Which flow and pressure constraints are required for sustainable operation of Water Distribution Systems?

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WHICH FLOW AND PRESSURE CONSTRAINTS ARE REQUIRED FOR SUSTAINABLE OPERATION OF WATER DISTRIBUTION SYSTEMS?

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Abstract

Water quantity and quality modelling tools are often used to understand the working and operate properly water distribution systems. In this paper, we discuss how to choose flow and pressure constraints at nodes and links in the distribution network graph for sustainable operation of these systems. Using practical and concrete examples, we show how the problem of choosing the appropriate flow and pressure constraints amounts to verifying that a certain algebraic condition of maximum rank holds (a constraint qualification condition), or equivalently that there is a corresponding spanning tree with unsaturated links and demand nodes. Some situations of non-convergence in the solution path are discussed.

Keywords
Sustainable operations; Convex programming; Interior point methods; Active set methods; Flow and pressure constraints; Primal and dual constraints; Pressure-driven analysis; Design; Water distribution systems.

1 BACKGROUND

Water Distribution Systems (WDSs) are complex and aging infrastructures that need to be operated properly, protected, and made more resilient to natural disasters. There are considerable preservation, health & safety, and sustainability issues at stake in being able to properly manage and understand the functioning of water networks. Sustainable management of the distribution system may involve redesigning the system to better control its hydraulic state (i.e., pressure and velocity) [1,2].

Recently, several authors [3,4] have proposed ad hoc formulation and active set or interior point methods to deal with linkflow constraints for Demand Driven Modelling (DDM) and Pressure Driven Modelling (PDM) steady-state problems. To guarantee a non-empty feasible solution set and the existence of a solution, they choose to constrain the flows, $\boldsymbol{q}$, only on a cotree of the network graph and zero flow must belong to the constraint interval. In this way, there is always an unconstrained path to supply the demand nodes, and zero flow in a disconnected component is possible. Nevertheless, it may be advantageous to consider certain flow and pressure constraints also in the spanning tree, while imposing zero flow in the cotree (as in the self-cleaning network concept), [1]).

For PDM modelling, it is classical to extend the graph of the network, and produce a virtual control equivalent network, for example, by adding additional reservoirs, Flow Control Valves (FCVs), check valves and a throttle-control valves connected to each demand node as in [5]. The flow rates on these additional links correspond to the outflows at junction nodes.

In the very general case, the spanning tree of interest and the corresponding cotree are not known beforehand or given. Theoretically, there could be flow and pressure constraints on any link, from which only a subset belonging to a cotree (outflow, **c**, and linkflow constraints) and spanning tree (pressure constraints) can be active at the same time. The final decomposition/partitioning in a spanning tree and cotree is no longer a property of the network graph. It is rather driven by the actual consumption and boundary conditions.

2 GRAPH REPRESENTATION OF PRESSURE AND FLOW CONSTRAINTS

As in [3], flow rate controls, check valves and pumps are modelled as min-max box constraints for links, $q_j^{min} \leq q_j \leq q_j^{max}$: 1) Choosing an upper limit, which is smaller than the link's unconstrained flow, models the action of a flow control valve; 2) Choosing a lower limit, which is larger than the link's unconstrained flow, models the action of a pump; 3) Fixing flow direction models check valves. Additionally, fixing the flow to zero models a closed valve, and choosing the capacities of the links amounts to determining linkflow characteristics in network design.

When calculating a DDM or PDM steady state where there are flow constraints, the flow rate can be assigned to its lower or upper limit in the active set method [3]. In such a case, we would say the link is saturated. In the path to the solution, the assignment (saturated or inactive) may change, causing a situation where some demand nodes are isolated (not connected to a fixed head node). If a linkflow constraint is saturated, its link is no longer relevant to the energy equation and the link is removed from the graph, and the assigned flow rate at its min or max bounds is subtracted or added at both end nodes of the link. The saturation of links at the origin of some isolated nodes is illustrated in Figure 1.

In the interior point method [4] as in other penalty methods (e.g. [6]), a penalty head loss is added to the total head loss link j, so the linkflow constraints are never saturated and removed from the network graph. In this paper, for the sake of simplicity, we choose not to illustrate with a penalty method.

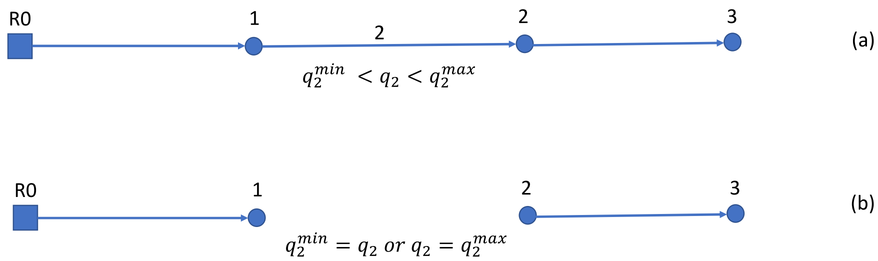

Figure 1. Graph representation of linkflow constraints; at top (a), linkflow constraint is inactive; at bottom (b), the constraint is saturated causing nodes 1 and 2 to be disconnected from source node R0.

In PDM simulation, c_i positive and bounded by above outflows are also modelled as min-max box constraints, $0 \leq c_i \leq d_i$, where d_i is the nominal demand at node i. In this paper, we also extend the network graph, but in simpler manner than in [5]. The purpose of this extension is to facilitate the analysis of existence and uniqueness of the solution. It is not used in finding the solution [3, 4, 7]. Here one additional virtual reservoir and a link are added to each demand node as in Figure 2. Figures 2a and 2b illustrate the cases when the outflow constraint at note 3 is inactive. Nodes 2 and 3 are not isolated, even if linkflow constraint 2 is saturated and link 2 removed from the network graph. In Figure 2c, nodes 2 and 3 are isolated; the heads at nodes 2 and 3 are not uniquely defined and if the flow bounds at link 2 and node 3 are not compatible, there is no flow solution.

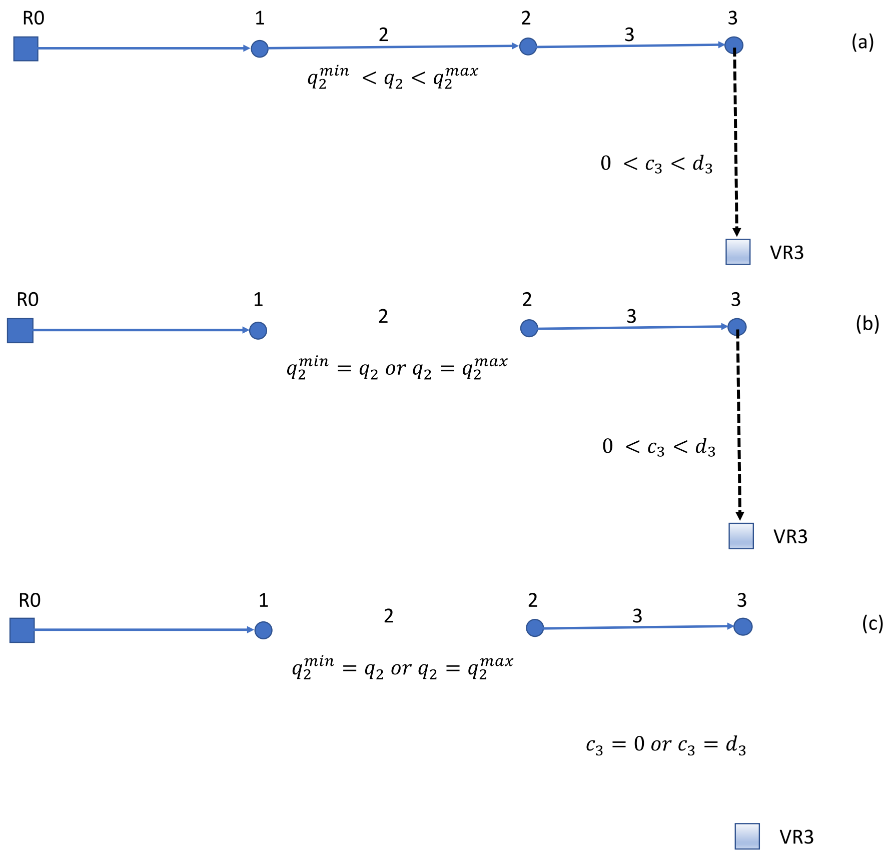

Figure 2. Graph representation of outflow constraints in PDM mode; at top (a), the linkflow constraint and outflow constraints are inactive; at middle (b), the linkflow constraint is saturated but nodes 2 and 3 are still connected to virtual reservoir VR3; at bottom (c), the linkflow and outflow constraints are saturated causing nodes 2 and 3 to be disconnected from source nodes R0 and VR3.

An unknown head loss (UHE) element is represented by a link whose head loss can be controlled to achieve a given set pressure at an assigned set pressure node (SPN). There is a one-to-one relationship between one UHE and its SPN. That means that there is exactly one SPN for each UHE. The UHE is assumed to be a unidirectional element that allows flow only in one direction ($q_j \geq 0$). In the very general case, the UHE headloss can have positive ($z_j \geq 0$) or negative sign (pumping, $z_j \leq 0$). The SPN can be upstream or downstream of the UHE; the SPN can also be at a distance from the UHE. UHE can be a pressure regulating device (PRV), or a pressure sustaining valve (PSV) or a variable speed pump (VSP).

Figure 3a shows the PRV V2 at equal distance between the two tank nodes R0 and R3. In Figure 3b, the PRV is open, the link V2 must be included in the network graph and in the conservation of energy ($h_1 = h_2$, neglecting the local head loss, or considering it $h_1 > h_2$); in Figure 3c, the PRV is active, node 2 is considered a fixed-head node, the head loss produced by the PRV explains the head difference: $z_2 = h_1 - h_2 > 0$, and the flow rate through the valve is not in one-to-one relationship with the head loss, so link 2 is removed from the graph; in Figure 3d, the PRV is closed, the heads at nodes 1 and 2 are separated.

It should be noted that having a demand at SPN 2 will not permit to satisfy the mass balance at this node, the outflow c_2 will not necessarily correspond to $q_2 - q_3$; this is because, the pressure head at node 2 is fixed by the pressure set (so the outflow would also be fixed) and does not reflect the characteristics of the system.

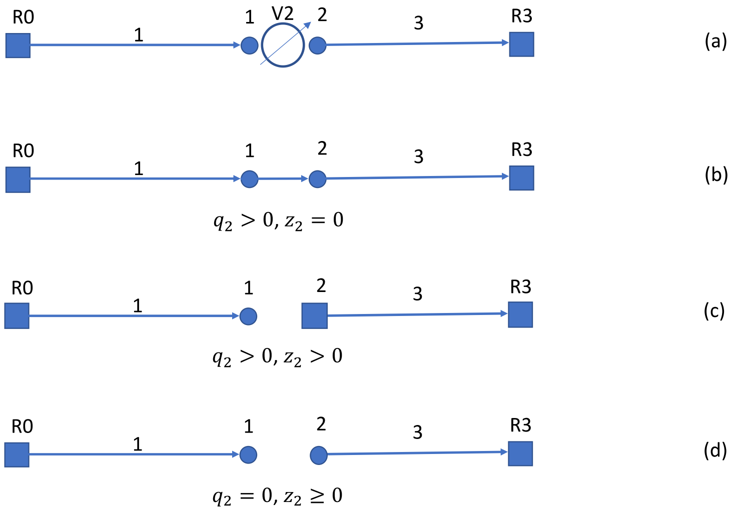

Figure 3. Graphical representations of a PRV depending on its different states; at top (a) configuration of the PRV V2 between the two tanks R0 and R3; at (b) V2 is open; at (c) V2 is active and its SPN is at the given set pressure; at (d), V2 is closed, and no water crosses the valve

The network graph may have small variations from one iteration to the next if the status of the output flow and linkflow constraints or the UHE pressure control devices change. If some junction nodes are disconnected from the source nodes, this causes a problem in the solution algorithm. We explain why below.

3 INCIDENCE MATRIX VS GRAPH REPRESENTATION

Incidence matrices are equivalent to the directed graph representations of the network. They are commonly used in the vectorised form of the steady state equations and are involved in the mass and energy equations.

Let $\mathbf{A}$ denote the $nl \times nj$, unknown-head arc-node incidence matrix (ANIM), with nl the number of links and nj the number of junction nodes. If some junction nodes are isolated from source/energy nodes then $rank(\mathbf{A}) < nj$. For the isolated junction nodes, the induced junction node incidence submatrix will therefore be rank-deficient (one of its columns is the sum of all others). As a consequence, the nodal head $\mathbf{h}$ is not defined uniquely [8]:

$$\text{if rank }(\mathbf{A}) < nj, \exists \mathbf{w} \neq \mathbf{0} / \mathbf{A}(\mathbf{h}+\mathbf{w}) = \mathbf{A}(\mathbf{h}) \tag{1}$$

Figure 4 shows the ANIMs for network variations in Figures 1 and 2. The rank of the ANIMs for network 1b and 2c are insufficient ($rank(\mathbf{A}) = 2 < 3$).

For the outflow and linkflow constraints and steady state equations, it is possible to formulate a convex minimization problem to solve [3,4]. The analysis of the existence and the search for an initial feasible solution can be achieved by solving a linear programming (LP) problem [9]; [10] used this LP problem to assess solvability in the presence of flow control devices. The verification of the full rank of the matrix, constructed from the Jacobian of the active inequality constraints and the Jacobian of the equality constraints, defines the so-called Linear Independence Constraint Qualification Condition (LICQ). Constraint qualifications such as LICQ, etc. are necessary to ensure that an optimal solution will satisfy the KKT conditions. If during the path to the solution, $rank(\mathbf{A}) < nj$, or some junction nodes are isolated, the LICQ conditions will not hold at the current iteration point, and there will be non-uniqueness of the KKT multipliers. Additionally, the Schur matrix (iteration matrix) for updating the heads has same structure/graph than $\mathbf{A}^T\mathbf{A}$, so there will be a problem solving this linear system.

The pressure control problem cannot be expressed as a single optimisation problem. Some authors have proposed a bilevel optimisation [6], other Nash equilibrium [11]. The condition $\text{rank}(\mathbf{A}) = nj$, or checking if there are zero disconnected junction nodes from sources, is very robust and holds also for pressure control or UHE problem.

$$\mathbf{A}^{(\text{Fig 1a})} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \underset{\text{removing row 2}}{\Rightarrow} \mathbf{A}^{(\text{Fig 1b})} = \left(\begin{array}{c|cc} -1 & 0 & 0 \\ \hline 0 & 1 & -1 \end{array}\right)$$

$$\mathbf{A}^{(\text{Fig 2a})} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \underset{\text{removing row 2}}{\Rightarrow} \mathbf{A}^{(\text{Fig 2b})} = \left(\begin{array}{c|cc} -1 & 0 & 0 \\ \hline 0 & 1 & -1 \\ 0 & 0 & 1 \end{array}\right)$$

$$\mathbf{A}^{(\text{Fig 2a})} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \underset{\text{removing rows 2 \& 4}}{\Rightarrow} \mathbf{A}^{(\text{Fig 2c})} = \left(\begin{array}{c|cc} -1 & 0 & 0 \\ \hline 0 & 1 & -1 \end{array}\right)$$

Figure 4. unknown-head ANIM for network variations Figures 1 to 2; partition lines in the matrix indicates diagonal block matrices for different connected components.

4 NON-CONVERGENCE EXAMPLE

With Epanet 2.2, the simple configuration with a PSV and PRV in series cannot be handled correctly. The two UHE are between two tanks at 60 m (R0) and 30 m (R5). The set pressure for the PSV is 58 m, and the set pressure for the PRV is 35 m. Epanet has calculated in 7 iterations, with the PSV open but it cannot deliver the correct head (it should be $h_1 \geq 58\ m$).

It can be seen from Figure 5 that when the PSV and PRV are active, the two junction nodes in the middle are disconnected from the sources. This could explain Epanet's wrong results.

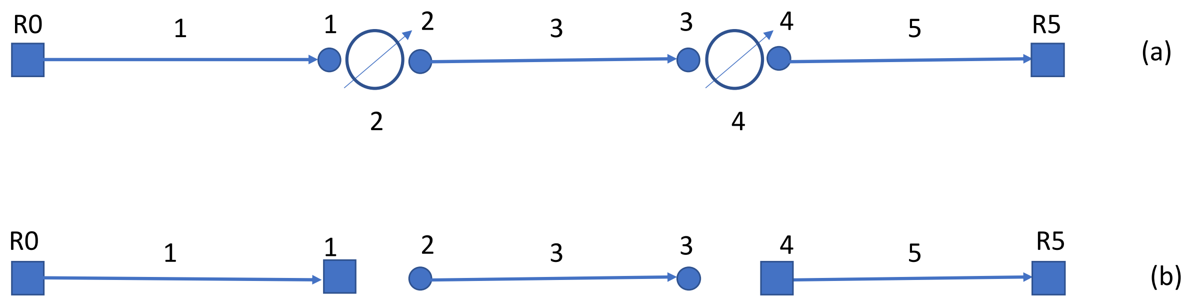

Figure 5. Graphical representation of a PSV and a PRV in series both active at same time; at top (a), configuration of the PSV/PRV between the two tanks R0 and R5; at bottom (b), the two UHEs are active causing junction nodes 2 and 3 to be disconnected from source nodes R0 and R5.

5 CONCLUSIONS

In this paper graph representation of pressure control and flow constraints are made explicit.

In the context of the PDM simulation, the original graph is extended with virtual elements, one virtual link on which is imposed the outflow constraint and a virtual reservoir at the elevation of the connected demand node.

Depending on the saturation of the outflow and linkflow constraint or the status of the UHE, the corresponding link has to be removed from the graph, and some junction nodes are transformed in fixed head nodes. This process is repeated on the path to the solution, generating several variations of the extended graph. Analysis of each variation may reveal some junction node(s) disconnected from source nodes; this may cause some problem of convergence or convergence to a wrong solution. Moreover, the head is not uniquely defined in such a case.

We establish the link between the presence of disconnected junction nodes from source and rank deficiency for the junction node incidence matrix.

Future work will consist of finding rapid and robust algorithm for analysing the connection of junction nodes to the source nodes. One way is to consider only the impacts of graph variations during the iteration. Similarly, the rank of the incidence matrix of the junction nodes can be analyzed. Additionally, when dealing with large networks and a lot of constraints, finding efficient algorithms that can relax some constraints that might not be appropriate in a certain sense remains an open question.

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