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# Model of steady-state ultrafiltration of colloidal suspension with formation of non-Newtonian concentration polarization layer and compressible deposit under laminar cross-flow

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- We extended approach of Gaddis (1992) and Bacchin et al. (2002) to model the cross-flow filtration of real colloidal suspensions
- Our model accounts for actual rheological properties of the system (either Newtonian or non-Newtonian concentration polarization layer), as well as local compressibility and permeability of the concentration polarization layer and the deposit
- The local filtrate flux and the local concentration polarization layer/ the local deposit structure can be calculated for given concentration of suspension, transmembrane pressure, and applied wall shear stress
- However, the model application is limited to the case of the laminar cross-flow filtration with fully retentive membrane and relatively thin concentration-polarization layer (as compared to the hydraulic diameter of the filter channel)
- All equations can be found in the extended abstract in the electronic version of Filtech 2023 proceedings; an example of the model application can be found in Loginov et al., *Journal of Membrane Science*, 2021; for everyone's convenience main model equations are provided below\*\*



\*Dear friend, if you know where to find a single channel ceramic membrane with the inner diameter of 4 mm or less, please, let me know. Thank you! Maksym [maksym.loginov@inrae.fr](mailto:maksym.loginov@inrae.fr)

\*\*Same thing, now with equations

Model accounts for (a) local cross-flow transport of particles that depends on local particle concentration and local cross-flow velocity

$$Q_0 \phi_0 = 2\pi R \int_{h_d(x)}^{h_{cp}(x)+h_d(x)} u(x,z) (\phi(x,z) - \phi_0) dz + Q(x) \phi_0$$

(b) local cross-flow velocity that depends on local *rheological properties* and local shear stress

$$u(x,z) = \int_{h_d(x)}^z \dot{\gamma}(\phi,\tau) dz$$

(c) and local filtrate flux that depends on local *compressibility and permeability*

$$\mu_f J(x) = -k(x,z) \frac{d\Pi(x,z)}{dz}$$

Combination of these equations yields

$$Q(x) = Q_0 - \frac{2\pi R}{\phi_0 \mu_f^2 J(x)^2} \int_{\phi_0}^{\phi_{sg}} (\phi - \phi_0) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} \left[ \int_{\phi}^{\phi_{sg}} \dot{\gamma}(\phi,\tau) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} d\phi \right] d\phi$$

that describes the mass balance over the membrane covered with the deposit, and

$$Q(x) = Q_0 - \frac{2\pi R}{\phi_0 \mu_f^2 J(x)^2} \int_{\phi_0}^{\phi_w(x)} (\phi - \phi_0) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} \left[ \int_{\phi}^{\phi_w(x)} \dot{\gamma}(\phi,\tau) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} d\phi \right] d\phi$$

that is applied, where there is no deposit on the membrane surface.

Each of these mass balance equations can be combined with

$$\frac{dQ(x)}{dx} = -2\pi R J(x)$$

that relates local cross-flow and filtrate flux,

$$\mu_f R_m J(x) = P(x) - \Pi(\phi_w(x)) - P_f(x)$$

that relates local filtrate flux and local pressure balance, and

$$P(x) = P_0 - \frac{2\tau}{R} x$$

that is the local pressure variation along the filter channel.

Combination of four last equations yields ordinary differential equation

$$\frac{d\phi_w}{dx} = \frac{-\frac{\phi_0}{\mu_f R_m^2} \left[ P_0 - \frac{2\tau}{R} x - \Pi(\phi_w) \right]^4 - \frac{4\tau}{R} M(\phi_w, \tau)}{\left[ P_0 - \frac{2\tau}{R} x - \Pi(\phi_w) \right] \frac{dM(\phi_w, \tau)}{d\phi_w} + 2M(\phi_w) \frac{d\Pi(\phi_w)}{d\phi_w}}$$

that can be solved numerically for given colloidal object (filtered suspension), i.e., for known *rheological properties, compressibility and permeability*. Obtained dependency of  $\phi_w(x)$  is used to calculate  $J(x)$ .

In the area, where  $\phi_w(x) > \phi_{sg}$  (the membrane is covered with the deposit) the local filtrate flux is calculated as

$$J(x) = \left[ J^{-3}(x_{cr1}) + \frac{3\phi_0 \mu_f^2}{2M(\phi_{sg}, \tau)} (x - x_{cr1}) \right]^{\frac{1}{3}}$$

The local variable  $M$  is defined as

$$M(\phi_w, \tau) = \int_{\phi_0}^{\phi_w} (\phi - \phi_0) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} \left[ \int_{\phi}^{\phi_w} \dot{\gamma}(\phi, \tau) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} d\phi \right] d\phi$$

when the deposit is not on the membrane surface, and

$$M(\phi_{sg}, \tau) = \int_{\phi_0}^{\phi_{sg}} (\phi - \phi_0) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} \left[ \int_{\phi}^{\phi_{sg}} \dot{\gamma}(\phi, \tau) k(\phi) \frac{\partial \Pi(\phi)}{\partial \phi} d\phi \right] d\phi$$

when it is.

Among other things, it means, that the steady-state filtration kinetics is not governed by deposit properties.

NOMENCLATURE

$h_{cp}$  local thickness of CP layer (m)

$h_d$  local thickness of deposit (m)

$J$  local filtrate flux ( $m \cdot s^{-1}$ )

$k$  local permeability ( $m^2$ )

$M$  filterability, material function that governs cross-flow filtration ( $m^4 \cdot Pa^2 \cdot s^{-1}$ )

$P$  total pressure in the filtration channel (Pa)

$P_0$  total pressure at the entrance to the filter channel (Pa)

$P_f$  pressure at filtrate side (Pa)

$Q$  average tangential flow rate in the filter channel ( $m^3 \cdot s^{-1}$ )

$Q_0$  average tangential flow rate at the entrance to the filter channel ( $m^3 \cdot s^{-1}$ )

$R$  inner radius of filter channel (m)

$R_m$  membrane resistance ( $m^{-1}$ )

$u$  local tangential flow rate ( $m \cdot s^{-1}$ )

$x$  axial distance from the entrance of the filter channel (m)

$x_{cr1}$  first critical distance, axial distance from the entrance of the filter channel, where the deposit appears on the membrane surface (m)

$z$  normal distance from the membrane surface (m)

GREEK LETTERS

$\gamma$  local shear rate ( $s^{-1}$ )

$\mu_f$  filtrate viscosity (Pa·s)

$\Pi$  local osmotic pressure of particles in CP layer or solid pressure in deposit (Pa)

$\tau$  shear stress, wall shear stress (Pa)

$\phi$  particle volume fraction (dimensionless)

$\phi_0$  particle volume fraction in feed suspension (dimensionless)

$\phi_{sg}$  particle volume fraction in point of sol-gel transition (dimensionless)

$\phi_w$  particle volume fraction on the membrane surface (dimensionless)