# Consideration of external uncertainties in the estimation of parameters of a water and pesticide transfer model at the watershed scale

Katarina Radišić, Claire Lauvernet, Arthur Vidard

# Prise en compte d'incertitudes externes dans l'estimation de paramètres d'un modèle de transfert d'eau et de pesticides à l'échelle du bassin versant

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*Fraternité*

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# Context : Landscape management in the service of water quality

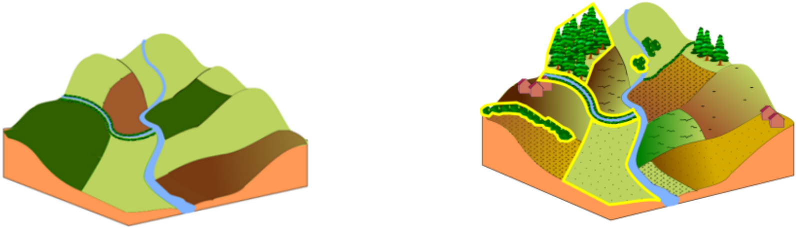

Figure: Landscape elements influence pesticide transfer dynamics,
image credit Rouzies et al. (2018) ⟨hal-02608211⟩

# Context : The PESHMELBA model

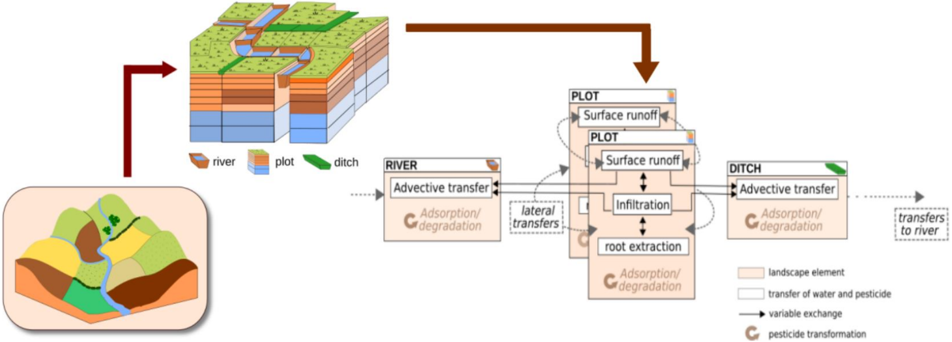


Figure: The catchment of assembled parcels, their vertical decomposition, and the main processes of water and pesticide transfer in PESHMELBA,
(Rouzies et al., 2019) doi : 10.1016/j.scitotenv.2019.03.060, image credit E. Rouzies

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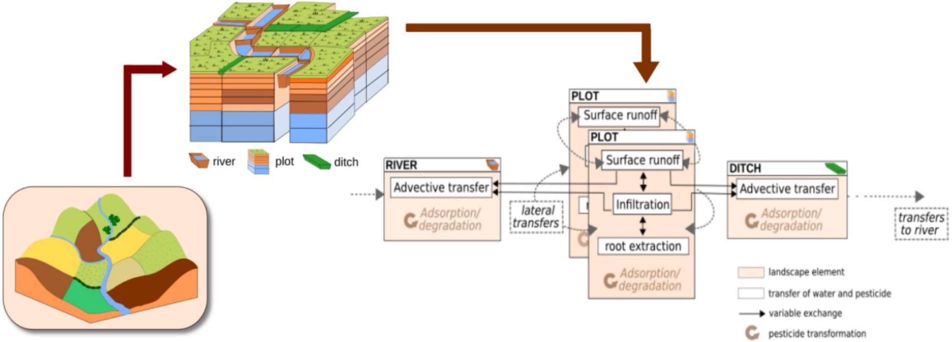


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Complex model with many parameters -> needs to be calibrated.

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# Introduction: From classical to robust calibration

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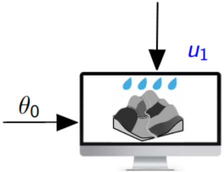

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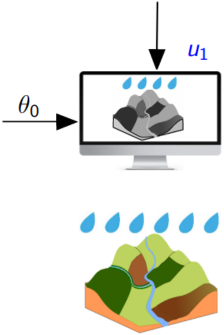

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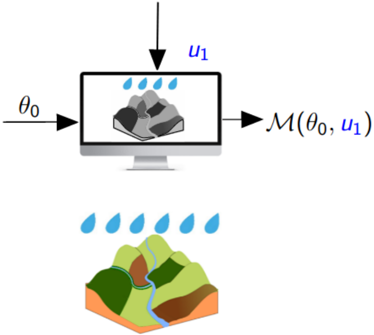

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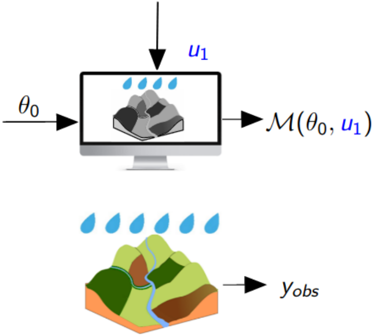

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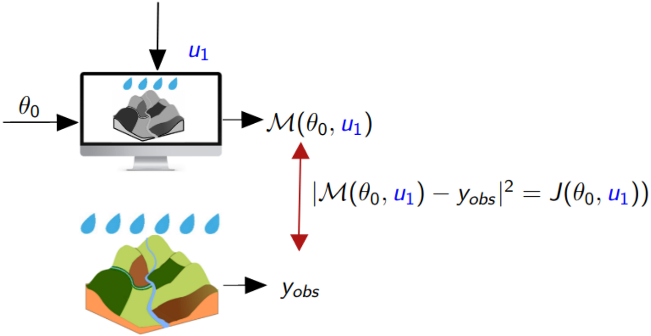

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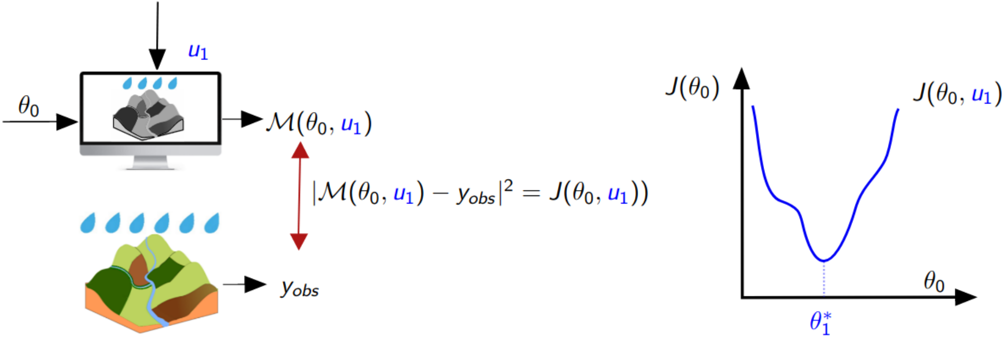

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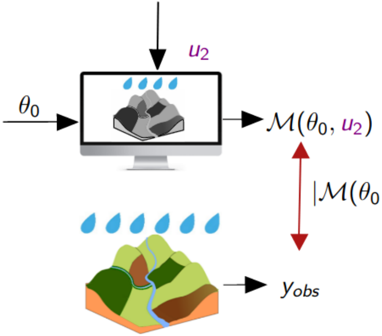
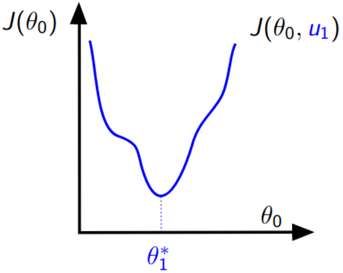



$$|\mathcal{M}(\theta_0, u_2) - y_{obs}|^2 = J(\theta_0, u_2))$$



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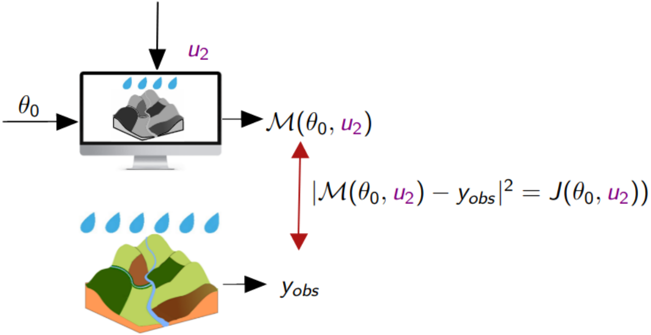


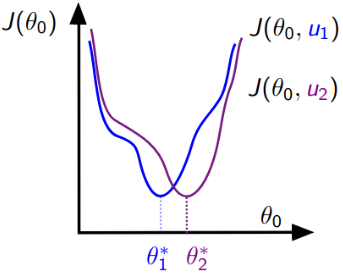

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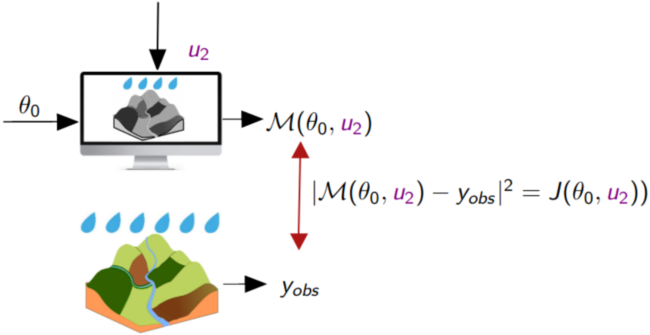


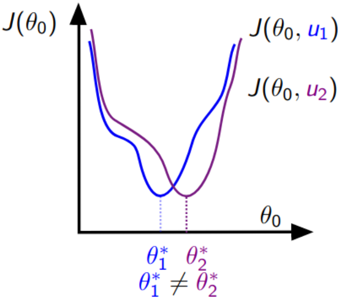

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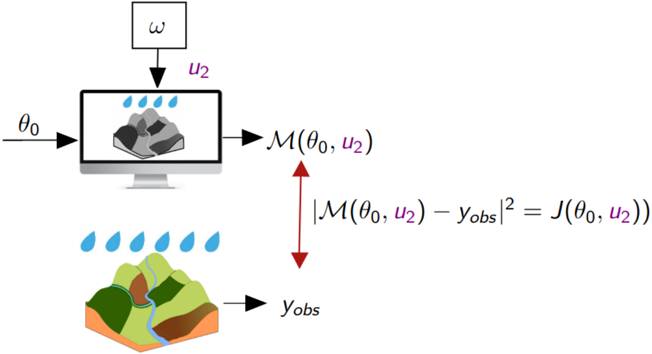


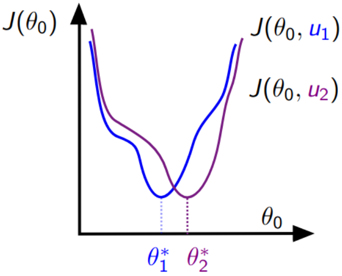

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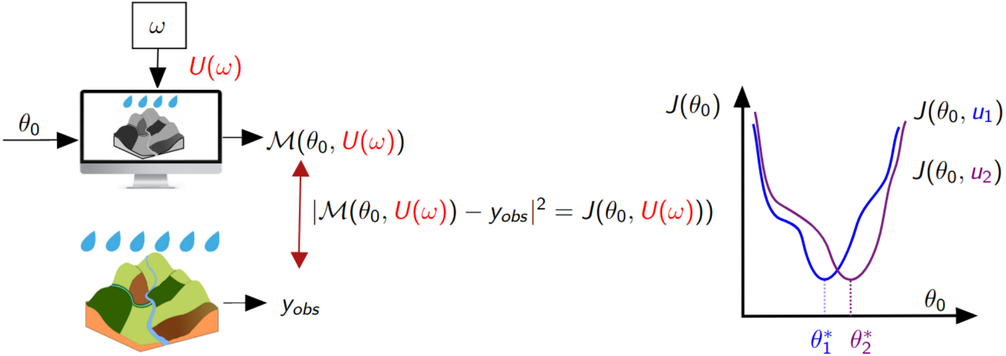

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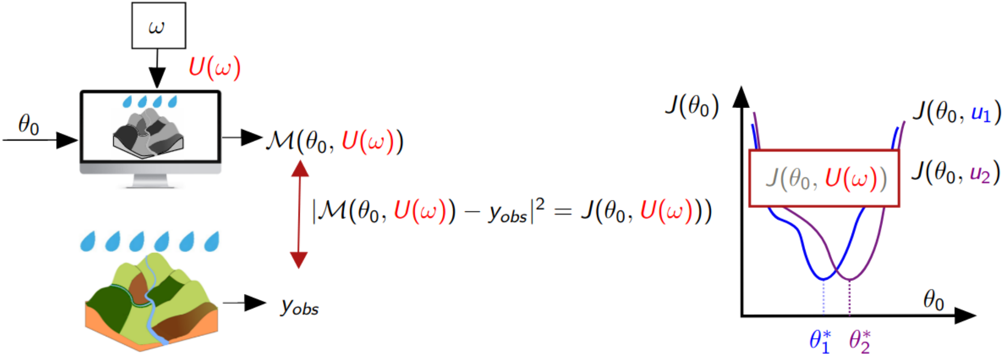

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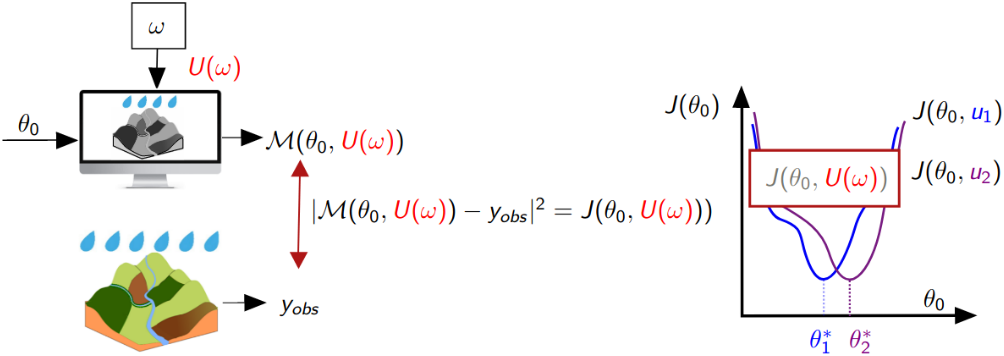


All hydrological model suffer from this.
Here, only rain uncertainties will be considered.

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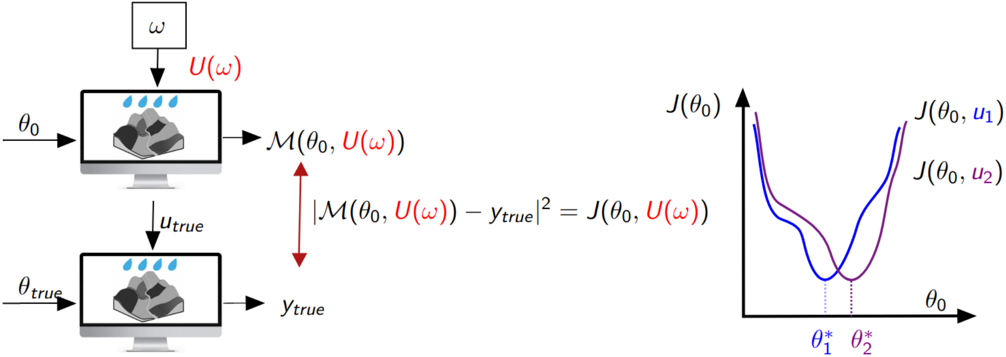


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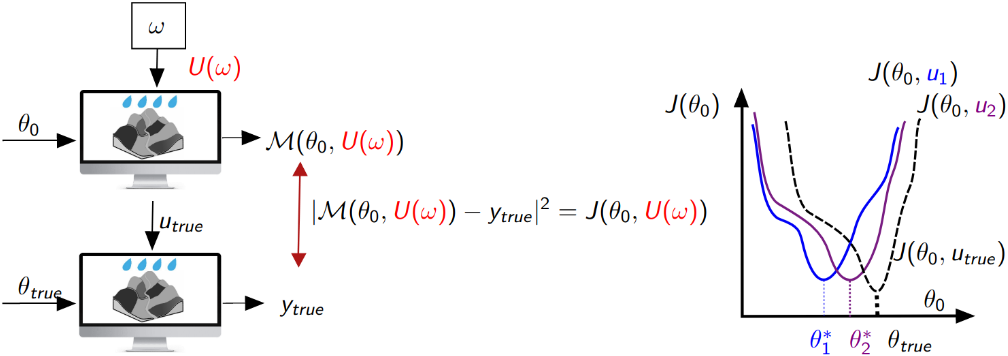


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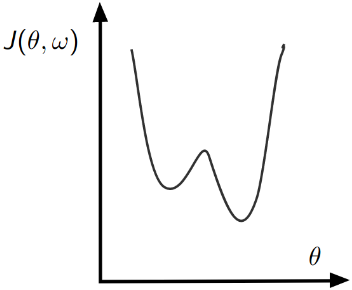
# Methods: Definition of robust estimators



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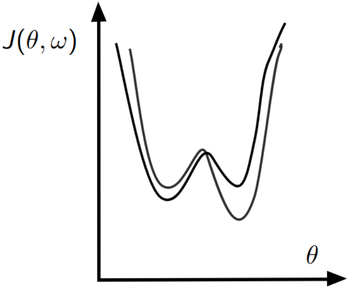

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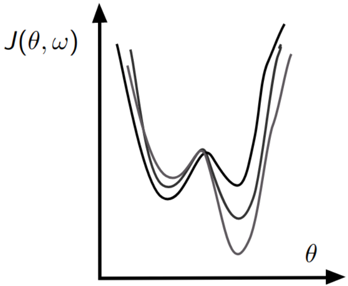

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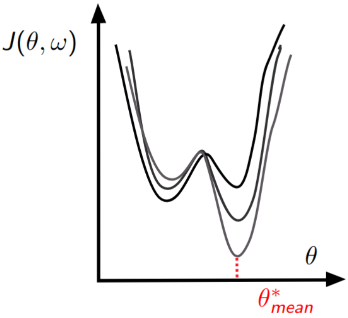


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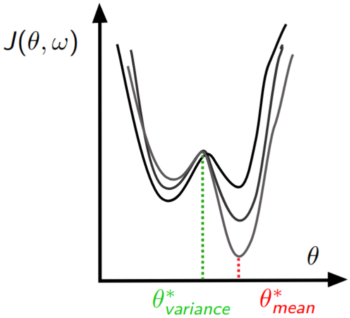


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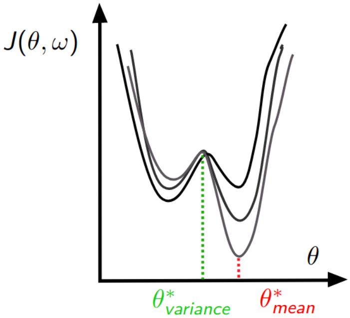


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The one we choose depends on our goal.
No matter the robustness we choose, they will all be computationally expensive.

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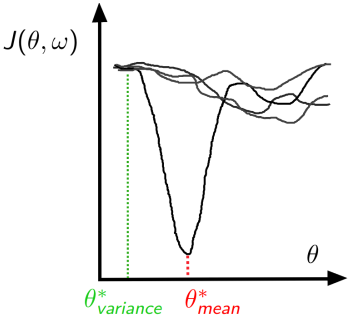


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# Case study: PESHMELBA virtual catchment

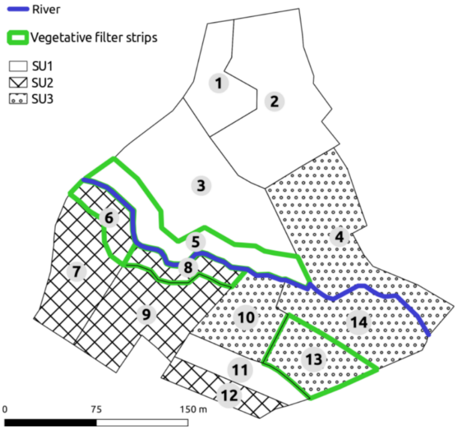



Figure: Rouzies et al. 2023
https://doi.org/10.5194/gmd-2021-425

- 3 soil types
- 2 vegetation types
- 4 soil horizons for each soil type
- 9 soil parameters for each horizon

- We focus on parcel 4.
- Calibrate parameters of parcel 4.
- Observations on parcel 4.
- Our observations are moisture profiles of the soil column after 100 hours of rain.

# Case study: Observations, cost function and rain perturbations

We calibrate only two parameters :

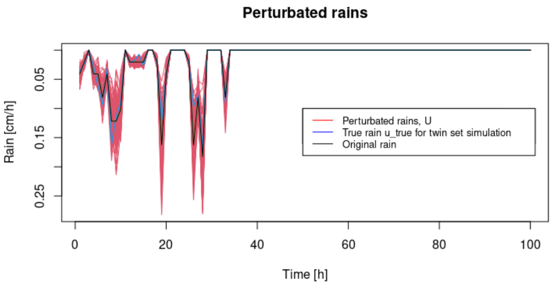
1. the water content in the soil at saturation **(thetas10)**
2. the parameter governing Van Genuchten infiltration equation **(mn10)**

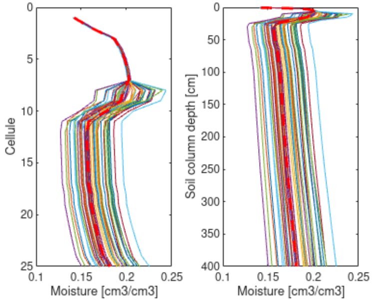
$$thetas_{10} \sim \mathcal{N}(0.3160, 0.0316^2),$$

$$mn_{10} \sim \mathcal{N}(0.1791, 0.0179^2),$$

$$\boldsymbol{\theta}_{true} = (0.164, 0.292).$$

# Case study: Observations, cost function and rain perturbations




$$J(\theta, u) = (\mathcal{M}(\theta, u) - y_{true})^2 = \sum_{cell_i=1}^{25} (\mathcal{M}(\theta, u)|_{cell_i} - y_{true}|_{cell_i})^2$$

$$y_{true} = \mathcal{M}(\theta_{true}, u_{true}) \in \mathbb{R}^{25}$$

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# Results:

We compare the following minimization problems :

1. $\boldsymbol{\theta}^*(u_{true}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} J(\boldsymbol{\theta}, u_{true})$
2. $\boldsymbol{\theta}^*(u_{false}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} J(\boldsymbol{\theta}, u_{false})$
3. Robust estimators
   3.1 $\boldsymbol{\theta}^*_{mean} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}(\mathbb{E}_U[J(\boldsymbol{\theta}, U)],$
   3.2 $\boldsymbol{\theta}^*_{variance} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}(\mathbb{V}ar_U[J(\boldsymbol{\theta}, U)],$

# Results: Calibration with $u_{true}$ vs $u_{false}$

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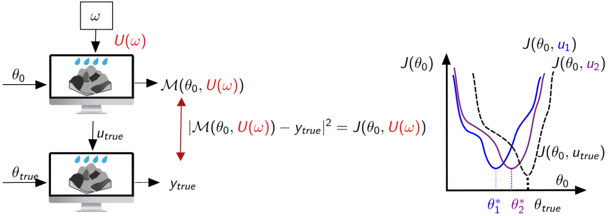

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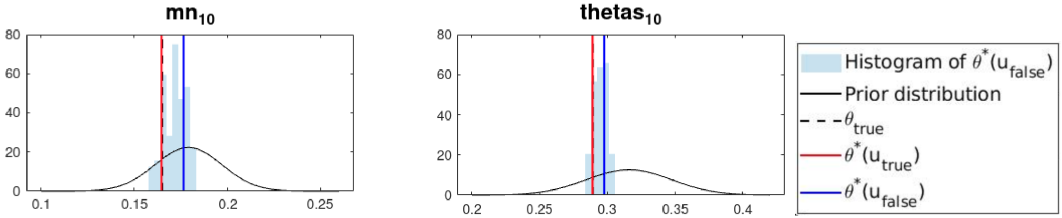


Figure: Minimizers when $u = u_{true}$ in red and when $u = u_{false}$ in blue.

# Results: Robust estimators

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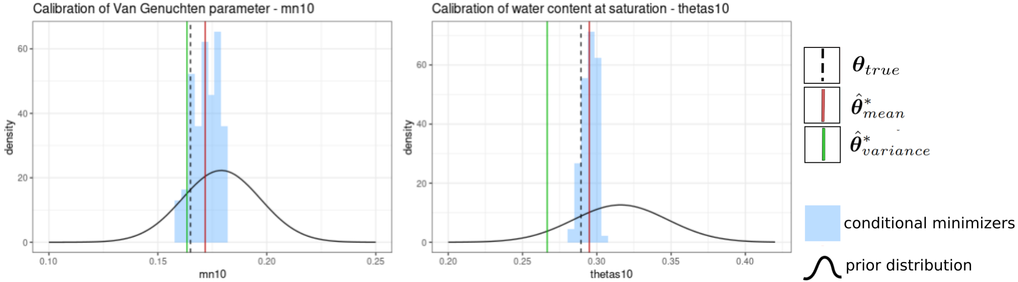


Robust estimators : the minimizer of the mean and variance for the two estimated parameters.

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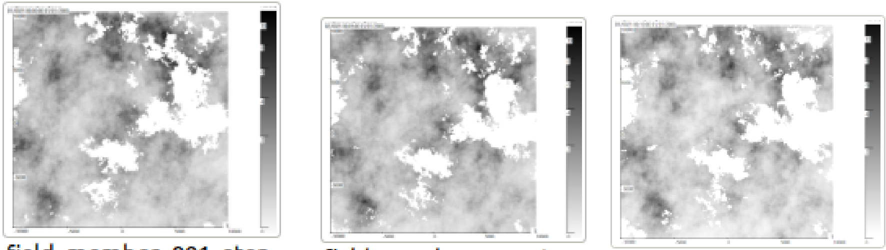
Suite :

- Différentes définitions de robustesse
- Différentes perturbations de pluie
- Le coût numérique de PESHMELBA -> metamodèles
- passage à plusieurs paramètres, plusieurs parcelles
- passage aux pluies spatialisées

# Pluies spatialisées



field_member_001_step_00001.png field_member_001_step_00002.png field_member_001_step_00003.png

Figure: Simulations de champs de pluie provenant de SAMPO, (Leblois and Creutin, 2013) doi : 10.1002/wrcr.20190.