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M. von Sperling, S.D. Wallace, Jaime Nivala. Representing performance of horizontal flow treatment wetlands: The Tanks In Series (TIS) and the Plug Flow with Dispersion (PFD) approaches and their application to design. Science of the Total Environment, 2023, 859, pp.160259. 10.1016/j.scitotenv.2022.160259 . hal-04044916

Science of the Total Environment 859 (2023) 160259

Contents lists available at ScienceDirect

Science of the Total Environment

journal homepage: www.elsevier.com/locate/scitotenv

Representing performance of horizontal flow treatment wetlands: The Tanks In Series (TIS) and the Plug Flow with Dispersion (PFD) approaches and their application to design

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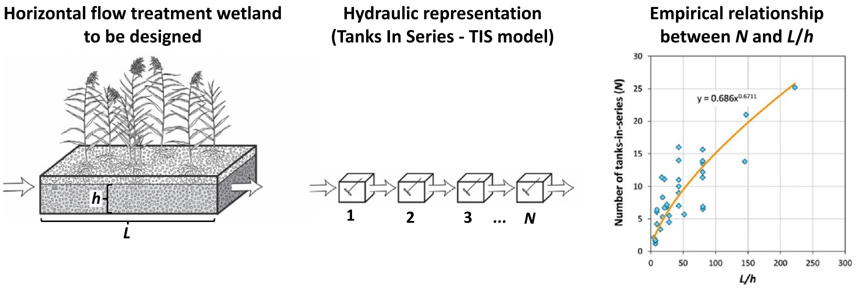
HIGHLIGHTS

- Utilization of the TIS model for the design of horizontal flow wetlands is proposed.
- Estimation of the number of TIS can be made based on the wetland ratio length/depth.
- Equations for areal-based and volumetric-based TIS and PFD models are presented.
- TIS and PFD give virtually the same estimation of effluent concentrations.
- An adaptation of removal rate coefficients from *P-k-C** to TIS and PFD models is made.

GRAPHICAL ABSTRACT



ARTICLE INFO

Editor: Paola Verlicchi

Keywords:
Constructed wetlands
Continuously stirred tank reactor
First-order removal rate coefficient
NTIS
Horizontal subsurface flow wetlands
Wastewater treatment

ABSTRACT

Horizontal flow wetlands have been designed using the so-called *P-k-C** approach, which has been largely embraced by the treatment wetlands literature. *P* is meant to represent the equivalent number of apparent tanks in series (hydraulic factor), but also incorporates the loss of biodegradability as the wastewater undergoes treatment (kinetic factor). For design purposes, literature proposes fixed values of *P*. The proposal of this paper is to decouple hydraulics from kinetics and use the traditional concept of *N* or *N*TIS (number of tanks in series) as a function of geometric relationships of the wetland to be designed, leaving kinetic elements to be dealt with solely by the first-order removal rate coefficient (*k*). From the literature, a database with 41 wetlands with data from tracer studies was used, and a novel regression-based equation was derived relating *N* with the ratio length/depth of horizontal wetlands. This equation can be used at the design stage for estimating *N* and, hence, the output concentration of the pollutant using the traditional structure of the TIS model, with a possible inclusion of background concentration (*C**). The paper presents all relevant equations, including those from the plug-flow with dispersion model (PFD), and it is shown how to convert from one hydraulic model to the other, what is also believed to be a novel approach in the treatment wetland literature. Finally, the area-based removal rate coefficients (k_A) proposed by Kadlec and Wallace (2009) for designs of horizontal wetlands treating domestic wastewater based on the *P-k-C** approach are converted into k_A values for the TIS model in the paper.

1. Introduction

Similar to many wastewater treatment technologies, treatment wetlands are biological reactors. Design equations for predicting effluent concentrations and removal efficiencies depend on a good representation of reactor hydraulics and reaction kinetics. The subject of reaction kinetics

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http://dx.doi.org/10.1016/j.scitotenv.2022.160259
Received 24 August 2022; Received in revised form 4 November 2022; Accepted 14 November 2022

and reactor hydraulics is covered in many chemical engineering textbooks, such as Levenspiel (1999), Fogler (2001) and Schmal (2014), and wastewater treatment and associated texts, such as Arceivala (1981), von Sperling and Chernicharo (2005), von Sperling (2007a), Metcalf and Eddy (2003, 2014), Mihelcic and Zimmerman (2014), and von Sperling et al. (2018, 2020). In the specific case of treatment wetlands, Kadlec and Wallace (2009) present a detailed coverage, which should be consulted for complementary information. Additionally, this topic has been extensively dealt with in a large number of scientific publications that present models for estimating effluent concentrations and removal rate coefficients for treatment wetlands. While this paper focusses on horizontal subsurface flow treatment wetlands, or simply horizontal flow (HF) treatment wetlands, the concepts apply to other saturated treatment wetland systems.

In the general theory of reactor hydraulics, the two idealized models are extensively used in the representation of reactors in different wastewater treatment processes: the completely stirred tank reactor (CSTR) model and the plug-flow reactor (PFR) model. These models are considered to represent idealized reactors because they cover the two extreme conditions that a reactor theoretically could exhibit: infinite dispersion (e.g., completely-stirred) or no dispersion (plug flow). In the case of HF treatment wetlands, early literature used the concept of plug-flow reactors in the modelling of constituent removal (Kadlec and Knight, 1996), but was later deemed inappropriate by Kadlec and Wallace (2009) due to the fact that tracer testing of multiple HF treatment wetlands revealed that these systems do not exhibit plug-flow reactor hydraulics. Removal rate coefficients assuming plug-flow hydraulics and first-order reactions for Biochemical Oxygen Demand (BOD_5, here simply denoted as BOD) and Chemical Oxygen Demand (COD) have been presented in several studies, highlighting the focus that has been given to this approach (Reed et al., 1988; Conley et al., 1991; Metcalf and Eddy, 1991; Rousseau et al., 2004; Brasil, 2005; Stein et al., 2006; Sandoval-Cobo and Peña, 2007; Fia, 2009; von Sperling and De Paoli, 2013; Crites et al., 2014).

It is not difficult to accept that existing horizontal flow wetlands do not exhibit this idealized behavior and do not act as ideal plug-flow reactors. Horizontal flow wetlands do not have the elongated geometry that is typically associated with plug flow. The presence of the filter medium and plant roots forces fluid elements to by-pass obstacles, inducing some lateral dispersion and possibly even some backward movement, meaning that the idealized presumption that the liquid moves as a perfect plug from inlet to outlet is not realistic.

Building on adaptations of the structure of these idealized models, other more applicable models have been developed, in such a way that they better represent the non-ideal hydraulics and mixing of a biological reactor, such as a horizontal flow wetland. These models include (a) Plug-Flow with Dispersion (PFD), also called dispersed flow and (b) Tanks In Series (TIS), also known as Completely Stirred Tank Reactor (CSTR) in series or complete-mix reactors in series (Fig. 1). The characteristic parameter in the equations associated with the PFD model is dispersion number (δ); for the TIS model it is the equivalent number of apparent tanks-in-series (N). In an idealized plug-flow reactor, $\delta = 0$ and $N = \infty$, while in an idealized completely mixed reactor, $\delta = \infty$ and $N = 1$. In the treatment wetland literature, the TIS model has also been termed *N*TIS, to differentiate it from the *P*TIS approach (Kadlec and Wallace, 2009), which is based on the parameter P, included in the *P*-*k*-*C** approach (described below). The terms TIS and PFD models are used in the remainder of this paper, and *N*TIS is used as an equivalent to *N*.

In horizontal flow treatment wetlands (and in other treatment wetland systems with a saturated flow regime), whose operating conditions are obviously not idealized, the values of δ (for the PFD model) and N (for the TIS model) are, of course, neither zero nor ∞. The representative values for δ and N will fall inside these boundary values associated with the idealized models. Estimates of δ and N are obtained from the results of tracer testing. However, for design of a new system, tracer testing obviously cannot be done, since the wetland does not yet exist. The difficulty in selecting a value of δ or N to be used in the design of horizontal flow wetlands has hindered the more widespread use of these two model approaches. Providing reasonable estimates of δ and N in the absence of tracer testing is the major motivation of this paper.

The PFD and TIS models are two choices for describing nonideal flow in treatment wetlands. For most practical applications in wetland design, the two options will yield similar predicted effluent concentrations, provided that there is a consistency between δ and N. The characteristic parameters δ and N can be calculated, one from the other, so that the designer may decide which model to use. When using such simplified approaches for design purposes, the field of treatment wetland design has embraced the TIS model more strongly than the PFD model (Kadlec and Wallace, 2009; Dotro et al., 2017). Interestingly, literature on design of waste stabilization ponds has historically favored the PFD model (Arceivala, 1981; von Sperling, 2007b).

In terms of kinetics, the equations for estimating effluent concentrations (C_{out}) from wetlands usually assume first-order reactions (Kadlec and Wallace, 2009). This assumption is also adopted in this paper. In the application of the PFD and TIS models for design, two approaches can be used: (a) design calculations based on the wetland liquid volume (wetland volume × porosity, or $V \times \varepsilon$), theoretical hydraulic retention time (HRT or τ) and volumetric removal rate coefficient (k_V) or (b) design calculations based on wetland surface area (A), areal hydraulic loading rate (HLR_A or q_A) and areal removal rate coefficient (k_A). All relevant equations are presented in this paper.

A subsequent development in the design of saturated treatment wetlands took place when Kadlec (2003) presented the concept of pollutant weathering in wetland modelling, which was subsequently used in Kadlec and Wallace (2009), and had a strong influence on the international literature (Dotro et al., 2017). Some pollutant measurements (such as BOD and COD) do represent a mixture of a variety of individual compounds, each with its own degradation rate. As the most highly degradable compounds are removed first, there is an apparent slowing down of the removal rate of the aggregated pollutant mixture. Kadlec's proposal was to replace the number of tanks-in-series (N) with a new parameter (P), which was created to account for both the hydraulics of the reactor (e.g., number of tanks-in-series, N) and the weathering of a pollutant as it undergoes treatment in the wetland, with the limitation of $P \leq N$. The value of P cannot be empirically measured and there is no explicit guidance for choosing the value of P that will replace N in the TIS approach. Kadlec and Wallace (2009) used the following global values of P for the design of horizontal flow wetlands: for BOD: $P = 3$; for Total Kjeldahl Nitrogen (TKN), Total Nitrogen (TN) and thermotolerant coliforms: $P = 6$, regardless of the geometry of the treatment wetland system.

A further advancement in the design of treatment wetlands was the incorporation of the concept of a background concentration (C^*) into the model equations. C^* could represent the conditions of residual, non-biodegradable, refractory and irreducible concentrations, or in some cases, analytical limits of detection. This concept was adopted by Kadlec and Wallace (2009) and has since then been highly influential in the treatment wetland literature. With P and C^*, together with the removal rate coefficient k, the overall approach was termed *P*-*k*-*C**, which was widely adopted in the international literature (e.g. textbooks such as Dotro et al., 2017; Alarcón Herrera et al., 2018; Vidal and Hormazábal, 2018).

In spite of the widespread use of the *P*-*k*-C* approach, the authors of this paper consider that further developments are still possible. The rationale is that it is more accurate to have the apparent number of tanks in series associated with only the hydraulic representation of the wetland, and not incorporating, in itself, the assumed kinetic elements of pollutant weathering, as initially proposed by the *P*-*k*-C* approach. Therefore, the proposal in this paper is to go back to the traditional structure of the TIS model, decoupling hydraulics from kinetics, with the adoption of N as a function only of the wetland hydraulics, and k as a function only of the reaction kinetics. Furthermore, it is believed that N should be variable with the wetland geometry, and not fixed, as it was in the *P*-*k*-C* approach. If a designer wants to explicitly take into account pollutant weathering or decrease of biodegradability along the wetlands longitudinal distance, a retardation model can be used, with the correction of the removal rate coefficient k, such as proposed by Shepherd et al. (2001) and Matos et al. (2018). In this way, kinetic

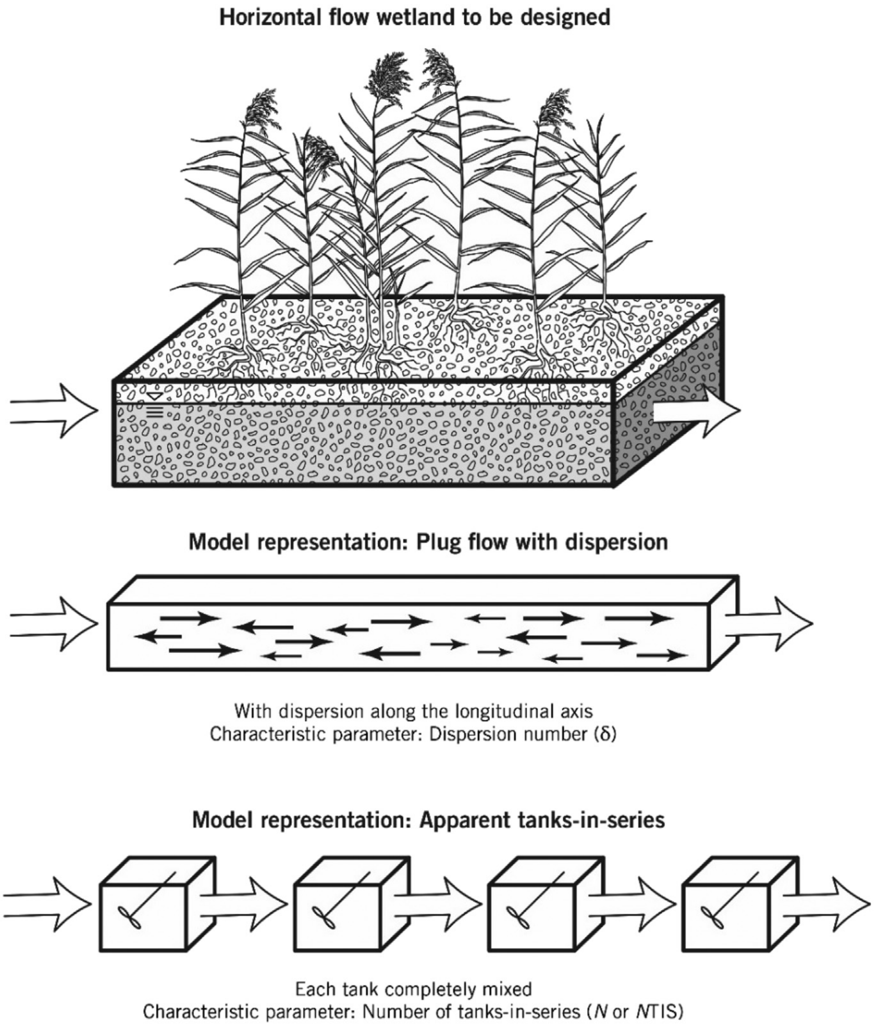

Fig. 1. Model representations of non-ideal reactors in treatment wetland hydraulics.

aspects are represented solely in the reaction rate coefficient *k*, and are kept separately from the hydraulics of the wetland (*N*).

In order to circumvent the difficulties associated with the prior estimation of *N* for design purposes of horizontal flow wetlands with different geometries, an investigation was performed, aiming to correlate the value of *N* obtained by tracer testing with the geometry of the wetland system. To be useful for design purposes, the equation relating these two parameters should be simple. Intuitively, the question was whether an elongated horizontal flow wetland would have a higher value of the apparent number of tanks in series *N* (indicating a lower longitudinal dispersion) and conversely, if a wetland with a shorter longitudinal length would exhibit a lower value of *N* (suggesting a greater longitudinal dispersion).

A review of the stabilization pond literature is instructive. Facultative ponds are the main variant of waste stabilization ponds, and, similarly to horizontal flow wetlands, are also natural extensive reactors that are subjected to departures from idealized hydraulic models. In the facultative pond literature, the utilization of the PFD model is consolidated for design purposes, thanks to the availability of empirical equations that relate the dispersion coefficient (δ), as determined from tracer tests performed in several ponds, with the geometry of the pond (including factors such as length, width, height and other variables) (Polprasert and Bhattarai, 1985; Agunwamba et al., 1992; Yanez, 1993; Nameche and Vasel, 1998; von Sperling, 1999).

Once a suitable equation for estimating tanks-in-series hydraulics (*N*) is obtained, kinetic coefficients (*k*) must be obtained to perform a design. The k_A values proposed by Kadlec and Wallace (2009) for their *P*-*k*-*C** approach, which are based on an extensive database, are adapted in this paper for use in the TIS model. As such, for compatibility with the *P*-*k*-C* approach, the model proposed here could be termed *N*-*k*-*C**. However, in order to use a terminology which is already accepted in the general wastewater treatment literature, the authors feel that the simple and traditional nomenclature of the TIS model should be used, without the need of specifying that *k* and *C** are included in the model structure.

Therefore, the objectives of this paper, aiming at the design of saturated horizontal flow treatment wetlands, are: (a) to derive a simple empirical equation for the estimation of *N* as a function of geometrical and physical characteristics of a treatment wetland, supported by a database from existing systems on which tracer testing has already been performed, (b) to present all relevant equations for the estimation of effluent concentrations from horizontal flow wetlands, assuming first-order kinetics, using the tanks in series (TIS) and plug-flow with dispersion (PFD) models, showing how to convert from one model to the other, and (c) to propose removal rate coefficients (k_A) based on adaptations to the k_A values suggested by Kadlec and Wallace (2009), for use in the proposed TIS and PFD models.

2. Methods

A literature survey was performed for publications that reported tracer tests on horizontal flow wetlands and also included basic information on the treatment units, such as flow (*Q*) and dimensions (length *L*, width *W*, saturated depth *h*), and other specificities for each tracer test. A database presented in Kadlec and Wallace (2009) was used and complemented with other references. In total, 79 tracer test results were obtained, of which 41 had sufficient information to be used in this analysis. The other 38 tests were discarded because either the wetlands were too small (<2.5 m^2), there was lack of basic information, or the results were considered inconsistent. In total, the 41 wetlands covered a broad range in terms of characteristics, since they were from eight different countries, encompassed seven different plant species, had experiments with eight different tracers and covered surface areas ranging from 2.5 to 605 m^2, theoretical hydraulic retention times from 1.3 to 6.1 days and aspect ratios (length/width) from 0.7 to 25.0. Table S 1 (Supplementary Material) includes the relevant information from each wetland, together with their respective original references.

Several linear and non-linear regression analyses with N as a dependent variable were performed against different combinations of independent variables. The independent variables included length (L), width (W), length/width (L/W ratio, or aspect ratio), length/depth (L/h ratio), surface hydraulic loading rate (Q/A), where A is the surface area), Q/h, $L \times (Q/A)$, $(Q/A) \times R_h$, where R_h is the hydraulic radius (ratio of wetted cross-sectional area divided by wetted perimeter, for a possible relation with the Reynolds number), $(Q/A)/(g \times h)^{0.5}$ (where g is the acceleration of gravity, for a possible relation with the Froude number). The goodness-of-fit was calculated by the Coefficient of Determination (CoD), as presented in von Sperling et al. (2020) and shown in Eq. 1. A CoD equal to 0 indicates a null fitting, while a CoD equal to 1 represents a perfect fitting.

$$\mathrm{CoD} = 1 - \frac{\sum_{i=1}^{n} \left(Y_{\mathrm{obs}i} - Y_{\mathrm{est}i}\right)^2}{\sum_{i=1}^{n} \left(Y_{\mathrm{obs}i} - Y_{\mathrm{obs}\,mean}\right)^2} \tag{1}$$

where: $Y_{\mathrm{obs\ i}}$ = observed value at position i in the data sequence; $Y_{\mathrm{est\ i}}$ = estimated value at position i in the data sequence; $Y_{\mathrm{obs}mean}$ = mean of observed values; n = number of datapoints

The conversion of the areal-based first-order removal rate coefficients (k_A) proposed by Kadlec and Wallace (2009) for the P-k-C^* approach to the TIS model proposed here is explained in Supplementary Material.

3. Results

3.1. Simple equation for estimating N based on a regression analysis

From the various regression analyses performed, the selected equation, presented in Eq. 2, has a simple structure and is supported by a conceptual background, in that the axial dispersion in horizontal flow wetlands is influenced by the length of the cell (here expressed by the L/h ratio). Simply stated, wetlands with larger L/h ratios are likely to be associated with a larger number of apparent tanks in series (N). Another point in favor of this concept is that L/h is equal to the ratio of the surface area ($L \times W$) divided by the cross-sectional area ($W \times h$), and both areas are influential in the hydraulics of a HF wetland. In addition to these points, Eq. 2 also provided a good fit with the experimental data, as discussed below; thus, it is considered that it can be directly used for design of subsurface horizontal flow wetlands.

$$N = 0.686 \left(\frac{L}{h}\right)^{0.671} \tag{2}$$

where: N = number of apparent tanks in series (dimensionless); L = wetland length (m); h = wetland saturated depth (m); L/h = length/depth ratio (dimensionless)

The number of data points in the regression analysis is n = 41, and the 95 % confidence interval for the exponent 0.671 is (0.531–0.811). Fig. 2 (left) shows the regression analysis between NTIS and L/h, using the data from the 41 horizontal flow wetlands surveyed. The results are conceptually coherent, and the goodness-of-fit can be considered satisfactory, as given by the Coefficient of Determination (CoD = 0.669). From the figure, the scatter of the data points is evident, as would be anticipated for field-scale systems. Of course, it cannot be expected that an empirical equation with such a simple structure, based on a single variable (L/h ratio), could be entirely representative of the multitude of factors that influence the hydraulic behavior of a horizontal flow wetland. Nevertheless, the trend depicted shows a clear increase of NTIS with L/h, which is conceptually sustainable and useful for design, because the designer will have already obtained the physical dimensions (L and h) of the wetland before the wetland hydraulics is investigated. As shown in Fig. 2 (right), the fitting was satisfactory for the entire range of N values and, of particular importance, for the range of small NTIS, <6 tanks-in-series. This lower range is most critical in terms of the estimation of output concentrations, assuming first-order reactions, because in most cases a variation of N from 2 to 4 has a stronger impact in the estimation of the effluent concentrations as compared with a variation of N from, for example, 10 to 20.

3.2. Estimation of the dispersion number δ for the PFD model based on N from the TIS model

It has already been mentioned that the representation of a reactor using the plug-flow with dispersion model (based on the dispersion number, δ)

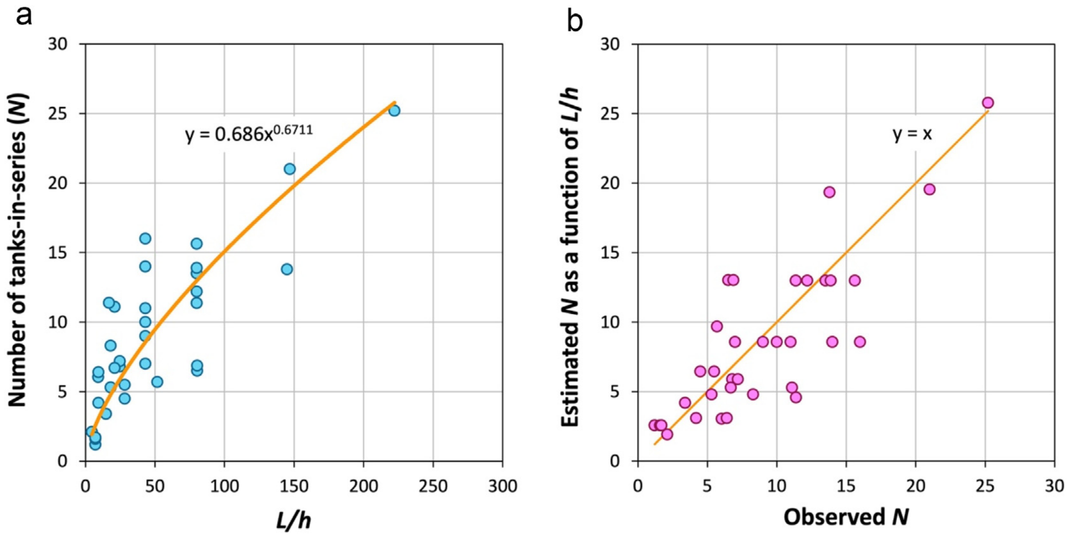

Fig. 2. Left: regression analysis between the number of tanks in series (NTIS) and the ratio (length/depth, L/h), based on 41 tracer tests performed in different passive horizontal wetlands. Markers: observed data; line: estimated values. Right: goodness-of-fit between observed and estimated values of N, with the line of perfect fit (slope 1:1). Coefficient of Determination: CoD = 0.669.

can be compared to the apparent tanks-in-series model (based on the number of tanks in series, *N*). In this regard, it is possible to estimate δ as a function of *N* and vice-versa, as given by Eqs. 3 and 4 (Elgeti, 1996; Fogler, 2001). Therefore, to use the plug-flow with dispersion model, one can use Eq. 2 to estimate *N* based on *L*/*h* of the wetland being designed. After that, the designer can use Eq. 3 to convert *N* into δ, and can then use the PFD model. To the authors' knowledge, this convergence of approaches between the two hydraulic models has not been covered in the treatment wetland literature, and it can be considered an important scientific contribution of this paper.

$$\delta = \frac{1}{2(N-1)} \tag{3}$$

$$N = \frac{1}{2\delta} + 1 \tag{4}$$

3.3. Summary of *N* and δ for different ranges of *L*/*h* ratios

The results from the utilization of Eq. 2 for the prediction of *N*TIS as a function of *L*/*h* in horizontal wetlands are shown in Table 1. In this table, the corresponding values of the dispersion number δ are calculated from *N*TIS using Eq. 3.

3.4. Summary of design equations for estimating output concentrations

This section provides a summary of the design equations to be used for estimating output concentrations from horizontal wetlands, considering the two main hydraulic models (TIS and PFD) and the two main approaches (volume- and area-based). First-order reactions are assumed in all equations. The number of apparent tanks in series (*N*) and the dispersion number (δ) can be estimated as shown in this paper. The areal removal rate coefficient (k_A) can be derived from the literature and is the same for both hydraulic models, what also applies to the volumetric removal rate coefficient (k_V).

Table 1

Values of equivalent *N*TIS and corresponding dispersion numbers (δ) for ranges of values of length/depth (*L*/*h*) ratios.

L/*h* ratio	Equivalent *N*TIS	Dispersion number (δ)
≤5	2	0.500
5 < *L*/*h* ≤ 9	3	0.250
9 < *L*/*h* ≤ 14	4	0.167
14 < *L*/*h* ≤ 20	5	0.125
20 < *L*/*h* ≤ 26	6	0.100
26 < *L*/*h* ≤ 32	7	0.083
32 < *L*/*h* ≤ 39	8	0.071
39 < *L*/*h* ≤ 46	9	0.063
46 < *L*/*h* ≤ 54	10	0.056
54 < *L*/*h* ≤ 62	11	0.050
62 < *L*/*h* ≤ 71	12	0.045
71 < *L*/*h* ≤ 80	13	0.042
80 < *L*/*h* ≤ 89	14	0.038
89 < *L*/*h* ≤ 99	15	0.036
99 < *L*/*h* ≤ 109	16	0.033
109 < *L*/*h* ≤ 120	17	0.031
120 < *L*/*h* ≤ 130	18	0.029
130 < *L*/*h* ≤ 141	19	0.028
141 < *L*/*h* ≤ 152	20	0.026

Notes:
1. In each row, the upper value in the *L*/*h* range is used for the estimation of *N*TIS
2. *N*TIS is calculated using empirical Eq. 2: *N*TIS $= 0.686(L/h)^{0.671}$
3. Dispersion number δ is calculated from *N*TIS using Eq. 3: $\delta = 1/[2(N\text{-}1)]$
4. For larger wetlands, with *L*/*h* > 152, there is no guarantee that the empirical equation will apply, because it is outside the range of experimental data used for deriving the equation. However, it could be expected that they would be associated with large *N*TIS values, and small deviations will have a small influence on the prediction of output concentrations.
5. There are not currently sufficient data for estimating *N*TIS for wetlands in which the inlet is the longer dimension, and the liquid travel is through the smaller dimension, that is, the distance from inlet to outlet is the smaller dimension (*L*/*W* < 1).

The volume-based and the area-based approaches are interlinked, and the designer may choose which is his/hers preferred way. The connections between the two coefficients may be made by the following equations, which are based on the well-known relationship between the theoretical hydraulic retention time (τ), liquid volume ($V \times \varepsilon$) and flow (*Q*): $\tau = (V \times \varepsilon)/Q = ((A \times h) \times \varepsilon)/Q = (h \times \varepsilon)/q$:

$$\frac{k_A}{q} = k_V \times \tau \text{ or } k_V \times \tau = \frac{k_A}{q} \tag{5}$$

$$k_A = k_V \times h \times \varepsilon \text{ or } k_V = \frac{k_A}{h \times \varepsilon} \tag{6}$$

$$q = \frac{h \times \varepsilon}{\tau} \text{ or } \tau = \frac{h \times \varepsilon}{q} \tag{7}$$

where: *q* = areal hydraulic loading rate, equal to flow divided by surface area [(m^3/(m^2· d)]; τ = theoretical hydraulic retention time, equal to liquid volume divided by flow (d); *h* = liquid depth (m); *V* = wetland volume (m^3); ε = medium porosity (dimensionless); k_A = areal removal rate coefficient (m/d); k_V = volumetric removal rate coefficient (d^{-1})

The volume- and area-based approaches presented in Table 2 lead to exactly the same values of output concentrations. Furthermore, the TIS and PFD models produce effluent concentrations that are virtually the same, at least for practical applications of design. For design of horizontal flow treatment wetlands, the data to be provided by the designer are: (1) estimated influent concentration (C_{in}); (2) parameters associated with the physical sizing of the wetland: theoretical hydraulic retention time (τ) or areal hydraulic loading rate (q_A); (3) hydraulic parameter *N* (estimated from values of *L* and *h* defined at the sizing stage, as proposed in Eq. 2) or δ (estimated from *N*, as indicated in Eq. 3) (4) background concentration (C^*), and (5) kinetic parameters k_V or k_A (adopted from literature).

3.5. First-order removal rate coefficients to be used for design

The first objective of this paper was to present the TIS approach and its applicability for the design of horizontal subsurface flow treatment wetlands, based on a simplified estimation of *N*. It is expected that improved estimations of *N* can be made in the future, as well as the development of other similar empirical equations, based on even larger databases than the one in this study. Additionally, for design purposes, robust values of removal rate coefficients (k_A or k_V) are necessary. Researchers can estimate new values of k_A or k_V from existing wetlands by making them the unknown variable in the equations provided in Table 2 for the TIS or PFD models, using measured values of C_{in} and C_{out} and estimated values of *N*, and thus further enhancing the available design information in the treatment wetland literature. However, *k* coefficients available in existing literature, based on other approaches, are not ready to be used for the TIS model, and thus must be adapted for the design of a new wetland.

To the authors knowledge, the dataset developed and used by Kadlec and Wallace (2009) for deriving k_A values for the *P*-*k*-*C** approach is the largest one available, being based on an extensive evaluation of more than one hundred existing horizontal flow wetlands, analyzed for each main wastewater constituent. Important elements are summarized in Table 3. The values of *P* used by Kadlec and Wallace (2009) were fixed for all horizontal subsurface flow wetlands, being *P* = 3 for BOD and *P* = 6 for TKN, TN and thermotolerant coliforms. The values of *C** considered in this analysis are also shown in Table 3. The original tables in Kadlec and Wallace (2009) expressed the median areal removal rates and the k_A coefficients on a yearly basis, but they have been converted to a daily basis here. From their original tables, which presented k_A distributions in increments of 10 percentiles, the median (50th percentile) was selected as the central tendency of k_A, and the 30th and 70th percentiles were

Table 2

Summary of the design equations for estimating the output concentration of a constituent, according to first-order kinetics, expressed on areal and volume basis, for the two main hydraulic models proposed (TIS and PFD).

Basis	Model	Estimation of effluent concentration (first-order reaction)	Equation
Areal	Tanks-in-series model (TIS)	$C_{\text{out}} = C^* + \frac{(C_{\text{in}} - C^*)}{(1+\frac{k_A}{N.q})^N}$	(8)
	Plug-flow with dispersion model (PFD)	$C_{\text{out}} = C^* + (C_{\text{in}} - C^*).\frac{4\text{a}\text{e}^{1/(2\delta)}}{(1+\text{a})^2\text{e}^{\text{a}/(2\delta)} - (1-\text{a})^2\text{e}^{-\text{a}/(2\delta)}}$ $\text{a} = \sqrt{1 + 4\frac{k_A}{q}.\delta}$	(9)
Volumetric	Tanks-in-series model (TIS)	$C_{\text{out}} = C^* + \frac{(C_{\text{in}} - C^*)}{(1+k_V.\frac{\tau}{N})^N}$	(10)
	Plug-flow with dispersion model (PFD)	$C_{\text{out}} = C^* + (C_{\text{in}} - C^*).\frac{4\text{a}\text{e}^{1/(2\delta)}}{(1+\text{a})^2\text{e}^{\text{a}/(2\delta)} - (1-\text{a})^2\text{e}^{-\text{a}/(2\delta)}}$ $\text{a} = \sqrt{1 + 4k_{\text{V}}.\tau.\delta}$	(11)

C_{in} = influent concentration to wetland (mg/L).
C_{out} = effluent concentration from wetland (mg/L).
C^* = background concentration (mg/L).
N = number of apparent tanks in series (NTIS) (dimensionless)
δ = dispersion number (dimensionless); $\delta = 1/[2 \times (N\text{-}1)]$.
k_{A} = areal removal rate coefficient (m/d).
k_{V} = volumetric removal rate coefficient (d^{-1}).
q = applied areal hydraulic loading rate $[\text{m}^3/(\text{m}^2 \cdot \text{d})]$.
τ = total theoretical hydraulic retention time in the wetland ($\tau = V \times \varepsilon/Q = h \times \varepsilon/q$).
Q = flow (m^3/d).
V = wetland volume (m^3).
h = liquid depth (m).
ε = medium porosity (dimensionless).

calculated and adopted here as the lower and upper values of a range considered typical for design.

BOD removal was analyzed in a more detailed manner in Kadlec and Wallace (2009) than it is in this paper. Four separate datasets were prepared in Kadlec and Wallace (2009), characterizing different pretreatment levels and influent BOD concentrations for horizontal flow wetlands, including: (a) tertiary effluent, C_{in} = 3–30 mg/L; (b) secondary effluent, C_{in} = 30–100 mg/L; (c) primary effluent, C_{in} = 100–200 mg/L; and (d) super: $C_{\text{in}} > 200$ mg/L. The major focus in this paper is the treatment of primary and secondary effluents, and so the two categories of 'tertiary' and 'super' have not been further analyzed. Additionally, it was observed from the dataset that the k_{A} values from the 'primary' and 'secondary' effluent categories have similar ranges. A Mann-Whitney test performed comparing the medians from both data samples indicated no significant difference between them at the 5 % significance level. Because of this, both datasets of k_{A} values have been merged here into a single set, 'primary + secondary', and this is the k_{A} dataset used here, whose design values (30th, 50th, and 70th percentiles) are presented in Table 3.

When using N instead of P, the value of the first-order removal rate coefficient k_{A} must be converted from the P-k-C* approach (because it had been affected by the adoption of a fixed P for all horizontal subsurface flow wetlands, which imbedded both hydraulics and kinetics) to the TIS model. Ideally, for this conversion, a value of N would have been used, together with a value of k_{A}, for each individual wetland in the large database of Kadlec and Wallace (2009). Unfortunately, this was not possible, so a fixed value of N had to be adopted but, in this case, aiming at reflecting purely the hydraulics of the HF wetlands. For this, the median value of N from the tracer database used to derive Eq. 2, also available in Kadlec and Wallace (2009), was adopted (N = 8). The full conversion procedure is explained and exemplified in the Supplementary Material.

Table 4 presents a summary of the resulting proposed design parameters for the TIS model applied to horizontal flow wetlands, after having made the conversion of k_{A} values from the P-k-C* approach. The parameters presented in the table are first-order areal removal rate coefficients (k_{A}) and background concentrations (C^*). It is expected that the range of typical k_{A} values, or median k_{A} values, can be used for design purposes for horizontal flow wetlands with different values of N (associated with the wetland dimensions). This is because the kinetic coefficient k_{A} in the TIS model is, in principle, independent from the reactor hydraulics, as compared with the P-k-C^* approach, in which P incorporated both hydraulics and kinetics.

Table 3

Summary of first-order areal removal rate coefficients k_{A} presented in Kadlec and Wallace (2009) for the P-k-C^* approach, based on a large dataset of horizontal flow wetlands treating domestic wastewater, together with their main operating conditions and model assumptions.

Constituent		BOD$^{(1)}$	TKN	TN	Thermotolerant coliforms
Number of wetland systems		103	123	123	130
Operating conditions	Median q ($\text{m}^3/\text{m}^2 \cdot \text{d}$)	0.036	0.049	0.049	0.051
	Median C_{in}	85	35	41	1.91×10^5
	Median C_{out}	19	23	26	3.31×10^3
	Median load removal	2.37 $\text{g/m}^2 \cdot \text{d}$	0.62 $\text{g/m}^2 \cdot \text{d}$	0.75 $\text{g/m}^2 \cdot \text{d}$	1.82 LRV
Model assumptions	P (dimensionless)	3	6	6	6
	C^* (mg/L)	5–10	1	1	0
k_{A} values (m/d) (20 °C)	30th percentile	0.058	0.013	0.013	0.153
	50th percentile (mean)	**0.079**	**0.025**	**0.023**	**0.282**
	70th percentile	0.121	0.040	0.039	0.496

Notes:
1. k_{A} (BOD): treatment of primary and secondary effluents together (merged original data presented in Kadlec and Wallace, 2009, which were originally separated for the treatment of primary and secondary effluent, with C^* = 10 mg/L for primary effluent and C^* = 5 mg/L for secondary effluent)
2. Concentrations in mg/L (except thermotolerant coliforms, in MPN/100 mL)
3. LRV = Log-Reduction Value

When observing the k_A values for nitrogen (TKN and TN) from the TIS model (Table 4), one can see that they are virtually the same as those reported in Table 3 for the *P*-*k*-C* approach. This is because the values of the k_A coefficients are small, and because $N = 8$ is numerically close to $P = 6$. For thermotolerant coliforms, they are also similar (difference <10 %), but the departure is slightly higher, because the coefficients themselves are higher. For BOD, the proposed value for the TIS model (median of 0.066 m/d) is lower than that for the *P*-*k*-C* approach (median of 0.079 m/d) because the value of $P = 3$ is much lower than $N = 8$ adopted for the TIS model. Still, the difference in k_A from the two approaches can be considered low (around only 20 %).

The k_A values shown in Table 4 for the TIS model can be used, without modifications, for the plug-flow with dispersion model (PFD) with background concentration (C^*) (see equations in Table 2). The hydraulic models for the reactor are different, but TIS is comparable with PFD, as previously stated. To use the equations for a volume-based coefficient (k_V, instead of k_A), the relevant equations in Table 2 can also be used, knowing that the theoretical hydraulic retention time in the wetland is given by $\tau = (V \times \varepsilon)/Q = (h \times \varepsilon)/q$, and that k_V can be calculated by $k_V = k_A/(h \times \varepsilon)$.

3.6. Comments on balancing organic cross-sectional loadings vs. hydraulic efficiency

As noted in Wallace and Knight (2006) and in Kadlec and Wallace (2009), there is an empirical relationship between the organic loading applied to the inlet cross section of a horizontal flow wetland and the potential for problematic accumulation of organic matter (solids). It was originally estimated that stable hydraulic performance in subsurface horizontal flow wetlands was for cross-sectional inlet loadings <250 g/m$^{2\cdot}$ d BOD for short-term loadings, and Wallace (2014) estimated that <100 g/m$^{2\cdot}$ d of BOD was a safer criterion for long-term loadings.

The criterion of a “clogging threshold” may impart inlet geometries for horizontal subsurface flow wetlands that seem at odds with hydraulic efficiency when high organic influent loads are introduced. It is also worth noting that the HF configuration places limitations on the hydraulic gradient that can be maintained in the system for subsurface flow according to Darcy's Law. However, low organic influent loads, or systems with multiple cells could likely be further optimized based on the work presented in this study.

Table 4

Summary of proposed design parameters for the TIS model applied to horizontal flow wetlands treating domestic wastewater, at the standard liquid temperature of 20 °C: first-order areal removal rate coefficients (k_{A20}) and background concentrations (C^*).

Design parameter	BOD	TKN	TN	Thermotolerant coliforms
Background concentration C*	7	1	1	0
30th percentile k_{A20} (m/d)	0.048	0.013	0.013	0.140
50th percentile k_{A20} (m/d) (median)	**0.066**	**0.025**	**0.023**	**0.258**
70th percentile k_{A20} (m/d)	0.100	0.040	0.039	0.453

Notes:
1. These values are based on Kadlec and Wallace (2009) database, converting k_A values from the *P*-*k*-*C** approach to the TIS model.
2. k_A values are reported as median values (typical value for design) and ranges comprising the 30th and 70th percentiles from the dataset (lower and upper values of typical design ranges)
3. k_A values are reported for the standard liquid temperature of 20 degrees Celsius, hence the subscript $_{20}$.
4. The same k_A values can be used for the apparent tanks in series (TIS) model and plug-flow with dispersion (PFD) model
5. Background concentration C^* for BOD: an intermediate value between 5 and 10 mg/L is adopted, to cover the combined dataset for treatment of primary and secondary effluents.
6. C^* concentrations in mg/L (except thermotolerant coliforms, in MPN/100 mL)

4. Concluding remarks

The proposal of a simplified approach for the estimation of the apparent number of tanks in series (N) for the TIS model, based on a regression analysis using data from existing horizontal subsurface flow treatment wetlands with different geometric relationships and available tracer data, is considered an advancement in the existing treatment wetland design literature. Prior equations assumed N to be infinite (idealized plug-flow models), and later on fixed (as P) and associated with kinetics (*P*-*k*-*C** approach). The simple equation derived here decouples hydraulics from kinetic removal rates by relating N solely to the ratio of length/depth of a wetland, facilitating the utilization of a model based on a more traditional structure (TIS). Naturally, there is room for future improvement, as data from more wetlands with tracer studies are made available in the treatment wetland literature.

Given the possibility of estimating N for the design of a new horizontal subsurface flow treatment wetland, the TIS model can be used for estimating effluent concentrations, assuming first-order removal rates. This paper also presents the plug-flow with dispersion model (PFD), and how to convert the characteristic hydraulic parameter (N) for the TIS model into the dispersion number (δ) used in the PFD model. This study shows that both model approaches lead, in practical terms, to the same estimation of effluent concentrations. The equations are presented for areal-based and volume-based design approaches, and, again, both calculation procedures lead to the same calculated values of effluent concentrations. This convergence of approaches is novel in the existing treatment wetland design literature.

Values of the first-order areal removal rate coefficient (k_A) for the design of horizontal flow wetlands treating domestic wastewater using the TIS model are also proposed, based on an adaptation of the k_A values suggested by Kadlec and Wallace (2009) for the *P*-*k*-*C** approach. Because these parameters are based on the largest available k_A dataset and are independent on the value of N, they are considered to be an advancement in the design procedure of horizontal flow wetlands.

It should not be expected that the approaches proposed here will faithfully represent ‘reality’, but this comment is also applicable to any other design method. The purpose of this paper is to present an advancement in estimating the expected behavior of saturated horizontal subsurface flow (HF) wetlands, during the design process and prior to the operating stage. The advantages of the equations provided in this paper are the support of large databases used in the derivation of N and k_A, as well as the simplicity in the calculations, which are usually much appreciated by treatment wetland designers.

CRediT authorship contribution statement

M. von Sperling: Methodology, Formal analysis, Investigation, Data curation, Writing – original draft, Visualization. **S.D. Wallace:** Investigation, Validation, Writing – review & editing. **J. Nivala:** Investigation, Validation, Writing – review & editing, Visualization, Resources.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

J. Nivala acknowledges support from the European Commission H2020 Project MULTISOURCE (Grant Agreement 101003527).

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.scitotenv.2022.160259.

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