Hotelling and Recycling

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Abstract

We study the exploitation of recyclable exhaustible resources such as metals that are crucial for the energy transition or phosphorus that is crucial for agricultural production. We use a standard Hotelling model of resource exploitation that includes a primary sector and a recycling sector. We study two polar cases: competitive and monopolistic extraction. We show that, when the primary sector is competitive, the Hotelling's rule holds and the price of the recyclable resource increases over time. We then show a new reason why the price of an exhaustible resource may decrease: when the primary sector is monopolistic, the primary producer has incentives to delay its production activities in order to delay recycling. As a consequence, the price path of the recyclable resource may be U-shaped. Numerical simulations reveal that the monopolist has an incentive to delay extraction when the recoverability rate is high (because more recycled goods are produced) or when the recoverability rate is low (when fewer recycled goods are expected to be produced in the future). As a consequence, the date of exhaustion of the virgin resource is further away in time for high and low levels of recoverability than for intermediate levels.

Keywords: non-renewable, recycling, monopoly, competition, market power, optimal control
JEL codes: Q31, Q53.

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1 Introduction

Recyclable exhaustible resources, such as metals (lithium, cobalt, rare earths, nickel, copper, manganese, etc.) and other elements like phosphorus, are increasingly important industrial inputs. Indeed, the aforementioned metals are important inputs for the production of many modern technologies, such as cell phones, light bulbs, automobiles, hybrid car batteries and gearboxes, and wind turbines (Chakhmouradian and Wall, 2012). Phosphorus, derived from phosphate rocks, is essential for soil fertility and has no substitute in agricultural production processes (Cordell at al., 2009).

Historically, the supply of these resources has often been highly concentrated.[1] Moreover, due to economic development and an increasing world population, demand for these resources has been growing rapidly and is expected to grow even more in the future (Alonso *et al.* 2012, Steen 1998).[2]

One strategy to increase supply and reduce the dependence of other countries on these resources is recycling. In order to assess the effect of recycling, it is necessary to understand how the primary sector may react. Since the main input to the production of recycled materials is the stock of scrap, the emergence of recycling activities may affect the dynamics of both the extraction of the exhaustible resource as well as the price of the final goods.

In this paper, we study the impact of a recycling sector in a stylized economic model of exhaustible resource extraction. We develop a Hotelling model of resource extraction in which the consumption good is produced from virgin or recycled materials. Virgin materials are extracted from a finite stock of a virgin resource and recycled materials are derived from the stock of scrap. The stock of recyclable scrap grows with current consumption of the final good at a given recoverability rate. We assume a competitive recycling sector in which production costs decrease with the stock of recyclable scrap. As a consequence, production in the primary sector generates a positive externality that benefits to the recycling sector. The inverse demand for the consumption good and the cost of recycling are linear. To ensure consistency with the various possible (future and present) market structures in the extraction sectors, we consider two polar cases: competitive and monopolistic

[1]Until 2010, China controlled 95% of the production of rare earths (Chakhmouradian and Wall, 2012), while a handful of countries, including Morocco, China, and the U.S.A, controlled most of the world's Phosphate rock production (IFDC, 2010)). However, prospects for the supply of rare earths and Phosphate rocks differ. Although China, which currently holds less than 40% of rare earth reserves, has maintained a very dominant position, supply has become less concentrated since 2010. The supply of Phosphate rocks has become more concentrated (85% of these reserves are currently located in Morocco and Western Sahara).

[2]Alonso *et al.* (2012) predict that the demand for rare earths will increase by 5 to 9 percent per annum until 2025. According to EFMA (2010) and Steen (1998), the demand for phosphorus may increase by as much as 50 to 100% by 2050 with increased global demand for food.

extraction.

Our first main result is the following. We show that, if the primary sector is competitive, the optimal level of production for firms in the primary sector is such that the price of the resource grows at the discount rate (this is the so-called Hotelling rule) because these firms assume that their production will not increase the stock of scrap.

Our second and third main results focus on the case of a monopolistic exctraction sector. Our second main result concerns the case where the recoverability rate is 100%. In this case, we are able to derive the following analytical results. The stock of scrap (as well as recycling) increases over time as long as the virgin resource is not exhausted and extraction decreases over time. The scrap stock remains constant once the virgin resource is exhausted, as 100% of used goods become scrap. The monopolistic firm has an incentive to postpone extraction compared to a situation without recycling. As a consequence, the price of the resource is U-shaped, that is the price first decreases and then increases.

Our third main result concerns cases where the recoverability rate is less than 100%. In this case, we are able to solve the model numerically. Our simulations provide some interesting insights. When the recoverability rate is sufficiently high, our results are qualitatively similar to those obtained in the previous case. The scrap stock first increases and then decreases (in contrast to the previous case, it starts to decrease before the exhaustion date of the virgin resource). Extraction decreases over time and recycling first increases and then decreases and the price path is U-shaped. When the recoverability rate is sufficiently low, our results are qualitatively affected. The stock of scrap decreases over time and extraction first increases and then decreases. Recycling first decreases (then increases and decreases again), so that the price of the final good can be always increasing. Interestingly, the date of depletion of the virgin resource is the highest for both high and low levels of recoverability. Indeed, in both cases, the monopolist has an incentive to delay extraction in order to face less competition in the future: when the recoverability rate is high, the monopolist delays extraction in order to slow the accumulation of scrap. Recyclers have fewer inputs and therefore produce fewer recycled goods. When the recoverability rate is low (and there is already a stock of scrap), the monopolist has an incentive to delay extraction and wait for recyclers to use some of that stock, and then produce fewer recycled goods when the stock of scrap is smaller.

The main takeaway of the present paper is a new reason why the price of a resource may decrease: a firm with market power in the extraction sector will (strategically) choose to delay extraction in order to reduce the opportunities for recycling. A monopolistic extracting firm will start at a lower level than in the competitive case, so the price will start at a higher level than in the competitive case and it may first decrease before increasing afterwards. The price will first decrease if the amount of recycled material increases over time at a greater rate than the decrease in the amount of extraction.

The remainder of the paper is structured as follows. Section 2 presents the literature review. Section 3 introduces our model in which we consider an exhaustible resource and a competitive recycling sector. Section 4 studies the price dynamics in the case of a competitive extraction sector. Section 5 focuses on the properties of the optimal path in the case of a monopolistic primary producer. Section 6 discusses corner solutions and Section 7 concludes.

2 Related Literature

The present paper relates to the literature that has been motivated by the Alcoa case and deals with the problem of a monopolistic extraction sector facing a competitive recycling sector (Gaskins 1974, Swan 1980, Martin 1982, Suslow 1986, Hollander and Lasserre 1988, Grant 1999) . This literature shows that, despite the presence of a competitive recycling sector, the extraction firm maintains (at least some of) its monopoly rents. Gaskins (1974) shows that recycling leads the monopolist to increase the price of the virgin resource in the short run and to slightly decrease it in the long run. Swan (1980) shows that the monopolist sets a price which approaches its marginal cost of production when there is price discrimination. Baksi and Long build a model of partial recycling and consider that consumers who participate to the recycling activity are heterogeneous in terms of their recycling cost. They show that the price set by the virgin producer will be close to the competitive price when the rate of recycling is close to one.

Gaudet and Long (2003) consider imperfect competition in the recycling sector and show that, when primary and secondary production decisions are made simultaneously, the presence of the recycling sector may increase the market power of the primary producer.[3] This literature focuses on the comparison between a competitive and a monopolistic recycling sector, while we focus on the comparison between a competitive versus monopolistic extraction sector. It is worth stressing that none of the above papers show that the price of the primary good can be U-shaped.

An important aspect of our work, which has not often been considered in the literature, is that we provide insights into the important role of scrap and the feedbacks between the cost of scrap and the market for final goods. An exception

[3] Weikard and Seyhan (2009), motivated by the case of phosphorus, consider a model of competitive resource extraction and the possibility of saturated demand (i.e. taking into account the possibility that soil can become saturated with phosphorus).

is Kaffine (2014), which considers a static model with perfect competition and focuses on very different, policy-driven research questions.

Recently, two papers have investigated the issue of recycling under an energy transition perspective. Pommeret *et al.* (2022) analyze how the possibility of recycling can affect the timing of the energy transition. They consider the presence of a depreciated green capital that can be recycled. They deal with a social planner's problem. They show that recycling influences the steady state in that it increases the stock of green capital and reduces its value. They also show that recycling induces a larger use of minerals (primary resources). Intuitively, this means that the social planner boosts the use of primary resources in order to increase the possibilities of recycling. Fabre *et al.* (2020) analyze the issue of energy production in the case where minerals and fossil resources are rare by considering that minerals are recyclable. They consider a social planner's problem. They show that the presence of recycling speeds up the investment in renewable capacity and makes the energy mix based on more renewable energy. They also show that a larger recycling rate induces a greater rate of extraction of minerals in the initial period. Our results differ from those two papers in that we show that the extraction rate can be reduced when there is a recycling sector. Another difference is that both papers consider a social planner's problem, while we postulate a competitive/monopolistic framework. It is well acknowledged that the social planner would want an increase in primary resources use to boost future recycling, while we show that the monopolist would strategically choose a reduction in primary resources use to limit the possibilities of recycling. This can explain the differences observed in terms of results.

In this paper, we do not explicitly consider the social or environmental motivations for the development of recycling. The environmental advantages of recycling have long been recognized in the economic literature (Smith 1972, Weinstein and Zeckhauser 1974, Hoel 1978).[4] There is an important literature that includes waste accumulation and environmental damage in their models, making recycling a multiple dividend activity (e.g. Fullerton and Kinnaman, 1995; Palmer *et al.*, 1997; Acuff and Kaffine 2013; Lafforgue and Lorang, 2022). In the present paper, our focus is on the effect of the existence of a recycling sector on the virgin resource extraction sector and not on social welfare.

The present paper is also linked to the literature dealing with durable resources. Levhari and Pindyck (1981) show that, in the case of a competitive industry that produces a durable good, the price of the resource first decreases and may increase thereafter. In contrast, we find that the price of the resource is always increasing in the context of a competitive extraction sector.

[4]Andre and Cerda (2006) provide a model that takes into account the interactions of the material composition of output and waste as potentially recyclable products.

There are other explanations for U-shaped price profiles of exhaustible resources. Pindyck (1978) shows that this may occur when exploration and reserve accumulation are taken into account. In a model with exogenous technical change and endogenous change in grades, Slade (1982) also finds that U-shaped price profiles may occur. These studies do not consider the possibility of recycling.

3 The Model

The economy produces a quantity Q of a consumption good. The consumption good can be produced from a non-renewable resource or from recycled materials. For simplicity we assume that the virgin and recycled materials are perfect substitutes. The primary sector faces a competitive sector of recycling firms.

Non-renewable resource and scrap dynamics

Let $X(t) \geq 0$ be the residual stock of virgin resource at time t, X^0 be the initial stock, with $X(0) \equiv X^0 > 0$, and $x(t) \geq 0$ be the extraction rate at time t, so that:

$$\dot{X}(t) = -x(t). \tag{1}$$

The unit cost of extraction of the virgin resource is assumed to be zero.

Let $S(t) \geq 0$ be the stock of (recyclable) scrap at time t, with an initial stock $S(0) = S^0$. Let $r(t) \geq 0$ be the quantity of recycled materials marketed at time t, so that the total quantity consumed at time t is $Q(t) = x(t) + r(t)$. Let $\alpha \in [0, 1]$ be the proportion of the output that becomes recyclable scrap. It represents the recoverability rate of the final good. Here, $1 - \alpha$ can be interpreted as "dissipated" materials Gloser *et al.* (2013) or as a "rate of retirement" (Gaskins 1974, Grant 1999)). The dynamics of the scrap material thus writes:

$$\dot{S}(t) = \alpha Q(t) - r(t) = \alpha x(t) - (1 - \alpha) r(t). \tag{2}$$

The recycling sector

The recycling sector is assumed to be competitive. The total cost of recycling includes the cost of collecting, processing and transporting waste (included in the price of waste if the recycler buys waste from specialized companies) in addition to the cost of the recycling operation itself. As such, the marginal cost of recycling, denoted $c(S, r)$, is assumed to be a decreasing function of the stock of scrap[5] and

[5]This assumption is natural if the cost to collect waste decreases with the total quantity of available waste recycler (it is easier to find scrap) or if recyclers buy waste from specialized companies and the price of waste decreases with available waste. See for instance Slade (1980) for the case of copper and Blomberg and Soderholm (2009) for the case of aluminum.

an increasing function of the quantity of recycled materials,[6] that is $\frac{\partial c(S,r)}{\partial S} < 0$ and $\frac{\partial c(S,r)}{\partial r} > 0$.

In equilibrium in the recycling sector, absent any corner solution,[7] the price of the consumption good must equal the marginal cost of recycling:

$$p\left(Q\left(t\right)\right) = c\left(S\left(t\right), r\left(t\right)\right). \tag{3}$$

The primary sector

The price of primary production is the same as the price of recyled materials. Thus, the discounted profits in the primary sector, with discount rate $\delta \geq 0$, are given by:

$$\int_{0}^{+\infty} e^{-\delta t} px dt, \tag{4}$$

In the following, we will consider two polar cases: the case of a competitive primary sector and the case of a monopolistic primary sector. In the case of a competitive primary sector, resource owners behave as price takers, and they consider the price of the resource to be a function of time, $p \equiv P\left(t\right)$. In the case of a monopolistic primary sector, the owner of the resource takes into account how extracted quantities affect the total quantity of material supplied (virgin as well as recycled) and the effect of this supply on the price of the resource, that is $p \equiv p\left(Q\left(t\right)\right)$.

Linear specification

For most of our analysis, we will use the following linear specifications of the demand for the consumption good and the recycling cost functions:

$$p\left(Q\left(t\right)\right) = 1 - Q\left(t\right) \text{ and } c\left(S\left(t\right), r\left(t\right)\right) = 1 - b - \beta\left(S\left(t\right) - r\left(t\right)\right), \tag{5}$$

with $\beta > 0$. Parameter $\underline{b} \leq b \leq \overline{b}$ is a measure of the efficiency of the recycling technology (the higher b, the lower the marginal cost of recycling). We further assume that $\overline{b} \leq 1 - \beta(S^0 + X^0)$ in order to ensure that the marginal cost c is always non negative. We also must have $\underline{b} \geq -\beta(S^0 + X^0)$, in order to avoid cases where (3) has never an interior solution.[8]

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[6] There may be economies of scales, at least for sufficiently low levels of recycling (e.g. see Bohm *et al.* 2010). However, assuming that the marginal cost function of recycling is increasing in recycled materials seems reasonable, and we follow Rosendahl and Rubiano (2019) and Gaudet and Long (2003) who make the same assumption.

[7] see Section 6 for a discussion about corner solutions.

[8] Indeed, if $b < -\beta(S^0 + X^0)$, then $p(Q) < C(S, r)$ for all $r \geq 0$ and all $S \leq S^0 + X^0$, that is recycling is never profitable.

Solving the recycling sector equilibrium condition (3), we characterize the equilibrium quantity of recycled material at time t as follows:

$$r(t) = \frac{b + \beta S(t) - x(t)}{1 + \beta}. \tag{6}$$

The condition implies that the quantity of recycled material at time t increases with the quantity of scrap and decreases with the quantity of extracted resource. This result is quite intuitive. Since recycling relies on scrap, the higher the stock of scrap, the larger the recycling firms' production. Recycling at time t decreases with the quantity of virgin product sold at time t because recycled and virgin products are substitutes.

4 Competitive primary sector

In this section, we consider the case of a competitive primary sector. In this case, producers take the price, P, as well as the total quantity, Q, as given.[9] They consider the following problem:

$$\underset{\{x\}}{Max} \int_0^{+\infty} e^{-\delta t} P(t)\, x(t)\, dt, \tag{7}$$

s.t.

$$\dot{X}(t) = -x(t), \tag{8}$$

$$X(t) \geq 0, \;\; x(t) \geq 0. \tag{9}$$

The Hamiltonian and the Lagrangian for this optimal control problem are as follows:[10]

$$H = Px + \lambda_X(-x), \tag{10}$$

$$L = H + \mu_X X + \mu_x x, \tag{11}$$

where λ_X is the co-state variable associated with the stock X, and, μ_X, μ_x are the multipliers associated with the non-negativity constraints $X \geq 0$, and $x \geq 0$. The competitive solution is found by solving problem (7) subject to (8) and (9) and then using (2) and (3) to determine the recycling level and the market clearing price. The Maximum Principle requires that the following conditions hold:

$$\frac{\partial L}{\partial x} = P - \lambda_X + \mu_x = 0, \tag{12}$$

[9] The same assumption is made in Levhari and Pindyck (1981). The producers therefore do not take into account the effect of x on the evolution of the stock of scrap and on recycling.

[10] We drop the time index when there is no possible confusion.

$$\dot{\lambda}_X = \delta\lambda_X - \frac{\partial L}{\partial X} = \delta\lambda_X - \mu_X, \tag{13}$$

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{14}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0. \tag{15}$$

When both extraction and residual stock levels, $x(t)$ and $X(t)$, are strictly positive, we have $\mu_x = 0$ and $\mu_X = 0$. Substituting these respective values into (12) and (13) yields:

$$P - \lambda_X = 0, \tag{16}$$

$$\dot{\lambda}_X = \delta\lambda_X \tag{17}$$

Differentiating (16) with respect to time gives:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \frac{\dot{P}}{P} \tag{18}$$

From (17), we have:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \delta \tag{19}$$

The combination of (18) and (19) yields:

$$\frac{\dot{P}}{P} = \delta \tag{20}$$

We can thus conclude the following:

Proposition 1: *If the extraction sector is competitive, the optimal extraction path is such that the Hotelling's rule holds: the price of the resource grows over time at a rate equal to the discount rate.*

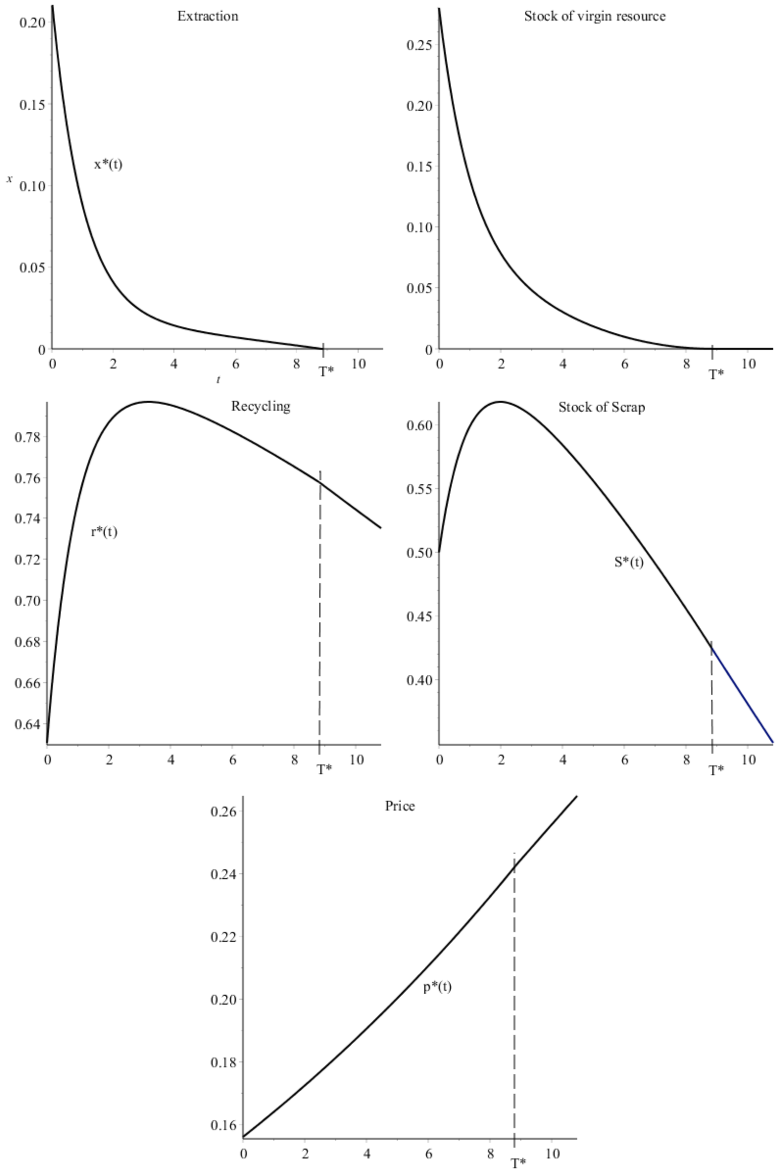
This proposition shows that the price of the resource increases over time when the extraction sector is competitive. This result reveals a major difference between recyclable goods and durable goods.

In the case of durable goods, such as cars for example, consumers arbitrate between keeping their car or reselling it, so if the price of cars increases (when the resource to produce them is abundant), there are opportunities for arbitrage: Consumers will buy cars to use them and resell them later at a higher price. Thus, the price of durable goods will first decrease. The price of a durable exhaustible resource is then either always decreasing or U-shaped when the resource extraction sector is competitive (Levhari and Pindyck 1981). In the case of recyclable goods, such as used batteries, there is no arbitration for consumers, since used batteries have to be reprocessed to have resale value.

The path of recycling and of the stock of scrap are determined by the recycling equilibrium condition (3) together with the dynamics of the stock of scrap (2). Thus, the rate at which the price increases is given by the Hotelling rule and does not depend on the recycling technology.

Figure 1 shows the result of a numerical simulation that illustrates the case of a competitive extraction sector. The shadow price of the exhaustible resource increases over time, so extraction and stock of this resource decrease over time until the resource is exhausted. Extraction is initially high, so recycling is low (due to the substitutability between extraction and recycling), and levels are such that, for this simulation, the scrap stock initially increases. Since the stock of scrap increases and extraction decreases, recycling profitability increases, so it increases. At some point, recycling becomes large enough and extraction small enough that the scrap stock decreases. Once the exhaustible resource is depleted, since the simulation uses a recoverability rate of less than 100%, the scrap stock decreases until it is completely exhausted. As a consequence, recycling also decreases at some point in time.

Figure 1: Optimal extraction path with a competitive extraction sector



Notes: We use the linear specification to plot these graphs. The parameter values are $\alpha = 0.95$, $X^0 = 0.28$, $\theta = 0.3$, $\delta = 0.05$, $b = 0.9$, $S^0 = 0.5$. T^* denotes the date of exhaustion of the virgin resource.

5 Monopolistic primary sector

In this section, we consider the case of a monopolistic primary sector and we focus on the linear specification of the model. We derive several properties regarding the optimal time path of virgin resource extraction, the stock of scrap, the equilibrium recycling quantity, and the price of the consumption good.

Using the equilibrium recycling condition (6) and substituting into the expression of the price given in (5), we have $p(Q) = \theta(a - x - S)$, where $a = (1 - b + \beta)/\beta$ and $\theta = \frac{\beta}{1+\beta}$. To find the optimal extraction path for the monopolist, we solve the following maximization problem:

$$\underset{\{x \geq 0\}}{Max} \int_{0}^{+\infty} e^{-\delta t}\theta\left(a - x(t) - S(t)\right)x(t)dt, \tag{21}$$

subject to the dynamic of the resource stock:

$$\dot{X}(t) = -x(t), \tag{22}$$

and to the dynamic of the stock of scrap:

$$\dot{S}(t) = \alpha' x(t) - (1-\alpha)\theta S(t) - b', \tag{23}$$

where $\alpha' = \alpha + \frac{1-\alpha}{1+\beta}$, $b' = \frac{1-\alpha}{1+\beta}b$, $X, S, x \geq 0$, $X^0 > 0$ and $S^0 \geq 0$ given.

The current value Hamiltonian H and Lagrangian L are defined as follows:

$$H = \theta(a - x - S)x + \lambda_X(-x) + \lambda_S\left(\alpha' x - (1-\alpha)\theta S - b'\right), \tag{24}$$

and,

$$L = H + \mu_X X + \mu_S S + \mu_x x, \tag{25}$$

where λ_X and λ_S are the co-state variables associated with the stocks X and S, and μ_X, μ_S, μ_x are the multipliers associated with the non-negativity constraints $X \geq 0$, $S \geq 0$, and $x \geq 0$.

Full resolution of the monopolist's programme yields the extraction and recycling paths as well as the evolution of the stock of virgin resource and the stock of scrap and the dynamics of the price of the consumption good. Solving the problem involves finding constants that are characterized by nonlinear equations, which limits our ability to study the properties of the solution for any value of the parameters. However, we are able to derive the main properties of the solution when the recoverability rate is 100%. We first focus on this case and then provide numerical results for different levels of the recoverability rate. The two cases have different qualitative implications.

5.1 Optimal extraction with perfect recoverability

Perfect recoverability seems to be a reasonable assumption for a number of materials, such as copper,[11] vanadium, iron, nickel, palladium, iridium, platinum or gold (see Ciacci *et al.* 2015). In this case, we can show that the optimal path has the following qualitative properties:

Proposition 2: *Under the linear specification and perfect recoverability ($\alpha = 1$), the optimal extraction path is such that:*
(i) *Extraction $x^*(t)$ is decreasing up to the date of depletion of the virgin resource T^*;*
(ii) *The stock of scrap $S^*(t)$ increases up to T^* and remains constant after;*
(iii) *Recycling $r^*(t)$ increases up to T^* and remains constant after.*

The optimal level of extraction decreases over time. This result is in line with the standard Hotelling model. Indeed, the extracting firm discounts time, choosing to extract more of the resource today and less tomorrow. The quantity of marketed recycled material, in contrast, increases over time up to the exhaustion of the virgin resource. The intuition of these results is as follows. The recoverability rate is 100%, thus the stock of scrap increases over time up to the exhaustion of the virgin resource, which reduces the unit cost of recycling. This, in turn, provides incentives for recycling firms to increase their production. At the same time, the quantity of extracted material decreases, also causing the level of recycling to increase (due to substitutability).

Once the virgin resource is exhausted, since the recoverability rate is perfect and there is no more extraction of virgin material, the stock of scrap and the level of recycling remain constant.

We are now in a position to state our main result. This one concerns the optimal price path:

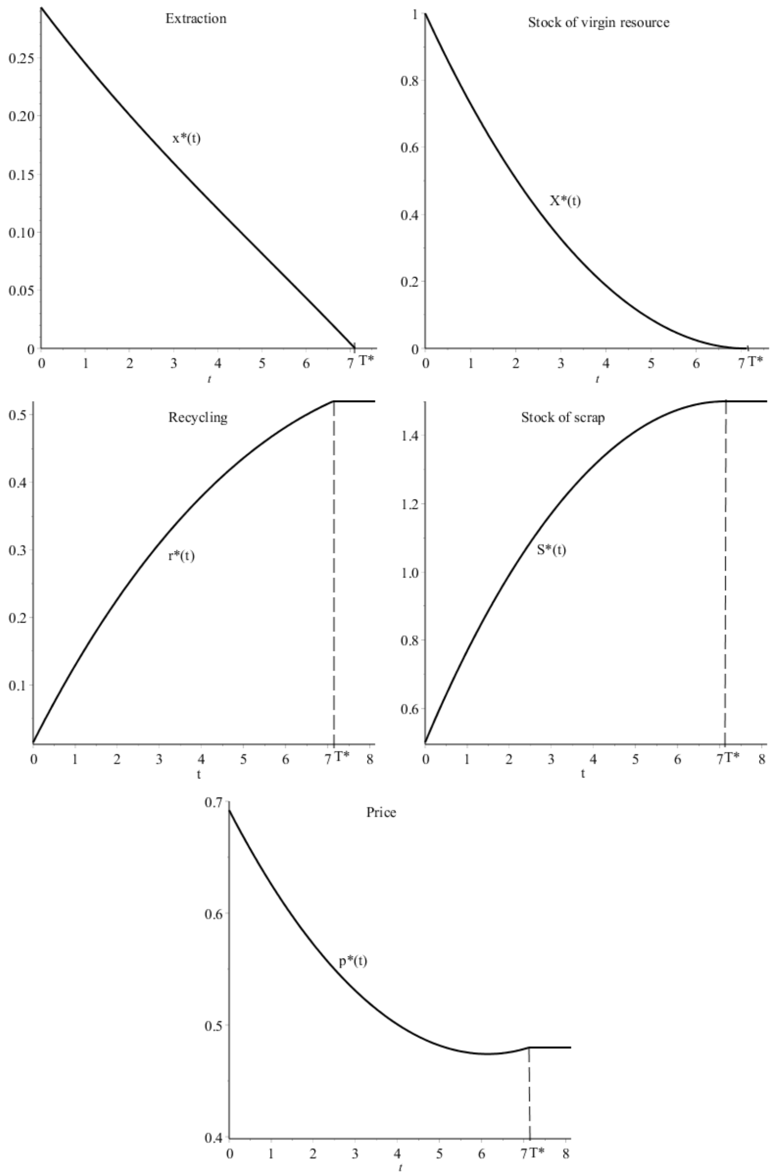
Proposition 3: *Under the linear specification and perfect recoverability ($\alpha = 1$), the optimal price path $p^*(t)$ is U-shaped.*

This result states that the standard result of an increasing resource price does not hold if the recoverability rate is 100%. In the first phase, the price decreases because the amount of scrap increases over time at a greater rate than the decrease in the amount of extraction ($\dot{p} = -\theta(\dot{x} + \dot{S})$). Intuitively, a low pace of extraction delays accumulation of scrap and then future recycling, which is beneficial to the monopoly. In the second phase, we are getting closer to the date of exhaustion

[11] Copper has a recoverabillity rate around 94-99% for most applications, see Table S5 in Gloser *et al.* (2013).

of the virgin resource. Consequently, the marginal cost of extraction becomes increasingly high and then, at some point in time, the price of the resource increases.

Figure 2: Optimal extraction path with a monopolistic extraction sector under perfect recoverability



Notes: The parameter values used to plot these graphs are $\alpha = 1$, $X^0 = 1$, $\theta = 0.3$, $\delta = 0.05$, $b = 0.1$, $S^0 = 0.5$. T^* denotes the date of exhaustion of the virgin resource.

Figure 2 illustrates the results of Propositions 2 and 3. Notice that after the date of exhaustion of the virgin resource, since there is no extraction of virgin resource and the level of recycling is constant, the price is also constant.

Before going further, it is important to understand why the case where the recoverability rate is perfect is specific and simpler to solve than the other cases.

The optimal extraction path is given by:

$$\frac{\partial L}{\partial x} = \theta \left(a - 2x - S \right) - \lambda_X + \alpha' \lambda_S + \mu_x = 0. \tag{26}$$

The pace of extraction is thus given by (assuming $x > 0$):

$$\dot{x} = -\frac{1}{2\theta} \dot{\lambda}_X - \frac{1}{2} \dot{S} + \left(\theta\alpha + 1 - \theta \right) \frac{1}{2\theta} \dot{\lambda}_S. \tag{27}$$

Condition (27) shows that extraction tends to decrease over time when the shadow price of the virgin resource increases over time, when the stock of scrap increases or when the shadow price of scrap decreases over time. The shadow price of the virgin resource always increases over time as the virgin resource becomes scarcer. The shadow price of scrap and the stock of scrap vary over time in opposite directions.

Thus, the evolution of the stock of scrap provides useful information as regards the evolution of extraction. When the stock of scrap increases over time, extraction necessarily decreases over time. When the stock of scrap decreases over time, extraction may increase or decrease over time.

We can now see why the case where the recoverability rate is 100% is simpler than the other cases. When the recoverability rate is perfect, each unit of virgin resource becomes, after consumption, a unit of scrap (in this case, condition (23) simplifies to $\dot{S} = x \geq 0$). Thus, the stock of scrap grows and then extraction of the virgin resource decreases over time. When the recoverability rate is not perfect (condition (23) writes $\dot{S} = \alpha x - (1 - \alpha) r$), the stock of scrap may decrease over time over some intervals of time, and thus it is more difficult to conclude as regards the evolution of extraction. It is therefore also more difficult to conclude about the qualitative properties of the optimal path and the evolution of the price of the final good.

To provide insight into cases where the recoverability rate is not 100%, we perform numerical simulations for different levels of recoverability in section 5.2 below.

5.2 Optimal extraction for different recoverability rates

For some materials, the recoverability rate is quite low. For recyclable elements such as cerium (a rare earth metal), the recoverability rate is as low as 10% (see Ciacci *et al.* 2015).

For cases where the recoverability rate is less than 100%, we are able to solve the problem numerically, using the linear specification and for parameter values compatible with our assumptions. Table 1 shows the simulation results for the dates of exhaustion of the virgin resource (T^*) and the stock of scrap (T'). We numerically solve for both dates for different values of the recoverability rate and hold the values of all other parameters constant. These simulations suggest that the date of depletion of the stock of scrap increases as the recoverability rate increases, which is intuitive. A less intuitive result is that the date of depletion of the virgin resource is a non monotonic function of the recoverability rate. Indeed, for sufficiently high levels of recoverability (above 50%), our results suggest that an increase in the recoverability rate leads to an increase in the date of exhaustion of the virgin resource. For sufficiently low levels of recoverability (below 40%), an increase in the recoverability rate leads to a decrease in the date of exhaustion.

Table 1: Date of exhaustion and recoverability rate with a monopolistic extraction sector for different recoverability rates

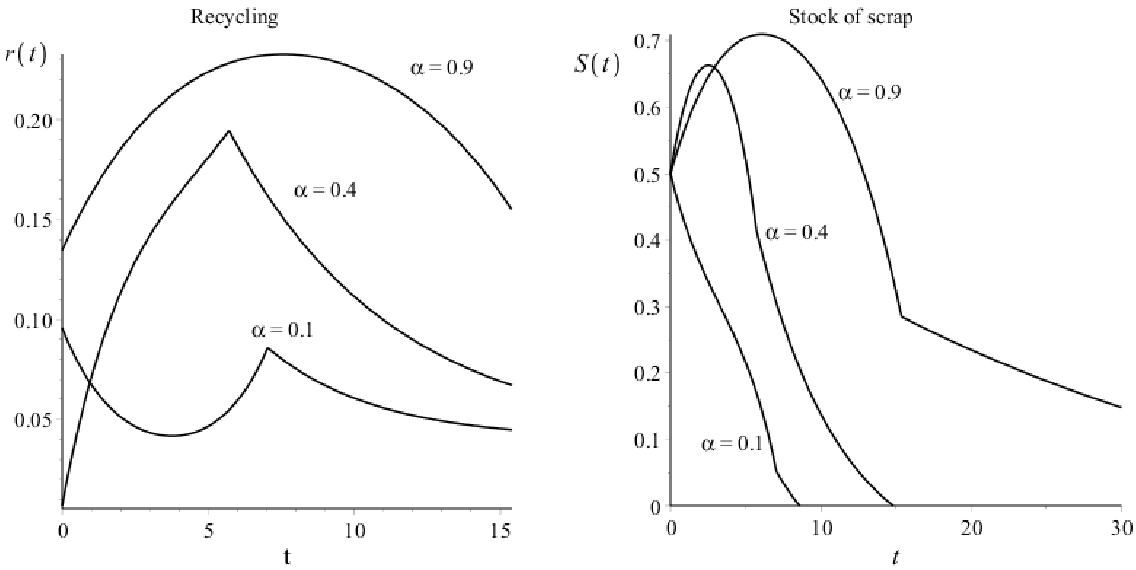
Parameters						Simulations	
α	X^0	S^0	δ	θ	b	T^*	T'
10%	1	0.5	2%	0.3	0.1	7.03	8.60
20%	1	0.5	2%	0.3	0.1	6.14	10.52
30%	1	0.5	2%	0.3	0.1	5.77	12.45
40%	1	0.5	2%	0.3	0.1	5.71	14.82
50%	1	0.5	2%	0.3	0.1	5.91	17.99
60%	1	0.5	2%	0.3	0.1	6.38	22.54
70%	1	0.5	2%	0.3	0.1	7.30	29.70
80%	1	0.5	2%	0.3	0.1	9.24	42.46
90%	1	0.5	2%	0.3	0.1	15.37	60.27

Notes: This Table presents the simulation results of the exhaustion dates T^* (virgin resource) and T' (scrap).

This counterintuitive result can be understood by examining the optimal extraction path for various levels of recoverability. Figure 3 shows the optimal monopoly extraction path of the virgin resource for $\alpha = 90\%$, $\alpha = 40\%$ and $\alpha = 10\%$. When the recoverability rate is low (and there is already a stock of waste), the monopolist has an incentive to delay extraction and wait for recyclers to use some of that stock, and then produce fewer recycled goods when the stock of waste is smaller. Thus, if the recoverability rate is low, an increase in the recoverability rate reduces the monopolist's incentive to delay recycling. Hence, in this case, an increase in the recoverability rate leads to a decrease in the exhaustion date of the virgin resource.

Figure 3 shows interesting features. When the recoverability rate is as high as 40% or 90%, the stock of scrap is first increasing and then decreasing. Notice that, differently from the case where $\alpha = 1$, it starts decreasing before the exhaustion date (it is $T^* = 15.37$, 5.71 and 7.03, for $\alpha = 0.9$, 0.4 and 0.1, respectively). The evolution of recycling is similar to the case where the recoverability rate is close to 100% (increasing and then decreasing). When the recoverability rate is as low as 10%, the stock of scrap decreases over time and recycling has a quite complex dynamics: it is first decreasing, then increasing, then decreasing again.

Figure 3: Recycling and stock of scrap paths with a monopolistic extraction sector for different recoverability rates

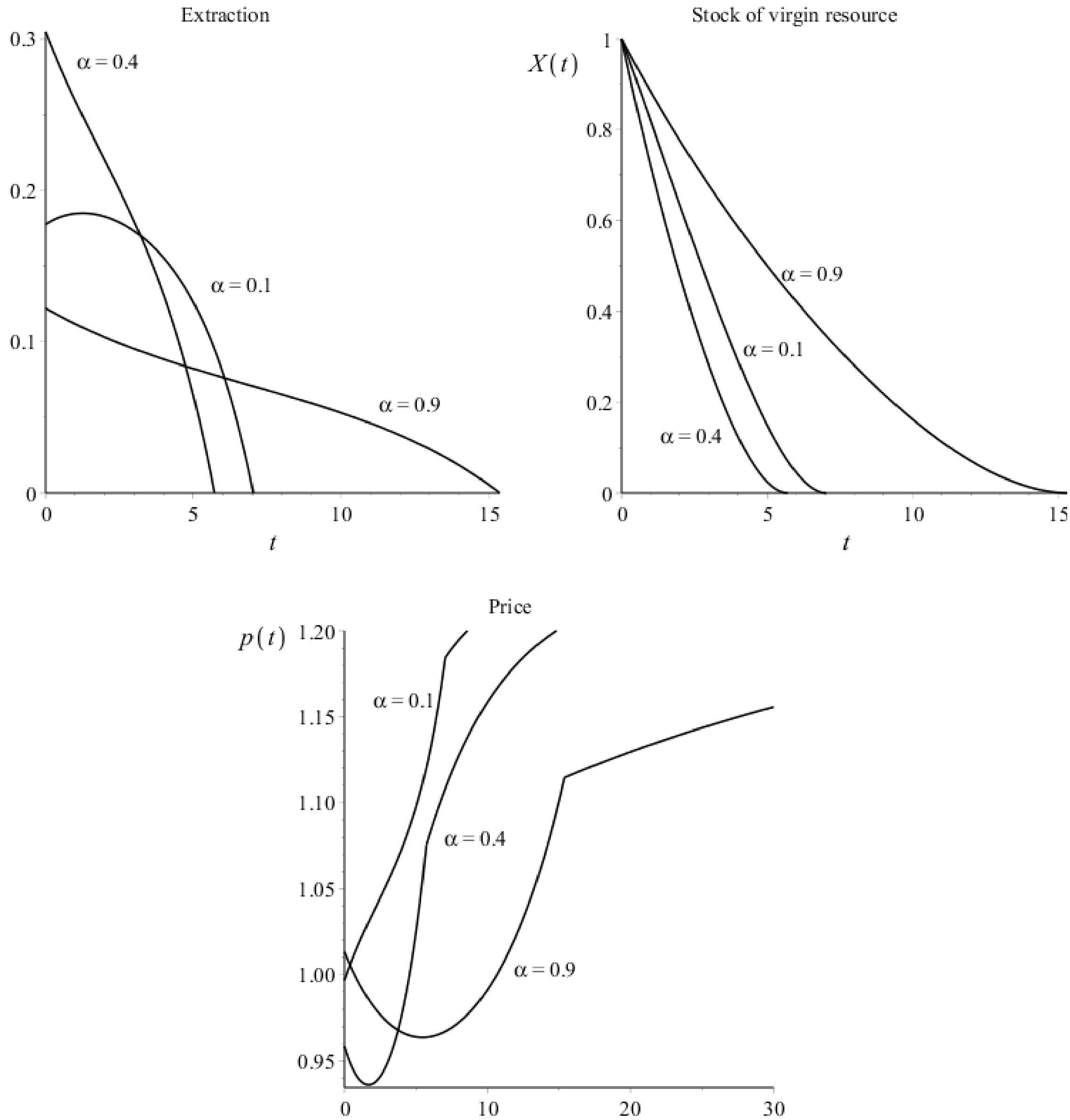


Notes: Parameter values: $X^0 = 1$, $\theta = 0.3$, $\delta = 0.02$, $b = 0.1$, $S^0 = 0.5$. T^* denotes the date of exhaustion of the virgin resource.

When looking at the evolution of extraction and the stock of virgin resource (see Figure 4), we can make the following observations. When the recoverability rate is sufficiently high ($\alpha = 0.4$ or $\alpha = 0.9$), extraction decreases over time. Moreover, the higher the recoverability rate, the lower the pace of extraction. This highlights the fact that a higher recoverability rate provides the monopoly more incentives to delay extraction. When the recoverability rate is low ($\alpha = 0.1$), the optimal extraction path is not always decreasing over time as in the previous cases, it first increases and then decreases.

We can now comment on the evolution of the price of the final good. When the recoverability rate is as large as 40% or 90%, the evolution of the price is similar to the case where the recoverability rate is close to 100%. The price is first decreasing and then increasing over time. When the recoverability rate is

Figure 4: Extraction, stock of virgin resource and price path with a monopolistic extraction sector for different recoverability rates



Notes: Parameter values: $X^0 = 1$, $\theta = 0.3$ and $\delta = 0.02$, $b = 0.1$, $S^0 = 0.5$. T^* denotes the date of exhaustion of the virgin resource.

only 10%, the price is always increasing over time. This is similar to the situation where there is no recycling (the Hotelling model), but the underlying reason why the price is increasing at the beginning is different. This initial increase in the price of the resource is not due to a decrease in extraction (which is first increasing), but to a decrease in recycling.

Finally, notice that the initial extraction level is lower for high and low recoverability rates ($\alpha = 0.9$ or 0.1) than for an intermediate recoverability rate ($\alpha = 0.4$).[12] The intuition is that when the recoverability rate is high, the monopoly has a strong incentive to extract little virgin resource initially, as more material extracted today means more scrap and therefore more recycling in the future. When the recoverability rate is low, the monopoly has a strong incentive to wait for recyclers to use the available scrap stock, as this stock will decrease quickly and recycling will then also decrease. Since recycling and extraction are strategic substitutes, we observe the opposite as regards recycling: it is initially higher for high and low recoverability rates than for an intermediate level of recoverability. Since an increase in extraction leads to a less than proportional decrease in recycling (see equation (6) and $\frac{1}{1+\beta} < 1$), we also observe that the price is initially higher for high and low recoverability rates than for an intermediate recoverability rate.

6 Discussion

6.1 Recycling deterrence

In the previous analysis, we focused on interior solutions as regards the recycling sector competitive equilibrium. Indeed, we focused on the case where the level of recycling is determined by condition (3), that is $p(Q(t)) = c(S(t), r(t))$. For some parameter values, it could be the case that the optimal extraction path is such that over some periods of time, there is some extraction and no recycling. Indeed, if the level of extraction is sufficiently high and the stock of scrap is sufficiently low, recycling is deterred.[13] Indeed, given our linear specification of the price and cost functions, $r(t) = 0$ if $p(x) \leq c(S, 0)$, which is equivalent to $b + \beta S \leq x$.

Let us discuss more deeply the possibility of recycling deterrence in the case of perfect recoverability ($\alpha = 1$). First, notice that deterrence can never occur if the initial stock of virgin resource X^0 is sufficiently small, because in this case extraction is sufficiently low so that recycling is not deterred.[14] In this case, the

[12] And, since $\dot{X} = -x$, a direct consequence is that the slope of the stock of virgin resource X is initially larger for high and low recoverability rates.

[13] See Ba and Mahenc (2019) for a two period model of recycling deterrence.

[14] It cannot happen as long as $r^*(0) = \frac{b}{\beta} + (a - S^0)\frac{\delta}{2\beta}\frac{e^{\gamma^- T^*} - e^{\gamma^+ T^*}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \geq 0$, as $r^*(t)$ given in condition (61) in Appendix A increases over time. Since T^*, which is implicitly characterized by condition (56), increases when X^0 increases and goes to 0 when X^0 goes to 0, we have that the condition holds when $X^0 = 0$ and for sufficiently low levels of X^0.

solution is the one we discussed in the previous section.

Second, under our assumptions, it is not possible that the monopoly optimally chooses to deter recycling up to the exhaustion date, that is to have only extraction in a first phase, followed by only recycling in a second one, since in this case $S(T^*) = S^0 + X^0 > 0$ and then $b + \beta S(T^*) > x(T^*) = 0$. Indeed, when going close to the exhaustion date, the extraction level has to become small and then the price will become sufficiently high so that recycling becomes profitable (i.e. condition $b + \beta S < x$ cannot hold for sufficiently small x).

Deterrence occurs when the stock of scrap is sufficiently low and the extraction level is sufficiently high ($b+\beta S \leq x$). In the case of perfect recoverability, the stock of scrap increases while extraction decreases over time (Proposition 2). Thus, if deterrence occurs at some point, it will be at the beginning of the horizon. Deterrence is a limit case of the phenomenon we highlight in our analysis: a monopolistic extractor has incentives to delay extraction in order to reduce recycling.

6.2 Extraction breaks

In the previous analysis, we focused on solutions where extraction takes place at the beginning and only stops when the primary resource is exhausted. Let us discuss here cases where the extraction level falls to zero before the virgin resource is depleted. Notice that the monopoly will never leave some resource unexploited for ever since this would simply reduce the monopoly's profits (we have assumed zero extraction costs). As a consequence, optimal extraction necessarily falls to zero at the exhaustion date. Still, can the monopoly find it optimal to make an extraction break, that is optimally stop extraction at one point and start extracting the resource again afterwards?

Under perfect recoverability ($\alpha = 1$), it can be shown that it is never optimal for the monopoly to momentarily stop extracting the resource. Indeed, in this case, the scrap can be recycled infinitely without loss. Thus, during an extraction break, the scrap stock and recycling remain constant. When the monopoly starts to extract the resource again, it faces the same level of recycling as when it started the pause. Thus, the monopoly has no incentive to momentarily stop extraction.

For recoverability rates lower than 100%, the monopoly might have incentives to momentarily stop extraction. The advantage of doing so is that, during the extraction break, the recycling sector uses scrap to produce recycled materials and then the stock of scrap decreases. At the time the monopoly starts extracting the resource again, the stock of scrap is lower than at the time it started the break and thus it faces lower recycling. Although we have excluded this type of corner solution from our analysis, we have allowed the possibility that the monopoly chooses arbitrarily low but positive extraction levels. Thus, we have considered the possibility of solutions as close as possible to this type of solution "with breaks".

However, none of the solutions we have been able to simulate suggest that a momentary pause in extraction can be optimal (see Figure 4). We cannot however conclude definitively that this can never be the case and we leave this investigation for future research.

7 Conclusion

Recycling appears to be a promising strategy to increase the supply of important exhaustible resources.

We have built a model of resource extraction in which the primary sector faces a recycling sector and we have considered two polar cases: competitive and monopolistic extraction. We have shown that, when the primary sector is competitive, the price of the recyclable resource increases over time. We have also shown that, when the primary sector is monopolistic, the price of the recyclable resource may be U-shaped when the recoverability rate is sufficiently large. This occurs because the primary producer has incentives to delay the extraction of the resource in order to limit recycling possibilities. We have also shown that virgin resource depletion occurs later when the recoverability rate is high or low than when it is intermediate.

Our results suggest that market power in the primary sector may lead to phases in which the price of the virgin resource decreases. To show this result, we considered a stylised model using simplifying assumptions. In particular, we assumed that there was only one grade of ore in the primary and recycling sectors, and we focused on the case of linear demand [15] and marginal cost functions. Further research is needed to explore the implications of changes in these assumptions.

[15] We have also assumed that the slope of the demand curve is equal to 1. This is not a critical assumption, see Appendix B for a discussion about the case where this slope is less than 1.

Appendix A: main computations and proofs

Computations for Figure 1 (competitive primary sector and linear specification):

Using (18) and (19) we have $P = \lambda_X = d_1 e^{\delta t}$, where d_1 is a constant to be determined latter. Using the linear specification of the inverse demand function, we have $1 - x - r = d_1 e^{\delta t}$ and then $x = 1 - r - d_1 e^{\delta t}$.

Let us assume that the solution is such that $x(t) > 0$ and $X(t) > 0$ over $[0, T^*)$ and $x(t) = X(t) = 0$ for $t \geq T^*$.

First consider the first phase in which $t \in [0, T^*)$. Using (6), we obtain $r = \frac{b+\beta S-1+d_1 e^{\delta t}}{\beta}$. Using the equation of the dynamic of the stock of scrap (2), we have $\dot{S} + S = \alpha + \frac{1-b}{\beta} - (\alpha + \frac{1}{\beta}) d_1 e^{\delta t}$. Solving for this differential equation, we find $S = d_2 e^{-t} - d_1 \frac{\alpha\beta+1}{(1+\delta)\beta} e^{\delta t} + \frac{\alpha\beta+1-b}{\beta}$. We thus have $x = 1 - \alpha - \frac{1+\beta}{\beta} d_1 e^{\delta t} - d_2 e^{-t} + d_1 \frac{\alpha\beta+1}{(1+\delta)\beta} e^{\delta t}$. Integrating x between 0 and t, we have $X = X^0 - (1-\alpha)t + \frac{1+\beta}{\beta}\frac{d_1}{\delta}(e^{\delta t} - 1) - d_2\left[e^{-t} - 1\right] - d_1 \frac{\alpha\beta+1}{(1+\delta)\delta\beta}\left[e^{\delta t} - 1\right]$.

Hence, using $S(0) = S^0$, $x(T^*) = 0$ and $X(T^*) = 0$, we have the following three conditions:

$$S^0 = d_2 - d_1 \frac{\alpha\beta + 1}{(1+\delta)\beta} + \frac{\alpha\beta + 1 - b}{\beta}, \tag{28}$$

$$0 = 1 - \alpha - \frac{1+\beta}{\beta} d_1 e^{\delta T^*} - d_2 e^{-T^*} + d_1 \frac{\alpha\beta + 1}{(1+\delta)\beta} e^{\delta T^*}, \tag{29}$$

$$0 = X^0 - (1-\alpha)T^* + \frac{1+\beta}{\beta}\frac{d_1}{\delta}\left(e^{\delta T^*} - 1\right) - d_2\left[e^{-T^*} - 1\right] - d_1 \frac{\alpha\beta + 1}{(1+\delta)\delta\beta}\left[e^{\delta T^*} - 1\right]. \tag{30}$$

Now consider the second phase in which $t \geq T^*$. Over this phase, we have $\dot{S} = -(1-\alpha)\frac{b+\beta S}{1+\beta}$. Solving for this differential equation, we obtain:

$$S(t) = \left[S(T^*) + \frac{b}{\beta}\right] e^{-(1-\alpha)\frac{\beta}{1+\beta}(t-T^*)} - \frac{b}{\beta}. \tag{31}$$

The recycled quantity over this phase is given by $r(t) = \frac{b+\beta S(t)}{1+\beta}$.

Necessary conditions for the monopoly problem (used to derive all the results in Section 5): The necessary conditions include the following.

$$\frac{\partial L}{\partial x} = \theta\left(a - 2x - S\right) - \lambda_X + \alpha' \lambda_S + \mu_x = 0, \tag{32}$$

$$\dot{\lambda}_X = \delta\lambda_X - \frac{\partial L}{\partial X} = \delta\lambda_X - \mu_X, \tag{33}$$

$$\dot{\lambda}_S = \delta\lambda_S - \frac{\partial L}{\partial S} = \delta'\lambda_S - \mu_S + \theta x, \tag{34}$$

where $\delta' = \delta + (1-\alpha)\theta$,

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{35}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{36}$$

$$S \geq 0, \mu_S \geq 0, \mu_S S = 0, \tag{37}$$

and S^0 and X^0 are given. Notice that our assumption that $b \leq \bar{b} \leq 1 - \beta(S^0 + X^0)$ implies that $a > S^0 + X^0$, which ensures that the price is always non negative.

Proof of Proposition 2: Let us assume that the solution is such that $x(t) > 0$ and $X(t) > 0$ over $[0, T^*)$ and $x(t) = X(t) = 0$ for $t \geq T^*$. Notice that since $\alpha = 1$, we have $\alpha' = 1$, $b' = 0$ and $\delta' = \delta$.
First consider the first phase in which $t \in [0, T^*)$. Since $x(t) > 0$, $X(t) > 0$ and $S(t) > 0$, using (35), (36), and (37), we have $\mu_x = \mu_X = \mu_S = 0$. Then (33) writes

$$\dot{\lambda}_X = \delta\lambda_X, \tag{38}$$

and then

$$\lambda_X = c_1 e^{\delta t}, \tag{39}$$

where c_1 is a constant to be determined later.

Conditions (32), and (34) write

$$\theta(a - 2x - S) - c_1 e^{\delta t} + \lambda_S = 0, \tag{40}$$

and,

$$\dot{\lambda}_S = \delta\lambda_S + \theta x, \tag{41}$$

Differentiating (40) with respect to time, we find

$$-2\theta\dot{x} - \theta\dot{S} - \delta c_1 e^{\delta t} + \dot{\lambda}_S = 0. \tag{42}$$

Using (40) and (42), we find

$$-2\theta\dot{x} - \theta\dot{S} - \delta c_1 e^{\delta t} - \delta\left(\theta a - 2\theta x - \theta S - c_1 e^{\delta t}\right) + \left(\dot{\lambda}_S - \delta\lambda_S\right) = 0. \tag{43}$$

Using (41) we obtain

$$-2\dot{x} - \dot{S} + \delta S + (1 + 2\delta)x - \delta a = 0, \tag{44}$$

Differentiating (23) with respect to time, we obtain

$$\ddot{S} = \dot{x} \tag{45}$$

Substituting (23) and (45) into (44), and rearranging, we have

$$\ddot{S} - \delta\dot{S} - \frac{1}{2}\delta S = -\frac{\delta}{2}a. \tag{46}$$

Solving for the stock of scrap S, we find

$$S = a + c_2 e^{\gamma^+ t} + c_3 e^{\gamma^- t}, \tag{47}$$

where c_2 and c_3 are two constants to be determined later, $\gamma^+ = \frac{\delta+\sqrt{\delta^2+2\delta}}{2}$, $\gamma^- = \frac{\delta-\sqrt{\delta^2+2\delta}}{2}$.

Differentiating (47) with respect to time, we obtain

$$\dot{S} = \gamma^+ c_2 e^{\gamma^+ t} + \gamma^- c_3 e^{\gamma^- t}. \tag{48}$$

Using (48) and (47), we obtain

$$x = \gamma^+ c_2 e^{\gamma^+ t} + \gamma^- c_3 e^{\gamma^- t}. \tag{49}$$

Using $X^0 - X(t) = \int_0^t x dt$ and integrating (49) between 0 and t, we find

$$X^0 - X(t) = c_2 \left(e^{\gamma^+ t} - 1 \right) + c_3 \left(e^{\gamma^- t} - 1 \right). \tag{50}$$

Now consider the second phase in which $t \geq T^*$. We have $x(t) = 0 = X(t)$ and $S(t) \geq 0$. We have $\dot{S} = 0$, and then $S(t) = S(T^*)$, $r^*(t) = \frac{b+\beta S^*(T^*)}{1+\beta}$, and $p^*(t) = 1 - \frac{b+\beta S^*(T^*)}{1+\beta}$.

It remains to solve for the constants. Using $x(T^*) = X(T^*) = 0$ and $S(0) = S^0$, we obtain the three following conditions:

$$0 = \gamma^+ c_2 e^{\gamma^+ T^*} + \gamma^- c_3 e^{\gamma^- T^*}, \tag{51}$$

$$X^0 = c_2 \left(e^{\gamma^+ T^*} - 1 \right) + c_3 \left(e^{\gamma^- T^*} - 1 \right), \tag{52}$$

$$S^0 = a + c_2 e^{\gamma^+ T^*} + c_3 e^{\gamma^- T^*} \tag{53}$$

This set of equations can be rearranged such that:

$$c_2 = (a - S^0) \frac{\gamma^- e^{\gamma^- T^*}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{54}$$

$$c_3 = -(a - S^0) \frac{\gamma^+ e^{\gamma^+ T^*}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{55}$$

$$X^0 = (a - S^0) \left(1 - \frac{\gamma^+ - \gamma^-}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} e^{\delta T^*} \right). \tag{56}$$

We conclude that the optimal extraction path is, for $t \in [0, T^*]$:

$$x^*(t) = \frac{(a - S^0)\delta}{2} \left(\frac{e^{\gamma^+ T^*} e^{\gamma^- t} - e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right), \tag{57}$$

the stock of virgin resource is, for $t \in [0, T^*]$,

$$X^*(t) = X^0 - (a - S^0) \left[1 - \frac{\gamma^+ e^{\gamma^+ T^*} e^{\gamma^- t} - \gamma^- e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right], \tag{58}$$

the stock of scrap is, for $t \in [0, T^*]$,

$$S^*(t) = a - (a - S^0) \frac{\gamma^+ e^{\gamma^+ T^*} e^{\gamma^- t} - \gamma^- e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{59}$$

and the market price, for $t \in [0, T^*]$,

$$p^*(t) = \theta(a - S^0) \left(\frac{\gamma^+ - \gamma^-}{2} \right) \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}. \tag{60}$$

Since $\gamma^+ > 0 > \gamma^-$, the extraction level $x^*(t)$ characterized in (57) decreases over time over $[0, T^*]$, while the stock of scrap increases over time over this interval, $\dot{S}^*(t) = x^*(t) \geq 0$. Using $a = \frac{1-b+\beta}{\beta}$, we find that recycling is given by, for $t \in [0, T^*]$:

$$r^*(t) = 1 - \theta(a - S^0) \left(\frac{\left(\gamma^+ + \frac{\delta}{2\beta}\right) e^{\gamma^+ T^*} e^{\gamma^- t} - \left(\gamma^- + \frac{\delta}{2\beta}\right) e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} \right), \tag{61}$$

and increases over time over $[0, T^*]$.□

Proof of Proposition 3:

The right hand side in condition (56) increases when T^* increases. Indeed, its derivative with respect to T^* is:

$$\frac{\theta(a - S^0)}{2} \frac{\sqrt{\delta^2 + 2\delta}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} > 0. \tag{62}$$

Moreover, the right hand side in condition (56) goes to 0 when T^* goes to 0 and it goes to $a - S^0$ when T^* goes to $+\infty$. Hence, there is a unique solution T^*. Moreover, T^* goes to 0 when X^0 goes to 0 and it goes to $+\infty$ when X^0 goes to a.

From (60), we know that the price of the consumption good is

$$p^*(t) = \theta \frac{(a - S^0)}{2} \sqrt{\delta (2 + \delta)} \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{63}$$

The sign of the derivative with respect to time is given by

$$\frac{\partial p^*}{\partial t} \propto \gamma^- e^{\gamma^+ T^*} e^{\gamma^- t} + \gamma^+ e^{\gamma^- T^*} e^{\gamma^+ t}, \tag{64}$$

which is positive if and only if

$$t \geq T^* + \frac{1}{\gamma^+ - \gamma^-} \ln \left(1 - \frac{\delta}{\gamma^+} \right). \tag{65}$$

Hence, $\frac{\partial p^*}{\partial t} \geq 0$ for all $t \in [0, T]$ if and only if

$$T^* \leq \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right). \tag{66}$$

Notice that the right hand side in condition (56) taken at $T^* = \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$ is equal to:

$$(a - S^0) \left(1 - \frac{\gamma^+ - \gamma^-}{\gamma^+ - \gamma^- \frac{\gamma^+ - \delta}{\gamma^+}} \right) \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)^{\frac{\gamma^+}{\gamma^+ - \gamma^-}} < 0 \tag{67}$$

Hence, we must have $T^* > \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$.

Hence, p^* is decreasing up to $t = T^* - \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$, and increasing after this date.□

Computations used for Table 1 and Figures 3 and 4 (Monopolistic primary sector for $\alpha < 1$):

Let us assume that the solution is such that $x(t) > 0$ and $X(t) > 0$ over $[0, T^*)$ and $x(t) = X(t) = 0$ for $t \geq T^*$. Also assume that $S(t) > 0$ over $[0, T')$ and $S(t) = 0$ for $t \geq T'$, where T'^*.

First consider the first phase in which $t \in [0, T^*)$. Since $x(t) > 0$, $X(t) > 0$ and $S(t) > 0$, using (35), (36), and (37), we have $\mu_x = \mu_X = \mu_S = 0$. Then (33) writes

$$\dot{\lambda}_X = \delta \lambda_X, \tag{68}$$

and then

$$\lambda_X = c_1 e^{\delta t}, \tag{69}$$

where c_1 is a constant to be determined later.

Conditions (32), and (34) write

$$\theta\left(a-2x-S\right)-c_1e^{\delta t}+\alpha'\lambda_S=0, \tag{70}$$

and,

$$\dot{\lambda}_S=\delta'\lambda_S+\theta x, \tag{71}$$

Differentiating (70) with respect to time, we find

$$-2\theta\dot{x}-\theta\dot{S}-\delta c_1e^{\delta t}+\alpha'\dot{\lambda}_S=0. \tag{72}$$

Using (70) and (72), we find

$$-2\theta\dot{x}-\theta\dot{S}-\delta c_1e^{\delta t}-\delta'\left(\theta a-2\theta x-\theta S-c_1e^{\delta t}\right)+\alpha'\left(\dot{\lambda}_S-\delta'\lambda_S\right)=0. \tag{73}$$

Using (71) we obtain

$$-2\dot{x}-\dot{S}+\delta'S+\left(\alpha'+2\delta'\right)x-\delta'a+(1-\alpha)c_1e^{\delta t}=0, \tag{74}$$

Differentiating (48) with respect to time, we obtain

$$\ddot{S}=\alpha'\dot{x}-(1-\alpha)\theta\dot{S}. \tag{75}$$

Substituting (48) and (75) into (74) and using $\delta'=\delta+(1-\alpha)\theta$, we have:

$$-2\ddot{S}+2\delta\dot{S}+\left[\alpha'\delta'+(\alpha'+2\delta')(1-\alpha)\theta\right]S+\alpha'(1-\alpha)c_1e^{\delta t}+(\alpha'+2\delta')b'-\alpha'\delta'a=0 \tag{76}$$

Notice that $\alpha'=\alpha+\frac{1-\alpha}{1+\beta}=1-(1-\alpha)\theta$. Using this expression and $\delta'=\delta+(1-\alpha)\theta$, we obtain:

$$\ddot{S}-\delta\dot{S}-\frac{1}{2}\left[\delta+(1-\alpha)\theta(2+\delta)\right]S=\frac{\alpha'(1-\alpha)}{2}c_1e^{\delta t}+\left(\frac{\alpha'}{2}+\delta'\right)b'-\frac{\alpha'\delta'}{2}a. \tag{77}$$

Solving for the stock of scrap S, we find

$$S=A+c_2e^{\gamma^+t}+c_3e^{\gamma^-t}-Be^{\delta t}, \tag{78}$$

where c_2 and c_3 are two constants to be determined later, $\gamma^+=\frac{\delta+\sqrt{\delta^2+2\delta+2(1-\alpha)\theta(2+\delta)}}{2}$, $\gamma^-=\frac{\delta-\sqrt{\delta^2+2\delta+2(1-\alpha)\theta(2+\delta)}}{2}$, $A=\frac{\alpha'\delta'a-(\alpha'+2\delta')b'}{\delta+(1-\alpha)\theta(2+\delta)}$ and $B=\frac{\alpha'(1-\alpha)}{\delta+(1-\alpha)\theta(2+\delta)}c_1$.

Differentiating (78) with respect to time, we obtain

$$\dot{S}=\gamma^+c_2e^{\gamma^+t}+\gamma^-c_3e^{\gamma^-t}-B\delta e^{\delta t}. \tag{79}$$

Using (79) and (78), we obtain

$$x = \frac{(1-\alpha)\theta A + b'}{\alpha'} + \left(\frac{\gamma^+ + (1-\alpha)\theta}{\alpha'}\right) c_2 e^{\gamma^+ t} + \left(\frac{\gamma^- + (1-\alpha)\theta}{\alpha'}\right) c_3 e^{\gamma^- t} - \left(\frac{\delta + (1-\alpha)\theta}{\alpha'}\right) B e^{\delta t}. \tag{80}$$

Using $X^0 - X(t) = \int_0^t x dt$ and integrating (80) between 0 and t, we find

$$X^0 - X(t) = \frac{(1-\alpha)\theta A + b'}{\alpha'} t + \left(\frac{\gamma^+ + (1-\alpha)\theta}{\alpha'}\right) \frac{c_2}{\gamma^+} \left(e^{\gamma^+ t} - 1\right) + \left(\frac{\gamma^- + (1-\alpha)\theta}{\alpha'}\right) \frac{c_3}{\gamma^-} \left(e^{\gamma^- t} - 1\right) - \left(\frac{\delta + (1-\alpha)\theta}{\alpha'}\right) \frac{B}{\delta} \left(e^{\delta t} - 1\right). \tag{81}$$

Now consider the second phase in which $t \in [T^*, T')$. We have $x(t) = 0 = X(t)$ and $S(t) > 0$. Using (37), we have $\mu_S = 0$. Condition (34) writes

$$\dot{\lambda}_S = \delta' \lambda_S, \tag{82}$$

and then

$$\lambda_S = c_5 e^{\delta' t}, \tag{83}$$

where c_5 is a constant to be determined later.

Notice that $\dot{S} = -(1-\alpha)\theta S - b'$, and then

$$S = c_4 e^{-(1-\alpha)\theta t} - \frac{b'}{(1-\alpha)\theta}. \tag{84}$$

where c_4 is a constant to be determined.

Now consider the third phase in which $t \geq T'$. We have $x = X = S = 0$. The remaining first order conditions are $\mu_x = -\theta a + \lambda_X - \alpha' \lambda_S \geq 0$, $\mu_X = \dot{\lambda}_X - \delta \lambda_X \geq 0$ and $\mu_S = \dot{\lambda}_S - \delta' \lambda_S \geq 0$.

Using (84) at $t = T'$, we obtain

$$S(T') = c_4 e^{-(1-\alpha)\theta T'} - \frac{b'}{(1-\alpha)\theta} = 0, \tag{85}$$

which implies that $T' = \frac{1}{(1-\alpha)\theta} \ln\left(\frac{(1-\alpha)\theta c_4}{b'}\right)$.

Using $x(T^*) = 0$ and (80), we have

$$(1-\alpha)\theta A + b' + \left(\gamma^+ + (1-\alpha)\theta\right) c_2 e^{\gamma^+ T^*} + \left(\gamma^- + (1-\alpha)\theta\right) c_3 e^{\gamma^- T^*} - (\delta + (1-\alpha)\theta) B e^{\delta T^*} = 0. \tag{86}$$

Using $X(T^*) = 0$ and (81), we have

$$\alpha'^0 = \left((1-\alpha)\theta A + b'\right)T^* + \left(\gamma^+ + (1-\alpha)\theta\right)\frac{c_2}{\gamma^+}\left(e^{\gamma^+ T^*} - 1\right) + \left(\gamma^- + (1-\alpha)\theta\right)\frac{c_3}{\gamma^-}\left(e^{\gamma^- T^*} - 1\right) - \left(\delta + (1-\alpha)\theta\right)\frac{B}{\delta}\left(e^{\delta T^*} - 1\right). \quad (87)$$

Using (78) at $t = 0$, we have

$$S^0 = c_2 + c_3 + A - B. \quad (88)$$

Using (78) and (84) at $t = T^*$, we have:

$$c_4 = \left(A + \frac{b'}{(1-\alpha)\theta} + c_2 e^{\gamma^+ T^*} + c_3 e^{\gamma^- T^*} - Be^{\delta T^*}\right)e^{(1-\alpha)\theta T^*}. \quad (89)$$

To get an additional condition, we use the following necessary condition:

$$\frac{dL}{dt} = \frac{\partial L}{\partial t} = 0 \text{ for all } t \quad (90)$$

Using (79), we have, for $t \in [0, T^*]$:

$$\left(-x - \lambda_S(1-\alpha)\right)\theta\dot{S} - x\dot{\lambda}_X + \dot{S}\dot{\lambda}_S = 0 \quad (91)$$

Using (71), we obtain:

$$-\lambda_S(1-\alpha)\theta\dot{S} - x\dot{\lambda}_X + \dot{S}\delta'\lambda_S = 0, \quad (92)$$

or,

$$\delta\dot{S}\lambda_S = x\dot{\lambda}_X. \quad (93)$$

Using (70), we have:

$$\delta\dot{S}\left[\theta(a - 2x - S) - \lambda_X\right] + \alpha' x\dot{\lambda}_X = 0. \quad (94)$$

At $t = T^*$, this condition is equivalent to:

$$\dot{S}(T^*)\left[\lambda_X(T^*) - \theta a + \theta S(T^*)\right] = 0. \quad (95)$$

Assume that $\dot{S}(T^*) = 0$. Together with (79), (86) and (89), this implies $c_4 = 0$. Hence, using (84), we have $S(t) < 0$ when $t \geq T^*$, which is impossible. Hence, the last condition is given by:

$$(A - a)\theta + \theta c_2 e^{\gamma^+ T^*} + \theta c_3 e^{\gamma^- T^*} + (c_1 - \theta B)e^{\delta T^*} = 0. \quad (96)$$

Appendix B: additional material

Discussion on the role of the slope of the demand curve (discussed in the conclusion): We have assumed in our main specification that the demand curve has a slope equal to 1. One may wonder how it changes our results to consider that the demand is such that $Q = 1 - \epsilon P$ with $\epsilon < 1$.

In this case, we can easily show that the problem of the monopolistic extracting firm is the following:

$$\underset{\{x\}}{Max} \int_0^{+\infty} e^{-\delta t} \frac{1}{\epsilon} \hat{\theta} \left(a - x(t) - S(t) \right) x(t) dt, \tag{97}$$

subject to the dynamic of the virgin resource stock:

$$\dot{X}(t) = -x(t), \tag{98}$$

and to the dynamic of the stock of scrap:

$$\dot{S}(t) = \hat{\alpha}' x(t) - (1 - \alpha) \hat{\theta} S(t) - \hat{b}', \tag{99}$$

where $\hat{\alpha}' = \alpha + \frac{1-\alpha}{1+\hat{\beta}}$, $\hat{b}' = \frac{1-\alpha}{1+\hat{\beta}} \hat{b}$, $X, S, x \geq 0$, $X^0 > 0$ and $S^0 \geq 0$ given. We have $\hat{\beta} = \epsilon \beta$, $\hat{b} = 1 - \epsilon + \epsilon b$ and $\hat{\theta} = \frac{\hat{\beta}}{1+\hat{\beta}}$.

We obtain the following necessary conditions:

$$\frac{\hat{\theta}}{\epsilon} (a - 2x - S) - \lambda_X + \hat{\alpha}' \lambda_S + \mu_x = 0, \tag{100}$$

$$\dot{\lambda}_X = \delta \lambda_X - \frac{\partial L}{\partial X} = \delta \lambda_X - \mu_X, \tag{101}$$

$$\dot{\lambda}_S = \hat{\delta}' \lambda_S - \mu_S + \frac{\hat{\theta}}{\epsilon} x, \tag{102}$$

where $\hat{\delta}' = \hat{\delta} + (1 - \alpha) \hat{\theta}$,

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{103}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{104}$$

$$S \geq 0, \mu_S \geq 0, \mu_S S = 0, \tag{105}$$

and S^0 and X^0 are given.

Let us focus on the case of perfect recoverability ($\alpha = 1$). In this case, the necessary conditions become:

$$\frac{\hat{\theta}}{\epsilon} (a - 2x - S) - \lambda_X + \alpha \lambda_S + \mu_x = 0, \tag{106}$$

$$\dot{\lambda}_X = \delta \lambda_X - \frac{\partial L}{\partial X} = \delta \lambda_X - \mu_X, \tag{107}$$

$$\dot{\lambda}_S = \hat{\delta}' \lambda_S - \mu_S + \frac{\hat{\theta}}{\epsilon} x, \tag{108}$$

where $\hat{\delta}' = \hat{\delta} + (1-\alpha)\hat{\theta}$,

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{109}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{110}$$

$$S \geq 0, \mu_S \geq 0, \mu_S S = 0, \tag{111}$$

and S^0 and X^0 are given.

Moreover, the dynamics of the stock of scrap is given by $\dot{S} = x$ and the dynamics of the stock of virgin resource by $\dot{X} = -x$.

Compared to the expressions we found in the case where $\epsilon = 1$ (conditions (57) to (61)), only the one of r^* and p^* are affected. We find, for $t \in [0, T^*]$:

$$p^*(t) = \frac{\beta}{1+\epsilon\beta}(a - S^0)\left(\frac{\gamma^+ - \gamma^-}{2}\right)\frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}. \tag{112}$$

and,

$$r^*(t) = 1 - \frac{\epsilon\beta}{1+\epsilon\beta}(a - S^0)\left(\frac{\left(\gamma^+ + \frac{\delta}{2\epsilon\beta}\right) e^{\gamma^+ T^*} e^{\gamma^- t} - \left(\gamma^- + \frac{\delta}{2\epsilon\beta}\right) e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}\right), \tag{113}$$

The intuitions are as follows. x^* does not depend on ϵ is due to the fact that the monopolist's objective is to maximize the value of sales. Since the stock of scrap depends only on extraction under perfect recoverability ($\dot{S} = x$), the stock of scrap does not depend on ϵ either. Thus, compared to the case where $\epsilon = 1$, with $\epsilon < 1$, the equilibrium price p^* is multiplied by $\frac{1+\beta}{1+\epsilon\beta} > 1$. As regards the recycling level, one can show that it is a decreasing function of ϵ, which is very intuitive. Indeed, a decrease in the slope of the demand function ϵ leads to an upward jump in the price of the output. Thus, recycling is more beneficial, so it is higher when ϵ is smaller.

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