Model of steady-state ultrafiltration of colloidal suspension with formation of non-Newtonian concentration polarization layer and compressible deposit under laminar cross-flow

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- We extended approach of Gaddis (1992) and Bacchin et al. (2002) to model the cross-flow filtration of real colloidal suspensions
- Our model accounts for actual rheological properties of the system (either Newtonian or non-Newtonian concentration polarization layer), as well as local compressibility and permeability of the concentration polarization layer and the deposit
- The local filtrate flux and the local concentration polarization layer/ the local deposit structure can be calculated for given concentration of suspension, transmembrane pressure, and applied wall shear stress
- However, the model application is limited to the case of the laminar cross-flow filtration with fully retentive membrane and relatively thin concentration-polarization layer (as compared to the hydraulic diameter of the filter channel)
- All equations can be found in the extended abstract in the electronic version of Filtech 2023 proceedings; an example of the model application can be found in Loginov et al., *Journal of Membrane Science*, 2021; for everyone's convenience main model equations are provided below**

*Dear friend, if you know where to find a single channel ceramic membrane with the inner diameter of 4 mm or less, please, let me know. Thank you! Maksym
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**Same thing, now with equations

Model accounts for (a) local cross-flow transport of particles that depends on local particle concentration and local cross-flow velocity

$$Q_0\phi_0 = 2\pi R\int_{h_d(x)}^{h_{CP}(x)+h_d(x)} u(x,z)(\phi(x,z)-\phi_0)\,\mathrm{d}z + Q(x)\phi_0$$

(b) local cross-flow velocity that depends on local *rheological properties* and local shear stress

$$u(x,z) = \int_{h_d(x)}^{z} \dot{\gamma}(\phi,\tau)\,\mathrm{d}z$$

(c) and local filtrate flux that depends on local *compressibility and permeability*

$$\mu_f J(x) = -k(x,z)\frac{\mathrm{d}\Pi(x,z)}{\mathrm{d}z}$$

Combination of these equations yields

$$Q(x) = Q_0 - \frac{2\pi R}{\phi_0\mu_f^2 J(x)^2}\int_{\phi_0}^{\phi_{sg}} (\phi-\phi_0)k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\left[\int_{\phi}^{\phi_{sg}} \dot{\gamma}(\phi,\tau)\,k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\mathrm{d}\phi\right]\mathrm{d}\phi$$

that describes the mass balance over the membrane covered with the deposit, and

$$Q(x) = Q_0 - \frac{2\pi R}{\phi_0\mu_f^2 J(x)^2}\int_{\phi_0}^{\phi_w(x)} (\phi-\phi_0)k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\left[\int_{\phi}^{\phi_w(x)} \dot{\gamma}(\phi,\tau)\,k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\mathrm{d}\phi\right]\mathrm{d}\phi$$

that is applied, where there is no deposit on the membrane surface.
Each of these mass balance equations can be combined with

$$\frac{\mathrm{d}Q(x)}{\mathrm{d}x} = -2\pi R J(x)$$

that relates local cross-flow and filtrate flux,

$$\mu_f R_m J(x) = P(x) - \Pi(\phi_w(x)) - P_f(x)$$

that relates local filtrate flux and local pressure balance, and

$$P(x) = P_0 - \frac{2\tau}{R}x$$

that is the local pressure variation along the filter channel.
Combination of four last equations yields ordinary differential equation

$$\frac{\mathrm{d}\phi_w}{\mathrm{d}x} = \frac{\frac{\phi_0}{\mu_f R_m^3}\left[P_0 - \frac{2\tau}{R}x - \Pi(\phi_w)\right]^4 - \frac{4\tau}{R}M(\phi_w,\tau)}{\left[P_0 - \frac{2\tau}{R}x - \Pi(\phi_w)\right]\frac{\mathrm{d}M(\phi_w,\tau)}{\mathrm{d}\phi_w} + 2M(\phi_w)\frac{\mathrm{d}\Pi(\phi_w)}{\mathrm{d}\phi_w}}$$

that can be solved numerically for given colloidal object (filtered suspension), i.e., for known *rheological properties, compressibility and permeability.* Obtained dependency of $\varphi_w(x)$ is used to calculate $J(x)$.
In the area, where $\varphi_w(x) > \varphi_{sg}$ (the membrane is covered with the deposit) the local filtrate flux is calculated as

$$J(x) = \left[J^{-3}(x_{cr1}) + \frac{3\phi_0\mu_f^2}{2M(\phi_{sg},\tau)}(x - x_{cr1})\right]^{-\frac{1}{3}}$$

The local variable M is defined as

$$M(\phi_w,\tau) = \int_{\phi_0}^{\phi_w}(\phi-\phi_0)k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\left[\int_{\phi}^{\phi_w}\dot{\gamma}(\phi,\tau)\,k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\mathrm{d}\phi\right]\mathrm{d}\phi$$

when the deposit is not on the membrane surface, and

$$M(\phi_{sg},\tau) = \int_{\phi_0}^{\phi_{sg}}(\phi-\phi_0)k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\left[\int_{\phi}^{\phi_{sg}}\dot{\gamma}(\phi,\tau)\,k(\phi)\frac{\partial\Pi(\phi)}{\partial\phi}\mathrm{d}\phi\right]\mathrm{d}\phi$$

when it is.
Among other things, it means, that the steady-state filtration kinetics is not governed by deposit properties.

NOMENCLATURE

h_{CP} local thickness of CP layer (m)
h_d local thickness of deposit (m)
J local filtrate flux ($m \cdot s^{-1}$)
k local permeability (m^2)
M filterability, material function that governs cross-flow filtration ($m^4 \cdot Pa^2 \cdot s^{-1}$)
P total pressure in the filtration channel (Pa)
P_0 total pressure at the entrance to the filter channel (Pa)
P_f pressure at filtrate side (Pa)
Q average tangential flow rate in the filter channel ($m^3 \cdot s^{-1}$)
Q_0 average tangential flow rate at the entrance to the filter channel ($m^3 \cdot s^{-1}$)
R inner radius of filter channel (m)
R_m membrane resistance (m^{-1})
u local tangential flow rate ($m \cdot s^{-1}$)
x axial distance from the entrance of the filter channel (m)
x_{cr1} first critical distance, axial distance from the entrance of the filter channel, where the deposit appears on the membrane surface (m)
z normal distance from the membrane surface (m)

GREEK LETTERS

γ local shear rate (s^{-1})
μ_f filtrate viscosity (Pa·s)
Π local osmotic pressure of particles in CP layer or solid pressure in deposit (Pa)
τ shear stress, wall shear stress (Pa)
ϕ particle volume fraction (dimensionless)
ϕ_0 particle volume fraction in feed suspension (dimensionless)
ϕ_{sg} particle volume fraction in point of sol-gel transition (dimensionless)
ϕ_w particle volume fraction on the membrane surface (dimensionless)