Optimal Control of Coffee Berry Borers: Synergy Between Bio-insecticide and Traps

Yves Fotso Fotso, Suzanne Touzeau, Frédéric Grognard, Berge Tsanou, Samuel Bowong

Optimal Control of Coffee Berry Borers: Synergy between Bio-insecticide and Traps *

Yves Fotso Fotso[1,5,†] **Suzanne Touzeau**[2,4] **Frédéric Grognard**[2]
Berge Tsanou[1,5] **Samuel Bowong**[3,5]

[1] Department of Mathematics and Computer science, University of Dschang, Cameroon
[2] Université Côte d'Azur, INRIA, INRAE, CNRS, Sorbonne Université, Biocore, France
[3] Department of Mathematics and Computer Science, University of Douala, Cameroon
[4] Université Côte d'Azur, INRAE, CNRS, ISA, France
[5] IRD, Sorbonne Université, UMMISCO, Bondy, France
[†] Corresponding author: fotsofyves@yahoo.fr

Date: April 20, 2022

Abstract. The coffee berry borer (CBB), *Hypothenemus hampei*, is the most destructive insect pest affecting coffee plantations in most coffee producing countries, hence causing major economic losses worldwide. The cryptic life cycle of CBB inside coffee berries makes their control extremely difficult. To tackle this problem, we use a dynamical model describing the plant–pest interactions during a cropping season, which includes a berry age structure to account for CBB preference for mature berries. We introduce two environmentally friendly control methods, consisting in applying a bio-insecticide to reduce berry infestation and in trapping the colonising CBB. Our objective is to maximise the profit generated by the harvest of healthy coffee berries, while minimising the CBB population for the next cropping season. The existence of an optimal control strategy is provided and necessary optimality conditions are established. Finally, the optimal control problem is solved numerically and simulations are provided. They show that combining the two control methods is a cost-effective strategy to protect coffee berries from CBB infestation.

Keywords. Population dynamics, Age-structured model, Plant epidemiology, Optimal control, Numerical simulations

AMS subject classifications. 49J20. 35L60. 92D30

1. Introduction.

Originating from East Africa, coffee (*Coffea Rubiaceae*) is an important agricultural commodity in tropical and subtropical countries [25], ensuring the livelihood of an estimated 25 million people in Latin America, Africa, and Asia [26]. There are several species of coffee, but commercial production is mainly based on two closely related species: *Coffea Canephora* (know as robusta) and *Coffea Arabica*. However, coffee production is threatened by numerous pests, the coffee berry borer (CBB) *Hypothenemus hampei* being the most damaging pest. It is an insect pest of African origin, which has spread to almost all coffee producing countries in the world. CBB use the coffee berries for food and shelter. Adult females bore a hole to enter a berry, where they feed and lay their eggs, and where their larvae develop. CBB infestation hence severely reduces the coffee yield and quality, causing more than 500 million US$ in damages annually [19]. Moreover, CBB cryptic life cycle within coffee berries protects them quite efficiently from pest control programs.

Control of CBB has long relied on the application of insecticides, which are harmful both for human health and the environment, and whose efficacy is limited by insecticide resistance [7, 18]. Alternative methods include cultural control, which mainly consists in removing all remaining berries from the trees and the soil [4, 7], as well as trapping, which allows to capture female CBB when they look for coffee berries to lay their eggs [8, 22]. More recently, research on biological control has led to the use of

*This work is supported by EPITAG, an Inria associated team part of the LIRIMA (https://team.inria.fr/epitag/), as well as the Collège doctoral régional de l'Afrique Centrale et des Grands Lacs "Mathématiques, Informatique, Biosciences et Géosciences de l'Environnement"(AUF, French Embassy in Cameroon, IRD).

natural enemies, including parasitoids, nematodes and entomopathogenic fungi such as *Beauveria bassiana* [4, 7, 26]. The latter is present and naturally infects CBB in many coffee growing countries. As a bio-insecticide, it can be safely sprayed in the plantation and significantly reduces CBB infestation [4]. However, there are still gaps to fill to develop successful biocontrol strategies [26, 19].

Tackling the issue of CBB biocontrol through mathematical modelling and optimal control is the approach we retained in this work. It is far less costly, both in terms of time and money, than field experiments and it can provide valuable insights on efficient control strategies. We use a dynamical model describing the infestation dynamics of coffee berries by CBB during a cropping season, which includes a berry age-structure to take into account the CBB marked preference for mature berries [14], demonstrated in several field and laboratory studies [26, 16, 24]. Optimal control theory, has been applied to several age-structured systems, to study for instance competing species [11], harvesting control [3, 23], birth control [21], or epidemic disease control [1, 2, 5, 17], but not in plant epidemiology.

As in [12], we implemented control methods combining the use of traps, to reduce the colonising female population, and the spraying of an entomopathogenic fungus *Beauveria bassiana*, to limit berry infestation. Since the ultimate goal of coffee farmers is the production of high quality coffee at the best market price and the lowest cost, we designed an optimal control problem whose objective is to maximise the yield while minimising both the control costs and the CBB population for the next cropping season. In the optimisation criterion, we took into account the fact that the berry price depends on the berry age, which was not possible in our previous works aiming at controlling CBB impact on coffee yield [12, 13].

This manuscript is structured as follows. Section 2 briefly presents the coffee berry–CBB interaction model developed in [14] and on which this control study is based. Section 3 is devoted to the optimal control problem, which is formalised in Subsection 3.1. The existence of an optimal cntrol pair is shown in Subsection 3.2 and characterised using the maximum principle in Subsection 3.3. Numerical simulations are provided in Section 4, to illustrate the theoretical results. The paper then ends with Conclusions in Section 5.

2. Model Overview. The model we use in this study describes the development of coffee berries and their infestation by CBB during a cropping season [14]. Coffee berries are characterised by their age $a \in [0, a_\dagger]$ and their epidemiological state: $s_b(t,a)$ and $i_b(t,a)$ are respectively the age-specific density of healthy and infested coffee berries at time t. The CBB population is divided in two: the colonising females, denoted by $y(t)$, which are adult fertilised females looking for healthy berries to lay their eggs; and the infesting females, denoted by $z(t)$, which feed and reproduce inside infested coffee berries. Males are not considered in the model, as only females disperse and take part in the infestation process of new healthy berries: males remain in the berry where they were born. Moreover, we assume that there are always enough males to fertilise the young females in the berry [7, 4].

Figure 1 represents the dynamics of the coffee berries and CBB females. New healthy coffee berries $s_b(t,0)$ are produced at time-dependent rate $\lambda(t)$. Healthy berries $s_b(t,a)$ can then be infested by colonising females $y(t)$ at rate $\beta(a)f(B,y)$, where $\beta(.)$ is the berry age-dependent infestation rate and f is an interaction function, which depends on the number of healthy berries $B(t) := \int_0^{a_\dagger} s_b(t,a)da$ and colonising females $y(t)$. Infestation results in a transfer from healthy to infested berries $i_b(t,a)$ and simultaneously, from colonising to infesting female $z(t)$. Inside the coffee berries,

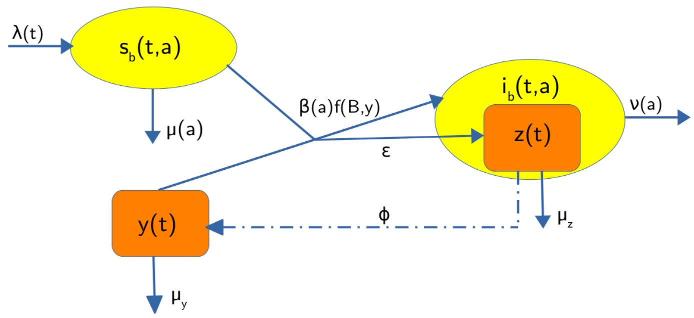

FIG. 1. *Diagram of the coffee berry–CBB interaction model.*

infesting females lay eggs which go through their development cycle until the emergence of new fertilised colonising females at rate ϕ. All these compartments undergo mortality at rates μ_y and μ_z for colonising and infesting females; $\mu(a)$ and $\nu(a)$ for healthy and infested coffee berries.

Figure 1 translates in the following system:

$$
\begin{cases}
\partial_t s_b(t,a) + \partial_a s_b(t,a) = -\beta(a) f(B(t), y(t)) s_b(t,a) - \mu(a) s_b(t,a) \\
\partial_t i_b(t,a) + \partial_a i_b(t,a) = \beta(a) f(B(t), y(t)) s_b(t,a) - \nu(a) i_b(t,a) \\
\dot{y}(t) = -\varepsilon f(B(t), y(t)) \|\beta s_b(t,.)\| - \mu_y y(t) + \phi z(t) \\
\dot{z}(t) = \varepsilon f(B(t), y(t)) \|\beta s_b(t,.)\| - \mu_z z(t)
\end{cases}
\tag{2.1}
$$

where ε is a scaling parameter that corresponds to the number of colonising females per infested berry (so usually, $\varepsilon = 1$ CBB/berry) and $\|\beta s_b(t,.)\| = \int_0^{a_\dagger} \beta(a) s_b(t,a) da$. The system is completed by the following boundary and initial conditions:

$$
s_b(t,0) = \lambda(t), \quad i_b(t,0) = 0, \quad s_b(0,a) = s_{b0}(a), \quad i_b(0,a) = i_{b0}(a). \tag{2.2}
$$

This model relies on the following assumptions.

Assumption 2.1. Positivity and smoothness of the functions and parameters of model (2.1–2.2)

1. Parameters ϕ, ε, μ_y, μ_z and initial conditions y_0 and z_0 are nonnegative.
2. $\lambda(.) \in L^\infty_+(0,\infty)$, $\beta(.) \in L^\infty_+(0,a_\dagger)$; boundary conditions $s_{b0}(.), i_{b0}(.) \in L^1_+(0,a_\dagger)$ and are bounded.
3. Mortality rates $\mu(.), \nu(.) \in L^\infty_+(0,a_\dagger)$, $\nu(a) \geq \mu(a)$, and there exists a real number $\widetilde{\mu} > 0$ satisfying : $\mu(a) \geq \widetilde{\mu}$ for almost every $a \in [0,a_\dagger]$.
4. Contact function $f(.,.)$ is bounded and C^1–Lipschitz continuous for both arguments. Moreover, $f(B,y)$ decreases with B and increases with y, with $f(B,0) = 0$ and for all $y > 0$, $\lim_{B\to+\infty} f(B,y)B$ is finite.

$(L^1_+(I), \|.\|)$ is the space of nonnegative measurable functions equiped by the product norm and by $(L^\infty_+(I), \|.\|_\infty)$ the space of nonnegative and Lebesgue integrable functions over the set $I \subset \mathbb{R}$.

This model is thoroughly analysed in [14]. In particular, it establishes that the model has a unique nonnegative and bounded solution.

3. Optimal Control. In this section, we describe and study an optimal control problem designed to maximise the yield of healthy coffee berries, while minimising the control costs and the CBB population for the next cropping season, based on model (2.1,2.2) described above.

3.1. Problem Statement. We consider a fixed time interval $[0, t_f]$, where t_f denotes the end of the cropping season. We assume that all coffee berries are picked at time t_f. Two interventions strategies, called controls, are included into initial system (2.1). These controls are functions of time, constrained by lower and upper bounds, and described as follows:

- First, control $u(t) \in [0, 1]$ denotes the effort made to reduce the infestation of healthy coffee berries. In practise, this control can represent the action of synthetic insecticides, which are widely used in some coffee growing countries, but pollute the environment and favour the development of CBB resistance [6]. It can also represent the action of the entomopathogenic fungus *Beauveria Bassiana*, which is as an environmentally friendly bio-insecticide, not toxic to workers, that is sprayed on the coffee berries and kills CBB when they drill their entry hole in the berries [15]. We denote by ξ the efficiency of this control $u(t)$, so $(1 - \xi u(t))\beta(a)$ represents the reduced infestation rate.
- Second, control $v(t) \in [0, 1]$ consists in reducing the colonising female population. It can represent traps set in the plantation to capture colonising females during their migration flight [22, 8]. We denote by η the efficiency of control $v(t)$, so $\eta v(t)$ represents the fraction of colonising females captured per unit of time.

Note that variable $i_b(t, a)$ representing infested berries does not appear in the remaining equations of system (2.1). Moreover, none of the two control methods described above targets infested berries. Therefore, one can ignore the $i_b(t, a)$-equation to study and control the system dynamics. Hence, implementing both controls in system (2.1) and dropping the $i_b(t, a)$-equation, we obtain the following controlled system:

$$\text{(3.1)} \quad \begin{cases} \partial_t s_b(t,a) + \partial_a s_b(t,a) = -f(B(t), y(t))(1 - \xi u(t))\beta(a) s_b(t,a) - \mu(a) s_b(t,a) \\ \dot{y}(t) = -\varepsilon f(B(t), y(t)) \|\beta s_b(t,.)\| - \mu_y y(t) - \eta v(t) y(t) + \phi z(t) \\ \dot{z}(t) = +\varepsilon f(B(t), y(t))(1 - \xi u(t)) \|\beta s_b(t,.)\| - \mu_z z(t) \\ s_b(t,0) = \lambda(t), \quad s_b(0,a) = s_{b0}(a), \quad y(0) = y_0, \quad z(0) = z_0. \end{cases}$$

The existence and uniqueness of a solution of controlled system (3.1) over a finite time interval is obtained as in the case without control presented in [14]. Moreover, the solution remains nonnegative and bounded for nonnegative initial conditions, since the controls are bounded (the proof consists in using the comparison principle, as controlled system (3.1) is upper-bounded by the system without control, whose solution was showed to be bounded [14]).

Let $\mathcal{K}$ be the space of admissible controls defined by:

$$\mathcal{K} := \left\{ (u, v) \in (L^\infty(0, t_f))^2 : \ u(.) : [0, t_f] \to [0, 1], \ v(.) : [0, t_f] \to [0, 1] \right\}.$$

The optimal control problem is then formulated below.

Problem 3.1. *Find an admissible control pair $(u^\star(.), v^\star(.)) \in \mathcal{K}$ maximising the following objective functional:*

$$\text{(3.2)} \qquad \mathcal{J}(u, v) = \int_0^{a_\dagger} \Theta(a) s_b(t_f, a) da - \int_0^{t_f} \left[C_u u^2(t) + C_v v^2(t) \right] dt - C_y y(t_f).$$

The first term of the objective functional represents the crop yield, where the coffee berry price $\Theta(.)$ is an increasing, bounded, continuous function depending on berry age. We assume that infested berries do not contribute to the yield, as their economic return is negligible compared to healthy berries. The second integral term corresponds to the control implementation costs, with coefficients C_u and C_v being the maximal costs per unit of time of controls u and v, respectively. Quadratic expressions of the controls are included to indicate the nonlinearity of the implementation costs, as it is more costly to increase the control efficiency when it is already high. Finally, the last term is a penalty on the CBB population that remains in the plantation after harvest, weighted by parameter C_y. The remaining population consists only of colonising females, as we assume that all berries are picked to limit the infestation of the next cropping season.

3.2. Existence of an Optimal Control Pair. We then prove the existence of the solution of the optimal control problem 3.1. Consider the function:

$$\chi(t,a,B,y,u) = e^{-\int_0^a [\mu(\eta)+(1-\xi u(\theta))\beta(\eta) f(B(\theta),y(\theta))]_{\theta=t-a+\eta} d\eta}.$$

Setting $Q := [0,t_f] \times [0,a_\dagger]$, then the following lemma holds.

Lemma 3.2. *Under Assumption 2.1, function χ is Lipschitz in the following sense: for (B^i, y^i, u^i), $i \in \{1,2\}$ and $(t,a) \in Q$, there exists a constant $K > 0$ such that:*

$$\left|\chi(t,a,B^1,y^1,u^1) - \chi(t,a,B^2,y^2,u^2)\right| \leq K(|B^1 - B^2| + |y^1 - y^2| + |u^1 - u^2|).$$

Proof. Lemma 3.2 is obtained by direct computation, based on the following arguments: $|e^{-m} - e^{-p}| \leq |m - p|$ for all $m, p \in \mathbb{R}_+$, the state variables $B(.)$ and $y(.)$ are uniformly bounded and $u(.) \in \mathcal{U}$. ∎

Using the method of characteristics, we get an explicit solution of the $s_b(t,a)$-equation in system (3.1) as follows:

$$s_b(t,a) = \begin{cases} s_{b0}(a-t)\dfrac{\chi(t,a,B,y,u)}{\chi(t,a-t,B,y,u)} & \text{if } a > t, \\ \lambda(t-a)\chi(t,a,B,y,u) & \text{if } a \leq t. \end{cases} \tag{3.3}$$

For $(t,a) \in Q$, we obtain from equation (3.3) the boundedness of the age-specific density of healthy coffee berries as follows:

$$|s_b(t,a)| \leq \max\left\{ \sup_{a\in[0,a_\dagger]} |s_{b0}(a)|; \|\lambda\|_\infty \right\}. \tag{3.4}$$

By integrating the $s_b(t,a)$-equation of controlled system (3.1) on the age interval $[0,a_\dagger]$, one gets:

$$\dot{B}(t) = \lambda(t) - (1-\xi u) f(B,t)\|\beta s_b(t,.)\| - \|\mu s_b(t,.)\| \leq \|\lambda\|_\infty - \widetilde{\mu} B(t). \tag{3.5}$$

Therefore, it follows that $B(t) \leq \max\left\{B(0), \frac{\|\lambda\|_\infty}{\widetilde{\mu}}\right\}$ for all $t \in [0,t_f]$. Since the state variables $B(t)$, $y(t)$ and $z(t)$ are bounded for $t \in [0,t_f]$, it follows from (3.1) and (3.5) that:

$$|\dot{B}(t)| + |\dot{y}(t)| + |\dot{z}(t)| \leq \mathcal{M}, \tag{3.6}$$

where $\mathcal{M}$ is a constant that does not neither depend on time nor on the control pair.

We now prove that there exists an optimal control strategy that maximises the objective functional (3.2) subject to the age-structured model (3.1).

Theorem 3.3. *Under Assumption 2.1, the optimal control Problem 3.1 admits a solution, i.e. there exists an optimal control pair $(u^\star(.), v^\star(.)) \in \mathcal{K}$ with associated optimal solution $(s_b^\star(.,.), y^\star(.), z^\star(.))$ which maximises the objective functional $\mathcal{J}(.,.)$.*

Proof. Since the state variables and the controls are uniformly bounded, then it follows that $\sup\{\mathcal{J}(u,v) : (u,v) \in \mathcal{K}\}$ is finite. Thus there exists a maximising sequence $(u_n, v_n)_n \subset \mathcal{K}$ such that:

$$\lim_{n\to+\infty} \mathcal{J}(u_n, v_n) = \sup\{\mathcal{J}(u,v) : (u,v) \in \mathcal{K}\}.$$

Since sequence $(u_n, v_n)_n$ is bounded, there exists a subsequence, still denoted (u_n, v_n), that converges to the limit $(u^\star, v^\star)$ in the weak topology of $L^\infty(0,t_f) \times L^\infty(0,t_f)$. The limit $(u^\star, v^\star)$ belongs to $\mathcal{K}$, since $\mathcal{K}$ is a closed convex subset of $L^\infty(0,t_f) \times L^\infty(0,t_f)$ and so it is weakly closed. Let (s_{bn}, y_n, z_n) be the state variables associated with control pair (u_n, v_n) and $B_n(t) = \int_0^{a_\dagger} s_{bn}(t,a)da$. From equation (3.3), these variables are also related to each other by:

$$s_{bn}(t,a) = \begin{cases} s_{b0}(a-t)\dfrac{\chi(t,a,B_n,y_n,u_n)}{\chi(t,a-t,B_n,y_n,u_n)} & \text{if } a > t, \\ \lambda(t-a)\chi(t,a,B_n,y_n,u_n) & \text{if } a \le t. \end{cases} \tag{3.7}$$

Thanks to inequality (3.6), the $B_n(t)$, $y_n(t)$ and $z_n(t)$ sequences are uniformly Lipschitz continuous, so they are bounded and equicontinuous. According to Arzela-Ascoli's theorem, we can extract a subsequence, still denoted $B_n(t)$, $y_n(t)$ and $z_n(t)$, which converges uniformly to the limit $B^\star(t)$, $y^\star(t)$ and $z^\star(t)$ respectively in $\mathcal{C}(0,t_f)$. Using inequality (3.4) and the uniqueness of the limit, we have $s_{bn}(t,a) \longrightarrow s_b^\star(t,a)$ when $n \to \infty$ almost everywhere in Q and $B^\star(t) = \int_0^{a_\dagger} s_b^\star(t,a)da$. As a consequence of Lemma 3.2, we have the convergence $\chi(t,a,B_n,y_n,u_n) \longrightarrow \chi(t,a,B^\star,y^\star,u^\star)$ as $n \to \infty$ almost everywhere in Q. So we can pass to the limit in equation (3.7) to obtain:

$$s_b^\star(t,a) = \begin{cases} s_{b0}(a-t)\dfrac{\chi(t,a,B^\star,y^\star,u^\star)}{\chi(t,a-t,B^\star,y^\star,u^\star)} & \text{if } a > t, \\ \lambda(t-a)\chi(t,a,B^\star,y^\star,u^\star) & \text{if } a \le t. \end{cases} \tag{3.8}$$

Moreover, passing to the limit in the differential equations satisfied by the sequences $y_n(t)$ and $z_n(t)$ in controlled system (3.1), we obtain:

$$\begin{aligned} \dot{y}^\star(t) &= \phi z^\star(t) - \varepsilon f(B^\star, y^\star)\|\beta s_b^\star(t,.)\| - \mu_y y^\star(t) - \eta v^\star(t) y^\star(t), \\ \dot{z}^\star(t) &= (1 - \xi u^\star(t))\varepsilon f(B^\star, y^\star)\|\beta s_b^\star(t,.)\| - \mu_z z^\star(t). \end{aligned}$$

By using the continuity and boundedness of function $\Theta(.)$ and passing to the limit in the objective functional (3.2), we obtain $\lim_{n\to+\infty} \mathcal{J}(u_n, v_n) = \mathcal{J}(u^\star, v^\star)$. So the pair $(u^\star, v^\star)$ satisfies Problem 3.1. Hence, there exists an optimal control pair $(u^\star, v^\star) \in \mathcal{K}$ and the corresponding state variables $s_b^\star$, $y^\star$ and $z^\star$ that maximise the objective functional $\mathcal{J}(.,.)$ in $\mathcal{K}$. ∎

3.3. Necessary Optimality Conditions. We use the maximum principle for general age-structured systems on a finite time horizon provided by Feichtinger *et al.* [9, 10] and references cited therein, to derive the first-order necessary conditions and characterise the optimal control of Problem 3.1. We set state vector $x(t) =$

$(y(t), z(t))^\top$ to rewrite system (3.1) in the following compact form:

$$
\begin{cases}
\partial_t s_b(t,a) + \partial_a s_b(t,a) = F(t,a,x(t),q(t),s_b(t,a),u(t)) =: F(t,a) \\
\dot{x}(t) = G(t,x(t),q(t),u(t),v(t)) =: G(t) \\
s_b(t,0) = \lambda(t), \quad s_b(0,a) = s_{b0}(a), \quad x(0) = (y(0),z(0))^\top,
\end{cases}
\tag{3.9}
$$

where $q(t) = (q_1(t), q_2(t))^\top$ derives from $h(t,a) = (h_1(t,a), h_2(t,a))^\top$, with:

$$
h_1(t,a) = s_b(t,a), \qquad \text{and } q_1(t) = \int_0^{a_\dagger} h_1(t,a)da,
$$
$$
h_2(t,a) = \beta(a)s_b(t,a) \quad \text{and } q_2(t) = \int_0^{a_\dagger} h_2(t,a)da.
$$

Functions $F(t,a)$ and $G(t) = (G_1(t), G_2(t))^\top$ represent the right hand side of system (3.1) for the s_b-, y- and z-compartment respectively. Let ∇_x denotes the differentiation with respect to state variable x. We introduce the adjoint functions $\Phi_b(t,a)$ and $\Phi_x(t) = (\Phi_y(t), \Phi_z(t))$ corresponding to the state variable $s_b(t,a)$ and vector $x(t)$, respectively. Then, from [9], the adjoint system is given by:

$$
\begin{cases}
-\partial_t \Phi_b(t,a) - \partial_a \Phi_b(t,a) = \Phi_b(t,a)\nabla_{s_b} F(t,a) + \Phi_x(t)\nabla_q G(t)\nabla_{s_b} h(t,a) \\
\qquad + \int_0^{a_\dagger} \Phi_b(t,a)\nabla_q F(t,a)da\nabla_{s_b} h(t,a) \\
\Phi_b(t_f,a) = \Theta(a), \quad \Phi_b(t,a_\dagger) = 0 \\
-\dot{\Phi}_x(t) = \Phi_x(t)\nabla_x G(t) + \int_0^{a_\dagger} \Phi_b(t,a)\nabla_x F(t,a)da \\
\Phi_x(t_f) = (-C_y, 0).
\end{cases}
$$

This adjont system can be rewritten as:

$$
\begin{cases}
\partial_t \Phi_b + \partial_a \Phi_b = (1-\xi u)[\beta f(B,y)(\Phi_b - \varepsilon\Phi_z) + f_B(B,y)(\|\beta s_b \Phi_b\| \\
\qquad - \varepsilon\|\beta s_b\|\Phi_z)] + [\varepsilon\beta f(B,y) + \varepsilon\|\beta s_b\| f_B(B,y)]\,\Phi_y + \mu\Phi_b \\
\dot{\Phi}_y = (1-\xi u) f_y(B,y)\,[-\varepsilon\|\beta s_b\|\Phi_z + \|\beta s_b \Phi_b\|] + \varepsilon\|\beta s_b\| f_y(B,y)\Phi_y \\
\qquad + (\mu_y + \eta v)\Phi_y \\
\dot{\Phi}_z = -\phi\Phi_y + \mu_z\Phi_z
\end{cases}
\tag{3.10}
$$

with the following tranversality conditions associated with the adjoint state variables for $(t,a) \in Q$:

$$
\Phi_b(t_f,a) = \Theta(a), \quad \Phi_b(t,a_\dagger) = 0, \quad \Phi_y(t_f) = -C_y, \quad \Phi_z(t_f) = 0. \tag{3.11}
$$

From the solution $s_b(t,a)$, $y(t)$ and $z(t)$ of system (3.1) and the corresponding solution $\Phi_b(t,a)$, $\Phi_y(t)$ and $\Phi_z(t)$ of adjoint system (3.10), we define the Hamiltonian functional associated with the control problem 3.1 by:

$$
\mathcal{H}(t,u(t),v(t)) := -C_u u^2(t) - C_v v^2(t) + \Phi_y(t)G_1(t) + \Phi_z(t)G_2(t) + \int_0^{a_\dagger} \Phi_b(t,a)F(t,a)da.
$$

Applying Pontryagin's maximum principle [9], which consists in solving equations $\partial_u \mathcal{H}(t,u,v) = \partial_v \mathcal{H}(t,u,v) = 0$, and taking into account the boundaries of each control, we obtain:

$$
u^\star(t) = \min\{\max\{0, u_{sing}(t)\}, 1\}, \quad v^\star(t) = \min\{\max\{0, v_{sing}(t)\}, 1\}, \tag{3.12}
$$

where

$$u_{sing}(t) = -\frac{\xi f(B(t), y(t))(-\|\beta s_b(t,.)\Phi_b(t,.)\| + \varepsilon\|\beta s_b(t,.)\|\Phi_z(t))}{2C_u},$$
$$v_{sing}(t) = -\frac{\eta y(t)\Phi_y(t)}{2C_v}.$$

4. Numerical Results.

4.1. Functions and parameters. In the numerical simulations, as in [14], a constant berry production rate $\lambda(t) \equiv \lambda$ and an age-independent berry mortality rate $\mu(a) \equiv \mu$ are chosen. Furthermore, the infestation rate is defined by the following function:

$$\beta(a) = \begin{cases} \beta_{\min} & 0 \le a < a_\beta, \\ \beta_{\min} + \beta_a(1 - e^{-k_\beta(a-a_\beta)}) & a_\beta \le a \le a_\dagger, \end{cases} \tag{4.1}$$

with $a_\beta = 90$ days and remaining parameters chosen such that the average value $\bar{\beta} = \frac{1}{a_\dagger}\int_0^{a_\dagger} \beta(a)da \approx 0.023$ day^{-1}. The CBB–berry interaction function f is modelled by:

$$f(B, y) = \frac{y}{y + \alpha B + 1},$$

More details see [14] for the description):

We choose a sigmoid function to model the price of healthy coffee berries according to berry age:

$$\Theta(a) = \frac{\overline{\Theta}a^n}{a^n + a_\Theta^n}, \tag{4.2}$$

where $\overline{\Theta}$ is the asymptotic price of healthy coffee berries, a_Θ is the age at which berries are at half asymptotic price and $n \in \mathbb{N}^\star$ is the Hill constant. Note that with a finite age bounded by $a_\dagger$, the maximum price that mature berries can reach is $\frac{\overline{\Theta}a_\dagger^n}{a_\dagger^n + a_\Theta^n}$.

We assume that the cost C_u of the infestation control, associated with the bio-insecticide, is higher than the cost C_v of traps, as regular spraying is more time consuming and labour intensive.

All parameter values used in the simulations are given in Table 1. Moreover, we use the following initial conditions. The initial time $t = 0$ corresponds to the very beginning of flowering, so $s_{b0}(a) = 0$ for all $a \in [0, a_\dagger]$. As infesting females were eliminated when berries were picked during the preceding harvest, there are none initially, so $s_{b0}(a) = 0$ for all $a \in [0, a_\dagger]$ and $z(0) = 0$. The number of initial colonising females is set at $y(0) = 10^4$ females.

4.2. Simulations. Using the parameters given in Table 1 and the initial conditions described above, we numerically solve optimal control Problem 3.1 associated with system (3.1). We approximate the optimal solution using an extension of the forward-backward sweep method [20]. Firstly, the state variables of system (3.1) are approximated using the forward semi-implicit finite difference scheme in time and backward semi-implicit difference in age described in [14] for the model without control, with an initial guess for the control pair $(u(.), v(.))$, for instance 1-valued functions. Secondly, the adjoint system (3.11) is solved by using the backward difference

TABLE 1
Model and control parameter values. Most of parameter values are based on biological data collected in the literature [4]; more information is available in [13].

Symbol	Description	Value
t_f	Duration of a cropping season	250 days
$a_\dagger$	maximum age of coffee berry	250 days
λ	Production rate of new coffee berries	1200 berries.day^{-1}
μ	Natural mortality rate of healthy coffee berries	0.002 day^{-1}
ε	Colonising CBB per berry (scaling factor)	1 female.berry^{-1}
$\beta(a)$	Infestation function (4.1):	day^{-1}
$\beta_{\min}$	minimum infestation rate	0.004 day^{-1}
β_a	age-dependent extra infestation rate	0.036 day^{-1}
k_β	infestation coefficient	0.035 day^{-1}
a_β	infestation threshold age	90 days
α	CBB–berry interaction constant	0.7 female.berry^{-1}
ϕ	Emergence rate of new colonising females	2 day^{-1}
μ_y	Natural mortality rate of colonising females	1/20 day^{-1}
μ_z	Natural mortality rate of infesting females	1/27 day^{-1}
ξ	Maximum efficiency of control u (bio-insecticide)	0.75 –
η	Maximum efficiency of control v (traps)	0.08 day^{-1}
C_u	Weight of control u	5 \$.day^{-1}
C_v	Weight of control v	0.65 \$.day^{-1}
$\Theta(a)$	Coffee berry price function (4.2):	\$.berry^{-1}
$\overline{\Theta}$	coffee berry asymptotic price	0.025 \$.berry^{-1}
a_Θ	berry age at half asymptotic price	120 days
n	Hill constant	7 –
C_y	Cost of remaining colonising females	10^{-4} \$.female^{-1}

in time and forward difference in age, using the solutions of the state equations. After these two steps, the control function values are updated with the new values of the state and adjoint variables, thanks to equations (3.12) that characterise the optimal control. This procedure is repeated until convergence is achieved.

Results are illustrated in Figure 2. Panels (a–c) represent the dynamics of the (integrated) state variables, *i.e.* the total healthy coffee berries B, the colonising y and infesting z CBB females (plain red curves); panel (d) represents the evolution of the optimal controls $u^\star$ (plain magenta curve) and $v^\star$ (plain orange curve). For comparison purposes, the solution of the system without control is also drawn in panels (a–c) (dashed blue curves), as well as the pest-free solution obtained with $y(0) = 0$ female in panel (a) (dash-dotted black curve). These results show that the optimal application of both controls, the bio-insecticide that hampers infestation (u) and the traps that capture colonising females (v), significantly reduce the CBB population (Figure 2(b) and (c), plain red curves), compared to the case without control (dashed blue curves). Moreover, the total healthy coffee berries regain the dynamics obtained in the pest-free case (Figure 2(a), dash-dotted black curve). During most of the cropping season, both controls are at their highest level so as to severely decrease the CBB population. From day 190, the population increases again, but remains several orders of magnitude below the case without control. When we decompose the impact of both controls on the CBB dynamics, the bio-insecticide and the traps (see Figure 5 in Appendix A), we observe that neither control is sufficient to reduce the CBB population when applied alone. In both cases, the CBB population growth is slowed down compared to the uncontrolled case, but similar values are reached at the end of the

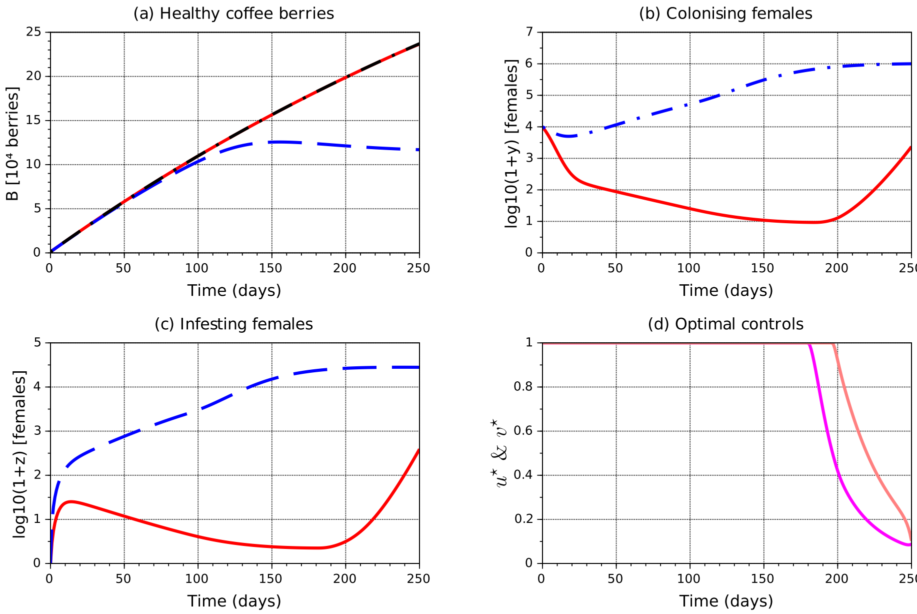

FIG. 2. *(a–c) Simulation of system (3.1) with optimal control pair $(u^\star, v^\star)$ (plain red curves), without controls ($u = v = 0$, dashed blue curves) and without pest ($y(0) = z(0) = 0$, dash-dotted black curve). (d) Evolution of optimal controls $u^\star$ (plain magenta curve) and $v^\star$ (plain orange curve). Parameter values are given in Table 1. Zero initial conditions are set, except for colonising CBB: $y(0) = 10^4$ females.*

cropping season. As a consequence, the healthy berries decline at the end of season, which reduces the harvest compared to the optimal case or pest-free case depicted in Figure 2(d). This shows that the controls have a synergistic effect. To further explore the impact of each control, we solve a modified version of control Problem 3.1: we still aim at maximising the objective functional (3.2), but we use a single control, either u or v, setting the other control to zero. The optimal controls obtained are denoted by $u^\dagger$ and $v^\dagger$. Results are shown in Figure 3. In both cases, the optimal strategy consists in applying the control at its maximal value during almost all (for the bio-insecticide u) or all (for the traps v) the cropping season. A single optimal control is less efficient to contain the CBB population than the optimal control pair (Figure 2), especially when only traps ($v^\dagger$) are deployed. In this latter case, $v^\dagger(t) = 1$ at all times, and the final healthy coffee berries are less abundant than in the optimal or pest-free cases depicted in Figure 2(d), which has an impact on the harvest. These results show that the bio-insecticide (u) alone can preserve the harvest, but not the traps (v). However, to control the CBB population, both controls are needed.

This synergistic effect of the controls appears even more clearly in Table 2, which splits the penalised profit $\mathcal{J}$ defined in equation (3.2) into its three components, for the various control strategies depicted above. The penalised profit $\mathcal{J}$ is based on the berry financial yield $\mathcal{Y}$, minus the control costs $\mathcal{C}$ and a penalty based on the colonising CBB population remaining after harvest, so as to limit the damages for the next cropping season. The bio-insecticide, especially when optimised ($u^\dagger$), is fairly efficient to preserve the yield and reduce the colonising CBB population after harvest, but its cost is high. In contrast, the traps (v) alone are poorly efficient, but not too

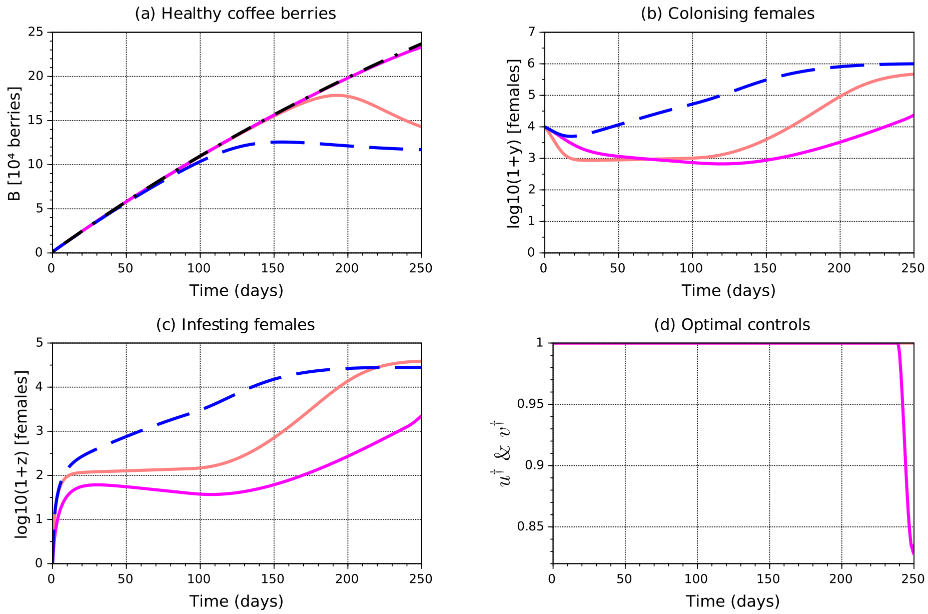

Fig. 3. *(a–c) Simulation of system (3.1) with optimal control* $u^\dagger$ *(*$v = 0$*, plain magenta curves), optimal control* $v^\dagger$ *(*$u = 0$*, plain orange curves), without control (*$u = v = 0$*, dashed blue curves) and without pest (*$y(0) = z(0) = 0$*, dash-dotted black curve). (d) Evolution of optimal controls* $u^\dagger$ *and* $v^\dagger$*. Parameter values are given in Table 1. Zero initial conditions are set, except for colonising CBB:* $y(0) = 10^4$ *females.*

Table 2

Penalised profit ($\mathcal{J}$ *in US\$), yield (*$\mathcal{Y}$ *in US\$) control costs (*$\mathcal{C}$ *in US\$) and colonising CBB females remaining after harvest at the end of the cropping season (*$t_f = 250$ *days). "control* $u^\star$ *only" corresponds to the application of* $u = u^\star$ *from optimal control pair* $(u^\star, v^\star)$ *with* $v = 0$*; "optimal control* $u^\dagger$*" corresponds to the optimisation of* u *when* $v = 0$ *(and vice versa for* v*).*

Intervention	$\mathcal{J}$	$\mathcal{Y}$	$\mathcal{C}$	$y(t_f)$
no control	311	412	0	1002008
optimal controls $(u^\star, v^\star)$	1528	2635	1106	2356
control $u^\star$ only	471	1503	967	649030
control $v^\star$ only	553	782	139	891815
optimal control $u^\dagger$	1316	2563	1244	23695
optimal control $v^\dagger$	628	838	163	469589

Penalised profit/financial gain in the pest-free case: $\mathcal{J} = \mathcal{G} = 2644$ US\$

costly. However, when applying traps ($v^\star$) on top of the bio-insecticide ($u^\star$), their effect is particular notable on all components of the penalised profit, which highlights the synergistic effect of both controls.

The yield $\mathcal{Y}$ is computed as the integral of the healthy berry age distribution at the end of the cropping season $s_b(t_f, a)$ multiplied by the age-dependent sigmoid-shaped price (4.2). These final distributions are depicted in Figure 4. As the season starts with no berries, which are then produced at a constant rate, there are less mature than young berries at the end of the season. Young berries do not contribute to the yield. As expected from the results above, the final berry and price distributions are

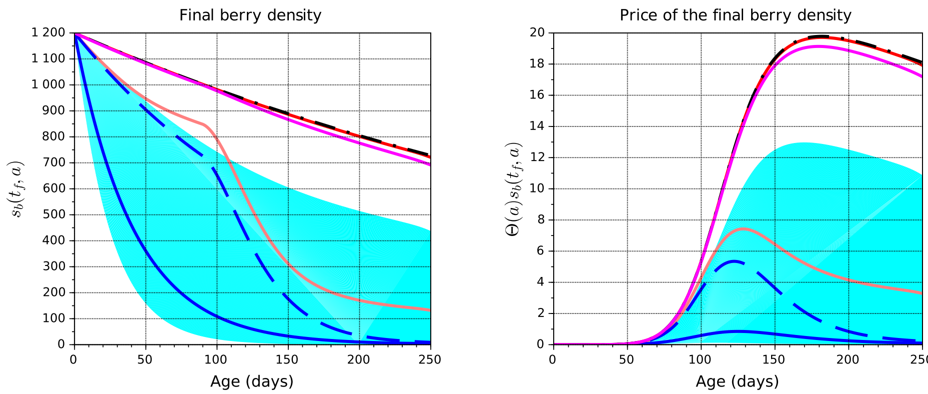

FIG. 4. *Age distribution of the healthy coffee berries and their price (yield) at the end of the simulation ($t = t_f$) with optimal control pair $(u^\star, v^\star)$ (plain red curves), optimal control $u^\dagger$ ($v = 0$, plain magenta curves), optimal control $v^\dagger$ ($u = 0$, plain orange curves), without control ($u = v = 0$, dashed blue curves) and without pest ($y(0) = z(0) = 0$, dash-dotted black curves). Cases without control and with constant infestation rates β are also represented (blue-shaded areas delimited by $\beta_{\min}$ and $\beta_{\max} = \beta_{\min} + \beta_a$, plain blue curves for average value $\bar{\beta}$). Parameter values are given in Table 1. Zero initial conditions are set, except for colonising CBB: $y(0) = 10^4$ females.*

similar in the pest-free (dash-dotted black curves) and optimal control cases when both controls are used (plain red curves). When the bio-insecticide only is optimised ($u^\dagger$, plain magenta curves), there are slightly less mature berries, which is reflected in the yield (Table 2). When only traps are used ($v^\dagger$, plain orange curves), there are markedly less berries, but still more than without control (dashed blue curves), hence a better yield. In these two latter cases, the age preference for berries older than 90 days is notable and reflected by the sharper decline occurring at this age. In the case without control (blue curves), we can compare the results with CBB preference for mature berries through function (4.1) (dotted curves) and without preference, setting the infestation rate β to a fixed value between $\beta_{\min}$ and $\beta_{\max}$ (shaded area). Using the mean value of function (4.1) results in a drastic reduction of berries of (almost) all ages at the end of the season and very low revenues (plain blue curves). Indeed, without preference, colonising females infest more immature berries. However, for lower values of the infestation rate β, there are more mature berries at the end of season, which generate notably higher revenues. It is therefore particularly relevant to take into account the CBB preference for mature berries.

5. Conclusions. In this study, we formulate a controlled PDE model describing the infestation dynamics of coffee berries by CBB during a cropping season taking into account two environmentally friendly control methods. An optimal control problem is then formulated, aiming at maximising the yield while minimising the control costs and the CBB population for the next cropping season. Two levers are considered to achieve this aim: a control which reduces the berry infestation, by spraying a bio-insecticide such as the entomopathogenic fungus *Beauveria Bassiana*, and a control which increases the colonising female mortality, based on traps. We establish the existence of an optimal solution, which we characterise using the maximum principle for general age-structured systems. The problem is solved numerically using a forward-backward sweep method.

Our main result is the synergistic effect of both controls: the bio-insecticide is

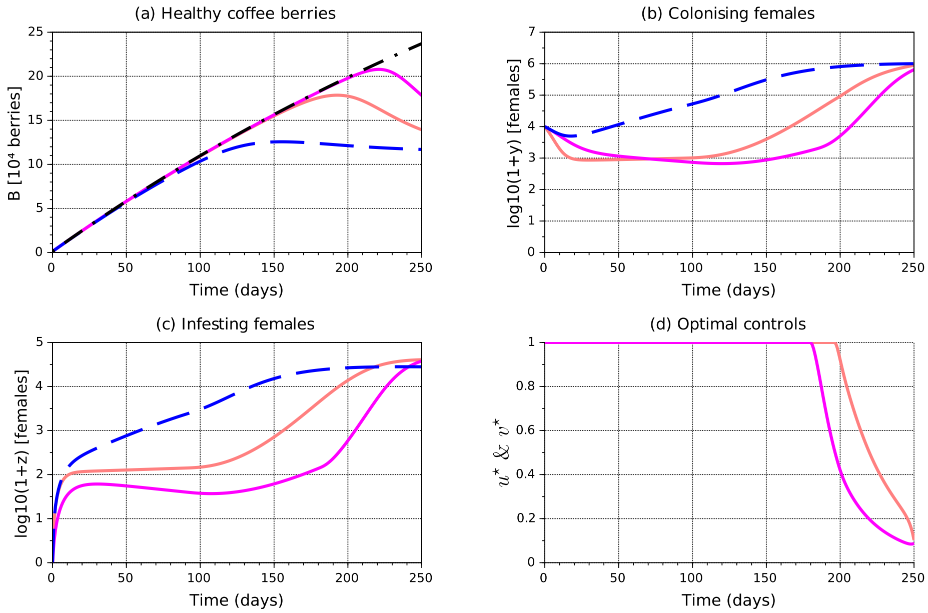

Fig. 5. *(a–c) Simulation of system (3.1 when only applying control $u^\star$ ($v = 0$, plain magenta curves) or control $v^\star$ ($u = 0$, plain orange curves) from optimal control pair $(u^\star, v^\star)$, without controls ($u = v = 0$, dashed blue curves) and without pest ($y(0) = z(0) = 0$, dash-dotted black curve). (d) Evolution of controls $u^\star$ and $v^\star$. Parameter values are given in Table 1. Zero initial conditions are set, except for colonising CBB: $y(0) = 10^4$ females.*

efficient but costly, the traps alone are poorly efficient but less costly; combining both allows to reach a cost-effective optimum.

Appendix A. Breakdown of the Optimal Control Impact.

Figures 5 and 6 are obtained by numerically solving optimal control Problem 3.1 to obtain the optimal control pair $(u^\star, v^\star)$, but then applying only one of the two controls and setting the other one to zero to integrate system (3.1). We hence observe the impact of the bio-insecticide ($u^\star$) alone and the traps ($v^\star$) alone on the crop–pest dynamics.

References

[1] B. Ainseba, Z. Feng, M. Iannelli, and F. Milner, *Control strategies for TB epidemics*, SIAM. J. Appl. Math., 77 (2017), pp. 82–107, https://doi.org/10.1137/15M1048719.

[2] B. Ainseba and M. Iannelli, *Optimal screening in structured SIR epidemics*, Math. Model. Nat. Phenom, (2012), pp. 12–27, https://doi.org/10.1051/mmnp/20127302.

[3] S. Anita, *Analysis and control of age-dependent population dynamics*, Kluwer, Boston, MA, 2000.

[4] L. F. Aristizábal, A. E. Bustillo, and S. P. Arthurs, *Integrated pest management of coffee berry borer: Strategies from Latin America that could be useful for coffee farmers in Hawaii*, Insects, 7 (2016), p. 6, https://doi.org/10.3390/insects7010006.

[5] H. Behncke, *Optimal control of deterministic epidemics*, Optim. Control Appl. Meth., 21 (2000), pp. 269–285, https://doi.org/10.1002/oca.678.

[6] L. O. Brun, C. Marcillaud, V. Gaudichon, and D. M. Suckling, *Cross resistance between insecticides in coffee berry borer, Hypothenemus hampei (Coleoptera: Scolytidae) from New Caledonia*, Bull. Entomol. Res., 84 (1994), pp. 175–178, https://doi.org/10.1017/S0007485300039651.

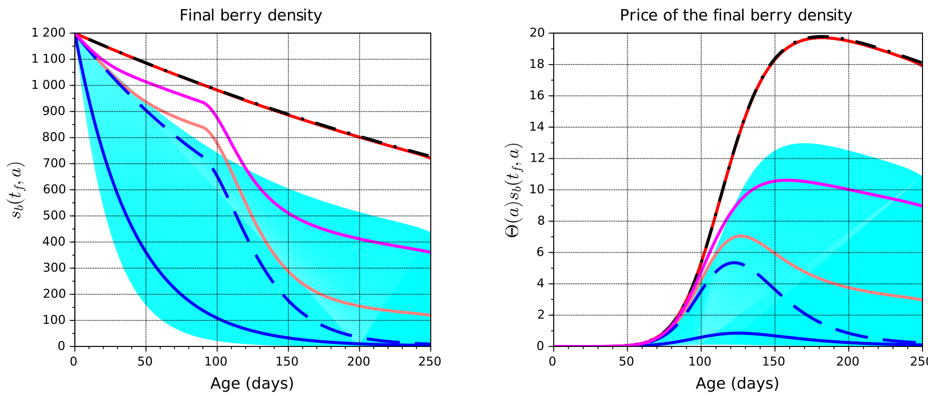

Fig. 6. *Age distribution of the healthy coffee berries and their price at the end of the simulation ($t = t_f$) with optimal control pair ($u^\star, v^\star$) (plain red curves), when only $u^\star$ ($v = 0$, plain magenta curves) or $v^\star$ ($u = 0$, plain orange curves) is applied, without control ($u = v = 0$, dashed blue curves) and without pest ($y(0) = z(0) = 0$, dash-dotted black curves). Cases without control and with constant infestation rates β are also represented (blue-shaded areas delimited by $\beta_{\min}$ and $\beta_{\max} = \beta_{\min} + \beta_a$, plain blue curves for mean value $\bar{\beta}$). Parameter values are given in Table 1. Zero initial conditions are set, except for colonising CBB: $y(0) = 10^4$ females.*

[7] A. Damon, *A review of the biology and control of the coffee berry borer, Hypothenemus hampei (Coleoptera: Scolytidae)*, Bull. Entomol. Res., 90 (2000), pp. 453–465, https://doi.org/10.1017/S0007485300000584.

[8] B. P. Dufour and B. Frérot, *Optimization of coffee berry borer, Hypothenemus hampei Ferrari (Col., Scolytidae), mass trapping with an attractant mixture*, J. Appl. Entomol., 132 (2008), pp. 591–600, https://doi.org/10.1111/j.1439-0418.2008.01291.x.

[9] G. Feichtinger, G. Tragler, and V. M. Veliov, *Optimality conditions for age-structured control systems*, J. Math. Anal. Appl., 288 (2003), pp. 47–68, https://doi.org/10.1016/j.jmaa.2003.07.001.

[10] G. Feichtinger, V. M. Veliov, and T. Tsachev, *Maximum principle for age and duration structured systems: A tool for optimal prevention and treatment of HIV*, Math. Popul. Stud., 11 (2004), pp. 3–28, https://doi.org/10.1080/08898480490422301.

[11] K. R. Fister and S. Lenhart, *Optimal control of a competitive system with age-structure*, J. Math. Anal. Appl., 291 (2004), pp. 526 – 537, https://doi.org/10.1016/j.jmaa.2003.11.031.

[12] Y. Fotso, F. Grognard, B. Tsanou, and S. Touzeau, *Modelling and control of coffee berry borer infestation*, in 14th African Conference on Research in Computer Science and Applied Mathematics, Stellenbosch, South Africa, Oct. 2018, pp. 54–63, https://www.cari-info.org/South-africa-2018/.

[13] Y. Fotso Fotso, S. Touzeau, B. Tsanou, S. Bowong, and F. Grognard, *Modelling and optimal strategy to control coffee berry borer*, Math. Meth. Appl. Sci., 44 (2021), pp. 14569–14592, https://doi.org/10.1002/mma.7726.

[14] Y. Fotso Fotso, S. Touzeau, B. Tsanou, F. Grognard, and S. Bowong, *Mathematical modelling of a pest in an age-structured crop model: the coffee berry borer case*. Submitted to Applied Mathematical Modelling, Dec. 2021, https://hal.inria.fr/hal-03505679.

[15] E. Greco and M. Wright, *Efficacy of Beauveria bassiana applications on coffee berry borer across an elevation gradient in Hawaii*, Biocontrol. Sci. Techn., (2018), https://doi.org/10.1080/09583157.2018.1493088.

[16] A. P. Gutierrez, A. Villacorta, J. R. Cure, and C. K. Ellis, *Tritrophic analysis of the coffee (Coffea arabica) – coffee berry borer [Hypothenemus hampei (Ferrari)] – parasitoid system*, An. Soc. Entomol. Bras., 27 (1998), pp. 357–385, https://doi.org/10.1590/S0301-80591998000300005.

[17] H. W. Hethcote, *Optimal ages of vaccination for measles*, Math. Biosci., 89 (1988), pp. 29 – 52, https://doi.org/10.1016/0025-5564(88)90111-3.

[18] J. Jaramillo, C. Borgemeister, and P. Baker, *Coffee berry borer Hypothenemus hampei (Coleoptera: Curculionidae): searching for sustainable control strategies*, Bull. Entomol.

Res., 96 (2006), pp. 223–233, https://doi.org/10.1079/BER2006434.

[19] M. A. Johnson, C. P. Ruiz-Diaz, N. C. Manoukis, and J. C. Verle Rodrigues, *Coffee berry borer (Hypothenemus hampei), a global pest of coffee: Perspectives from historical and recent invasions, and future priorities*, Insects, 11 (2020), p. 882, https://doi.org/10.3390/insects11120882.

[20] S. Lenhart and J. T. Workman, *Optimal Control Applied to Biological Models*, Mathematical and Computational Biology Series, Chapman & Hall/CRC, 2007.

[21] Z. Luo, Z.-R. He, and W.-T. Li, *Optimal birth control for predator–prey system of three species with age-structure*, Appl. Math. Comput., 155 (2004), pp. 665 – 685, https://doi.org/10.1016/S0096-3003(03)00808-7.

[22] R. Messing, *The coffee berry borer (Hypothenemus hampei) invades Hawaii: Preliminary investigations on trap response and alternate hosts*, Insects, 3 (2012), pp. 640–652, https://doi.org/10.3390/insects3030640.

[23] E. J. Park, M. Iannelli, M. Y. Kim, and S. Anita, *Optimal harvesting for periodic age-dependent population dynamics*, SIAM J. Appl. Math, 58 (1999), pp. 1648–1666, https://doi.org/10.1137/S0036139996301180.

[24] D. Rodríguez, J. R. Cure, A. P. Gutierrez, J. M. Cotes, and F. Cantor, *A coffee agroecosystem model: II Dynamics of coffee berry borer*, Ecol. Model., 248 (2013), pp. 203–214, https://doi.org/10.1016/j.ecolmodel.2012.09.015.

[25] USDA, *Coffee: World markets and trade*, 2020, https://www.fas.usda.gov/data/coffee-world-markets-and-trade (accessed 2020-10-05).

[26] F. Vega, F. Infante, A. Castillo, and J. Jaramillo, *The coffee berry borer Hypothenemus hampei (Ferrari) (Coleoptera: Curculionidae): A short review, with recent findings and future research directions*, Terr. Arthropod Rev., 2 (2009), pp. 129–147, https://doi.org/10.1163/187498209X12525675906031.