Design of asymptotic observers to estimate parameters of systems that are not asymptotically identifiable

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Abstract: We propose the derivation of asymptotic observers for the estimation of parameters of systems whose solutions converge to a set of steady-states that are not identifiable, under some hypotheses. The proposed observer generalizes a former work for batch bioprocess. It is illustrated on a two dimensional models, and its performance is compared with the least squares method.

Keywords: Parameter estimation, identifiability, detectability, observability singularity, asymptotic observers, biological systems, mechanical systems.

1. INTRODUCTION

System identification has received a great attention in the literature (see e.g. Walter and Pronzato (1997)). Once the system is identifiable, the least squares technique is the widely-used method to reconstruct unknown parameters from measurements. Other approaches based on observers (e.g. Gauthier and Kupka (1994); Krener (2004)) have also some interest in terms of robustness and noise sensitivity, especially when the state is only partially observed (It might important to briefly recall that in that case the unknown parameters are considered as additional state variables with dynamics equal to zero, and therefore the estimation problem is considered as a pure observability problem). In any case, the requirement of identifiability/observability imposes the system to be identifiable at any steady-state that can be reached by the system (according to the definition of identifiability (Walter and Pronzato (1997)) or observablity (Gauthier and Kupka (1994))). However, in some applications, systems are identifiable only away from steady states, while the trajectory solutions converge asymptotically to one of these steady-state (and therefore the system is not detectable neither). This is typically the case of batch processes, which stop when all the resources are consumed (Bastin and Dochain (1990); Rapaport and Dochain (2020)). In practice, those situations lead to several issues when applying the usual estimation techniques, especially when the initial condition is not known to be closed or not to an attractive equilibrium. In this context, there is no theoretical ground to ensure that a least squares estimation is reliable. A Luenberger-like observer (Luenberger (1971); Kazantzis and Kravaris (1998)) is no longer guaranteed to converge with a null error (in the best case one may expect to obtain a "practical" convergence). The gains of the high-gain observer (Gauthier et al (1992)) explode when the estimation is close to the singularities of the observability map (which are precisely the non-identifiable steady-states).

Recently, the authors have proposed an asymptotic observer for the state reconstruction of a bioprocess model, whose solutions converge asymptotically to a set of non-observable states (Rapaport and Dochain (2020)). This observer have been shown to converge asymptotically without any bias, whatever is the initial condition away from steady-state, differently to classical observers. Numerical implementations have shown the benefit of this observer. The aim of the present work is to generalize this approach to the parameters estimation with lack of asymptotic identifiability for a large class of systems. We shall consider systems for which observation variables consist in time derivatives of some functions of the state. This is typically the case of batch process when one measures the flux of a product of the reaction, but this can be also the case for other systems when it is possible to write (less directly) the observation variables in this way, as we shall illustrate it on a mechanical example. If some state variables of the system converge to a steady-state value, one may also have the convergence of their derivative to zero when applying Barbalat's Lemma. Then, any observation variable which is the derivative of a function of these variables will also converge to zero, whatever is the asymptotic steady-state of the system. When these steady-states are indistinguishable, the system is thus non asymptotically identifiable, which is exactly the framework we targeted to investigate in this work. This lack of asymptotic identifiability prevents the usual approaches, which give weight on incoming measurements and progressively forget the initial measurements, to ensure an unbiased estimation of the parameters. We propose here another approach which on the contrary give a permanent weight on the initial measurement, because in some sense we look for a backward filtering i.e. from present time to initial one.

In the next section, we define more precisely the problem to be investigated. in Section 3, we expose the estimator construction. Section 4 is devoted to examples along with numerical illustrations and discussions.

2. DESCRIPTION OF THE PROBLEM

Consider a dynamics in $\mathbb{R}^n$

$$\dot{x} = f(x,p),\ x(t_0) = x_0 \tag{1}$$

where x_0 is unknown, and p is an unknown vector of parameters which belongs to a subset P of $\mathbb{R}^m$. Along the solutions, we consider an observation variable in $\mathbb{R}^q$

$$y(t) = h(x(t),p), \quad t \geq t_0, \tag{2}$$

The maps f and h are smooth, say C^∞.

Assumption 1. There exists a domain $\mathcal{D}$ of $\mathbb{R}^n$ which contains 0 that is forward invariant by (1) for any $p \in P$. Moreover, for any $p \in P$, one has $f(0,p) = 0$ and $h(0,p) = 0$, and any solution of (1) in $\mathcal{D}$ converges asymptotically to 0.

This assumption implies that the system is not identifiable at 0 i.e. the observation y does not allow to reconstruct the parameters vector p when the system is at the steady state 0, and moreover its solutions converge locally to this steady state. In such situations, it is well known that even if the system is identifiable and observable everywhere away from steady state, a construction with a classical smooth observer (such as Luenberger, extended Kalman or high gain) cannot guarantee an exact asymptotic estimation of the parameters because of the asymptotic lack of identifiability and observability. Non-smooth observers (such as sliding mode observers (Spurgeon (2008))) or numerical differentiators (Levant (1998)) could be alternatives that provide finite time convergence. However, these constructions are known to be poorly robust to noise, especially when the system is close to a singular point of the observability map, which is the case of the steady state.

Our objective is to construct, under some conditions, a smooth estimator that provides an exact asymptotic estimation of x_0 and p from the single measurements $y(\cdot)$, for unknown initial conditions $x_0 \neq 0$ in $\mathcal{D}$ (even arbitrarily close to 0).

3. ASSUMPTIONS AND CONSTRUCTION OF THE ESTIMATOR

We first assume the following condition on the dimension of the observation set, which will plays a crucial role in the method.

Assumption 2. One has $n + m = 2q$.

We also assume that each coordinate y_i of the observation vector y can be written as a Lie derivative of a certain function F_i with respect to the vector field $f(\cdot,p)$, for any $p \in P$, as expressed in the assumption below.

Assumption 3. There exists a smooth map

$$F : (x,p) \in \mathbb{R}^n \times \mathbb{R}^m \mapsto F(x,p) \in \mathbb{R}^q$$

with $F(0,p) = 0$ for any $p \in P$ that fulfills

$$h(x,p) = \frac{\partial F}{\partial x}(x,p).f(x,p), \quad (x,p) \in \mathcal{D} \times P. \tag{3}$$

Under these two assumptions, we define the map

$$\Gamma : (x,p) \in \mathbb{R}^n \times \mathbb{R}^m \mapsto (h(x,p), F(x,p)) \in \mathbb{R}^{2q} \tag{4}$$

Let us underline that under Assumption (2), Γ can be considered as a map from $\mathbb{R}^{n+m}$ to itself.

Here is our main result.

Proposition 4. Let $\mathcal{Q}$ be a subset of $\mathcal{D} \times P$ such that Γ is onto on $\mathcal{Q}$, and Ψ a continuous map on $\mathcal{W} \subset \mathbb{R}^{2q}$ such that $\Gamma(\mathcal{Q}) \subset \mathcal{W}$ with $\Psi = \Gamma^{-1}$ on $\Gamma(\mathcal{Q})$. For any $x_0 \in \mathcal{D}$ and $t_0 \geq 0$ such that for any $p \in P$, the solution of (1) with $x(t_0) = x_0$ satisfies

$$\left(h(x(t_0),p), -\int_{t_0}^{t} h(x(\tau),p)d\tau \right) \in \mathcal{W}, \quad t > t_0 \tag{5}$$

and

$$\left(h(x(t_0),p), -\int_{t_0}^{+\infty} h(x(\tau),p)d\tau \right) \in \Gamma(\mathcal{Q}) \tag{6}$$

the following system

$$\begin{cases} \dot{v} = y(t), \quad v(t_0) = 0 \\ (\hat{x}_0(t), \hat{p}(t)) = \Psi(y(t_0), -v(t)), \quad t > t_0 \end{cases} \tag{7}$$

is an asymptotic exact estimator of x_0 and p i.e. one has

$$\lim_{t\to+\infty} \hat{x}_0(t) - x_0 = 0, \tag{8}$$

$$\lim_{t\to+\infty} \hat{p}(t) - p \ = \ 0. \tag{9}$$

Proof. Take $p \in P$ and an initial condition (t_0, x_0) with $x_0 \in \mathcal{D}$. Let $x(\cdot)$ be the corresponding solution of (1) and $y(\cdot)$ the output given by (2). Then the solution of (7) verifies, from Assumption 3

$$v(t) = \int_{t_0}^{t} h(x(\tau),p)\, d\tau \tag{10}$$

$$= \int_{t_0}^{t} \frac{\partial F}{\partial x}(x(\tau),p).f(x(\tau),p)\, d\tau \tag{11}$$

$$= F(x(t),p) - F(x_0,p) \tag{12}$$

for any $t \geq t_0$. Under condition (5), $(y(t_0), -v(t))$ belongs to $\mathcal{W}$ for any $t > t_0$ and thus the pair $(\hat{x}_0(t), \hat{p}(t)) = \Psi(y(t_0), -v(t))$ is well defined for $t > t_0$.

From Assumptions 1 and 3, we get

$$\lim_{t\to+\infty} F(x(t),p) = 0$$

and thus $v(\cdot)$ is bounded with

$$\lim_{t\to+\infty} v(t) = -F(x_0,p).$$

By continuity of Ψ on $\mathcal{W}$ and condition (6), one obtains

$$\lim_{t\to+\infty} \Psi(y(t_0), -v(t)) = \Psi(y(t_0), F(x_0,p)) = \Gamma^{-1}(x_0,p)$$

that is the desired convergence

$$\lim_{t\to+\infty}(\hat{x}_0(t),\hat{p}(t))=(x_0,p).$$

In the next section, we present examples for which the observation has the structure (3) and show how to define the set $\mathcal{W}$ and construct the map Ψ that fulfill conditions (5) and (6) of Proposition 4.

4. EXAMPLES

We first show that the model treated in Rapaport and Dochain (2020) is a particular case of application of Proposition 4. Then, we develop a new example in dimension two with two unknown parameters and an observation of dimension two.

4.1 A bioreactor model

We revisit the state estimation problem in the batch bioreactor model when measuring the biogas:

$$\left\{\begin{array}{l}\dot{x}=\mu(s)x\\ \dot{s}=-\mu(s)x\end{array}\right. \tag{13}$$

$$y=\mu(s)x \tag{14}$$

Here the function μ, which is null at 0 only, is assumed to be known, while the initial condition is not known. We show that the asymptotic observer proposed in Rapaport and Dochain (2020) for this system is a particular case of the methodology that we propose here. Let us pose

$$z=x+s \tag{15}$$

one has

$$\dot{z}=0 \tag{16}$$

Therefore, the system can be expressed equivalently as the one dimensional dynamics

$$\dot{s}=f(s,z)=-\mu(s)(z-s)$$

with observation

$$y(t)=h(s(t),z)=\mu(s(t))(z-s(t)),\quad t\geq 0$$

where z is an unknown parameter. Clearly all positive solutions with $s<z$ converges asymptotically to the steady state $s=0$, where parameter z is not identifiable. Here one has

$$h(s,z)=-f(s,z) \tag{17}$$

which leads to

$$F(s,z)=-s \tag{18}$$

as a function that verifies

$$h(s,z)=\frac{\partial F}{\partial s}F(s,z)f(s,z) \tag{19}$$

with $F(0,z)=0$ whatever is z. Then we consider the map

$$\Gamma(s,z)=\begin{bmatrix}\mu(s)(z-s)\\ -s\end{bmatrix}$$

and the set

$$\mathcal{Q}=\{(s,z)\in\mathbb{R}^2;\ z>s>0\} \tag{20}$$

For any $\xi\in\Gamma(\mathcal{Q})$, one has

$$s=-\xi_2,\quad z=s+\frac{\xi_1}{\mu(s)}$$

which shows that Γ is invertible on $\Gamma(\mathcal{Q})$. We define the function

$$\Psi(\xi)=\begin{bmatrix}-\xi_2\\ \dfrac{\xi_1}{\mu(-\xi_2)}-\xi_2\end{bmatrix}$$

that is well-defined and continuous on the set

$$\mathcal{W}=\{\xi\in\mathbb{R}^2;\ \xi_2<0\} \tag{21}$$

which contains $\Gamma(\mathcal{Q})$, and coincides with Γ^{-1} on $\Gamma(\mathcal{Q})$. Note that for any initial condition (t_0,s_0) with $s_0\in(0,z)$, one has $-\int_{t_0}^{t}y(\tau)d\tau<0$ for any $t>t_0$. Conditions of Proposition 4 are thus fulfilled for any initial condition (t_0,s_0) such that $s_0>0$ and $s_0<z$.

Finally the estimator of z

$$\left\{\begin{array}{l}\dot{v}(t)=y(t),\quad v(t_0)=0\\ \hat{z}(t)=\dfrac{y(t_0)}{\mu(v(t))}+v(t),\quad t>t_0\end{array}\right.$$

is exactly the one proposed in Rapaport and Dochain (2020). In this last reference, the benefits of the estimator are discussed in comparison with classical Luenberger or high gain observers for various growth functions μ.

4.2 A mechanical model

Consider the classical harmonic oscillator with damping

$$m\ddot{z}+kz+c\dot{z}=0$$

where the mass m and the damping coefficient c are unknown positive parameters (the spring constant k is assumed to be known). We assume that the position and the damping force are measured. The system with $x=(z,\dot{z})^\top$ in $\mathbb{R}^2$ writes

$$\dot{x}=f(x,p)=\underbrace{\begin{bmatrix}0 & 1\\ -\dfrac{k}{m} & -\dfrac{c}{m}\end{bmatrix}}_{A(p)}x,\quad p=\begin{bmatrix}m\\ c\end{bmatrix}$$

$$y=h(x,p)=\begin{bmatrix}x_1\\ cx_2\end{bmatrix}$$

One can check that the matrix $A(p)$ is Hurwitz and thus all solutions converges to the steady state 0, whatever is the positive vector p. At steady state, one has $f(0,p)=0$ and $h(0,p)=0$ and the system is thus not identifiable at 0. However, let us show that the system is infinitesimally identifiable away from 0.

- If $y_2\neq 0$, $\dot{y}_1=y_2/c$ is non null and one has $c=\dfrac{y_2}{\dot{y}_1}$. If moreover $\dot{y}_2\neq 0$, one gets $m=-\dfrac{(ky_1+y_2)y_2}{\dot{y}_1\dot{y}_2}$. If

$\dot{y}_2 = 0$, then $\dot{y}_1 = x_2$ is non null and $\ddot{y}_2 = -\frac{kc}{m}\dot{y}_1$ as well. We get $m = -\frac{ky_2}{\ddot{y}_2}$.

- If $y_2 = 0$, y_1 is non null away from steady state. then $\dot{y}_2 = -\frac{kc}{m}y_1$ is non null and $\ddot{y}_1 = -\frac{ky_1}{m}$ as well. We get $m = -\frac{ky_1}{\ddot{y}_1}$ and $c = \frac{\dot{y}_2}{\ddot{y}_1}$.

We show now how to apply the methodology exposed in Section 3. The map h can be written as

$$h(x,p) = \underbrace{\begin{bmatrix} -\frac{c}{k} & -\frac{m}{k} \\ c & 0 \end{bmatrix}}_{J(p)} f(x,p)$$

and then

$$F(x,p) = \begin{bmatrix} -\frac{c}{k}x_1 - \frac{m}{k}x_2 \\ cx_1 \end{bmatrix}$$

verifies

$$\frac{\partial F}{\partial x}(x,p) = J(p) \quad (22)$$

with $F(0,p) = 0$ for any p. The next step is to define

$$\Gamma : (x,p) \mapsto \begin{bmatrix} x_1 \\ cx_2 \\ -\frac{c}{k}x_1 - \frac{m}{k}x_2 \\ cx_1 \end{bmatrix}$$

and consider the set

$$\mathcal{Q} = \{(x,p) \in \mathbb{R}^2 \times \mathbb{R}^2;\ x_1 \neq 0,\ x_2 \neq 0,\ m > 0,\ c > 0\}.$$

For $\xi = \Gamma(x,p)$ in $\Gamma(\mathcal{Q})$, one obtains

$$x_1 = \xi_1, \quad x_2 = \frac{\xi_2}{c},\ c = \frac{\xi_4}{\xi_1},\ m = -\frac{k\xi_3 + \xi_4}{x_2}$$

showing that Γ is invertible on $\Gamma(\mathcal{Q})$. However, the map

$$\Psi(\xi) = \begin{bmatrix} \xi_1 \\ \frac{\xi_2\xi_4}{\xi_1} \\ -\frac{(k\xi_3 + \xi_4)\xi_4}{\xi_1\xi_2} \\ \frac{\xi_4}{\xi_1} \end{bmatrix}$$

is well-defined and continuous on the set

$$\mathcal{W} = \{\xi \in \mathbb{R}^4;\ \xi_1 \neq 0,\ \xi_2 \neq 0\}$$

which contains $\Gamma(\mathcal{Q})$ and coincides with Γ^{-1} on $\Gamma(\mathcal{Q})$. Conditions of Proposition 4 are thus verified, provided that the initial condition verifies $x_1(t_0) \neq 0$ and $x_2(0) \neq 0$. However, note that for any solution $x(\cdot)$ of (1) that is not at steady state, $x_1(t)$ and $x_2(t)$ are both non null for almost any t. Therefore, one can initialize the system (7) with t_0 as closed as desired to the initial time, such that the observation at t_0 verifies $y_1(t_0) \neq 0$ and $y_2(t_0) \neq 0$.

Finally, the estimator is as follows

$$\begin{cases} \dot{v}_i(t) = y_i(t), \quad v_i(t_0) = 0 \quad (i = 1,2) \\ \hat{m}(t) = -\frac{(kv_1(t) + v_2(t))v_2(t)}{y_1(t_0)y_2(t_0)} \\ \hat{c}(t) = -\frac{v_2(t)}{y_1(t_0)} \end{cases}$$

We have compared this asymptotic observer with a least-square estimation of the parameters, provided from 1000 measurement points over the same time window $[0,T]$ using the Levenberg-Marquardt algorithm (`lsqrsolve` function in `Scilab` software). Initial condition and parameters values used for the simulations are given in Table 1. We have compared the estimations on different time horizons T, without and with measurement noise (normal distribution with 0 mean and 0.01 standard deviation).

Table 1. Initial condition and parameter values used for the simulations

$x_1(0)$	$x_2(0)$	m	k	c
0.2	-2	1	1	0.4

Time-varying estimations provided by the asymptotic observer are depicted on Fig. 1 and 2. The final estimation errors are given on Tables 2 and 3.

Table 2. Estimation errors on parameters m and c (without noise)

T	least square	observer
20	$(e_m, e_c) = (3.3\%, 1.7\%)$	$(e_m, e_c) = (9.7\%, 11.1\%)$
30	$(e_m, e_c) = (7.0\%, 3.2\%)$	$(e_m, e_c) = (2.0\%, 2.1\%)$
40	$(e_m, e_c) = (1.75\%, 0.67\%)$	$(e_m, e_c) = (0.33\%, 0.33\%)$
50	$(e_m, e_c) = (4.3\%, 2.6\%)$	$(e_m, e_c) = (0.05\%, 0.04\%)$
70	$(e_m, e_c) = (3.7\%, 2.3\%)$	$(e_m, e_c) = (0.0006\%, 0.0005\%)$

Table 3. Estimation errors on parameters m and c (without noise)

T	least square	observer
20	$(e_m, e_c) = (3.1\%, 1.9\%)$	$(e_m, e_c) = (8.2\%, 9.6\%)$
30	$(e_m, e_c) = (2.0\%, 1.9\%)$	$(e_m, e_c) = (0.69\%, 0.81\%)$
40	$(e_m, e_c) = (2.7\%, 0.60\%)$	$(e_m, e_c) = (0.057\%, 0.13\%)$
50	$(e_m, e_c) = (4.2\%, 2.3\%)$	$(e_m, e_c) = (1.7\%, 1.6\%)$
70	$(e_m, e_c) = (4.2\%, 1.5\%)$	$(e_m, e_c) = (0.5\%, 0.5\%)$

The comparisons leads to the following comments.

- the asymptotic observer has a relatively slow convergence: it needs $T > 30$ to give an accurate estimation which improves with larger time windows even with presence of noise.
- the least square estimator does not provide an estimation as good as the asymptotic observer on large time horizons.
- the least square estimator is less affected by noise, but the quality of its estimation is deteriorating over large time horizon, when the state get closer to the equilibrium point.

Remark 5. Note that the dynamics can be written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\theta(ky_1(t) + y_2(t)) \end{cases}$$

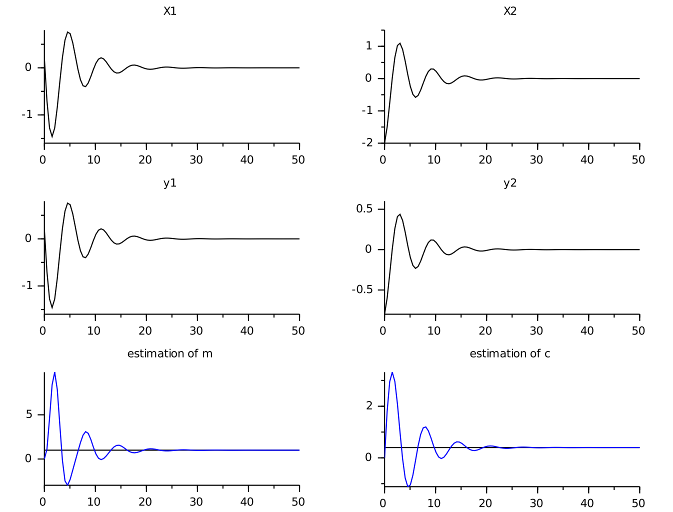

Fig. 1. Simulation of the system and the estimation provided by the observer (without noise)

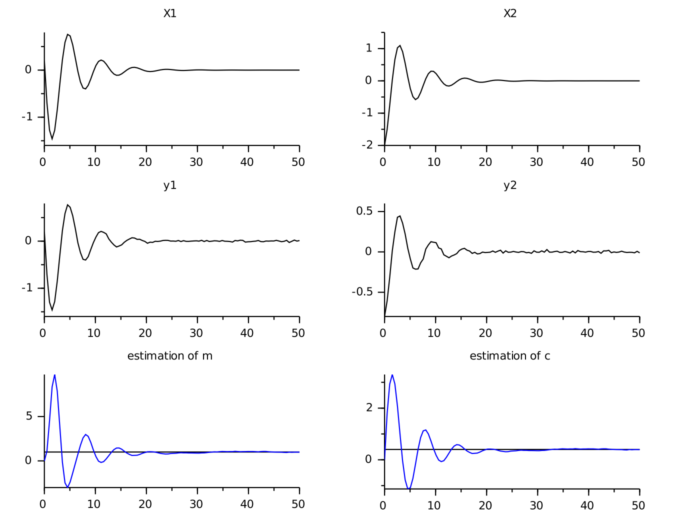

Fig. 2. Simulation of the system and the estimation provided by the observer (with noise)

where we pose $\theta = \frac{1}{m}$. Therefore, one may consider the "naive" Luenbeger observer

$$\begin{cases} \dfrac{d}{dt}\hat{x}_1 &= \hat{x}_2 + G_1(\hat{x}_1 - y_1(t)) \\ \dfrac{d}{dt}\hat{x}_2 &= -\hat{\theta}(ky_1(t) + y_2(t)) + G_2(\hat{x}_1 - y_1(t)) \\ \dfrac{d}{dt}\hat{\theta} &= -G_3(\hat{x}_1 - y_1(t)) \end{cases}$$

with

$$\hat{m} = \frac{1}{\hat{\theta}}, \quad \hat{c} = \frac{y_2}{\hat{x}_2}$$

to estimate parameters m and c. However, the error dynamics

$$\frac{d}{dt}e = \underbrace{\begin{bmatrix} G_1 & 1 & 0 \\ G_2 & 0 & -ky_1(t) - y_2(t) \\ G_3 & 0 & 0 \end{bmatrix}}_{M(t)} e$$

has a matrix $M(\cdot)$ that is periodically singular and that converges to a singular matrix, whatever is the choice of the gains G_1, G_2, G_3. Then, the convergence of the error cannot be obtained. As an illustration, we have simulated this observer without measurement noise, for the gains $G_1 = -13$, $G_2 = 32$, $G_3 = -20$ which gave the best results for the estimation of m. As one can see on Fig. 3, the estimation of m is biased. While the innovation $\hat{x}_1 - x_1$ is rapidly small, we did not obtain a fast enough convergence of the error on x_2 with this observer, which explains the bad behavior of the estimation of c.

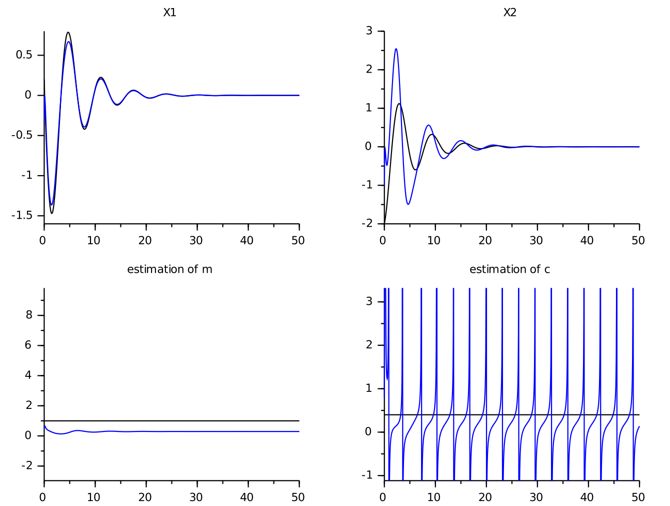

Fig. 3. Simulation with the Luenberger observer (without noise). Estimations are in blue

5. CONCLUSION

We have proposed the derivation of an asymptotic observer for the estimation of parameters of systems that are not asymptotically identifiable. Although it takes the form of an observer, its philosophy is quite different from the conventional observers, in the sense that it can be considered as an expression that combines initial observation and integrals of the observed variables, without innovation (and gain) terms. As a consequence, its convergence speed cannot be tuned. We believe that it is the price to pay to obtain an asymptotic estimation without bias, in this precise context of lack of asymptotic identifiability.

Numerical simulations show that this approach is more reliable than a classical least-squares method, especially in the long term, with or without noise. However, the structure of the proposed observer relies strongly on the initial observation which is prone to noise measurement. A future work will deal with robustness issues of this observer under stronger noises that the ones considered in this preliminary work.

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