# Invited session proposal submitted to IFAC World Congress 2023

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## Title

**Control theory perspectives on mathematical epidemiology**

## Abstract

Mathematical epidemiology may benefit from techniques and concepts from Control theory. As a matter of fact, understanding and controlling the spread of an infectious epidemic necessitates modeling of the phenomena underlying the propagation, estimation of the characteristic parameters and of the state of the epidemic, scheduling and scaling of the control measures. These are generic questions, for which tools have been developed in Control, admittedly in quite different applicative contexts. The aim of this session is precisely to explore in more details potential profit, as well as limitations, of this cross-fertilization.

Several common features of the problems arising from the application of Control theory to the handling of epidemics are worth mentioning here. First, the transmission of infection is inherently a nonlinear phenomenon. Second, the usual measurements of prevalence leads to a lack of observability and identifiability of the studied state space models at equilibria, a point which complicates the handling of these issues when the state is closed from these singular points (beginning of outbreak or final stabilization). Last, the cost functions of interest are usually non-conventional, typically proportional to the effort made (and not quadratic), and state constraint are not uncommon. All this, plus the important uncertainty present on the models and on the data, makes the corresponding problems rather sensitive. The six contributions that constitute this session offer a partial vision of issues to be adressed. They are more especially related to observability and identifiability (papers 1 and 2), and to optimal control applied to epidemic processes (papers 2 to 6).

## Choice of an IFAC technical committee for review

**TC 8.2. Biological and Medical Systems**

## Detailed description of the topic

Paper 1. is concerned with identifiability and observability issues for a compartmental SIR model with quarantine, where the flows of quarantined people are measured. Paper 2. proposes a simultaneous parameter and state estimation scheme for a class of general epidemic models, and solves associated optimal control problems. Paper 3. provides, for a class of positive compartmental systems, general results that characterize the solutions of some pertinent optimal control problems. Paper 4. studies optimal isolation strategies for epidemic models where, differently from the classical exponential distribution, the infectious period obeys an Erlang probability distribution. Paper 5. intends to study in a stochastic framework notions of herd immunity and of viable zones for non-pharmaceutical interventions in presence of constraints on the intensive care occupancy rate. Last, paper 6. presents several qualitative results on the optimal control that minimizes the epidemic final size of SIR model by social distancing.

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## Titles, authors, and abstracts of all six contributions

**1. About the identifiability and observability of the SIR model with quarantine,** by Frédéric Hamelin[1], Abderrahman Iggidr[2], Alain Rapaport[3], Gauthier Sallet[2], Max O. Souza[4].

We consider a SIRQ model which is an extension of the well-known SIR epidemic model with an additional compartment Q that represents the population that has been removed from the infected population and placed in quarantine. We analyze the identifiability and observability of this model assuming that the flow of individuals placed in quarantine and the size of the quarantine population are known at any time. We also derive some methods to deal with the practical identification of the model parameters and the estimation of the infected population size.

**2. Feedback Design for Optimal Control of Epidemic Processes,** by Muhammad Umar B. Niazi[5], Phillip E. Paré[6], Karl Henrik Johansson[5].

For reliable epidemic monitoring and control, this paper proposes a feedback mechanism design to effectively cope with data and model uncertainties through. Using the past epidemiological data, we propose a simultaneous parameter and state estimation method of general epidemic models. We provide necessary and sufficient conditions under which the parameter and state estimation problems are well-posed and feasible. Then, we devise optimal control policies by minimizing a predicted cost functional based on the estimated parameters and state of the model, and discuss under what condition the solution turns out to be feasible.

**3. Optimal control of compartmental models: the exact solution,** by Franco Blanchini[6], Paolo Bolzern[7], Patrizio Colaneri[8], Giuseppe De Nicolao[9], Giulia Giordano[10].

We formulate a control problem for positive compartmental systems formed by nodes (buffers) and arcs (flows). Our main result is that, on a finite horizon, we can solve the Pontryagin equations in one shot without resorting to trial and error via shooting. As expected, the solution is bang-bang and the switching times can be easily determined. We are also able to find a cost-to-go-function, in an analytic form, by solving a simple nonlinear differential equation. On an infinite horizon, we consider the Hamilton-Jacobi-Bellman theory and we show that the HJB equation can be solved exactly. Moreover, we show that the optimal solution is constant and the cost-to-go function is linear and copositive. This function is the solution of a nonlinear equation. We propose an iterative scheme for solving this equation, which converges in finite time. We also show that an exact solution can be found if there is a positive external disturbance affecting the process and the problem is formulated in a min sup framework. We finally provide illustrative examples related to flood control and epidemiology.

**4. Optimal isolation strategies in an SIR model with Erlang–distributed infectious period,** by Luca Bolzoni[11], Rossella Della Marca[12].

Mathematical models are formal and simplified representations of the knowledge related to a phenomenon. In classical epidemic models, a major simplification consists in assuming that the infectious period is exponentially distributed, then implying that the chance of recovery is independent of the time since infection. Here, we first attempt to investigate the consequences of relaxing this assumption on the performances of time–variant disease control strategies by using optimal control theory. In the

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framework of a basic susceptible–infected–removed (SIR) model, an Erlang distribution of the infectious period is considered and optimal isolation strategies are searched for. The objective functional to be minimized takes into account the cost of the isolation efforts per time unit and the sanitary costs due to the incidence of the epidemic outbreak. Applying the Pontryagin's minimum principle, we prove that the optimal control problem admits only bang–bang solutions with at most two switches. In particular, the optimal strategy could be postponing the starting intervention time with respect to the beginning of the outbreak. Finally, by means of numerical simulations, we show how the shape of the optimal solutions is affected by the different distributions of the infectious period, by the relative weight of the two cost components, and by the initial conditions.

**5. On the Design Techniques for Safety Zones In Brownian-Driven Epidemic Models,** by Dan Goreac[13], Juan Li[14], Yi Wang[15].

This paper is concerned with the mathematical description of *herd immunity* and *feasible zones* for *non-pharmaceutically controlled* Brownian-driven compartmental models when *intensive care units* restrictions are imposed. First, we focus on *consistency* considerations leading to non-negative and sub-unitary components for possibly non-constant population models. Second, based on viscosity approaches, we give a family of conditions allowing to envisage regular-barrier safety zones. Relevant examples of families of noise coefficients are presented.

**6. On the problem of minimizing the epidemic final size for SIR model by social distancing,** by Alain Rapaport[3], Pierre-Alexandre Bliman[17].

We revisit the problem of minimizing the epidemic final size in the SIR model through social distancing. In the existing literature, this problem has been considered imposing a priori interval structure on the time period when interventions are enforced. We show that when considering the more general class of controls with an $L^1$ constraint on the confinement effort that reduces the infection rate, the support of the optimal control is still a single time interval. This shows that, for this problem, there is no benefit in splitting interventions on several disjoint time periods.

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