How does residual fertility impact the effectiveness of the sterile insect technique in controlling Ceratitis capitata?

Marine Courtois, Kévan Rastello, Frédéric Grognard, Ludovic Mailleret, Suzanne Touzeau, Louise van Oudenhove

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Mathematical Population Dynamics, Ecology and Evolution - **MPDEE 2023**
CIRM - 26 April 2023

Pest Insects

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Ceratitis capitata

Damages

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📉 **Crop yields**

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Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

Means of control

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Preventive control

- Picking up and crushing fallen fruit
- Shallow tillage during winter

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Alternative control

- Mass trapping
- « Attract and kill »
- Parasitoids
- SIT: Sterile Insect Technique

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Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

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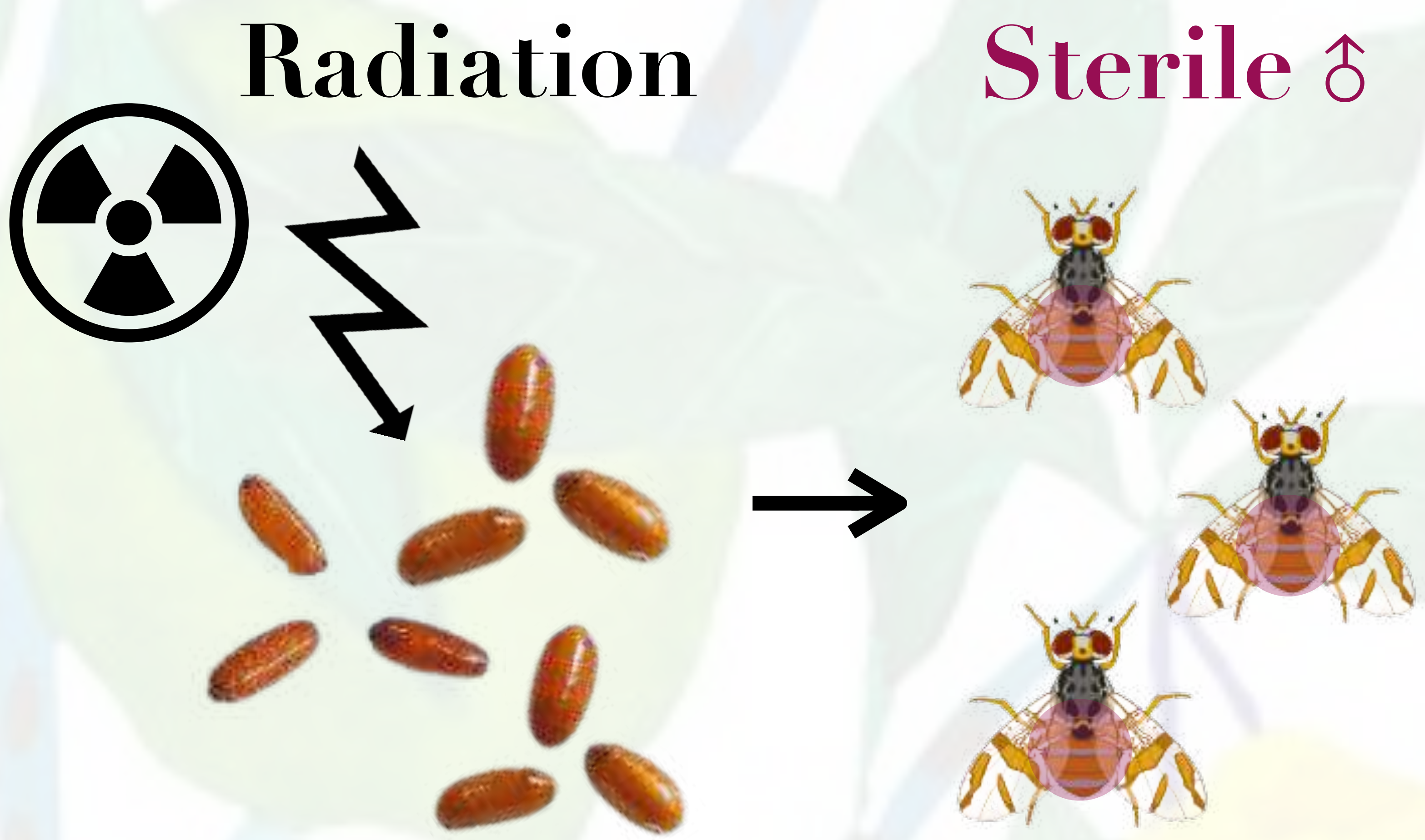

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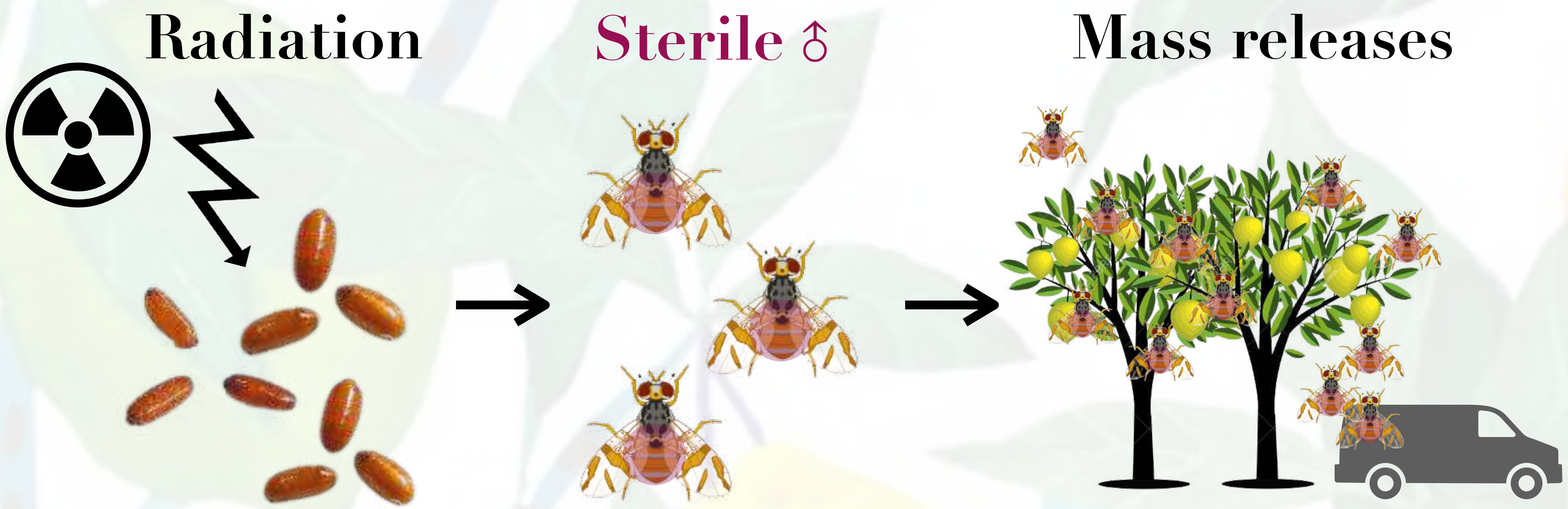

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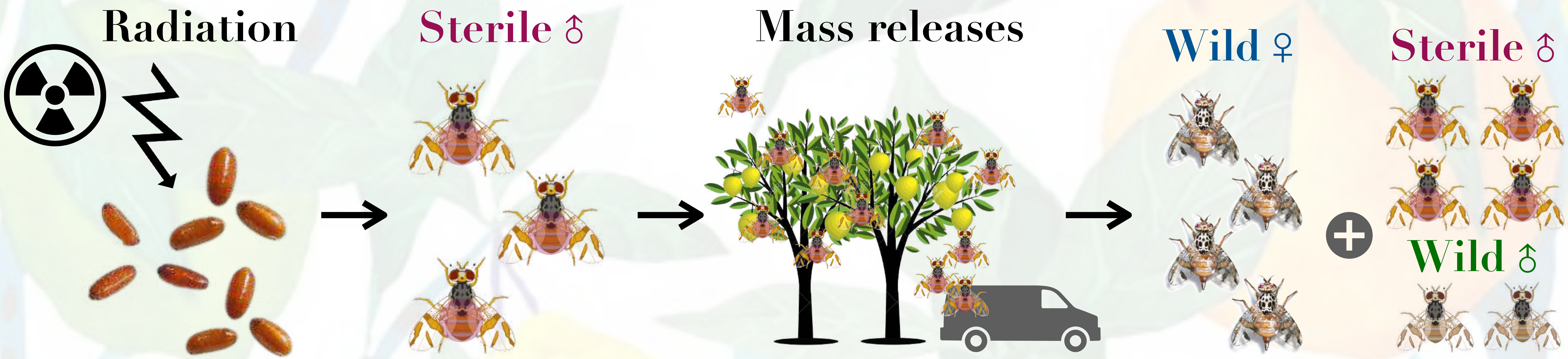

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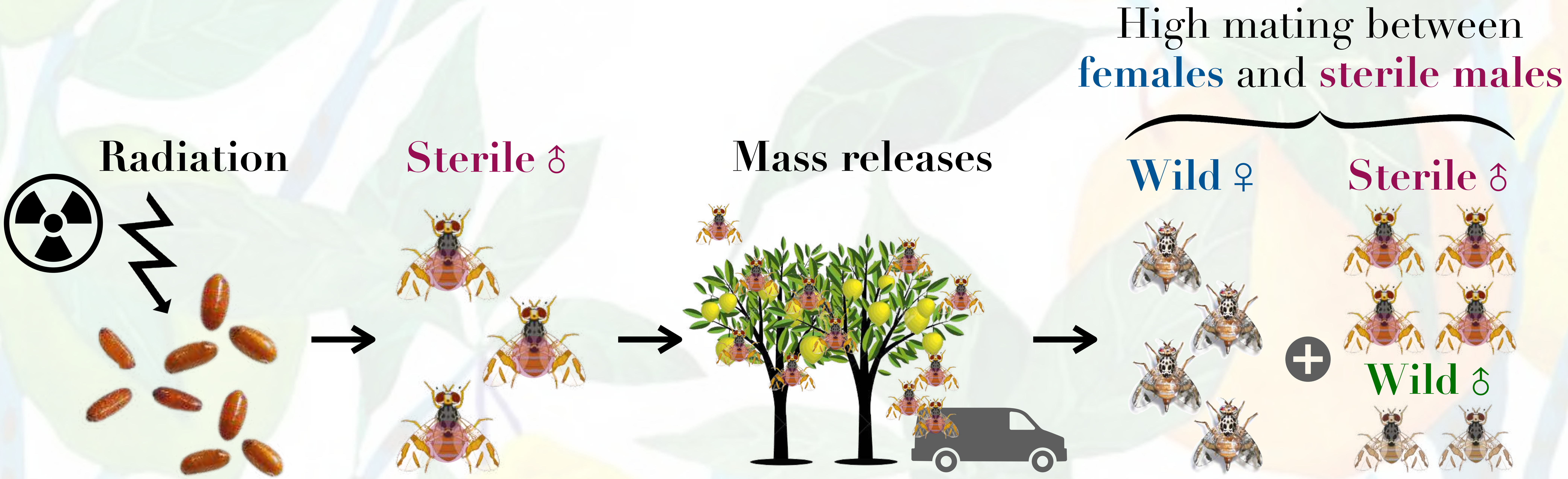

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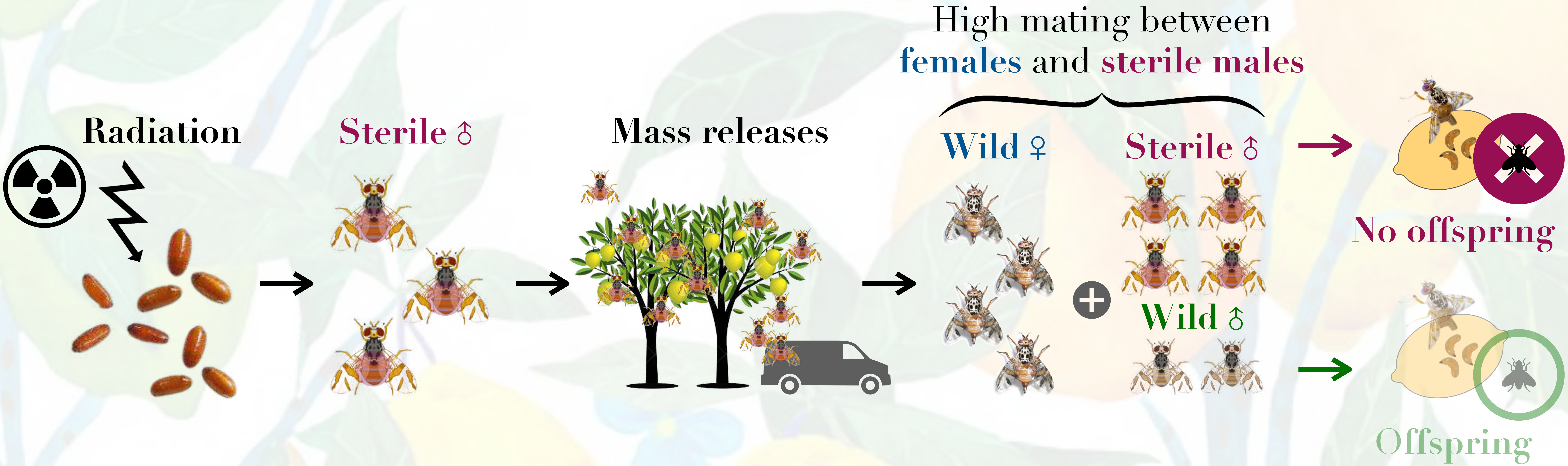

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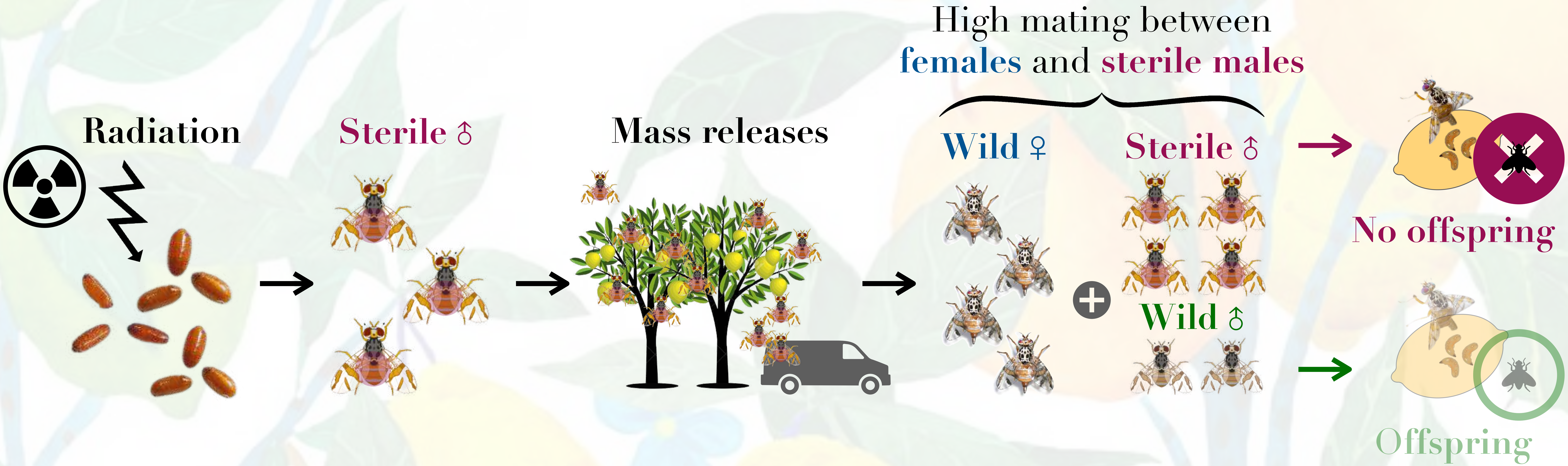

How many insects to release? How often?

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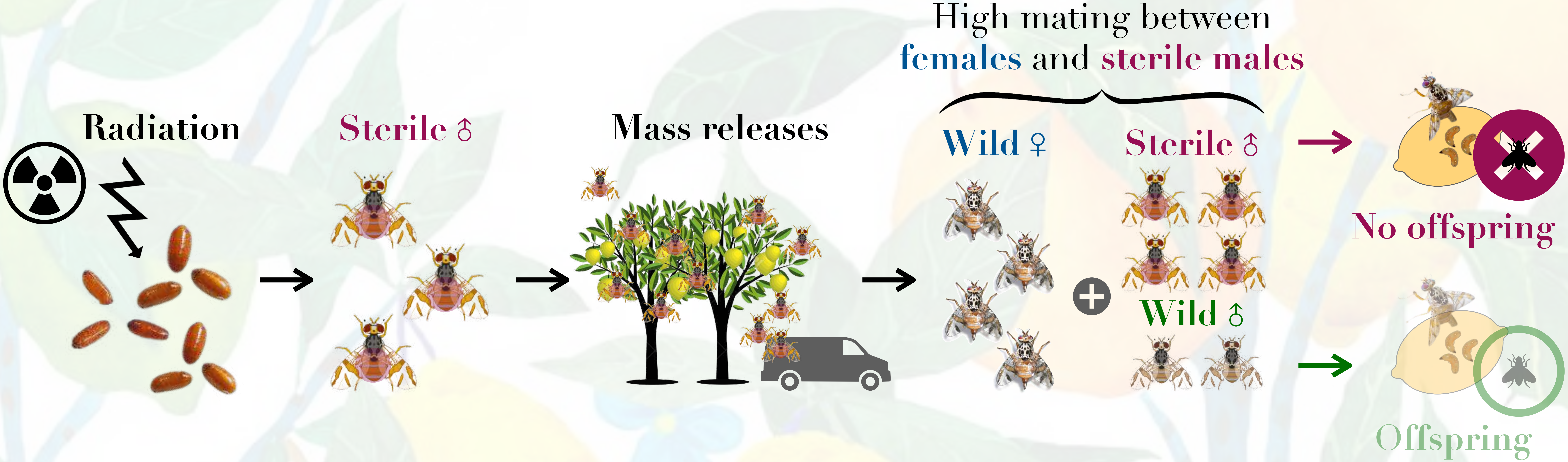

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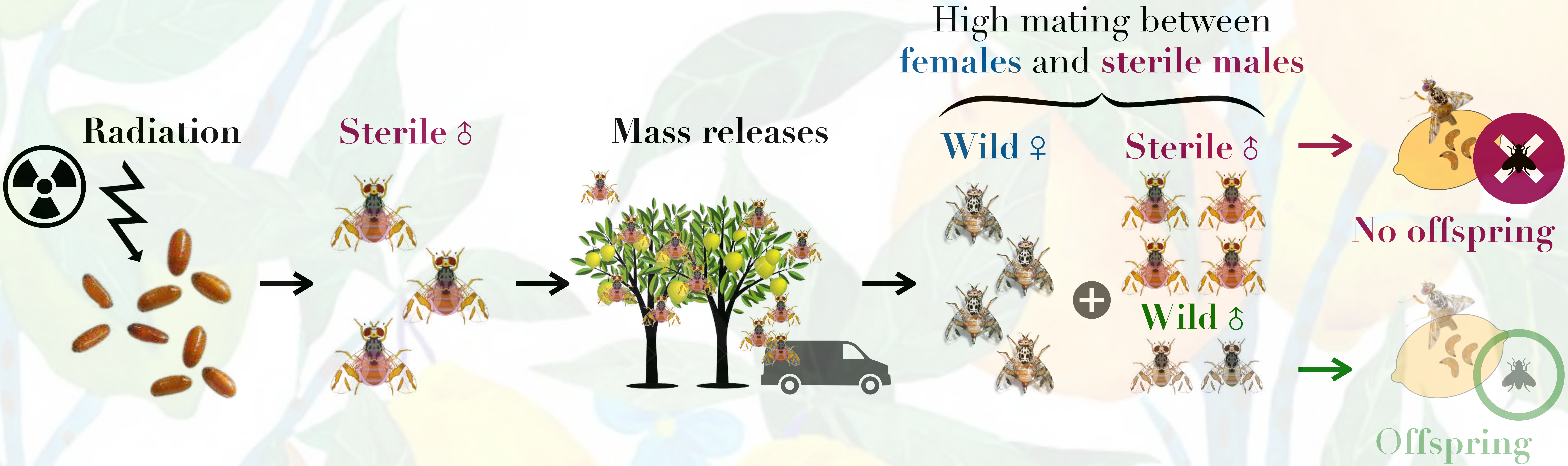

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Population decrease?

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High mating between

- **Main challenge: to determine how high the sterility rate should be to ensure pest control in the field**
- **Modeling represents an essential and efficient tool to tackle this issue (limitation of economic and temporal costs)**

Are all released insects sterile?

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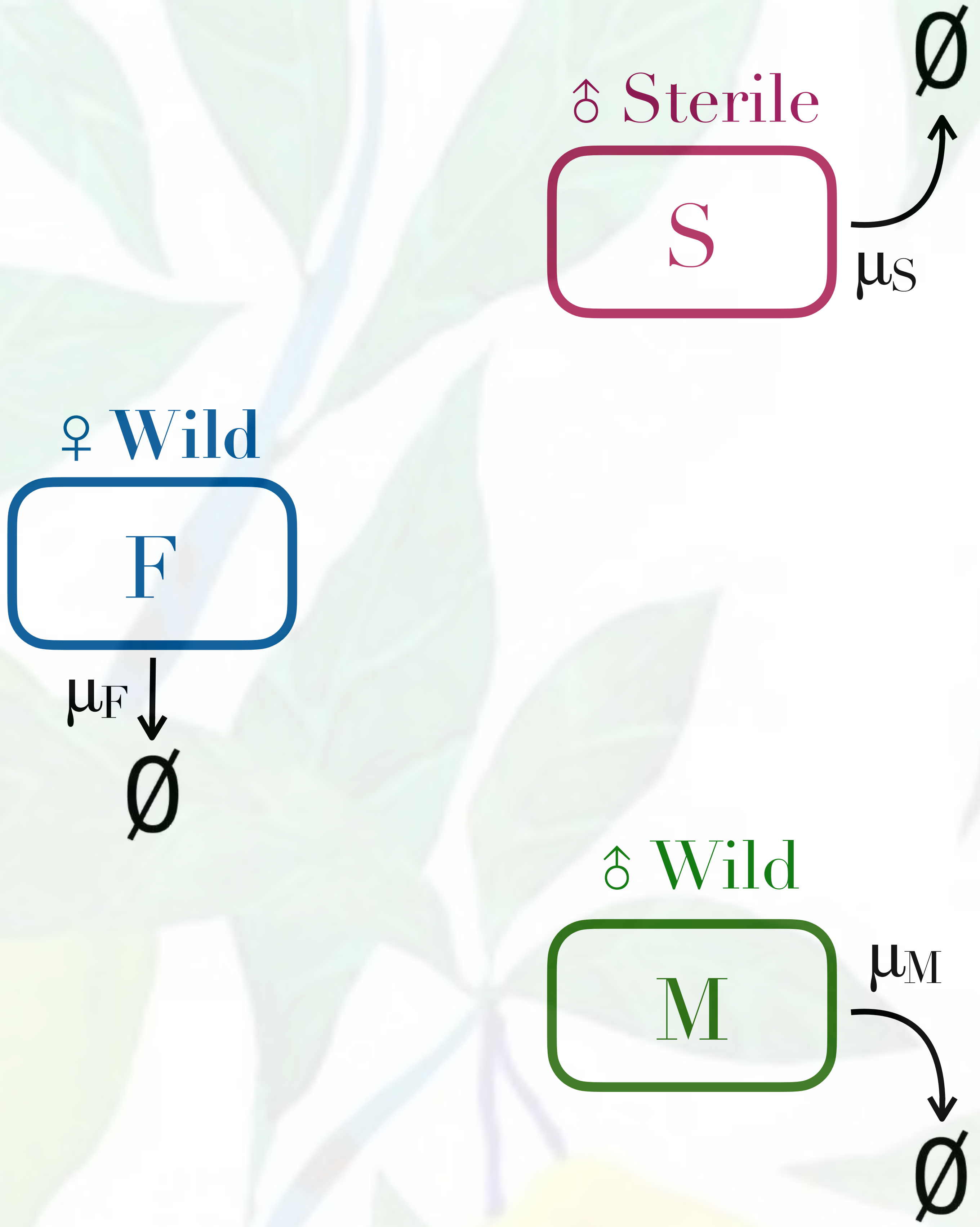
Population decrease?

$$\begin{cases} \dot{S} = \\ \dot{F} = \\ \dot{M} = \end{cases}$$



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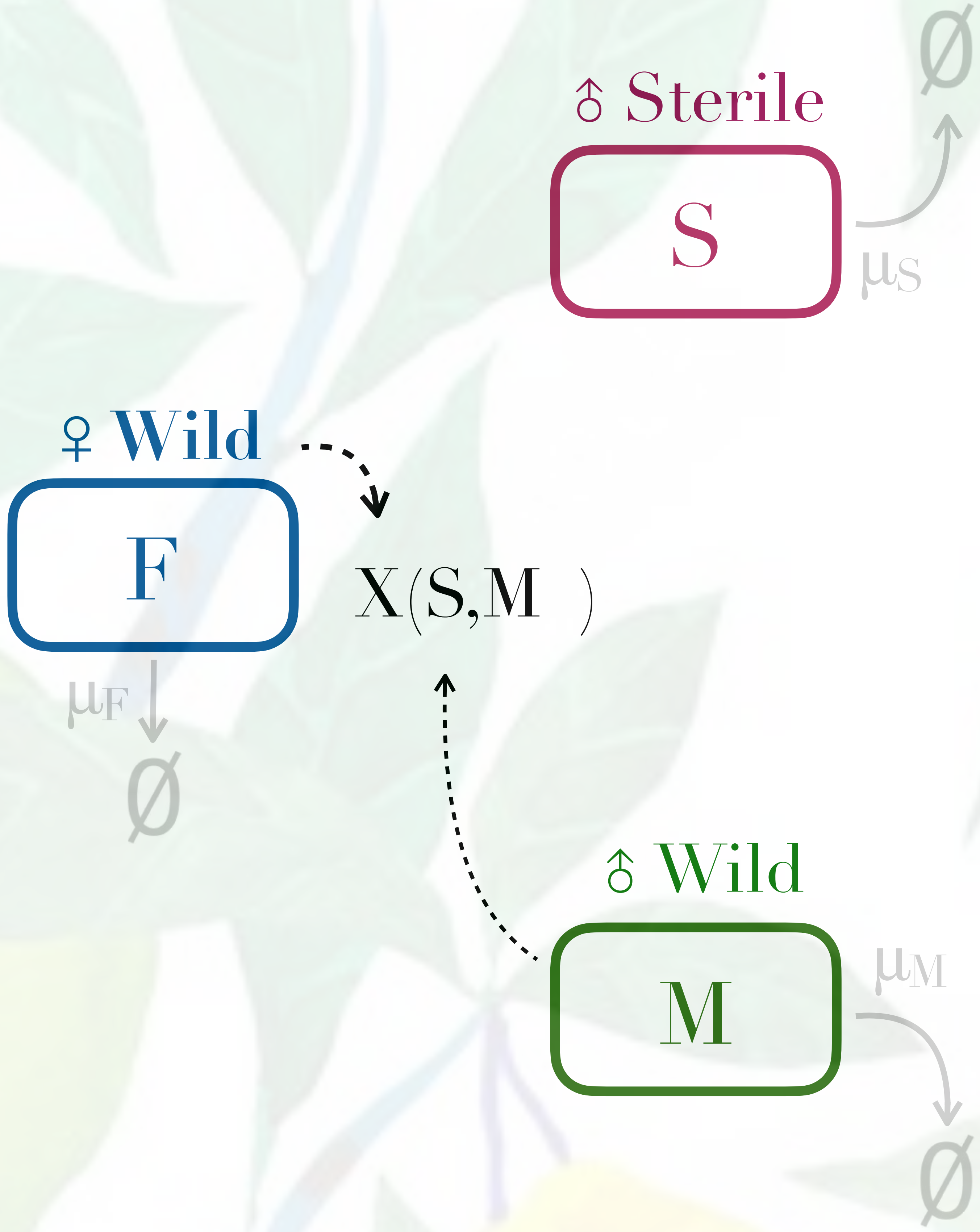
μ: mortality rate



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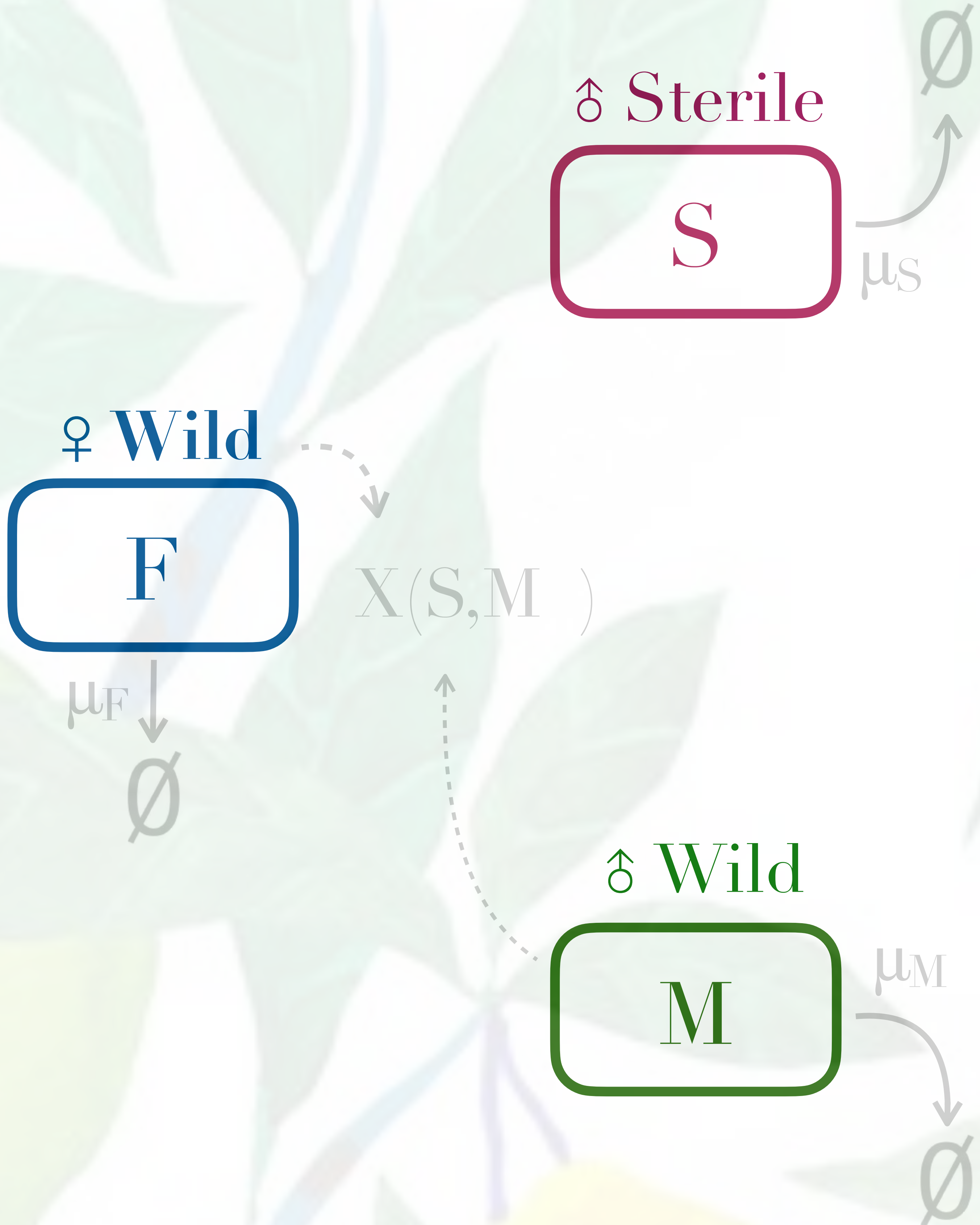

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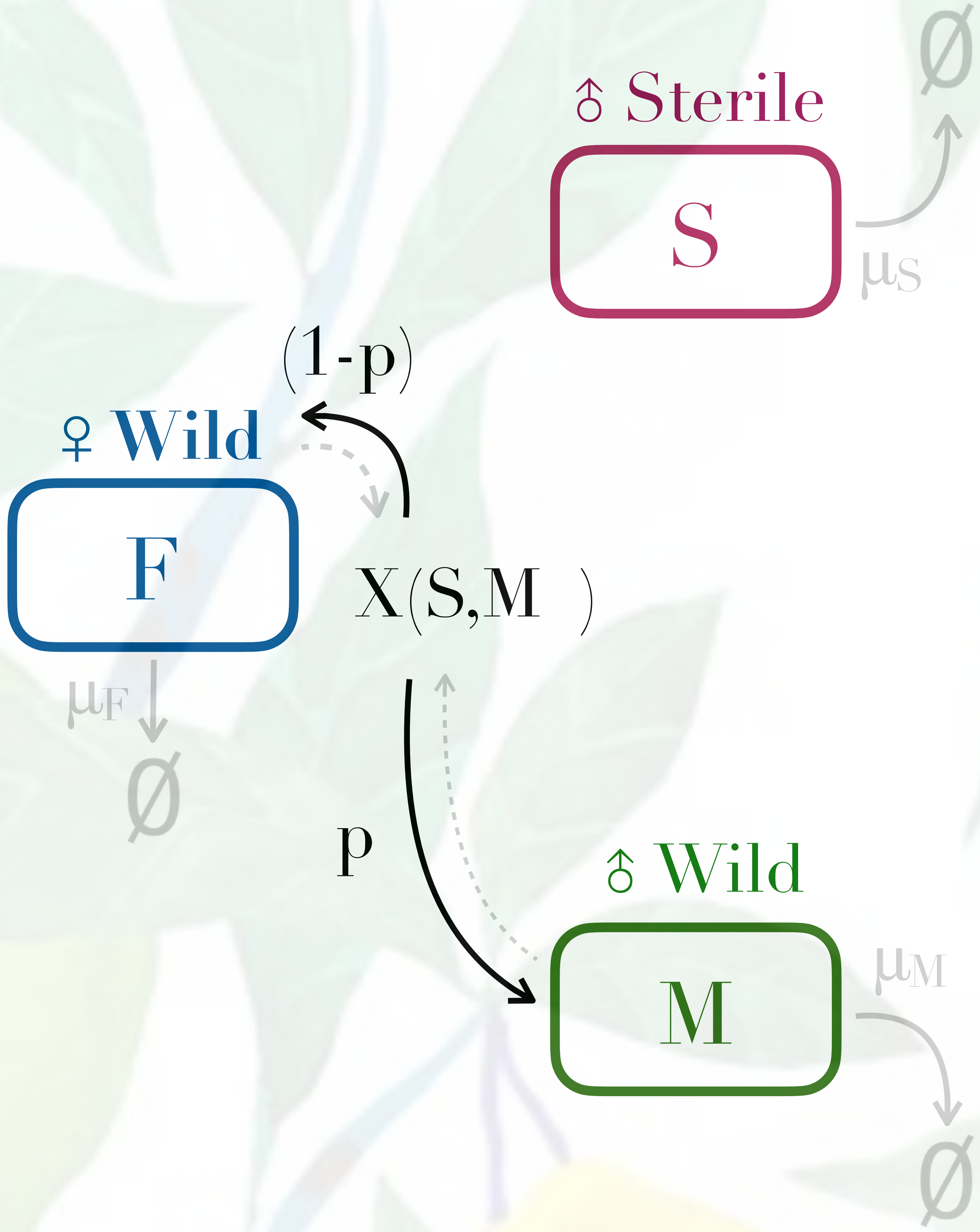

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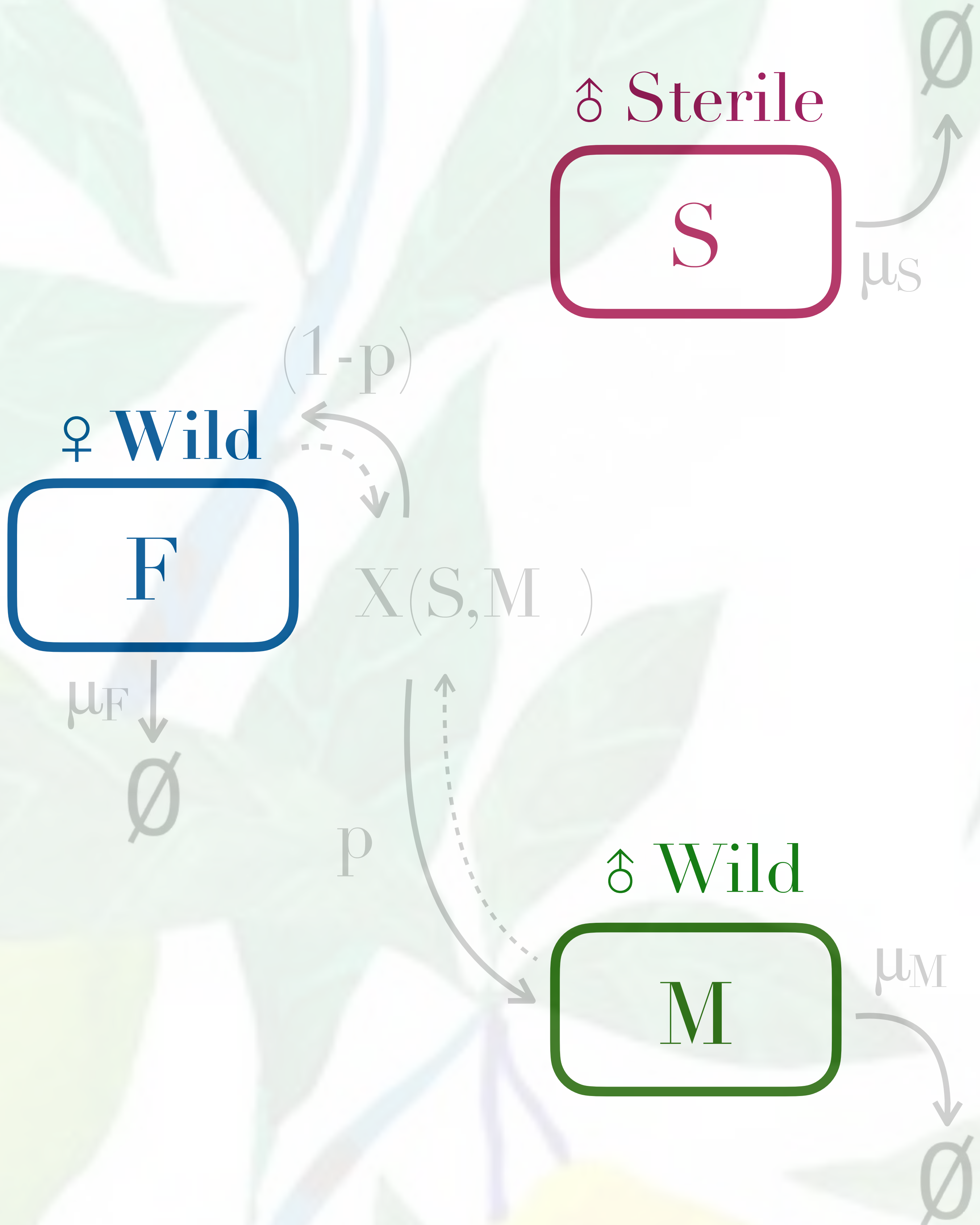

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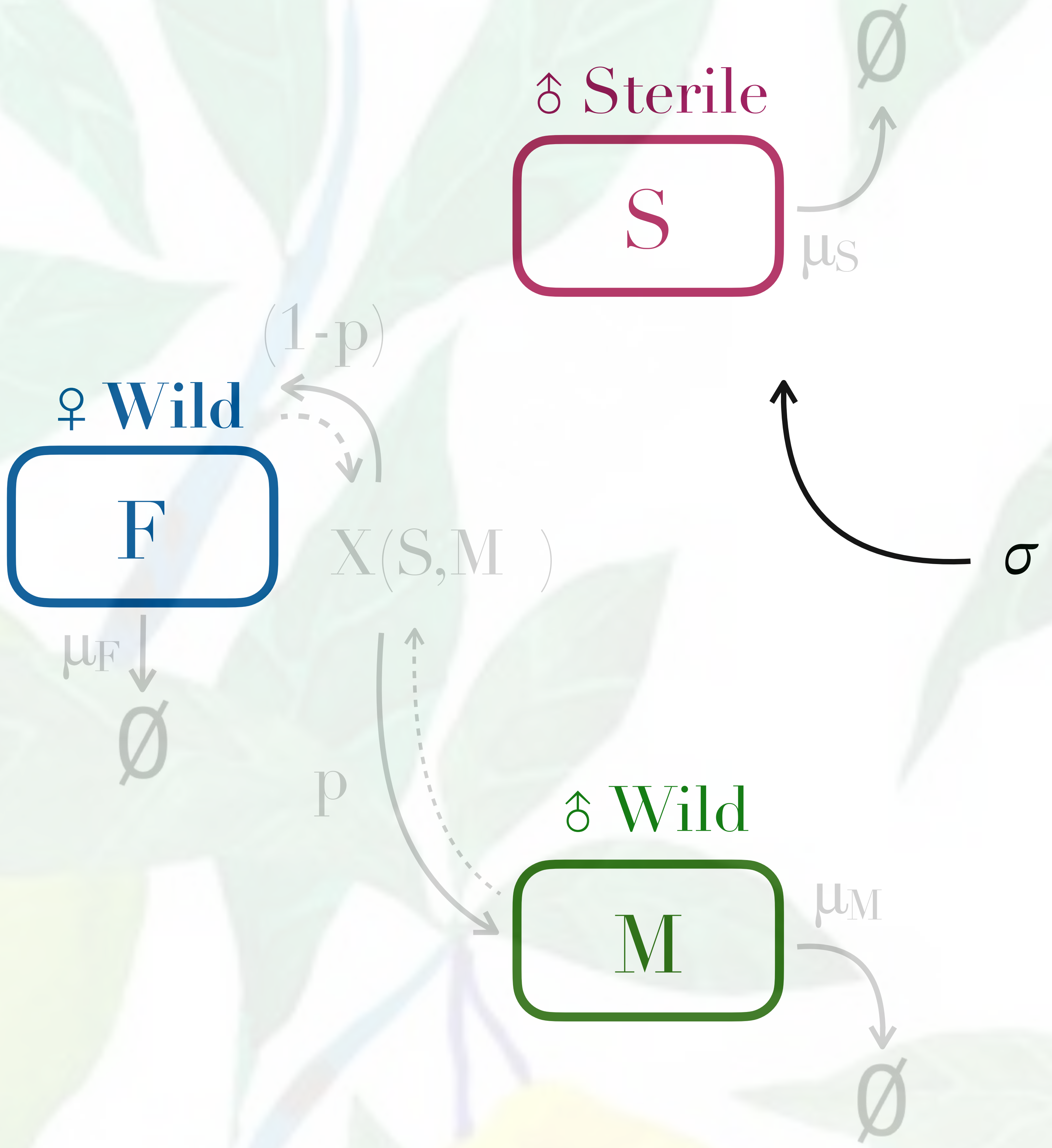

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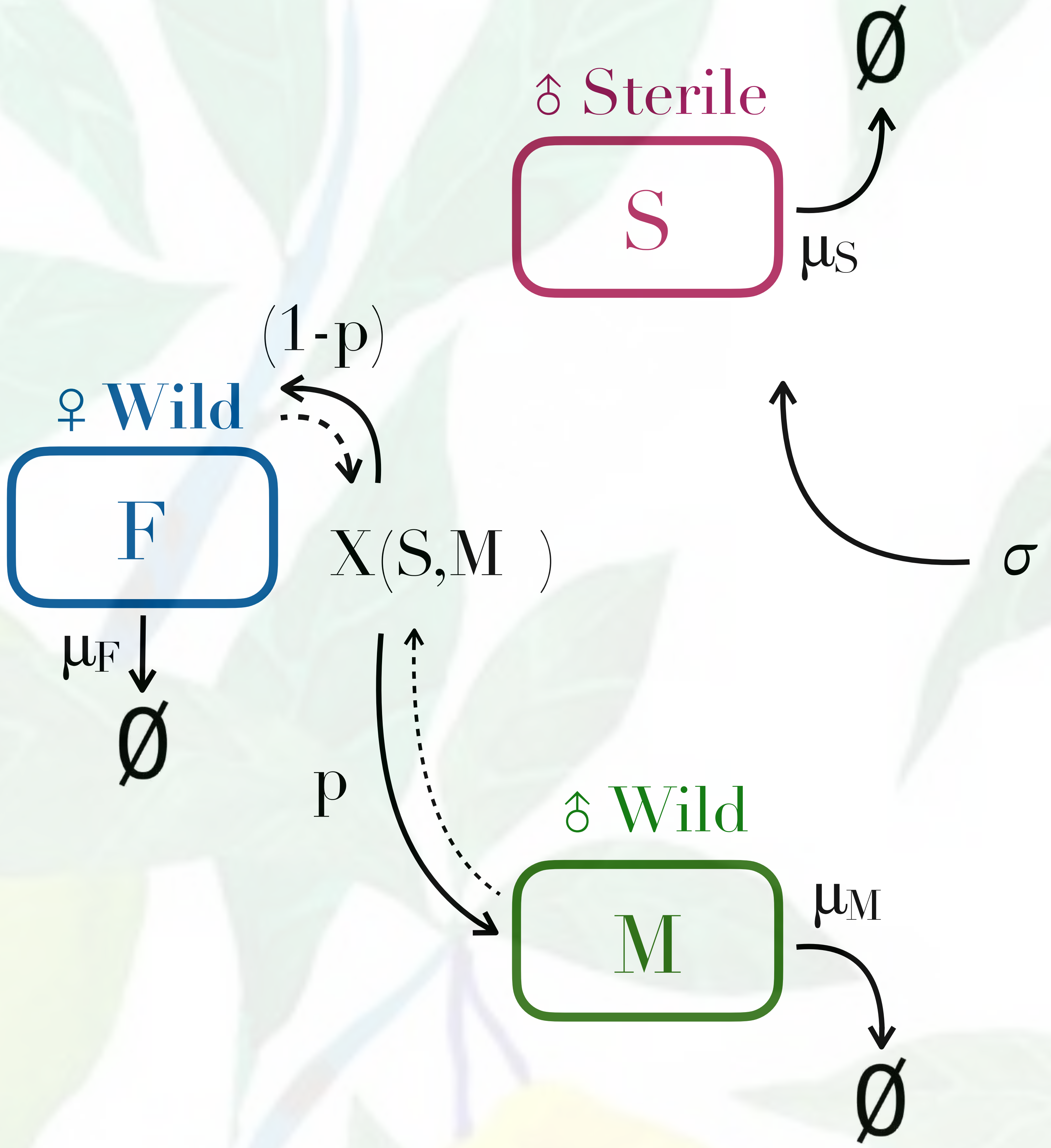

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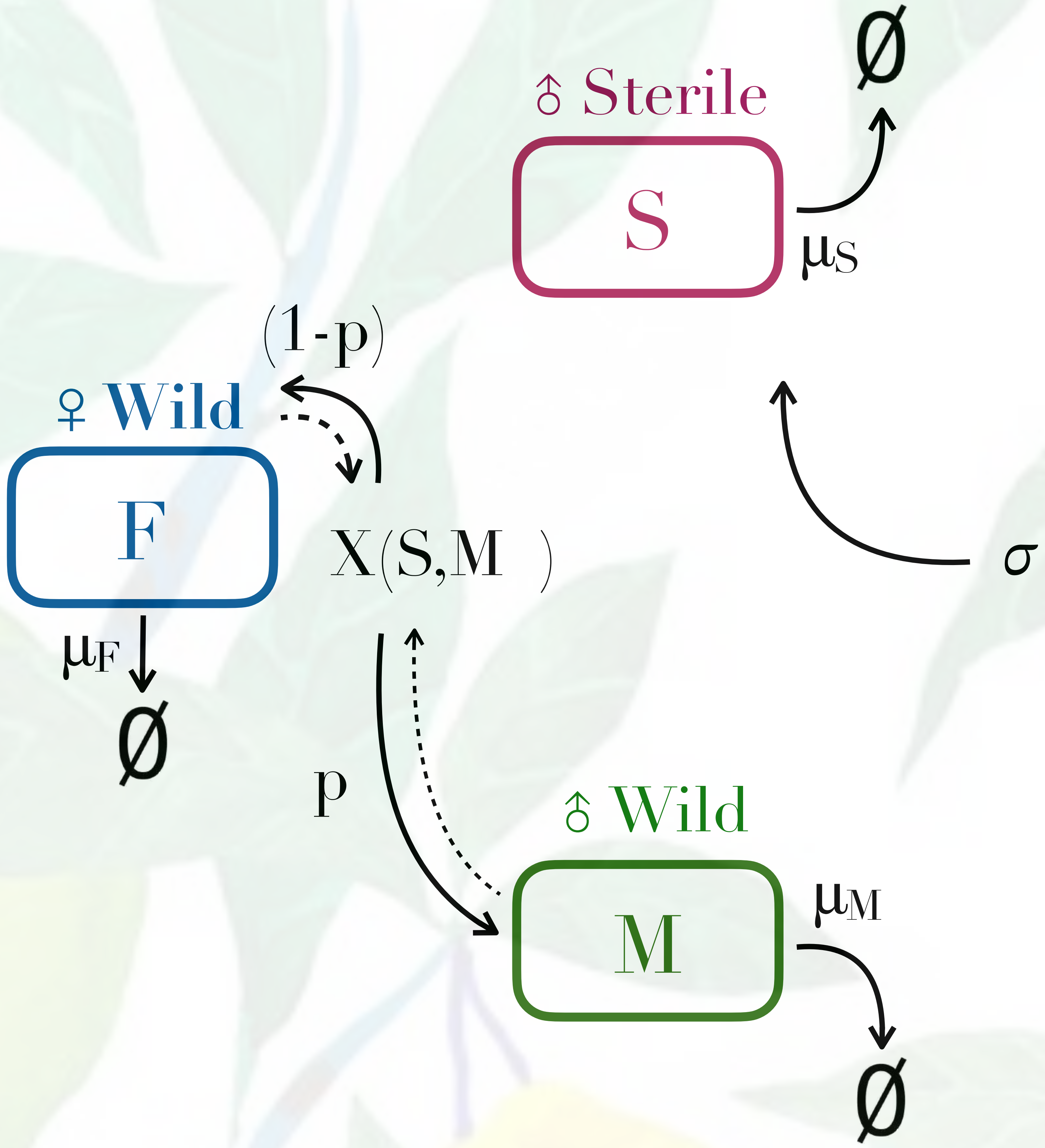

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Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

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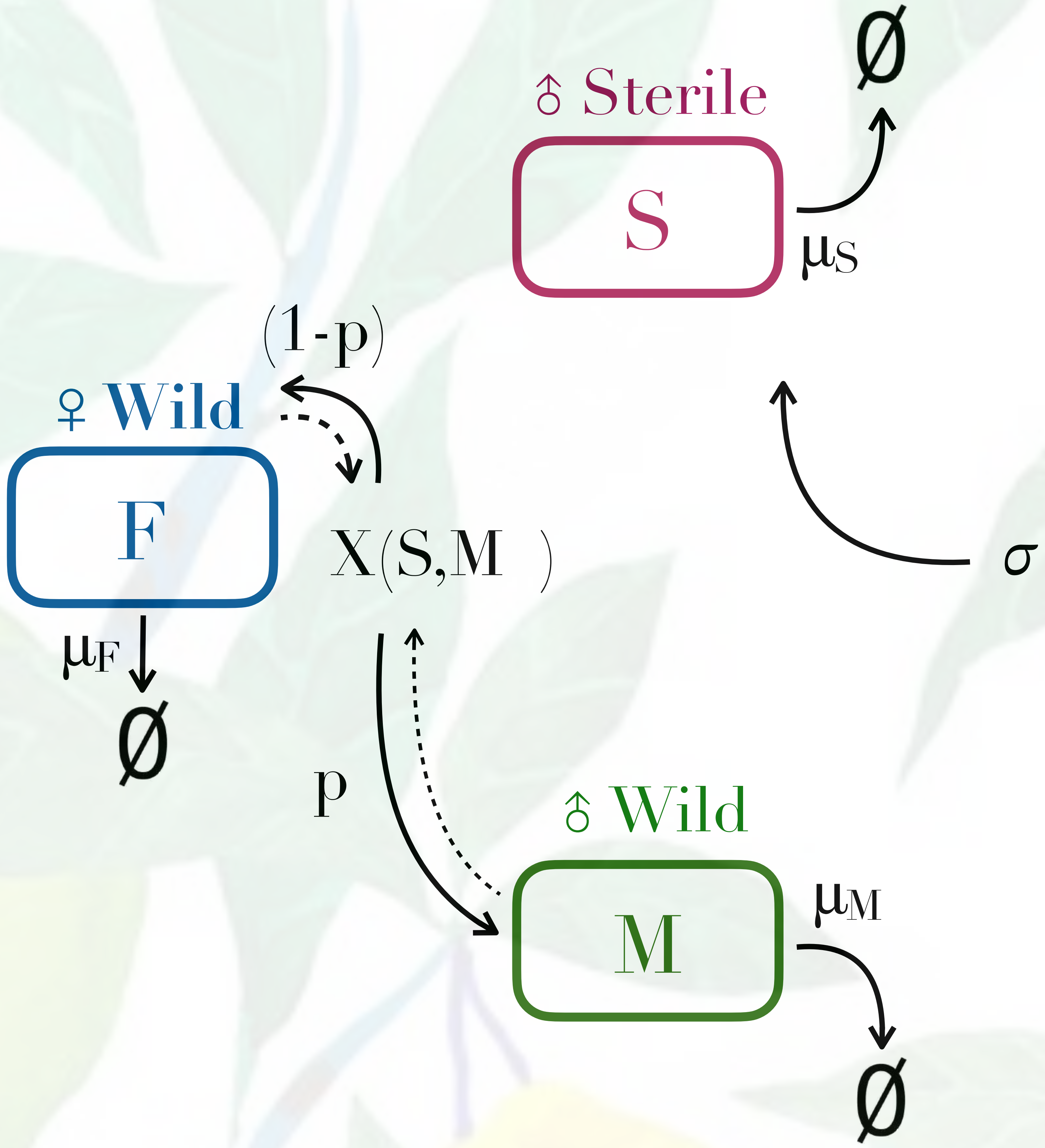

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No ~~Residual fertility~~
$\delta = 0, \epsilon = 0$

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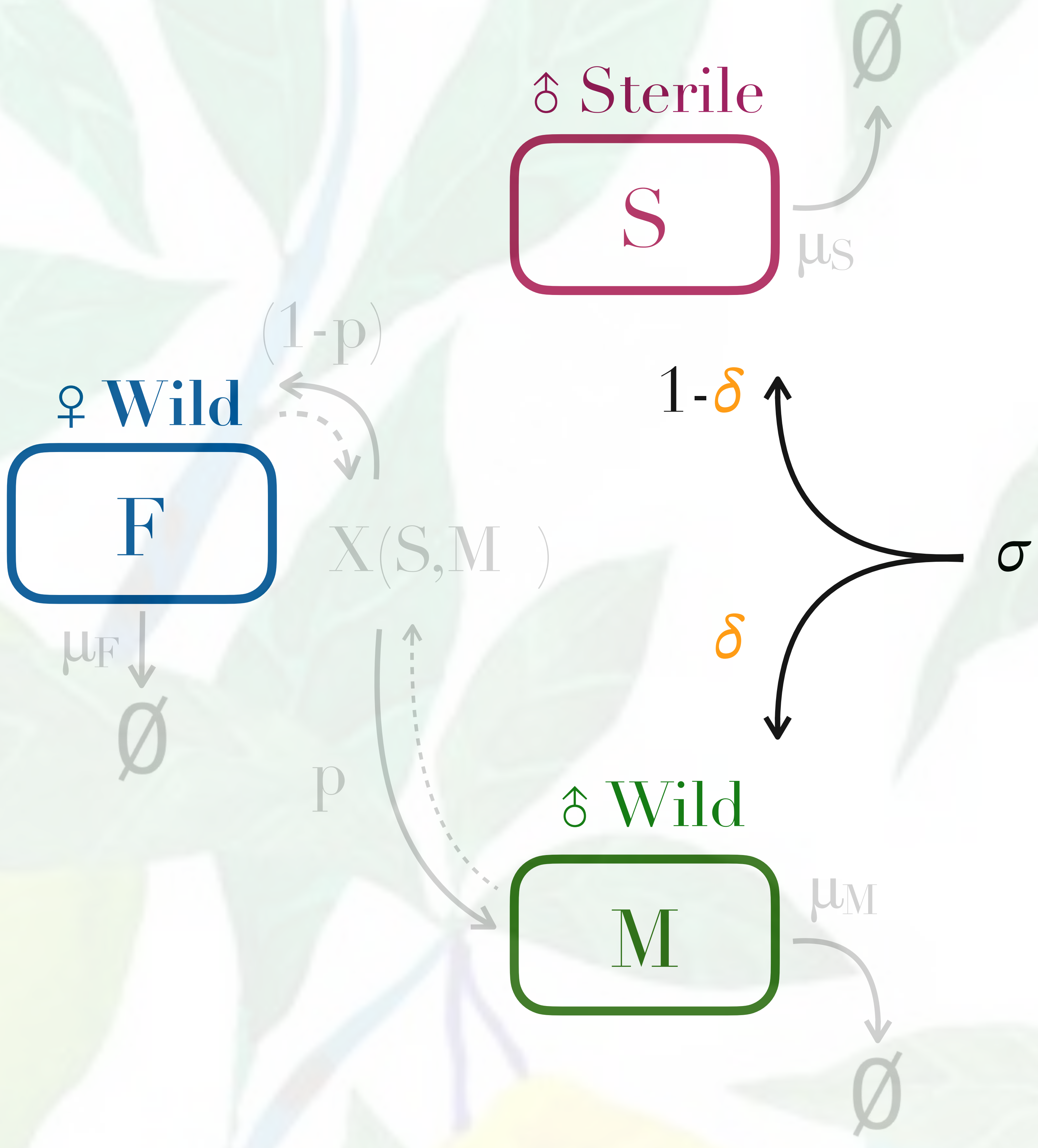

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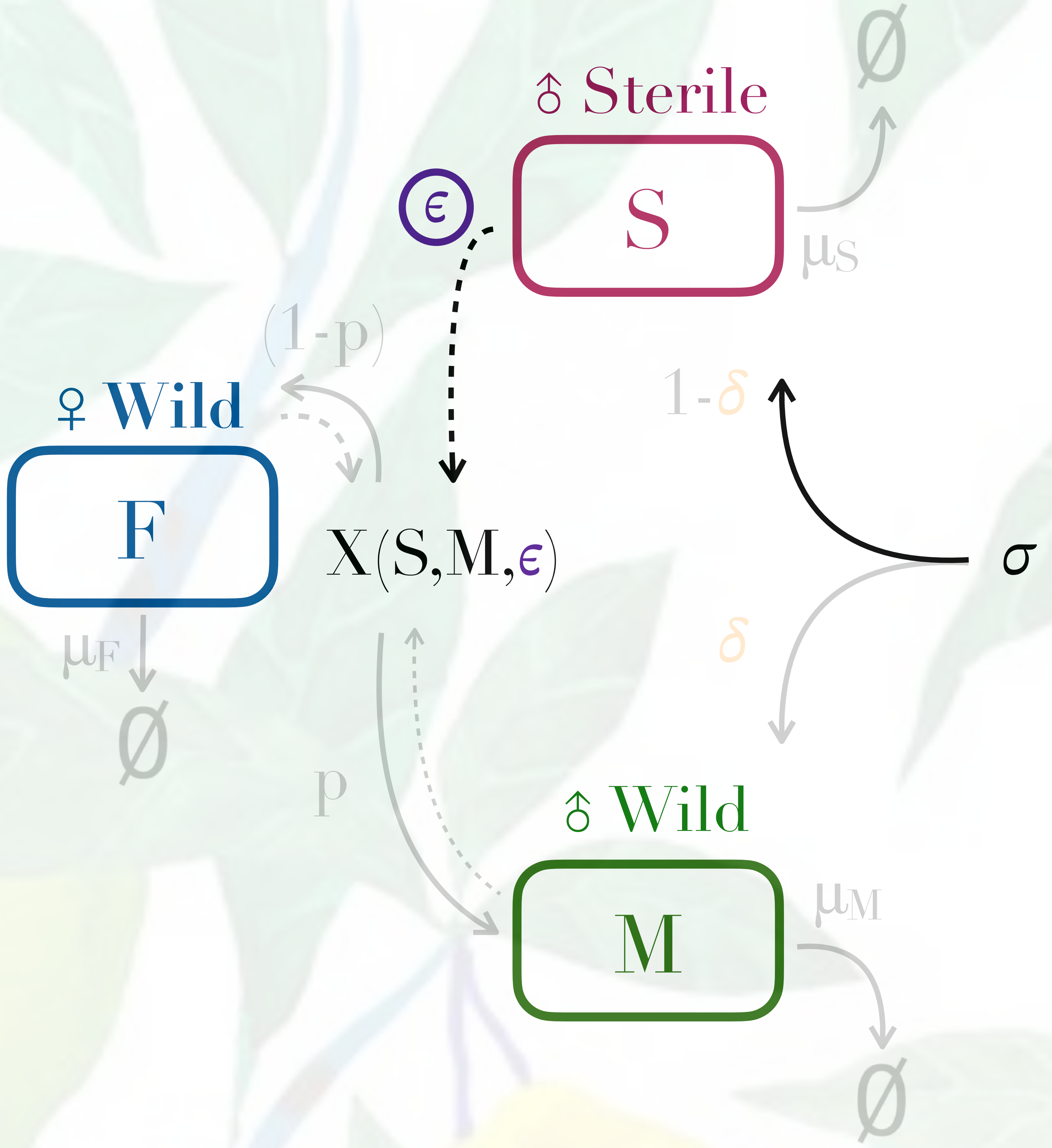

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(2) **Costly fertility model** $\delta = 0, \epsilon \neq 0$

X(**S**,**M**,ϵ): mating probability $\dfrac{M+\epsilon\eta S}{k+M+\eta S}$

η: Fitness cost

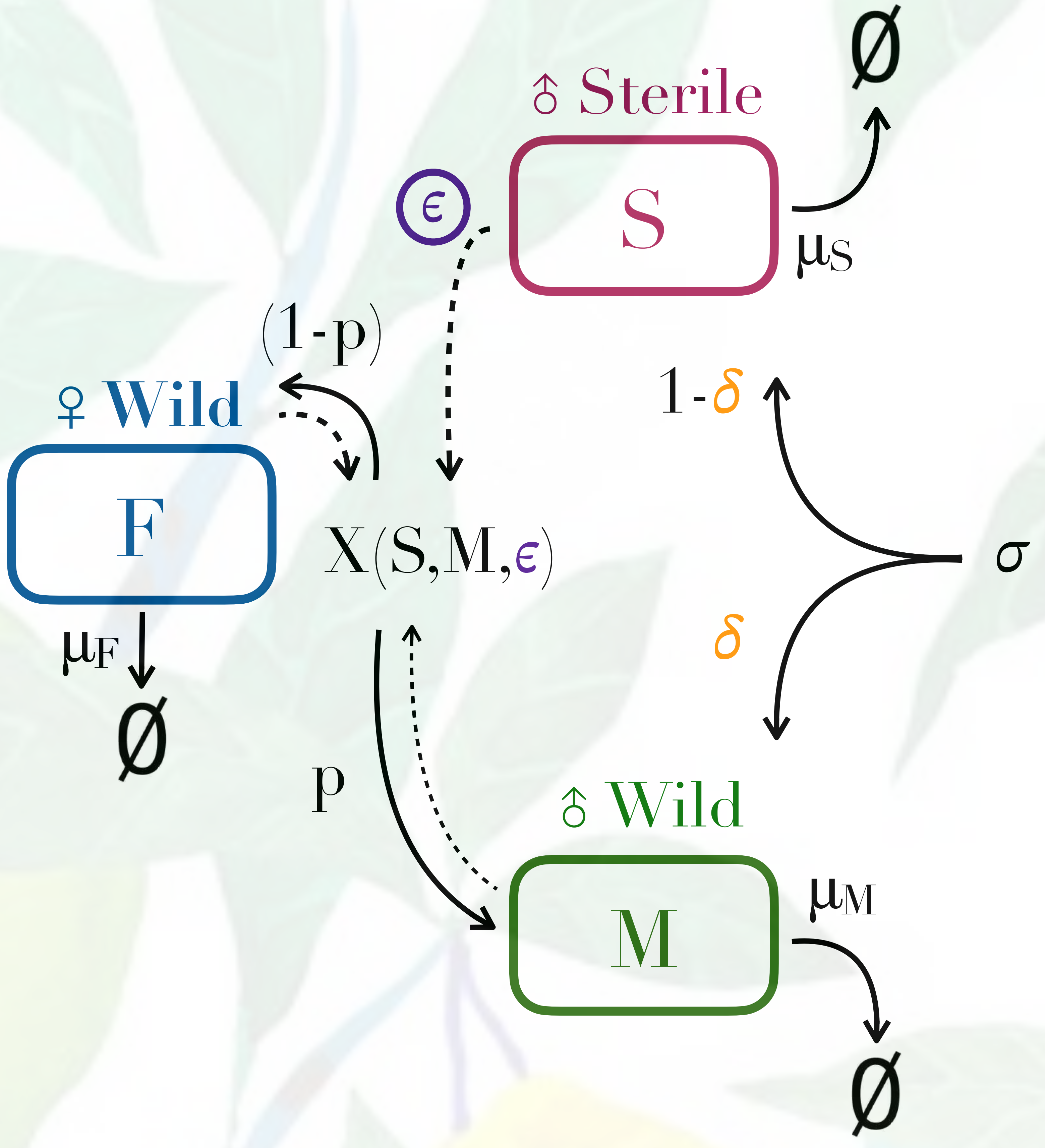

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Table 1: Model parameters

Parameters	Descriptions	Values	Units	References
μ_F	Female mortality rate	0.050	day^{-1}	Vargas *et al.* (2000) Pieterse *et al.* (2020)
μ_M	Male mortality rate	0.036	day^{-1}	Vargas *et al.* (2000) Pieterse *et al.* (2020)
μ_S	Sterile male mortality rate	0.057	day^{-1}	Calibrated value
p	Sex ratio	0.50	-	Pieterse *et al.* (2020)
r	Emergence rate (mean number of eggs leading to the adult stage per female)	1.19	eggs.♀$^{-1}$.day^{-1}	Shoukry and Hafez (1979) Carey (1982, 1984) Vargas *et al.* (1984, 2000) Krainacker *et al.* (1987) Duyck *et al.* (2002) Papadopoulos *et al.* (2002) Diamantidis *et al.* (2011)
k	Coupling half-saturation constant	1	♂ density	Calibrated value
β	Oviposition competition between females	0.85	(♀ density)$^{-1}$	Calibrated value
σ	Sterile male release rate	Variable	♂ density.day^{-1}	
$1-\eta$	Sterilization cost	0.8	-	Calibrated value
δ	Proportion of non-sterile males among the releases (cost-free fertility)	Variable	-	Studied value
ϵ	Proportion of non-sterile males among the releases (costly fertility)	Variable	-	Studied value

The values noted as "Calibrated values" were determined from laboratory data.

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$\boxed{F^* = 0}$ $\quad X(S^*,M^*,\epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

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Pest-free equilibrium

$(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$

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$$\Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0$$

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$$\Rightarrow \quad \dot{M} = 0 \iff -\mu_M M^* + rpX(S,M^*)C(F^*)F^* + \delta\sigma = 0$$

$$\boxed{M^* = \frac{\delta\sigma}{\mu_M} = M_0^*} \qquad \boxed{M^* = M(F^*) = \frac{p\mu_F}{(1-p)\mu_M}F^* + \frac{\delta\sigma}{\mu_M}}$$

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

Infestation equilibria are the F* values solutions of:

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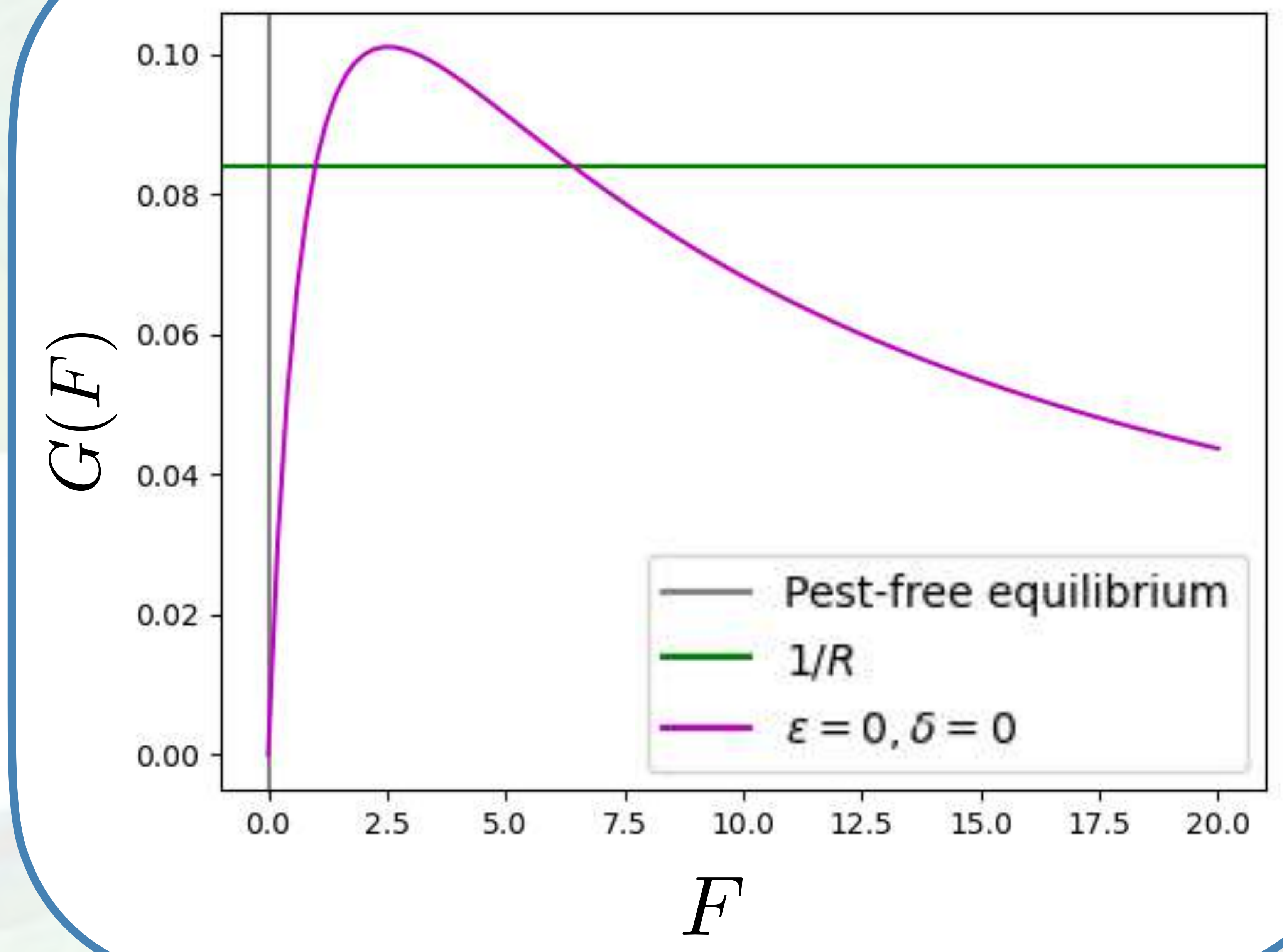
And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



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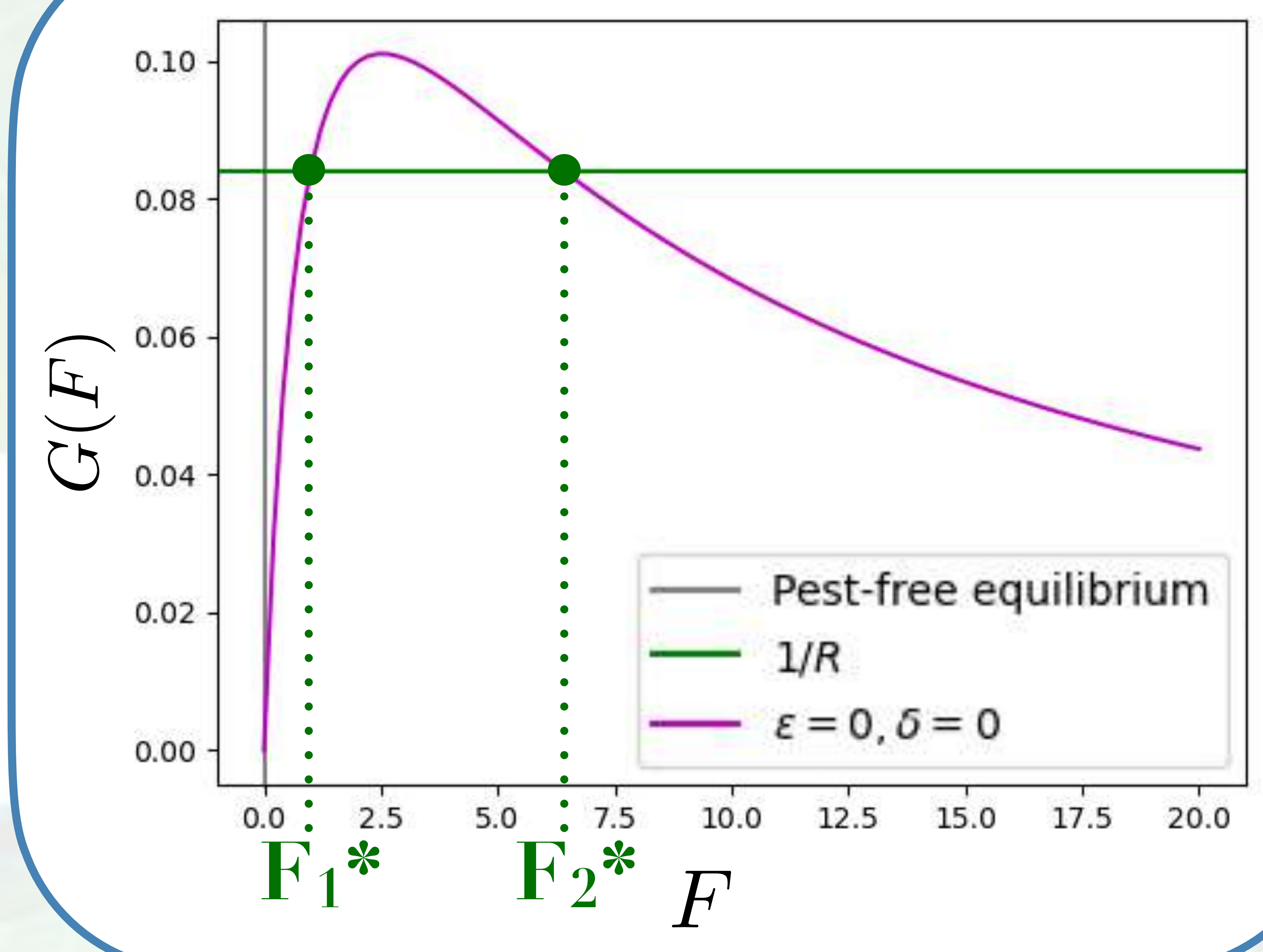

At least **2 solutions** for which **F_2*** > **F_1*** > 0 such that:

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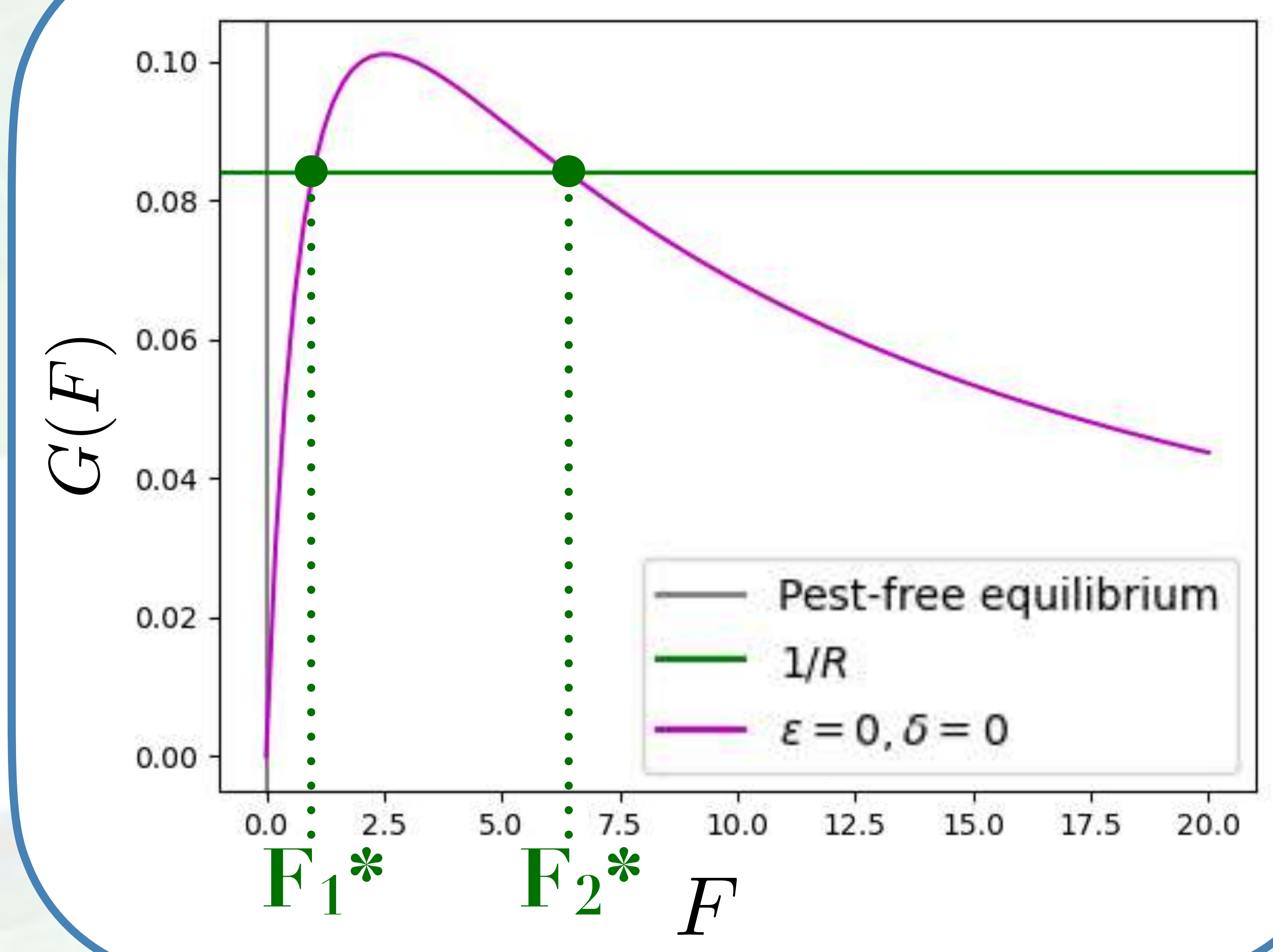

At least **2 solutions** for which $\mathbf{F_2^*} > \mathbf{F_1^*} > 0$ such that:

$$\frac{dG}{dF}(\mathbf{F_2^*}) < 0 \text{ and } \frac{dG}{dF}(\mathbf{F_1^*}) > 0$$

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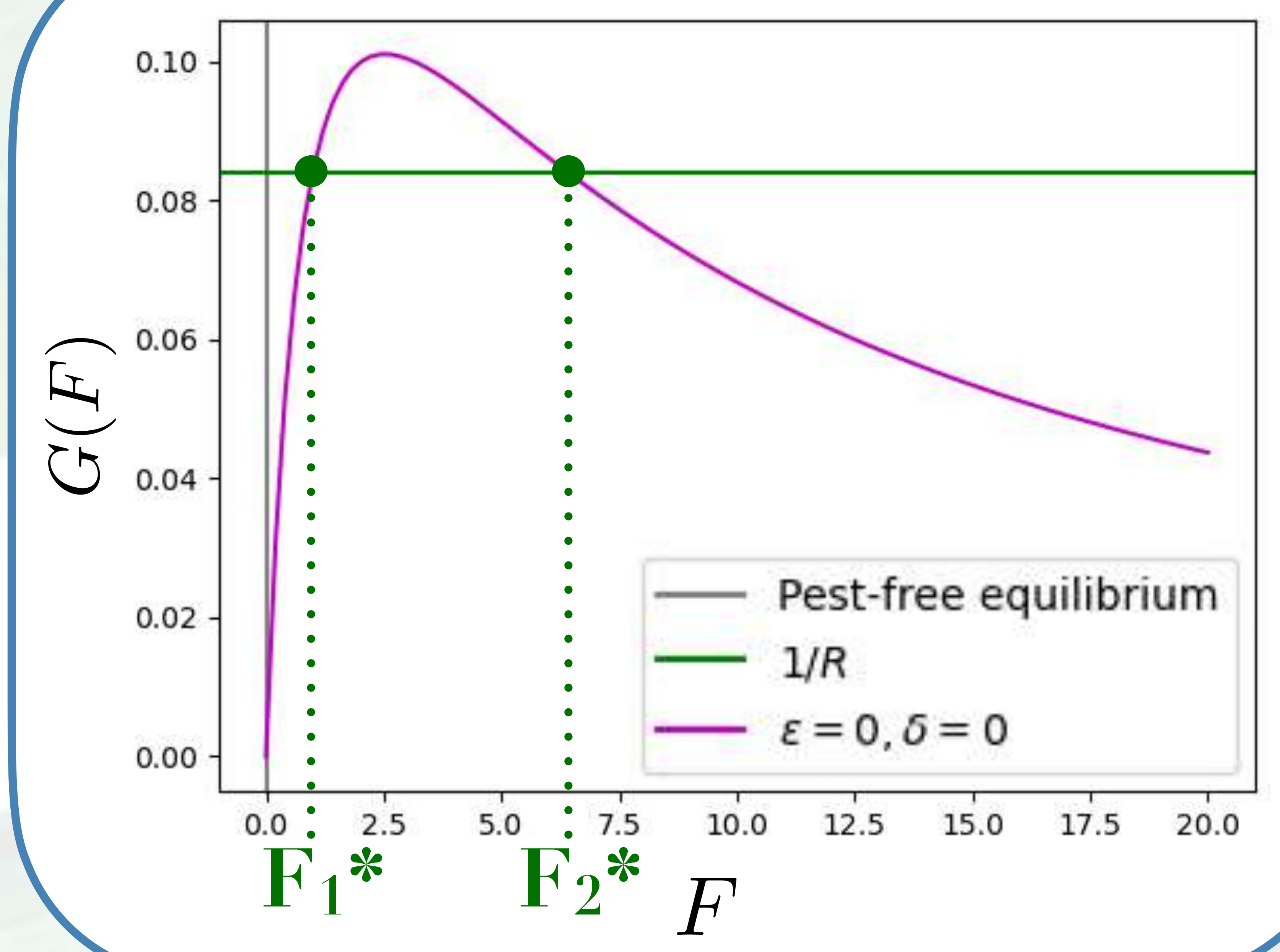

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The equilibria are: (S^*, F_1^*, M_1^*), (S^*, F_2^*, M_2^*).

Infestation equilibria are the F* values solutions of:

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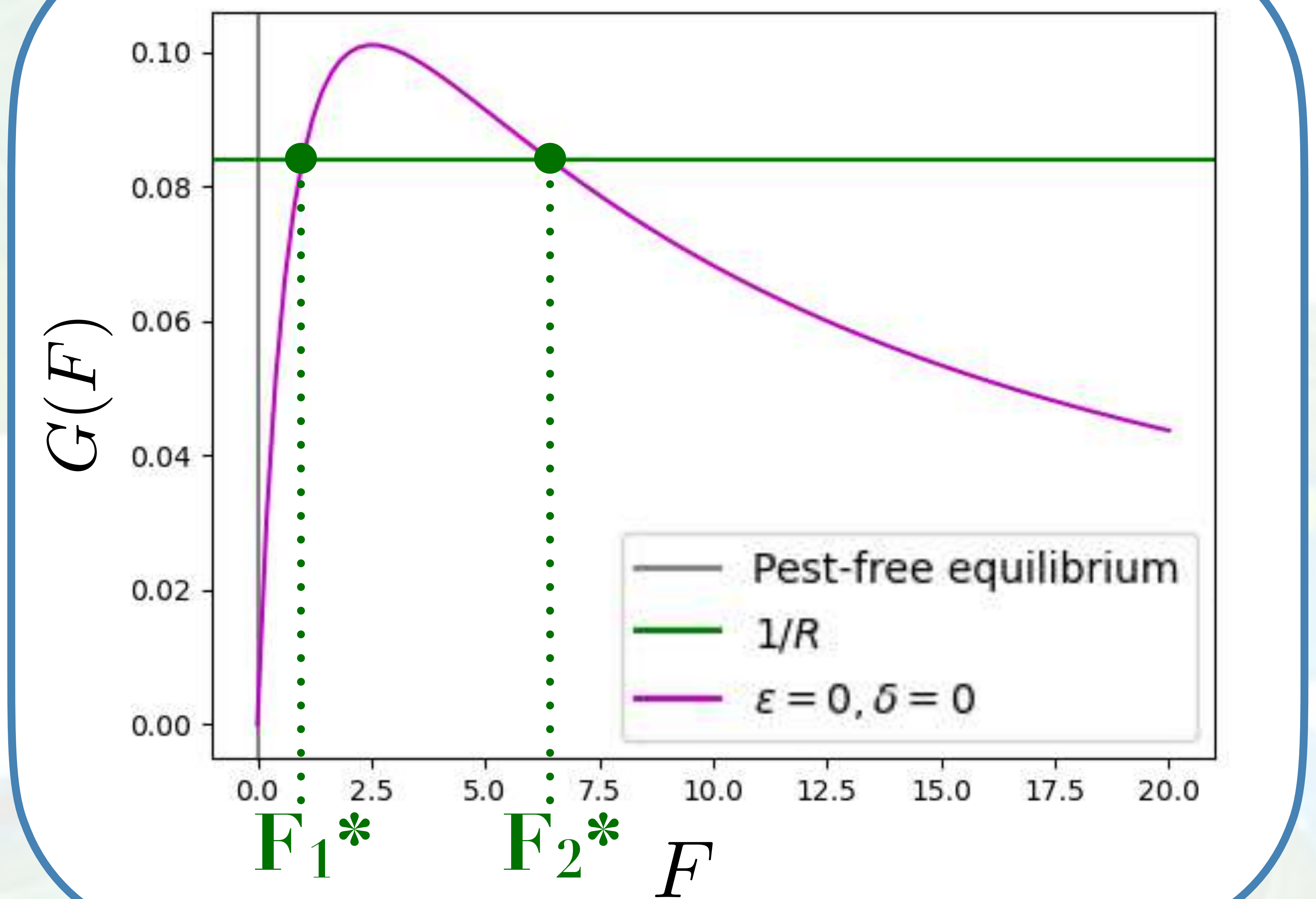
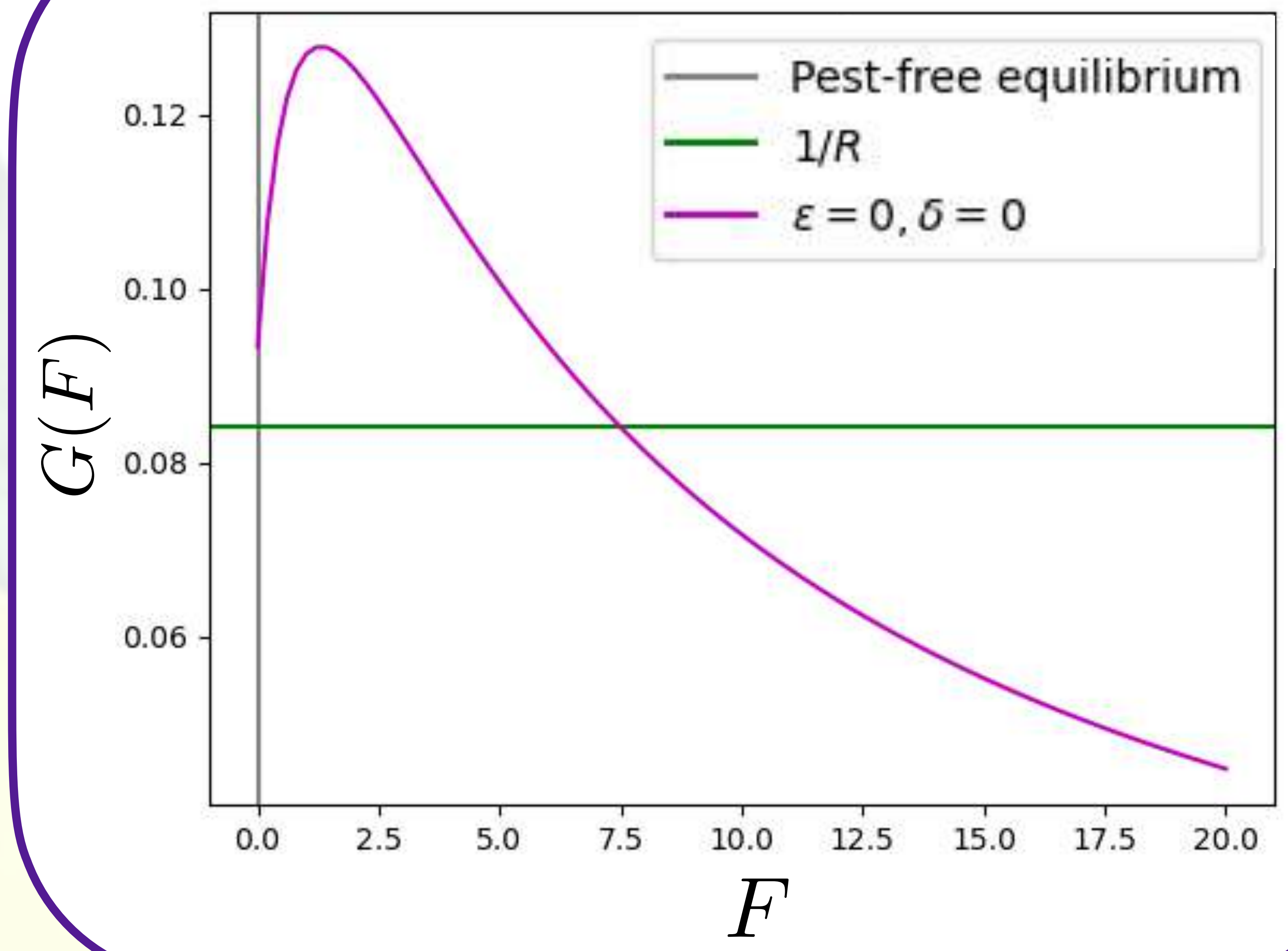


If $G(0) > \frac{1}{\mathcal{R}}$

(2)



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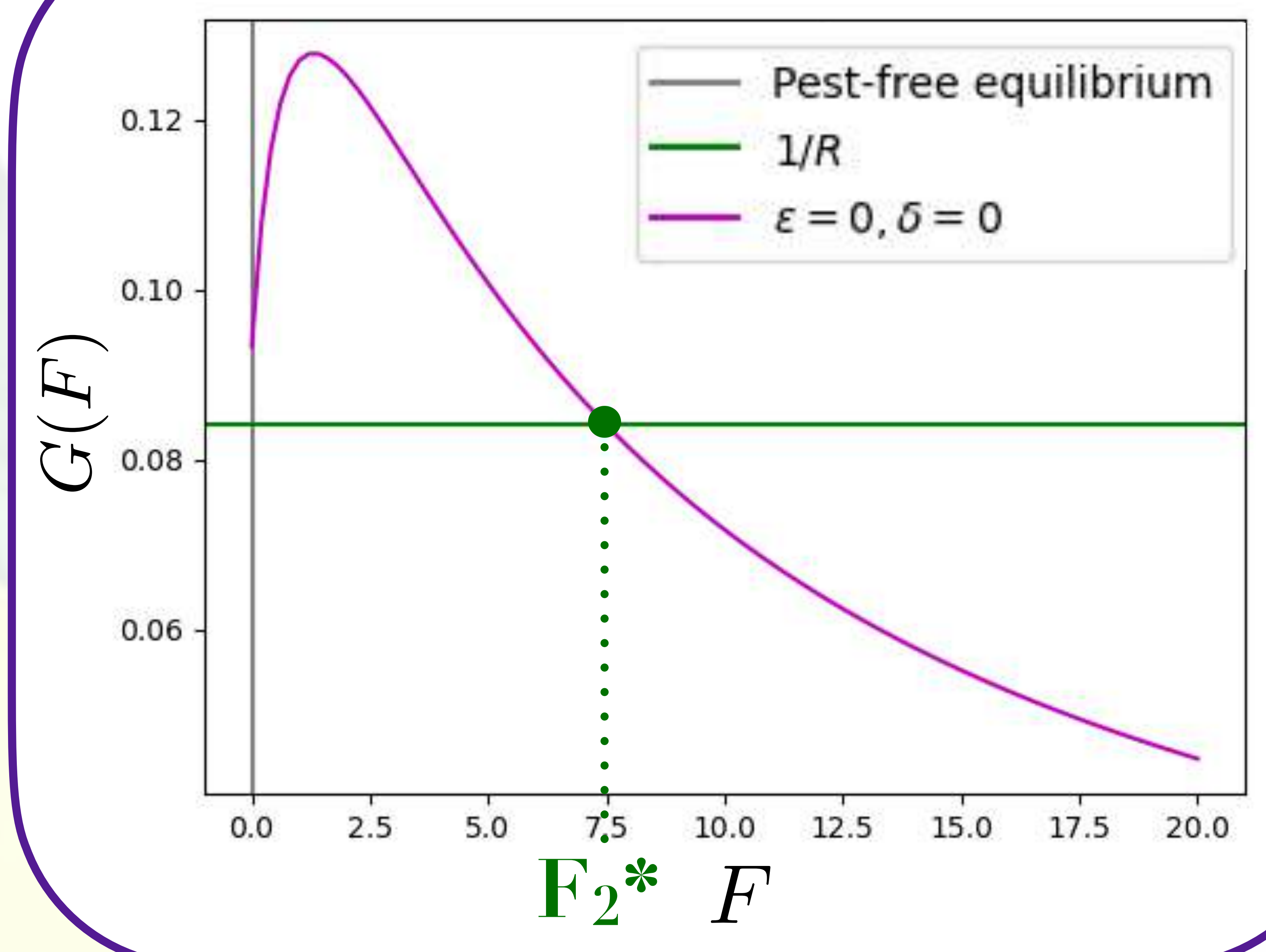
If $G(0) \cdot > \frac{1}{\mathcal{R}}$

At least 1 **solution** $\mathbf{F_2^*}$ such that:

$$\frac{dG}{dF}(\mathbf{F_2^*}) < 0$$

The equilibrium is: $(\mathbf{S^*}, \mathbf{F_2^*}, \mathbf{M_2^*})$.

(2)



Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

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Pest-free equilibrium

$$(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

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➡️ calculation of the trace and determinant

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➡️ calculation of the trace and determinant

$Tr(J) < 0$

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➡️ calculation of the trace and determinant

$Tr(J) < 0$

When $\frac{dG}{dF} > 0$, $Det(J) < 0$ (S*, F_1*, M_1*) is **UNSTABLE**

When $\frac{dG}{dF} < 0$, $Det(J) > 0$ (S*, F_2*, M_2*) is **STABLE**

In summary

If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$		

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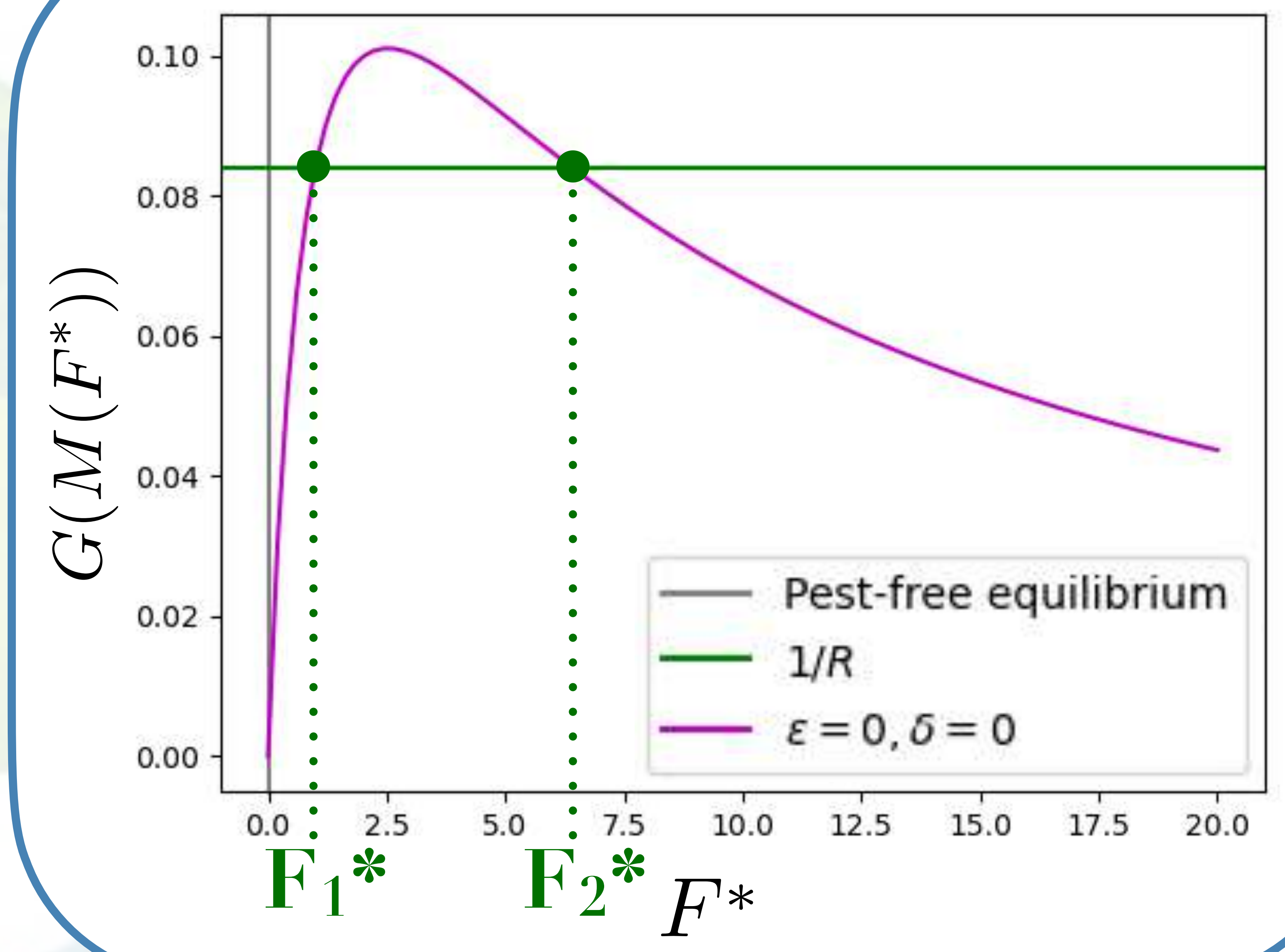
(0) 		

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Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE** Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**		

In summary

If $G(0) < \frac{1}{\mathcal{R}}$ And $max(G(F)) > \frac{1}{\mathcal{R}}$	If $G(0) \cdot > \frac{1}{\mathcal{R}}$	
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Pest-free equilibrium

$(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ STABLE

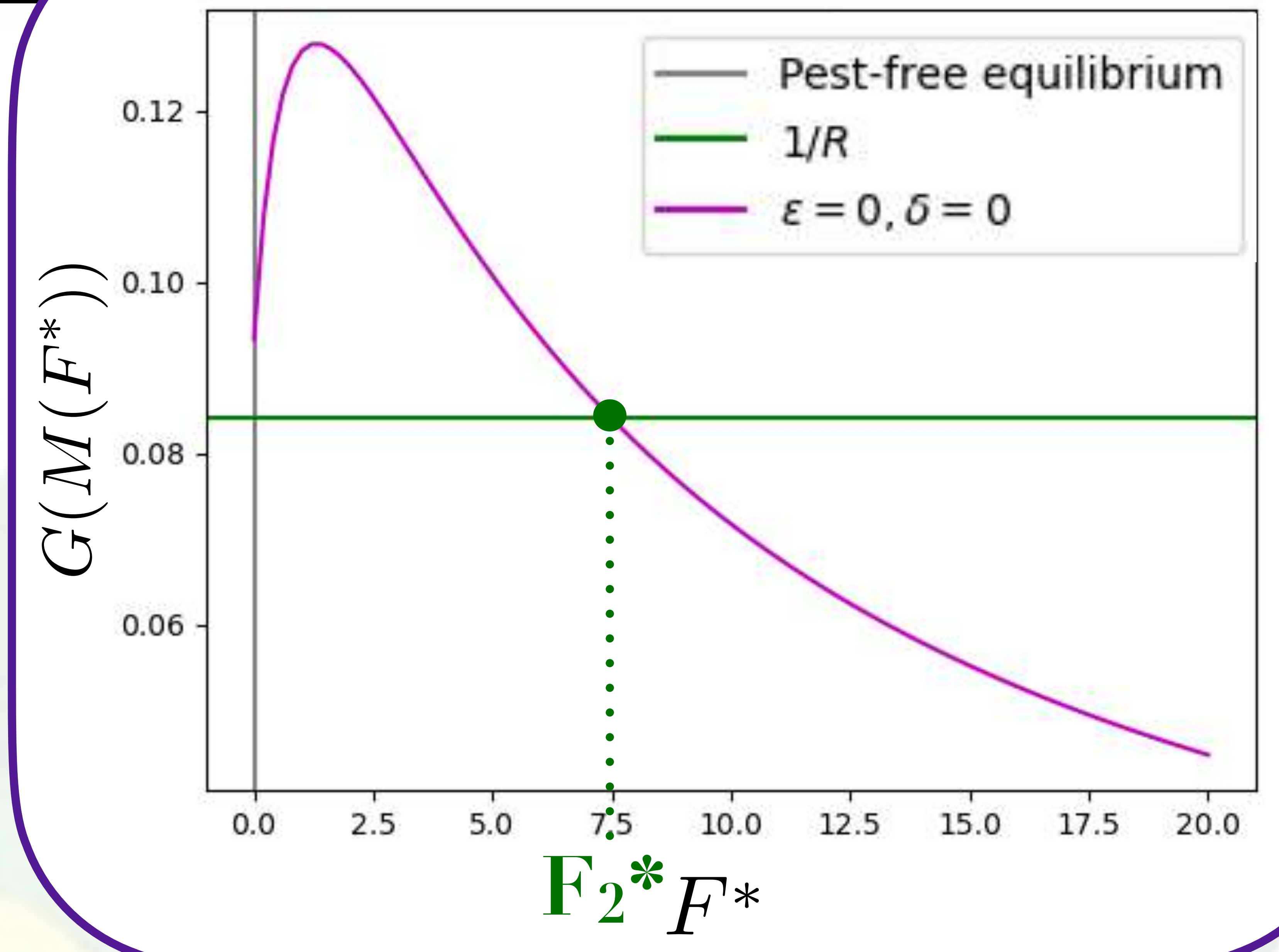
Infestation equilibria

(S^*, F_1^*, M_1^*) UNSTABLE

(S^*, F_2^*, M_2^*) STABLE

If $G(0) \cdot > \frac{1}{\mathcal{R}}$

(2)



In summary

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In summary

If $G(0) < \frac{1}{\mathcal{R}}$ And $max(G(F)) > \frac{1}{\mathcal{R}}$	If $G(0) \cdot > \frac{1}{\mathcal{R}}$	If $G(F) < \frac{1}{\mathcal{R}}$
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Study of the bifurcation diagram in σ

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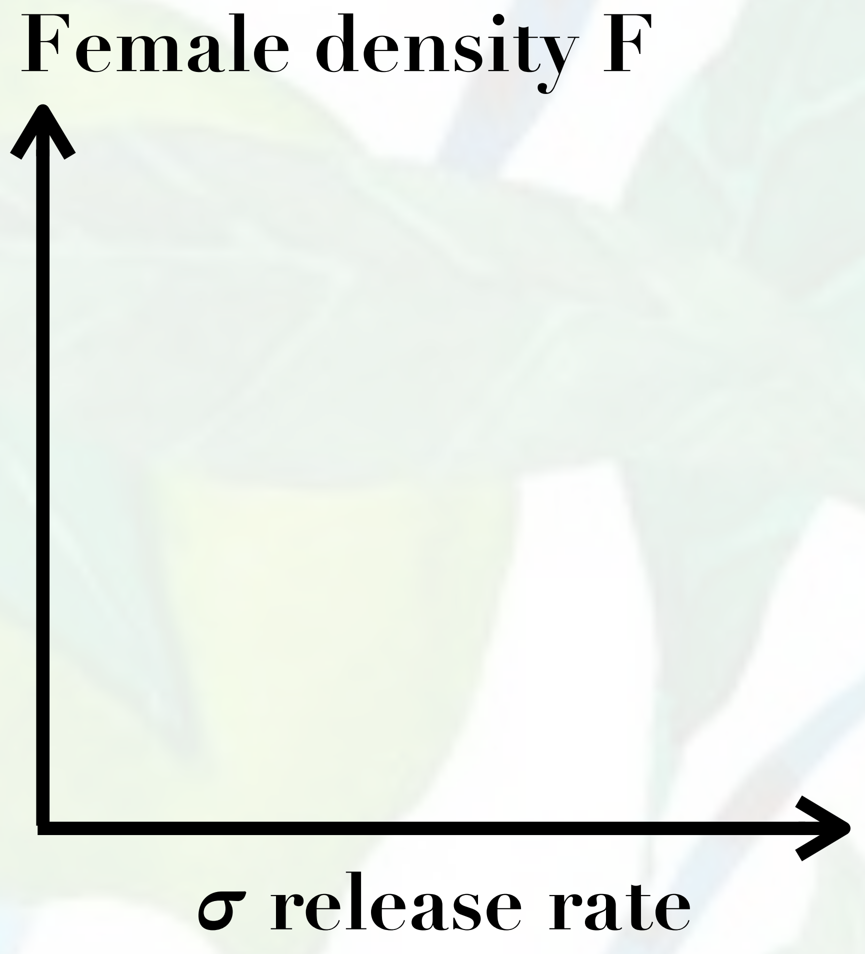


σ bifurcation diagram

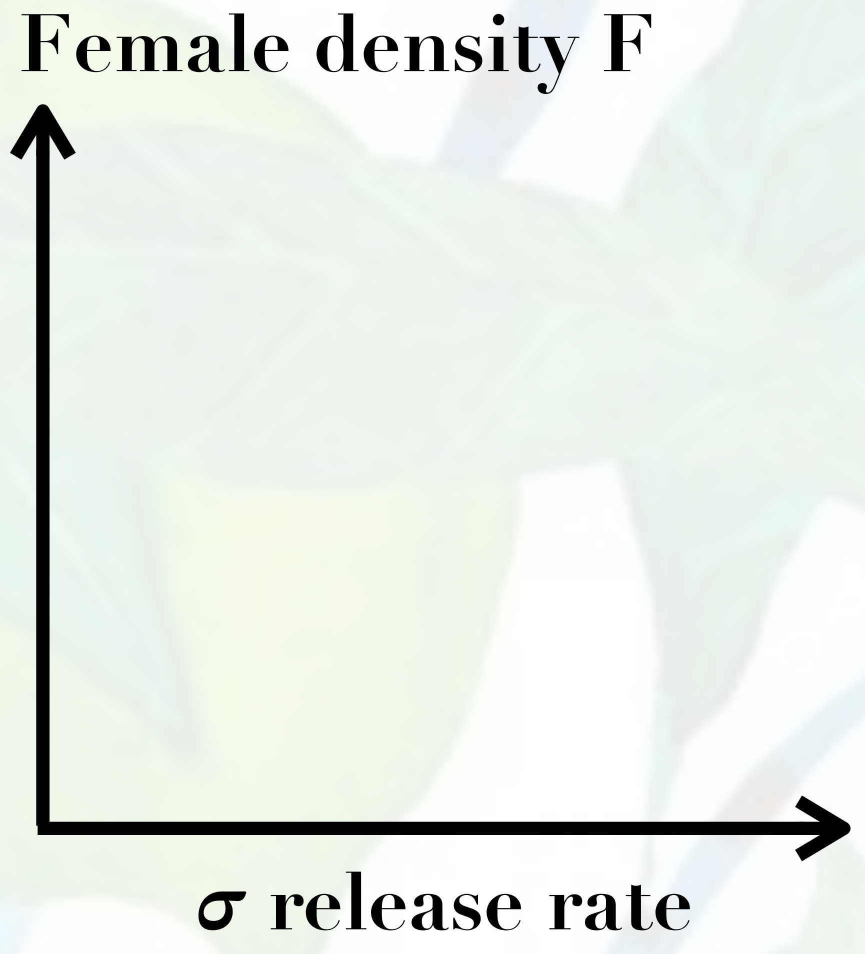
Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

$\downarrow$

σ bifurcation diagram



Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = \quad X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \quad \Longleftrightarrow \quad \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

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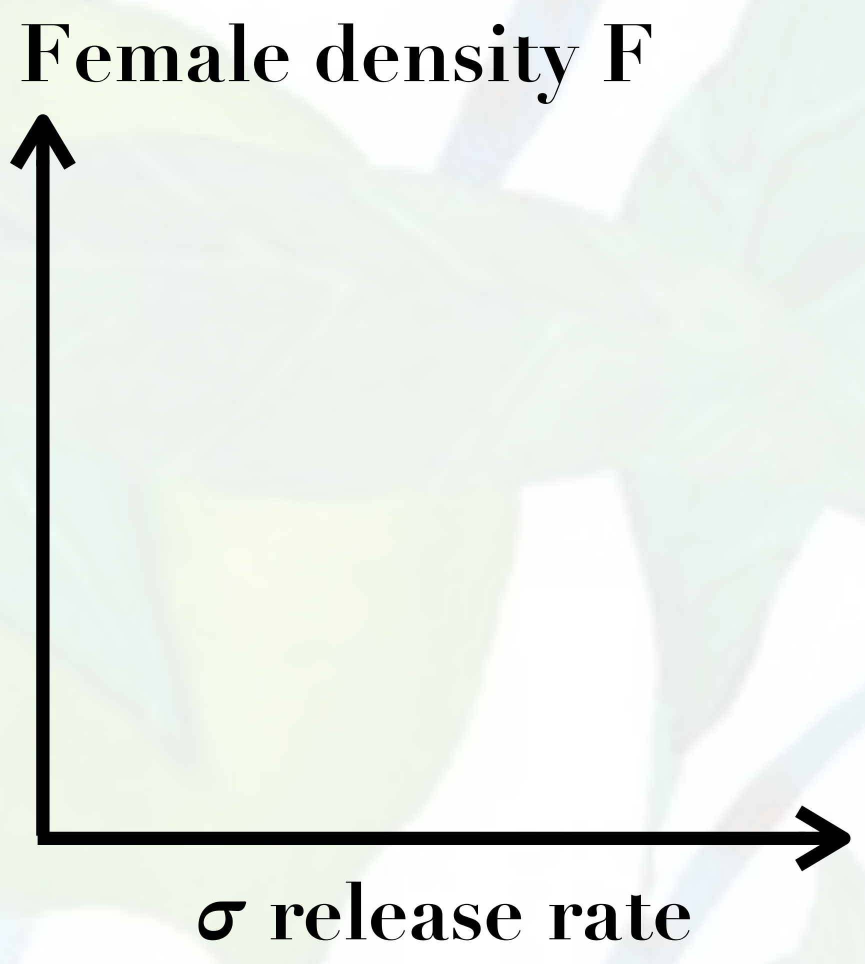

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$\text{G(F)} = \quad X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

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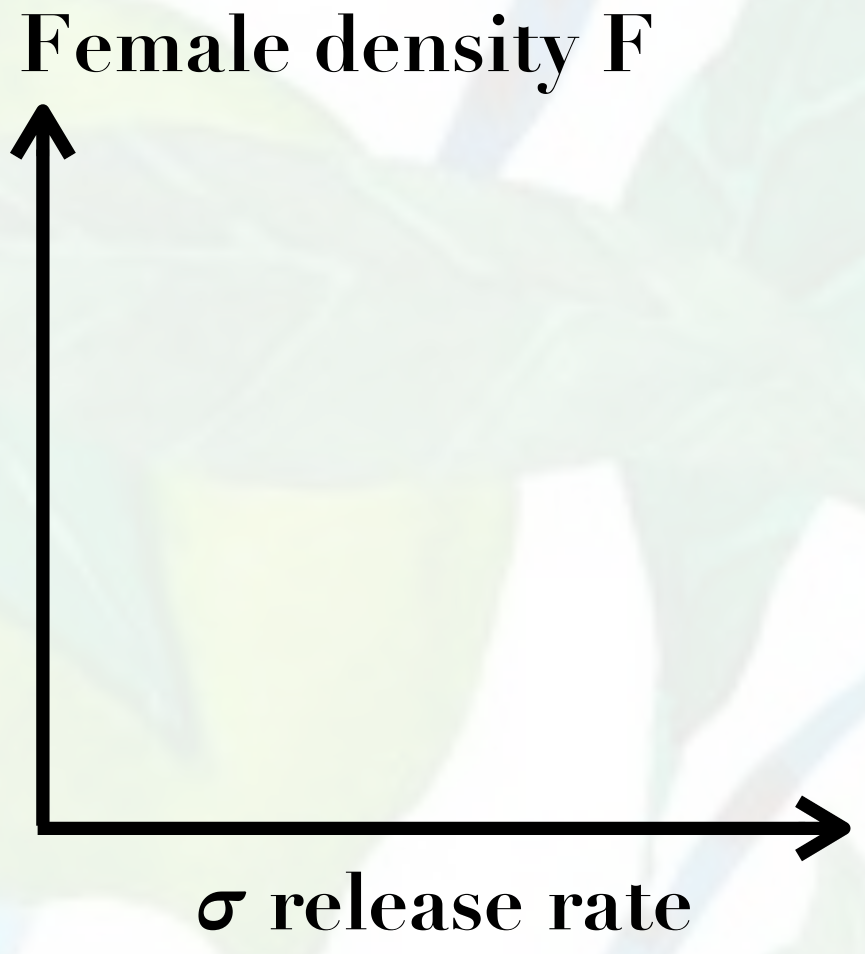

Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

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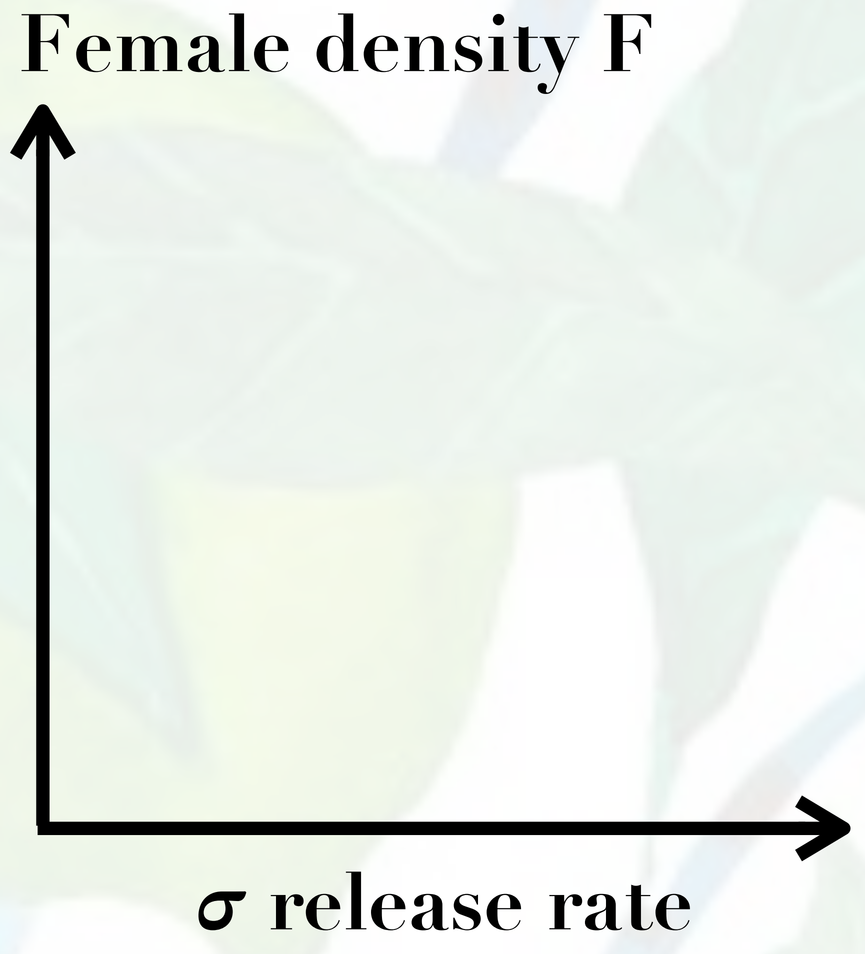

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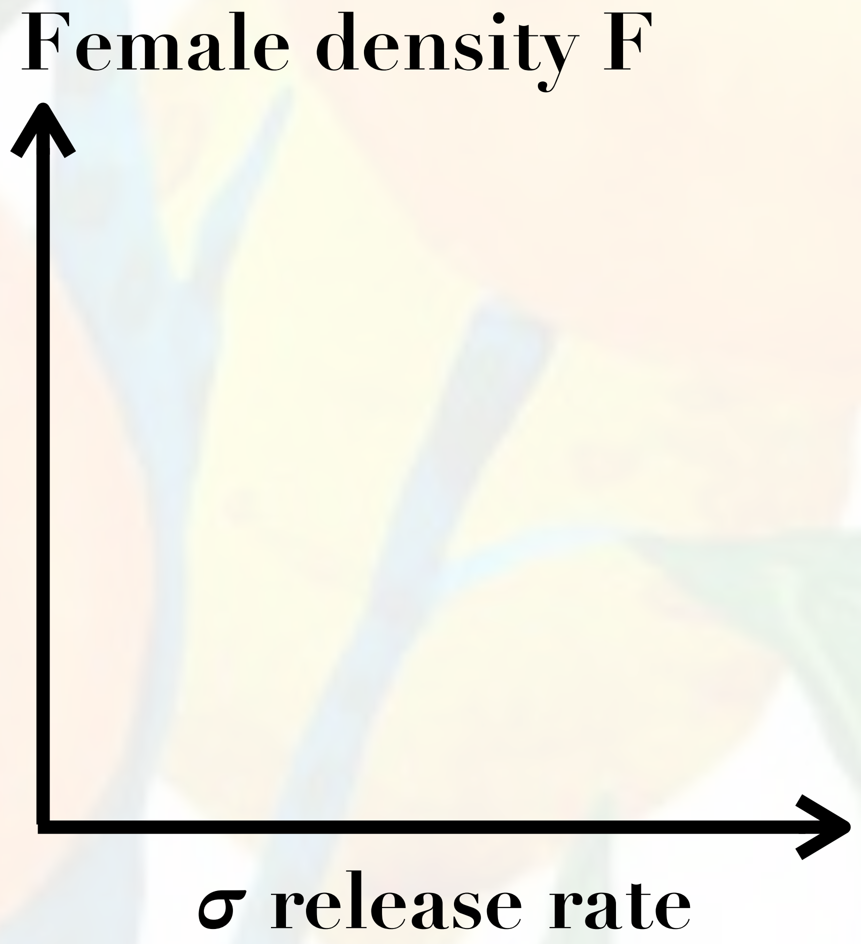
Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

Existence of an **asymptote** when the denominator cancels

Polynomial of degree 2 not depending
on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{\boxed{(1+\beta F)\left(k + \frac{p\mu_F}{(1-p)\mu_M}F\right) - \frac{Rp\mu_F}{(1-p)\mu_M}F}}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)\left(\eta\frac{(1-\delta)}{\mu_S}\right)}$$



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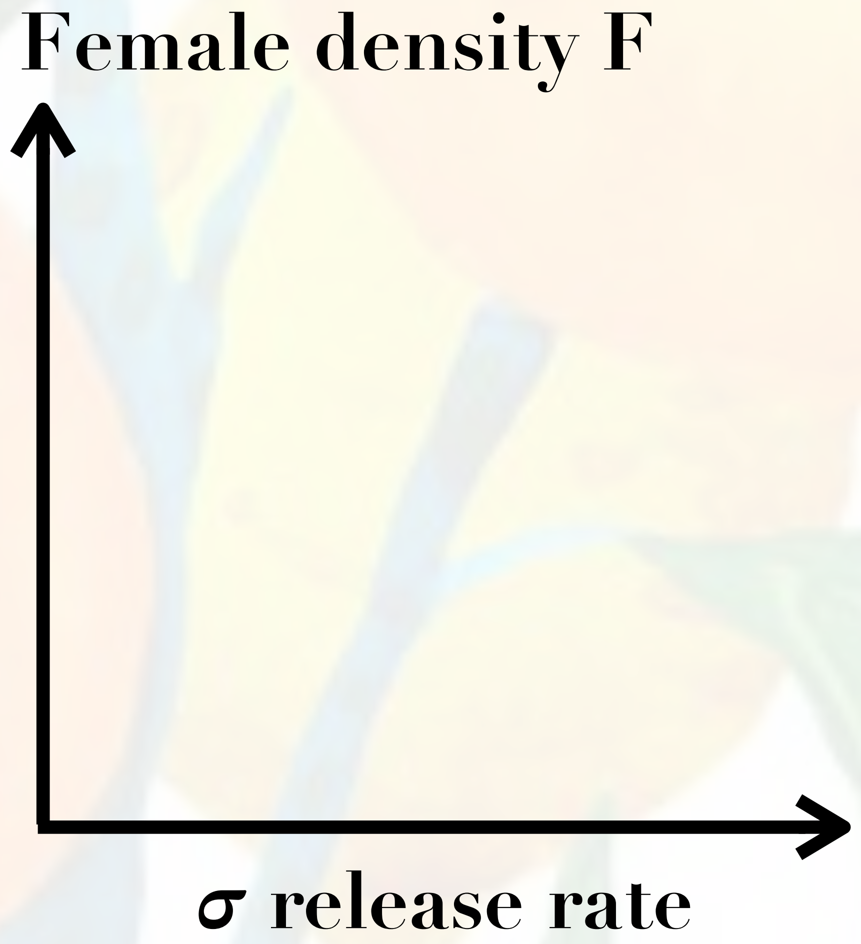
$$\iff \frac{\beta p\mu_F}{(1-p)\mu_M}F^2 + (\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k)F + k = 0$$



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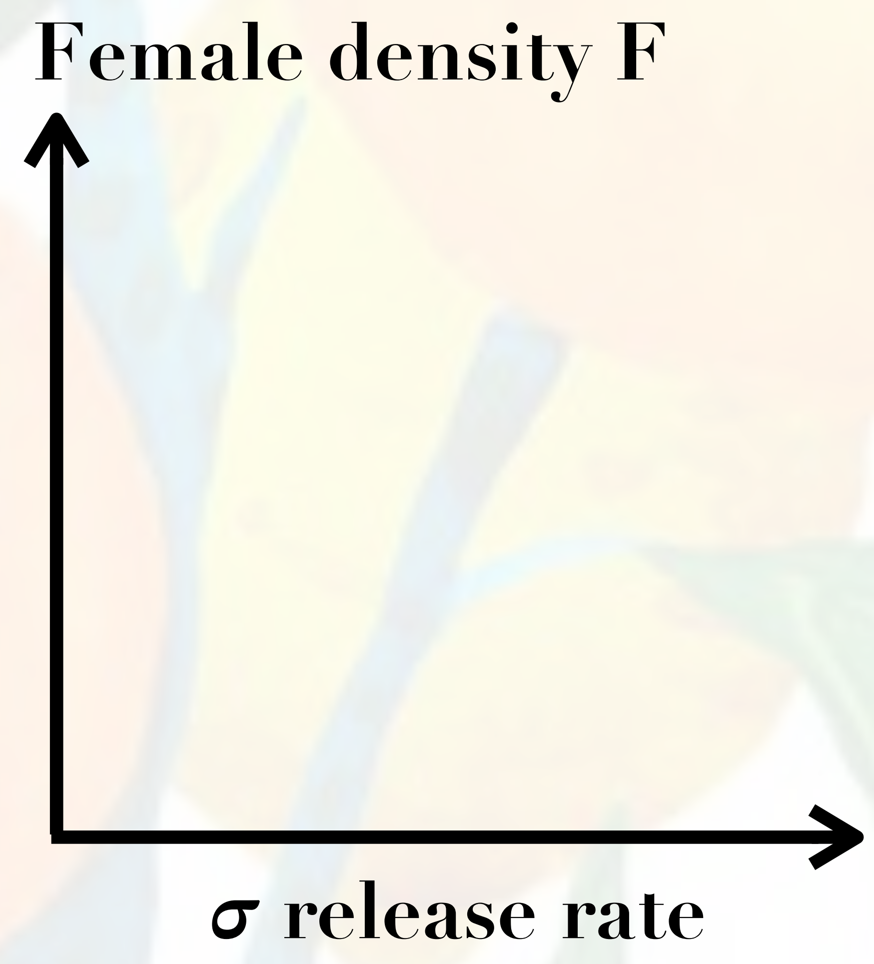

Sum of the roots >0 and therefore the 2 roots **Root₁** and **Root₂** are >0 if: $\frac{p\mu_F(R-1)}{\beta k(1-p)\mu_M} > 1$

Polynomial of degree 2 not depending
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$$\sigma = \frac{\boxed{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

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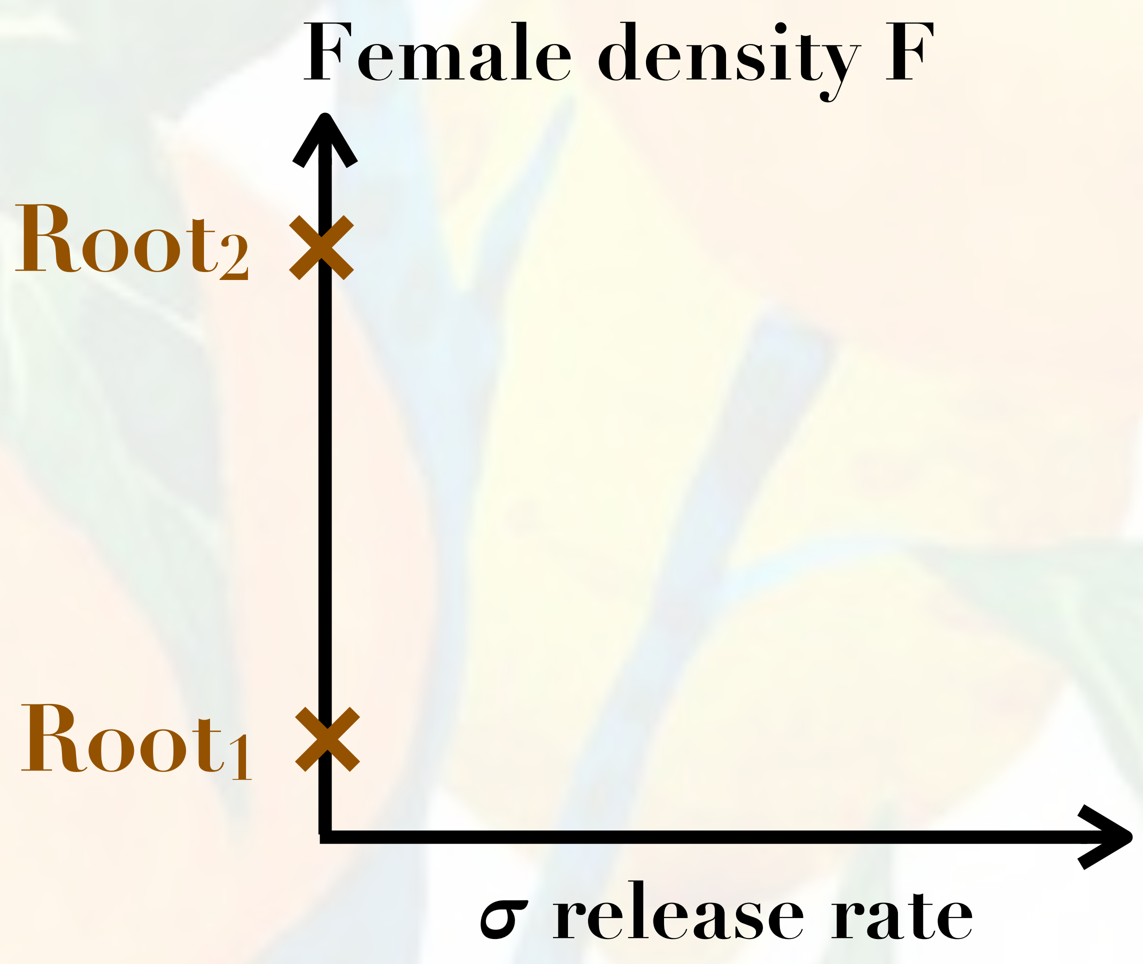

Sum of the roots > 0 and therefore the 2 roots **Root$_1$** and **Root$_2$** are > 0 if: $\frac{p\mu_F(R-1)}{\beta k(1-p)\mu_M} > 1$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\boxed{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}}$$

Existence of an **asymptote** when the denominator cancels

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

General case

$\boldsymbol{\delta \neq 0, \epsilon \neq 0}$

The denominator cancels for:

$$F = \frac{1}{\beta}\left(\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} - 1\right)$$

But F > 0 so existence when:

$$\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} > 1$$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\boxed{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}}$$

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(1)

Cost-free fertility model

$\delta \neq 0, \epsilon = 0$

The denominator cancels for:

$$F = \frac{1}{\beta}\left(\frac{R\delta\mu_S}{\delta\mu_S + \eta(1-\delta)\mu_M} - 1\right)$$

But F > 0 so existence when:

$$\delta > \frac{\eta\mu_M}{R\mu_S - \mu_S + \eta\mu_M}$$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\boxed{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}}$$

Existence of an **asymptote** when the denominator cancels

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(1)

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(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

The denominator cancels for:

$$F = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

The denominator cancels for:

$$\mathbf{F}_{\mathbf{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$

(2)
Costly fertility model
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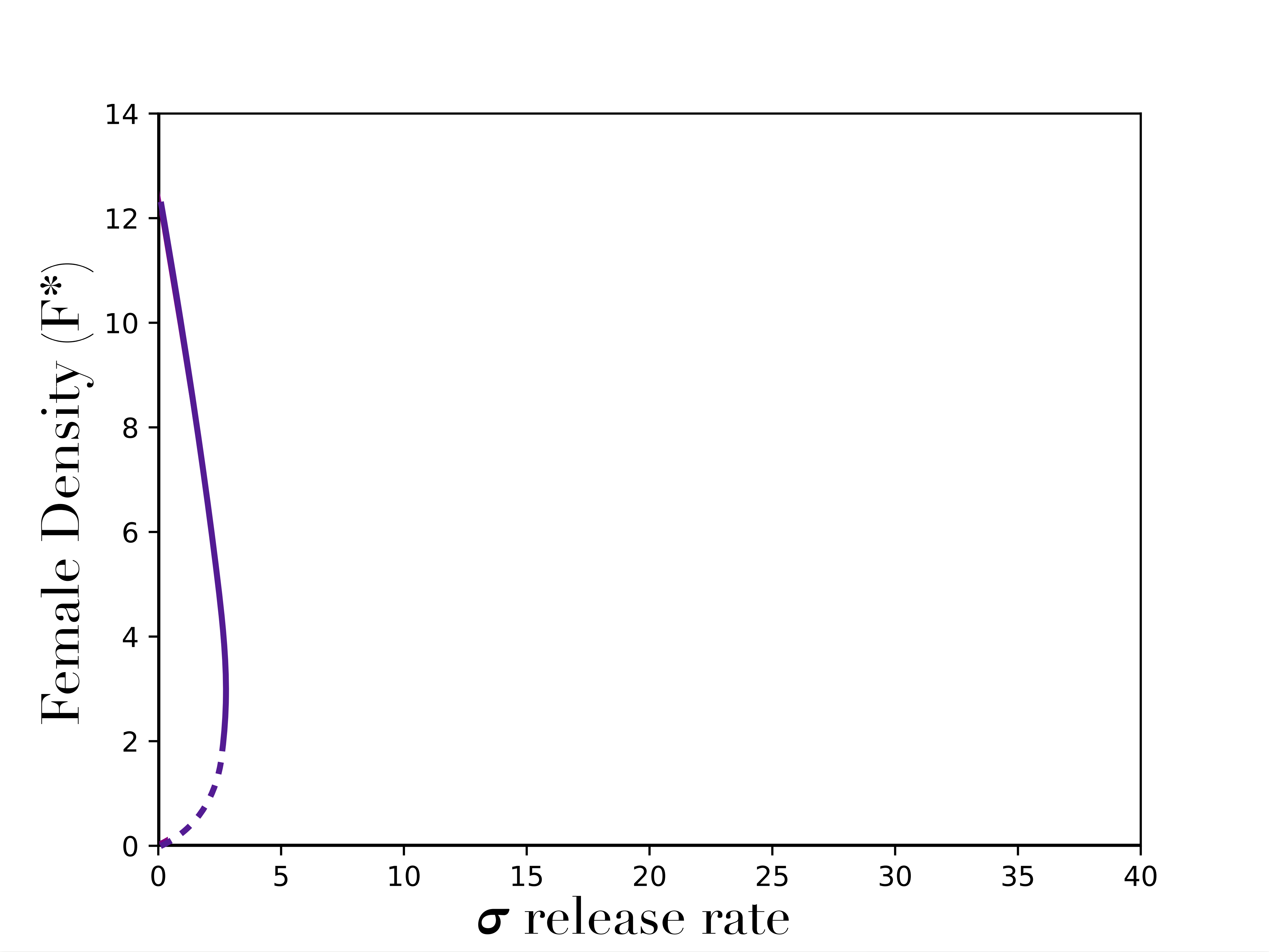

— Stable
- - Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

The denominator cancels for:

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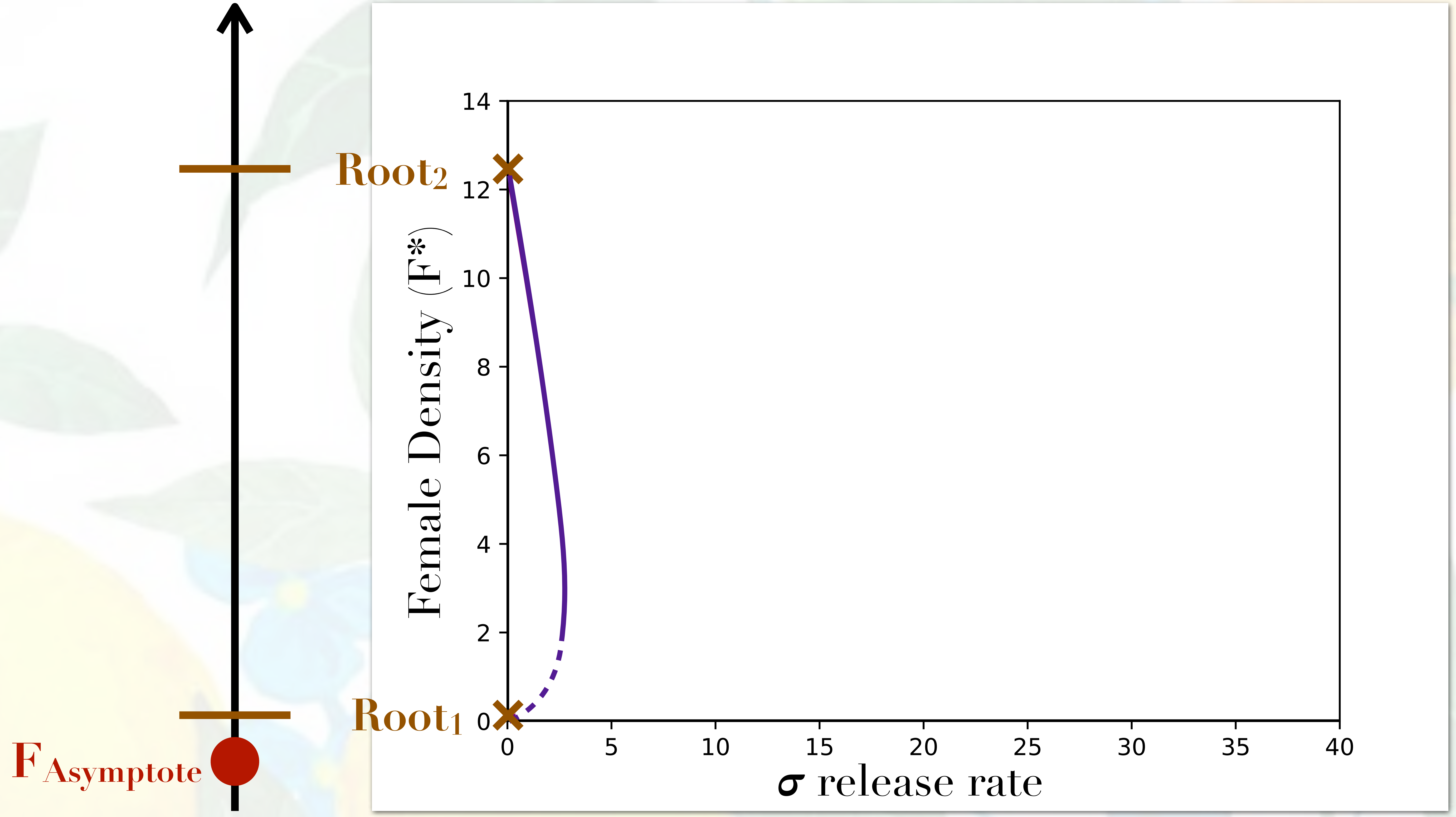

— Stable
-- Unstable

(2)

Costly fertility model

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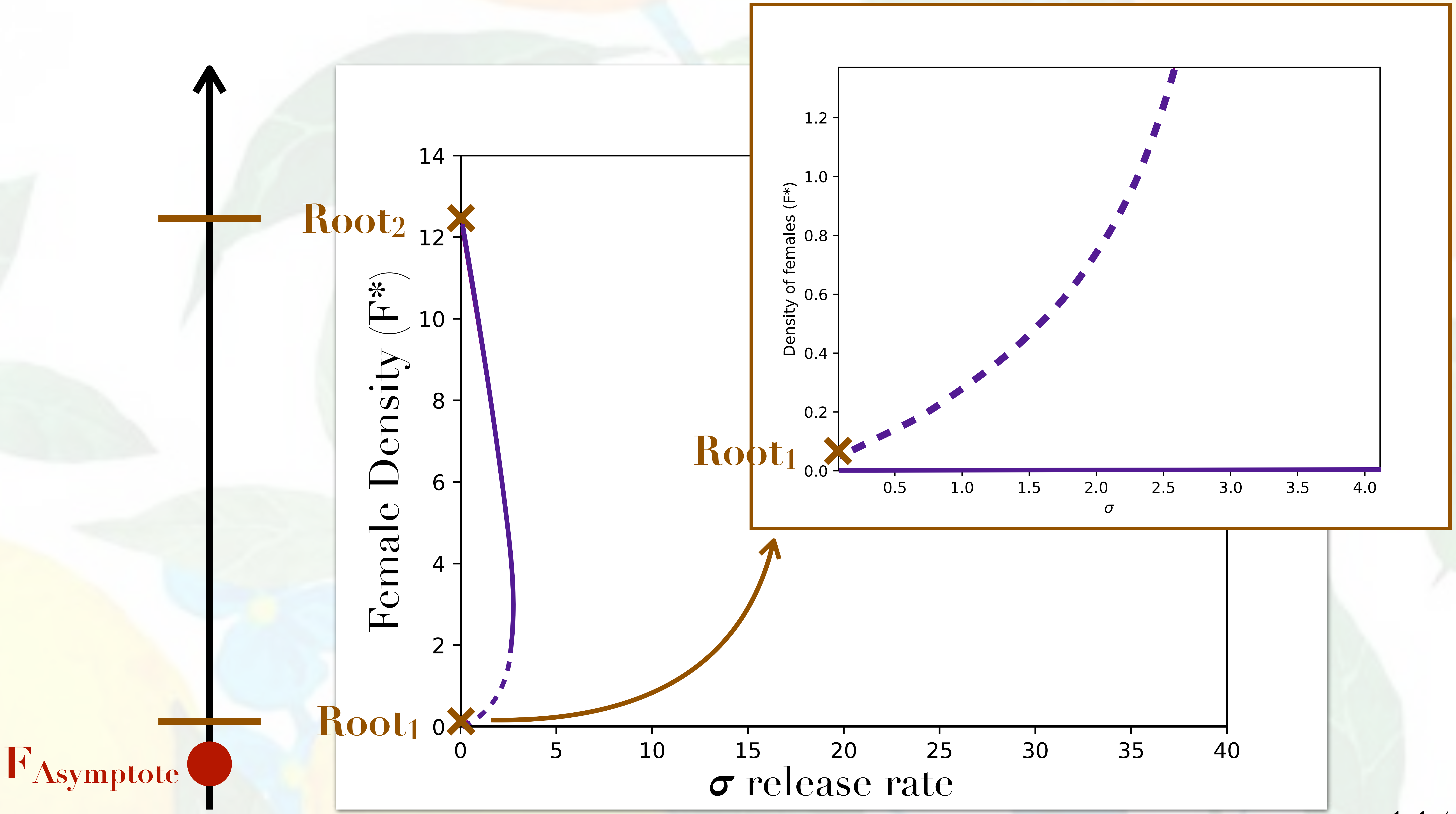

— Stable
- - Unstable

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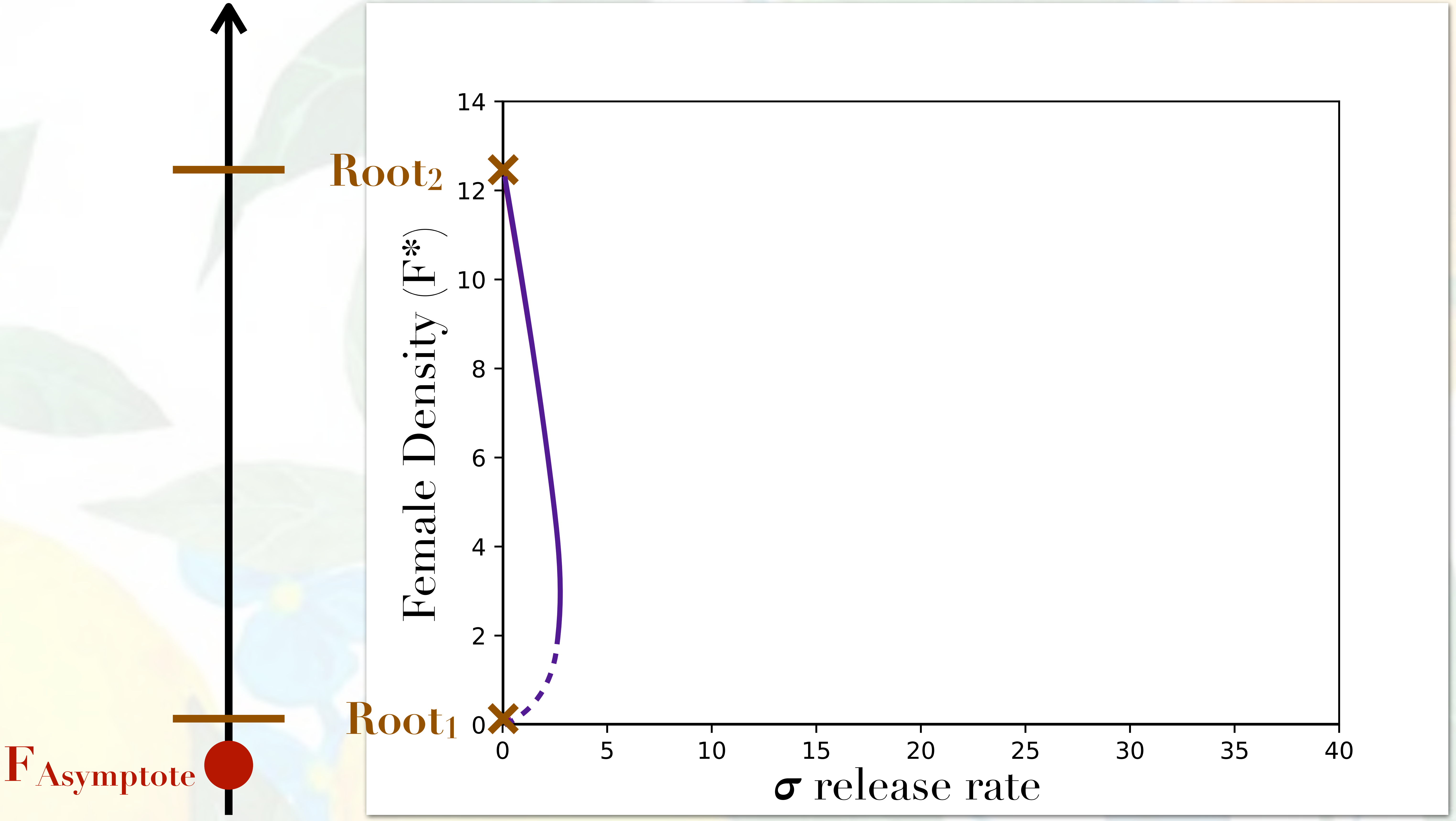

— Stable
- - Unstable

0.010

0.084

Residual Fertility rate ϵ

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

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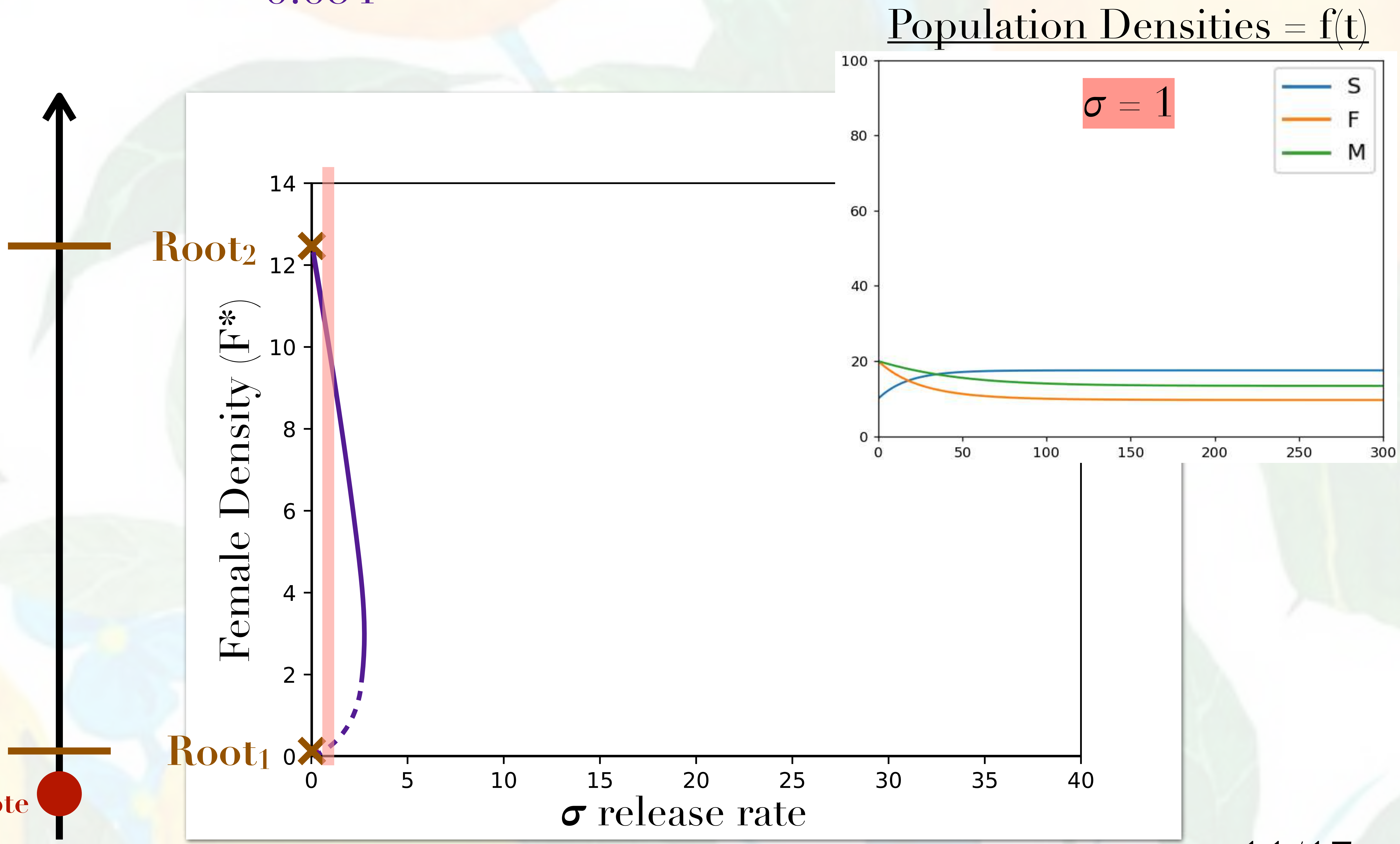
— Stable

- - Unstable



(2)

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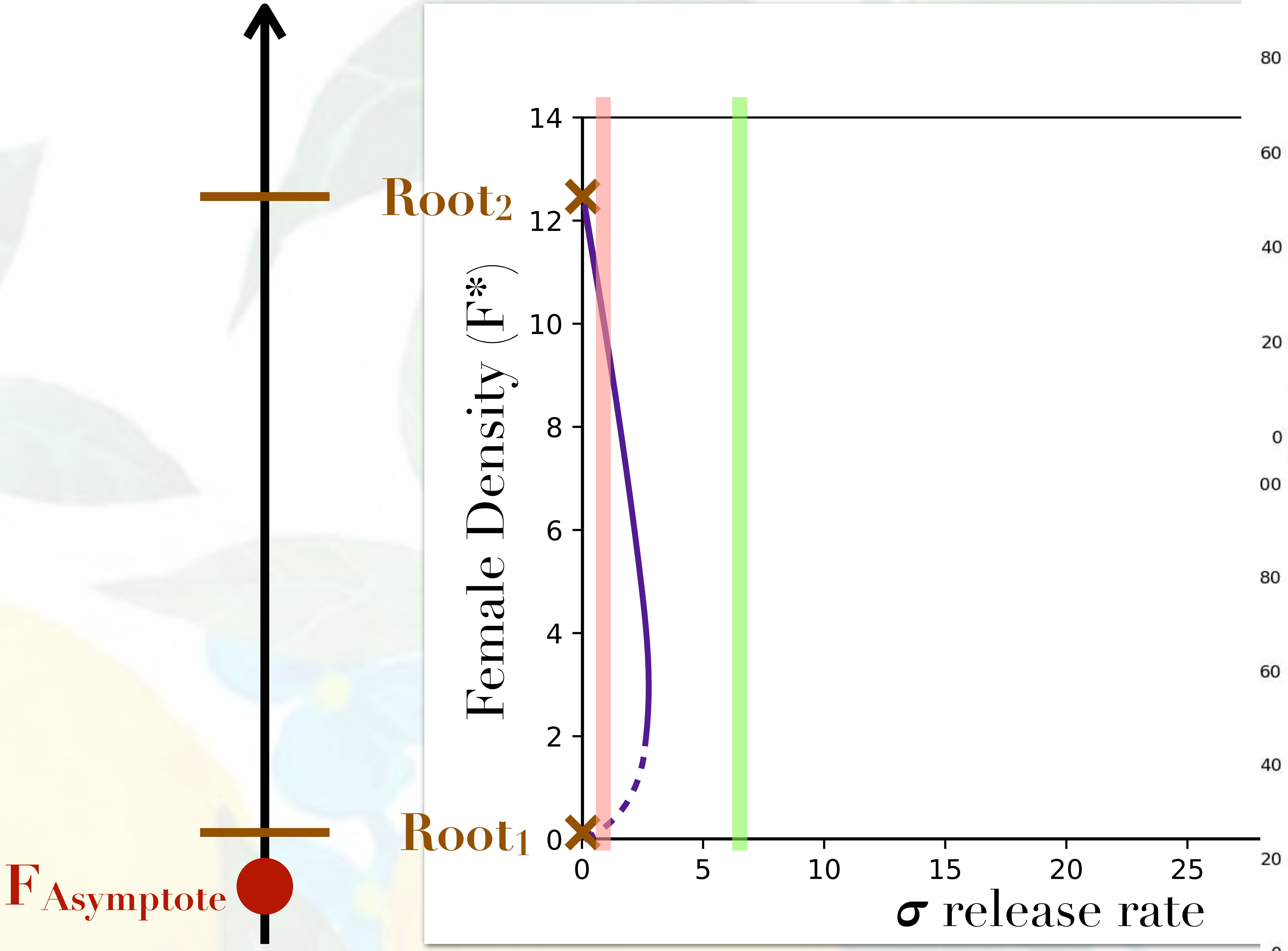
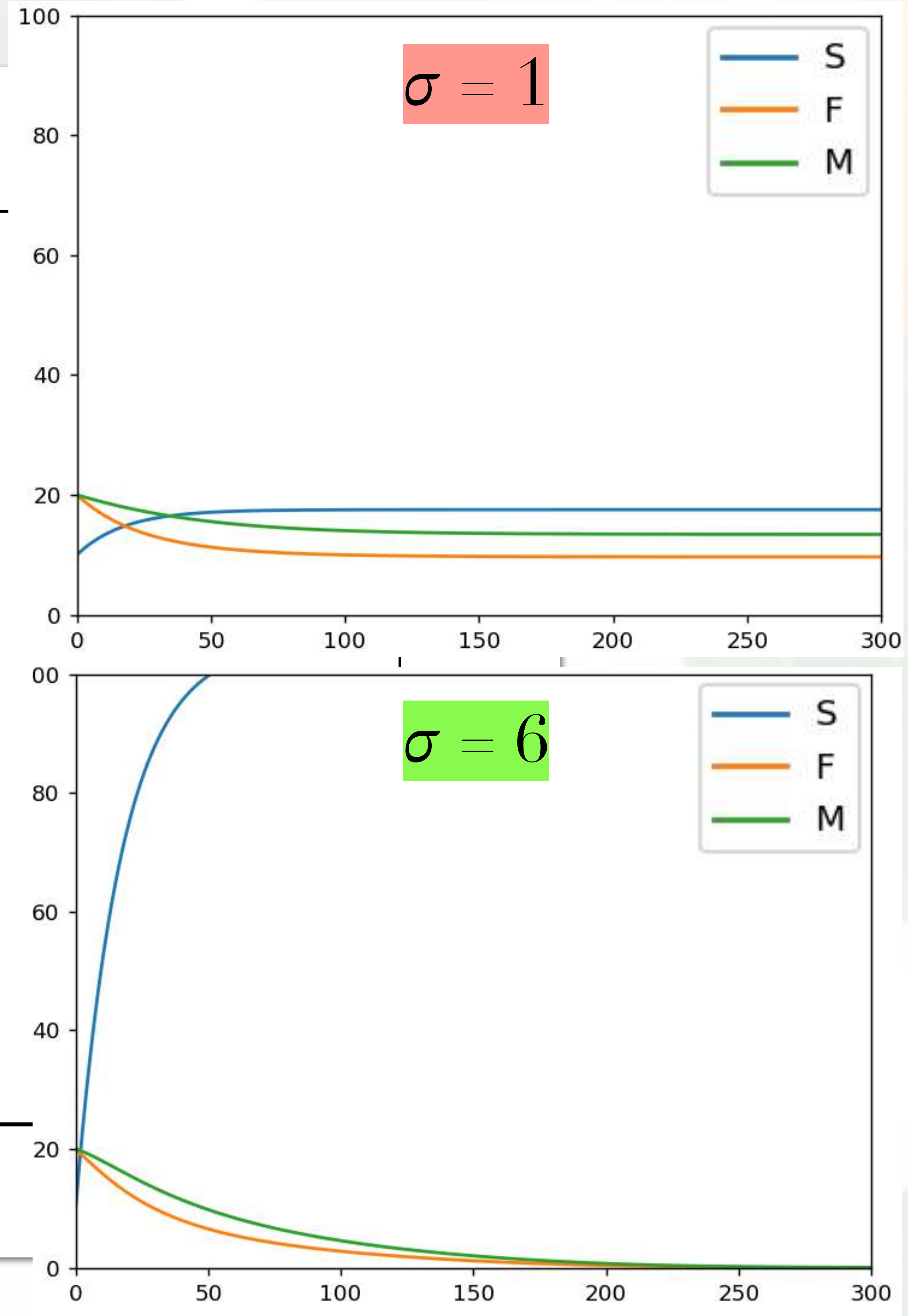
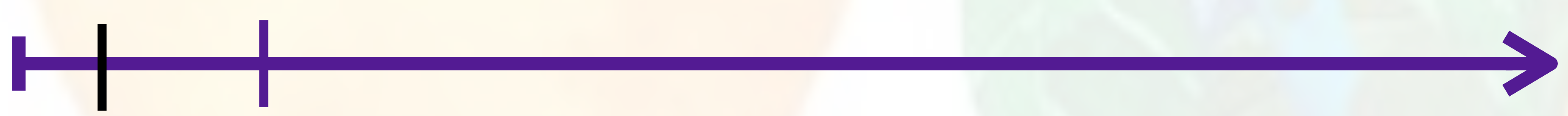


Population Densities = f(t)





(2)
Costly fertility model
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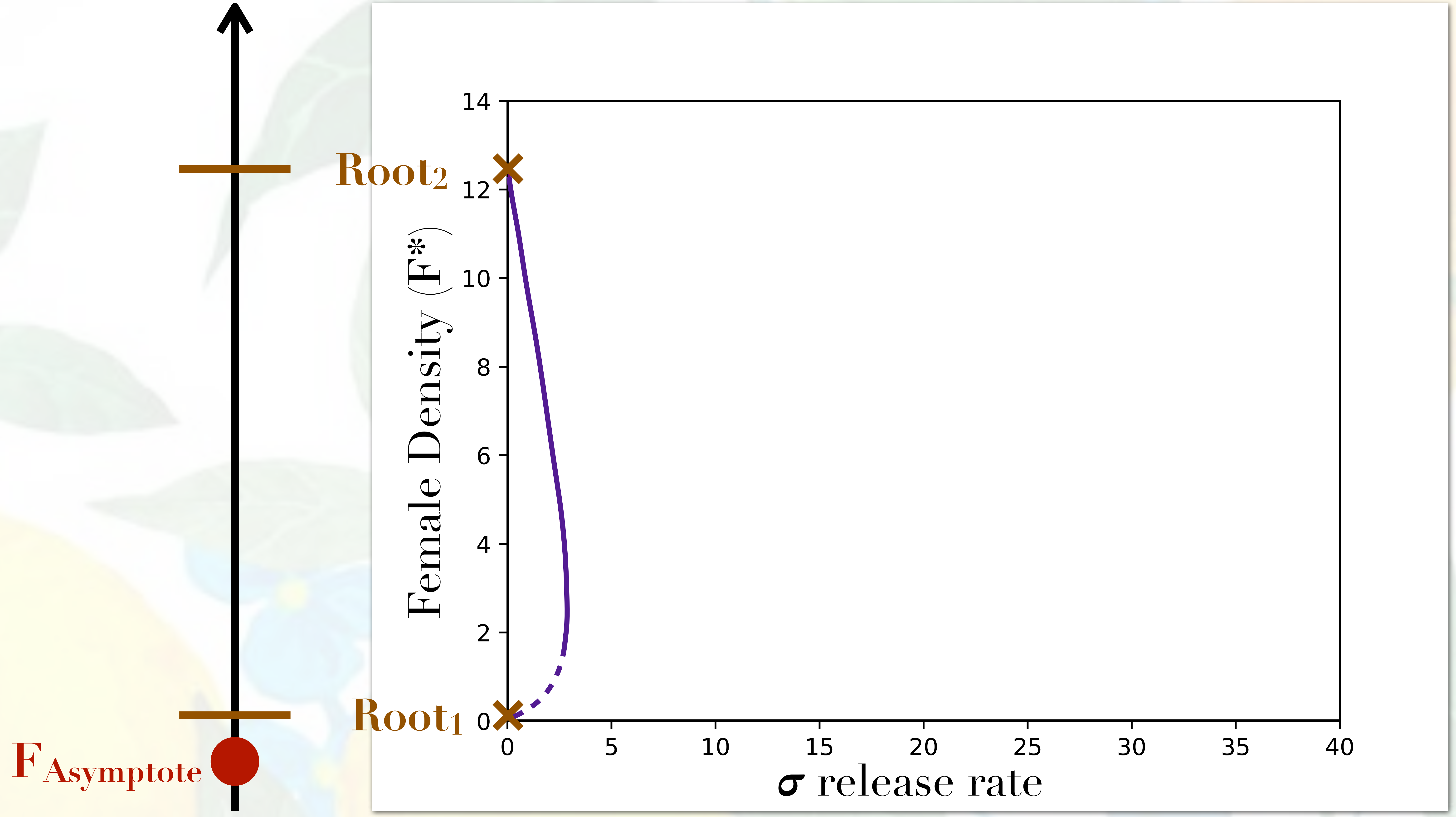

— Stable
- - Unstable

(2)
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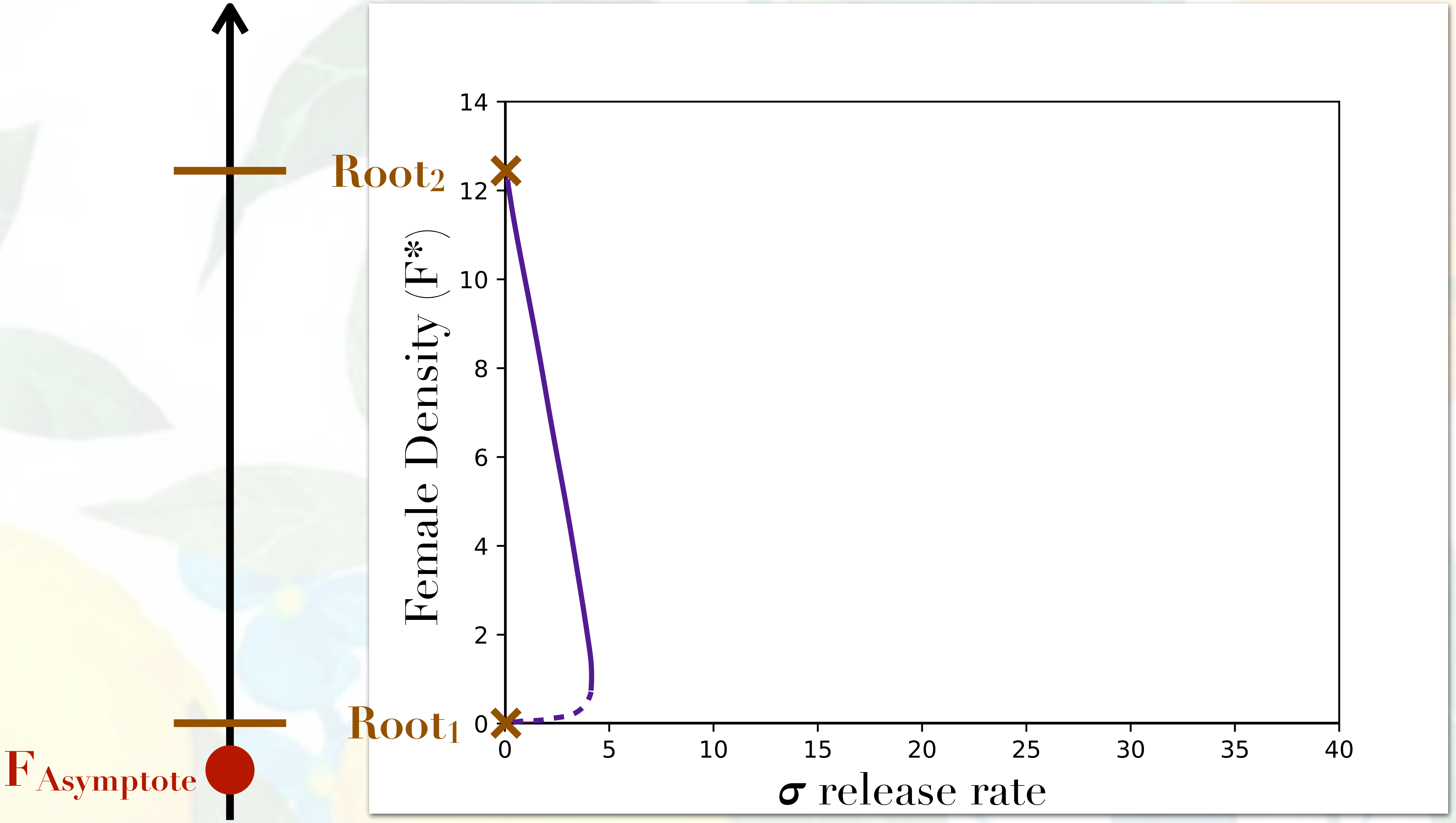

— Stable
- - Unstable

0.084

0.084

Residual Fertility rate ϵ

(2) Costly fertility model

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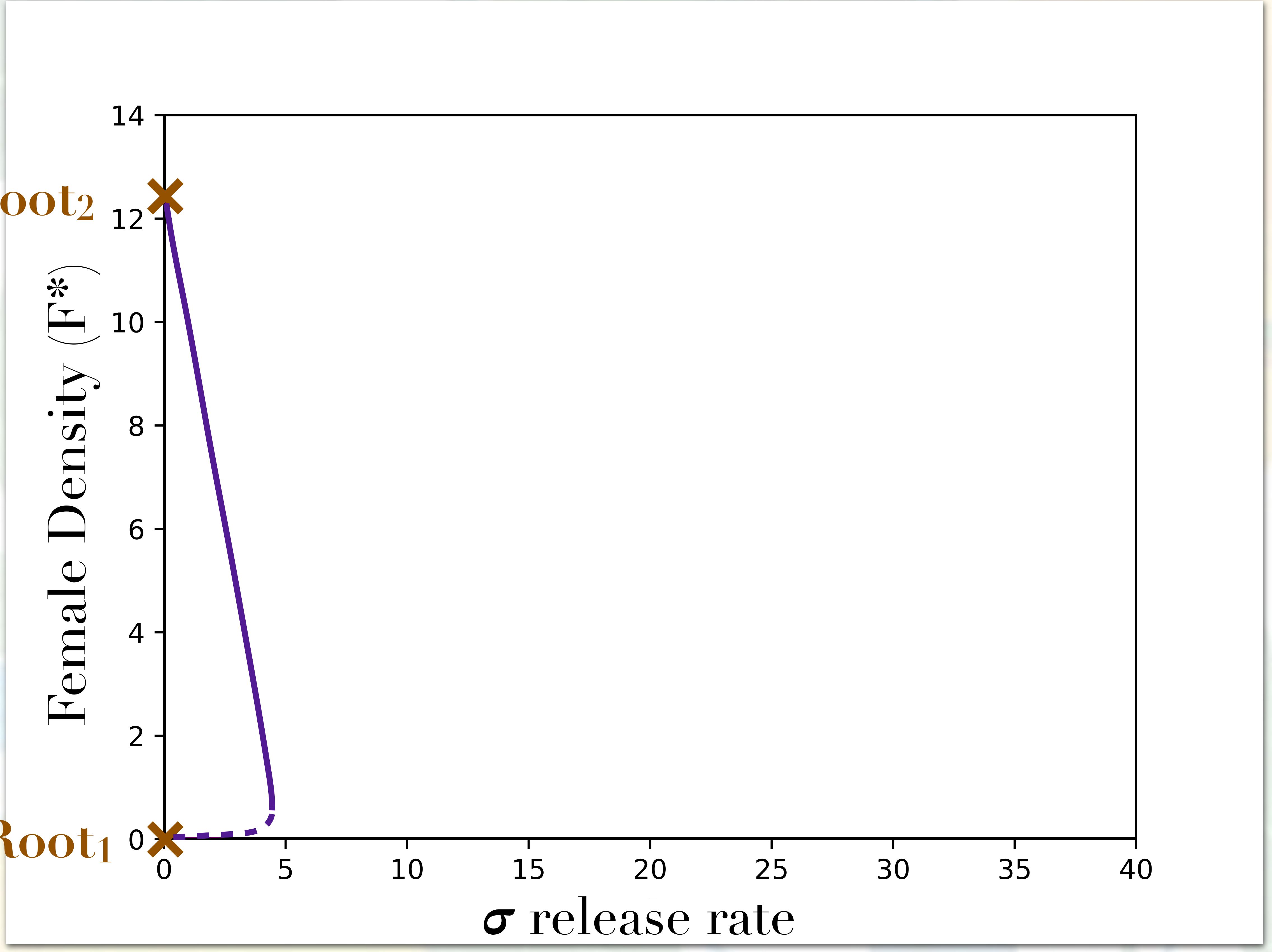
— Stable

- - Unstable

Root$_2$

Root$_1$

$F_{\text{Asymptote}}$



(2)
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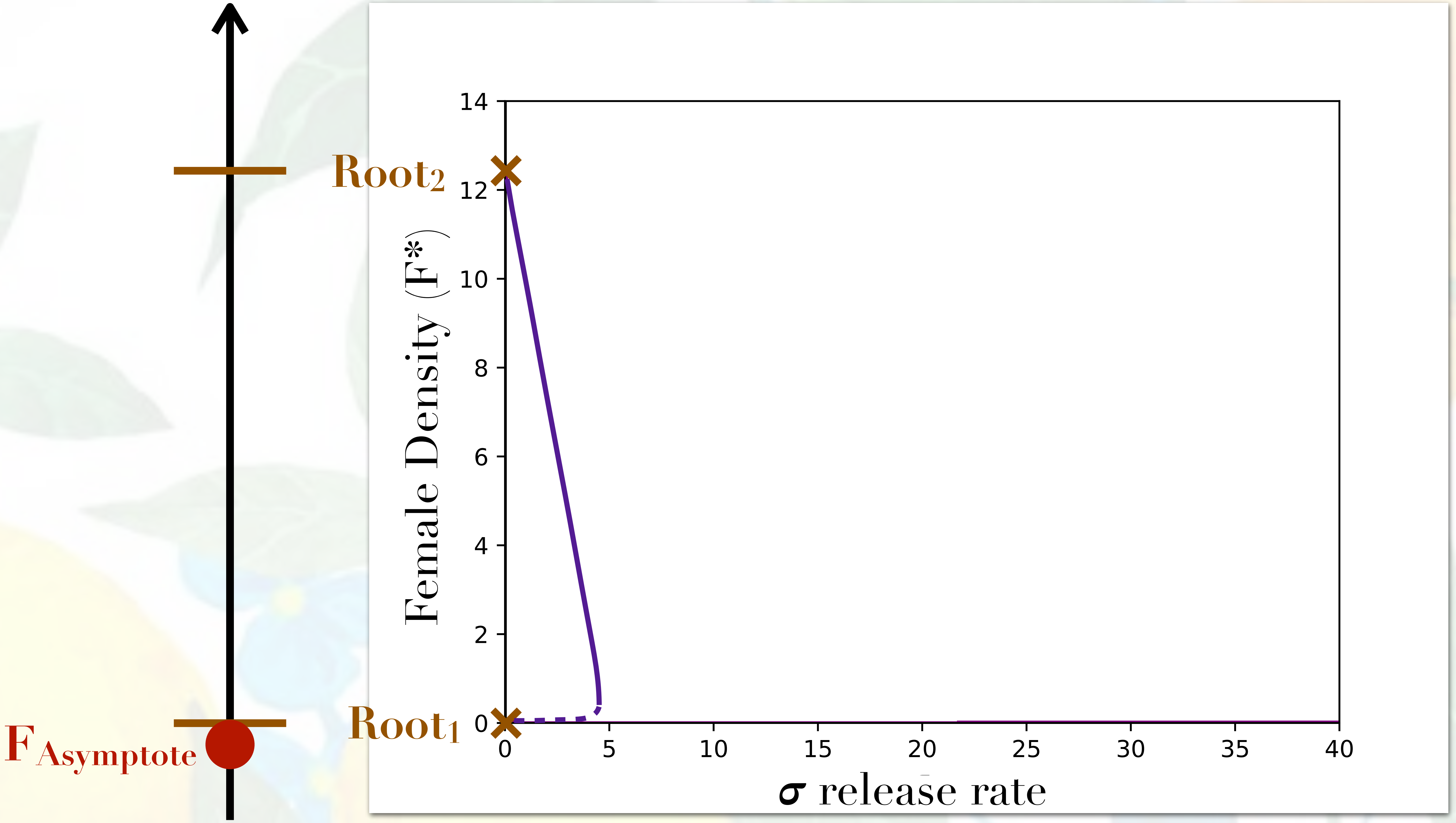

— Stable
- - Unstable

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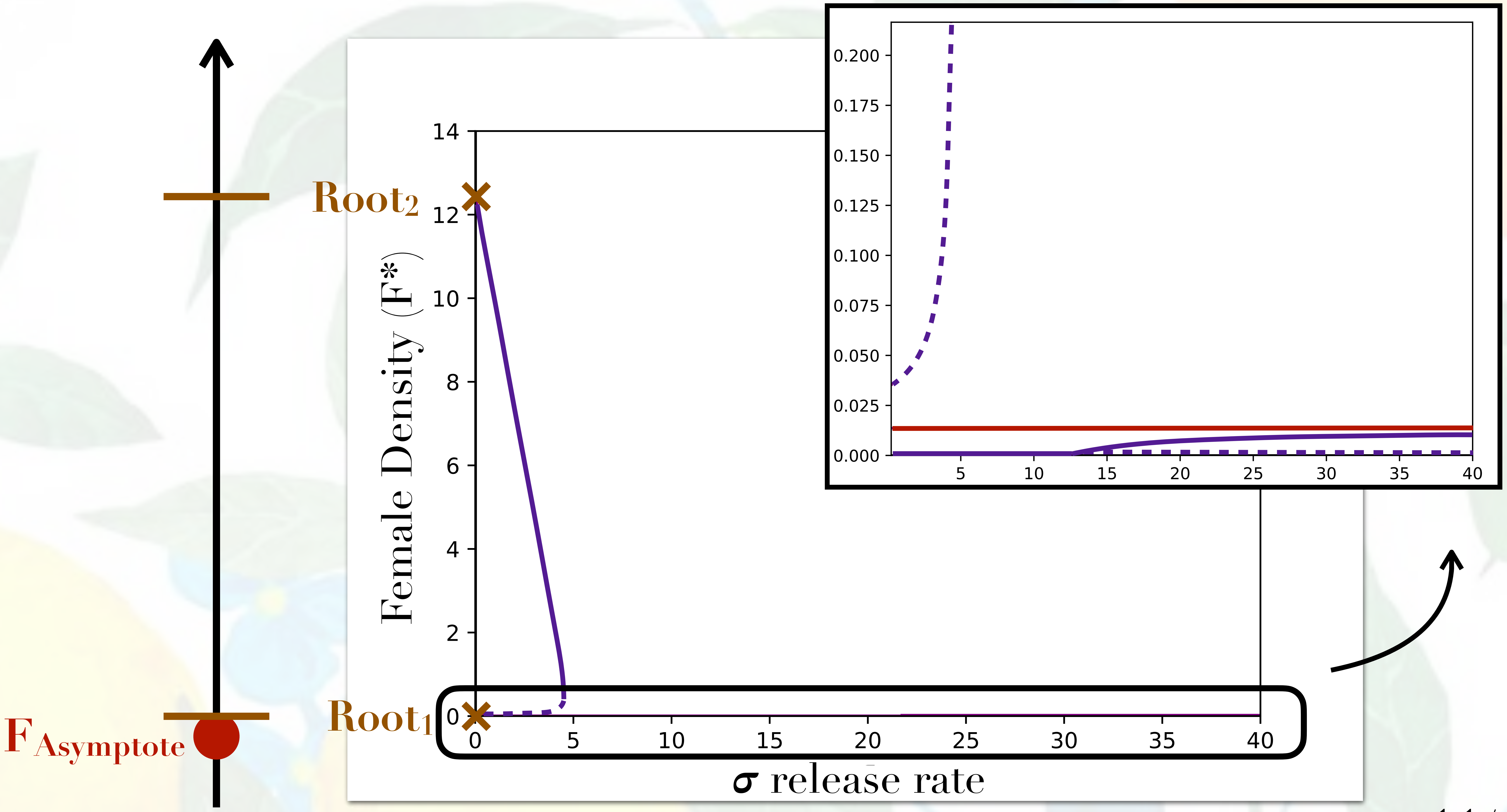

— Stable
- - Unstable

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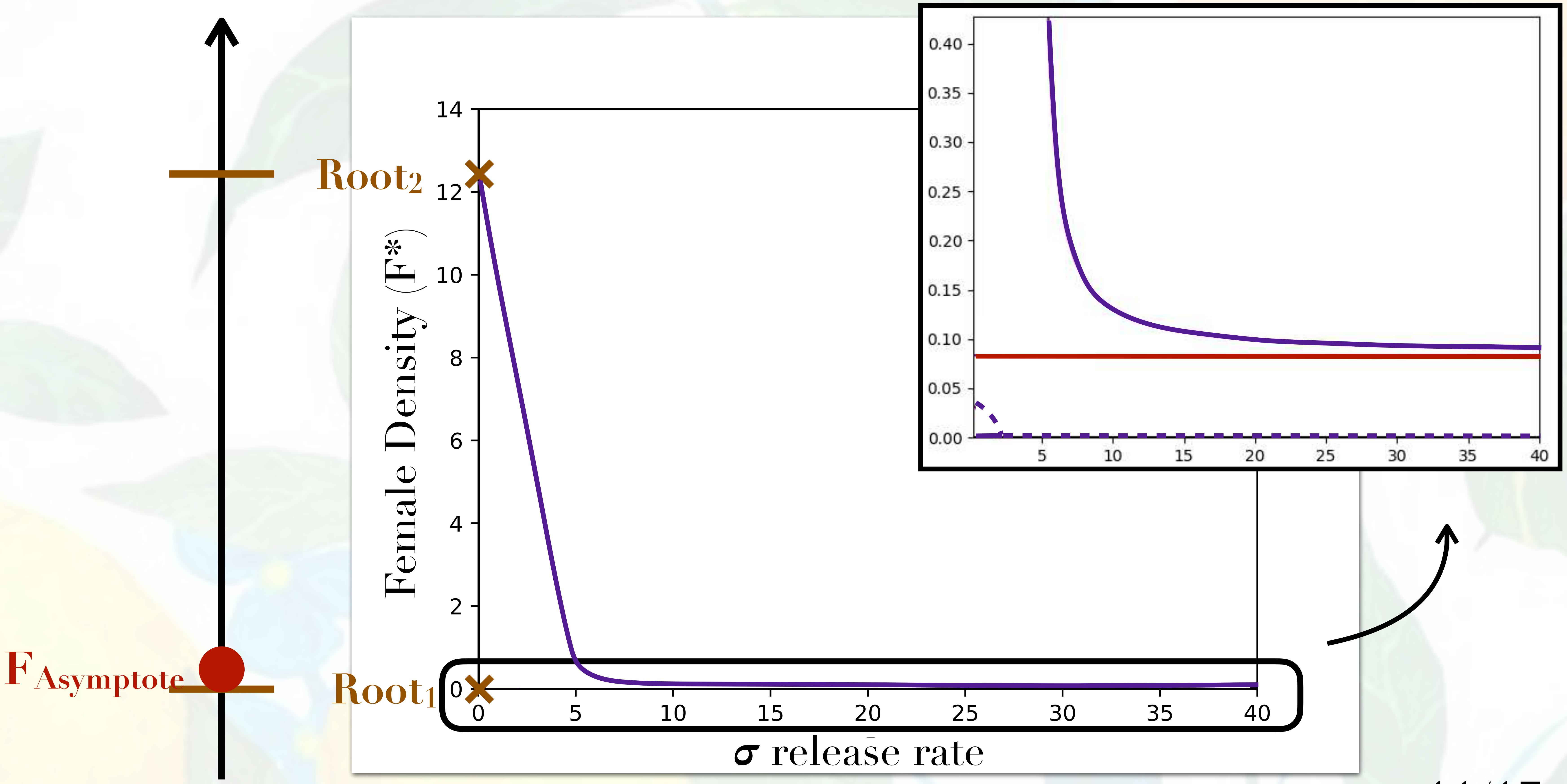

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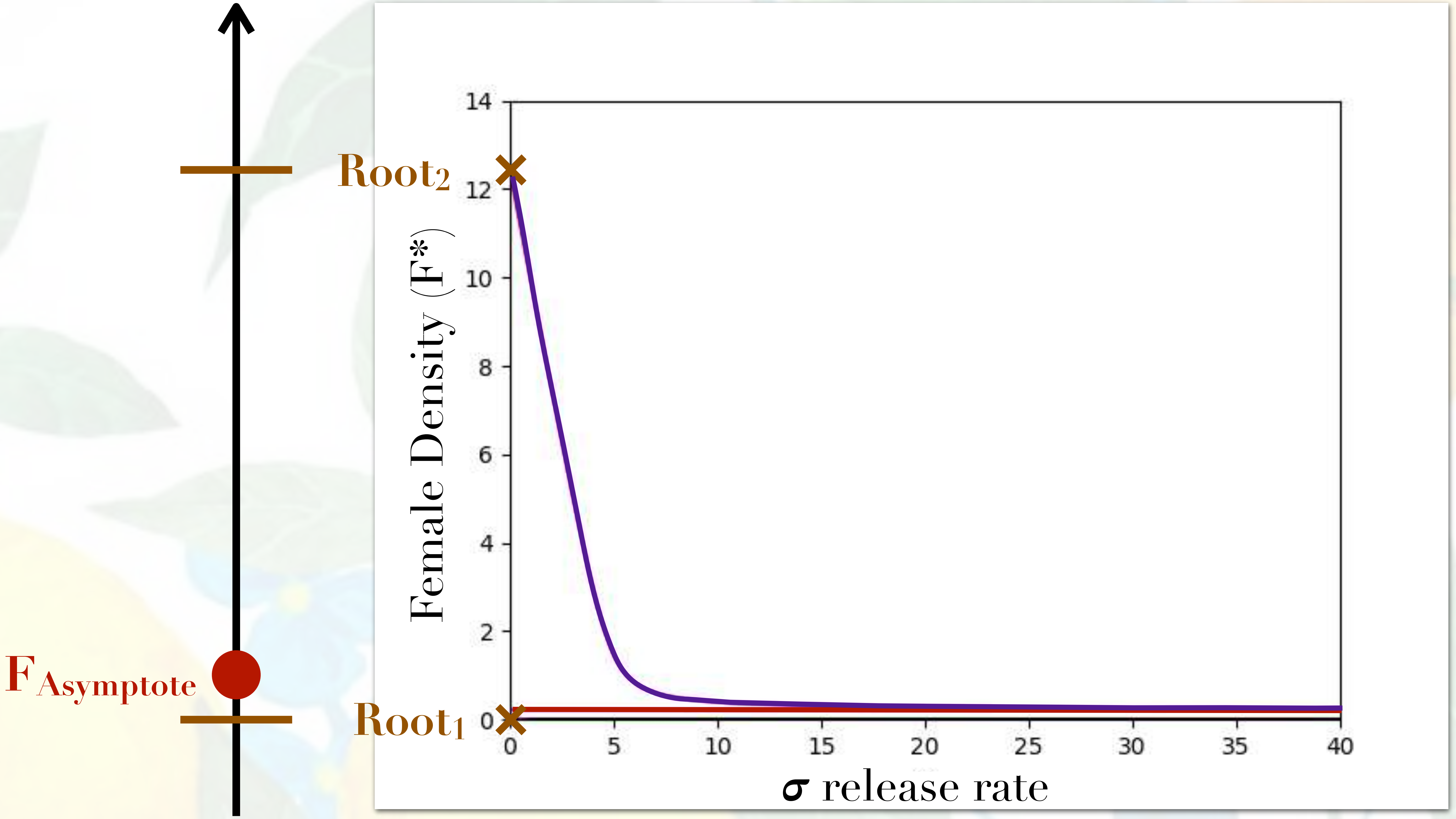

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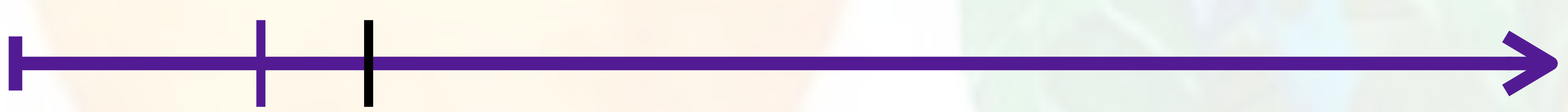

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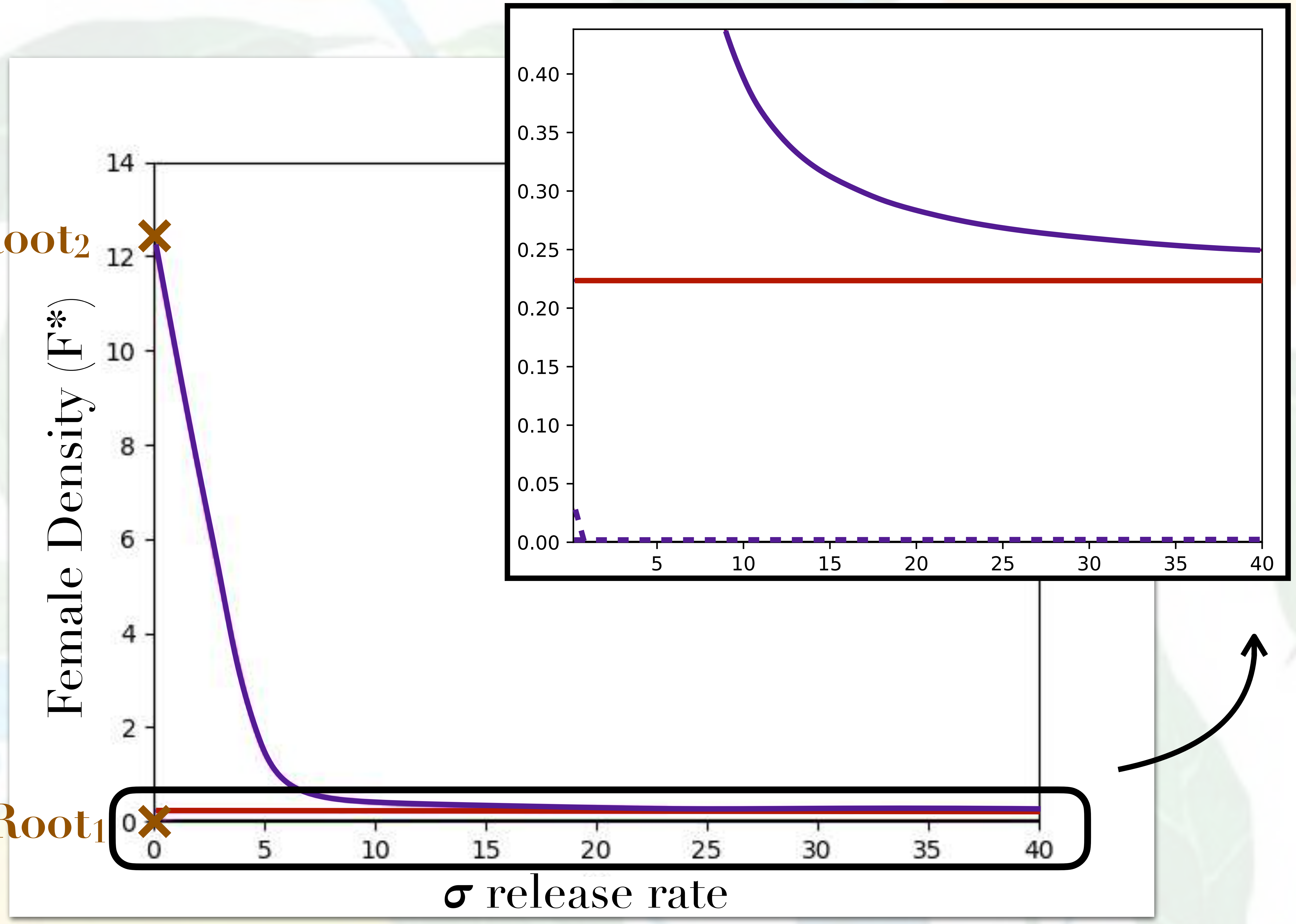

— Stable
- - Unstable

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$\delta = 0, \epsilon \neq 0$

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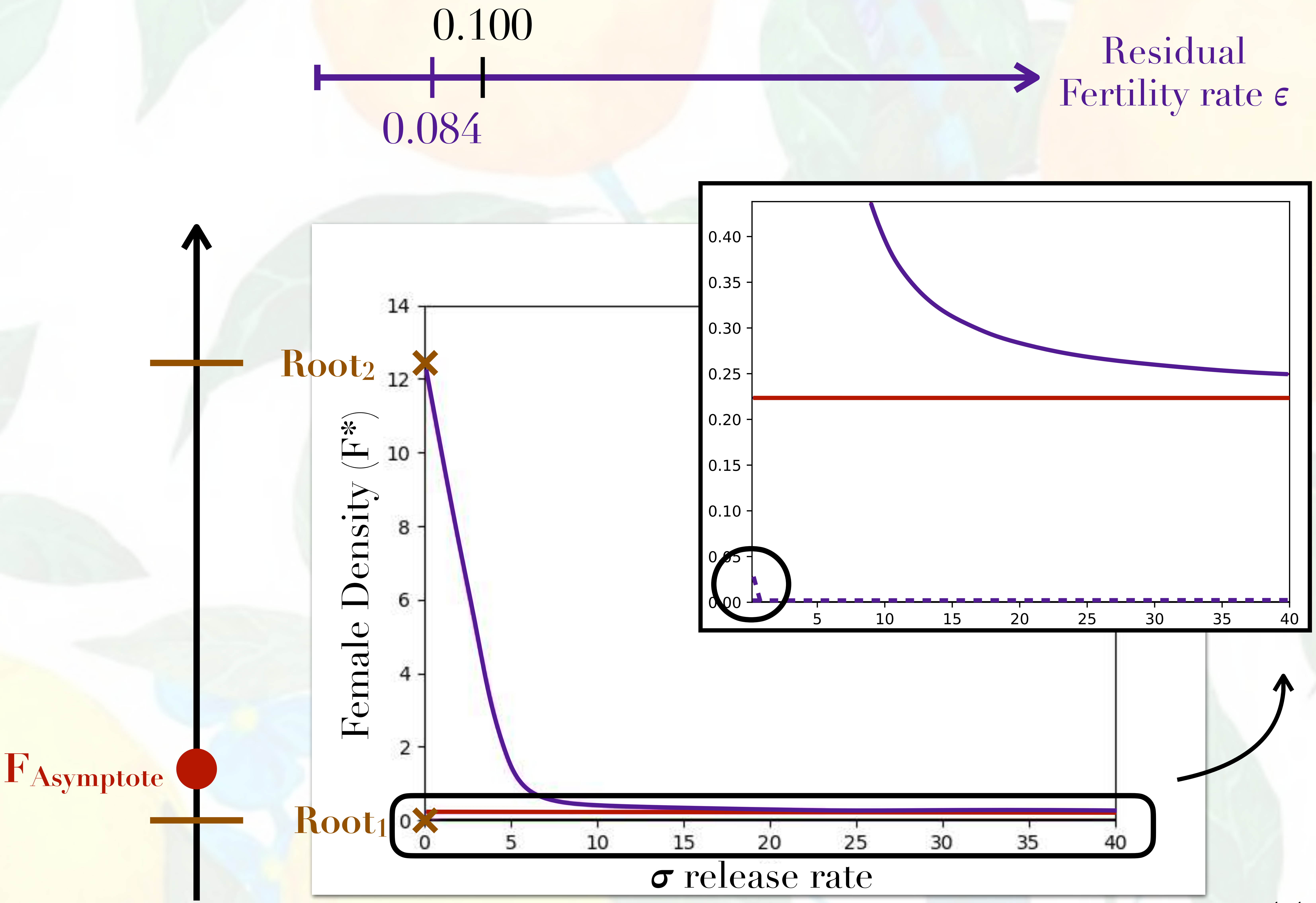

— Stable
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(2)
Costly fertility model
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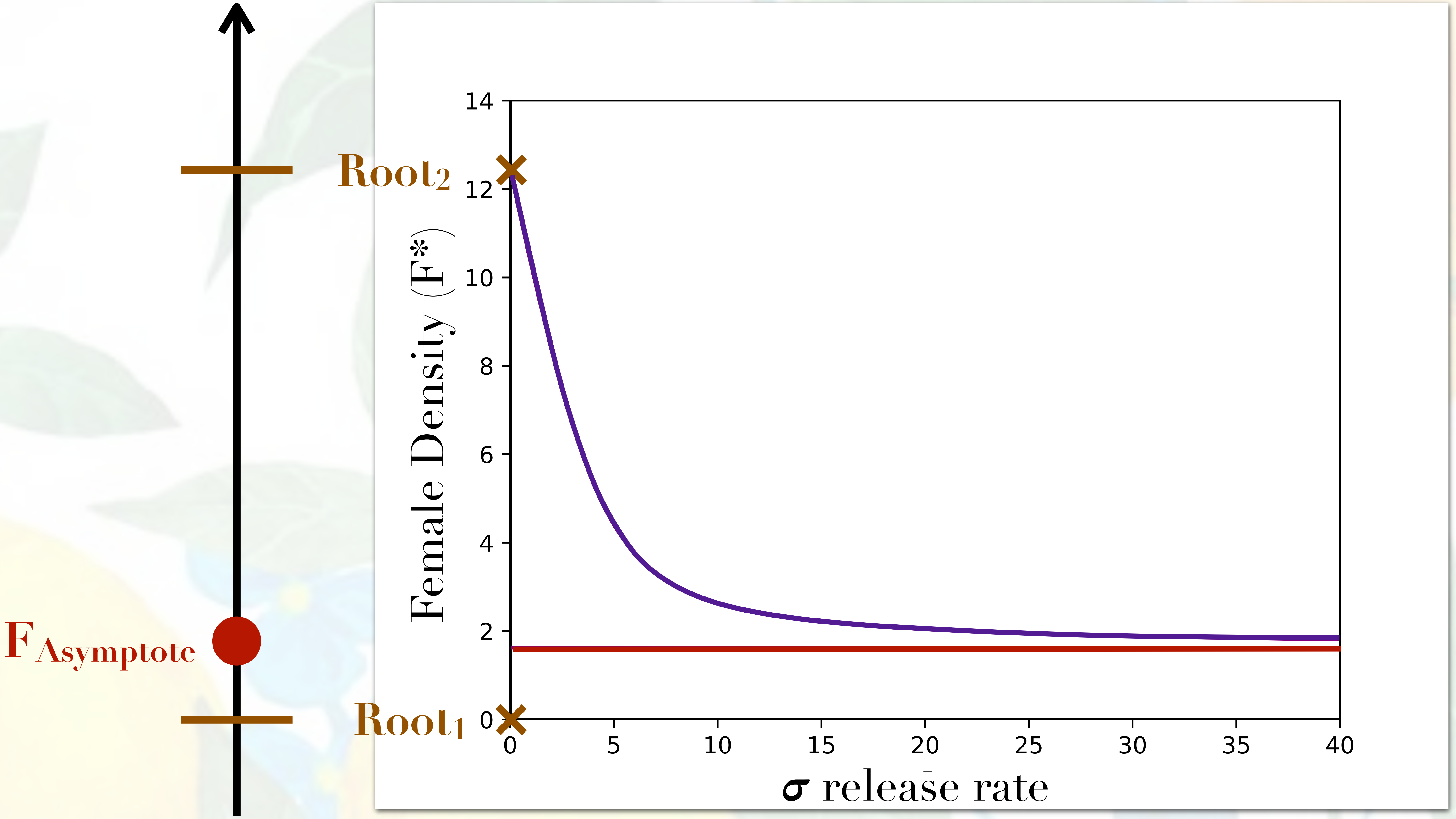

— Stable
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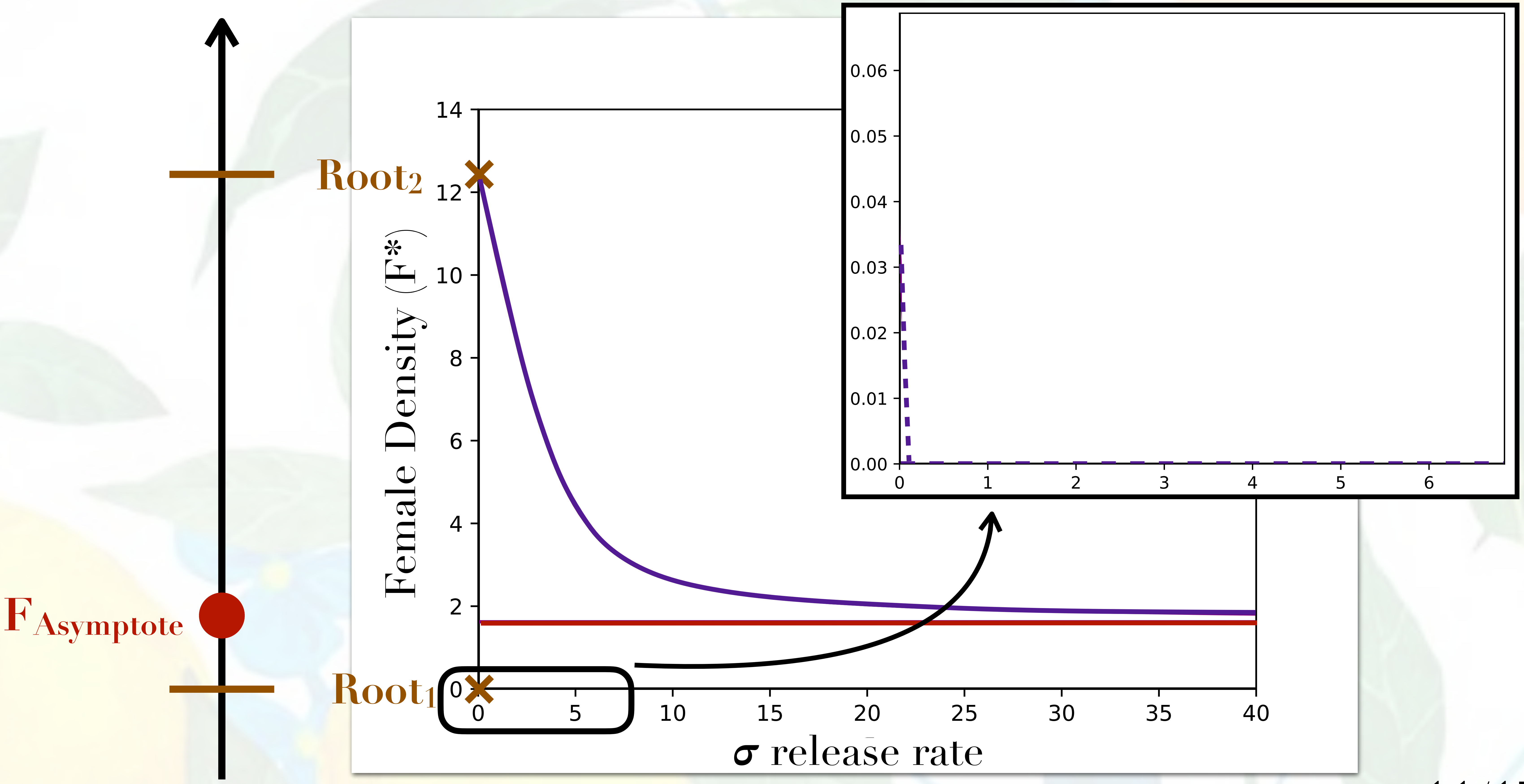

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(2)
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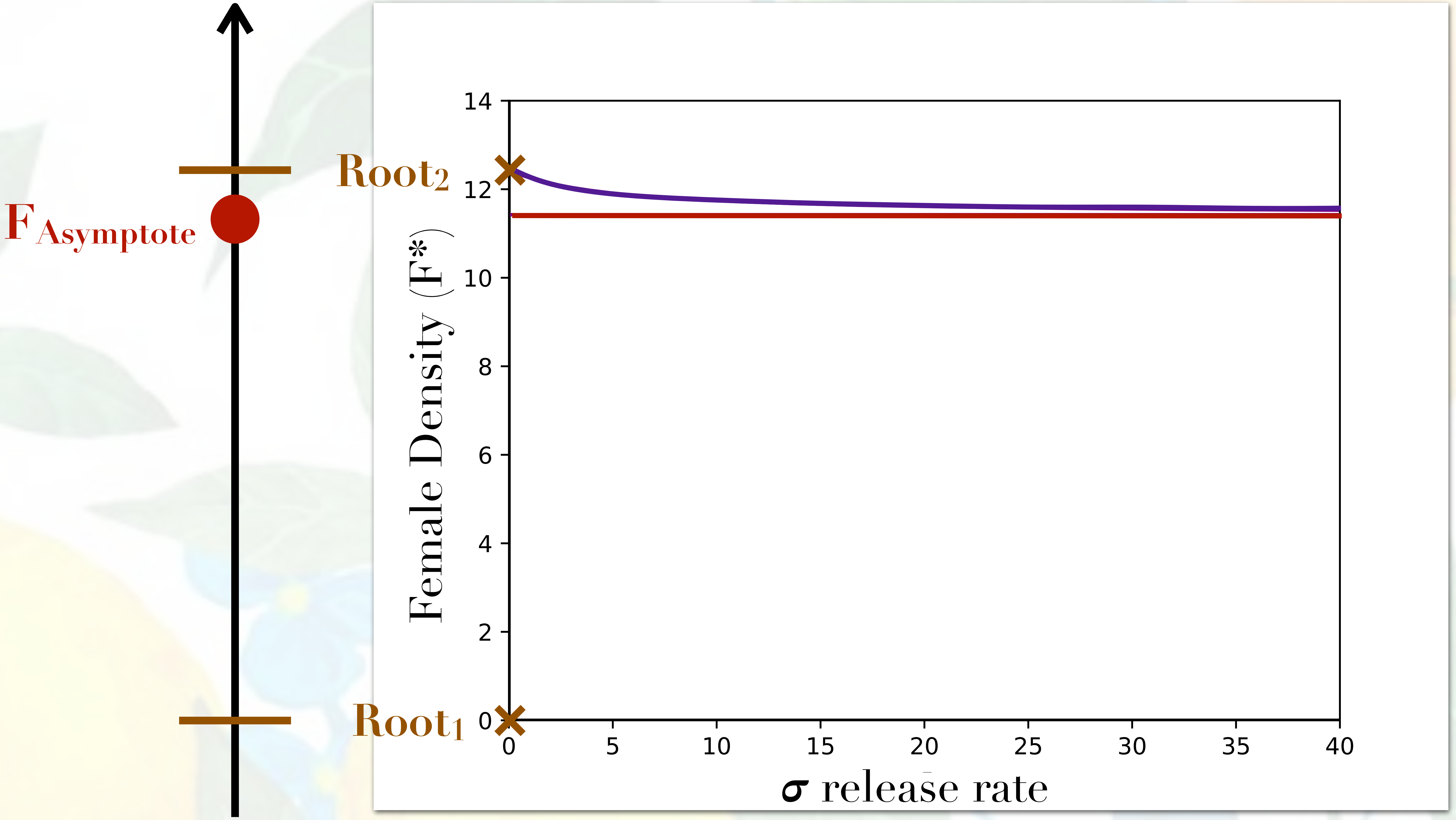

— Stable
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Residual Fertility rate ϵ

(2)

Costly fertility model

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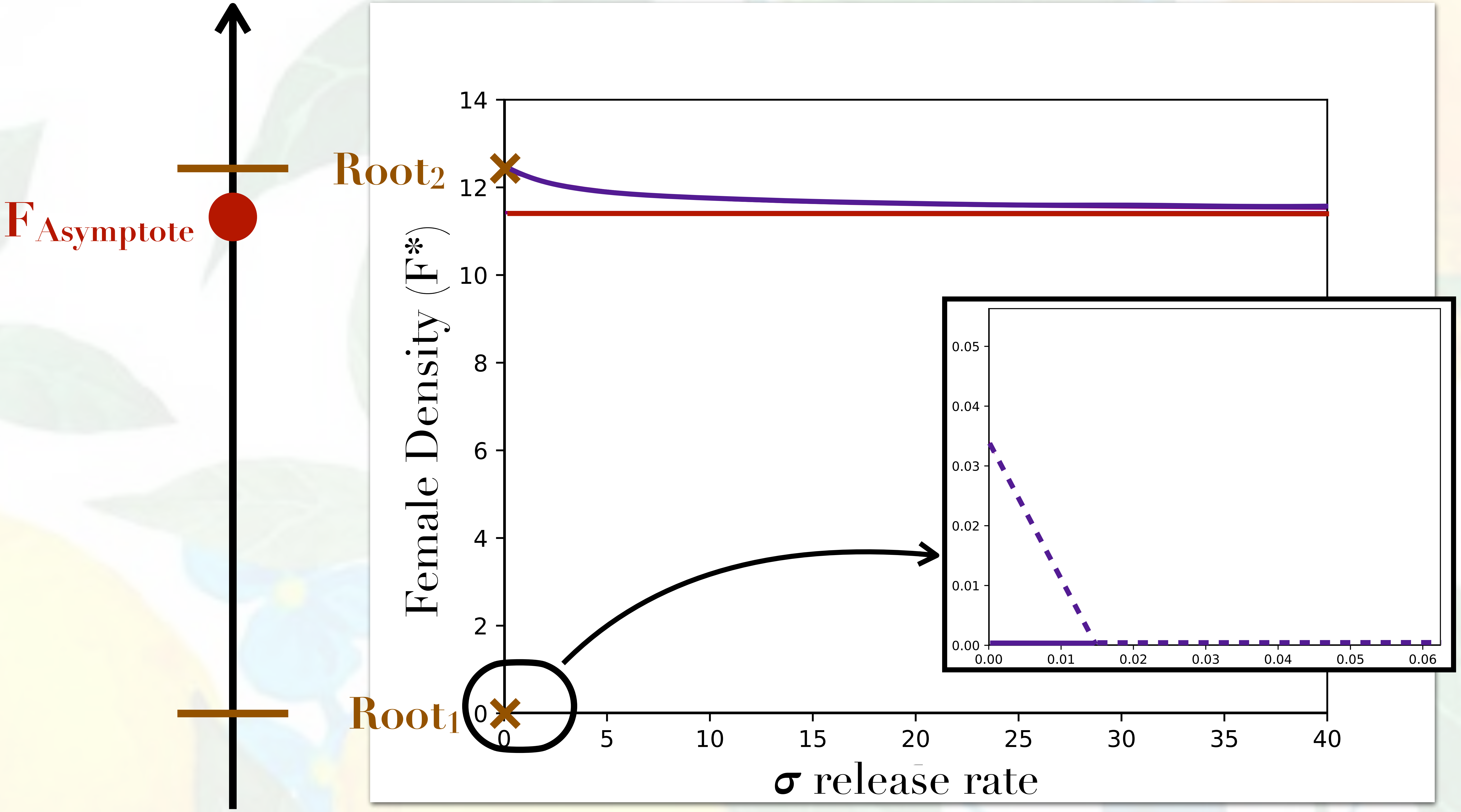

— Stable
-- Unstable

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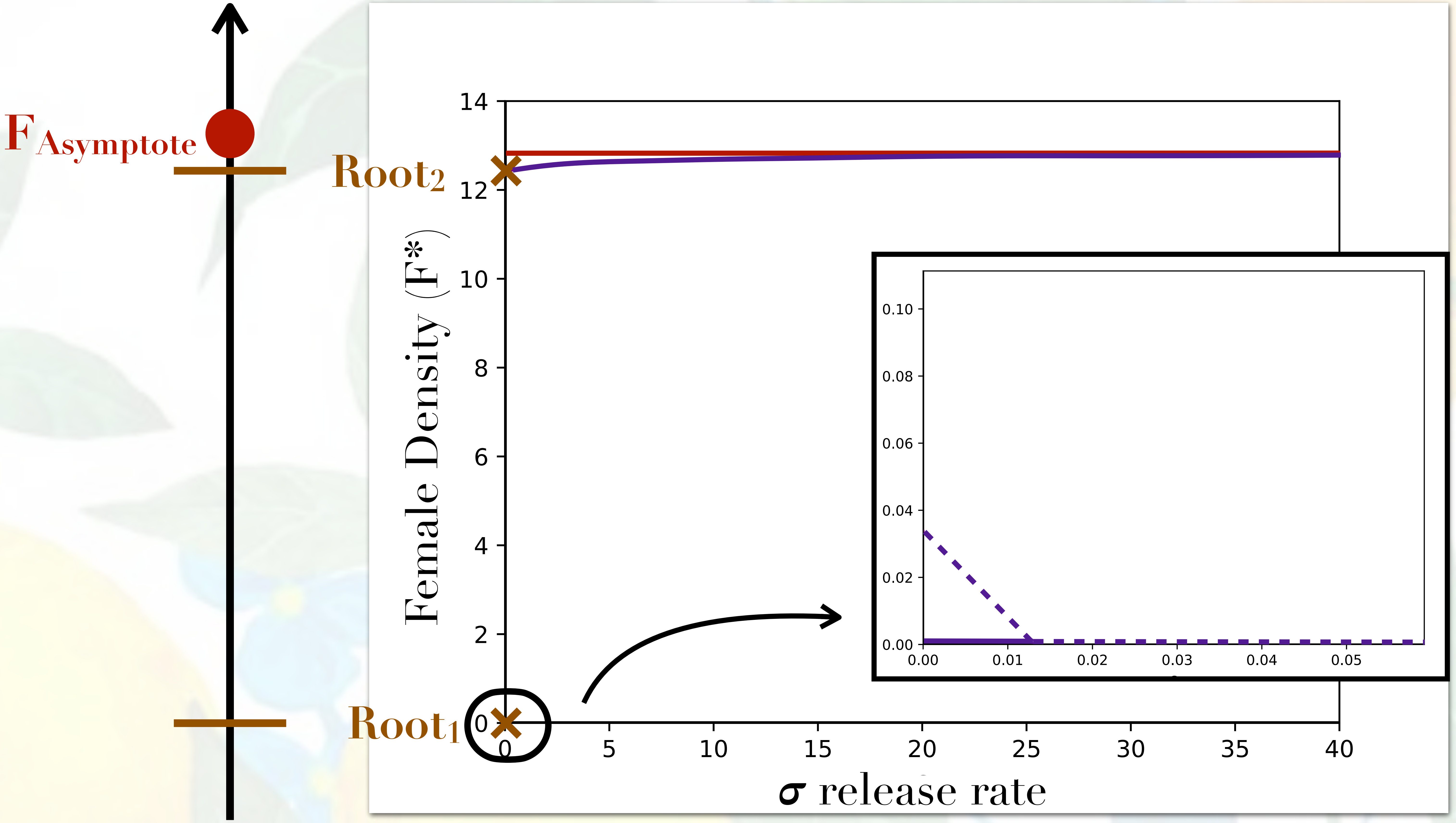

— Stable
-- Unstable

(2)
Costly fertility model
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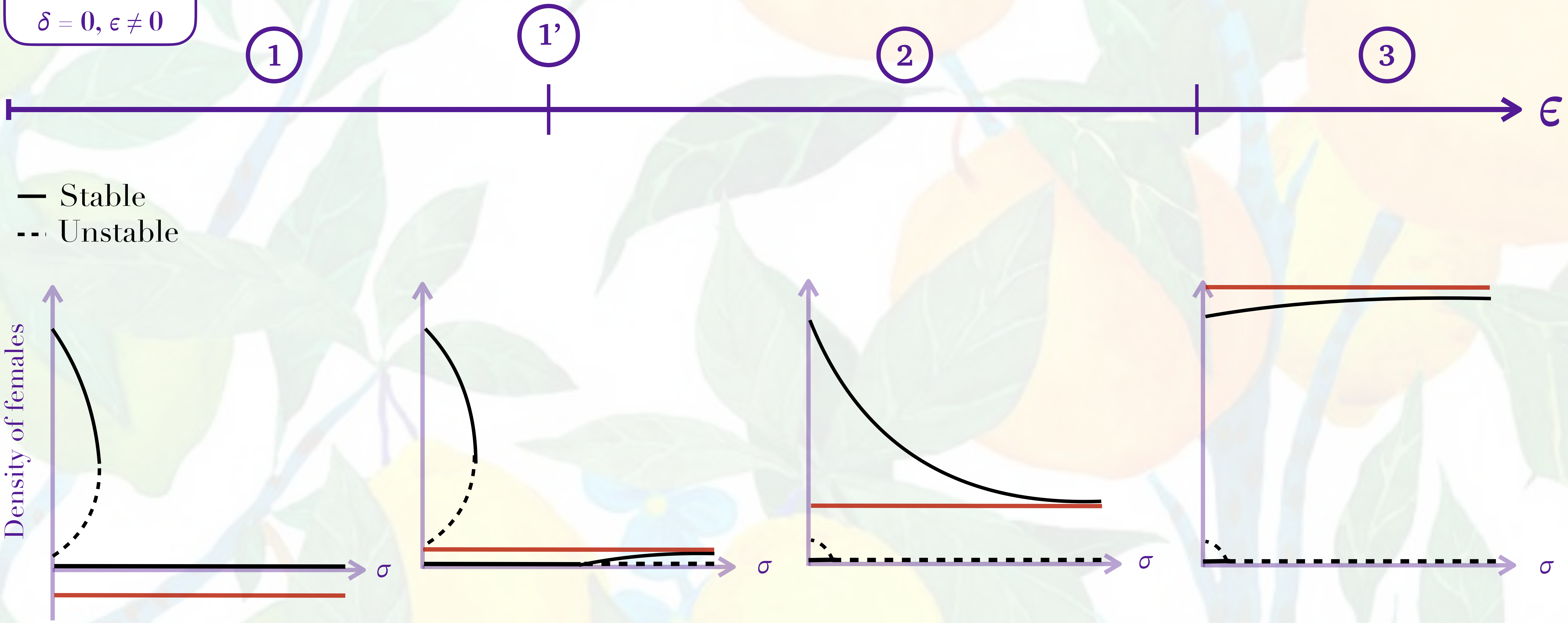

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Costly fertility model
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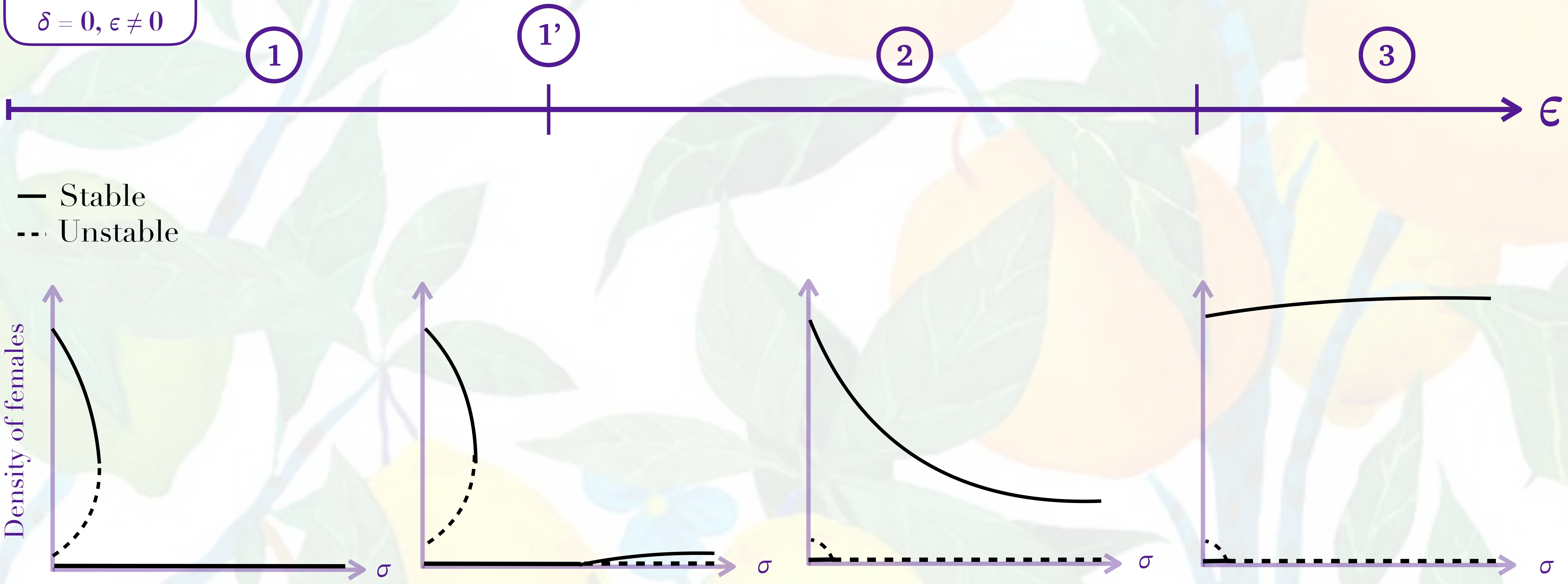

(2)
Costly fertility model
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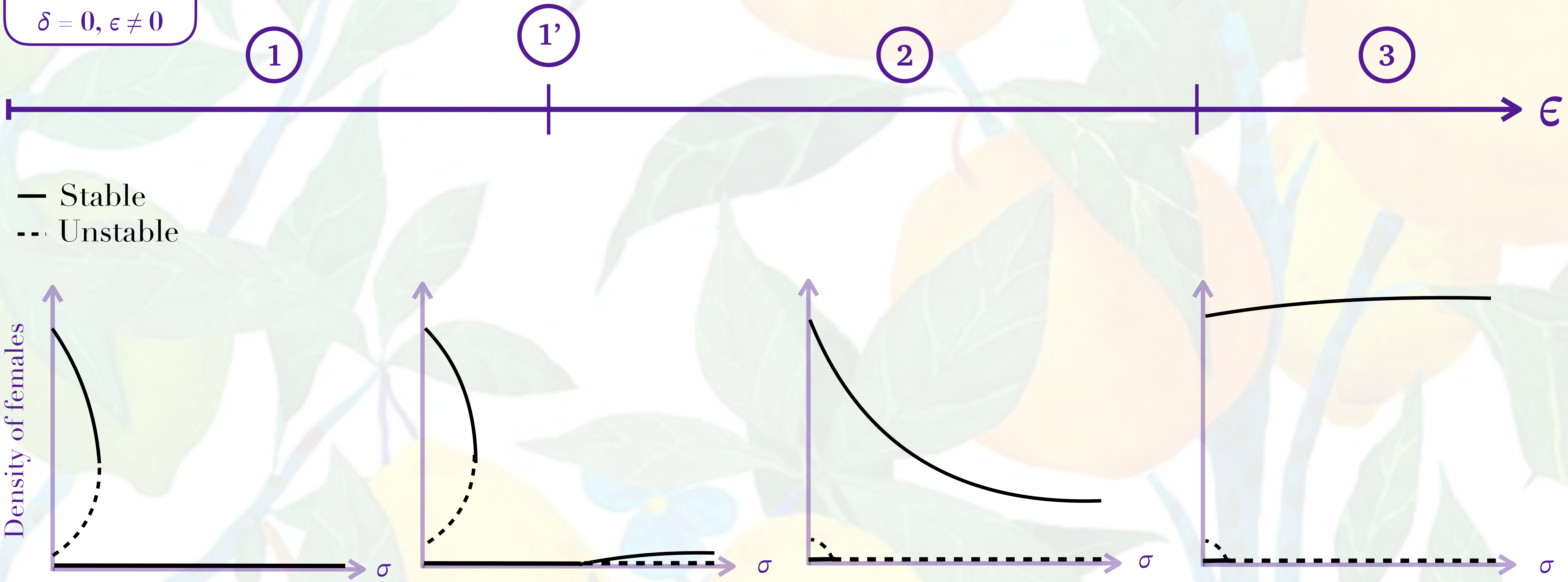
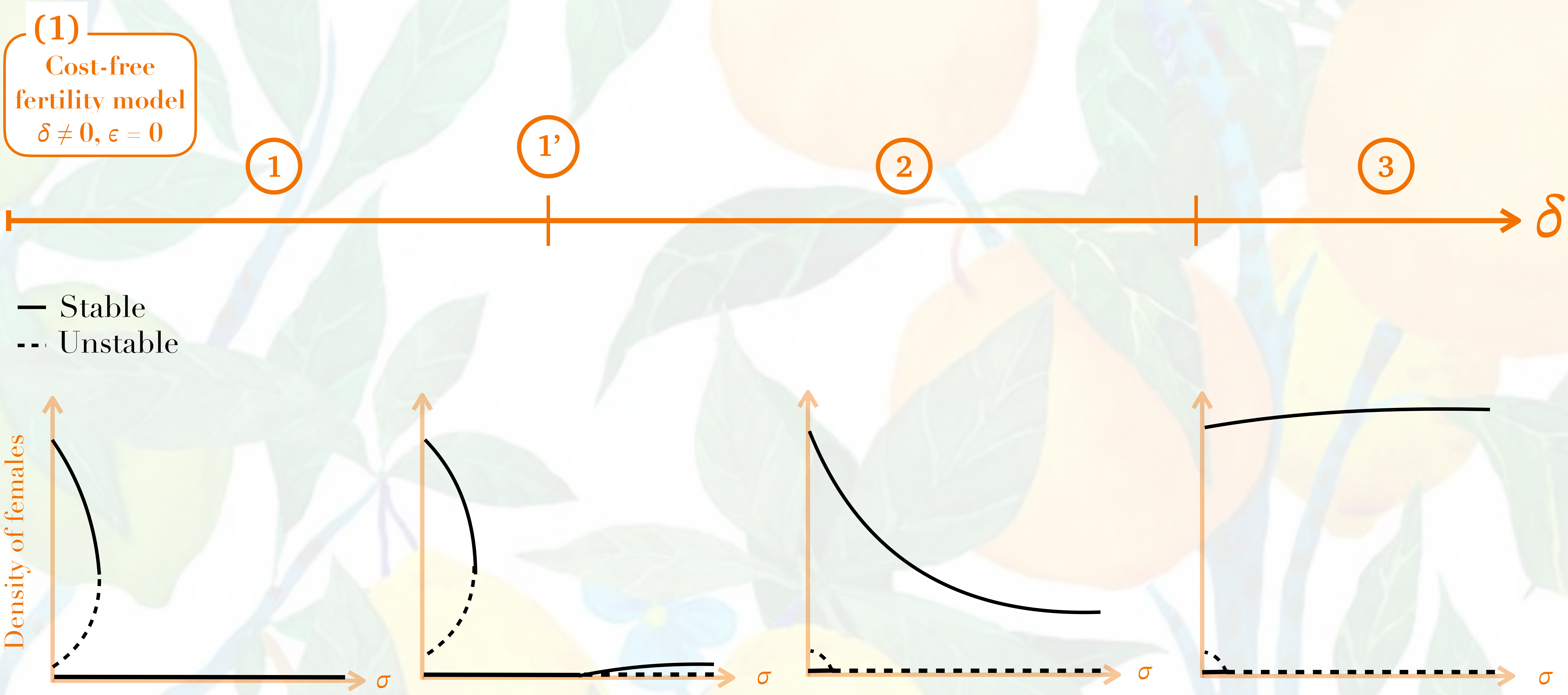


➡️ Different control capabilities depending on the shape of the bifurcation diagram



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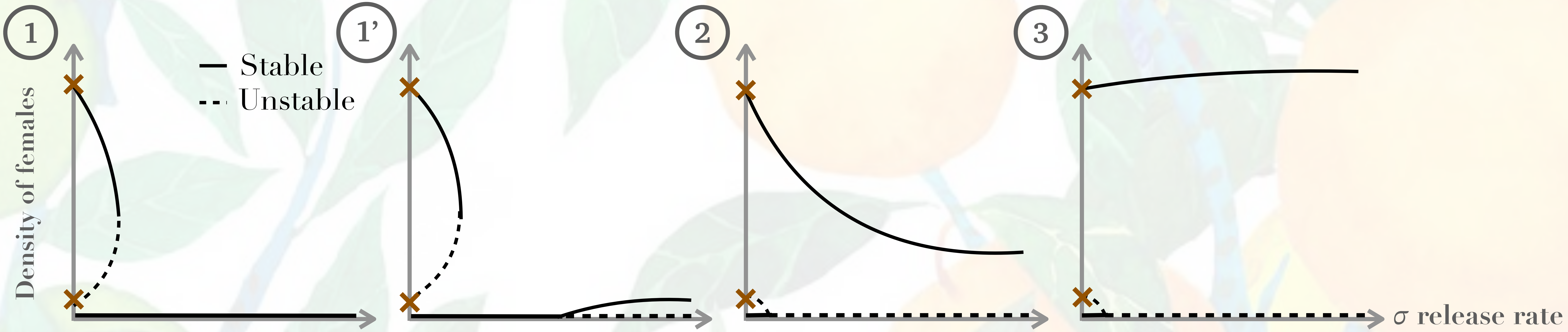
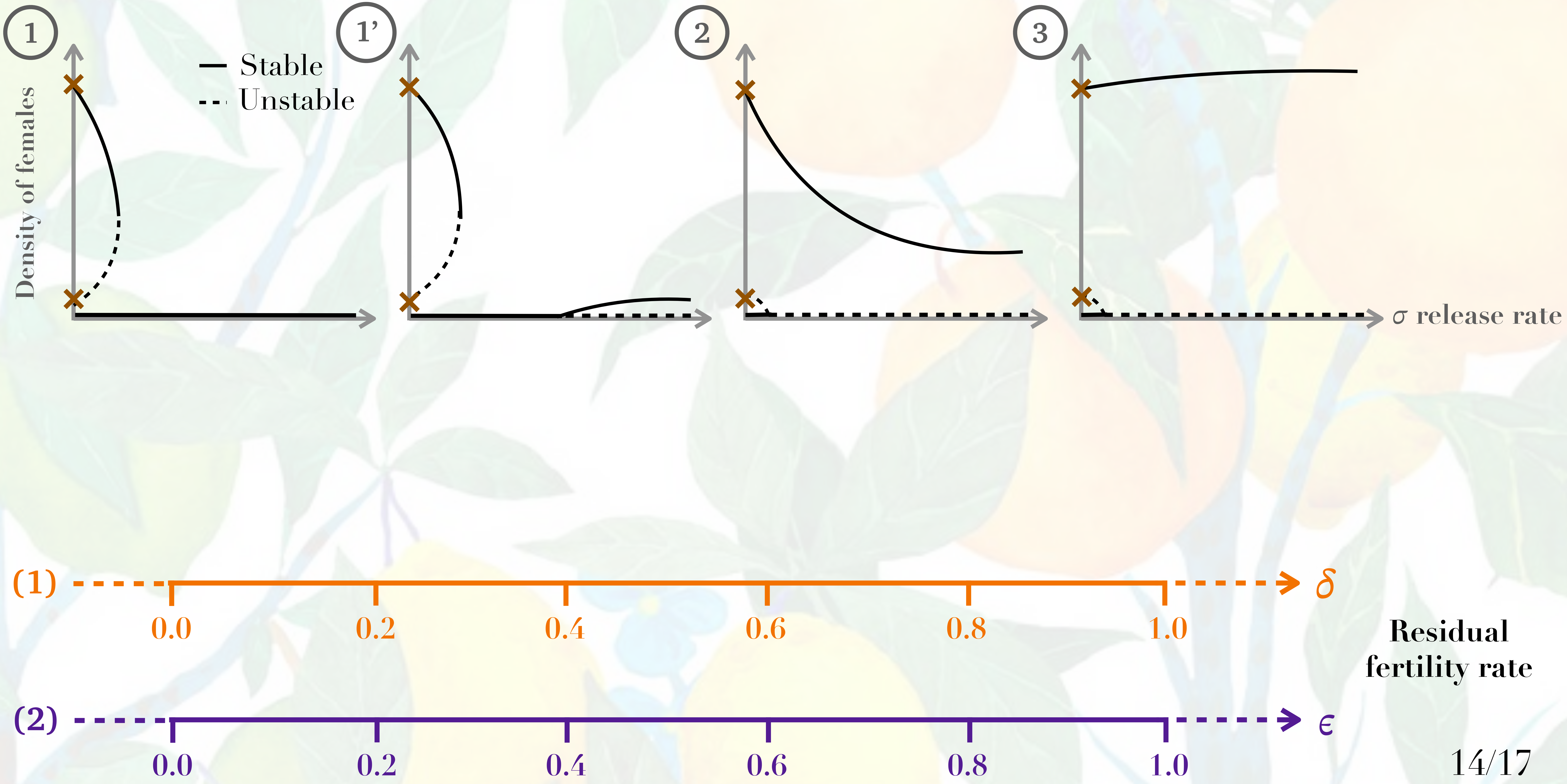
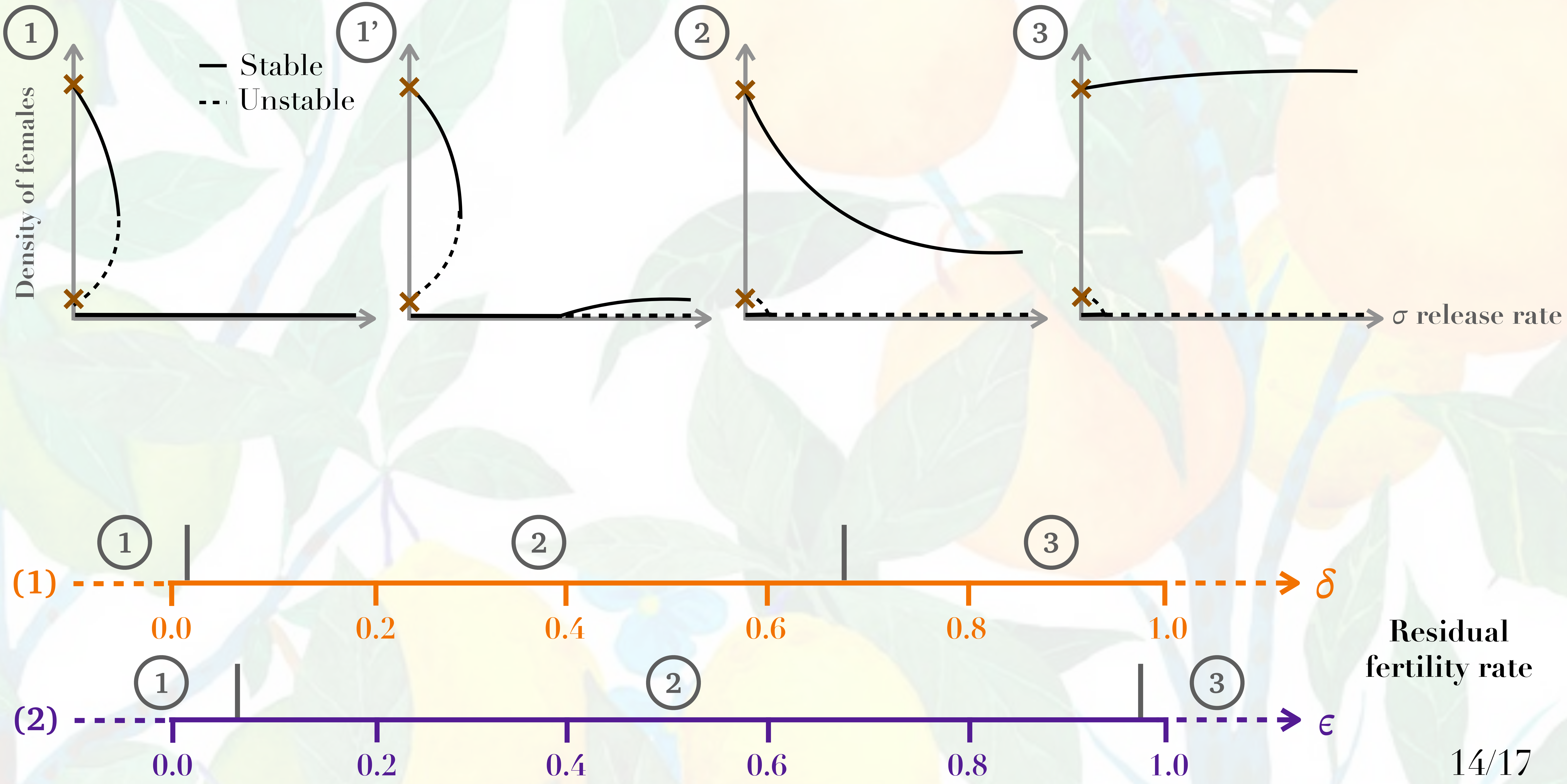
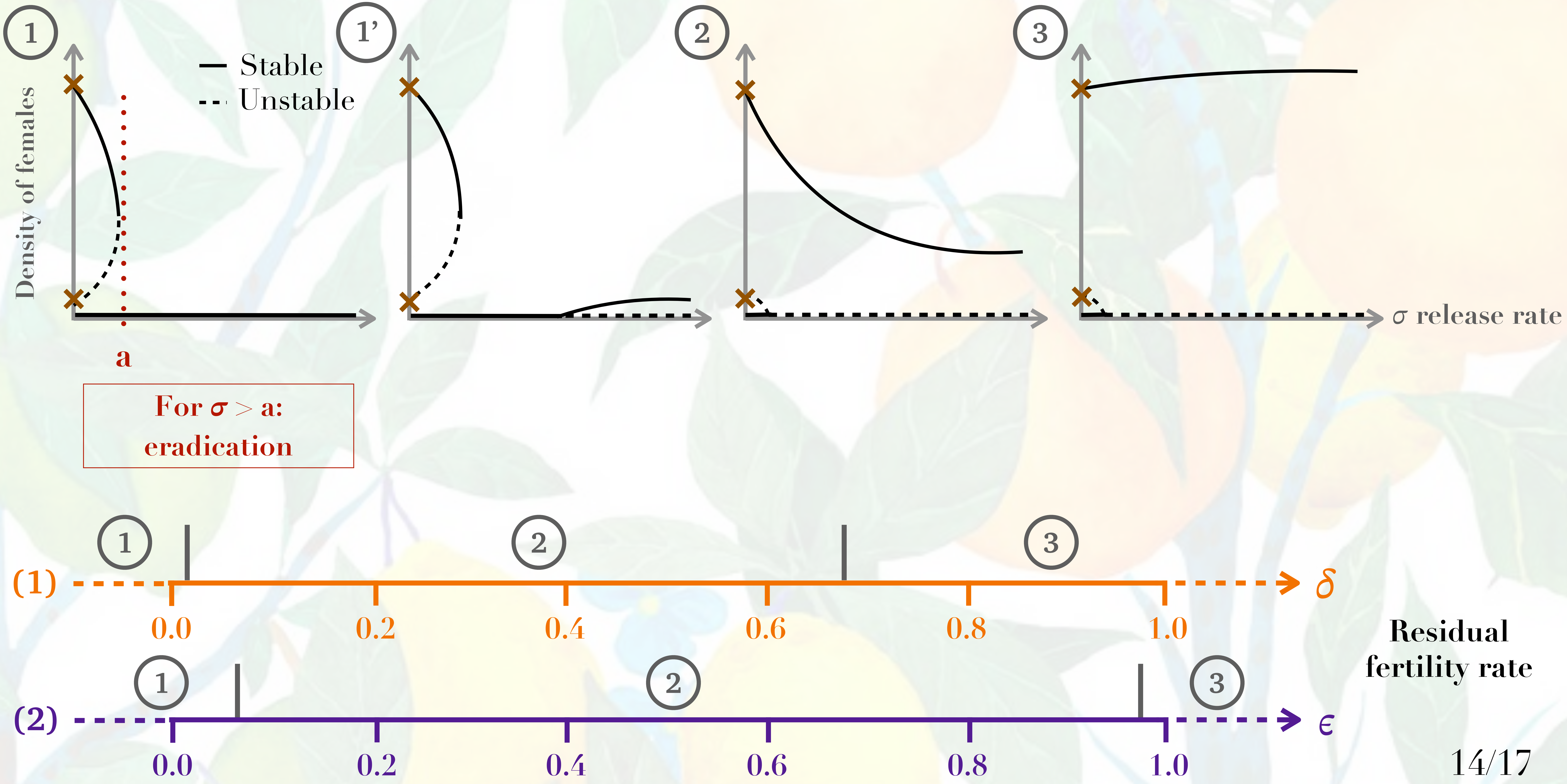
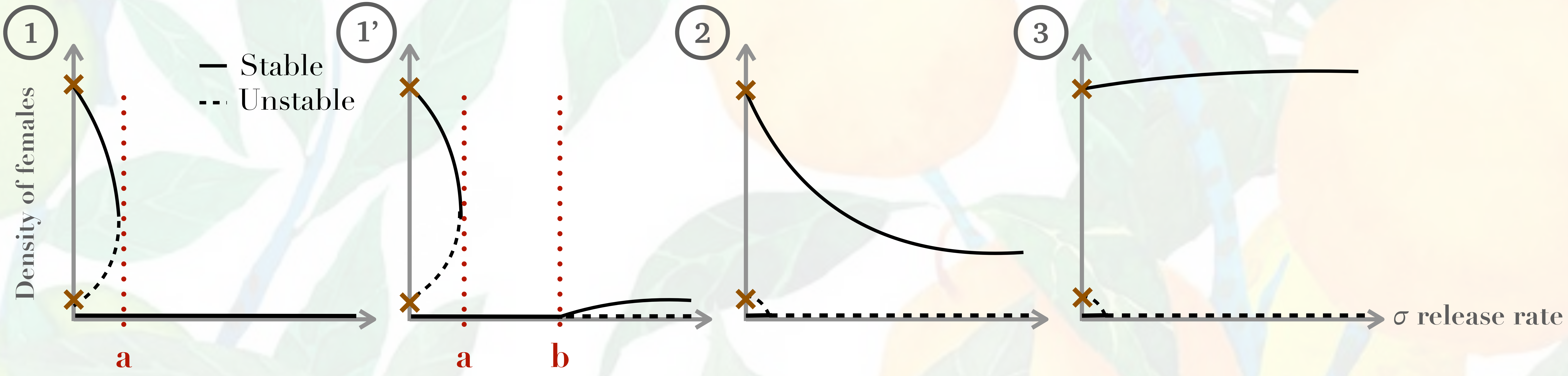

1
1'
2
3
Stable
Unstable
Density of females
σ release rate


1
1'
2
3
Stable
Unstable
Density of females
σ release rate
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
Residual fertility rate
(2)
ϵ
0.0
0.2
0.4
0.6
0.8
1.0

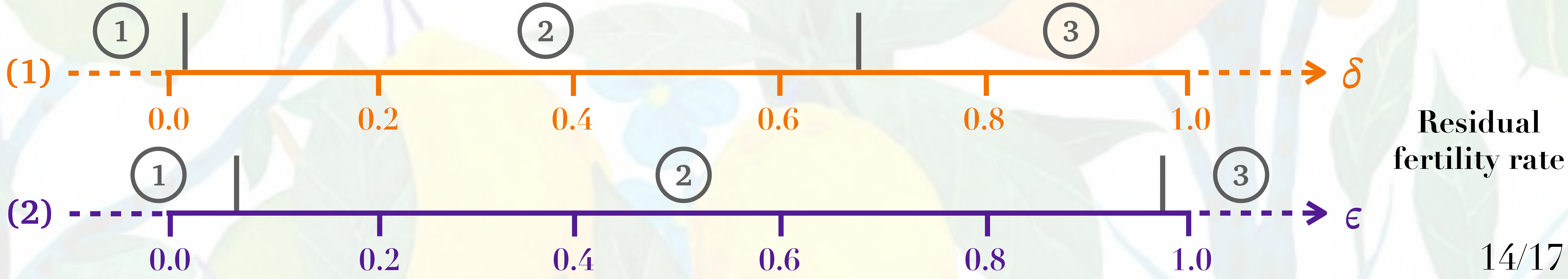

1
1'
2
3
Density of females
Stable
Unstable
σ release rate
1
2
3
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
Residual fertility rate
1
2
3
(2)
ϵ
0.0
0.2
0.4
0.6
0.8
1.0

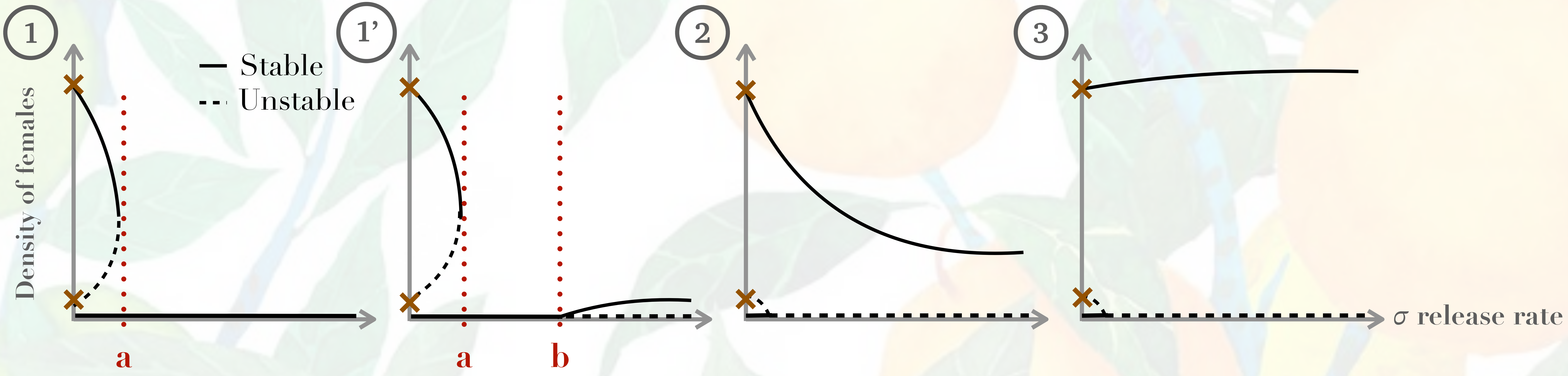

1
1'
2
3
Density of females
Stable
Unstable
σ release rate
a
For σ > a:
eradication
1
2
3
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
Residual
fertility rate
1
2
3
(2)
ϵ
0.0
0.2
0.4
0.6
0.8
1.0



For σ > a:
eradication

For a < σ < b: eradication
For σ >b: quasi eradication





For σ > a: eradication

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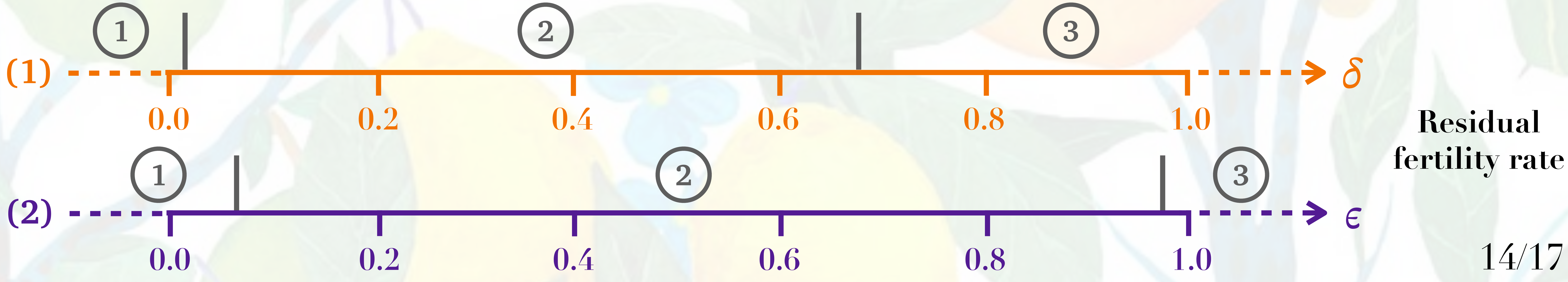
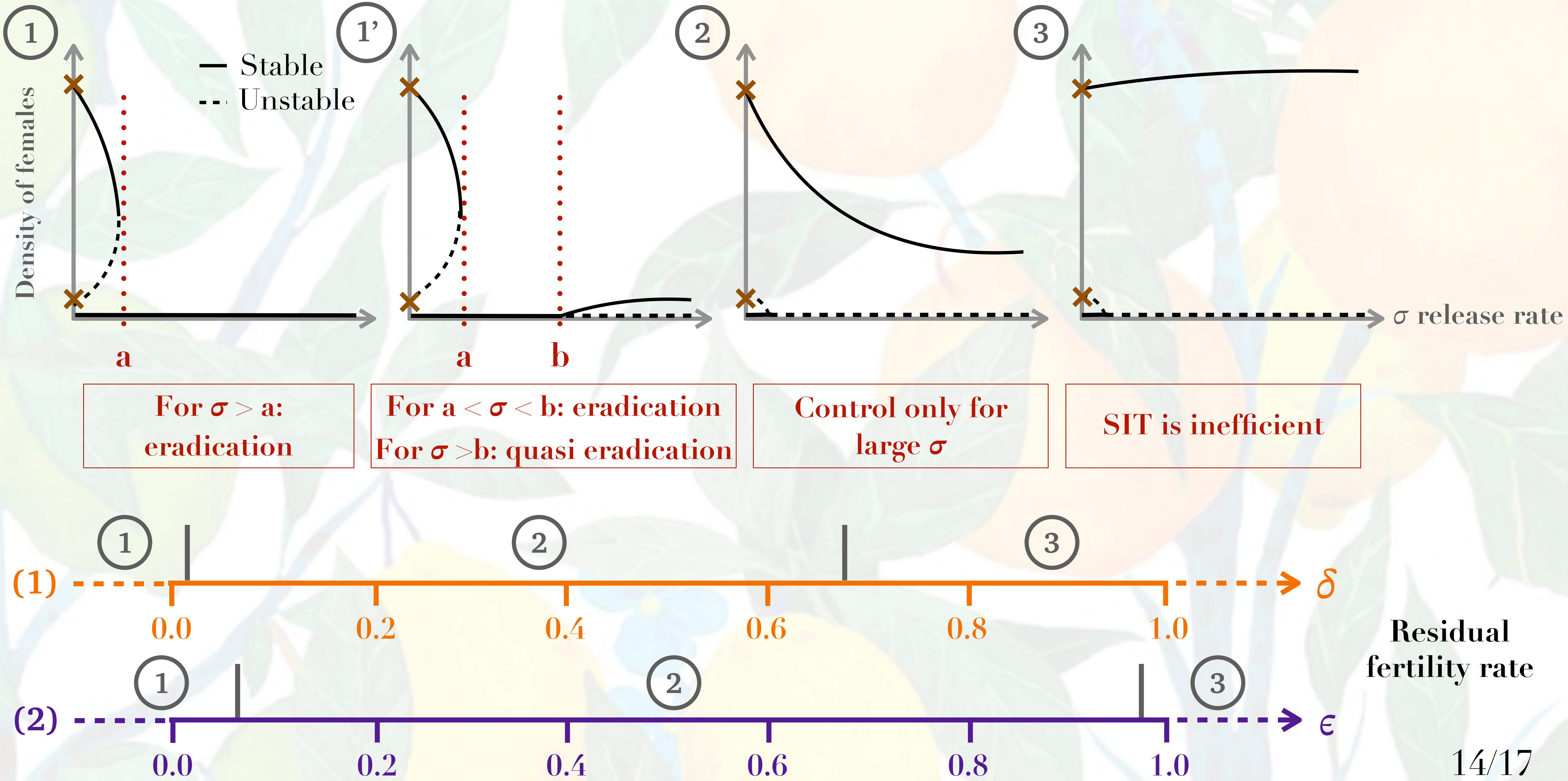
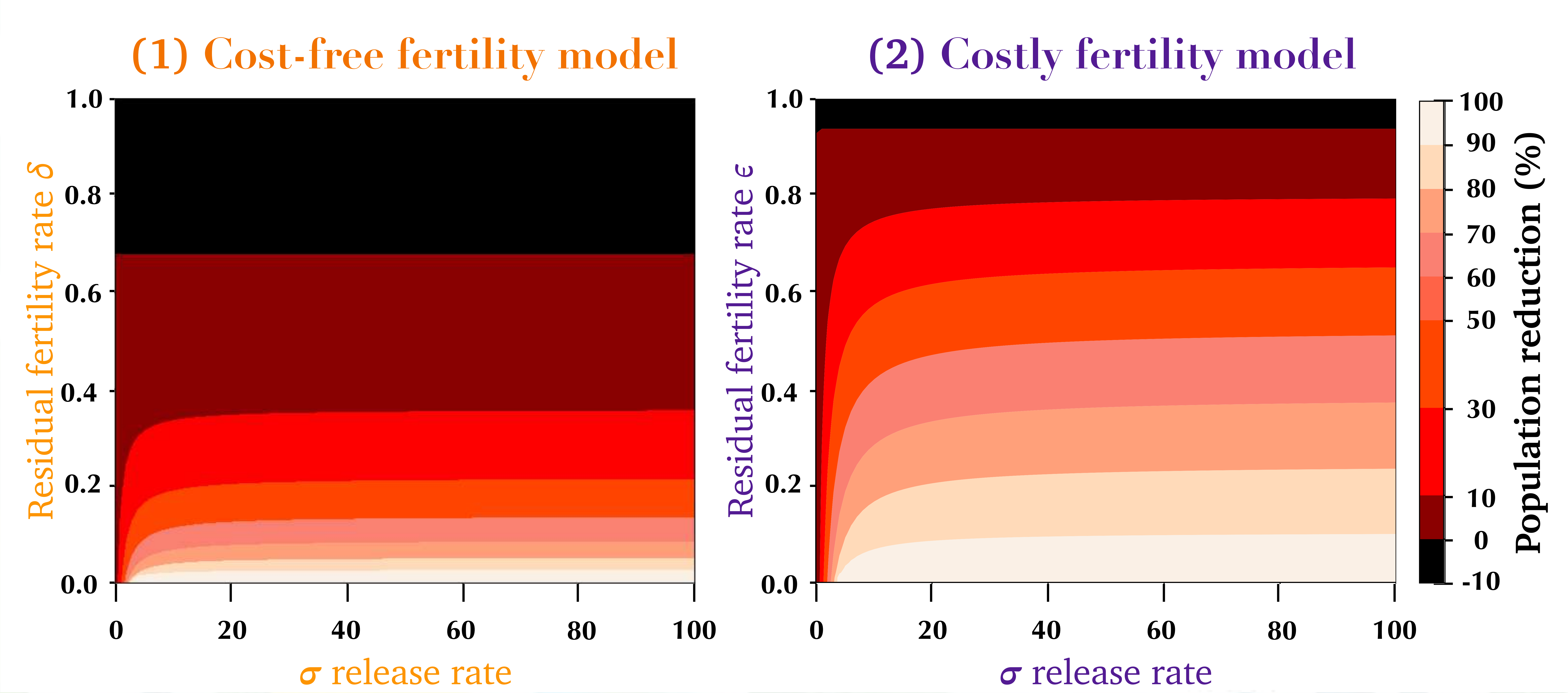
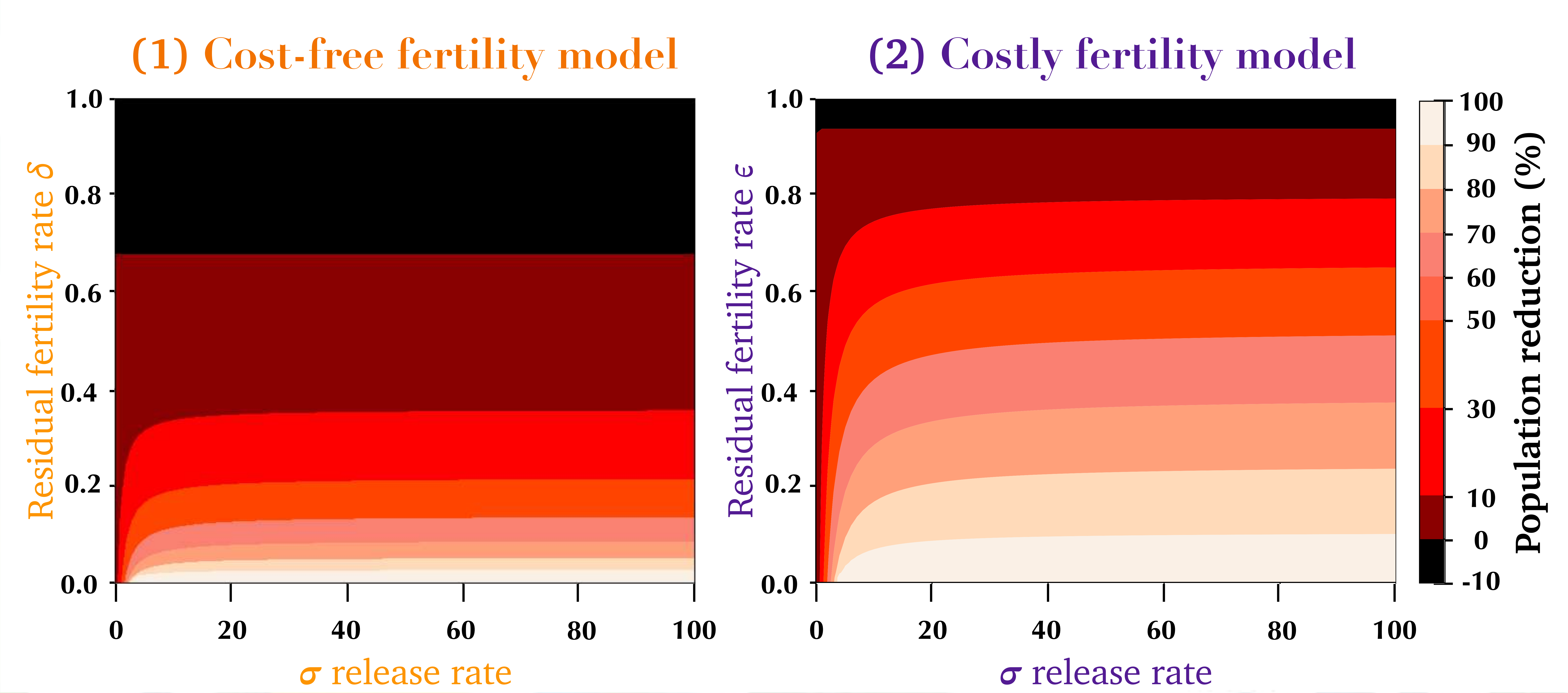
For σ >b: quasi eradication

Control only for large σ

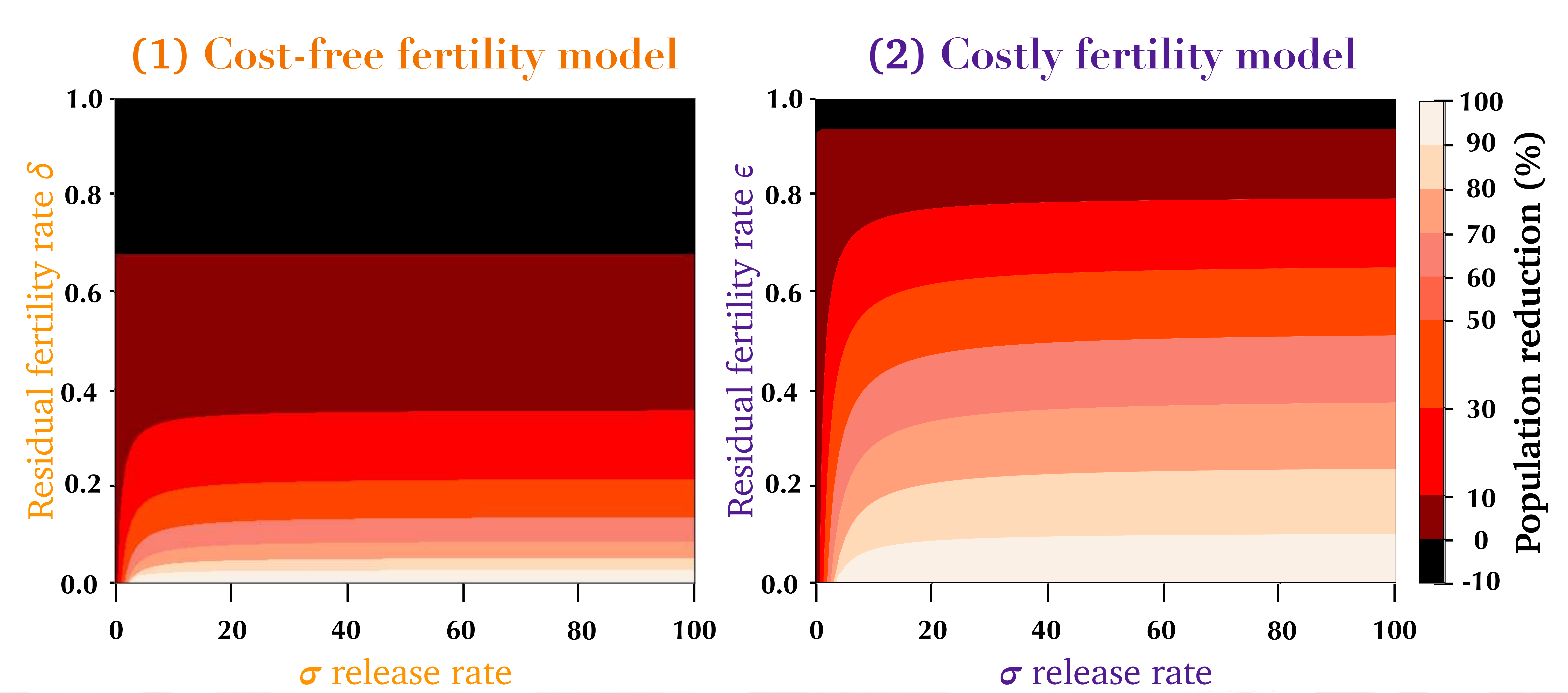



1
1'
2
3
Stable
Unstable
Density of females
a
a
b
σ release rate
For σ > a: eradication
For a < σ < b: eradication
For σ >b: quasi eradication
Control only for large σ
SIT is inefficient
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
(2)
ϵ
Residual fertility rate

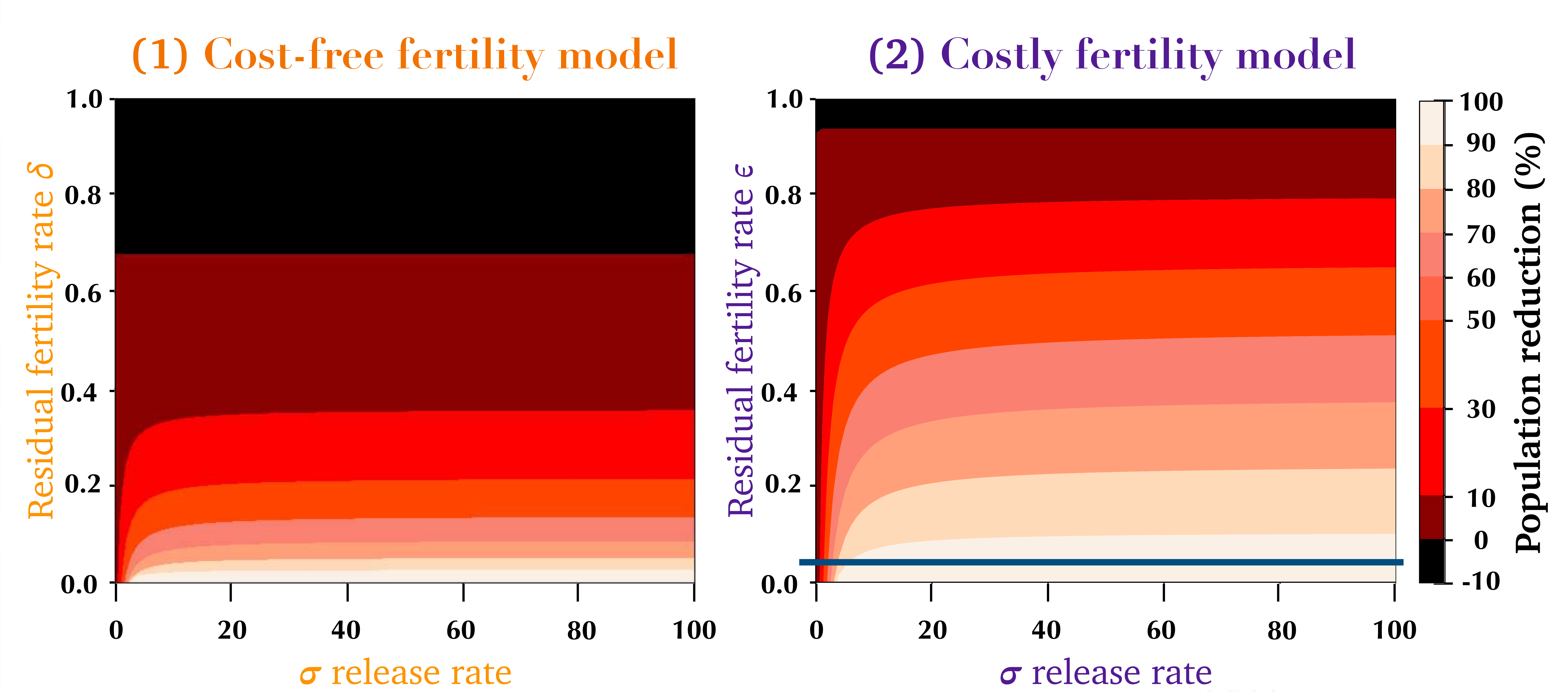

(1) Cost-free fertility model
(2) Costly fertility model
Residual fertility rate δ
Residual fertility rate ε
σ release rate
σ release rate
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
Population reduction (%)
100
90
80
70
60
50
30
10
0
-10



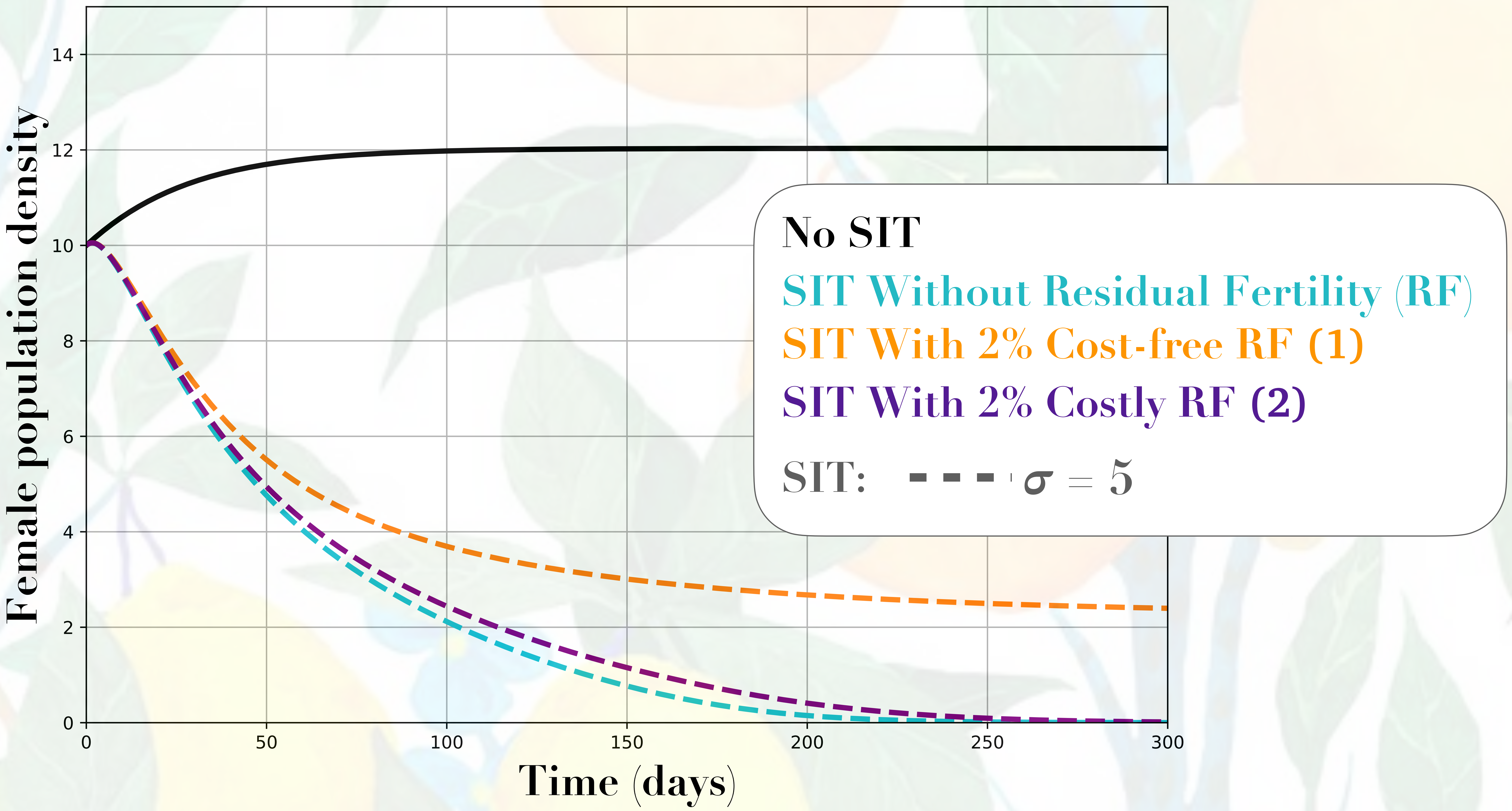
- Strong difference between the two sub models (1) and (2)



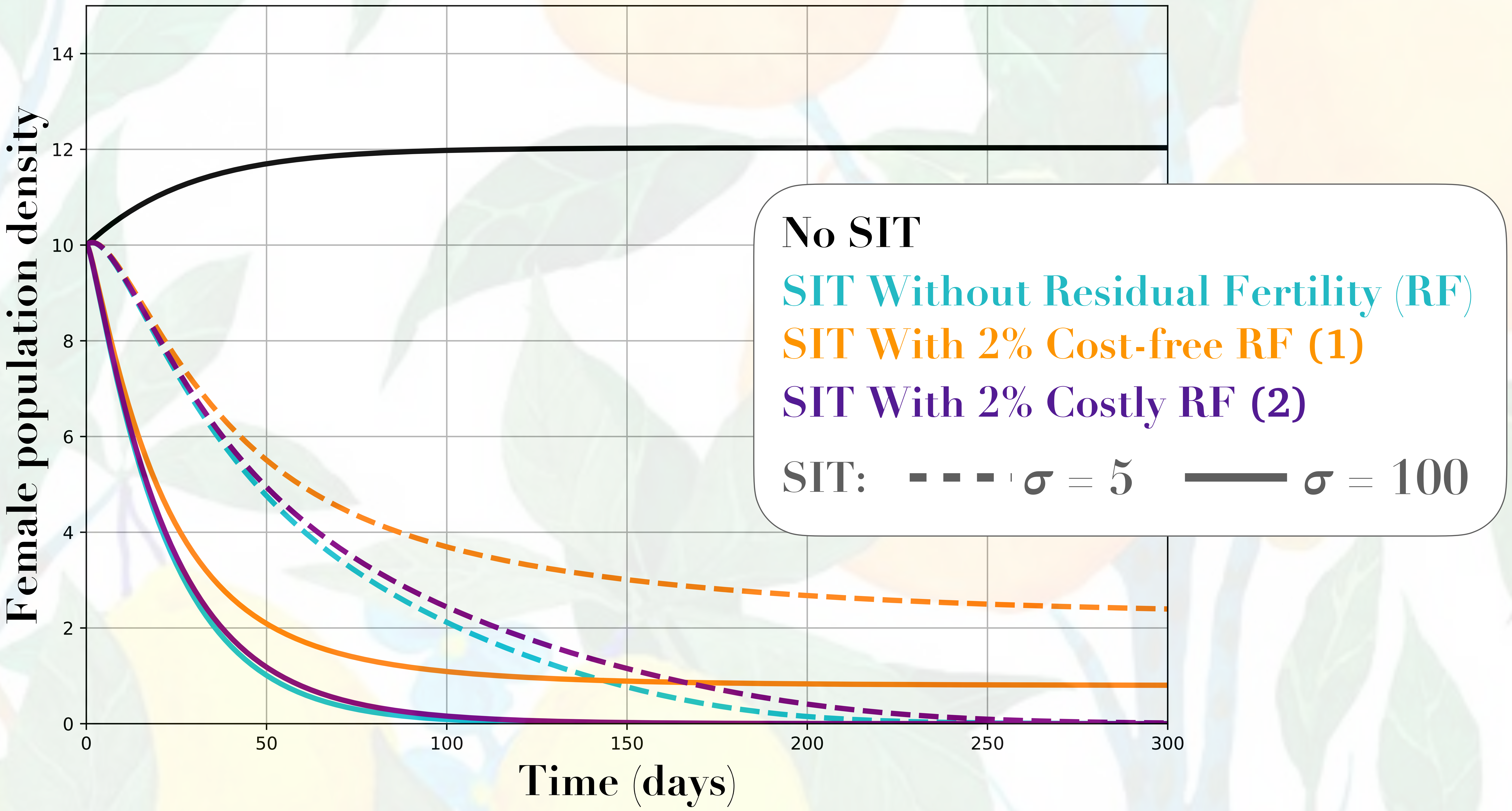
- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates



- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates
- Minimum $\boldsymbol{\sigma}$? If we increase it there is no more consequence ?



Female population density
Time (days)
0
2
4
6
8
10
12
14
0
50
100
150
200
250
300
No SIT
SIT Without Residual Fertility (RF)
SIT With 2% Cost-free RF (1)
SIT With 2% Costly RF (2)
SIT: $\sigma = 5$


Female population density
Time (days)
0
2
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No SIT
SIT Without Residual Fertility (RF)
SIT With 2% Cost-free RF (1)
SIT With 2% Costly RF (2)
SIT: σ = 5 σ = 100

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- For **costly residual fertility**, SIT is effective at higher residual fertility rates

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  - $\boldsymbol{\sigma}$: number of individuals released per day per 100 m^2
  - Example: For SIT effectiveness with **2% of residual fertility**
    - Releases of **at least 500 sterile males per day per hectare** ($\boldsymbol{\sigma} = 5$)

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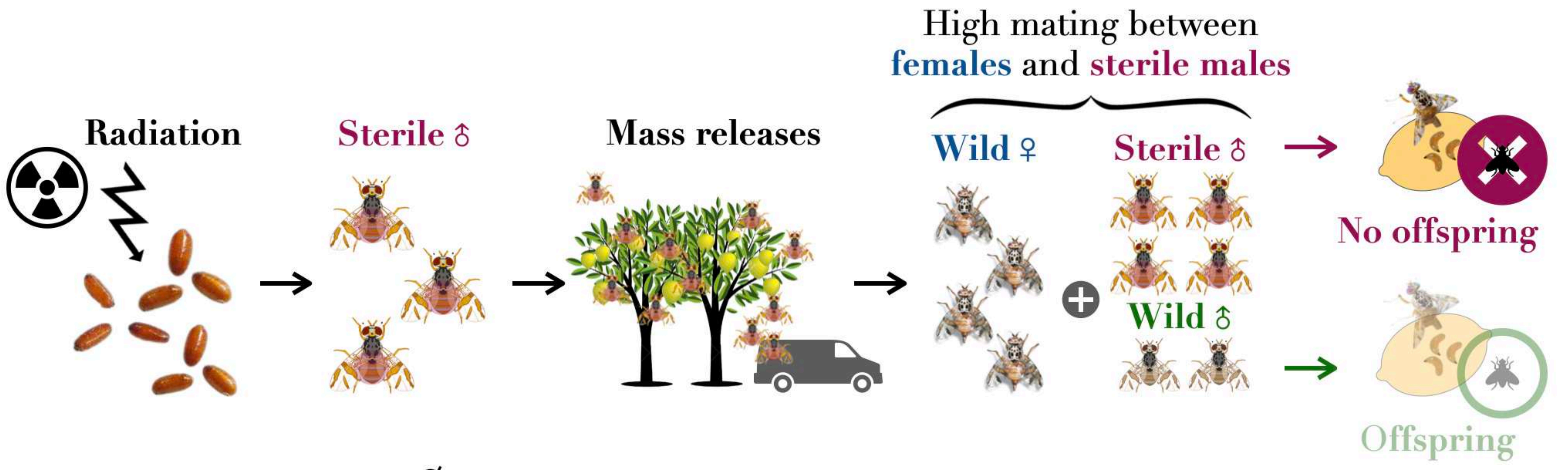
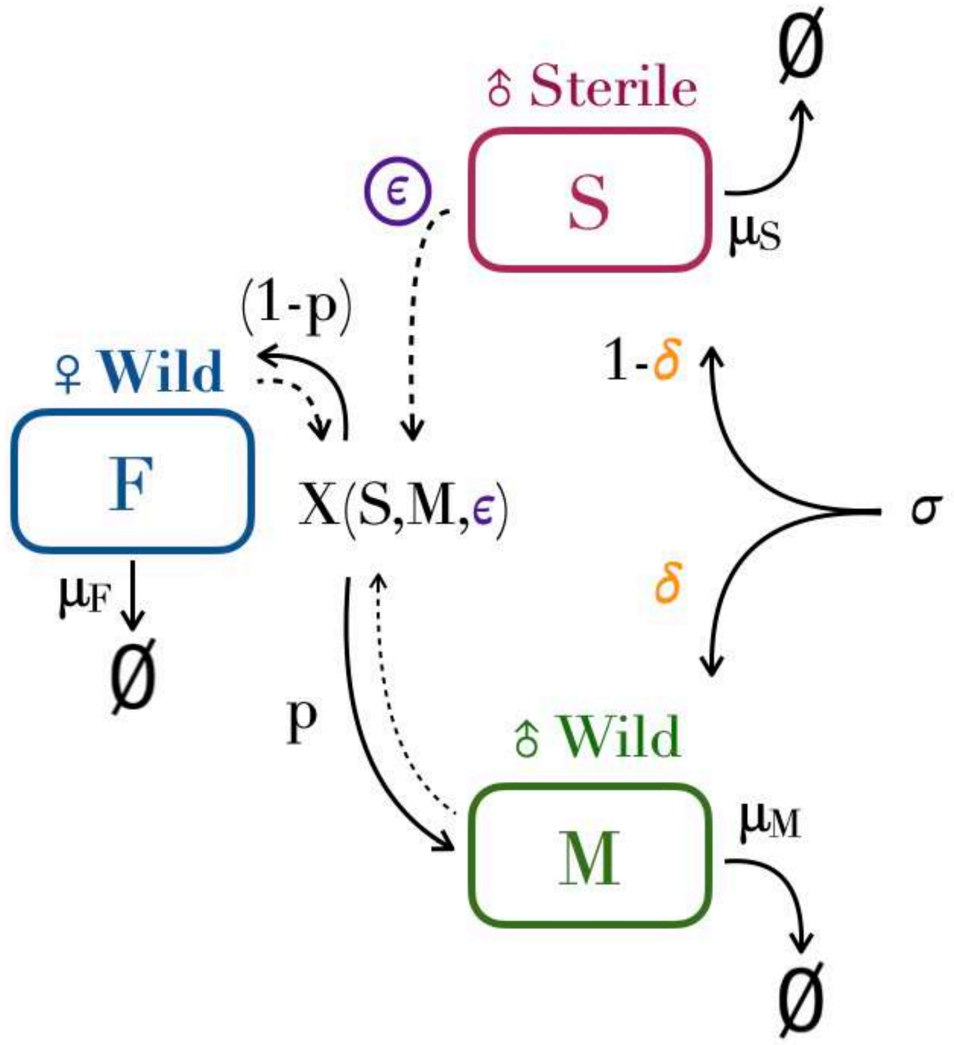
- **Ethics**
- **Control vs eradication**

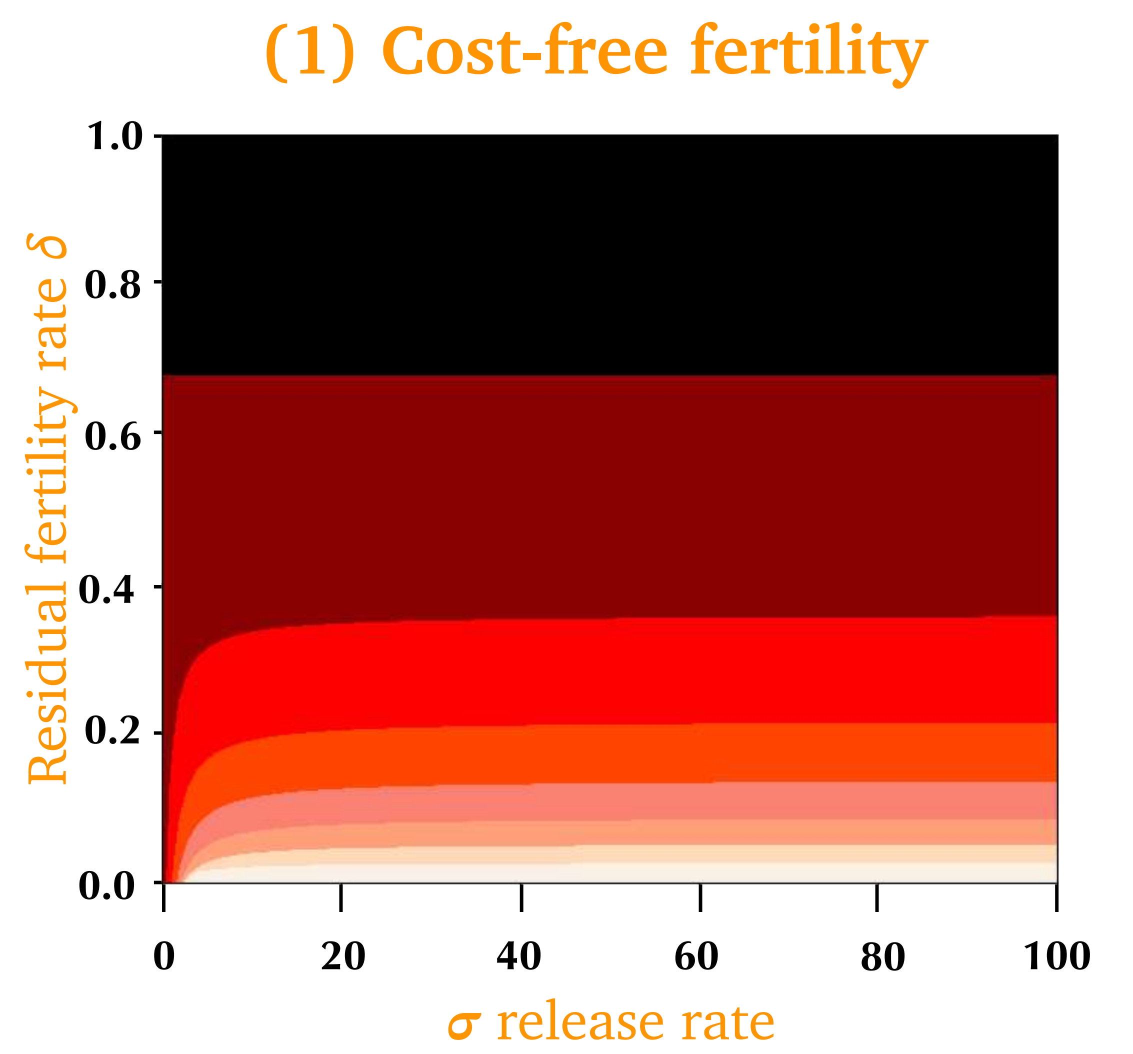
Thresholds: $\epsilon = \frac{1}{R} + q(1 - \frac{1}{R})$ | $\epsilon = \frac{1}{R}$

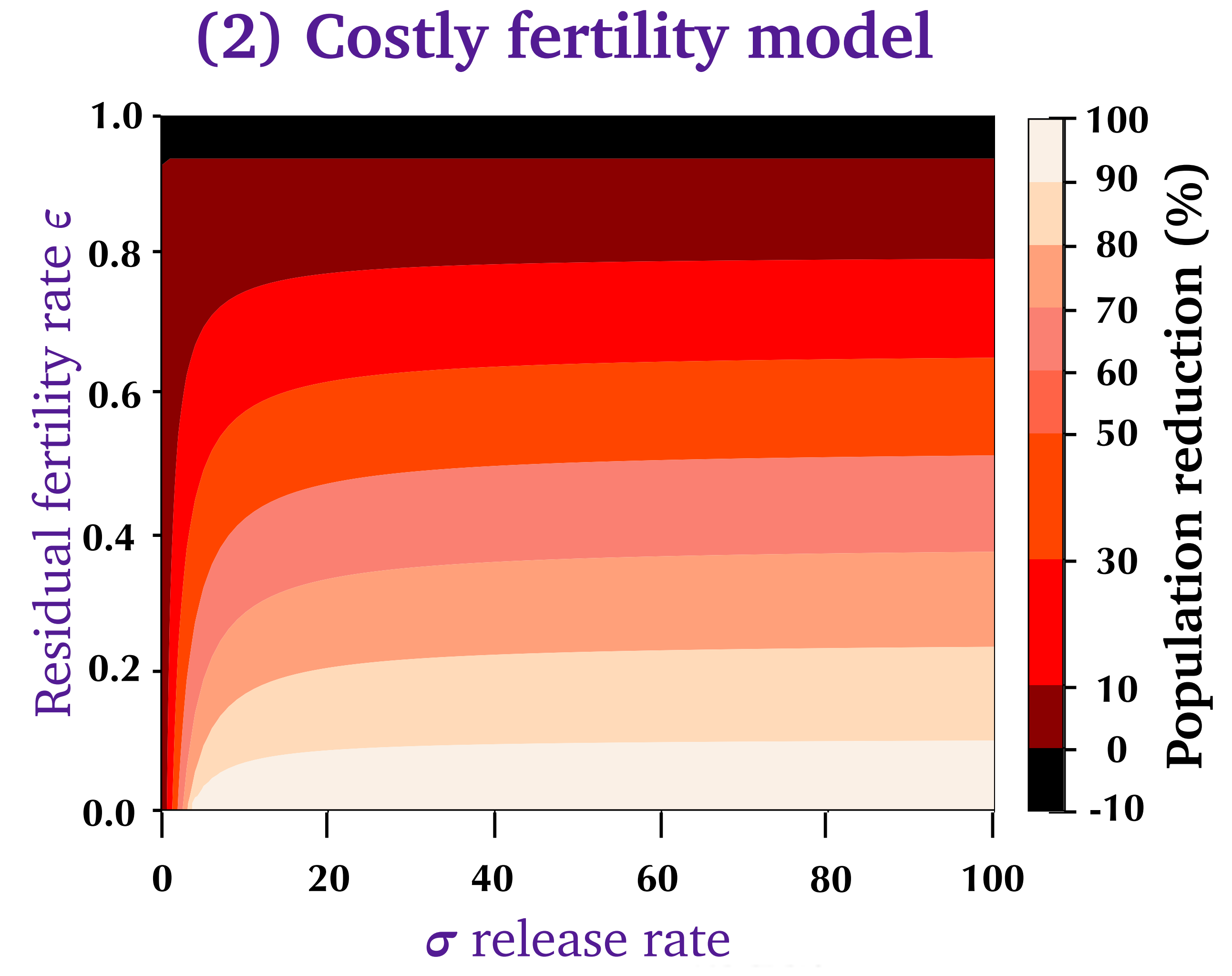
Thank you for your attention !





$$
\begin{cases}
\dot{S} = -\mu_S S + (1-\delta)\sigma \\
\dot{F} = -\mu_F F + r\,(1-p)\,X(S,M,\epsilon)\,C(F)\,F \\
\dot{M} = -\mu_M M + r\,p\,X(S,M,\epsilon)\,C(F)\,F + \delta\,\sigma
\end{cases}
$$





$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma \end{cases}$$

➡️ **Parameters** have been estimated **from the literature**

✅ **Easy parameters to estimate**

μ: mortality rate

r: emergence rate

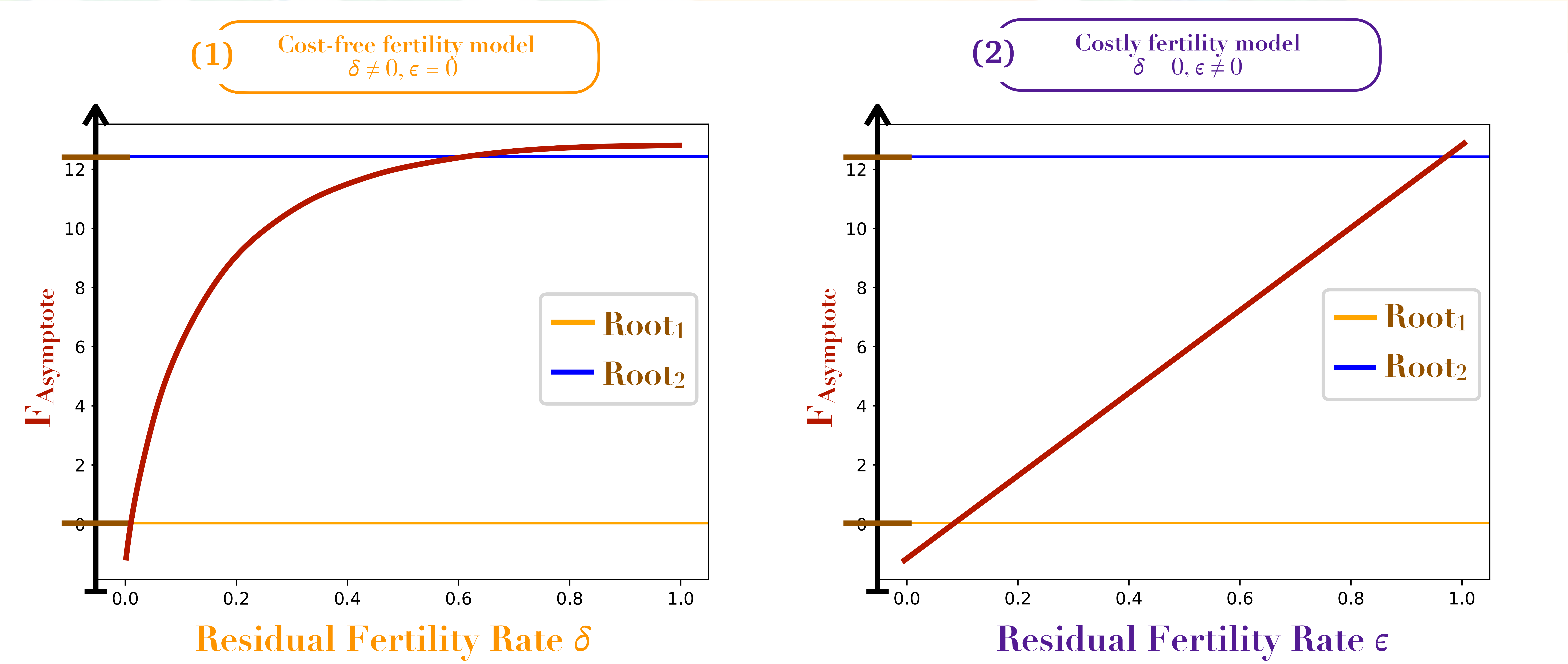
p: proportion of males

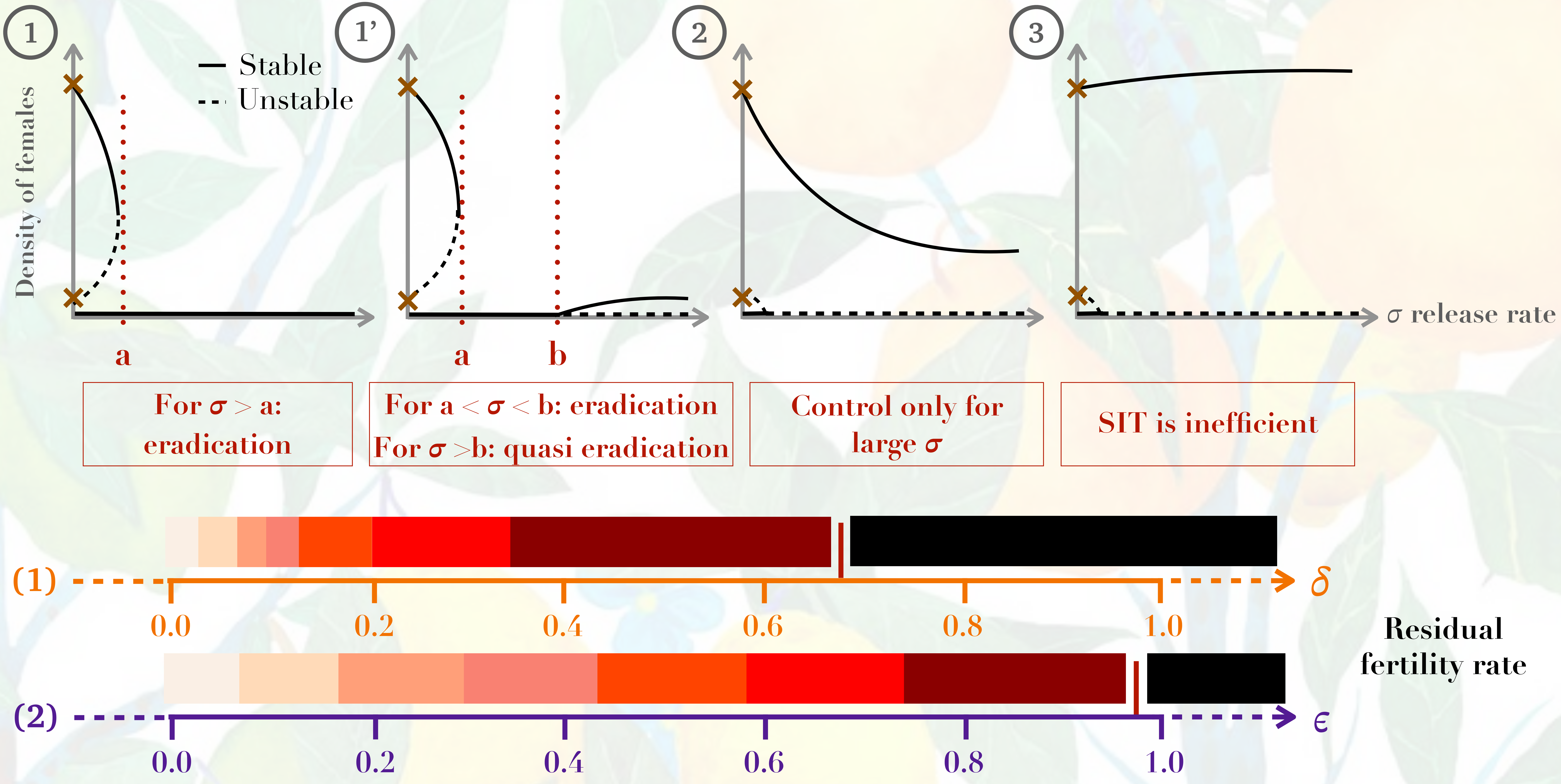
❌ **Not easy parameters to estimate**

C(F): competition $\frac{1}{1+\beta F}$

β: oviposition competition between females

η: Sterilisation cost


(1)
Cost-free fertility model
$\delta \neq 0, \epsilon = 0$
$F_{Asymptote}$
Root$_1$
Root$_2$
Residual Fertility Rate δ
(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$
$F_{Asymptote}$
Root$_1$
Root$_2$
Residual Fertility Rate ϵ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