{array}{l}\left[D_V^{t+1}\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_V^t\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right)\right]+\\ \left[D_V^{t+1}\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_V^t\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right)\right]\end{array}\right\} \qquad (15).$$

The next subsection introduces the *LFL* indicator for total price performance and its decomposition into an efficiency component and a global environment change component.

3.3 A Luenberger Färe-Lovell indicator for total price performance and its decomposition along the main economic drivers

Given that both Luenberger indicators ($LV^{t,t+1}$ and $LQ^{t,t+1}$) are expressed in common units (points of profit margin rate change), we can define the Luenberger price performance indicator as the difference between the two of them.

$$LP^{t,t+1} = LV^{t,t+1} - LQ^{t,t+1} \qquad (16).$$

[3] Another possibility is that the cost has decreased more rapidly than the decrease in the output revenue or that the output revenue has increased more rapidly than the cost.

This is an indirect price effect recovered from value-base and quantity-base models as a residual term. A positive value for $LP^{t,t+1}$ indicates that the DMU improves its price performance between the two periods.

Next, we show that the Luenberger price performance change indicator defined in equation (16) decomposes as the sum between a DMU individual characteristic (price advantage change) and a market, exogenous price shock component.

Given the equation of the Luenberger value-efficiency change (*LVEC*) indicator in equation (14), we observe that this measure can be rewritten as:

$$\begin{aligned} LVEC = & \left[D_V^t\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_Q^t\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right)\right] + D_Q^t\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) \\ & - \left[D_V^{t+1}\left(\mathbf{V}^{t+1};\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_Q^{t+1}\left(\mathbf{Q}^{t+1};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right)\right] + D_Q^{t+1}\left(\mathbf{Q}^{t+1};\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) \end{aligned} \qquad (17).$$

Ayouba et al. (2019) introduced the static price advantage measure as the increase in a DMU's profit margin rate resulting from a favourable input or output price environment. In the same spirit, we define it as the gap between technical inefficiency scores measured with value data and physical quantity data:

$$PA^t = D_V^t\left(\mathbf{V}^t;\mathbf{G}^V;\boldsymbol{\alpha}^V\right) - D_Q^t\left(\mathbf{Q}^t;\mathbf{G}^Q;\boldsymbol{\alpha}^V\right) \qquad (18).$$

A positive value for the price advantage measure is interpreted here as the sign that the observed DMU suffered from a disadvantageous price environment compared with its peers since the distance to the value benchmark is greater than its distance to the quantity benchmark. Thus, given the DMU's price system, its value inefficiency is greater than its technical inefficiency.

We further define the Luenberger price advantage change as the difference between period *t* and period *t+1* price advantage measures:

$$LPAC = PA_t - PA_{t+1} \qquad (19).$$

A positive value for the *LPAC* is interpreted as an improvement in the price advantage over the two periods, while a negative value is interpreted as a deterioration of the price advantage. A null value observed for this indicator is a sign that the DMU observed neutral price advantages, in other words, that it was technical and value efficient over the two periods (or, that technical and value inefficiency scores were equal to one another in both periods).

Rearranging terms in equation (17) and using the definitions in equations (11) and (19) for technical efficiency change (*LQEC*) and price advantage change (*LPAC*), we have,

$$\begin{aligned} LVEC &= PA_t - PA_{t+1} + \left[D_Q^t\left(\mathbf{Q}^t; \mathbf{G}^Q; \boldsymbol{\alpha}^V\right) - D_Q^{t+1}\left(\mathbf{Q}^{t+1}; \mathbf{G}^Q; \boldsymbol{\alpha}^V\right) \right] \\ &= LPAC + LQEC \end{aligned} \qquad (20).$$

Substituting this decomposition into the Luenberger overall performance change indicator (in equation 13), we find that the latter can be split as the sum between the Luenberger productivity indicator, the price advantage change and a new term that is the difference between the value-technology change and quantity-technology change:

$$\begin{aligned} LV^{t,t+1} &= LPAC + LQEC + LVTC \\ &= LPAC + \left(LQEC + LQTC\right) + \left(LVTC - LQTC\right) \\ &= LPAC + LQ^{t,t+1} + \left(LVTC - LQTC\right) \\ &= LQ^{t,t+1} + LPAC + \left(LVTC - LQTC\right) \end{aligned} \qquad (21).$$

Finally, we define the Luenberger price-global environment change (*LPGC*) as the difference between the Luenberger value-technology change and the Luenberger quantity-technology, that is to say, the change in the gap between the value-based technology and the quantity-base technology frontiers:

$$LPGC = LVTC - LQTC \qquad (22).$$

A positive value for *LPGC* will be interpreted as a positive price environment change affecting all DMUs since the shift in the value-based frontier is greater than the shift in the quantity-base technology.

Thus, we obtain a decomposition for the price performance indicator as the sum between price advantage change (*LPAC*) and price-global environment change (*LPGC*). While the first component reflects each DMU's individual efforts to improve their price advantage, the latter (*LPGC*) reflects a general shock in prices affecting the entire set of DMUs, even though the capacity to absorb this shock may be different among the DMUs.

$$LP^{t,t+1} = LPAC + LPGC \qquad (23).$$

Obviously, the price performance change indicator and its components are fit mainly for situations in which DMUs can, for different reasons, set distinct input and output prices. However, nothing prevents our indicator from being applied even in a price-taking situation, but in this case, the price indicators should be interpreted as arising from the relative system of input and/or output prices. To illustrate this point, suppose all DMUs are price takers and thus face the same input and output prices.

Obviously, efficient DMUs will obtain neutral price performance indicators, so a non-null price indicator can only be observed for inefficient DMUs as a sign that there exists a gap between a quantity indicator and its equivalent, value one. Let us suppose that we are dealing with a DMU which is both technical and value inefficient in the first period, with the value inefficiency being greater than the technical inefficiency. This means that the DMU has experienced a price disadvantage in that period. Suppose that, in the next period, the DMU has corrected for some of its technical inefficiency while prices remained constant throughout the two periods. Consider a case where we observe that the *LQEC* is positive and greater than the *LVEC*. We conclude that in the second period, the price disadvantage increased, leading to a negative value for the *LPAC*. How can we account for this event knowing that the market prices have been constant throughout the two periods? The answer lies in the fact that the DMU's quantity changes (here, a reduction in the technical inefficiency) are priced according to the market prices. Therefore, if the DMU concentrated its efforts towards a specific input or output for which the relative price was less advantageous, the increase in the *LQEC* indicator is more important than that of the *LVEC* indicator, and the final effect on the price advantage change is a negative one. In a price-taker context, technically inefficient DMUs' price indicators reflect the advantage or the disadvantage that affects the DMU with regards to the input and/or output system of relative prices. Therefore, even in a context of given prices, our indicator can be of help in order to target those inputs and/or outputs for which the relative price is more advantageous and seek to concentrate all efforts on the reduction of the technical inefficiency related to them.

3.4 Operationalisation of the Färe-Lovell distance functions in an intertemporal perspective

In order to propose a time evolution of productivity and overall performance indicators, distance functions must be defined for consecutive periods. In Luenberger's (1992) original work, the indicator is calculated in relation to a basket of commodities, which can be completely disconnected from the evaluated DMU. Chambers, Färe, et al. (1996) proposed several measures for their proportional (radial) distance function with regards to the direction in which DMUs are evaluated and denoted ($\boldsymbol{g}_{\mathrm{x}}$, $\boldsymbol{g}_{\mathrm{y}}$), where $\mathbf{x}$ is the input quantity vector, and $\mathbf{y}$ is the output quantity vector. Thus, they show that the Shephard distance function is only a special case in their model if the direction is taken only in the evaluated unit's output space: $(\boldsymbol{g}_{\mathrm{x}}, \boldsymbol{g}_{\mathrm{y}}) = (\mathbf{0}, \mathbf{y})$. This approach can be generalised to both inputs and outputs by considering that $(\boldsymbol{g}_{\mathrm{x}}, \boldsymbol{g}_{\mathrm{y}}) = (\mathbf{x}, \mathbf{y})$, thus retrieving the proportional distance function introduced by Briec (1997). The authors also consider a symmetrical approach for inputs and outputs, where the direction of evaluation becomes the unit, $(\boldsymbol{g}_{\mathrm{x}}, \boldsymbol{g}_{\mathrm{y}}) = (\mathbf{1}, \mathbf{1})$, but for this, data needs to be mean-deflated. However, it is important to highlight that, in order to ensure commensurability of direction functions, especially when they refer to different period technologies, the direction for evaluation needs to be identical, as it follows very clearly from Chambers' (2002) study.

In some works published more recently (e.g. Briec & Kerstens (2004); Mahlberg & Sahoo (2011)), the direction of evaluation is either symmetric (both inputs and outputs vectors are considered) or asymmetric (only one vector is considered), but, more importantly, it is also period specific, which for generally stated directions, may lead to serious comparison or interpretation issues.[4]

In the present paper, where the goal is to evaluate and compare individual DMU's performance, it seemed natural to consider a direction of evaluation that is symmetric with regards to the DMU's input and output space. From this point, two subsequent questions arose: the first choice of the researcher is to decide in which period should the evaluated DMU be observed? Should the direction of the evaluation be the evaluated DMU's production plan observed in period t'? In period t+1? This issue is settled by considering both possibilities and then calculating the arithmetic mean of the resulting efficiency scores in the vein of a Bennet indicator. The second choice of the researcher or the decision maker concerns the weighing system $\boldsymbol{\alpha}^V$, which allows to aggregate individual input and output quantities (or values) and that leads to interpreting the objective function as the optimal profit margin rate (equations (6) and (7)). Again, one has to choose between the period *t* and the period *t+1* weighing system. In order to remain as general as possible, both possibilities are considered, and then their arithmetic mean is considered for actual calculations of the final productivity indicators.

Under these considerations, we now introduce the following notations:

- $t' = \{t, t+1\}$ is the time period in which the reference period technology is considered;
- $t'' = \{t, t+1\}$ is the time period pertaining to the evaluated DMU;
- $t''' = \{t, t+1\}$ is the time period associated with the direction considered, and finally;
- $t^{iv} = \{t, t+1\}$ is the time period related to the weighting system in the objective function;
- $m = \{Q, V\}$ is the space (quantity or value) used, and
- $\left(\mathbf{Y}^{m}, \mathbf{X}^{m}\right)$ designates the general vector characterising a DMU, depending on whether it is evaluated in the quantity space or the value space.

Note that given the different values that can be taken by the different parameters introduced above, a total of $2^5 = 32$ models are obtained that lead to the calculation of the *LFL* quantity and overall performance indicators specified above for each DMU, respectively. Computing time can be drastically optimised by solving an aggregate model for all firms together. Thus, whatever the number of DMUs, at most, 32 models are solved. This aggregation follows from the fact that each *LP* is

[4] In these works, the authors avoided this trap by using *à la* Shephard distance functions, which lead to proportional indicators with a convenient percentage change interpretation.

independent from the others, and thus, constraints can be pooled, and objective functions can be summed over.

$$\mathbf{D}^{m,t'}\left(\left(\mathbf{Y}^{m,t''},\mathbf{X}^{m,t''}\right);\mathbf{g}^{m,t'''};\boldsymbol{\alpha}^{V^{t^{iv}}}\right)=$$

$$\max_{\lambda,\beta O,\beta I}\sum_{a=1}^{N}\left[\left(\sum_{k=1}^{K}\beta O_{a,k}\alpha_{a,k}^{VO^{t^{iv}}}+\sum_{j=1}^{J}\beta I_{a,j}\alpha_{a,j}^{VI^{t^{iv}}}\right)\right]$$

$$\begin{array}{lll}
\sum_{n=1}^{N}\lambda_{a,n}Y_{n,k}^{m,t'}\geq Y_{a,k}^{m,t''}+GO_{a,k}^{m,t'''}\;\beta O_{a,k} & a=1,...,N,k=1,...,K & \\
\sum_{n=1}^{N}\lambda_{a,n}X_{n,j}^{m,t'}\leq X_{a,j}^{m,t''}-GI_{a,j}^{m,t'''}\;\beta I_{a,j} & a=1,...,N,j=1,...,J & , \\
\lambda_{a,n}\geq 0 & a=1,...,N,\; n=1,...,N & \\
\beta I_{a,j}\geq 0 & a=1,...,N, j=1,...,J & \\
\beta O_{a,k}\geq 0 & a=1,...,N,k=1,...,K &
\end{array}
\qquad
\begin{array}{l}
m=\{Q,V\}, \\
t'=\{t,t+1\}, \\
t''=\{t,t+1\}, \\
t'''=\{t,t+1\}, \\
t^{iv}=\{t,t+1\}
\end{array}$$

(LP1).

LP1 will always have a feasible solution for contemporaneous evaluated DMUs and technologies (*t*' = *t*''). However, when intertemporal comparisons are made, the traditional Luenberger approach is to allow for negative efficiency scores in order to measure technical progress. In this case, the evaluated DMU can be outside the production possibility set of the technology, and negative efficiency score must be allowed to project the DMU onto the frontier. In our *LFL* framework, we cannot apply this approach since the positivity of the efficiency score is mandatory in the Färe-Lovell approach. Fortunately, there is a solution, which consists of reversing the direction instead of allowing negative efficiency scores. For intertemporal comparisons, whenever *LP1* is infeasible, we use *LP*2 to get the solution. By inverting the direction $(\mathbf{GO},\mathbf{GI})$ we can keep the positivity constraint on $\boldsymbol{\beta}\mathbf{O}$ and $\boldsymbol{\beta}\mathbf{I}$ to satisfy the *LFL* requirement. Obviously, the maximisation is transformed into a minimisation since the direction is reversed.

$$\mathbf{D}^{m,t'}\left(\left(\mathbf{Y}^{m,t''},\mathbf{X}^{m,t''}\right);\mathbf{g}^{m,t'''};\boldsymbol{\alpha}^{V^{t^{iv}}}\right)=$$

$$\min_{\lambda,\beta O,\beta I}\sum_{n=1}^{N}\left[\left(\sum_{k=1}^{K}\beta O_{n,k}\alpha_{n,k}^{VO^{t^{iv}}}+\sum_{j=1}^{J}\beta I_{n,j}\alpha_{n,j}^{VI^{t^{iv}}}\right)\right]$$

$$\begin{array}{lll}
\sum_{n'=1}^{N}\lambda_{n,n'}Y_{n',k}^{m,t'}\geq Y_{n,k}^{m,t''}+\left(-GO_{n,k}^{m,t'''}\right)\beta O_{n,k} & n=1...,N,k=1,...,K & \\
\sum_{n'=1}^{N}\lambda_{n,n'}X_{n',j}^{m,t'}\leq X_{n,j}^{m,t''}-\left(-GI_{n,j}^{m,t'''}\right)\beta I_{n,j} & n=1...,N,j=1,...,J & , \\
\lambda_{n,n'}\geq 0 & n=1,...,N,\ n'=1,...,N & \\
\beta I_{n,j}\geq 0 & n=1,...,N,j=1,...,J & \\
\beta O_{n,k}\geq 0 & n=1,...,N,k=1,...,K &
\end{array}
\qquad
\begin{array}{l}
m=\{Q,V\},\\
t'=\{t,t+1\},\\
t''=\{t,t+1\},\\
t'''=\{t,t+1\},\\
t^{iv}=\{t,t+1\}
\end{array}$$

(LP2).

Of course, depending on the research objectives, this very general setting can be simplified. For example, if the research objectives impose the direction of evaluation as contemporaneous with the technology considered, then it suffices to set $t'=t'''$ in the LPs 1-2 above. Appendix B. gives a detailed illustration of how this model applies to obtain all LFL indicators introduced in this section.

The two linear programs (LP1 and LP2) have been defined under the constant returns to scale assumption. Indeed, previous literature states that, for productivity analysis, through Luenberger or other productivity indices, the CRS assumption is the accurate assumption as it controls for both technical and scale inefficiencies. As emphasized by Grifell-Tatjé and Lovell (1995), measuring productivity under variable returns to scale leads to systematic biased measures. Indeed, this idea has been shared by many authors who all insist on the fact that constant returns to scale are required for productivity measurement (Ray and Desli, 1997; Balk, 2001; Lovell, 2003). Moreover, Balk (2001) goes even further and says that independently of the returns to scale assumption behind the production technology, the productivity analysis should always be carried under the CRS assumption. This being said, it is at least possible to include a decomposition of the productive efficiency component into a technical and a scale component by using a variable returns to scale technology for computing the former.

Finally, an important property of this model, relevant for interpretations of the empirical results is that one is capable of decomposing the change in the profit margin rate calculated in the objective function for each input and output. Indeed, for a *DMU* a, $\beta O_{a,k}\alpha_{a,k}^{VO^{t^{iv}}}$ and $\beta I_{a,j}\alpha_{a,j}^{VI^{t^{iv}}}$ are the specific contributions of each output k and input j to its profit margin rate. This property is useful for decomposing all Luenberger indicators introduced above by specific variables (input and output) following the idea introduced by Kapelko et al. (2015).

4. Illustration

As mentioned above, the analysis introduced here is mainly fit to characterise situations in which DMUs can set their output prices and in which they can have an impact on their input prices. Indeed, the best frame for application of our indicators is that of imperfect competition, where prices are not homogenous among firms. While this is a frequent observation for commodity markets, different reasons can justify this state of facts. Indeed, the less competitive the factors markets and/or the produces markets, the higher the chances that firms enjoy different input costs and can set different output prices. Thus, the size of the firm, the geographical location, the ability to control key resources represent an incentive for firms to seek and depart from a strictly competitive pricing scheme.

In the following, we illustrate the applicability of our method and provide clear economic interpretations of the proposed model. To simplify the understanding of the different price-quantity interactions that may occur we introduce, starting from a fictitious market setting, different scenarios of price/quantity change policies. In a first paragraph we analyse the case where only one firm in the market modifies its output quantity, *ceteris paribus*, then only its price policy (*ceteris paribus*) and finally, both its output price and quantity strategy (*ceteris paribus*). We show that our proposed indicators react to these changes in the expected way. In a second paragraph, all firms in the market modify their input and output price and quantity strategies (which is the most likely case that can be observed in real markets) and we show how, with the help of the proposed indicators, one can synthetise these phenomena in productivity and performance changes for each firm.

The market illustration deals with situations in which DMUs are price makers. For this, we introduce a fictitious market setting made up of six DMUs (N=6, named A, B, …, F) producing two outputs (K=2) out of two inputs (J=2). Let us assume that in the first period, the six DMUs start with the input and output prices and quantities depicted in Table 1. Period 1 profit $\left(\Pi^1\right)$ is defined as the difference between total revenue (VO) to total cost (VI) $\left(\Pi = VO - VI\right)$.

Table 1. DMUs' inputs and outputs quantities and prices for period 1

DMU	**A**	**B**	**C**	**D**	**E**	**F**
QO_1^1	10	12	8	6	10	8
QO_2^1	10	8	12	10	8	10
QI_1^1	2	4	5	6	8	10

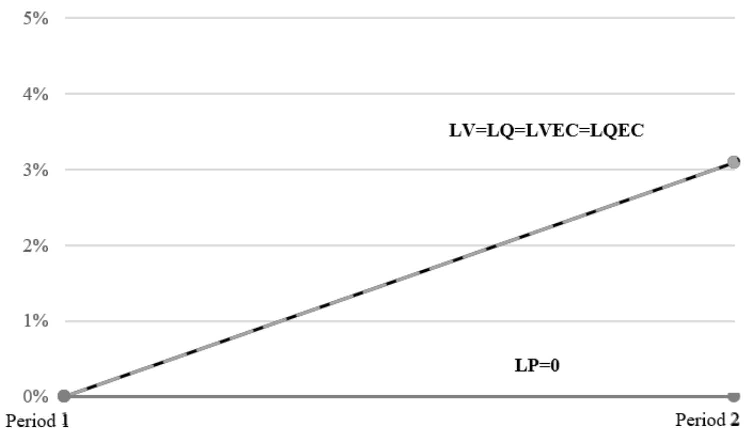
QI^{1}_{2}	12	4	10	6	6	2
PO^{1}_{1}	3	3	2.75	3	2,5	2
PO^{1}_{2}	4	2	4	3	3	3
PI^{1}_{1}	3	2	3	3	2	1
PI^{1}_{2}	1	2	1.5	2	1.75	4
Π^{1}	52	36	40	18	22.5	28
TE1	0%	0%	39%	86%	74%	0%
VE1	0%	0%	33%	70%	47%	0%

Legend:
The superscript '1' stands for the period of analysis. The subscripts '1' or '2' stand for the type of output/input quantity/price.

4.1 Case figures where only one DMU is affected by prix and/or quantity changes

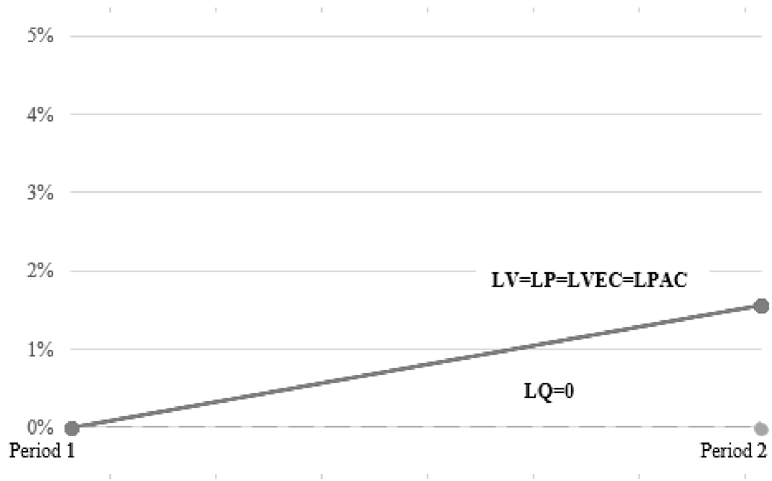
In this paragraph, we are analysing the case of only one DMU, here C, under three scenarios. In Scenario 1, it is the quantity of the output 1 that increases by 10%, *ceteris paribus*. This increase impacts positively the value performance indicator (*LV)* by 3 percentage points (pp), an increase that is entirely explained by the improvement observed by the productivity indicator (*LQ*). This evolution is illustrated in Figure 1 below. For the other DMUs, since the situation between the two periods has remained unchanged, one observes that all their productivity indicators, value and respectively price performance indicators are null (see Table 8 in Appendix C.1).

Figure 1. Overall performance, productivity and price performance evolutions for DMU C, under an output 1 quantity increase, ceteris paribus



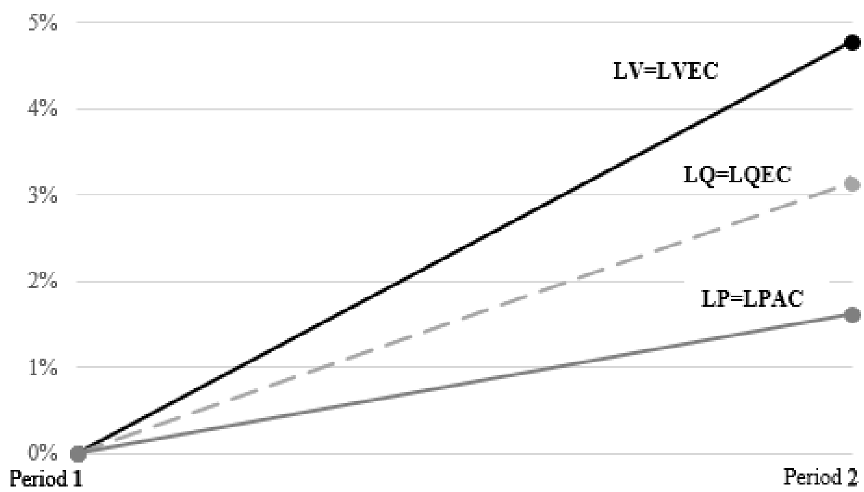
In Scenario 2, the price of the output 1 produced by DMU C increases by 5%, ceteris paribus. As expected, all quantity-related indicators remain unchanged, and the positive effect observed on the value performance indicator (*LV*, increase by 2pp) is fully explained by an increase in the price performance indicator (see Figure 2). Moreover, the transmission channel is via an improvement in the efficiency change indicators (*LVEC=LPAC*). Again, all performance indicators for the remaining DMUs are null between the two periods (see Table 9 in the Appendix C.1).

Figure 2. Overall performance, productivity and price performance evolutions for DMU C, under an output 1 price increase, ceteris paribus



In Scenario 3, both the quantity and the price of the output 1 produced by the DMU C increase (by 10 and respectively 5%), ceteris paribus. In Figure 3, we observe an increase in the total value performance (*LV*) by 5 pp, which is explained by a positive increase in the productivity indicator (*LQ*, by 3pp) and a positive increase in the price performance indicator (*LP*, by 2pp). Within each global indicator, the observed rise in performance is obtained via an improvement in the efficiency change. Thus, quantity efficiency change (*LQEC*) improves by 3pp while the quantity technology change is null. Similar evolutions are noticed in the price dimension, where the price advantage change indicator (*LPAC*) increased by 2pp, with the price-global environment change (*LPGC*) staying null. In this case figure too, all performance indicators are null for the rest of the DMUs (see Table 10 in the Appendix C.1).

Figure 3. Overall performance, productivity and price performance evolutions for DMU C, under an output 1 simultaneous quantity and price increases, ceteris paribus



4.2 General case figure where the entire market is affected by price and quantity changes

We adopt here a more general viewpoint where, in period 2 all DMUs' prices and quantities for inputs and outputs change in different proportions. Table 2 shows the resulting input and output quantities and prices (for each variable, the inter-period growth rate is calculated beneath). All DMUs' output quantities increased, and all input quantities decreased between the two periods. Similar phenomena affected output prices and, respectively, input prices. DMUs' technical and value inefficiency scores for both periods are also given. In the second period inefficiencies observed DMUs C, D in both the quantity and the value dimensions deepened. Defined as the gap between value and technical inefficiency scores (equation (18)), price advantages appear to be neutral for DMUs A, B and F in both periods. However, the price advantage measure is positive for DMUs C, D and E in period 1, meaning that these DMUs have a greater latitude to improve profits by absorbing their technical inefficiency than by absorbing their value efficiency. However, these three DMUs lost their price advantage from the initial period and experienced a price disadvantage in the second period.

Table 2. DMUs' inputs and outputs quantities and prices for period 2

DMU	A	B	C	D	E	F
QO_1^2 *(% change)*	12 *(20%)*	13.8 *(15%)*	8.8 *(10%)*	6.3 *(5%)*	11 *(10%)*	9.6 *(20%)*
QO_2^2 *(% change)*	11.5 *(15%)*	9.6 *(20%)*	13.2 *(10%)*	10.5 *(5%)*	8.4 *(5%)*	11.5 *(15%)*
QI_1^2 *(% change)*	1.6 *(-20%)*	3.2 *(-20%)*	4.75 *(-5%)*	5.4 *(-10%)*	7.6 *(-5%)*	8.5 *(-15%)*
QI_2^2 *(% change)*	10.2 *(-15%)*	3.4 *(-15%)*	9 *(-10%)*	5.7 *(-5%)*	5.7 *(-5%)*	1.6 *(-20%)*
PO_1^2 *(% change)*	3.6 *(20%)*	3.45 *(15%)*	2.8875 *(5%)*	3.3 *(10%)*	2.625 *(5%)*	2.5 *(25%)*

PO_2^2 *(% change)*	4.6 *(15%)*	2.5 *(25%)*	4.4 *(10%)*	3.15 *(5%)*	3.15 *(5%)*	3.45 *(15%)*
PI_1^2 *(% change)*	2.55 *(-15%)*	1.7 *(-15%)*	2.85 *(-5%)*	2.85 *(-5%)*	1.8 *(-10%)*	0.85 *(-15%)*
PI_2^2 *(% change)*	0.8 (-20%)	1.7 (-15%)	1.425 (-5%)	1.8 (-10%)	1.575 (-10%)	3.2 (-20%)
Π^2 *(% change)*	83.86 *(61%)*	60.39 *(68%)*	57.13 *(43%)*	28.21 *(57%)*	32.68 *(45%)*	51.33 *(83%)*
TE1	0%	0%	39%	86%	74%	0%
VE1	0%	0%	33%	70%	47%	0%
TE2	0%	0%	75%	138%	123%	0%
VE2	0%	0%	100%	156%	118%	0%

a. Decomposition by economic drivers

Luenberger productivity, overall performance and price performance indicators and their respective decompositions can be used to analyse concurrent price and quantity changes affecting DMUs. Table 4 shows that the changes in quantities and prices have had a positive impact on the overall performance change (*LV*) for all DMUs. For each DMU, we are further able to disentangle the quantity effect from the price effect.[5]

Table 4. Luenberger productivity indicators and their respective decompositions into efficiency change and technology change (general case scenario)

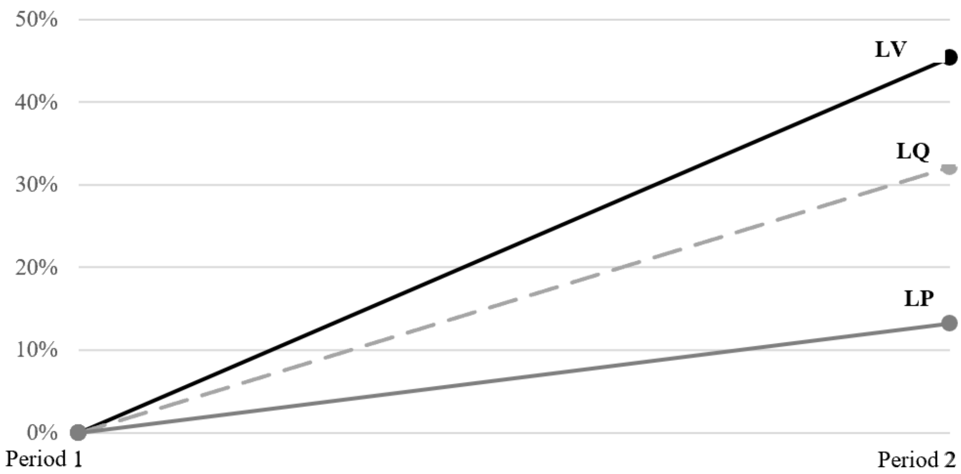
DMU	**A**	**B**	**C**	**D**	**E**	**F**
LVEC	0%	0%	-67%	-86%	-71%	0%
LVTC	53%	49%	113%	133%	120%	60%
LV	**53%**	**49%**	**45%**	**47%**	**49%**	**60%**
LQEC	0%	0%	-36%	-52%	-49%	0%
LQTC	23%	23%	68%	73%	66%	24%
LQ	**23%**	**23%**	**32%**	**21%**	**18%**	**24%**
LPAC	0%	0%	-31%	-34%	-22%	0%
LPGC	30%	26%	44%	60%	53%	36%
LP	**30%**	**26%**	**13%**	**26%**	**31%**	**36%**

The general situation for DMU A is depicted in the Figure 1. One notices that the increase in the overall performance by 53 percentage points (pp here after), as measured by *LV*, can be traced back to

[5] A more comprehensive table is presented in Table 11 in the Appendix C.2 where a scale component is added to the quantity effect. This table includes, within the quantity efficiency change dimension (*LQEC*) a component related to the pure technical efficiency change (*LQEC pure*) and a scale efficiency change indicator (*LScEC*) where the former is obtained with a VRS technology. The latter term is the result of the difference between the two *LQEC* measures obtained with a CRS technology and, a VRS technology respectively.

an improvement in its productivity (*LQ*) by 23 pp and an improvement in its price performance by 30 pp (*LP)*. The former increase is entirely due to the effect of technical change (*LQTC=LQ*), while the DMU has been technically efficient (*LQEC=0*) over the two periods. At the same time, the improvement in its price performance is due to the positive and exogenous shock on the market prices (*LPGC=LP, LPAC=0*).

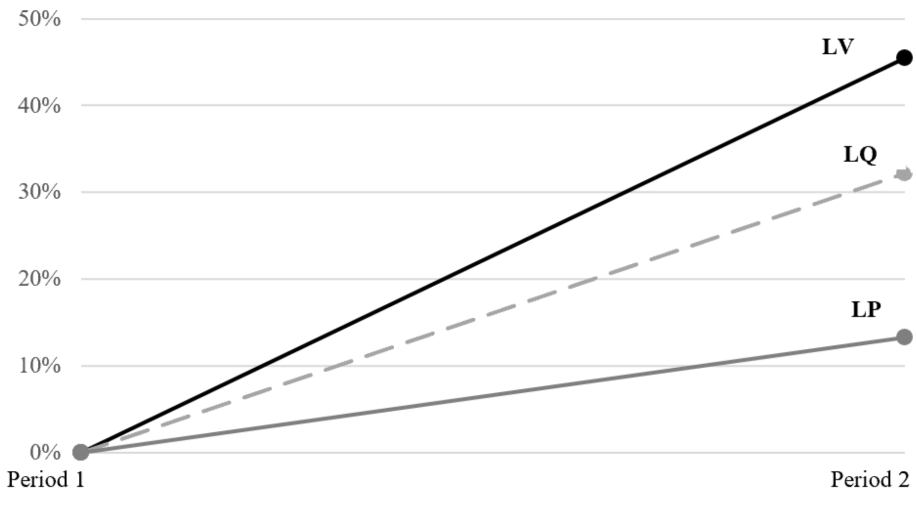
Figure 4. Overall performance, productivity and price performance evolutions for DMU A (general scenario)



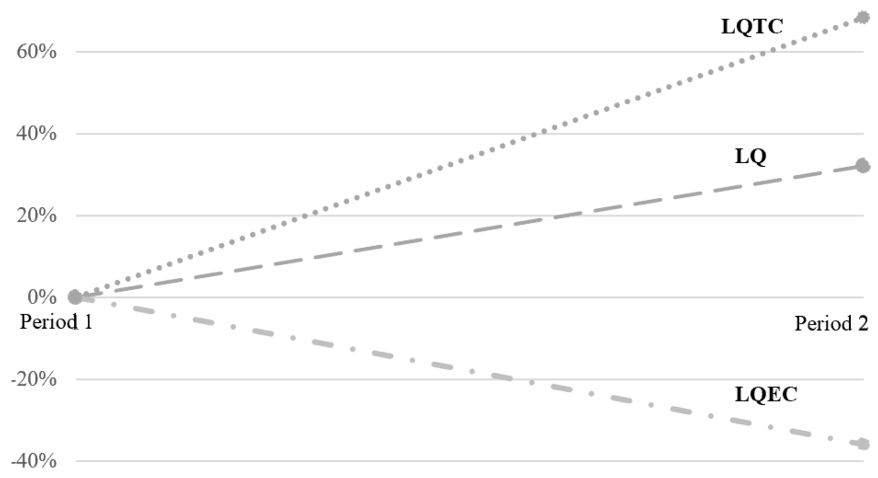
For DMU C, as shown in the Figure 5.a, the progression of the overall performance (*LV*) is 45 pp, with a contribution of 32 pp from productivity progression (*LQ*) and a contribution of 13 pp from the price performance (*LP*). Note that this DMU has enjoyed positive productivity change due to an important impact of the technical change (*LQTC*, by +68 pp), while its technical efficiency has had a negative impact (*LQEC*, -36 pp), as depicted in the Figure 5.b. Furthermore, as mentioned above, this DMU has lost its price advantage in the second period, which explains the negative value observed by the price advantage change indicator (*LPAC*, -31 pp) as shown in the Figure 5.c. However, the positive increase in the two output prices has materialised in a positive contribution for the price-environment change (*LPGC*) by 44 pp.

Figure 5. General situation for DMU C[5]

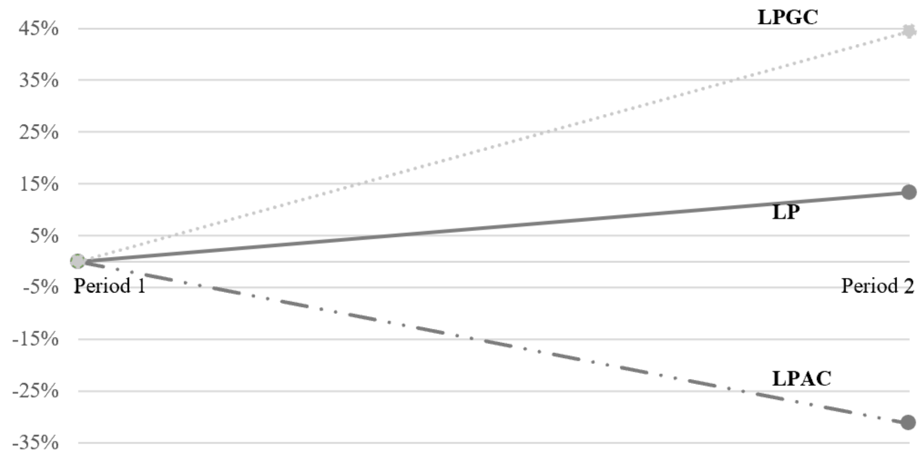
5.a Overall performance, productivity and price performance evolutions



5.b Decomposition of productivity chang into quantity-efficinecy change and quantity-tehcnical change



5.c Decomposition of price performance chang into price advantage change and price global environment change



b. Decomposition by input/output specific effects

The analysis can be detailed further by identifying the specific input/output performance indicators within their respective scores, weighed by their shares in the total revenue. Thus, decision makers and practitioners should be able to draw on these indicators and the relationships established between them to come up with meaningful applications where concurrent short- to mid-run price and quantity changes lead to very complex situations and where it is of paramount importance to be able to disentangle one effect from the other. Table 5 proposes to decompose by the variables of interest (input/output) all six performance indicators for DMUs A and C, which is an efficient DMU and an inefficient one.[6] As one should expect, the sum of all specific input and output indicators always amounts to the equivalent indicator calculated at the DMU level and presented in Table 4 above.

[6] Obviously, similar decompositions can be obtained for all DMUs.

Table 5. Decompositions of the Luenberger productivity indicators and their respective economic drivers by specific input and output variables for the DMUs A and C

	DMU A					**DMU C**				
Indicator	Output 1	Output 2	Input 1	Input 2	**TOTAL, DMU A**	Output 1	Output 2	Input 1	Input 2	**TOTAL, DMU C**
LVEC	0%	0%	0%	0%	**0%**	-73%	0%	6%	0%	**-67%**
LVTC	21%	23%	3%	5%	**53%**	103%	9%	-3%	2%	**113%**
LV	**21%**	**23%**	**3%**	**5%**	**53%**	**30%**	**9%**	**4%**	**2%**	**45%**
LQEC	0%	0%	0%	0%	**0%**	-19%	-17%	0%	0%	**-36%**
LQTC	9%	10%	2%	0%	**23%**	40%	27%	1%	0%	**68%**
LQ	**9%**	**10%**	**2%**	**0%**	**23%**	21%	10%	1%	0%	**32%**
LPAC	0%	0%	0%	0%	**0%**	-55%	18%	6%	0%	**-31%**
LPGC	12%	13%	1%	4%	**30%**	64%	-18%	-4%	2%	**44%**
LP	**12%**	**13%**	**1%**	**4%**	**30%**	**9%**	**0%**	**2%**	**2%**	**13%**

5. Conclusions

The main contribution of this paper is to propose a relevant decomposition of price performance change into two components: a firm-specific price advantage change and a global price environment change. To our knowledge, this decomposition has not been put forward in previous work dealing with other theoretical or empirical price indexes. Moreover, our method allows to identify these components at the level of each specific input and/or output for which we dispose of the necessary price and quantity information. These indicators can prove extremely useful to a practitioner who can use them to appraise their own total price performance change and dispatch it by economic drivers (individual efforts vs general market changes) for each specific input/output. Thanks to its high degree of adaptability to existing data, we anticipate that the present methodology can find a large scope for applications.

These price performance indicators result from the comparison between a productivity indicator based on changes in quantities and overall performance indicator based on value data. As a result, we obtain an indirect measure of performance change based on prices. More precisely, following the usual decomposition of productivity gains into an efficiency change effect (catching up to the frontier) and a technical progress component (shift of frontier), an equivalent decomposition of value overall performance changes is developed. At the same time, the gap between the value overall inefficiency and technical inefficiency changes leads to a measure of the price advantage variation.

The total price performance indicator is then split into a price advantage change (the time evolution of the price effect defined in the vein of Ayouba et al. (2019)), and a global price environment change (obtained from the difference between the shift in the value-base benchmark and the shift in the quantity-base benchmark). While the first component (price advantage change) is related to the

DMU's efforts to improve its prices compared to its peers, the second one (global price environment change) can be associated to an exogenous price shock impacting to a different extent all peers evolving in the same market.

The Färe-Lovell framework enabled us to interpret all previous indicators and their respective economic drivers in terms of profit margin rate change and ultimately to trace down their origins by specific inputs and outputs.

An illustration of this methodology has been performed to emphasise its operational side for practitioners and decision makers. Their interest can be roused by the complete picture this analysis provides of the evolution of business performances in a context of concurrent price and quantity effects. While business performance analyses are primarily based on changes in indicators expressed in monetary terms such as turnover, cost, profit, etc., their respective decompositions into specific input and/or output quantity and price effects represents a noteworthy refinement. Such a decomposition can prove to be crucial in order to identify what relates, on the one hand, to a better management of input and output quantities and, on the other hand, to a search for better market price opportunities by improving prices over time. While the quantity effects and their variations can lead to an improved process management, the price effects and their changes over time are likely to reveal the improvement or the deterioration of the competitive advantages of the firm compared to its peers.

6. References

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7. Appendix

A. Classical profit maximisation program

$$
\begin{aligned}
&\max \tilde{\Pi}_a = \max\left(\sum_k PO_{a,k}\tilde{QO}_{a,k} - \sum_j PI_{a,j}\tilde{QI}_{a,j}\right), \forall n = 1,\ldots,N \\
&\sum_n \lambda_n QO_{n,k} \geq \tilde{QO}_{a,k}, \forall k = 1,\ldots,K \\
&\sum_n \lambda_n QI_{n,j} \leq \tilde{QI}_{a,j}, \ \forall j = 1,\ldots,J \\
&\sum_n \lambda_n = 1 \\
&\tilde{QO}_{a,k} \geq 0 \\
&\tilde{QI}_{a,j} \geq 0
\end{aligned}
\qquad (LP3).
$$

One can define profit efficiency as the gap between the potential profit and the observed profit, $\Delta\Pi_a = \tilde{\Pi}_a - \Pi_a$. The profit gap can be directly computed from a slack-based program as in *LP4*. For this, we consider that the optimal output is the sum between the observed quantity output and its potential increase $\tilde{QO}_{a,k} = \left(QO_{a,k} + so_{a,k}\right)$ + $\tilde{QI}_{a,j} = \left(QI_{a,j} + si_{a,j}\right)$

$$
\begin{aligned}
&\max \Delta\Pi_a = \max\left(\sum_k PO_{a,k} so_{a,k} - \sum_j PI_{j,a} si_{a,j}\right), \forall a = 1,\ldots,N \\
&\sum_n \lambda_n QO_{n,k} \geq QO_{a,k} + so_{a,k}, \forall k = 1,\ldots,K \\
&\sum_n \lambda_n QI_{n,j} \leq QI_{a,j} + si_{a,j}, \forall j = 1,\ldots,J \\
&\sum_n \lambda_n = 1 \\
&so_{a,k} \textit{ unconstrained} \\
&si_{a,j} \geq -QI_{a,j}
\end{aligned}
\qquad (LP4).
$$

The above program (*LP4*) is the equivalent to (*LP5*) below, where Färe-Lovell distance functions are used instead of slacks.

$$
\begin{aligned}
&\max \Delta\prod_a = \max\left(\sum_k PO_{a,k}\delta^{QO}_{a,k}GO_{a,k} - \sum_j PI_{j,a}\delta^{QI}_{a,j}GI_{a,j}\right), \forall a = 1,...,N \\
&\sum_n \lambda_n QO_{n,k} \geq QO_{a,k} + \delta^{QO}_{a,k}QO_{a,k}, \forall k = 1,...,K \\
&\sum_n \lambda_n QI_{n,j} \leq QI_{a,j} + \delta^{QI}_{a,j}QI_{a,j}, \forall j = 1,...,J \\
&\sum_n \lambda_n = 1 \\
&\delta^{QO}_{a,k} \ unconstrained \\
&\delta^{QI}_{a,j} \geq -1
\end{aligned}
\qquad (LP5).
$$

B. LFL indicators operationalisation

Concerning the quantity-related productivity model $(m = Q, \text{ in } LP1-2)$, we derive 2^4=16 distance functions. The first eight distances (D_1-D_8) have in common that the technology considered is observed in period 1, whereas for the remaining ones, (D_9-D_{16}) the technology considered is observed in period 2 $(t^{'} = 2, \text{in } LP1-2)$. Besides that, the distance functions obtained and detailed in Table 6 represent all possible combinations for the remaining parameters $\left(t^{''} = \{1,2\}, t^{'''} = \{1,2\} \text{ and } t^{iv} = \{1,2\} \text{in } LP\right)$.

Table 6. Distance functions obtained from the quantity-base model (m=Q in LP1-2)

Distance functions based on the period 1 technology $(t^{'} = 1, \text{in } LP)$	***Distance functions based on the period 2 technology*** $(t^{'} = 2, \text{in } LP)$
$D_1 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^1;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^1}\right)$	$D_9 = \mathbf{D}^{Q,2}\left(\mathbf{Q}^1;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^1}\right)$
$D_2 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^1;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^2}\right)$	$D_{10} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^1;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^2}\right)$
$D_3 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^1;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^1}\right)$	$D_{11} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^1;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^1}\right)$
$D_4 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^1;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^2}\right)$	$D_{12} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^1;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^2}\right)$
$D_5 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^2;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^1}\right)$	$D_{13} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^2;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^1}\right)$
$D_6 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^2;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^2}\right)$	$D_{14} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^2;\mathbf{Q}^1;\boldsymbol{\alpha}^{V^2}\right)$
$D_7 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^2;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^1}\right)$	$D_{15} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^2;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^1}\right)$
$D_8 = \mathbf{D}^{Q,1}\left(\mathbf{Q}^2;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^2}\right)$	$D_{16} = \mathbf{D}^{Q,2}\left(\mathbf{Q}^2;\mathbf{Q}^2;\boldsymbol{\alpha}^{V^2}\right)$

Legend:
i) Superscript notation indicates that the distance is calculated in the quantity space, and it is based on either the period 1 technology (superscript *Q,1*) or the period 2 technology (superscript *Q,2*).

ii) The first argument of D refers to the evaluated production plan which is the quantity production plan of the evaluated DMU in either period 1 $\left(\mathbf{Q}^1 = \left(\mathbf{QI}^1, \mathbf{QO}^1\right)\right)$ or in period 2 $\left(\mathbf{Q}^2 = \left(\mathbf{QI}^2, \mathbf{QO}^2\right)\right)$.

iii) The second argument refers to the direction of evaluation **G**, which is the evaluated DMU's production plan in either period 1 $\left(\mathbf{G}^Q = \mathbf{Q}^1 = \left(\mathbf{QI}^1, \mathbf{QO}^1\right)\right)$ or in period 2 $\left(\mathbf{G}^Q = \mathbf{Q}^2 = \left(\mathbf{QI}^2, \mathbf{QO}^2\right)\right)$.

iv) The last argument refers to the period for which the weighing system (input cost shares and output revenue shares in the total revenue) is considered and which can be either the first period system $\left(\boldsymbol{\alpha}^{V^1}\right)$ or the second period system $\left(\boldsymbol{\alpha}^{V^2}\right)$.

Under this general frame, all DMU's quantity-related indicators are computed with the help of the distance functions in Table 6. For example, period 1 technical inefficiency is the arithmetic mean between scores obtained with distance functions D_1-D_4, which have in common their calculation with quantity-base data (the technology and evaluated production plan are considered for the same period), whereas all possible combinations between the period for the direction of evaluation and the period of the weighing system are taken into account. In this perspective, the period 1 technical inefficiency score is interpreted as the potential increase in the DMU's profit margin rate by adopting input and output optimal quantities, DMU's respective prices, mix and scale held 'almost' constant. Likewise, technical inefficiency score in period 2 is calculated as the arithmetic mean of scores obtained with distance functions D_{13}-D_{16}.

$$\begin{aligned} TE_1 &= \frac{1}{4}\left(D_1 + D_2 + D_3 + D_4\right) \\ TE_2 &= \frac{1}{4}\left(D_{13} + D_{14} + D_{15} + D_{16}\right) \end{aligned} \tag{24}$$

In the same way, the corresponding distance functions for the value models are given in Table 7 below where technologies, evaluated production plans and directions are expressed in value data and by considering all combinations of periods.

Table 7. Distance functions obtained from the value-based model (m=V in LP1-2)

Distance functions based on the period 1 technology $\left(t' = 1, \text{in } LP\right)$	***Distance functions based on the period 2 technology*** $\left(t' = 2, \text{in } LP\right)$
$D_{17} = \mathbf{D}^{V,1}\left(\mathbf{V}^1; \mathbf{V}^1; \boldsymbol{\alpha}^{V^1}\right)$	$D_{25} = \mathbf{D}^{V,2}\left(\mathbf{V}^1; \mathbf{V}^1; \boldsymbol{\alpha}^{V^1}\right)$
$D_{18} = \mathbf{D}^{V,1}\left(\mathbf{V}^1; \mathbf{V}^1; \boldsymbol{\alpha}^{V^2}\right)$	$D_{26} = \mathbf{D}^{V,2}\left(\mathbf{V}^1; \mathbf{V}^1; \boldsymbol{\alpha}^{V^2}\right)$
$D_{19} = \mathbf{D}^{V,1}\left(\mathbf{V}^1; \mathbf{V}^2; \boldsymbol{\alpha}^{V^1}\right)$	$D_{27} = \mathbf{D}^{V,2}\left(\mathbf{V}^1; \mathbf{V}^2; \boldsymbol{\alpha}^{V^1}\right)$
$D_{20} = \mathbf{D}^{V,1}\left(\mathbf{V}^1; \mathbf{V}^2; \boldsymbol{\alpha}^{V^2}\right)$	$D_{28} = \mathbf{D}^{V,2}\left(\mathbf{V}^1; \mathbf{V}^2; \boldsymbol{\alpha}^{V^2}\right)$
$D_{21} = \mathbf{D}^{V,1}\left(\mathbf{V}^2; \mathbf{V}^1; \boldsymbol{\alpha}^{V^1}\right)$	$D_{29} = \mathbf{D}^{V,2}\left(\mathbf{V}^2; \mathbf{V}^1; \boldsymbol{\alpha}^{V^1}\right)$
$D_{22} = \mathbf{D}^{V,1}\left(\mathbf{V}^2; \mathbf{V}^1; \boldsymbol{\alpha}^{V^2}\right)$	$D_{30} = \mathbf{D}^{V,2}\left(\mathbf{V}^2; \mathbf{V}^1; \boldsymbol{\alpha}^{V^2}\right)$

$D_{23} = \mathbf{D}^{V,1}\left(\mathbf{V}^2;\mathbf{V}^2;\boldsymbol{\alpha}^{V^1}\right)$	$D_{31} = \mathbf{D}^{V,2}\left(\mathbf{V}^2;\mathbf{V}^2;\boldsymbol{\alpha}^{V^1}\right)$
$D_{24} = \mathbf{D}^{V,1}\left(\mathbf{V}^2;\mathbf{V}^2;\boldsymbol{\alpha}^{V^2}\right)$	$D_{32} = \mathbf{D}^{V,2}\left(\mathbf{V}^2;\mathbf{V}^2;\boldsymbol{\alpha}^{V^2}\right)$

Legend:
i) Superscript notation indicates that the distance is calculated in the value space and is based on either the period 1 value technology (superscript *V,1*), or the period 2 value technology (superscript *V,2*).
ii) The first argument of D refers to the evaluated production plan, which is the value plan of the evaluated DMU in either period 1 $\left(\mathbf{V}^1 = \left(\mathbf{VI}^1, \mathbf{VO}^1\right)\right)$ or in period 2 $\left(\mathbf{V}^2 = \left(\mathbf{VI}^2, \mathbf{VO}^2\right)\right)$.
iii) The second argument refers to the direction of evaluation **G**, which is the evaluated DMU's production plan in either period 1 $\left(\mathbf{G}^V = \mathbf{V}^1 = \left(\mathbf{VI}^1, \mathbf{VO}^1\right)\right)$ or in period 2 $\left(\mathbf{G}^V = \mathbf{V}^2 = \left(\mathbf{VI}^2, \mathbf{VO}^2\right)\right)$.
iv) The last argument refers to the period for which the weighing system (input cost shares and output revenue shares in the total revenue) is considered and which can be either the first period system $\left(\boldsymbol{\alpha}^{V^1}\right)$ or the second period system $\left(\boldsymbol{\alpha}^{V^2}\right)$.

In this setting, a DMU's value inefficiency score for period 1 is computed as the mean of the scores obtained with distance functions D_{17}-D_{20} which share a number of common features (they are based on value data, the technology and evaluated production plan are considered for the same period), while they spin over all possible combinations between the period for the direction of evaluation and the period of the weighing system. The resulting mean value inefficiency score is interpreted as the potential increase in the DMU's profit margin ratio by adopting optimal values, DMU's respective mix and scale held 'almost' constant. The value efficiency score in period 2 is computed as the mean of scores obtained with functions D_{29}-D_{32}.

$$VE_1 = \frac{1}{4}\left(D_{17} + D_{18} + D_{19} + D_{20}\right)$$
$$VE_2 = \frac{1}{4}\left(D_{29} + D_{30} + D_{31} + D_{32}\right) \quad (25).$$

We now apply the directional distance functions introduced in Table 6 above, to determine the Luenberger indicator in base period t $\left(LQ^1\right)$. According to its definition in the equation (9), this indicator is obtained as the difference between two direction functions computed with reference to the technology observed in period 1, where the evaluated production plan is observed first in period 1 and then in period 2, holding the direction of evaluation constant. Given that the direction of evaluation is alternatively considered either in period 1 or in period 2, and that additionally, the weighing system is also a parameter in our model, we obtain that the period 1 Luenberger productivity is the arithmetic mean of four differences in distance functions as follows:

$$LQ^1 = \frac{1}{4}\left[\left(D_1 - D_5\right) + \left(D_2 - D_6\right) + \left(D_3 - D_7\right) + \left(D_4 - D_8\right)\right].$$

In the same way, the Luenberger Färe-Lovell productivity indicator for period *2* is defined as

$$LQ^2 = \frac{1}{4}\left[\left(D_9 - D_{13}\right) + \left(D_{10} - D_{14}\right) + \left(D_{11} - D_{15}\right) + \left(D_{12} - D_{16}\right)\right]$$

With our notations, we are now able to express the Luenberger quantity productivity indicator which becomes:

$$LQ^{1,2} = \frac{1}{8}\left\{\begin{matrix}\left[\left(D_1 - D_5\right) + \left(D_9 - D_{13}\right)\right] + \left[\left(D_2 - D_6\right) + \left(D_{10} - D_{14}\right)\right] + \\ \left[\left(D_3 - D_7\right) + \left(D_{11} - D_{15}\right)\right] + \left[\left(D_4 - D_8\right) + \left(D_{12} - D_{16}\right)\right]\end{matrix}\right\}.$$

LQEC takes into account all possible combinations between the direction and weighing system, which leads to four possibilities for which we calculate the arithmetic mean. We also verify that this formulation corresponds to the traditional definition as the difference between periods 1 and 2 technical inefficiency scores defined in equation (24).

$$\begin{aligned} LQEC &= \frac{1}{4}\left[\left(D_1 - D_{13}\right) + \left(D_2 - D_{14}\right) + \left(D_3 - D_{15}\right) + \left(D_4 - D_{16}\right)\right] \\ &= TE_1 - TE_2. \end{aligned}$$

At the same time, once we consider all possibilities for the direction of the evaluation and weighting system, *LQTC* becomes equal to the following:

$$LQTC = \frac{1}{8}\begin{bmatrix}\left(D_9 - D_1\right) + \left(D_{10} - D_2\right) + \left(D_{11} - D_3\right) + \left(D_{12} - D_4\right) + \\ \left(D_{13} - D_5\right) + \left(D_{14} - D_6\right) + \left(D_{15} - D_7\right) + \left(D_{16} - D_8\right)\end{bmatrix}.$$

Following the notations in Table 6, the overall performance indicator $LV^{1,2}$, which is obtained as the average between period 1 and period 2 Luenberger value overall performance levels, corresponds to:

$$LV^{1,2} = \frac{1}{8}\begin{bmatrix}\left(D_{17} - D_{21}\right) + \left(D_{25} - D_{29}\right) + \left(D_{18} - D_{22}\right) + \left(D_{26} - D_{30}\right) + \\ \left(D_{19} - D_{23}\right) + \left(D_{27} - D_{31}\right) + \left(D_{20} - D_{24}\right) + \left(D_{28} - D_{32}\right)\end{bmatrix}$$

Its two components (the overall efficiency change and the overall technology change) can be obtained as follows:

$$\begin{aligned} LVEC &= \frac{1}{4}\left[\left(D_{17} - D_{29}\right) + \left(D_{18} - D_{30}\right) + \left(D_{19} - D_{31}\right) + \left(D_{20} - D_{32}\right)\right] \\ &= VE_t - VE_{t+1} \end{aligned}$$

$$LVTC = \frac{1}{8}\begin{bmatrix}\left(D_{25} - D_{17}\right) + \left(D_{26} - D_{18}\right) + \left(D_{27} - D_{19}\right) + \left(D_{28} - D_{20}\right) + \\ \left(D_{29} - D_{21}\right) + \left(D_{30} - D_{22}\right) + \left(D_{31} - D_{23}\right) + \left(D_{32} - D_{24}\right)\end{bmatrix}.$$

Turning now to the price advantage measure for each period, we have

$$PA^1 = \left(VE_1 - TE_1\right) = \frac{1}{4}\left[\left(D_{17} + D_{18} + D_{19} + D_{20}\right) - \left(D_1 + D_2 + D_3 + D_4\right)\right],$$

$$PA^2 = \left(VE_2 - TE_2\right) = \frac{1}{4}\left[\left(D_{29} + D_{30} + D_{31} + D_{32}\right) - \left(D_{13} + D_{14} + D_{15} + D_{16}\right)\right].$$

Obviously, the *LPAC* is obtained as

$$LPAC = \frac{1}{4}\begin{bmatrix} (D_{17}+D_{18}+D_{19}+D_{20})-(D_1+D_2+D_3+D_4) \\ -(D_{29}+D_{30}+D_{31}+D_{32})+(D_{13}+D_{14}+D_{15}+D_{16}) \end{bmatrix}$$
$$= PA_1 - PA_2$$
$$= LVEC - LQEC$$

Finally, the Luenberger price global environment change is obtained as residue as follows:

$$LPGC = \frac{1}{8}\left\{ \begin{bmatrix} (D_5-D_{17})+(D_{26}-D_{18})+(D_{27}-D_{19})+(D_{28}-D_{20})+ \\ (D_{29}-D_{21})+(D_{30}-D_{22})+(D_{31}-D_{23})+(D_{32}-D_{24}) \end{bmatrix} - \begin{bmatrix} (D_9-D_1)+(D_{10}-D_2)+(D_{11}-D_3)+(D_{12}-D_4)+ \\ (D_{13}-D_5)+(D_{14}-D_6)+(D_{15}-D_7)+(D_{16}-D_8) \end{bmatrix} \right\}$$
$$= LVTC - LQTC$$
$$= LP^{1,2} - LPAC$$

C. Illustration- additional results

C.1. Scenarios 1-3

Table 8. Luenberger productivity indicators and their respective decompositions into efficiency change and technology change (Scenario 1)

DMU	**A**	**B**	**C**	**D**	**E**	**F**
LVEC	0%	0%	3%	0%	0%	0%
LVTC	0%	0%	0%	0%	0%	0%
LV	0%	**0%**	**3%**	0%	**0%**	**0%**
LQEC	0%	0%	3%	0%	0%	0%
LQTC	0%	0%	0%	0%	0%	0%
LQ	**0%**	**0%**	**3%**	**0%**	**0%**	**0%**
LPAC	0%	0%	0%	0%	0%	0%
LPGC	0%	0%	0%	0%	0%	0%
LP	0%	0%	**0%**	0%	0%	0%

Table 9. Luenberger productivity indicators and their respective decompositions into efficiency change and technology change (Scenario 2)

DMU	**A**	**B**	**C**	**D**	**E**	**F**
LVEC	0%	0%	2%	0%	0%	0%
LVTC	0%	0%	0%	0%	0%	0%
LV	0%	**0%**	**2%**	0%	**0%**	**0%**
LQEC	0%	0%	0%	0%	0%	0%
LQTC	0%	0%	0%	0%	0%	0%
LQ	**0%**	**0%**	**0%**	**0%**	**0%**	**0%**
LPAC	0%	0%	2%	0%	0%	0%
LPGC	0%	0%	0%	0%	0%	0%
LP	0%	0%	**2%**	0%	0%	0%

Table 10. Luenberger productivity indicators and their respective decompositions into efficiency change and technology change (Scenario 3)

DMU	A	B	C	D	E	F
LVEC	0%	0%	5%	0%	0%	0%
LVTC	0%	0%	0%	0%	0%	0%
LV	0%	**0%**	**5%**	0%	**0%**	**0%**
LQEC	0%	0%	3%	0%	0%	0%
LQTC	0%	0%	0%	0%	0%	0%
LQ	**0%**	**0%**	**3%**	**0%**	**0%**	**0%**
LPAC	0%	0%	2%	0%	0%	0%
LPGC	0%	0%	0%	0%	0%	0%
LP	0%	0%	**2%**	0%	0%	0%

C.2. General case scenario, with the consideration of a scale component

In our general presentation, the productivity indicator *LQ* was obtained using a CRS specification in LPs 1-2 (for *m=Q*) and therefore includes, besides a pure quantity component, a scale component as well. If one wishes to distinguish between the two, then LPs 1-2 (with *m=Q*) can be estimated with a VRS technology by adding a constraint on the sum of activity variables $\left(\sum_{n=1}^{N} \lambda_n = 1 \right)$. Thus, all indicators obtained with the VRS technology correspond to a "pure" quantity effect whereas scale-related indicators (noted *Sc*) can be obtained as the difference between the CRS-technology indicator and its VRS-technology indicator as follows

$$LScEC = LQEC^{CRS} - LQEC^{VRS}$$

The following Table 11 presents the complete decomposition in the general case scenario. We notice in this case figure that the quantity effect (*LQEC*) is shared between a pure technical effect (*LQEC pure*) and a scale effect (*LScEC*) for DMUs *D* and *E*, whereas, for DMU *C*, its quantity inefficiency change indicator (LQEC) is entirely explained by its scale inefficiency (*LScEC)*, with *LQEC pure*=0.

Table 11. Luenberger productivity indicators including a scale dimension and their respective decompositions into efficiency change and technology change (general case scenario)

DMU	A	B	C	D	E	F
LVEC	0%	0%	-67%	-86%	-71%	0%
LVTC	53%	49%	113%	133%	120%	60%
LV	**53%**	**49%**	**45%**	**47%**	**49%**	**60%**
LQEC (pure)	0%	0%	0%	-29%	-13%	0%
LScEC	0%	0%	-36%	-23%	-35%	0%
LQTC	23%	23%	68%	73%	66%	24%
LQ	**23%**	**23%**	**32%**	**21%**	**18%**	**24%**

LPAC	0%	0%	-31%	-34%	-22%	0%
LPGC	30%	26%	44%	60%	53%	36%
LP	**30%**	**26%**	**13%**	**26%**	**31%**	**36%**