Lagrangian diffusion properties of the wake behind a cylinder using time-resolved particle tracking velocimetry

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LAGRANGIAN DIFFUSION PROPERTIES OF THE WAKE BEHIND A CYLINDER USING TIME-RESOLVED PARTICLE TRACKING VELOCIMETRY

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ABSTRACT

In the present study, we investigate the computation of the Lagrangian second-order structure-function to characterise the multiscale dynamics of turbulence from measured particle trajectories. We performed time-resolved three-dimensional particle tracking velocimetry (4D-PTV) to study the anisotropic and inhomogeneous flow field of the wake behind a cylinder at a Reynolds number equal to 3900. We performed Lagrangian statistical analysis on nearly 12000 trajectories for 4000 time steps.

LAGRANGIAN DIFFUSION PROPERTIES

Taylor's turbulent diffusion theory (Taylor, 1922) has been used widely to study homogeneous isotropic turbulent (HIT) flows. In a given time τ, Taylor's theory computes the Lagrangian two-point correlation function $R^L_{uu}(\tau)$ for an ensemble of particle trajectories based on the mean square displacements of particles $\sigma^2(\tau)$ that can be written as,

$$\frac{\mathrm{d}^2\sigma^2}{\mathrm{d}\tau^2}(\tau) = 2R^L_{uu}(\tau). \quad (1)$$

The turbulent diffusion process to Lagrangian statistical properties of particle trajectories can be linked if we assume that particles in the present study act as a tracer (Viggiano *et al.*, 2021). This means that all the inertial effects are neglected, and particles perfectly follow the flow motion. Therefore, we can compute the Lagrangian second-order structure-function as,

$$S^L_2(\tau) = \langle[u(t+\tau) - u(t)]^2\rangle = 2\left(R^L_{uu}(0) - R^L_{uu}(\tau)\right), \quad (2)$$

where the Lagrangian trajectories were obtained from three-dimensional particle tracking velocimetry (4D-PTV). Therefore, we can compute the Lagrangian universal constant C_0 for the Lagrangian second-order structure function. This constant was found to be strongly sensitive to the Reynolds number, large-scale anisotropy and inhomogeneity of flow (Viggiano *et al.*, 2021). This means that computation of the C_0 constant is complicated in anisotropic and inhomogeneous turbulent cases. In the wake behind a cylinder, turbulent length and time scales evolve as flow goes downstream, which creates non-stationary anisotropic and inhomogeneous dynamics. Batchelor's diffusion theory (Batchelor, 1957) as an extension of Taylor's theory, proposed using the Lagrangian stationarisation idea for inhomogeneous cases such as the wake flow. Stationarisation is a process based on Eulerian self-similarity properties that stationarises the Lagrangian dynamics. Recently, Viggiano *et al.* (2021) investigated the highly anisotropic and inhomogeneous case in a free shear turbulent jet. Viggiano *et al.* (2021) characterised the inertial-range dynamics and the Lagrangian universal constant C_0 by the Lagrangian stationarisation idea. Two Lagrangian second-order structure function and two-point correlation function statistics needs to be computed. The Lagrangian second-order structure function can be written as,

$$S^L_2(\tau) = \left\langle [u_i(t+\tau) - u_i(t)]^2 \right\rangle = C_0 \frac{\varepsilon_i \tau}{\sigma^2_{u_i}}, \quad (3)$$

where ε is the turbulent energy dissipation. To stationarise the Lagrangian instationarity of the wake flow, as proposed by Viggiano *et al.* (2021), we compute the Eulerian mean velocity. Then the deviation between the instantaneous and the mean components non-dimensionalised by the Reynolds stress terms as follows,

$$\tilde{u}_i(\tau) = \frac{u_i(\tau) - \bar{u}_i(x(\tau))}{\sigma_{u_i}(x(\tau))}. \quad (4)$$

We can achieve a stationarised flow field with non-dimensionalised fluctuations through the entire spatial domain for evert time step. With the same derivation spirit discussed in Viggiano *et al.* (2021); Ouellette (2021), we are interested in exploring Lagrangian properties for the wake behind a cylinder. We will characterise the Eulerian turbulent properties as well as self-similarity behaviour, followed by estimation of the Lagrangian universal constant C_0.

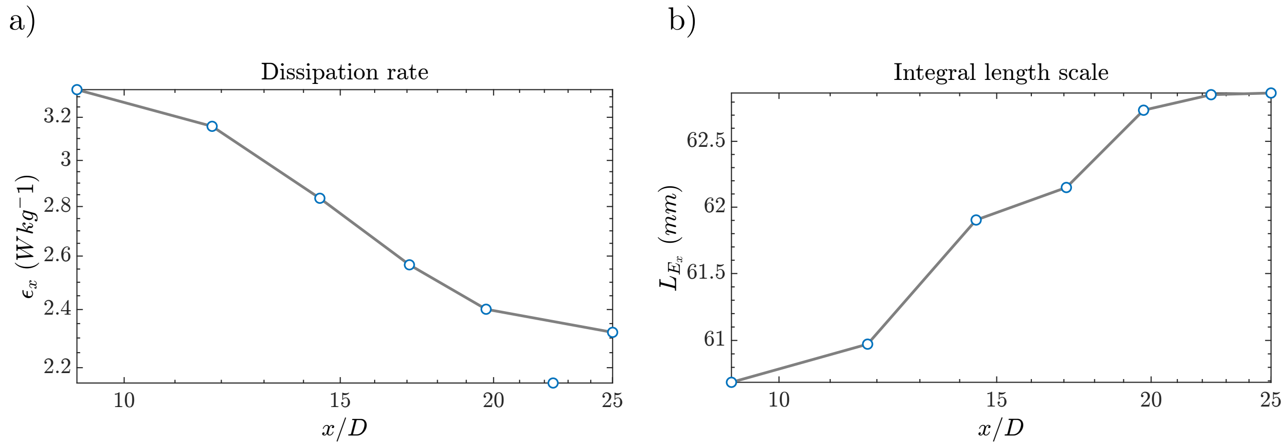

Figure 1. Computation of Eulerian statistics. (a) Decay of the dissipation rate ε_x. (b) Evolution of the integral length scale L_{E_x}.

4D-PTV EXPERIMENTAL STUDY

Recent advances in time-resolved particle tracking velocimetry (4D-PTV) have led to accurate and long trajectory reconstruction in turbulent flows (Schanz *et al.* (2016); Khojasteh *et al.* (2021*b*)). We performed an experimental study of the cylinder wake flow at Reynolds number equal to 3900. We designed an experimental setup with four CMOS SpeedSense DANTEC cameras and spatial resolutions of 1280×800 pixels. We used the camera's maximum frequency at 3 kHz in the present study to achieve the maximum reachable temporal resolution. Four Nikon 105 mm lenses were installed on the cameras. Considering the illumination direction, the first two cameras received particle signals in backward light scattering, while the second two cameras received maximum intensity signal in forward scattering. The calibration error reduced down to 0.04 pixel after the volume self-calibration. We captured roughly volume of 280 mm $\times$ 160 mm $\times$ 46 mm starting from roughly 4D downstream of the cylinder, knowing that the vortex formation zone ends at 4D where the cylinder diameter was $D = 12$ mm. Details of the experiment setup can be found in Khojasteh *et al.* (2021*a*). The velocity component of each trajectory was computed after fitting a curve over noisy reconstructed positions. The Eulerian fields were computed using fine-scale reconstruction (VIC#, Jeon *et al.* (2018)) between trajectories to achieve gridded velocity fields. Thereafter, time-averaged mean $\bar{u}_i(x(\tau))$ and Reynolds stress $\sigma_{u_i}(x(\tau))$ terms can be computed by averaging all instantaneous Eulerian velocity fields to start the process in equation (4).

WAKE FLOW STATISTICS

Due to the loss of momentum cases by the cylinder, the wake velocities are smaller than the free stream region. The wake thickness increases as the flow travels downstream of the cylinder. We can compute the Eulerian statistics within the area of the wake. Ensemble of all trajectories passing a virtual volume inside the wake at a certain x/D is considered to achieve statistically converged Eulerian properties. The Eulerian volume has the dimension of $\delta(x/D) \times 2D$ in y and z directions with 0.5mm depth in x direction, as suggested by Viggiano et al. Viggiano *et al.* (2021). Therefore, we can compute the Eulerian second order structure function over the ensemble of spatial velocity increments of each pair trajectories passing the Eulerian volume. Velocity components were stationarised with equation (4). We computed the Eulerian turbulent properties in seven downstream positions varying from $x/D = 9$ to 25 as listed in table 1. Nearly constant values of the Taylor microscale Re_λ shows that the stationarisation process suggested by Batchelor Batchelor (1957) is valid for the self similar wake flow far downstream of the wake flow. Decay of the dissipation rate ε_{u_x} toward downstream is also in with power-law decay in self-similar flows. Both Kolmogorov η_x, and integral L_{E_x} length scales are growing as flow goes far downstream.

Table 1. Statistical results of Eulerian parameters at various x/D positions.

x/D	σ_{u_x} (m s^{-1})	ε_x (W kg^{-1})	η_x (μm)	τ_{η_x} (ms)	λ_x (μm)	Re_λ	L_{E_x} (mm)	T_{E_x} (ms)
9	0.31	3.33	17.65	2.1	255.2	53.97	60.7	19.39
11	0.30	3.15	17.90	2.2	255.3	52.54	61	20.02
14	0.29	2.83	18.38	2.3	258.8	51.15	61.9	21.16
17	0.28	2.56	18.85	2.4	267.1	51.81	62.1	21.65
19	0.27	2.40	19.16	2.5	267.8	50.40	62.7	22.52
22	0.26	2.15	19.70	2.6	273.6	49.78	62.9	23.34
25	0.26	2.31	19.33	2.5	266.9	49.21	62.9	23.04

The evolution of Eulerian dissipation rate ε_{u_x}, Kolmogorov length scale η_x, and integral scale T_{E_x} will be used to compute Lagrangian statistical properties. To compute the Lagrangian second order statistics, we assume a small cube volume inside the wake with the length of $\delta(x/D)/3$ (suggested by Viggiano et al. Viggiano *et al.* (2021)) and index all trajectories passing the volume. Following equation (3), we compute temporal velocity increments on each individual trajectory. The ensemble of computed temporal increments is then averaged to compute the Lagrangian second order structure function. By solving the left side of equation (3), we can compute the C_0 constant where ε_i and $\sigma^2_{u_i}$ are the Eulerian dissipation rate and the averaged velocity standard deviation over the Lagrangian volume.

The Lagrangian second order structure function at four downstream locations is plotted in figure 1. The result of non-dimensional $S^2_{2,x}$ in far downstream shows that the Lagrangian statistics becomes independent of the downstream position ($x/D > 10$). In the present study, we were unable to reach converged statistics for $x/D < 10$. We also observe that the non-dimensional $S^2_{2,x}$ has linear relation with τ where

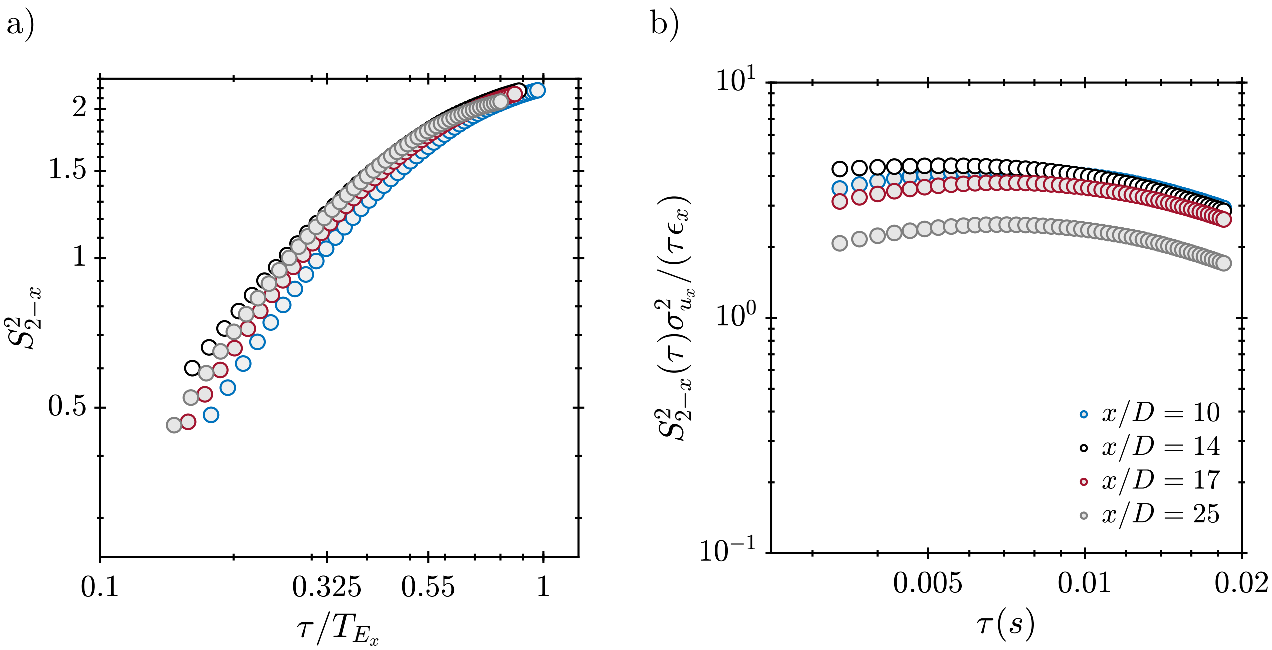

Figure 2. Lagrangian second order structure function of the streamwise direction at four downstream locations. (a) Non-dimensional $S^L_{2,x}$ as a function of non-dimensional timescale. (b) Re-dimensionalised structure function representing the C_0 constant.

$\eta_x < \tau < T_{E_x}$. These findings are in agreement with the free jet self-similar case Viggiano *et al.* (2021).

CONCLUSION AND OUTLOOK

In the present study, we are interested in computing the Lagrangian universal constant C_0 for the wake behind a cylinder at subcritical Reynold number. This constant in the Lagrangian framework is in a similar role as the Kolmogorov constant in the Eulerian framework (Viggiano *et al.*, 2021). We acquired 4000 time steps to achieve the converged statistics using in-house time-resolved three-dimensional particle tracking velocimetry (4D-PTV) code. Lagrangian statistics of the wake flow experiment showed that the Lagrangian statistics becomes independent of the downstream position in far downstream $x/D > 10$ (see figure 2). These findings are in agreement with the free jet self-similar case (Viggiano *et al.* (2021)). Figure 2.b suggests the C_0 value, which should stay nearly constant in the inertial range. C_0 is found to be between $2-4$ for the selected downstream locations. Based on the modelling suggested by Sawford Sawford (1991), C_0 should be around 2.6 for the corresponding Taylor microscale Reynolds number. Therefore, the estimated C_0 of the present study is in the same order as Sawford's findings.

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