How does residual fertility impact the effectiveness of the sterile insect technique in controlling Ceratitis capitata?

Marine Courtois, Kévan Rastello, Frédéric Grognard, Ludovic Mailleret, Suzanne Touzeau, Louise van Oudenhove

How does residual fertility impact the effectiveness of the Sterile Insect Technique (SIT) in controlling *Ceratitis capitata*?

Marine Courtois, Kévan Rastello, Frédéric Grognard, Ludovic Mailleret, Suzanne Touzeau, Louise van Oudenhove

Mathematical Population Dynamics, Ecology and Evolution - **MPDEE 2023**
CIRM - 26 April 2023

Pest Insects

Pest Insects

Ceratitis capitata

Damages

Ceratitis capitata

Damages

Ceratitis capitata

Damages

Ceratitis capitata

Crop yields

Damages

Ceratitis capitata

Crop yields

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

D

Means of control

↓

Crop yields

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

Crop yields

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

D

Means of control

Preventive control

- Picking up and crushing fallen fruit
- Shallow tillage during winter

📉 Crop yields

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

D

Means of control

Preventive control

- Picking up and crushing fallen fruit
- Shallow tillage during winter

Alternative control

- Mass trapping
- « Attract and kill »
- Parasitoids
- SIT: Sterile Insect Technique

Means of control

Preventive control

- Picking up and crushing fallen fruit
- Shallow tillage during winter

Alternative control

- Mass trapping
- « Attract and kill »
- Parasitoids
- SIT: Sterile Insect Technique

📉 **Crop yields**

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)

Sterile Insect Technique

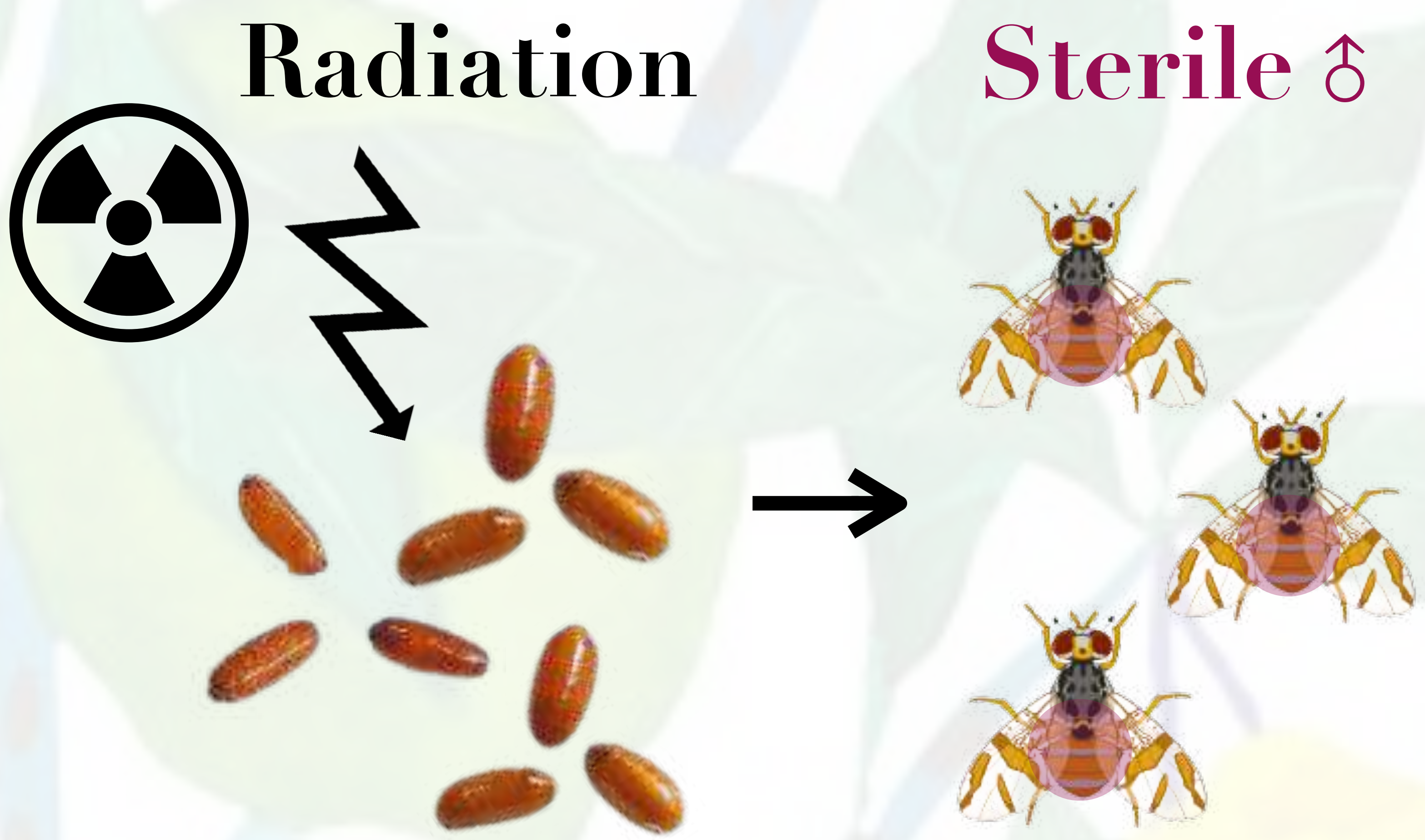
- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

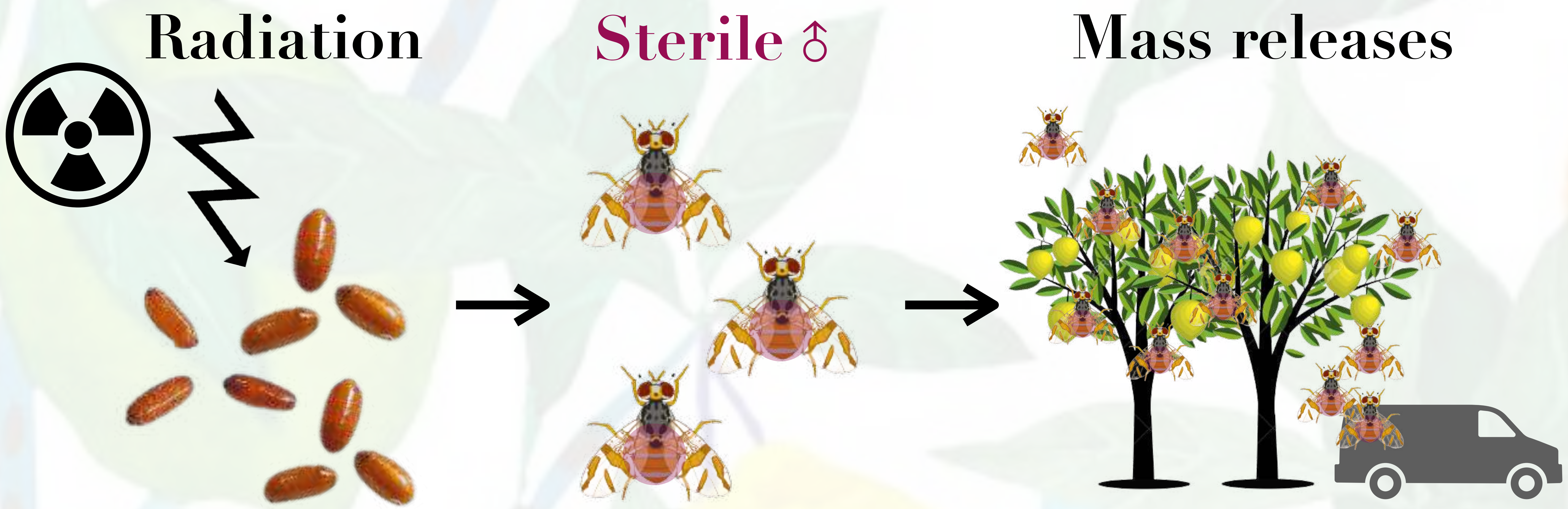
Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



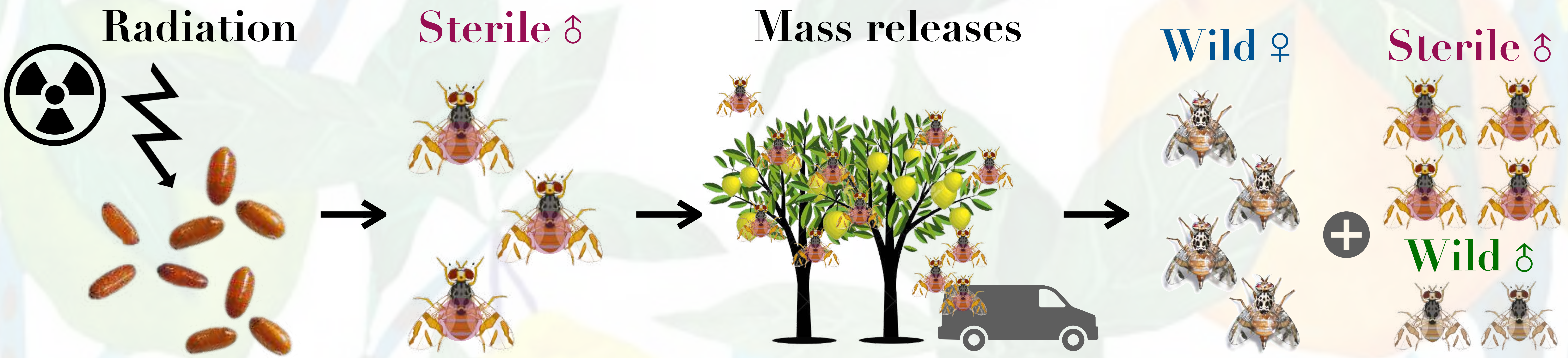
Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



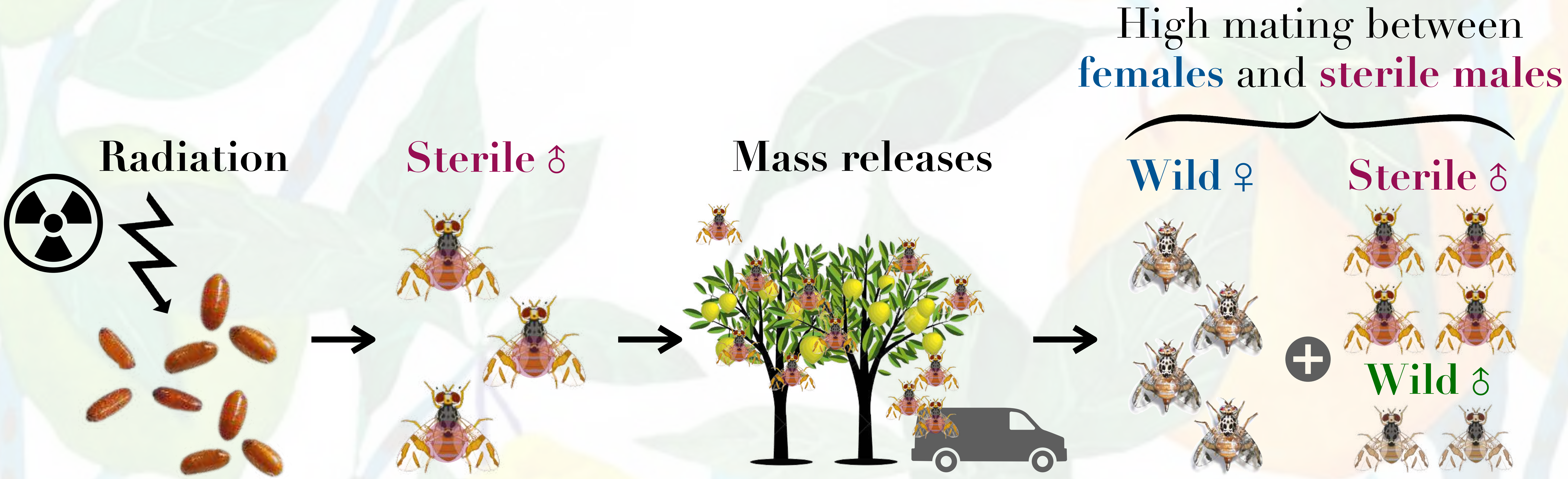
Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



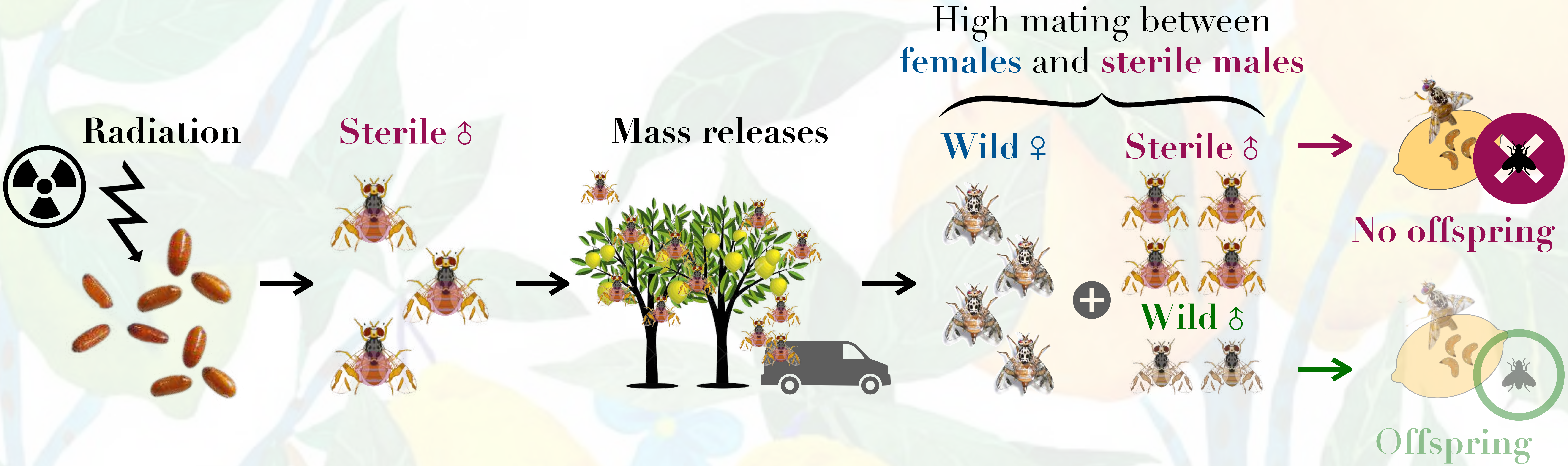
Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



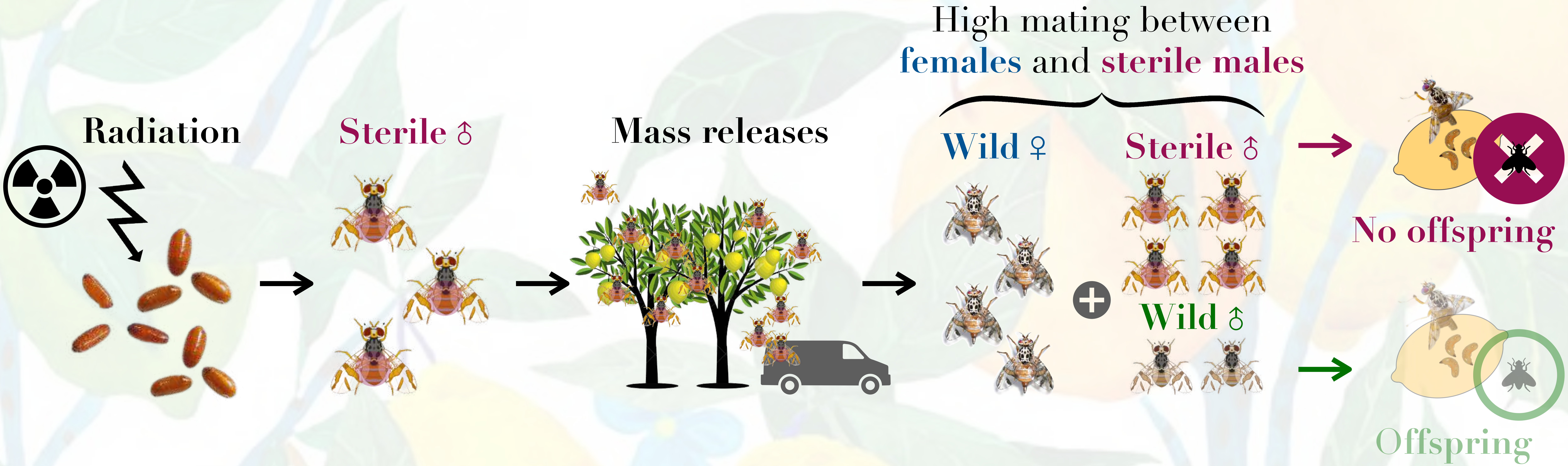
Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



Sterile Insect Technique

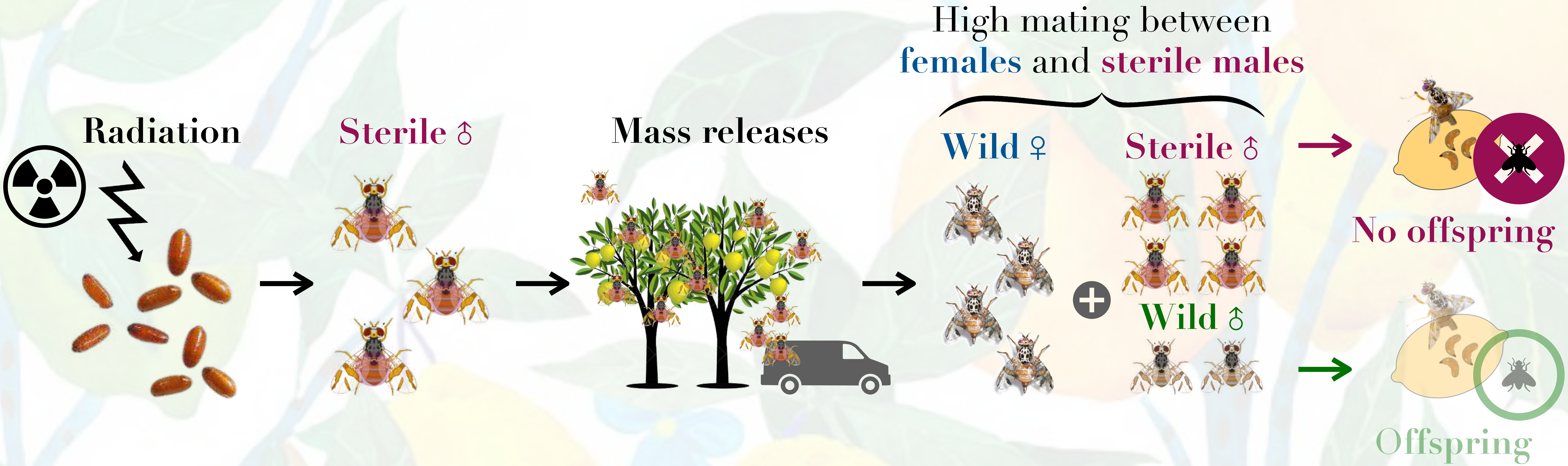
- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



How many insects to release? How often?

Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

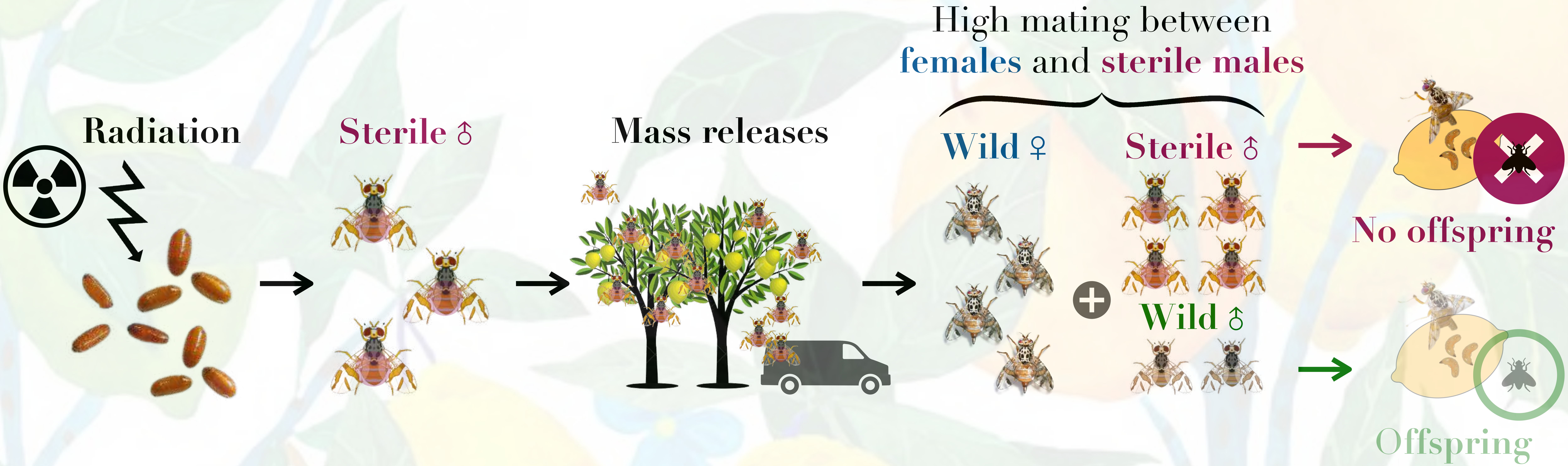


Are all released insects sterile?

How many insects to release? How often?

Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)



Are all released insects sterile?

How many insects to release? How often?

Population decrease?

Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

High mating between

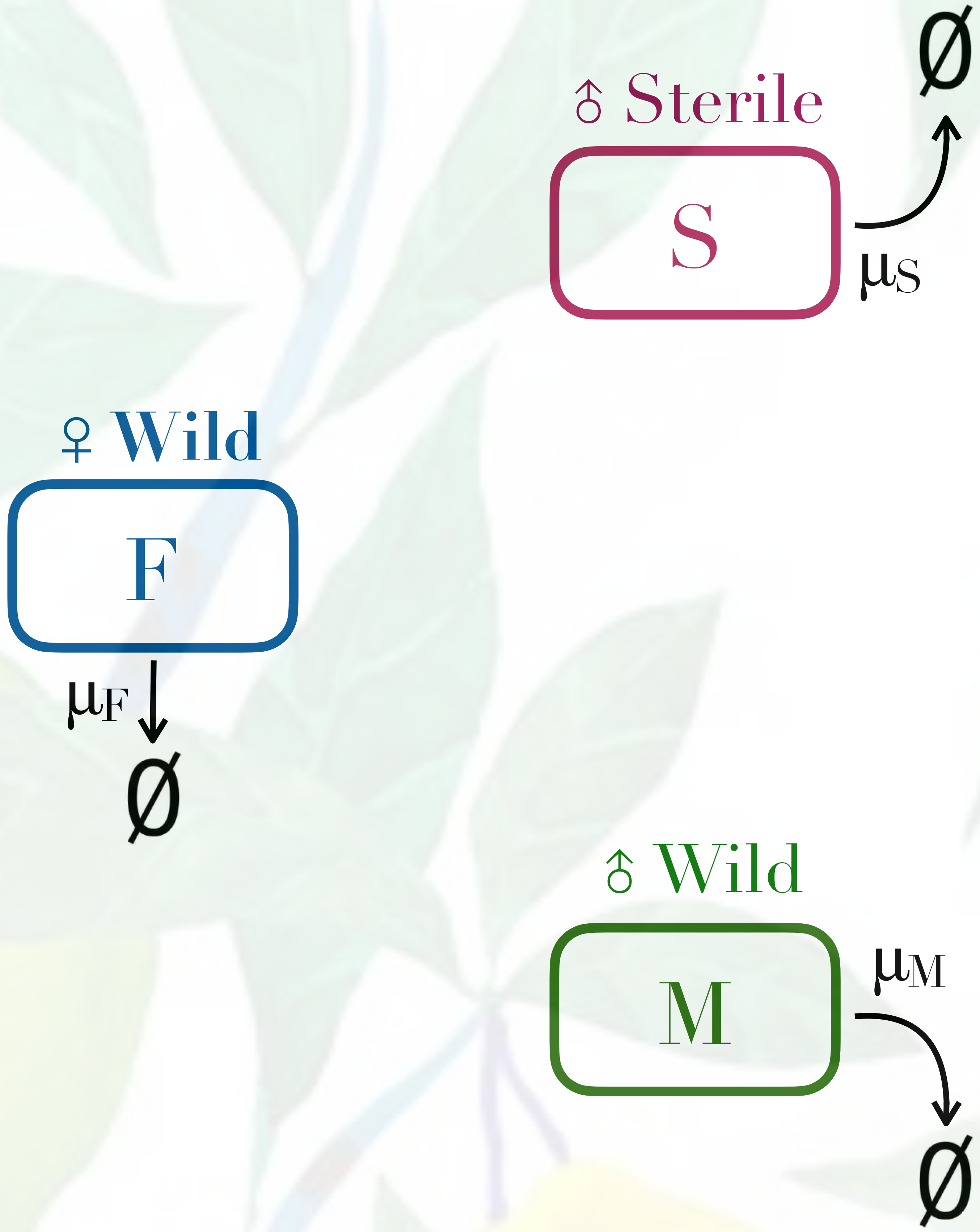
- **Main challenge: to determine how high the sterility rate should be to ensure pest control in the field**
- **Modeling represents an essential and efficient tool to tackle this issue (limitation of economic and temporal costs)**

Are all released insects sterile?

How many insects to release? How often?

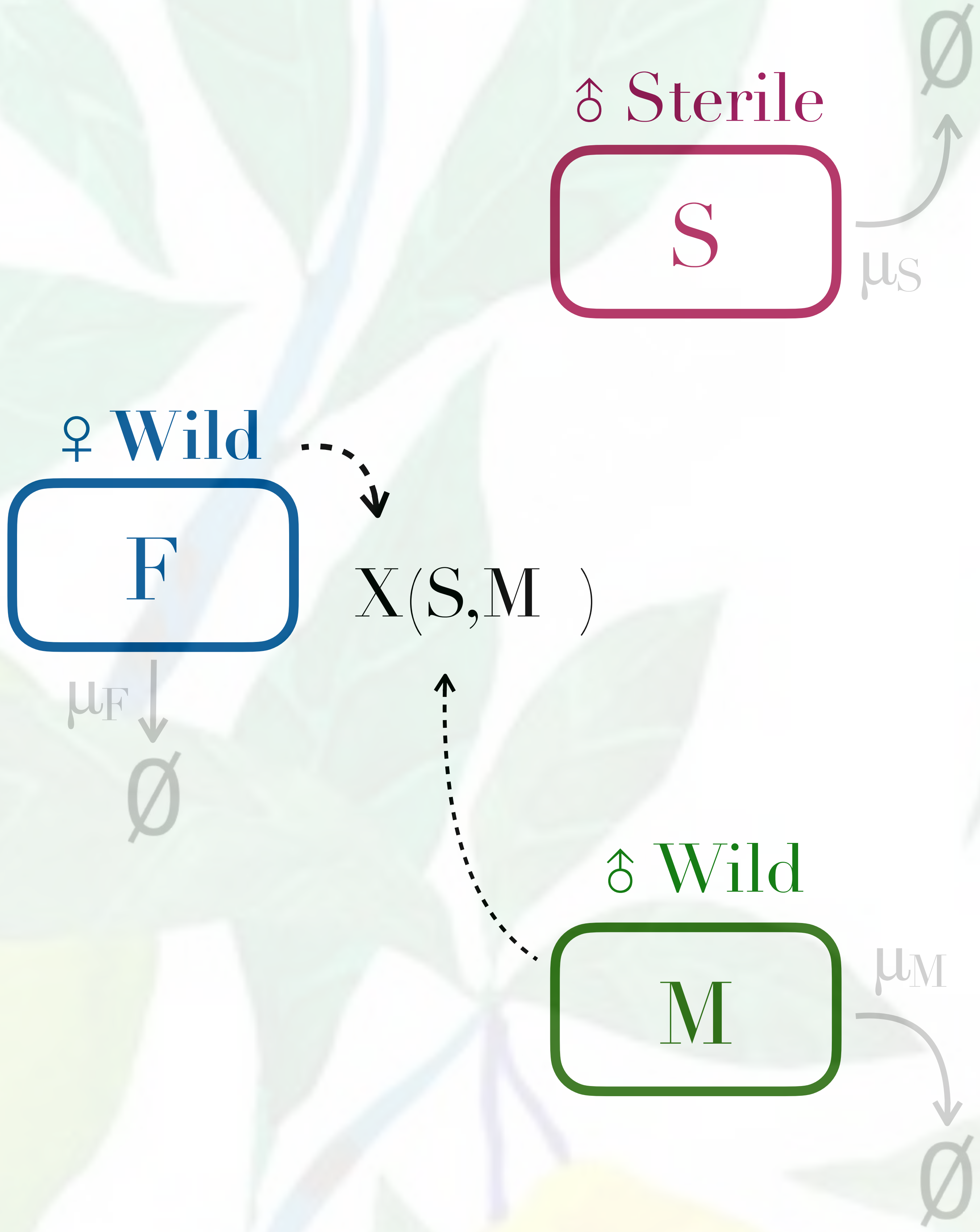
Population decrease?

$$\begin{cases} \dot{S} = \\ \dot{F} = \\ \dot{M} = \end{cases}$$



$$\begin{cases} \dot{S} = -\mu_S S \\ \dot{F} = -\mu_F F \\ \dot{M} = -\mu_M M \end{cases}$$

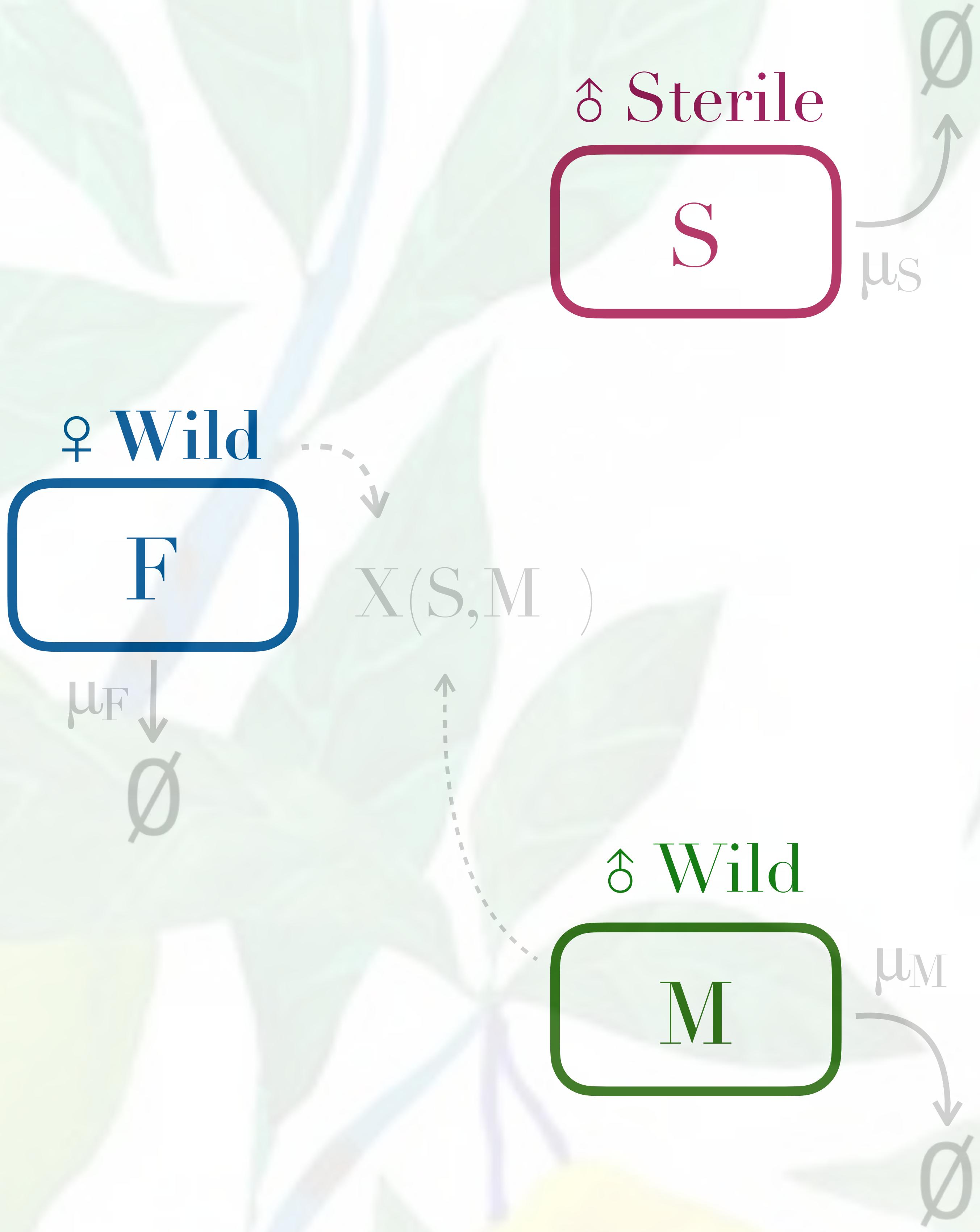
μ: mortality rate



$$\begin{cases} \dot{S} = -\mu_S S \\ \dot{F} = -\mu_F F + r(1-p)\, X(S,M)\, C(F)\, F \\ \dot{M} = -\mu_M M + r\, p\, X(S,M)\, C(F)\, F \end{cases}$$

μ: mortality rate

X(S,M): mating probability $\dfrac{M}{k+M+S}$

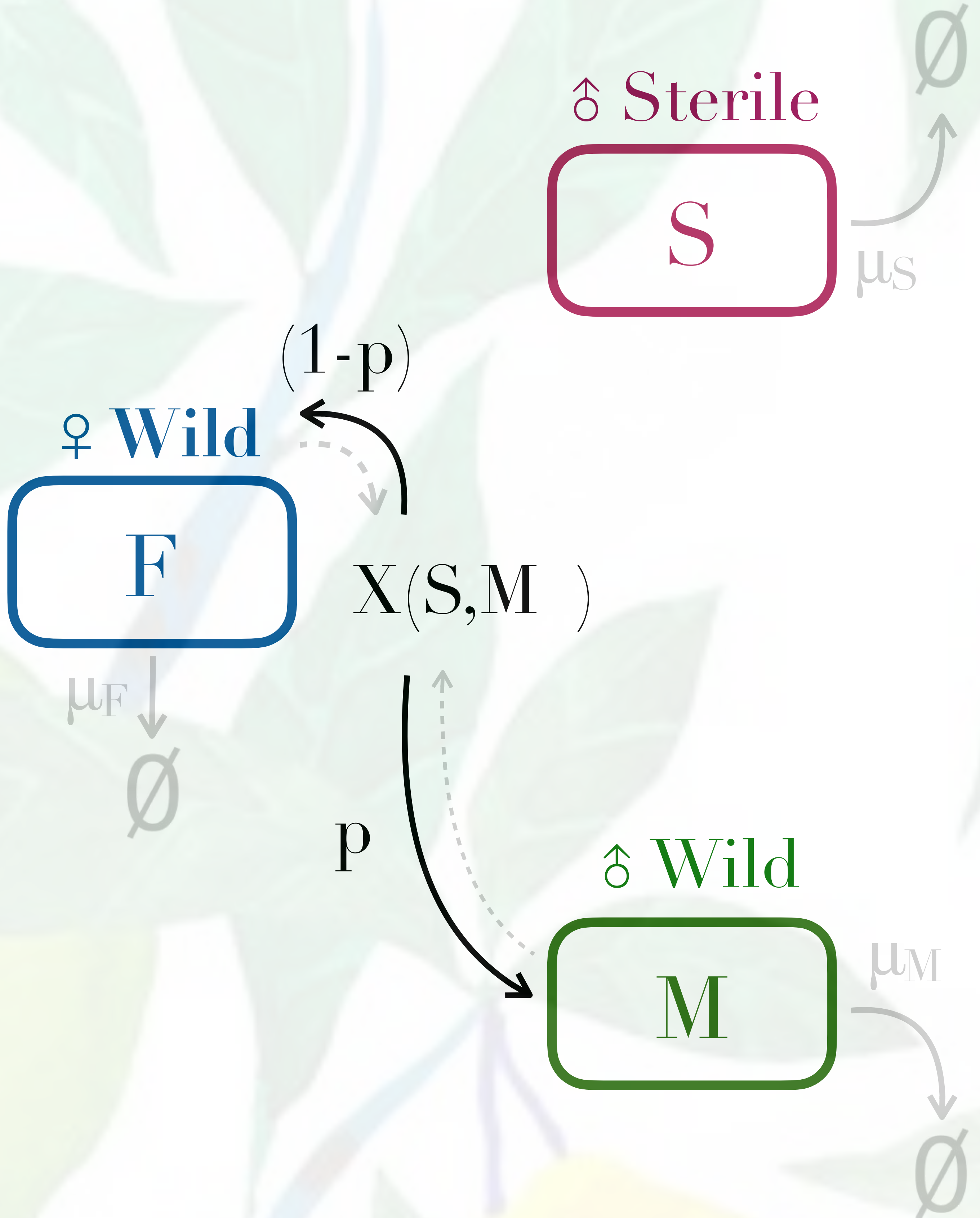


$$
\begin{cases}
\dot{S} = -\mu_S S \\
\dot{F} = -\mu_F F + \mathbf{r}(1-p)\, X(S,M)\, C(F)\, F \\
\dot{M} = -\mu_M M + \mathbf{r}\, p\, X(S,M)\, C(F)\, F
\end{cases}
$$

μ: mortality rate

X(S,M): mating probability $\frac{M}{k+M+S}$

r: emergence rate



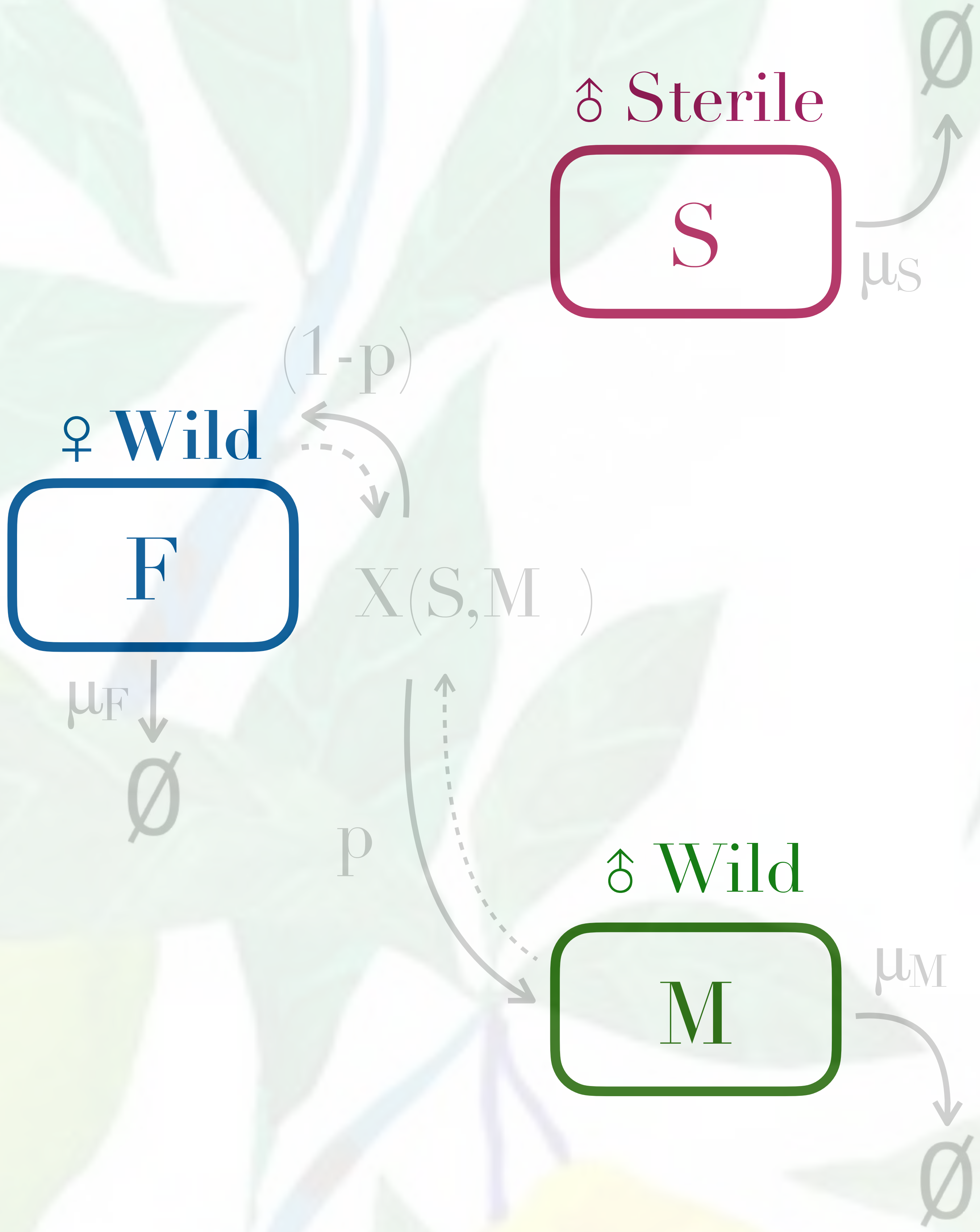
$$\begin{cases} \dot{S} = -\mu_S S \\ \dot{F} = -\mu_F F + r(1-p)\, X(S,M)\, C(F)\, F \\ \dot{M} = -\mu_M M + r\, p\, X(S,M)\, C(F)\, F \end{cases}$$

μ: mortality rate

r: emergence rate

p: proportion of males

X(S,M): mating probability $\frac{M}{k+M+S}$



$$\begin{cases} \dot{S} = -\mu_S S \\ \dot{F} = -\mu_F F + r(1-p)\,X(S,M)\,C(F)\,F \\ \dot{M} = -\mu_M M + r\,p\,X(S,M)\,C(F)\,F \end{cases}$$

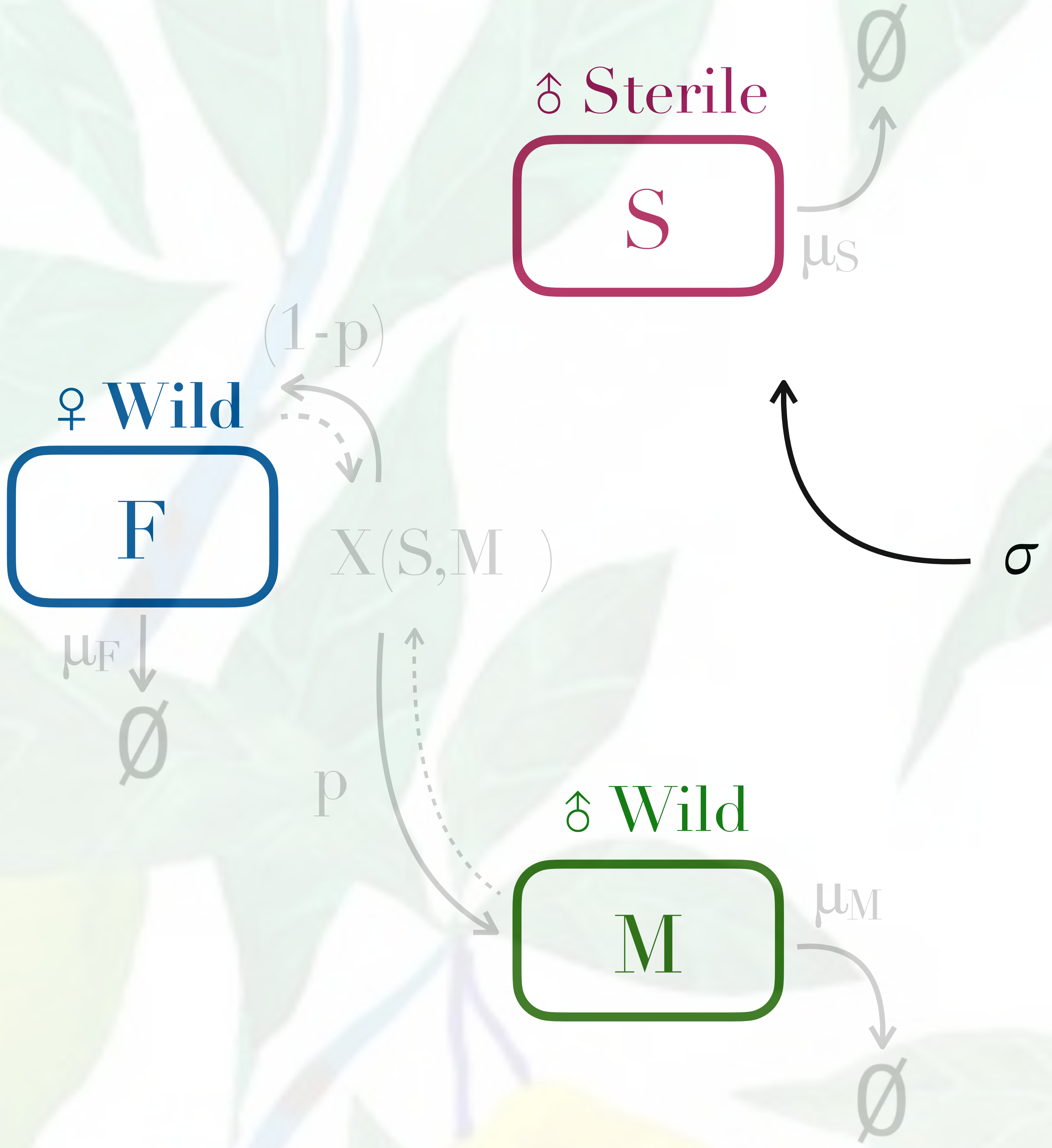
μ: mortality rate

r: emergence rate

p: proportion of males

X(S,M): mating probability $\frac{M}{k+M+S}$

C(F): competition $\frac{1}{1+\beta F}$



$$\begin{cases} \dot{S} = -\mu_S S + \sigma \\ \dot{F} = -\mu_F F + r(1-p)\, X(S,M)\, C(F)\, F \\ \dot{M} = -\mu_M M + r\, p\, X(S,M)\, C(F)\, F \end{cases}$$

μ: mortality rate

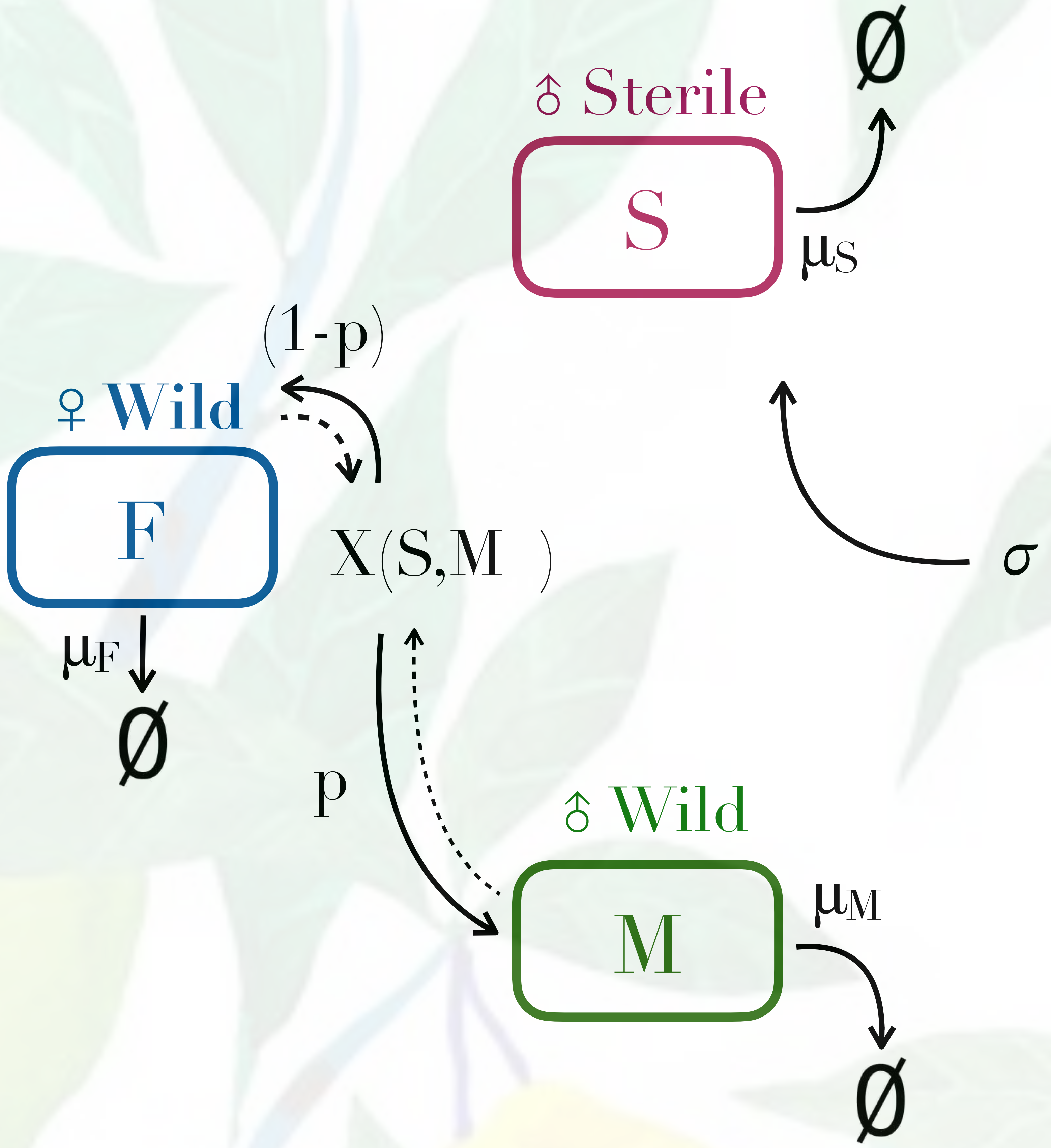
r: emergence rate

p: proportion of males

$X(S,M)$: mating probability $\frac{M}{k+M+S}$

$C(F)$: competition $\frac{1}{1+\beta F}$

σ: sterile male release rate



$$
\begin{cases}
\dot{S} = -\mu_S S + \sigma \\
\dot{F} = -\mu_F F + r(1-p)\, X(S,M)\, C(F)\, F \\
\dot{M} = -\mu_M M + r\, p\, X(S,M)\, C(F)\, F
\end{cases}
$$

μ: mortality rate

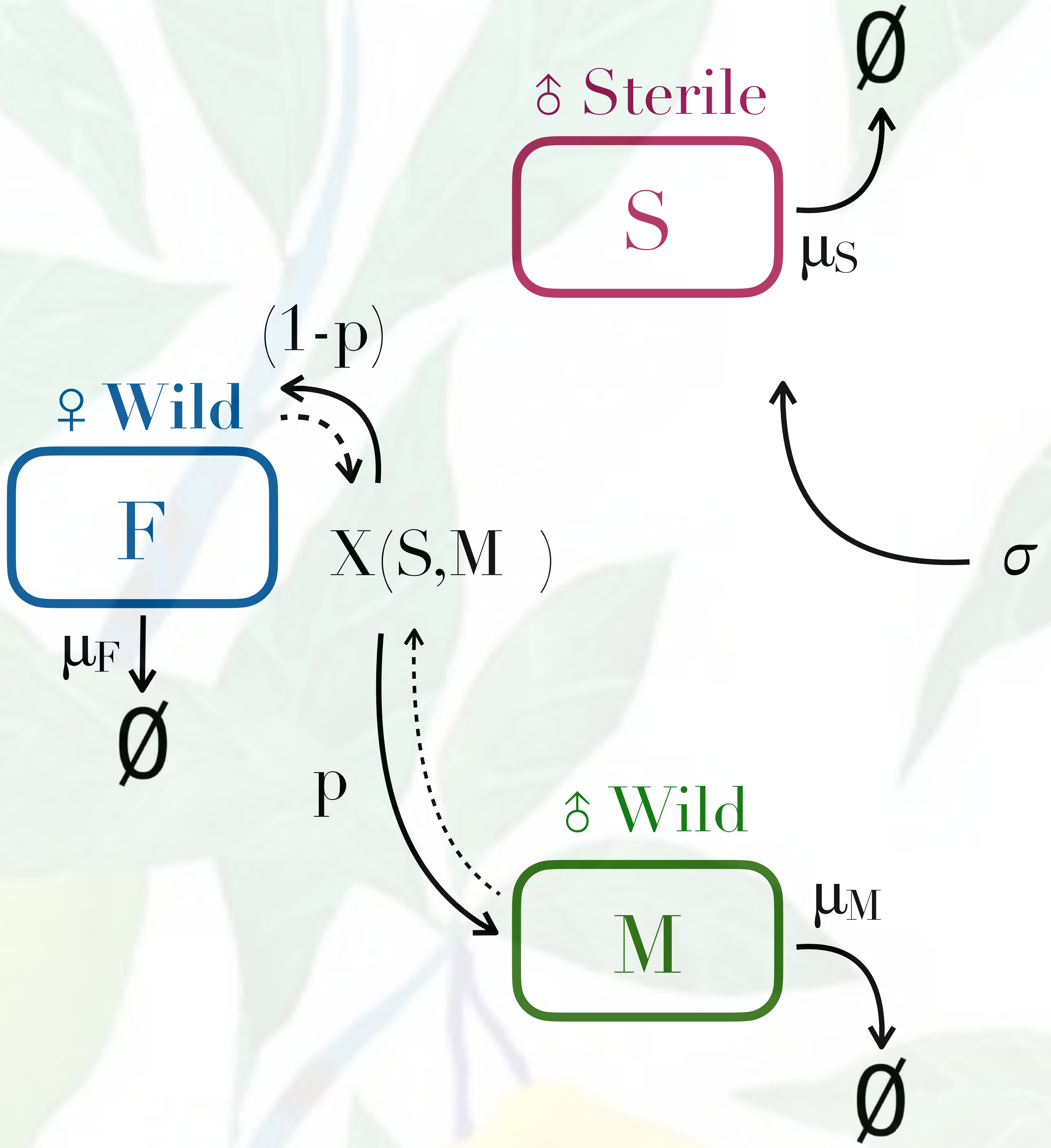
r: emergence rate

p: proportion of males

X(S,M): mating probability $\frac{M}{k+M+S}$

C(F): competition $\frac{1}{1+\beta F}$

σ: sterile male release rate

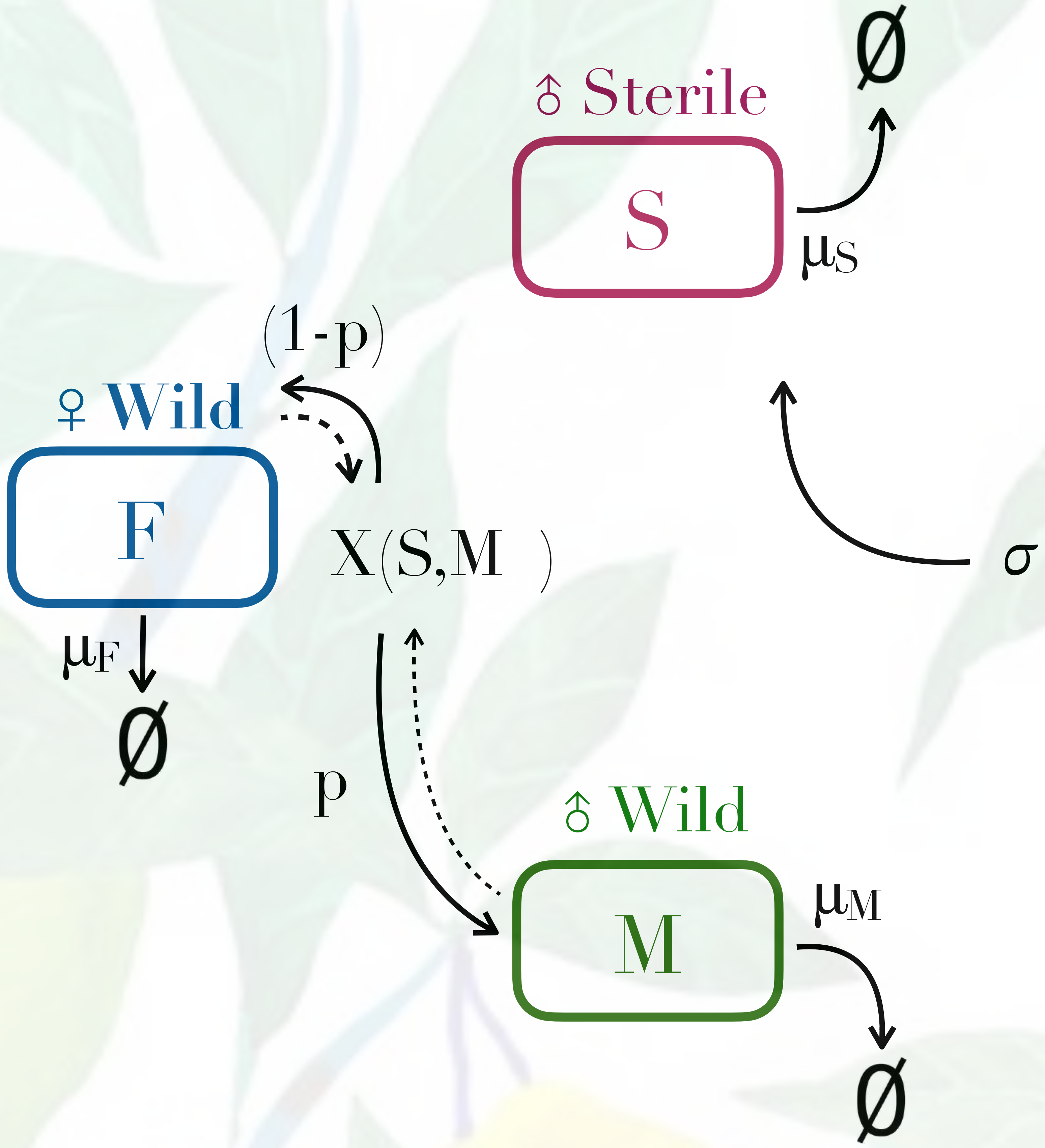


$$\begin{cases} \dot{S} = -\mu_S S + \sigma \\ \dot{F} = -\mu_F F + r(1-p)\,X(S,M)\,C(F)\,F \\ \dot{M} = -\mu_M M + r\,p\,X(S,M)\,C(F)\,F \end{cases}$$

Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

X(S,M): mating probability $\frac{M}{k+M+S}$



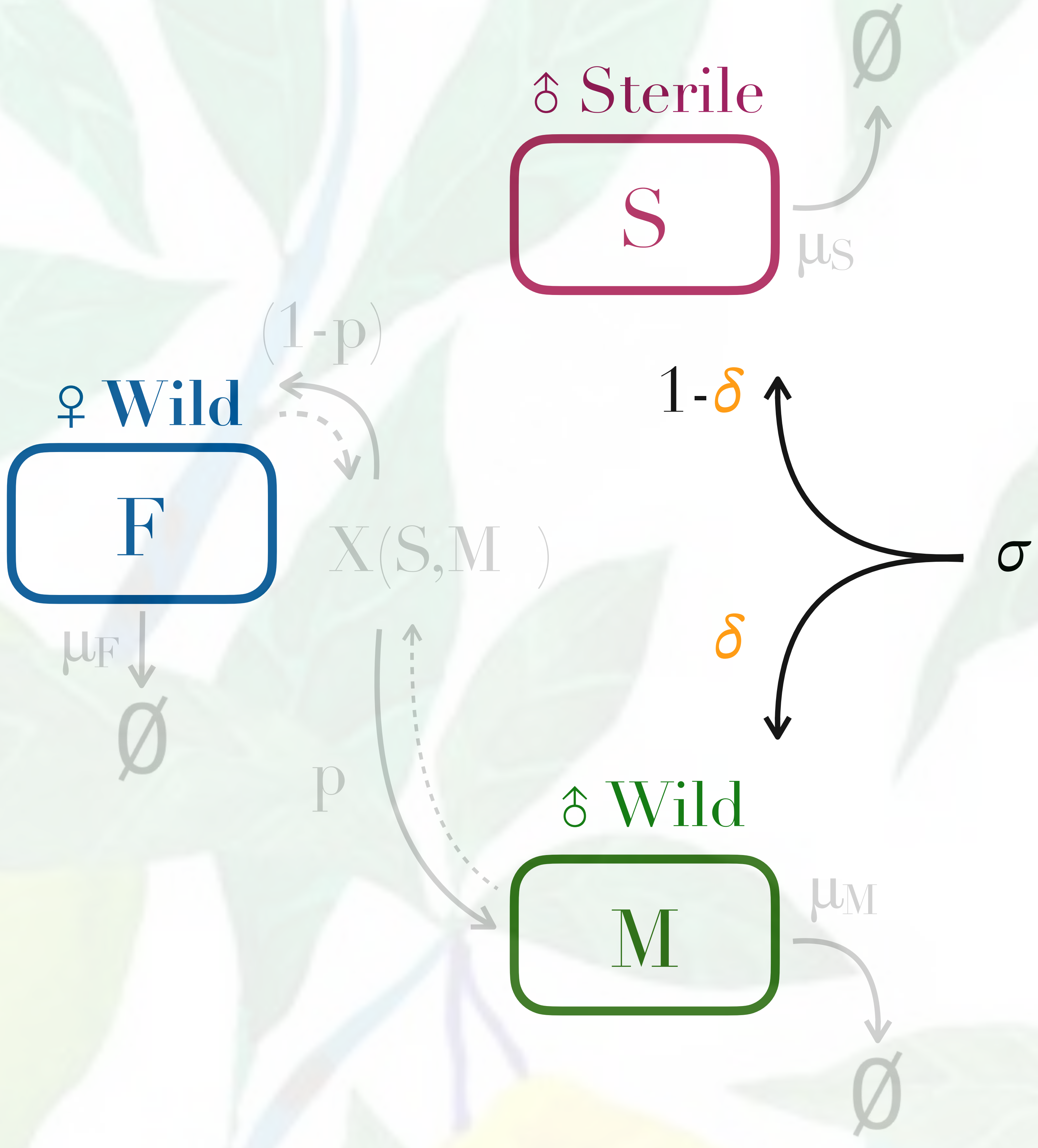
$$\begin{cases} \dot{S} = -\mu_S S + \sigma \\ \dot{F} = -\mu_F F + r(1-p)\,X(S,M)\,C(F)\,F \\ \dot{M} = -\mu_M M + r\,p\,X(S,M)\,C(F)\,F \end{cases}$$

Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

(0)
No ~~Residual fertility~~
$\delta = 0, \epsilon = 0$

X(S,M): mating probability $\frac{M}{k+M+S}$



$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)\,X(S,M)\,C(F)\,F \\ \dot{M} = -\mu_M M + r\,p\,X(S,M)\,C(F)\,F + \delta\sigma \end{cases}$$

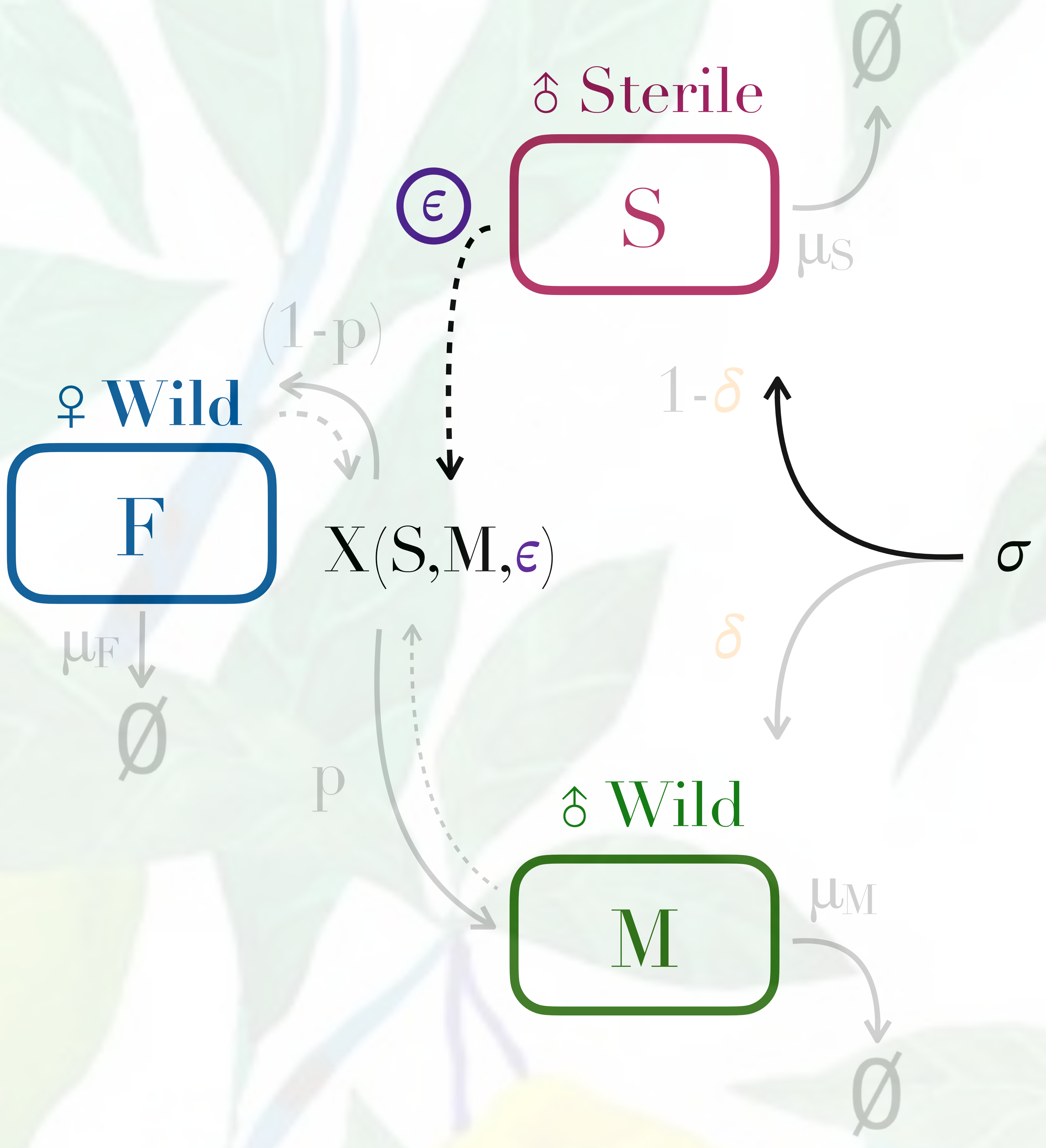
Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

(0) ~~No Residual fertility~~ $\delta = 0, \epsilon = 0$

(1) **Cost-free fertility model** $\delta \neq 0, \epsilon = 0$

$X(S,M)$: mating probability $\dfrac{M}{k+M+S}$



$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)\, X(S,M,\epsilon)\, C(F)\, F \\ \dot{M} = -\mu_M M + r\, p\, X(S,M,\epsilon)\, C(F)\, F + \delta\,\sigma \end{cases}$$

Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

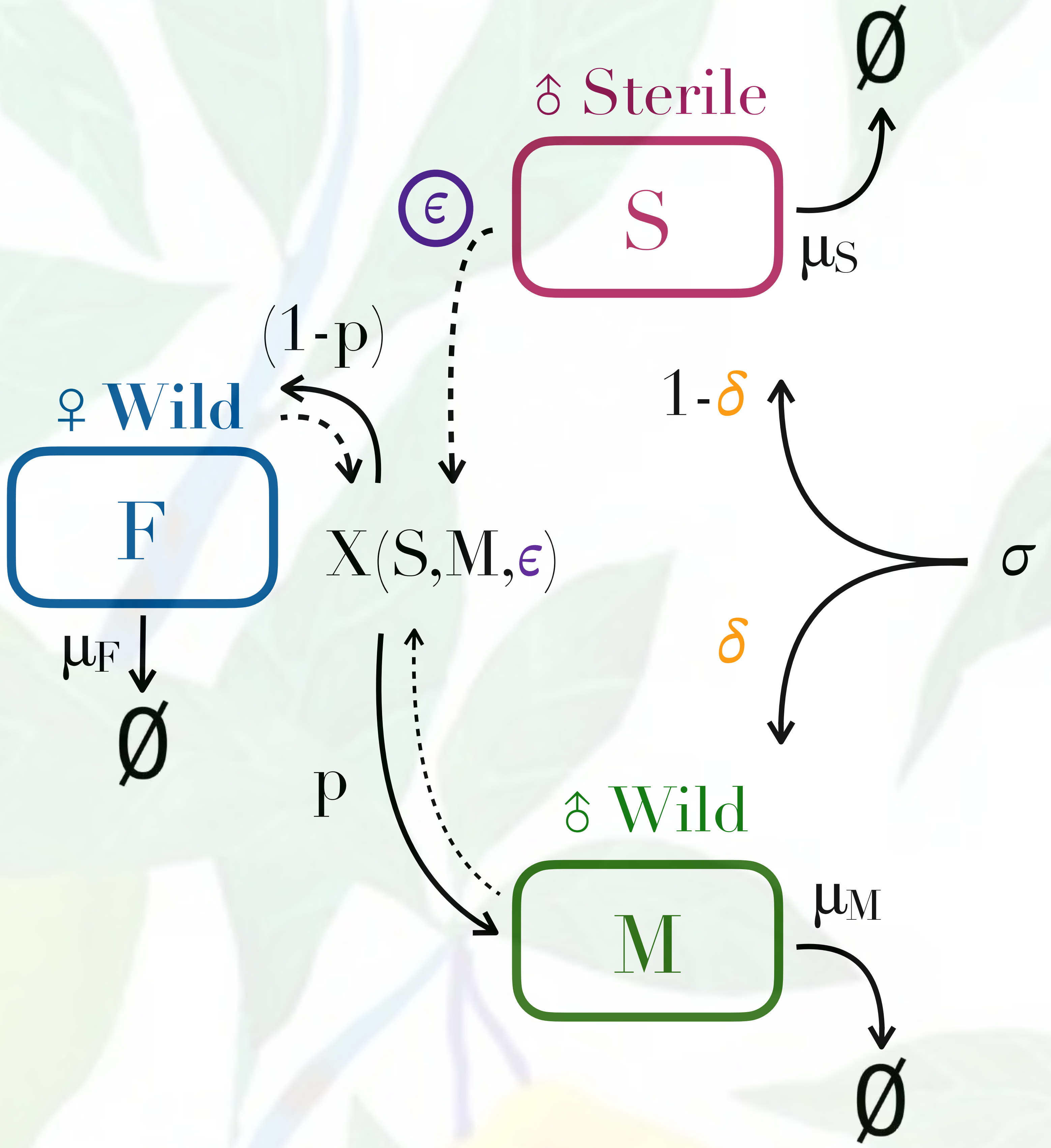
(0) No Residual fertility $\delta = 0, \epsilon = 0$

(1) Cost-free fertility model $\delta \neq 0, \epsilon = 0$

(2) **Costly fertility model** $\delta = 0, \epsilon \neq 0$

X(S,M,ϵ): mating probability $\dfrac{M+\epsilon\eta S}{k+M+\eta S}$

η: Fitness cost



$$
\begin{cases}
\dot{S} = -\mu_S S + (1-\delta)\sigma \\
\dot{F} = -\mu_F F + r(1-p)\, X(S,M,\epsilon)\, C(F)\, F \\
\dot{M} = -\mu_M M + r\, p\, X(S,M,\epsilon)\, C(F)\, F + \delta\, \sigma
\end{cases}
$$

Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

(0) No ~~Residual fertility~~ — $\delta = 0, \epsilon = 0$

(1) Cost-free fertility model — $\delta \neq 0, \epsilon = 0$

(2) Costly fertility model — $\delta = 0, \epsilon \neq 0$

X(S,M,ϵ): mating probability $\dfrac{M+\epsilon\eta S}{k+M+\eta S}$

η: Fitness cost

Table 1: Model parameters

Parameters	Descriptions	Values	Units	References
μ_F	Female mortality rate	0.050	day^{-1}	Vargas *et al.* (2000) Pieterse *et al.* (2020)
μ_M	Male mortality rate	0.036	day^{-1}	Vargas *et al.* (2000) Pieterse *et al.* (2020)
μ_S	Sterile male mortality rate	0.057	day^{-1}	Calibrated value
p	Sex ratio	0.50	-	Pieterse *et al.* (2020)
r	Emergence rate (mean number of eggs leading to the adult stage per female)	1.19	eggs.♀$^{-1}$.day^{-1}	Shoukry and Hafez (1979) Carey (1982, 1984) Vargas *et al.* (1984, 2000) Krainacker *et al.* (1987) Duyck *et al.* (2002) Papadopoulos *et al.* (2002) Diamantidis *et al.* (2011)
k	Coupling half-saturation constant	1	♂ density	Calibrated value
β	Oviposition competition between females	0.85	(♀ density)$^{-1}$	Calibrated value
σ	Sterile male release rate	Variable	♂ density.day^{-1}	
$1-\eta$	Sterilization cost	0.8	-	Calibrated value
δ	Proportion of non-sterile males among the releases (cost-free fertility)	Variable	-	Studied value
ϵ	Proportion of non-sterile males among the releases (costly fertility)	Variable	-	Studied value

The values noted as "Calibrated values" were determined from laboratory data.

➡️ Search for equilibria to see when the population can't settle.

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \end{cases}$$

➡️ Search for equilibria to see when the population can't settle.

$$
\begin{cases}
\dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\
\dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \\
\dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0
\end{cases}
$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0 \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \end{cases}$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0 \\ & \qquad\qquad \swarrow \\ & \qquad F^* = 0 \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \end{cases}$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0 \\ & \qquad F^* = 0 \qquad X(S^*,M^*,\epsilon)C(F^*) = \dfrac{\mu_F}{r(1-p)} = \dfrac{1}{\mathcal{R}} \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \end{cases}$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0 \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \iff -\mu_M M^* + rpX(S,M^*)C(F^*)F^* + \delta\sigma = 0 \end{cases}$$

↙ ↘

$$F^* = 0 \qquad X(S^*,M^*,\epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma \end{cases}$$

$\Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S}$

$\Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0$

↙ $F^* = 0$

↘ $X(S^*,M^*,\epsilon)C(F^*) = \dfrac{\mu_F}{r(1-p)} = \dfrac{1}{\mathcal{R}}$

$\Rightarrow \quad \dot{M} = 0 \iff -\mu_M M^* + rpX(S,M^*)C(F^*)F^* + \delta\sigma = 0$

↙ $M^* = \dfrac{\delta\sigma}{\mu_M} = M_0^*$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma & \Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S} \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F & \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0 \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma & \Rightarrow \quad \dot{M} = 0 \iff -\mu_M M^* + rpX(S,M^*)C(F^*)F^* + \delta\sigma = 0 \end{cases}$$

From $\dot{F} = 0$: ↙ ↘

$$F^* = 0 \qquad X(S^*,M^*,\epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

From $\dot{M} = 0$: ↙ ↘

$$M^* = \frac{\delta\sigma}{\mu_M} = M_0^* \qquad M^* = M(F^*) = \frac{p\mu_F}{(1-p)\mu_M}F^* + \frac{\delta\sigma}{\mu_M}$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma \end{cases}$$

$\Rightarrow \quad \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S}$

$\Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*,M^*,\epsilon)C(F^*)) = 0$

$\swarrow$ $\boxed{F^* = 0}$

$\searrow$ $X(S^*,M^*,\epsilon)C(F^*) = \dfrac{\mu_F}{r(1-p)} = \dfrac{1}{\mathcal{R}}$

$\Rightarrow \quad \dot{M} = 0 \iff -\mu_M M^* + rpX(S,M^*)C(F^*)F^* + \delta\sigma = 0$

$\swarrow$ $\boxed{M^* = \dfrac{\delta\sigma}{\mu_M} = M_0^*}$

$\searrow$ $M^* = M(F^*) = \dfrac{p\mu_F}{(1-p)\mu_M}F^* + \dfrac{\delta\sigma}{\mu_M}$

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

➡️ Search for equilibria to see when the population can't settle.

$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma \end{cases}$$

$\Rightarrow \dot{S} = 0 \iff S^* = \dfrac{(1-\delta)\sigma}{\mu_S}$

$\Rightarrow \dot{F} = 0 \iff F^*(-\mu_F + r(1-p)X(S^*, M^*, \epsilon)C(F^*)) = 0$

↙ $F^* = 0$

↘ $X(S^*, M^*, \epsilon)C(F^*) = \dfrac{\mu_F}{r(1-p)} = \dfrac{1}{\mathcal{R}}$

$\Rightarrow \dot{M} = 0 \iff -\mu_M M^* + rpX(S, M^*)C(F^*)F^* + \delta\sigma = 0$

↙ $M^* = \dfrac{\delta\sigma}{\mu_M} = M_0^*$

↘ $M^* = M(F^*) = \dfrac{p\mu_F}{(1-p)\mu_M}F^* + \dfrac{\delta\sigma}{\mu_M}$

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

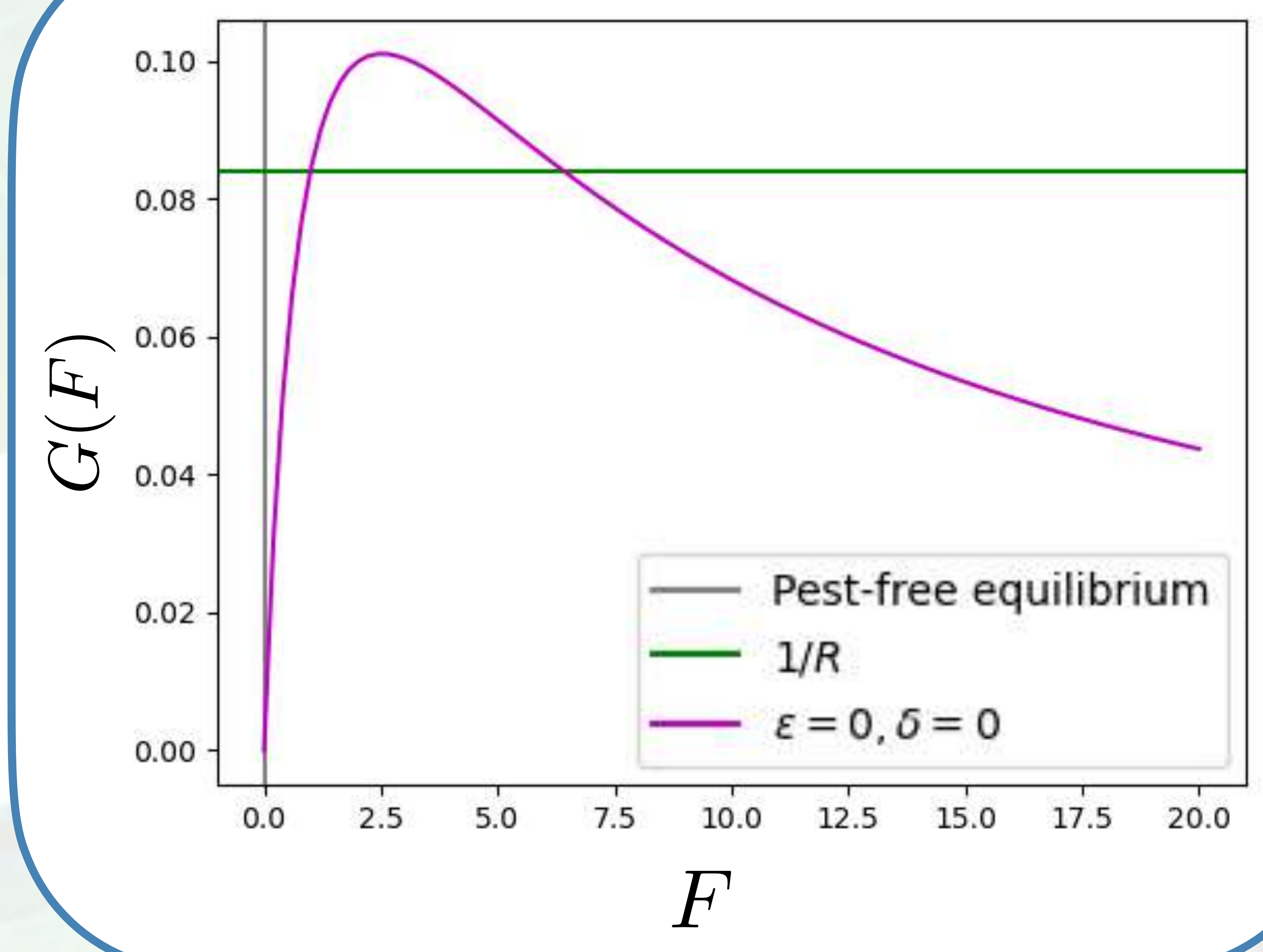
Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



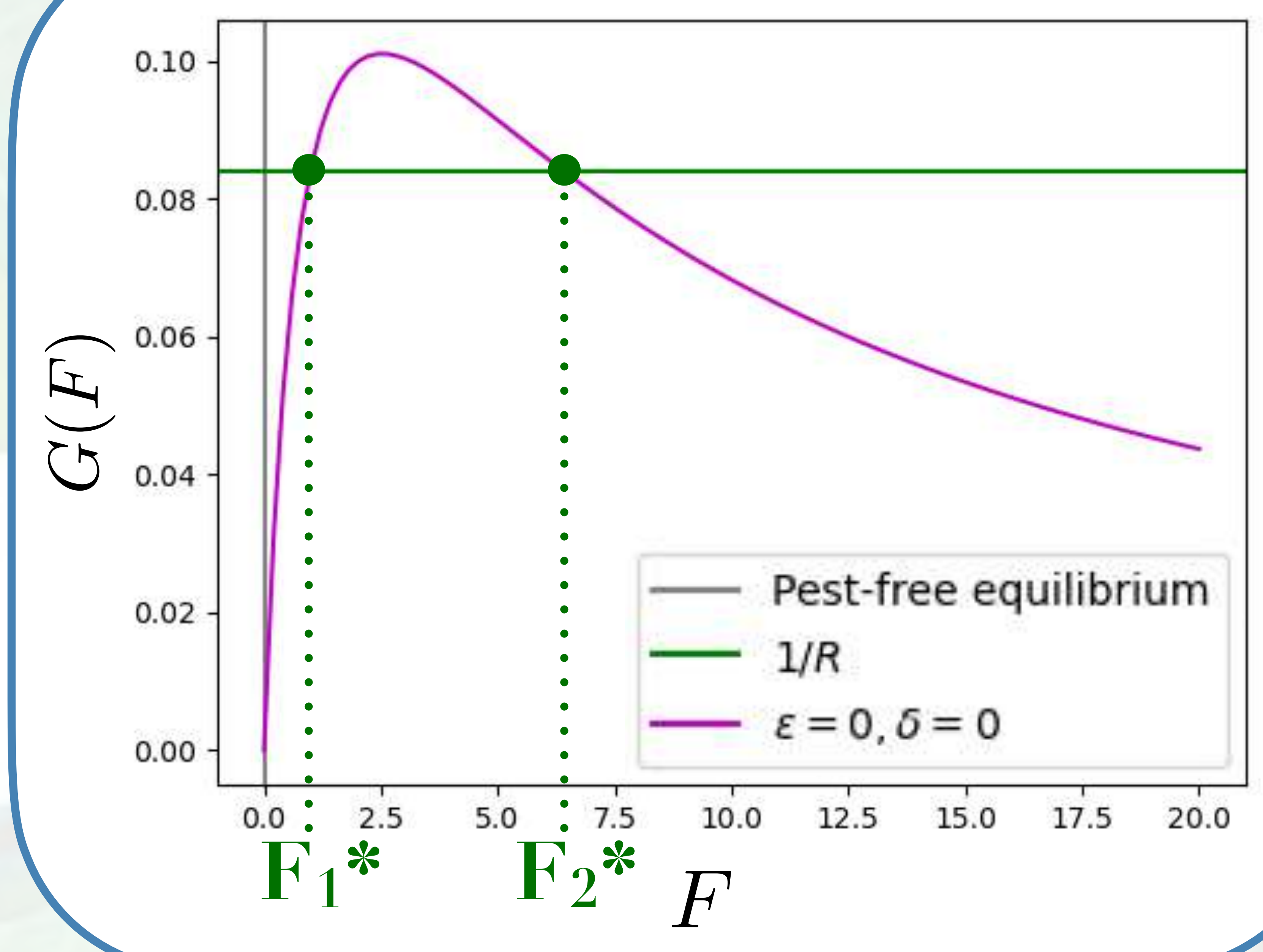
Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



At least **2 solutions** for which $F_2^* > F_1^* > 0$ such that:

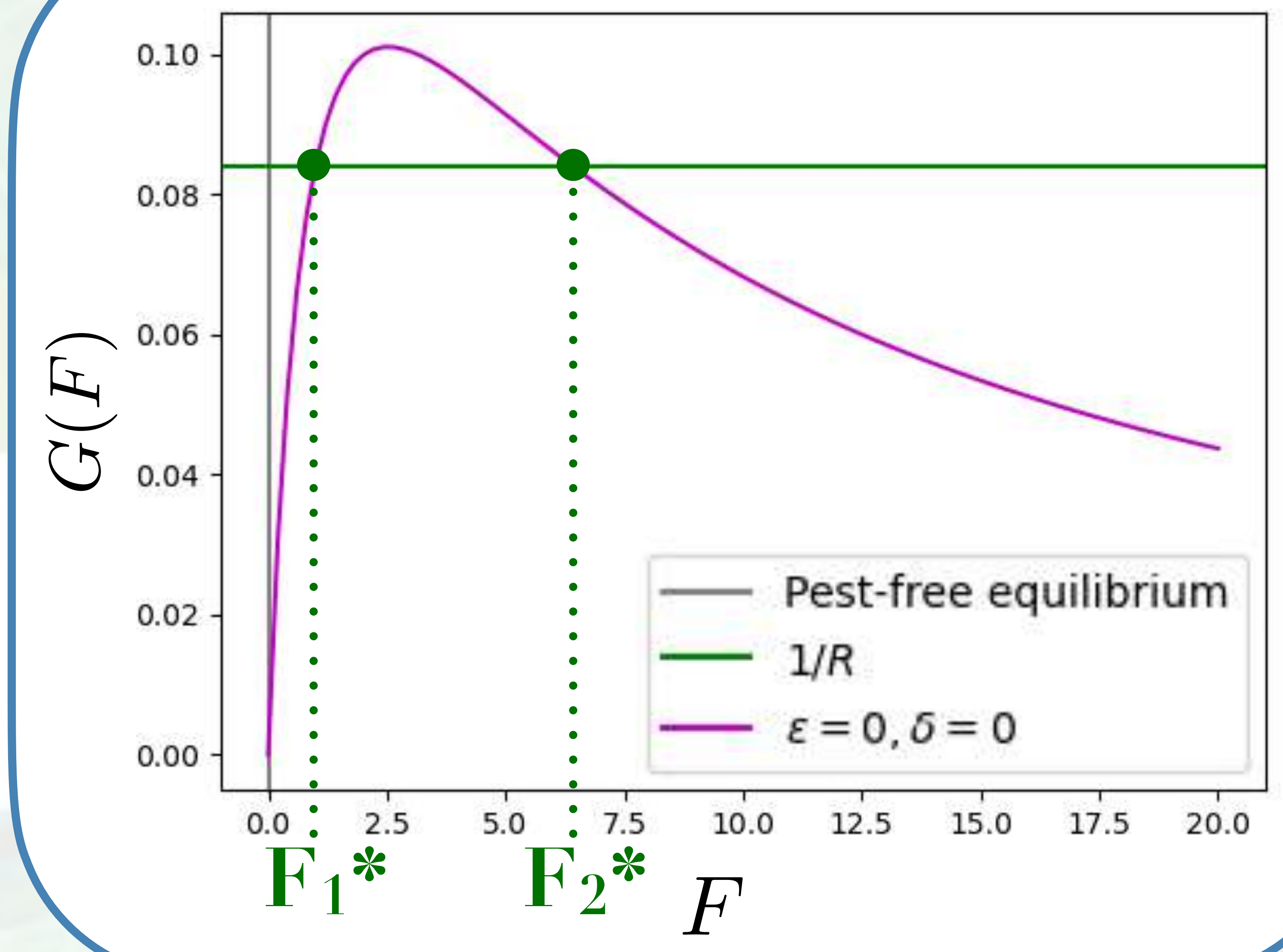
Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



At least **2 solutions** for which $\mathbf{F_2^*} > \mathbf{F_1^*} > 0$ such that:

$$\frac{dG}{dF}(\mathbf{F_2^*}) < 0 \text{ and } \frac{dG}{dF}(\mathbf{F_1^*}) > 0$$

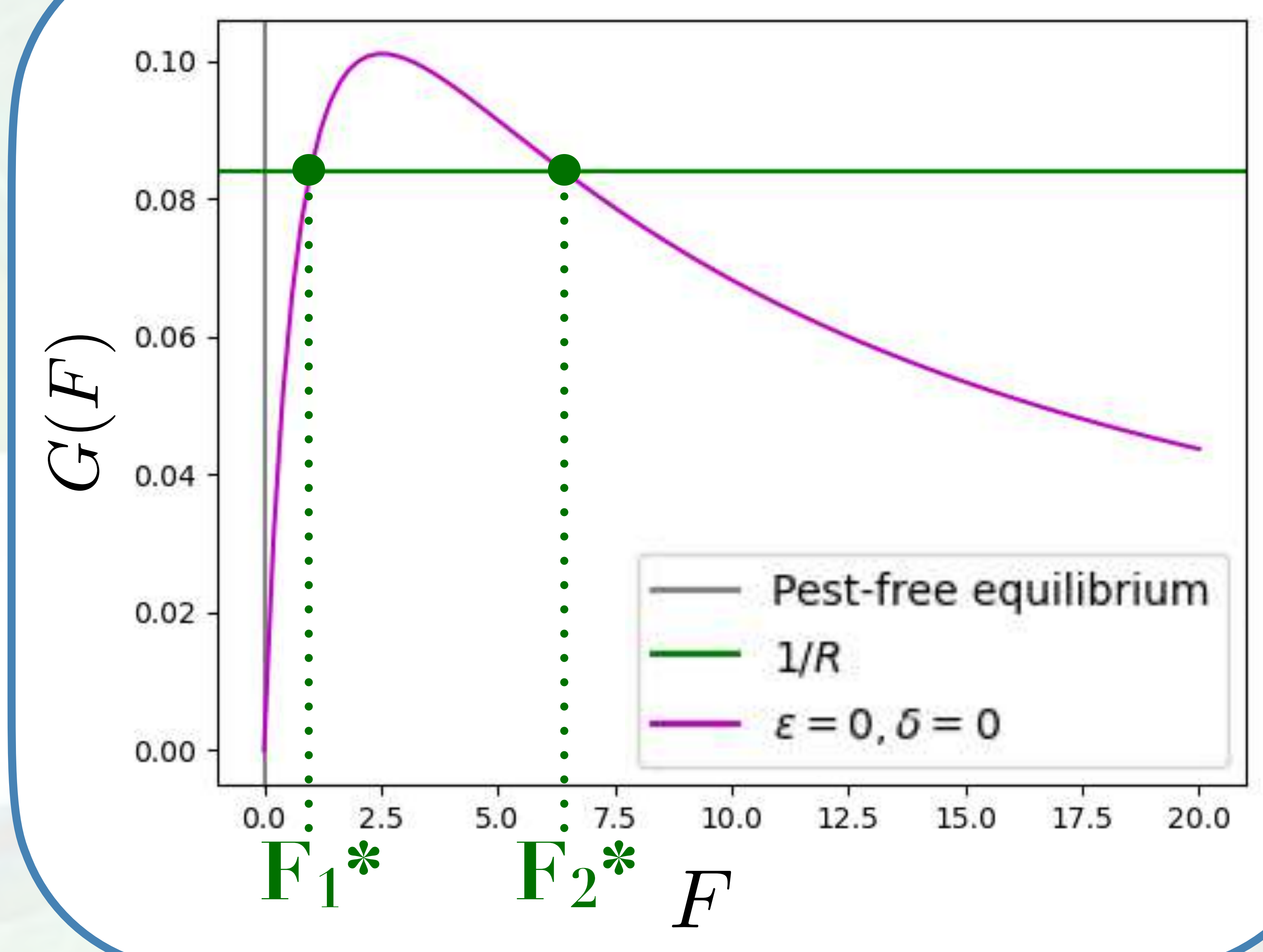
Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



At least **2 solutions** for which $F_2^* > F_1^* > 0$ such that:

$$\frac{dG}{dF}(F_2^*) < 0 \text{ and } \frac{dG}{dF}(F_1^*) > 0$$

The equilibria are: (S^*, F_1^*, M_1^*), (S^*, F_2^*, M_2^*).

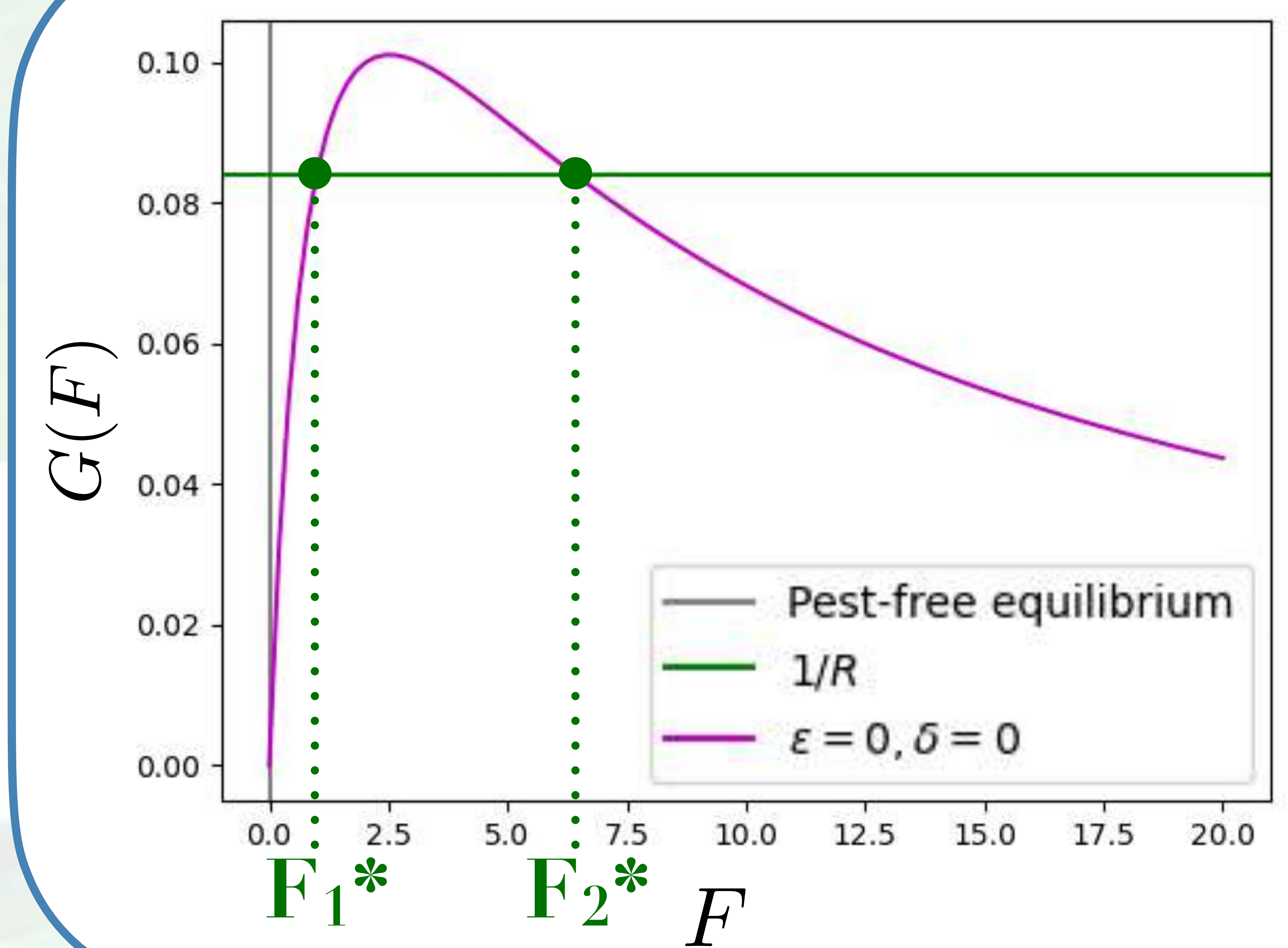
Infestation equilibria are the F* values solutions of:

$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) < \frac{1}{\mathcal{R}}$

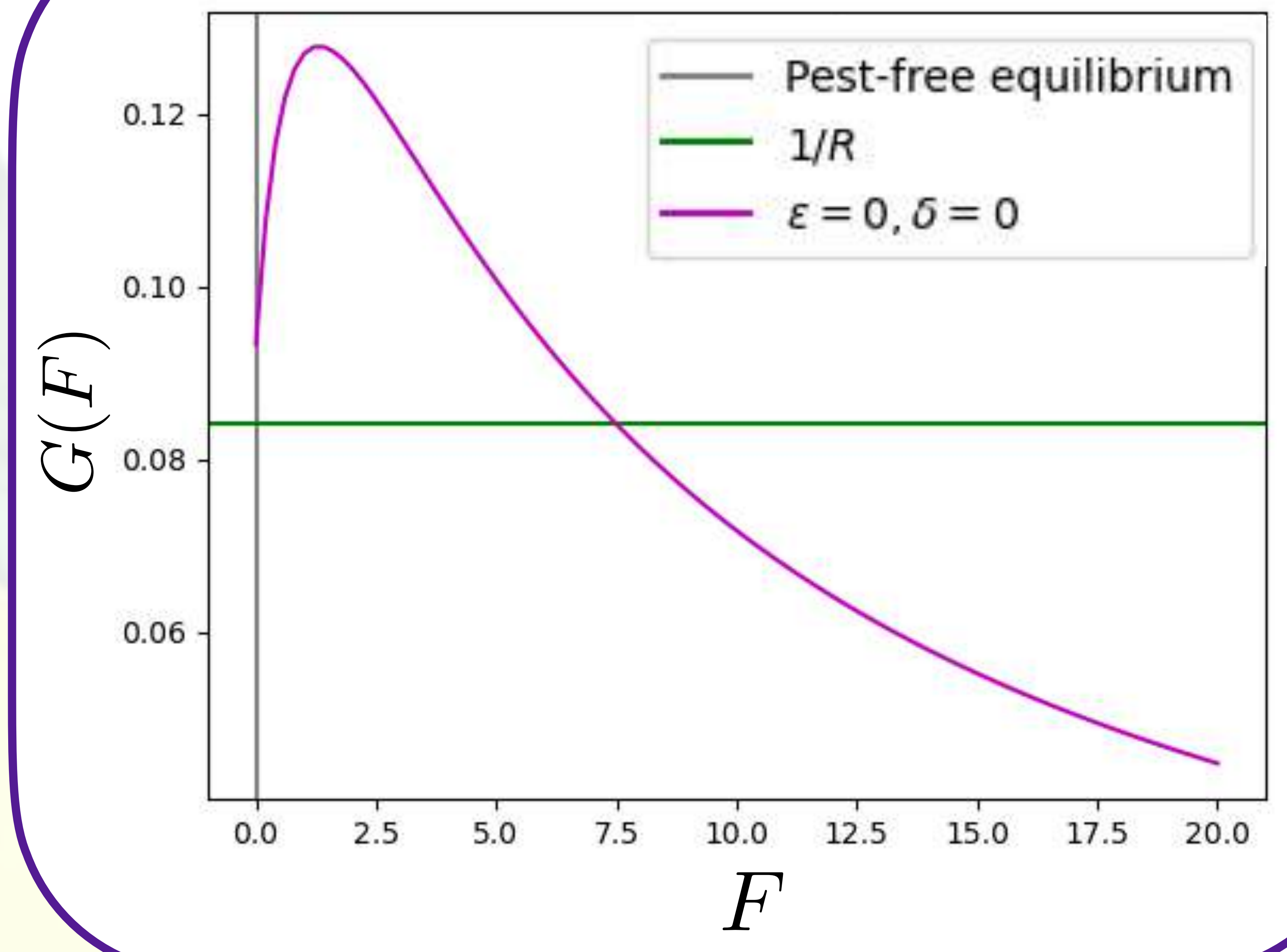
And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



If $G(0) > \frac{1}{\mathcal{R}}$

(2)



Infestation equilibria are the F* values solutions of:

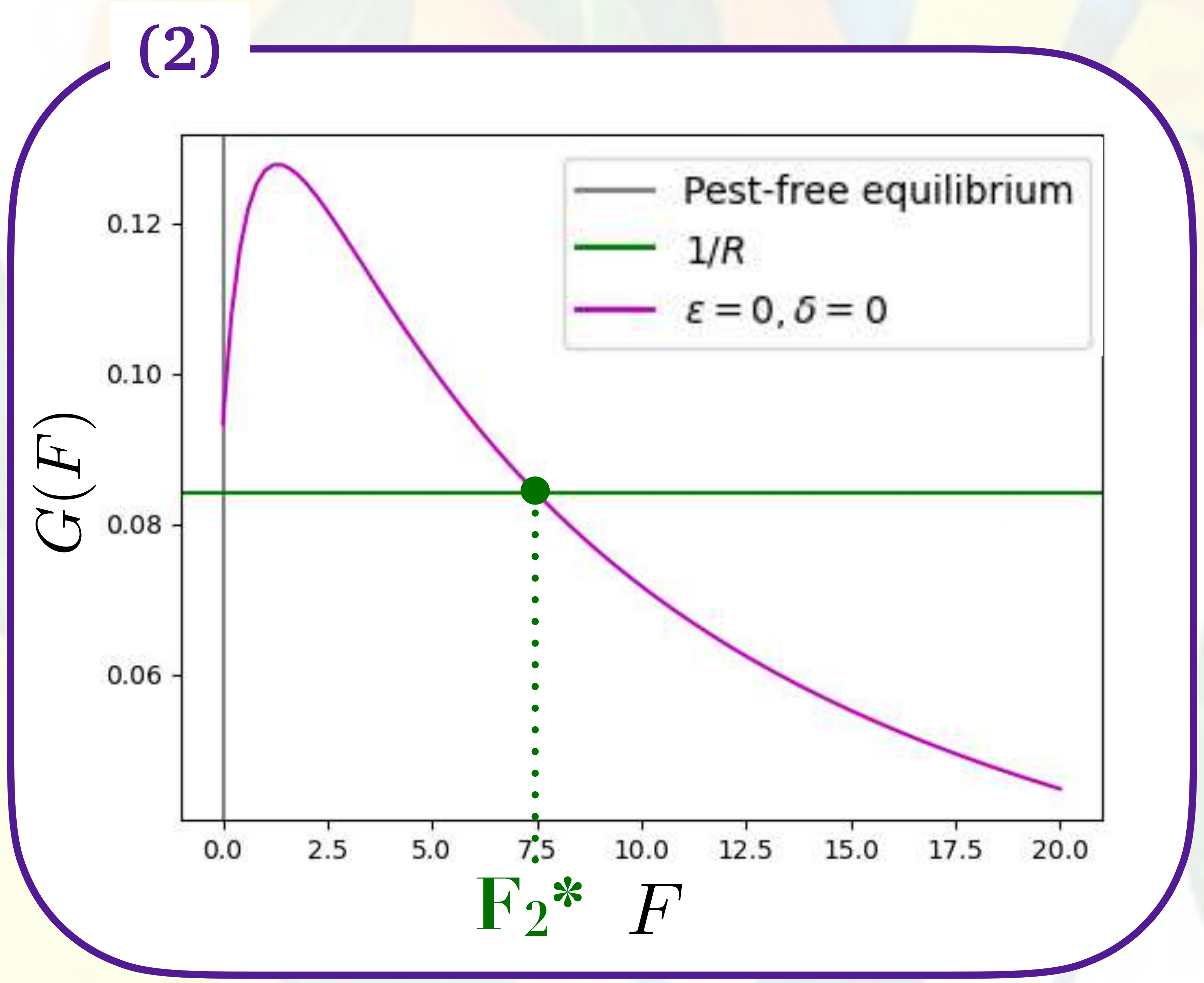
$$G(F) = X(S^*, M(F), \epsilon)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

If $G(0) \cdot > \frac{1}{\mathcal{R}}$

At least 1 **solution** $\mathbf{F_2^*}$ such that:

$$\frac{dG}{dF}(\mathbf{F_2^*}) < 0$$

The equilibrium is: $(\mathbf{S^*}, \mathbf{F_2^*}, \mathbf{M_2^*})$.



Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

$$\iff G(0) < \frac{1}{\mathcal{R}}$$

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

$$\iff G(0) < \frac{1}{\mathcal{R}}$$

Infestation equilibria are the F* values solutions of:

$$X(S^*, M(F^*), \epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} (1-p)X(M^*)rC'(F^*)F^* & (1-p)C(F^*)rF^*X'(M^*) \\ pX(M^*)r(C'(F^*)F^* + C(F^*)) & -\frac{pX(M^*)C(F^*)rF^*+\delta\sigma}{M^*} + pC(F^*)rF^*X'(M^*) \end{pmatrix}$$

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

$$\iff G(0) < \frac{1}{\mathcal{R}}$$

Infestation equilibria are the F* values solutions of:

$$X(S^*, M(F^*), \epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} (1-p)X(M^*)rC'(F^*)F^* & (1-p)C(F^*)rF^*X'(M^*) \\ pX(M^*)r(C'(F^*)F^* + C(F^*)) & -\frac{pX(M^*)C(F^*)rF^*+\delta\sigma}{M^*} + pC(F^*)rF^*X'(M^*) \end{pmatrix}$$

➡️ calculation of the trace and determinant

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

Infestation equilibria are the F* values solutions of:

$$X(S^*, M(F^*), \epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

$$\iff G(0) < \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} (1-p)X(M^*)rC'(F^*)F^* & (1-p)C(F^*)rF^*X'(M^*) \\ pX(M^*)r(C'(F^*)F^* + C(F^*)) & -\frac{pX(M^*)C(F^*)rF^*+\delta\sigma}{M^*} + pC(F^*)rF^*X'(M^*) \end{pmatrix}$$

➡️ calculation of the trace and determinant

$Tr(J) < 0$

Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at S = S*)

Pest-free equilibrium

$$\left(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M}\right)$$

Infestation equilibria are the F* values solutions of:

$$X(S^*, M(F^*), \epsilon)C(F^*) = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

STABLE if $X(M_0^*) < \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$

$$\iff G(0) < \frac{1}{\mathcal{R}}$$

$$J = \begin{pmatrix} (1-p)X(M^*)rC'(F^*)F^* & (1-p)C(F^*)rF^*X'(M^*) \\ pX(M^*)r(C'(F^*)F^* + C(F^*)) & -\frac{pX(M^*)C(F^*)rF^*+\delta\sigma}{M^*} + pC(F^*)rF^*X'(M^*) \end{pmatrix}$$

➡️ calculation of the trace and determinant

$Tr(J) < 0$

When $\frac{dG}{dF} > 0$, $Det(J) < 0$ (S*, F_1*, M_1*) is **UNSTABLE**

When $\frac{dG}{dF} < 0$, $Det(J) > 0$ (S*, F_2*, M_2*) is **STABLE**

In summary

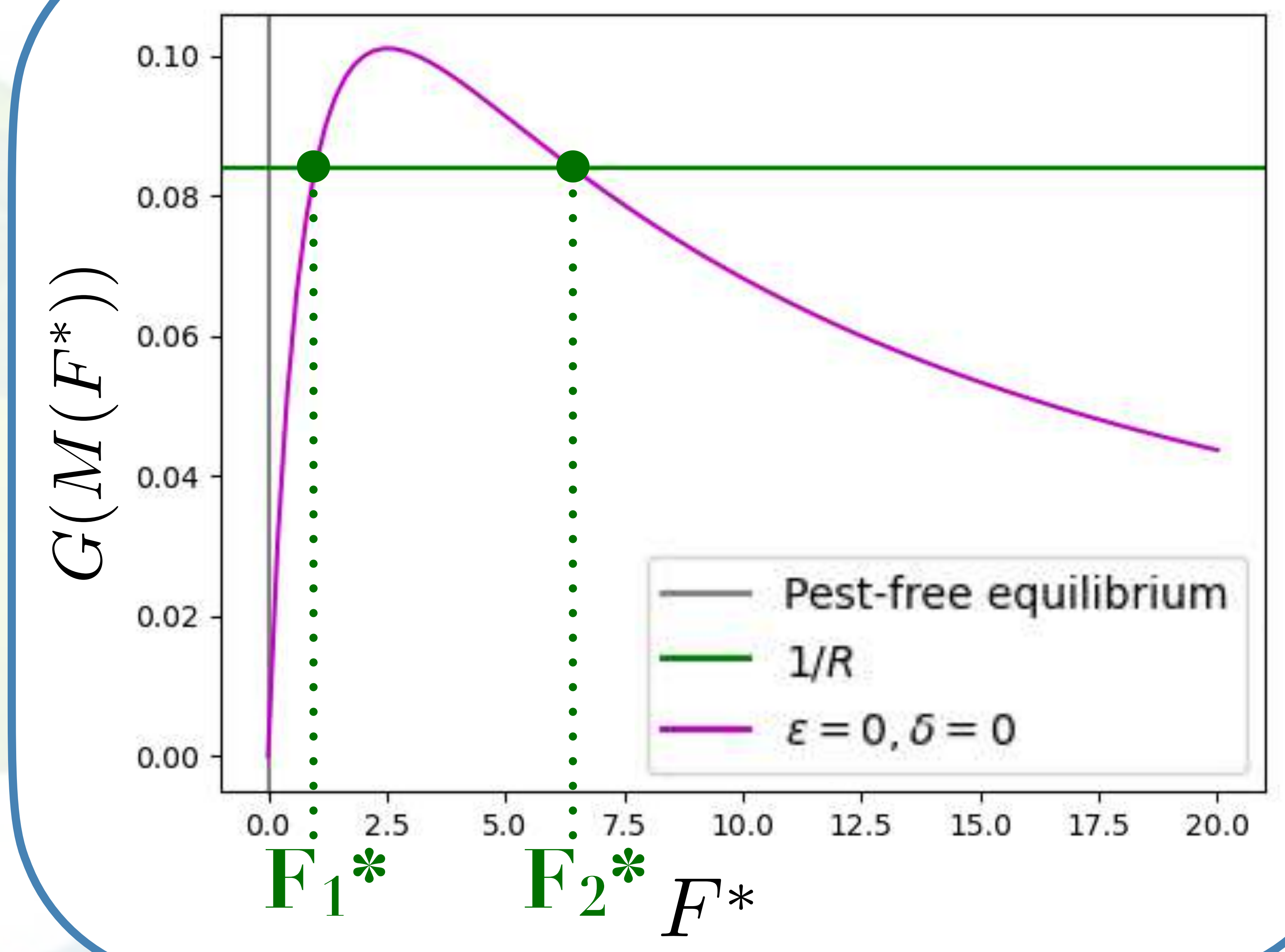
If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$		

In summary

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

(0)



In summary

If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$		
Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE** Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**		

In summary

If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$	**If** $G(0) \cdot > \frac{1}{\mathcal{R}}$	
Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE** Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**		

In summary

If $G(0) < \frac{1}{\mathcal{R}}$

And $max(G(F)) > \frac{1}{\mathcal{R}}$

If $G(0) \cdot > \frac{1}{\mathcal{R}}$

Pest-free equilibrium

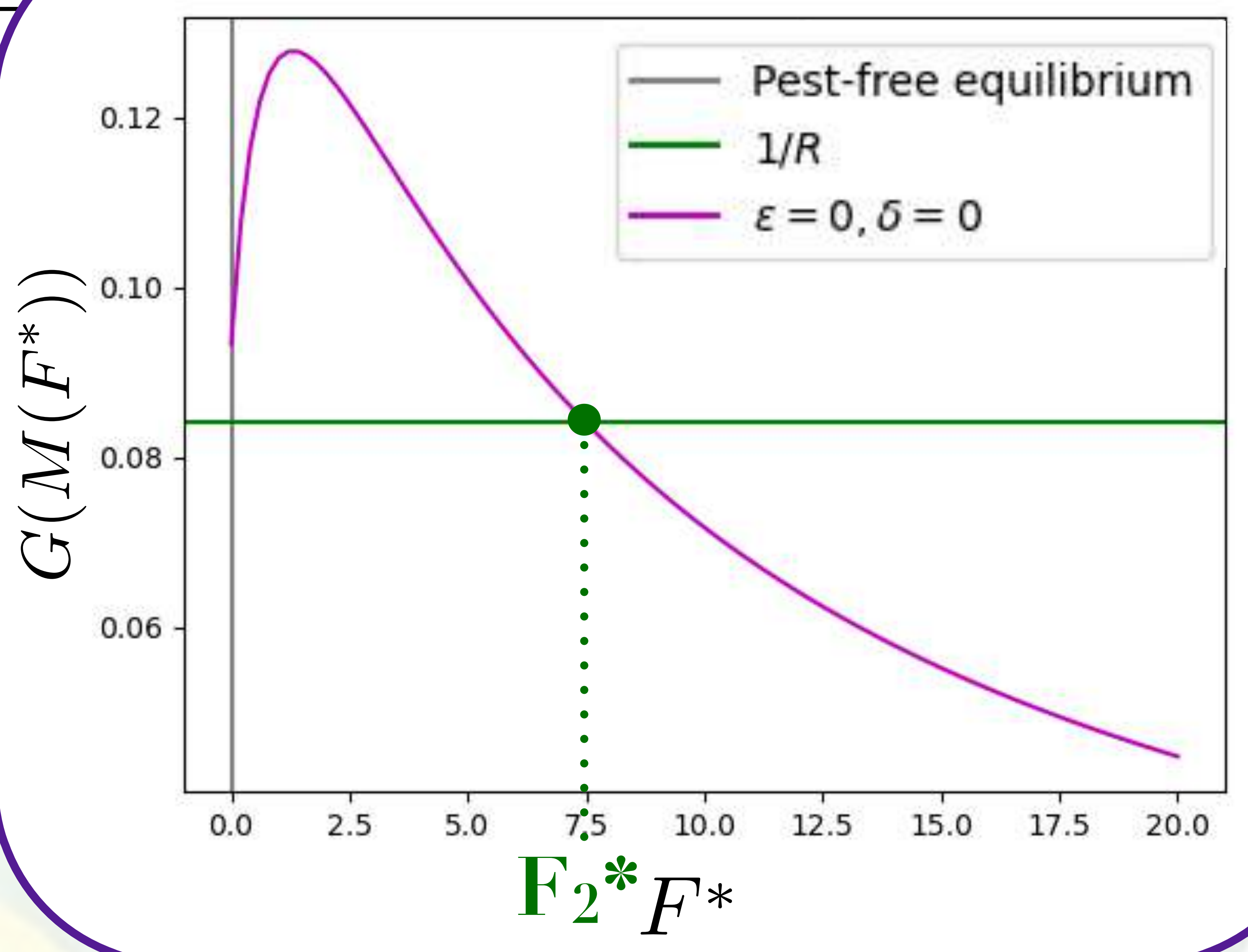
$(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE**

Infestation equilibria

(S^*, F_1^*, M_1^*) **UNSTABLE**

(S^*, F_2^*, M_2^*) **STABLE**

(2)



In summary

If $G(0) < \frac{1}{\mathcal{R}}$ And $max(G(F)) > \frac{1}{\mathcal{R}}$	If $G(0) \cdot > \frac{1}{\mathcal{R}}$	
Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE** Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**	Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **UNSTABLE** Infestation equilibrium (S^*, F_2^*, M_2^*) **STABLE**	

In summary

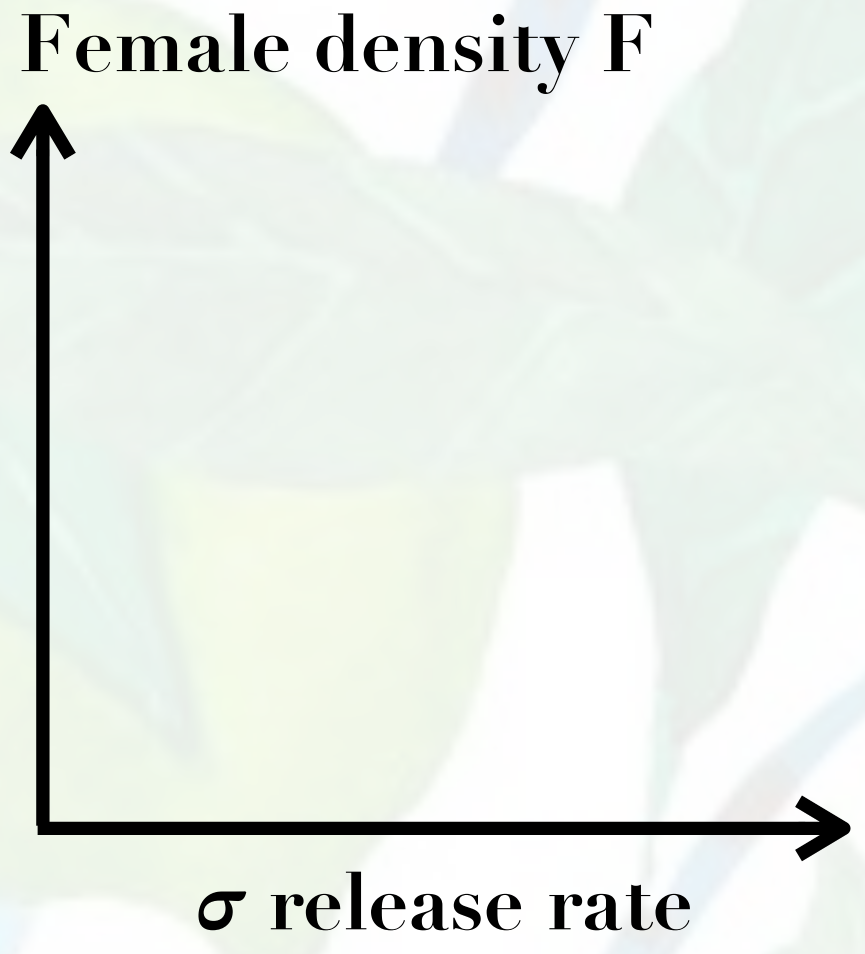
If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$	**If** $G(0) \cdot > \frac{1}{\mathcal{R}}$	**If** $G(F) < \frac{1}{\mathcal{R}}$
Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE** Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**	Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **UNSTABLE** Infestation equilibrium (S^*, F_2^*, M_2^*) **STABLE**	

In summary

If $G(0) < \frac{1}{\mathcal{R}}$ **And** $max(G(F)) > \frac{1}{\mathcal{R}}$	**If** $G(0) \cdot > \frac{1}{\mathcal{R}}$	**If** $G(F) < \frac{1}{\mathcal{R}}$
Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE**	Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **UNSTABLE**	Pest-free equilibrium $(\frac{(1-\delta)\sigma}{\mu_S}, 0, \frac{\delta\sigma}{\mu_M})$ **STABLE**
Infestation equilibria (S^*, F_1^*, M_1^*) **UNSTABLE** (S^*, F_2^*, M_2^*) **STABLE**	Infestation equilibrium (S^*, F_2^*, M_2^*) **STABLE**	

Study of the bifurcation diagram in σ

Study of the bifurcation diagram in σ



σ bifurcation diagram

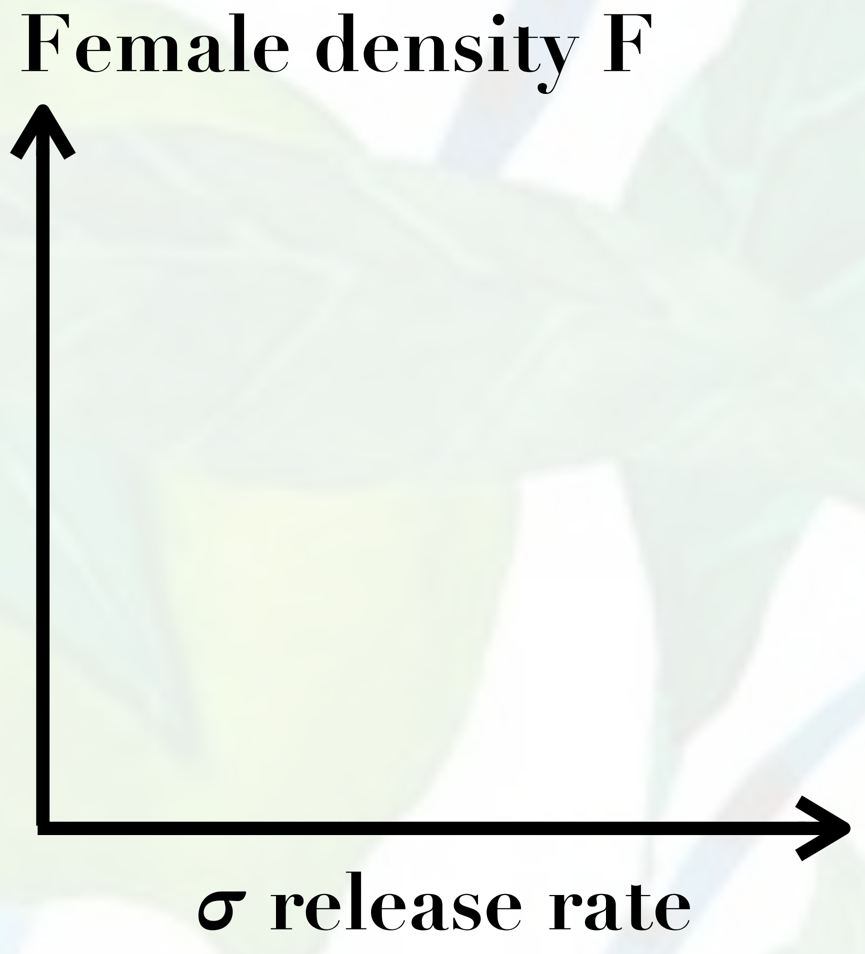
Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

↓

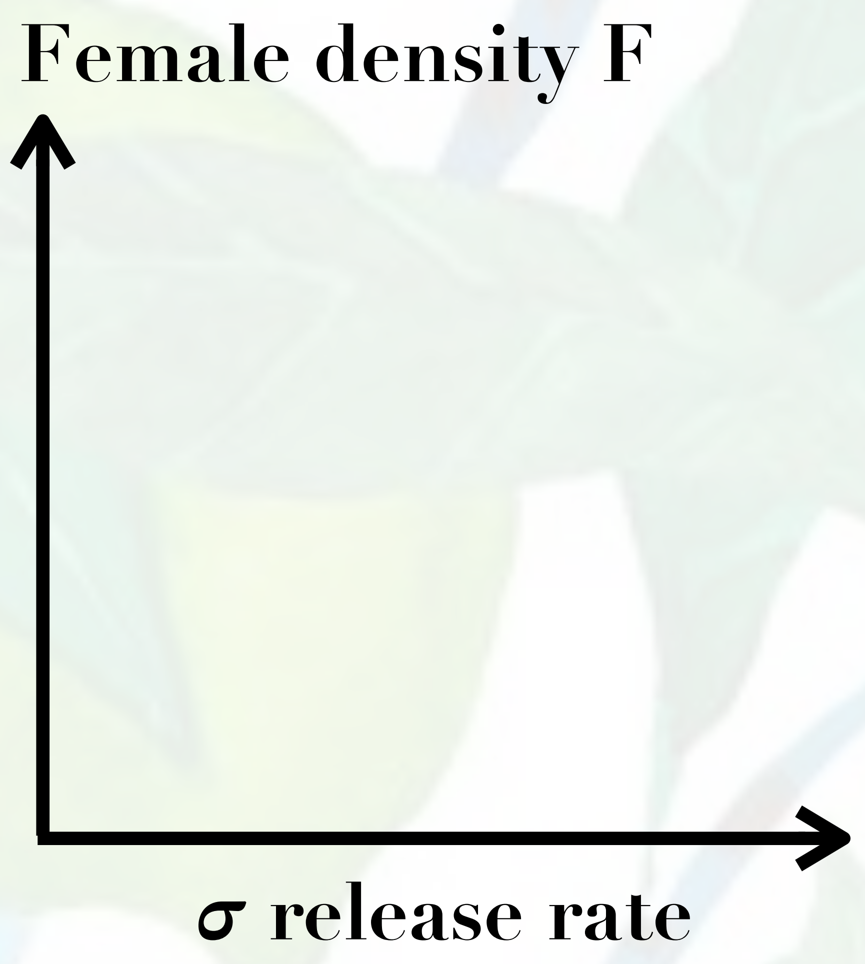
σ bifurcation diagram



Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$



↓

σ bifurcation diagram

↓

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Study of the bifurcation diagram in σ

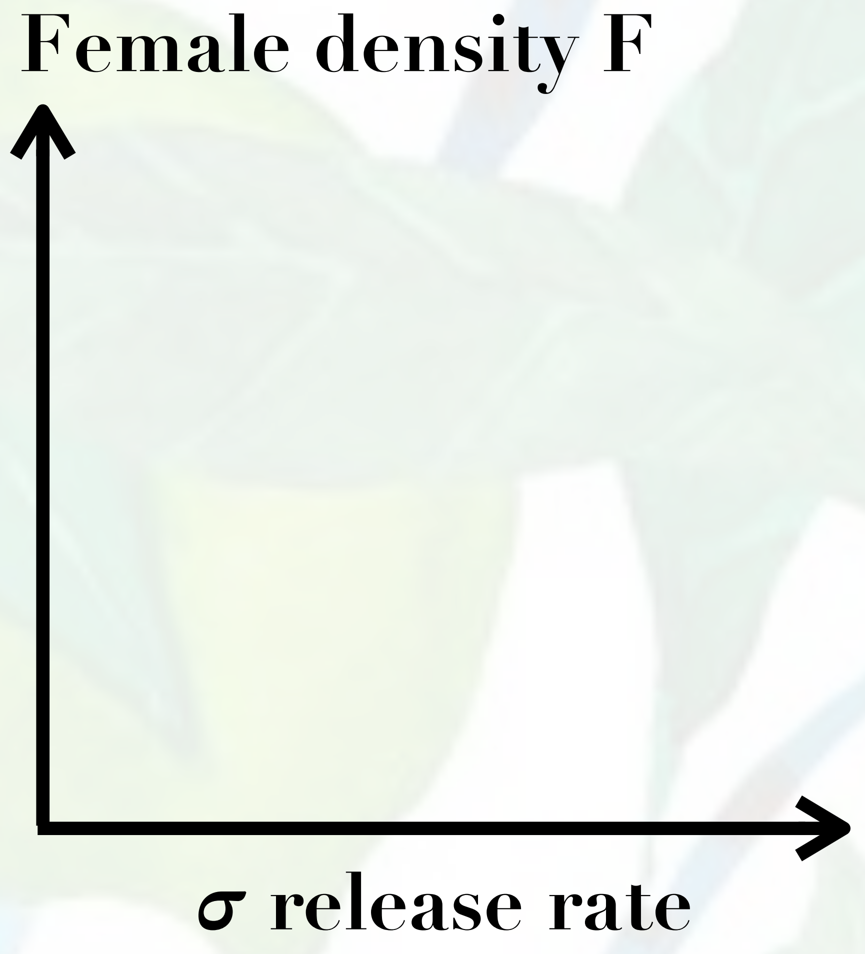
Infestation equilibria are the F values solutions of:

$$G(F) = \quad X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$

↓

σ bifurcation diagram

↓



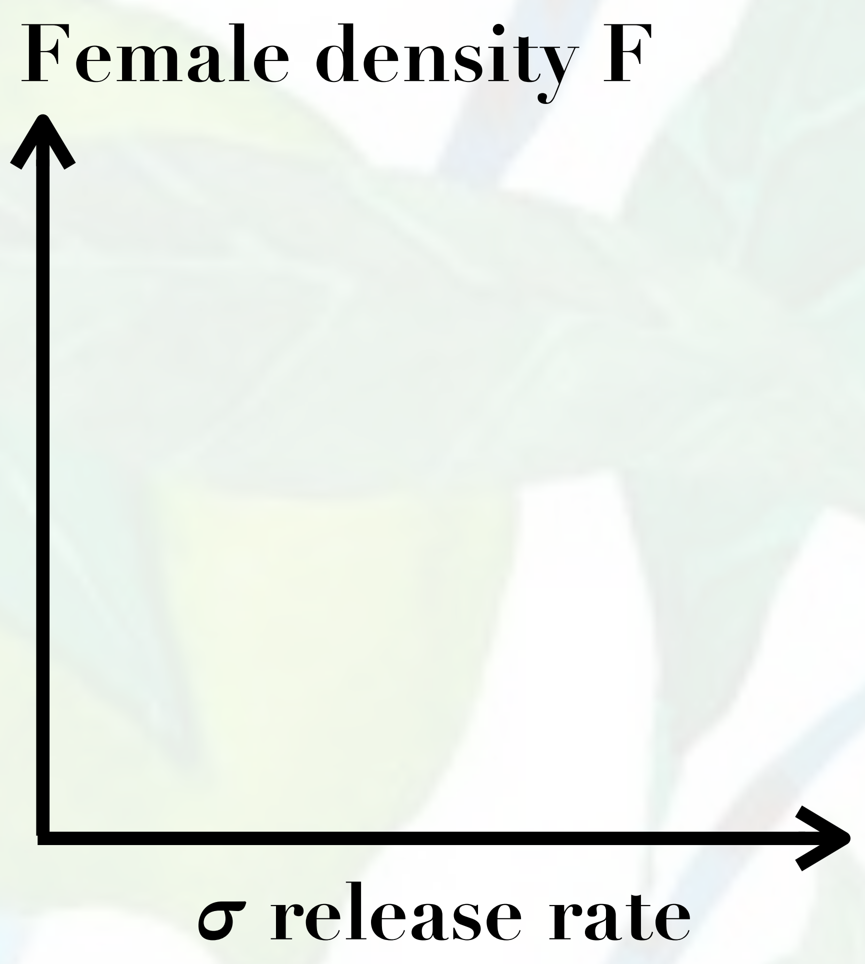
Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = X(S,M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1+\beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{\mathcal{R}}$$



σ **bifurcation diagram**

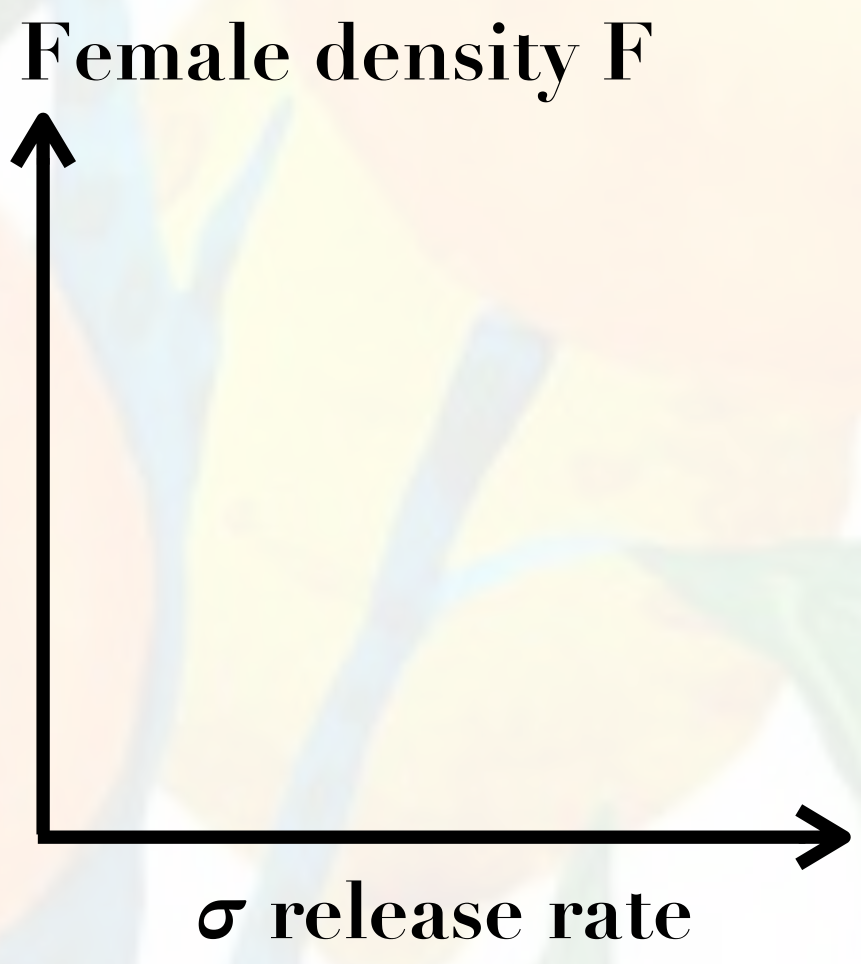
Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

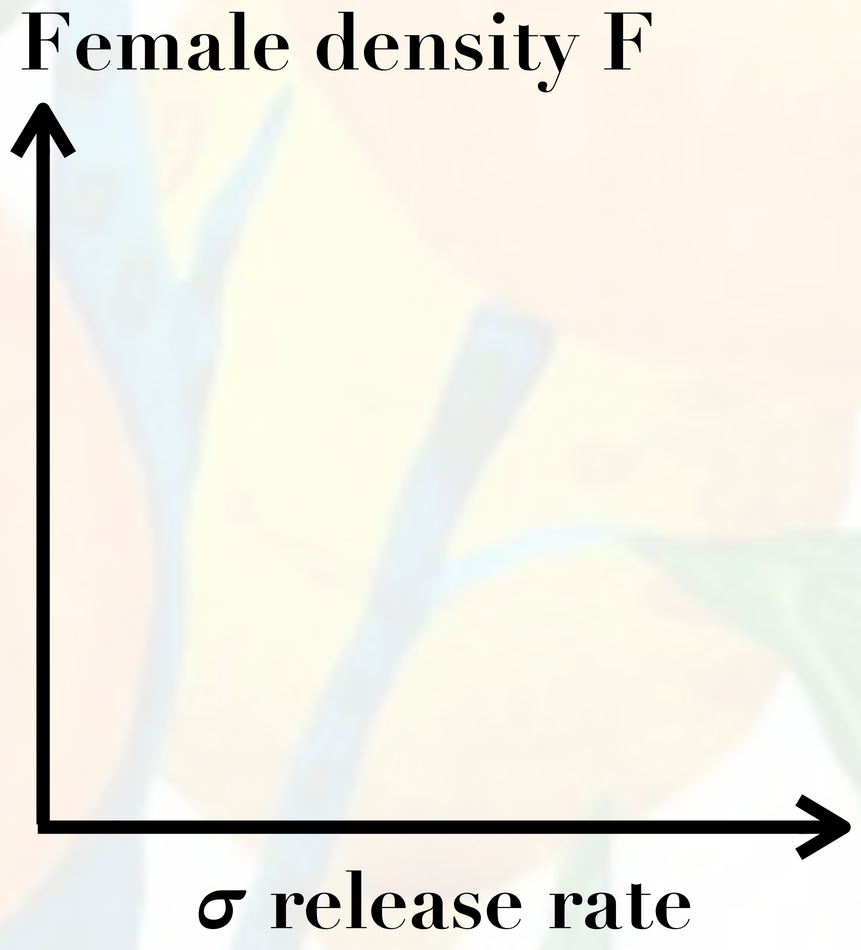
$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$



Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

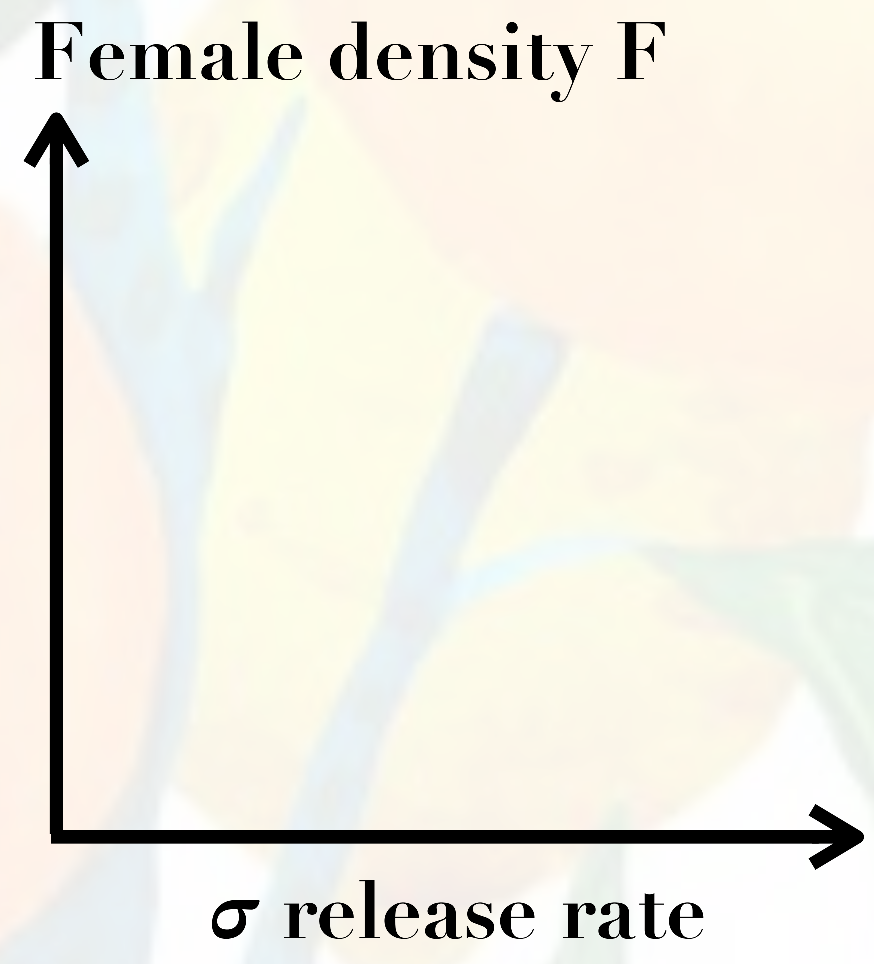
$$\iff \frac{\beta p \mu_F}{(1-p)\mu_M}F^2 + (\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k)F + k = 0$$



Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

$$\iff \frac{\beta p\mu_F}{(1-p)\mu_M}F^2 + (\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k)F + k = 0$$

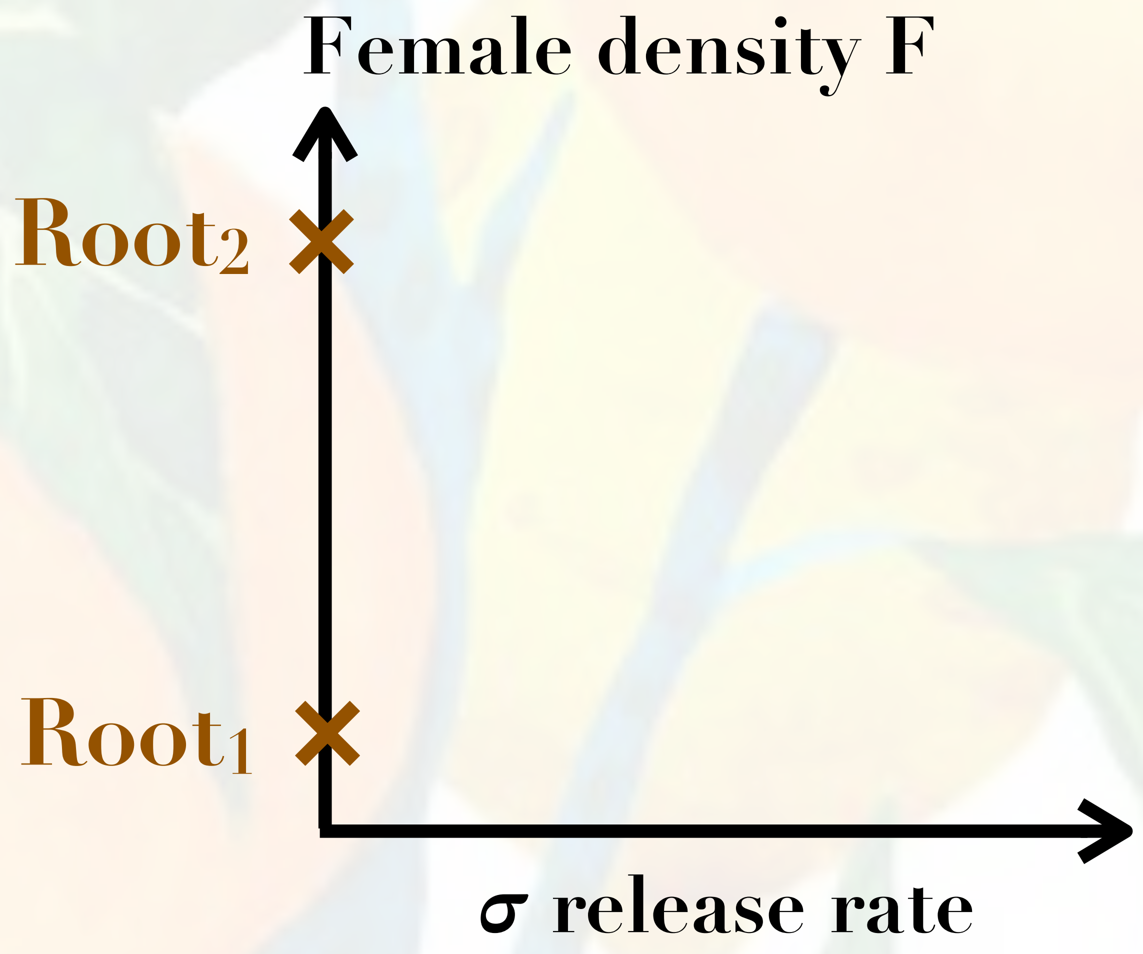


Sum of the roots > 0 and therefore the 2 roots **Root_1** and **Root_2** are > 0 if: $\frac{p\mu_F(R-1)}{\beta k(1-p)\mu_M} > 1$

Polynomial of degree 2 not depending on **residual fertility rate** (δ, ϵ)

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

$$\iff \frac{\beta p\mu_F}{(1-p)\mu_M}F^2 + (\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k)F + k = 0$$



Sum of the roots > 0 and therefore the 2 roots **Root₁** and **Root₂** are > 0 if: $\frac{p\mu_F(R-1)}{\beta k(1-p)\mu_M} > 1$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

General case

$\delta \neq 0$, $\epsilon \neq 0$

The denominator cancels for:

$$F = \frac{1}{\beta}(\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} - 1)$$

But F $>$ 0 so existence when:

$$\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} > 1$$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

General case

$\delta \neq 0, \epsilon \neq 0$

The denominator cancels for:

$$F = \frac{1}{\beta}(\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} - 1)$$

But F > 0 so existence when:

$$\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} > 1$$

(1)

Cost-free fertility model

$\delta \neq 0, \epsilon = 0$

The denominator cancels for:

$$F = \frac{1}{\beta}(\frac{R\delta\mu_S}{\delta\mu_S + \eta(1-\delta)\mu_M} - 1)$$

But F > 0 so existence when:

$$\delta > \frac{\eta\mu_M}{R\mu_S - \mu_S + \eta\mu_M}$$

$$\sigma = \frac{(1+\beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1+\beta F)\frac{\delta}{\mu_M} - (1+\beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an **asymptote** when the denominator cancels

General case

$\delta \neq 0, \epsilon \neq 0$

The denominator cancels for:

$$F = \frac{1}{\beta}(\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} - 1)$$

But F > 0 so existence when:

$$\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} > 1$$

(1)

Cost-free fertility model

$\delta \neq 0, \epsilon = 0$

The denominator cancels for:

$$F = \frac{1}{\beta}(\frac{R\delta\mu_S}{\delta\mu_S + \eta(1-\delta)\mu_M} - 1)$$

But F > 0 so existence when:

$$\delta > \frac{\eta\mu_M}{R\mu_S - \mu_S + \eta\mu_M}$$

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

The denominator cancels for:

$$F = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

↓

$$\epsilon > 0.084$$

0.010

0.084

Residual Fertility rate ϵ

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

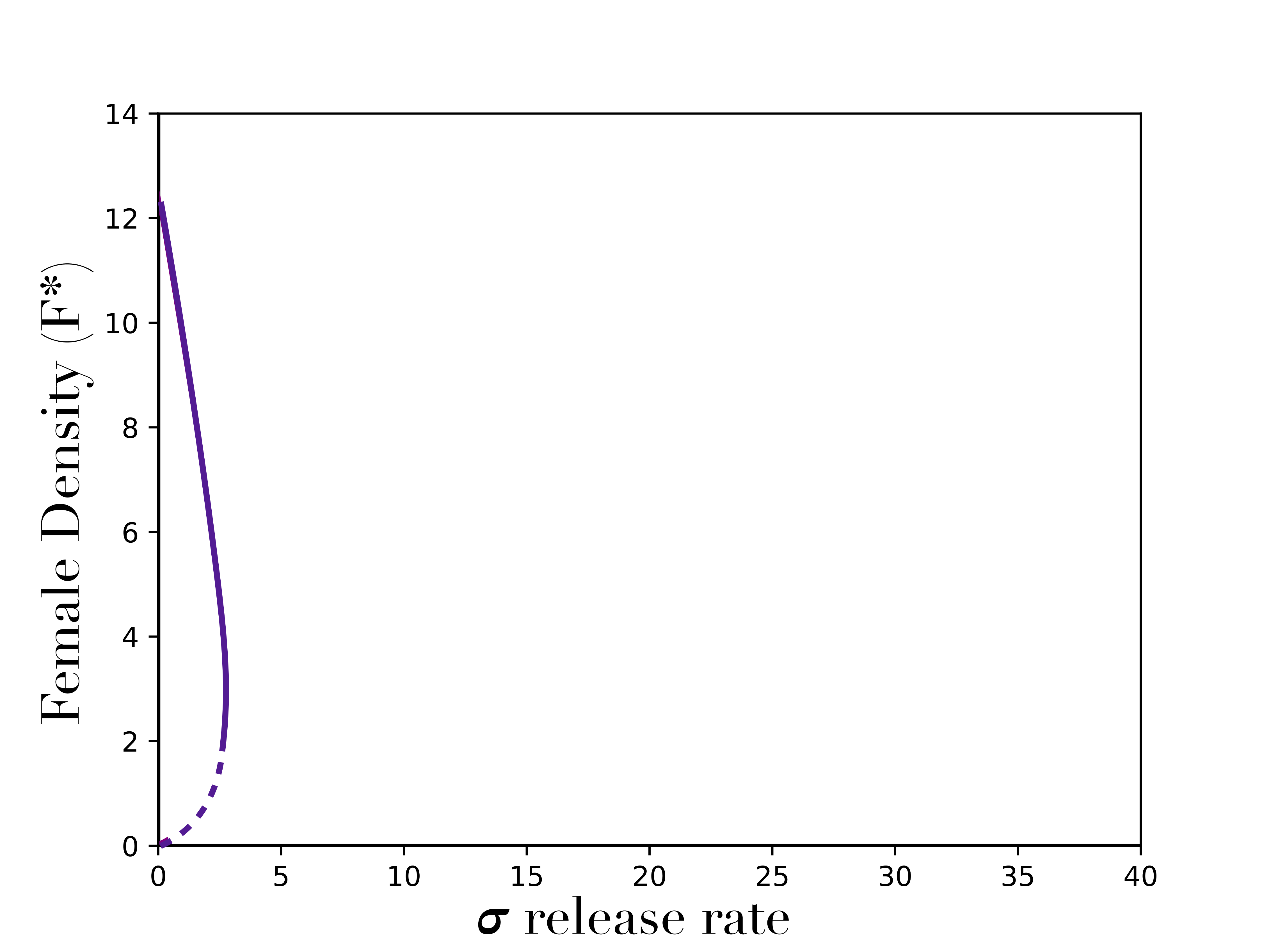
The denominator cancels for:

$$\mathbf{F}_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

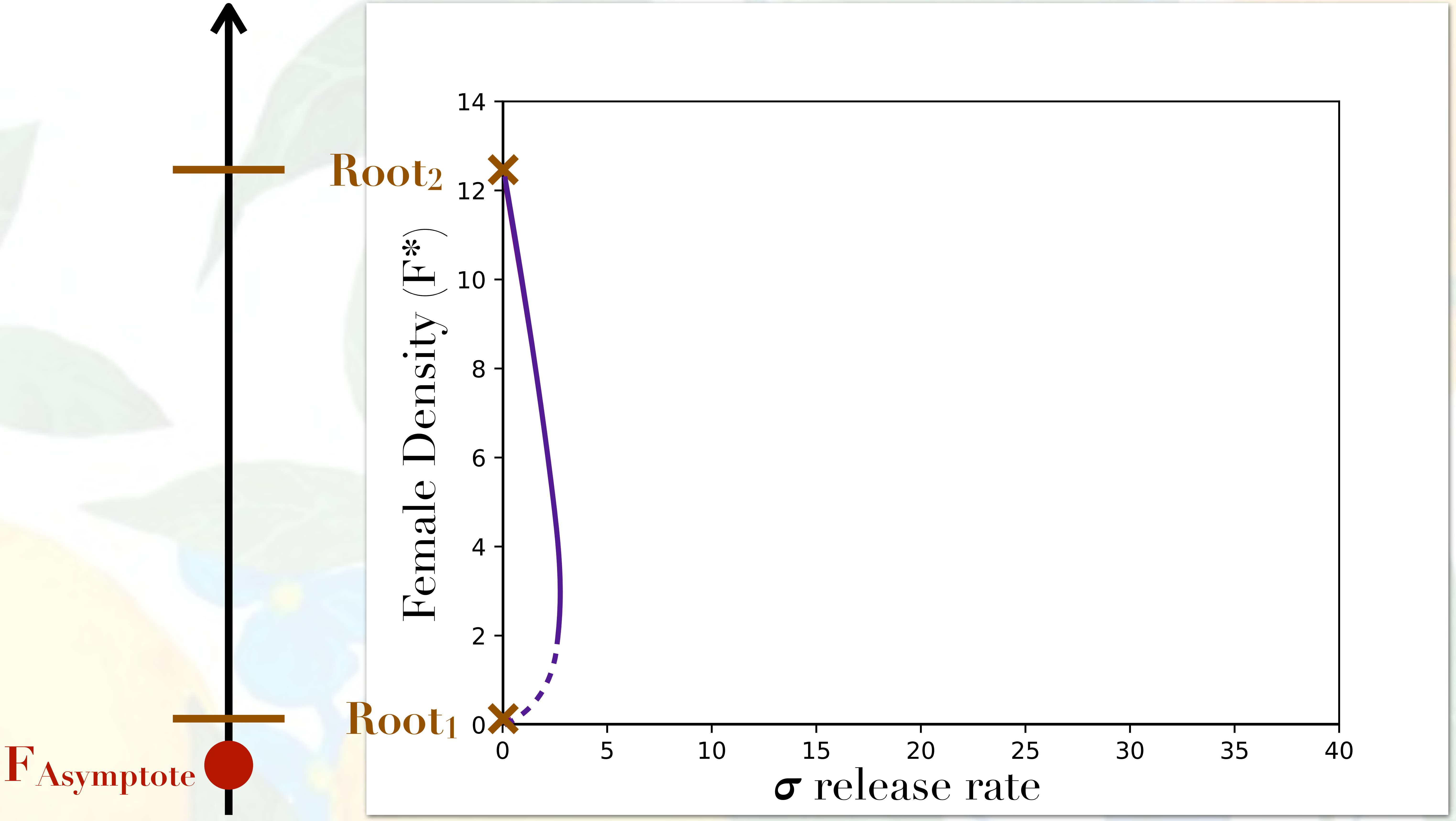
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2) Costly fertility model

$\delta = 0, \epsilon \neq 0$

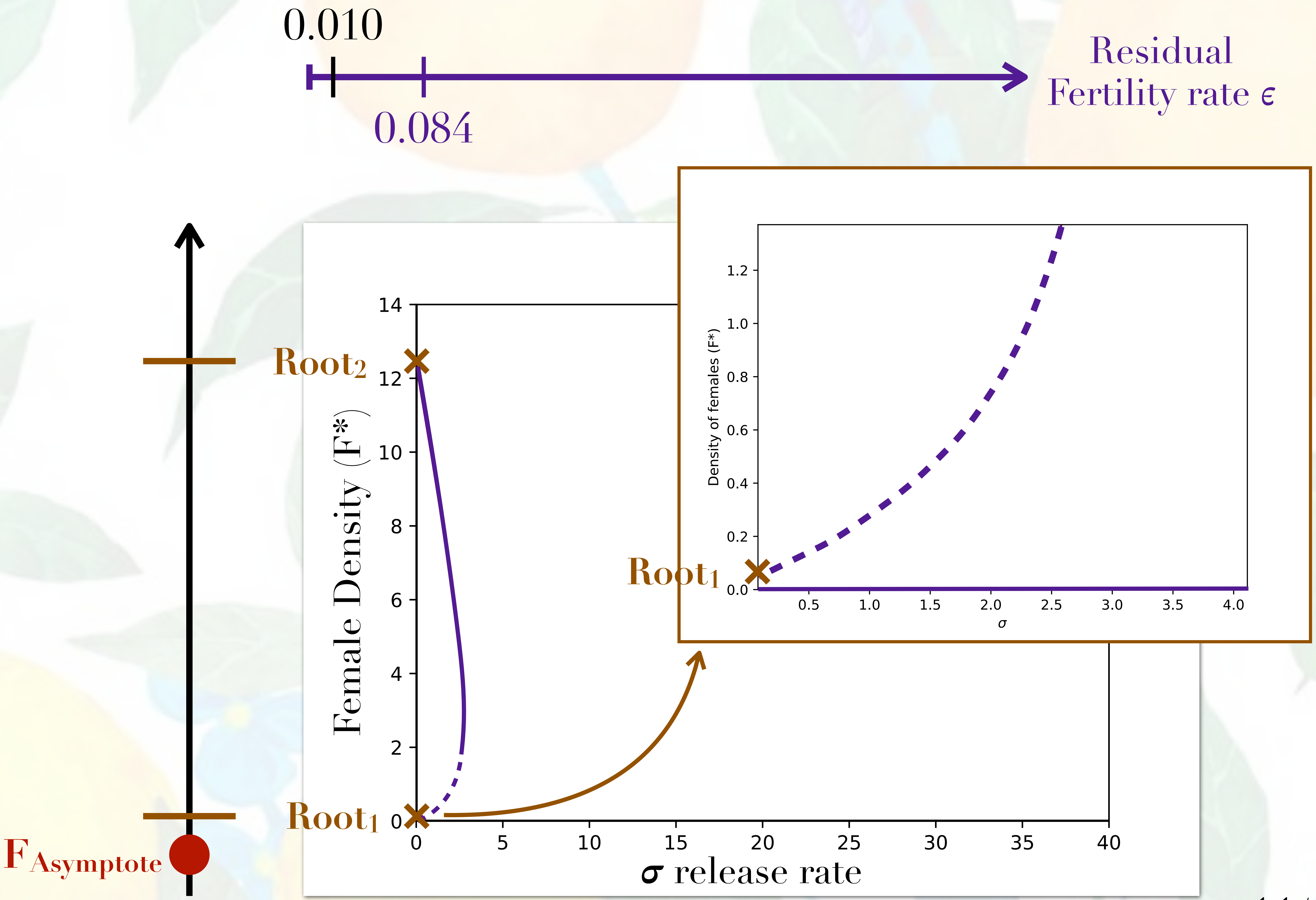
The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

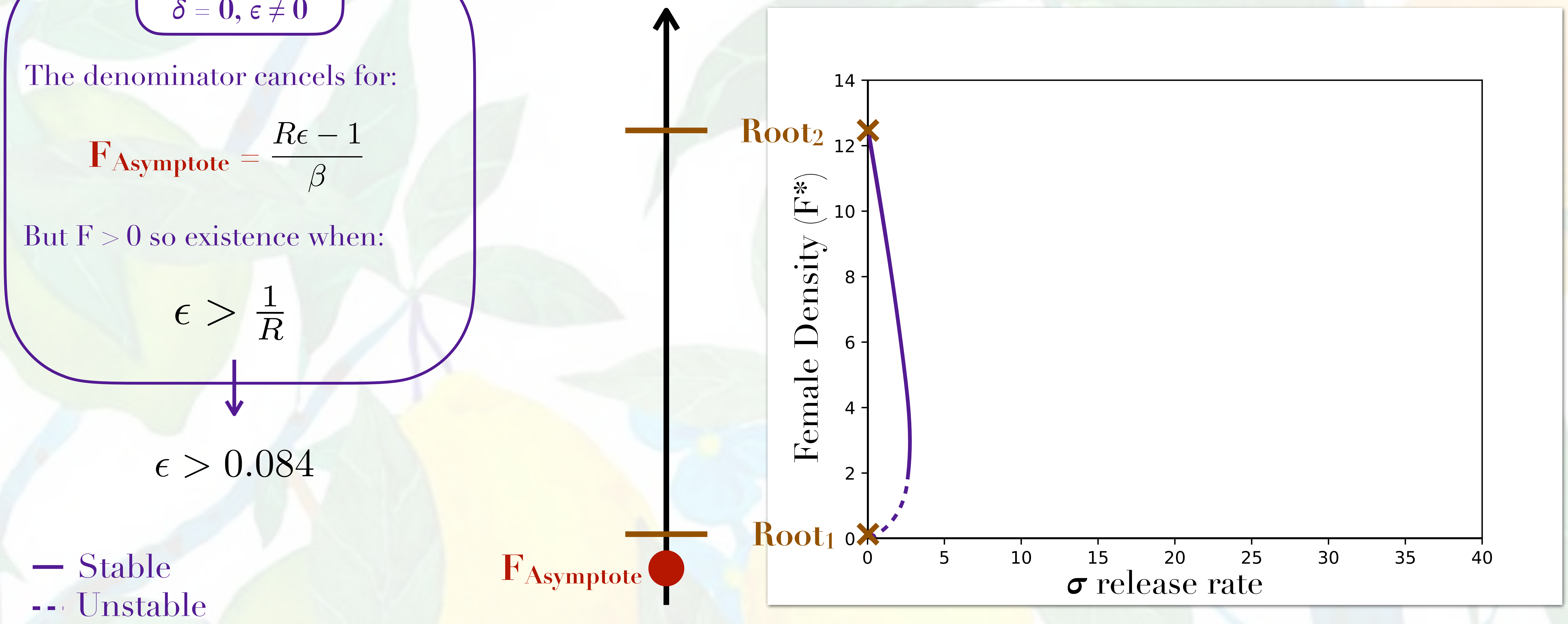
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



(2) Costly fertility model

$\delta = 0, \epsilon \neq 0$

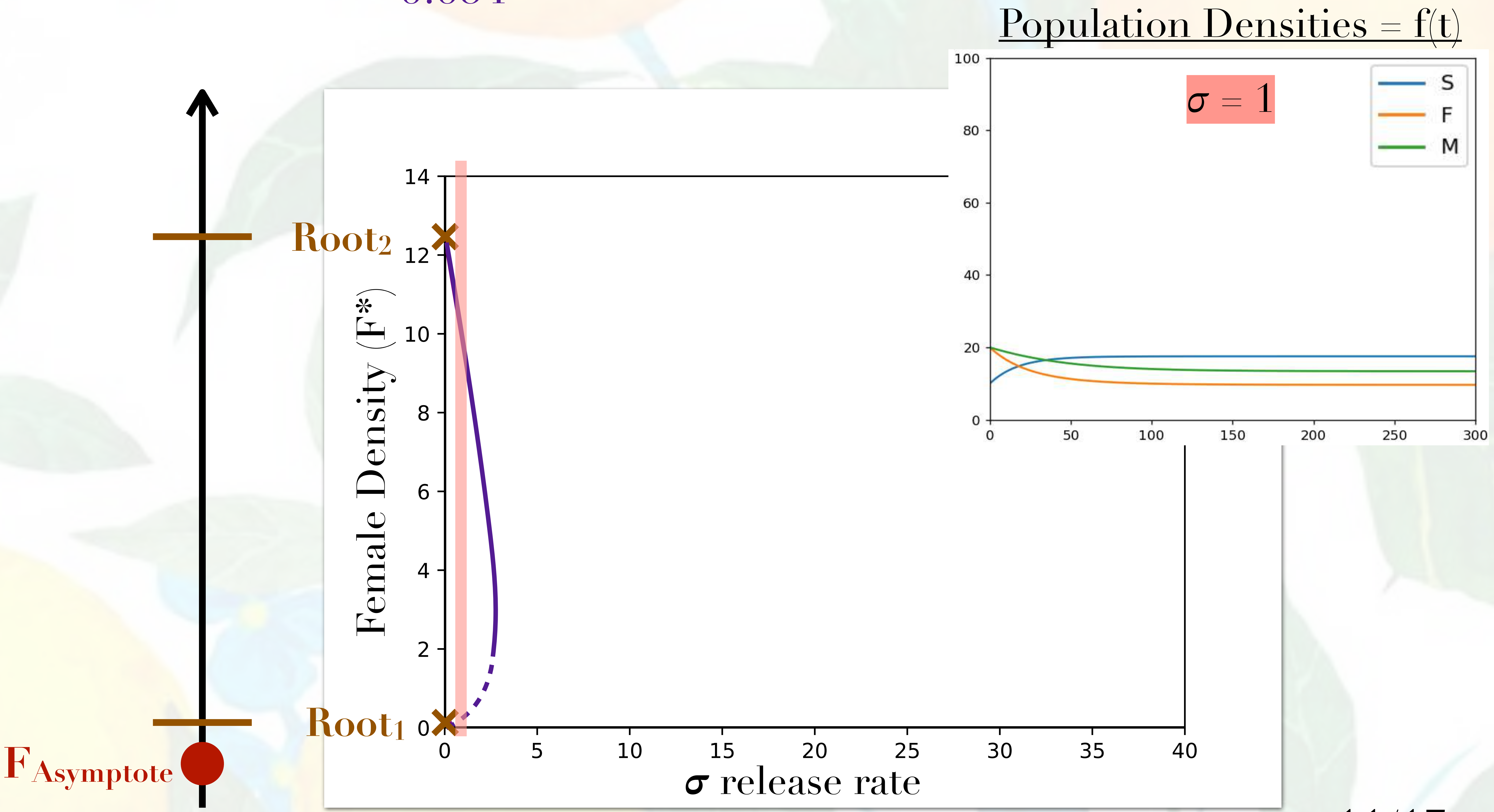
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

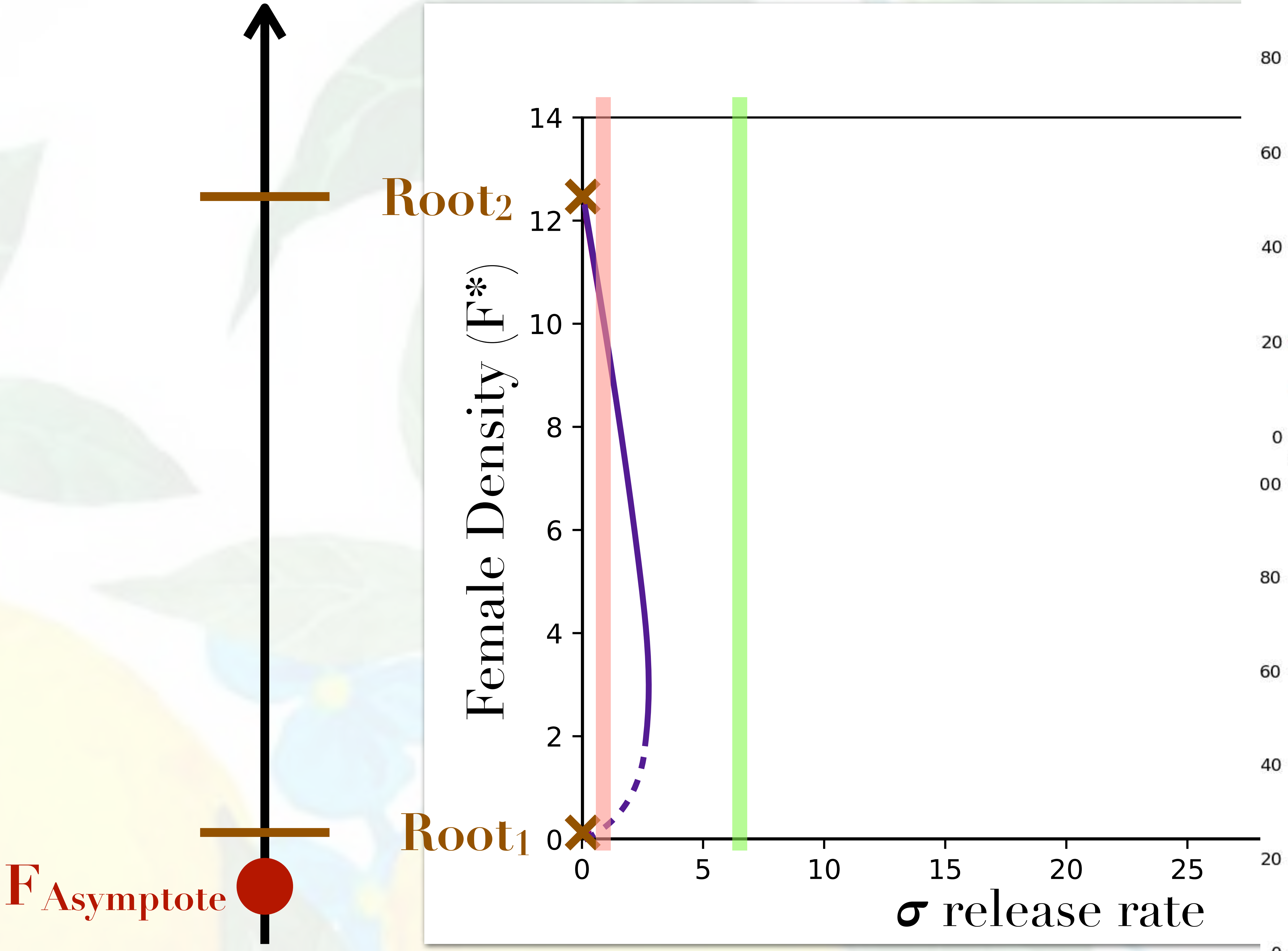
The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

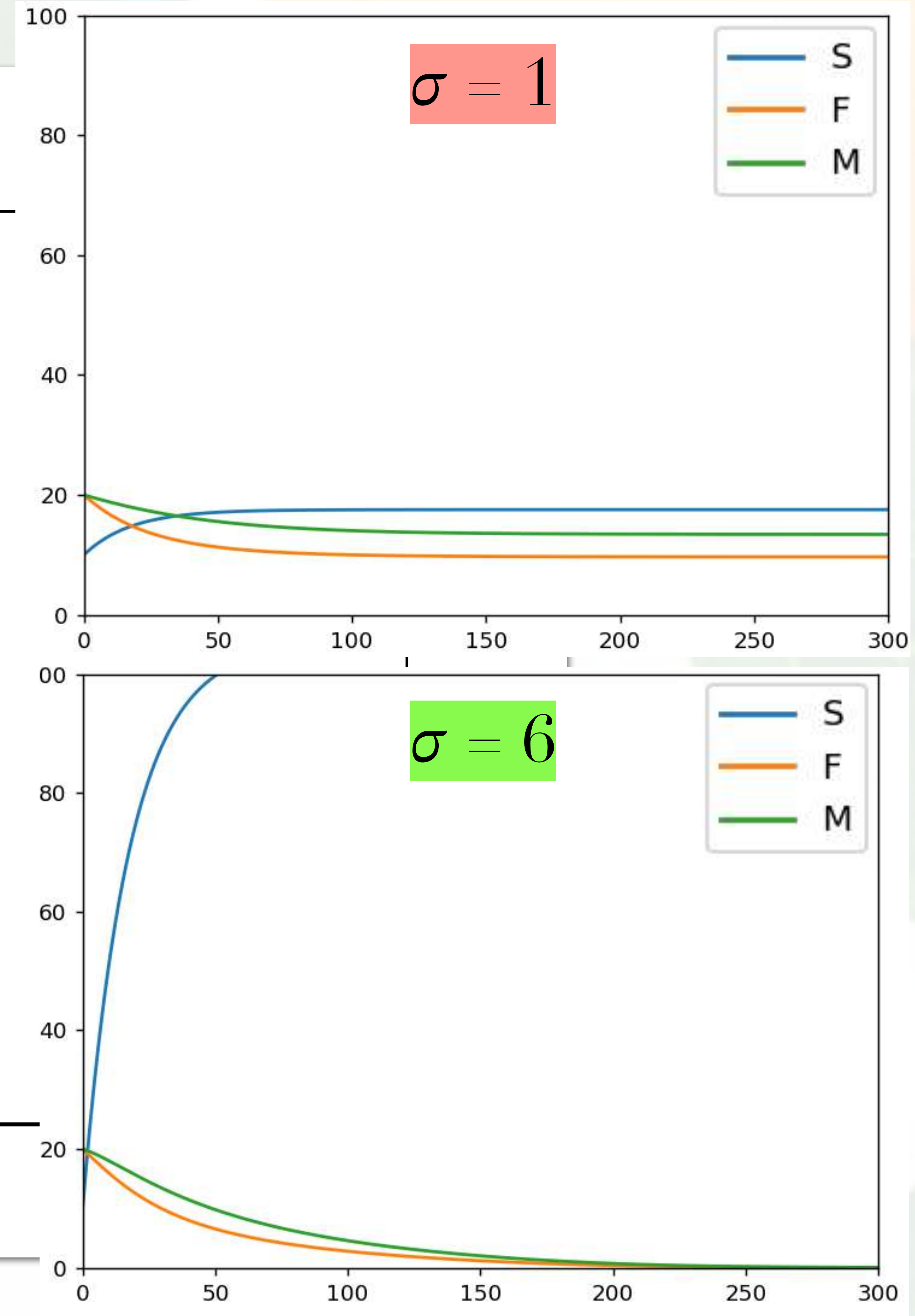
But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



Population Densities = f(t)



— Stable

- - Unstable

(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$

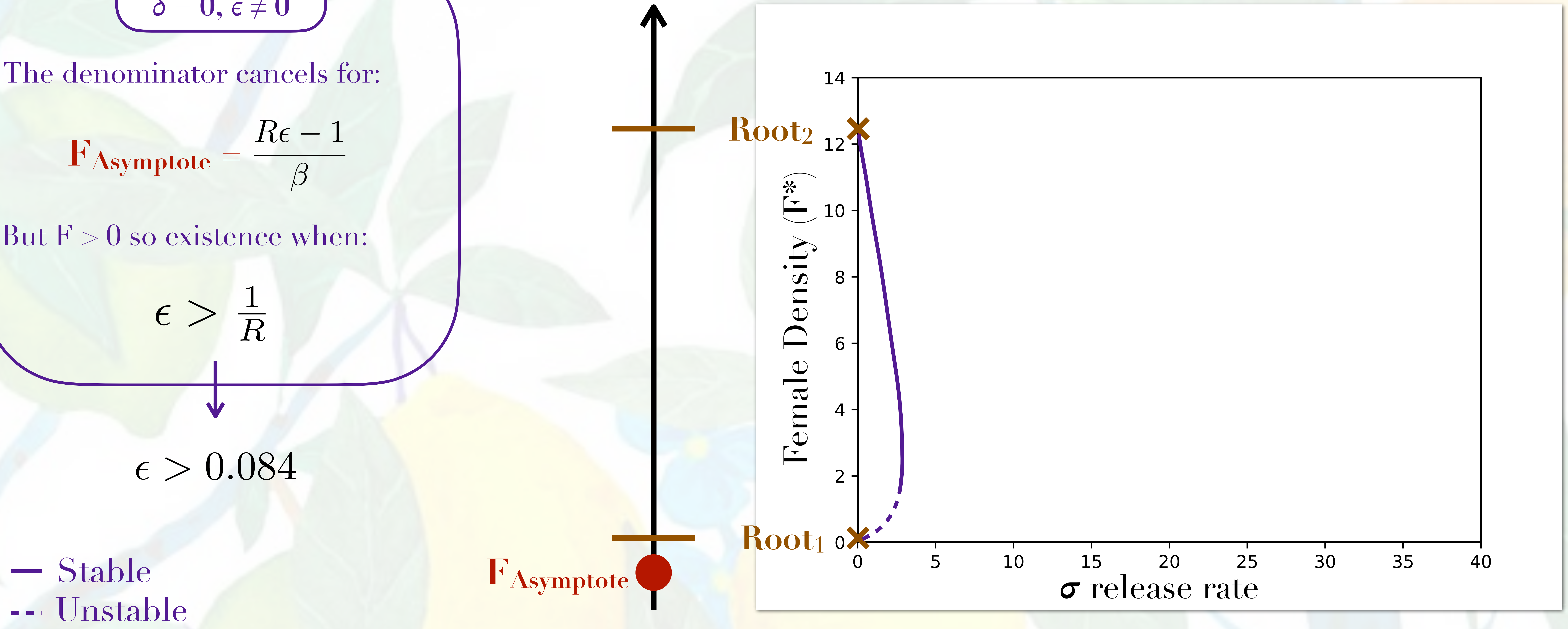
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$

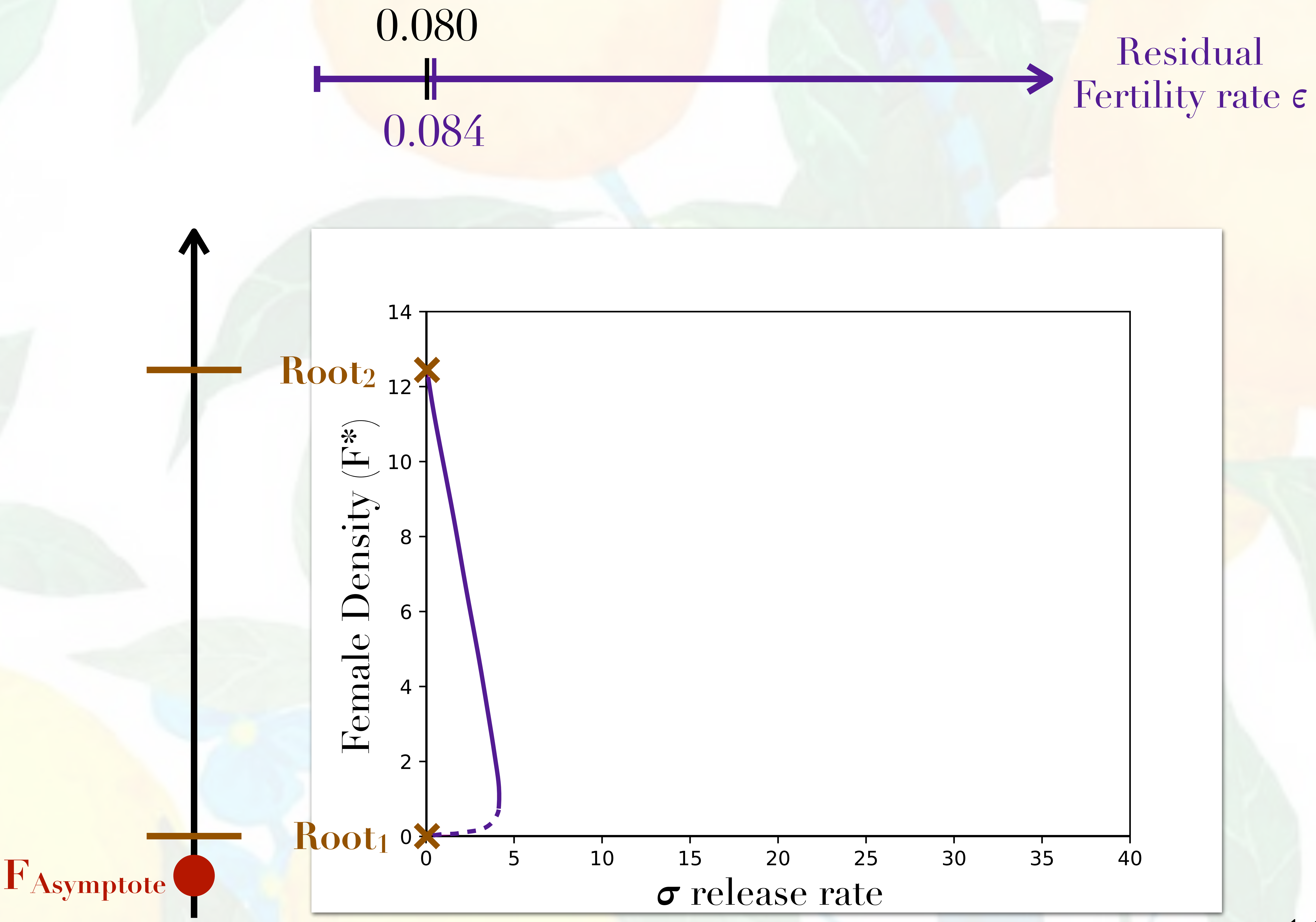
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



0.084

Residual Fertility rate ϵ

0.084

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

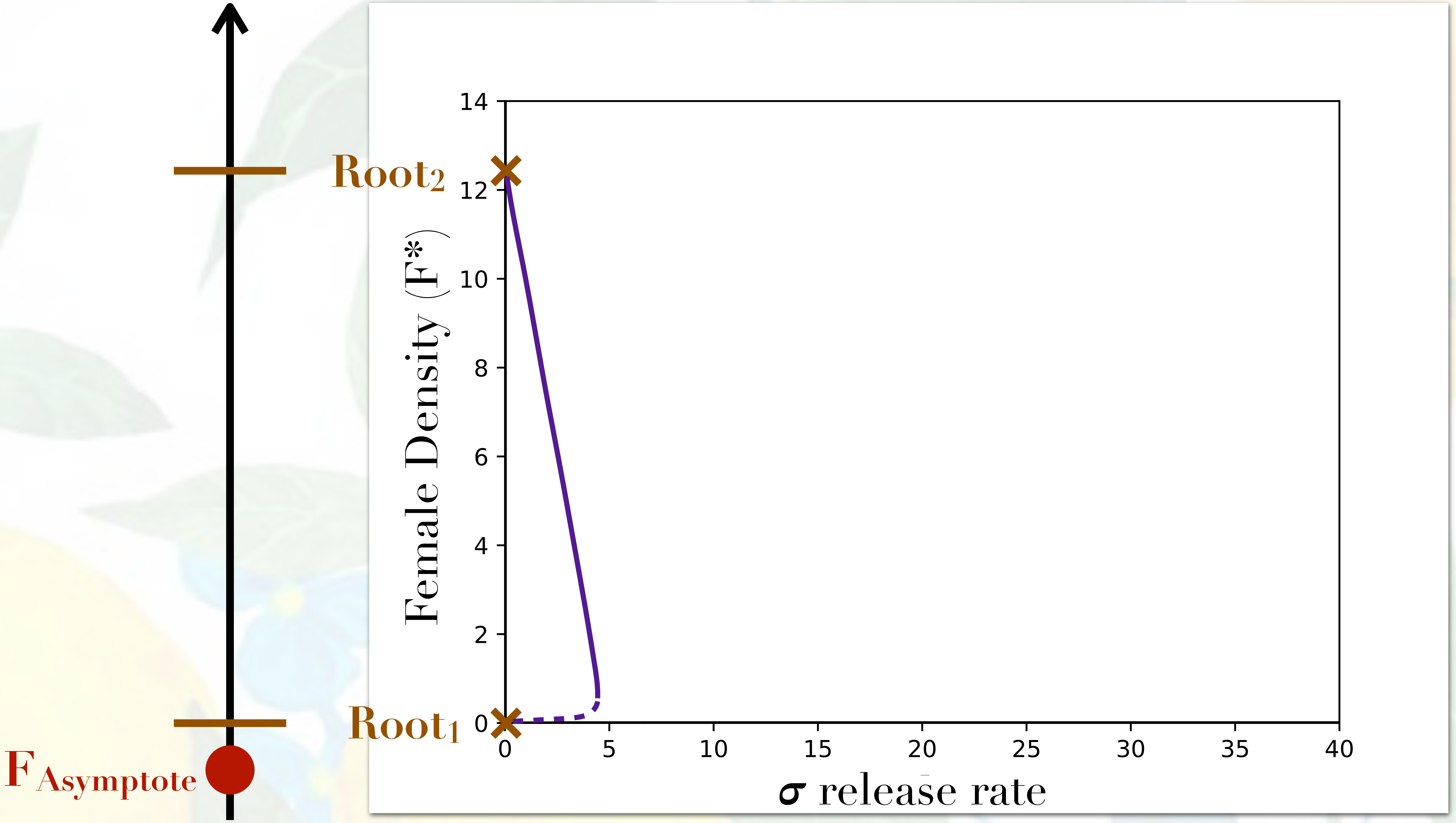
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

0.085

0.084

Residual Fertility rate ϵ

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

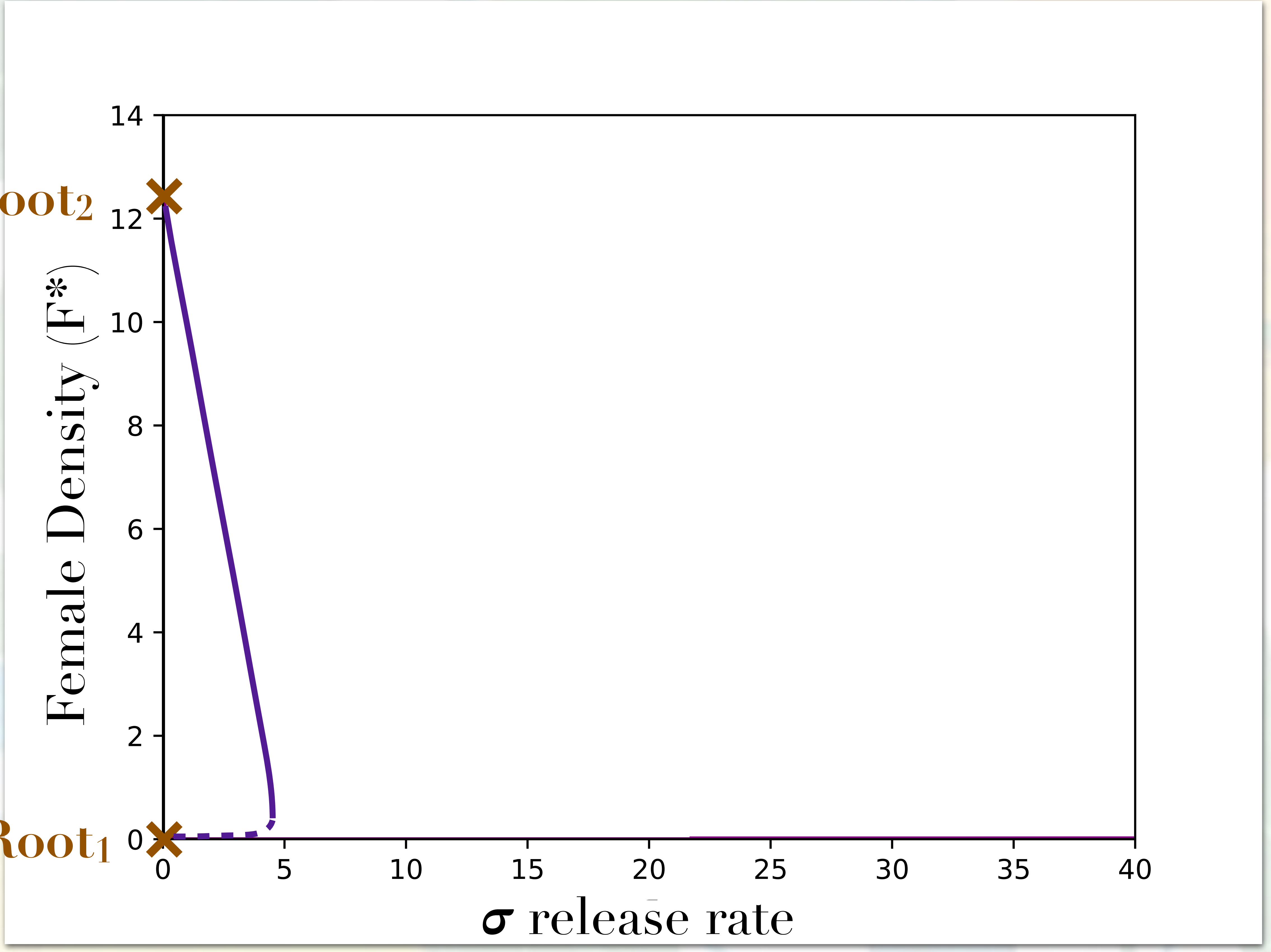
The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

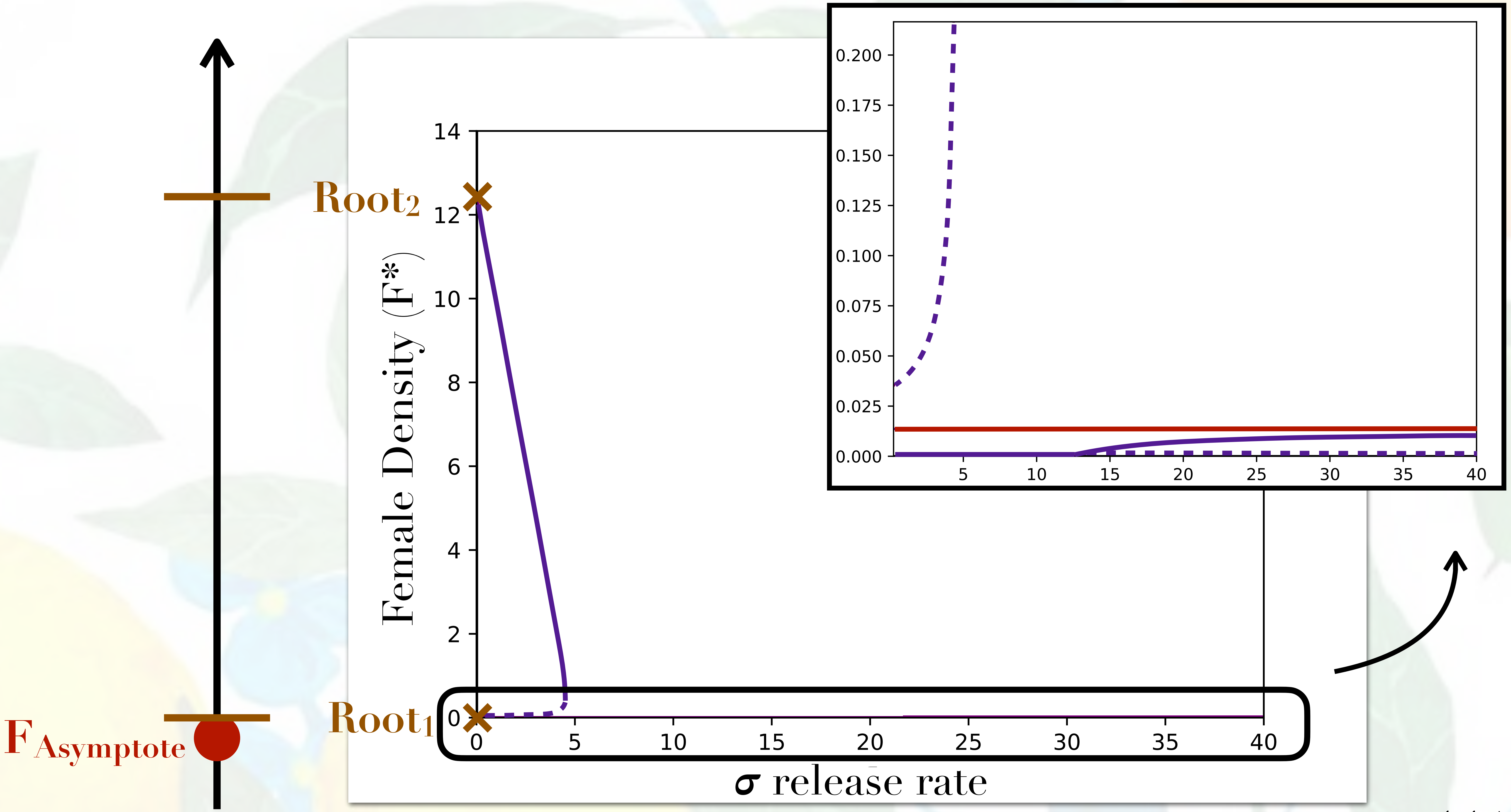
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2) Costly fertility model

$\delta = 0, \epsilon \neq 0$

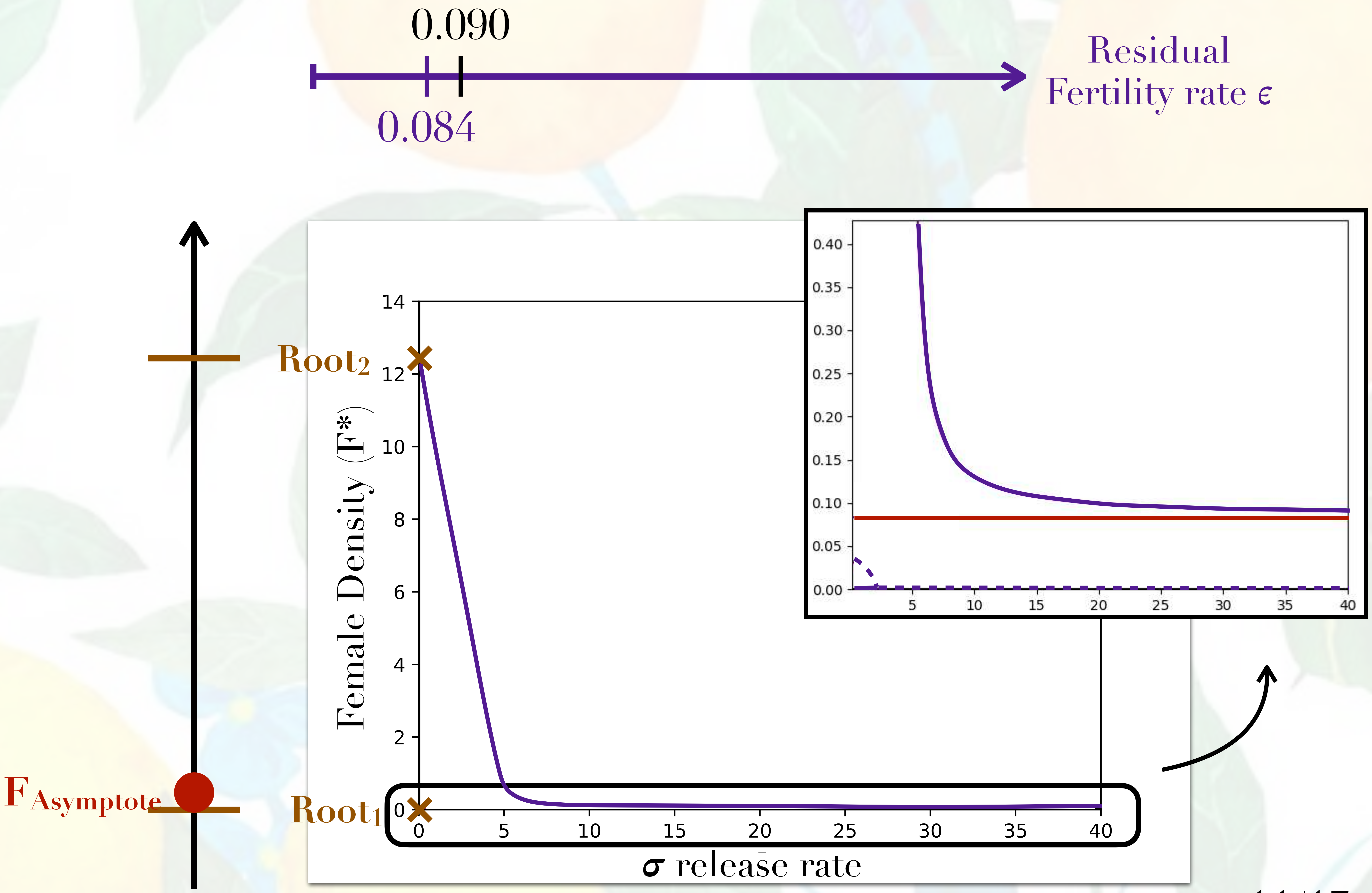
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$

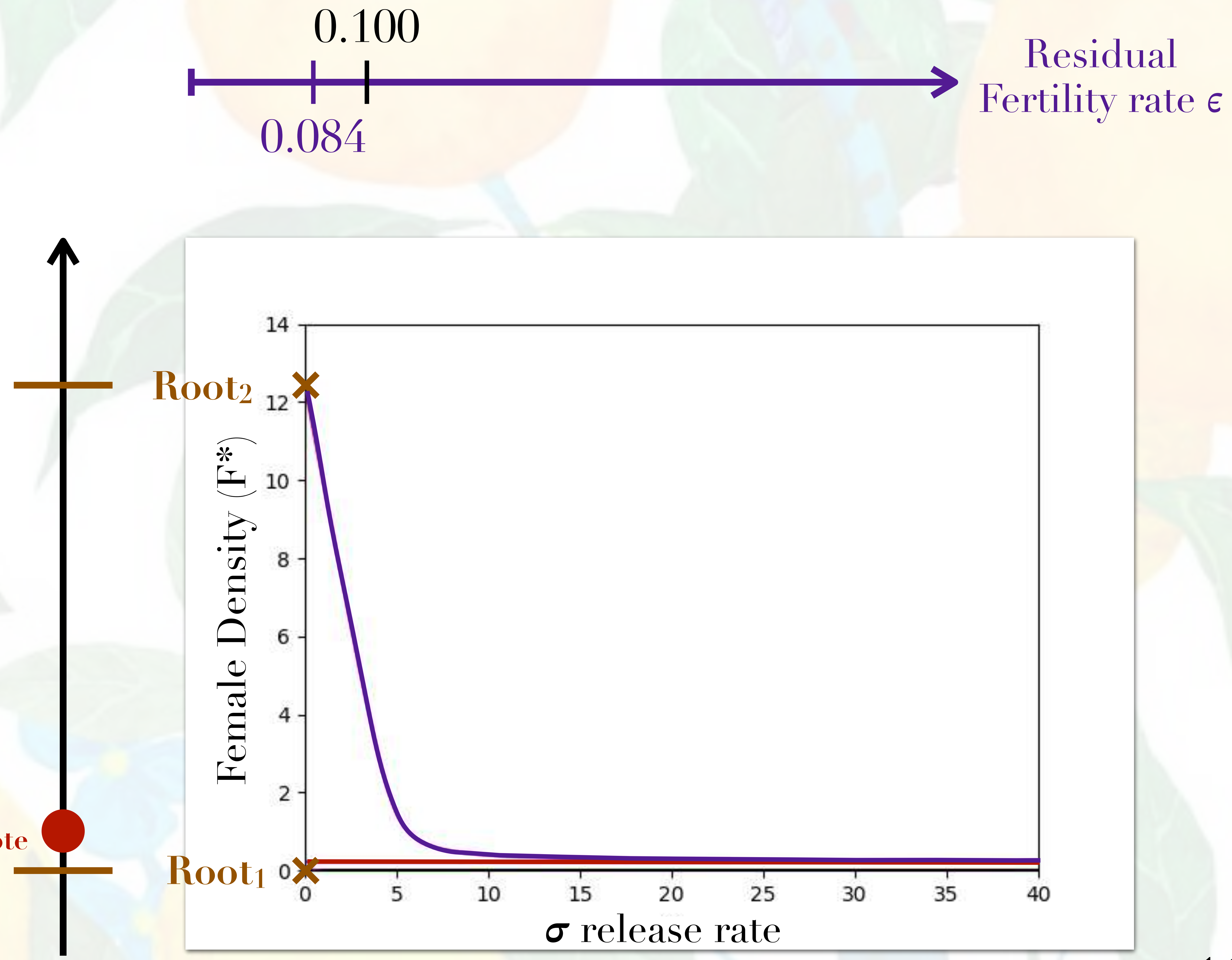
The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2) Costly fertility model

$\delta = 0, \epsilon \neq 0$

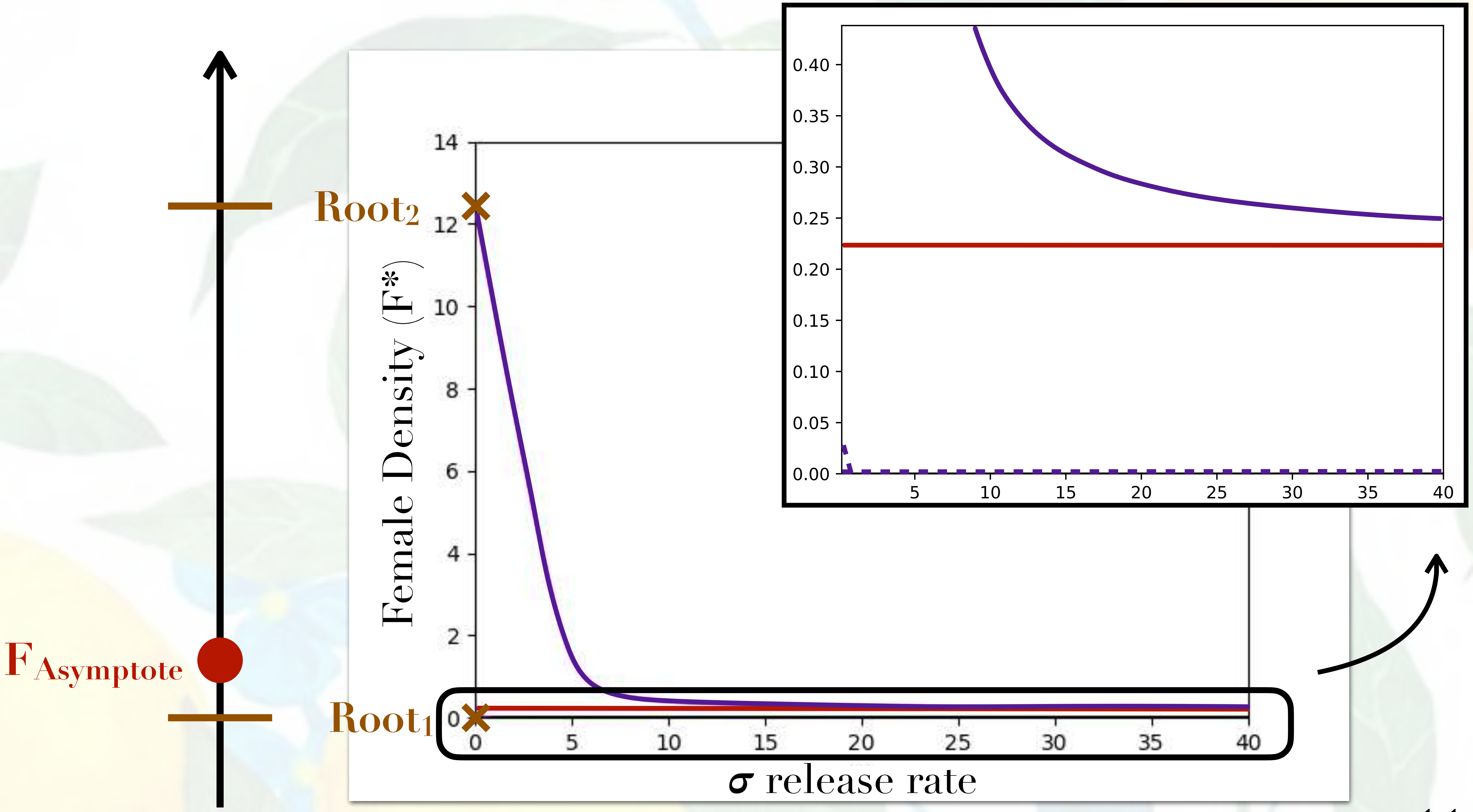
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2)

Costly fertility model

$\delta = 0, \epsilon \neq 0$

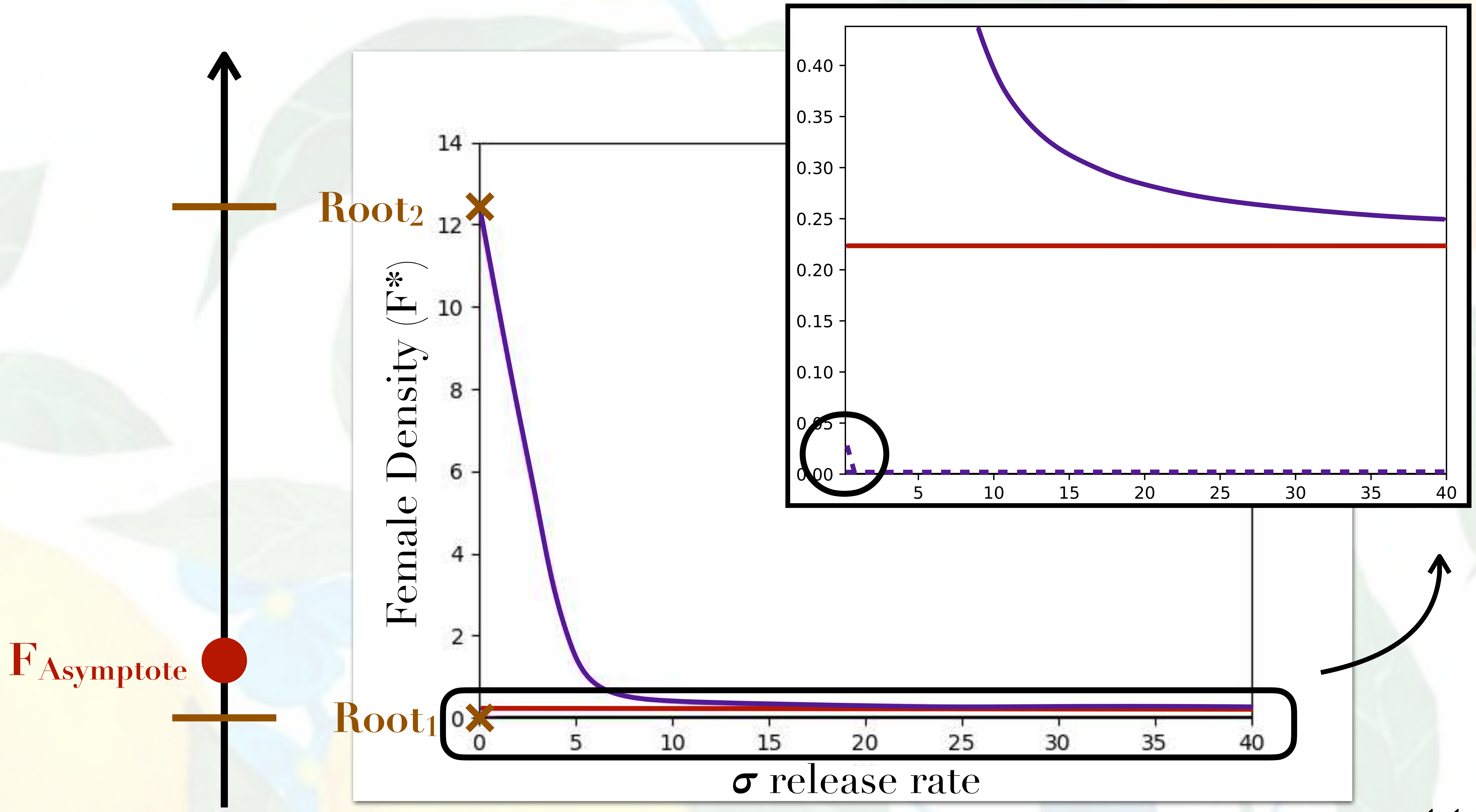
The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
- - Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

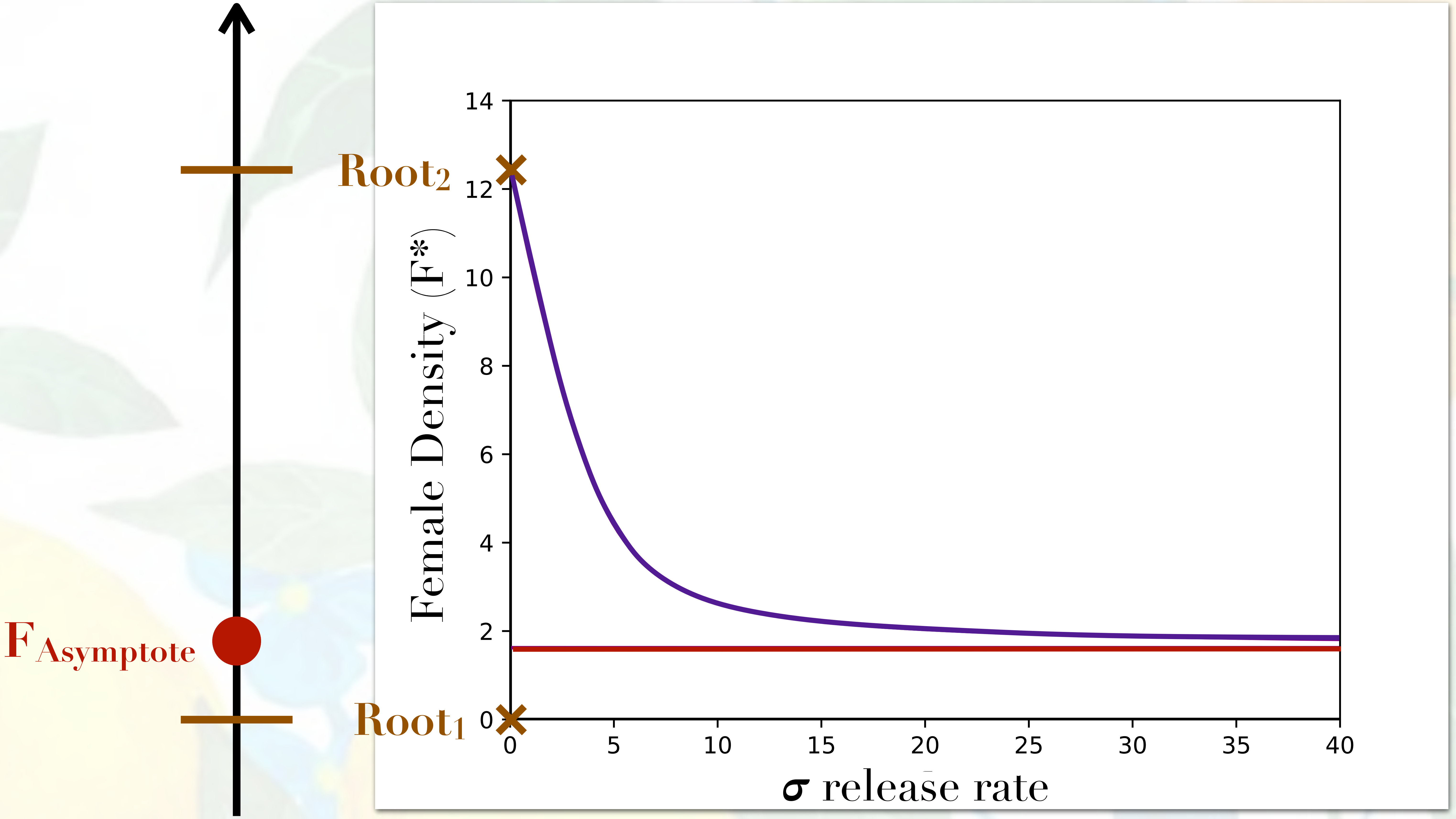
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$

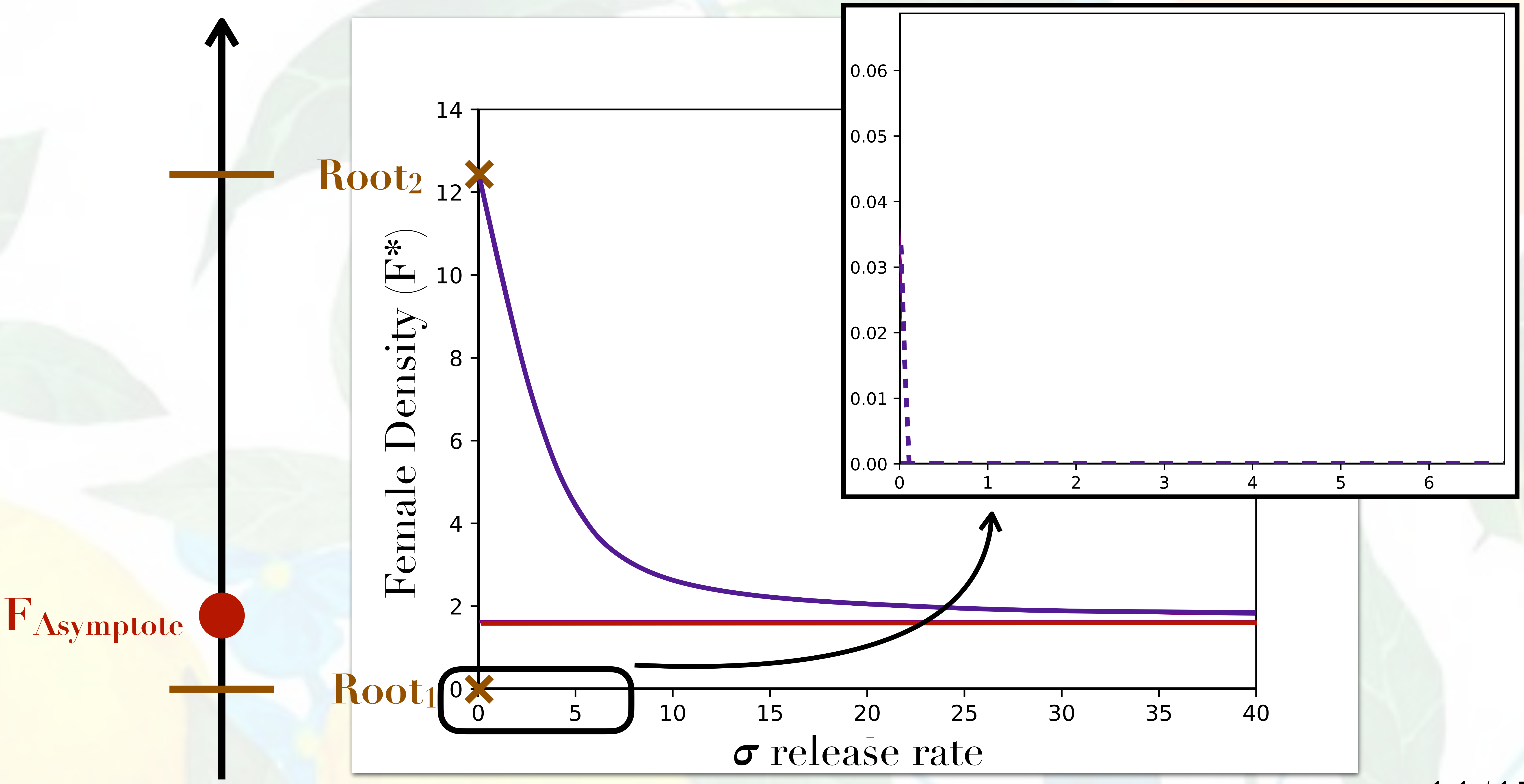
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

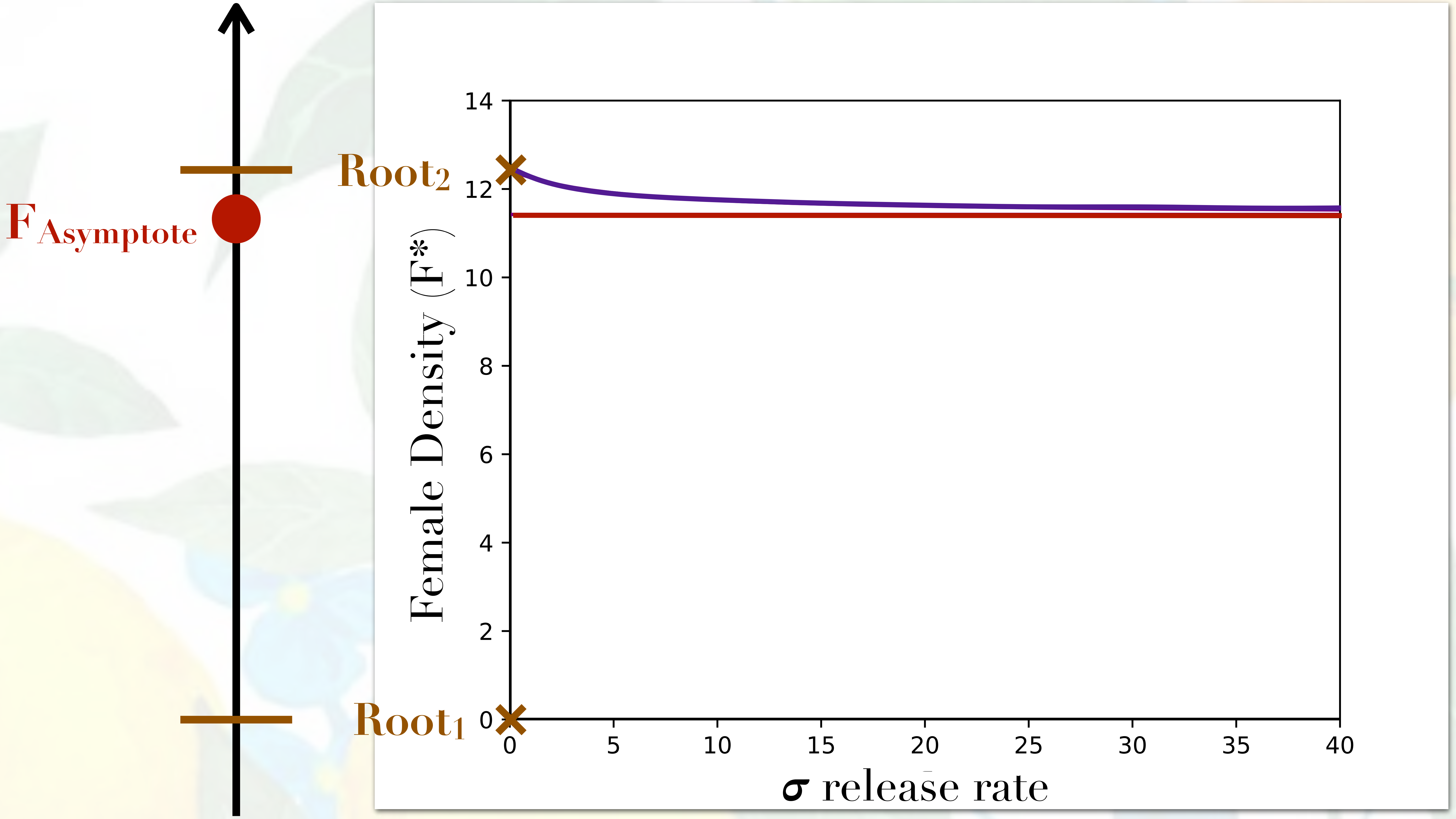
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

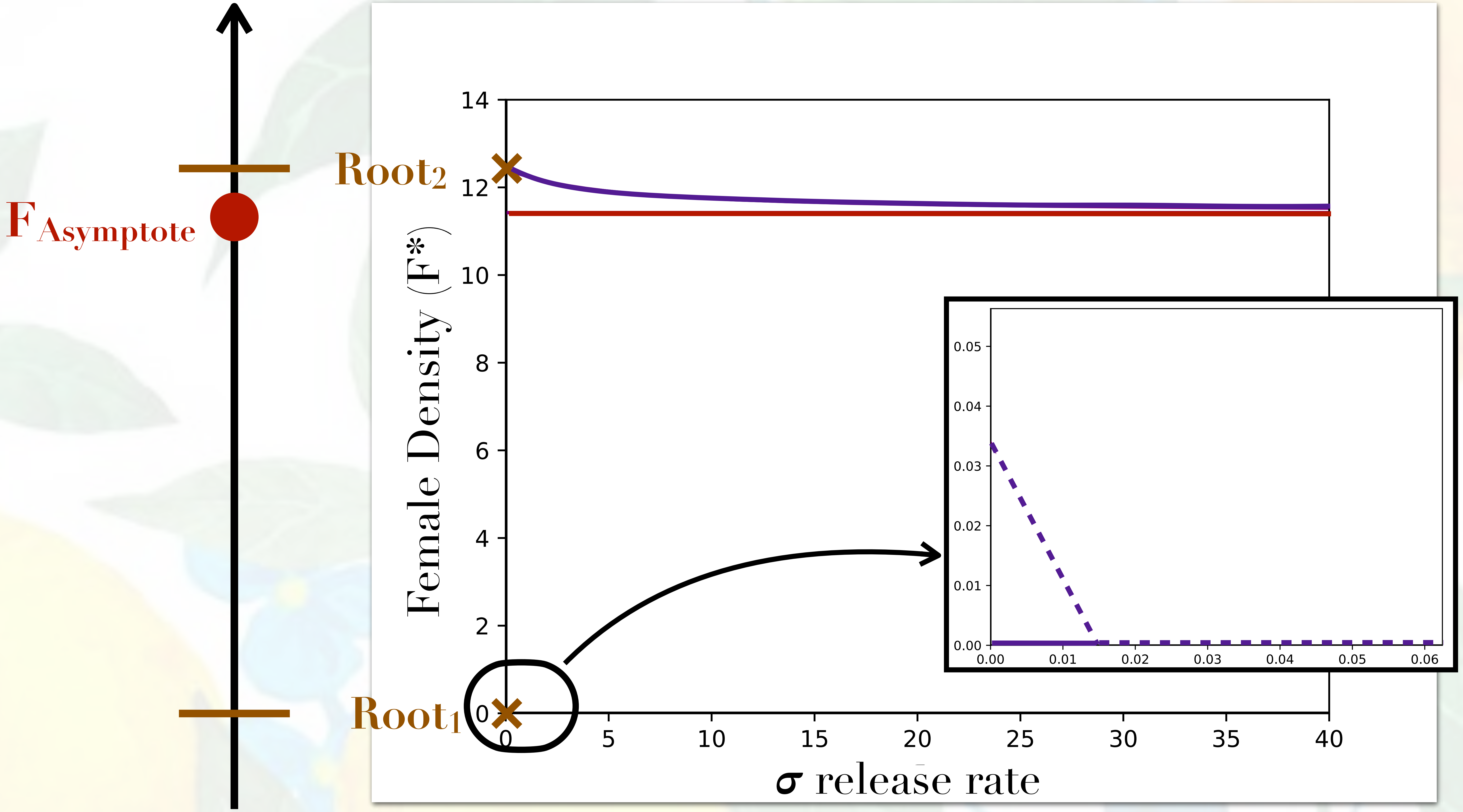
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$



— Stable
-- Unstable

1.000 Residual Fertility rate ϵ

0.084

(2) Costly fertility model

$\delta = 0, \epsilon \neq 0$

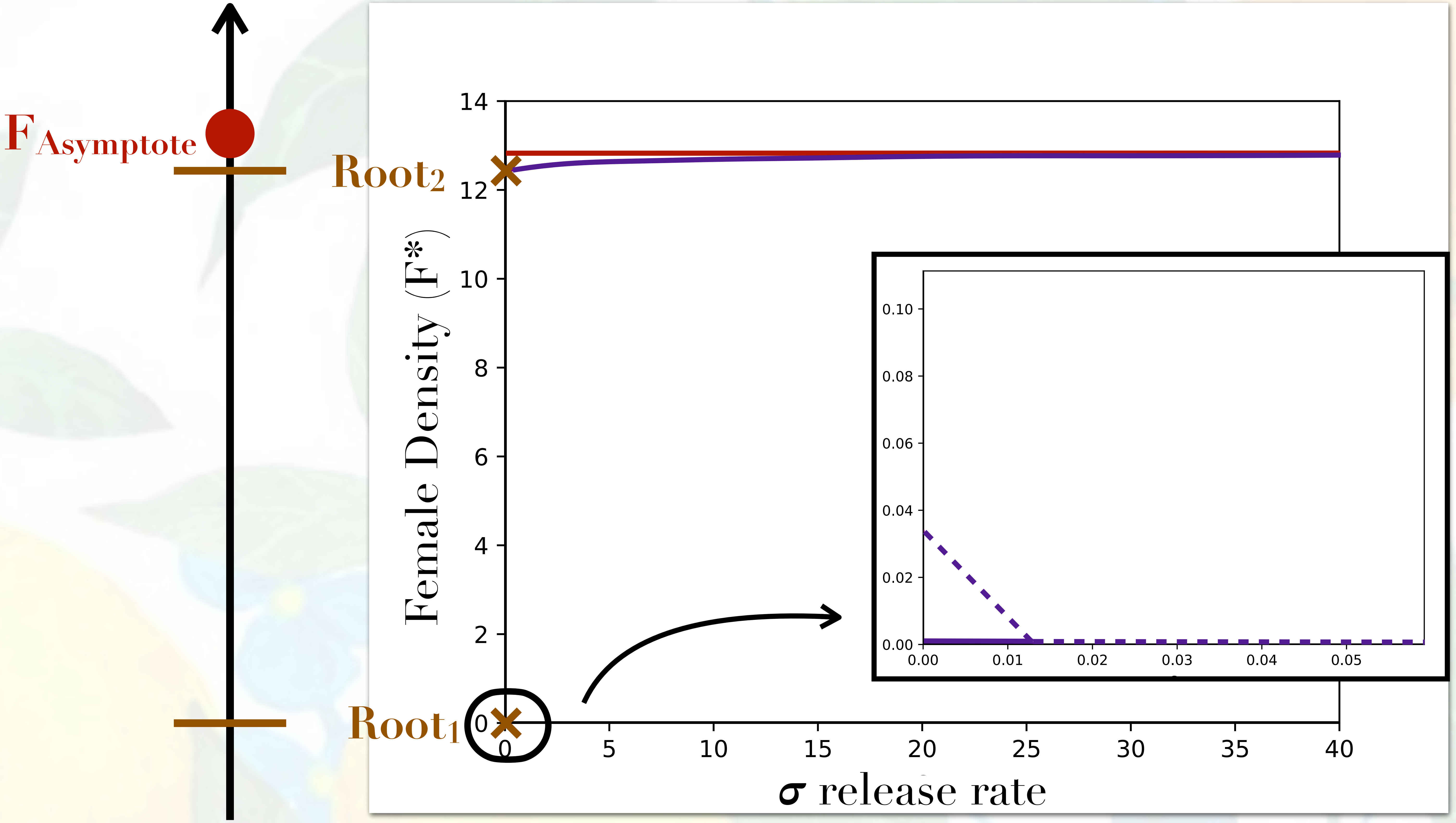
The denominator cancels for:

$$F_{Asymptote} = \frac{R\epsilon - 1}{\beta}$$

But F > 0 so existence when:

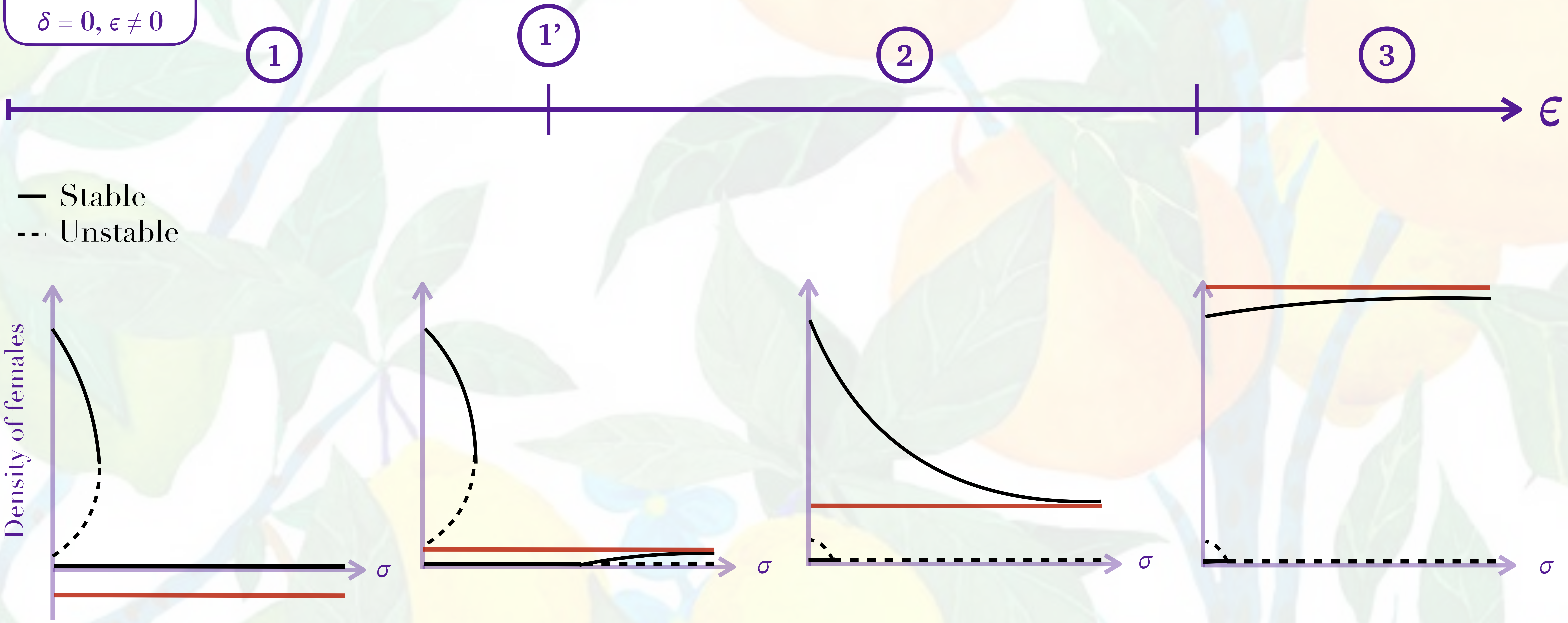
$$\epsilon > \frac{1}{R}$$

$$\epsilon > 0.084$$

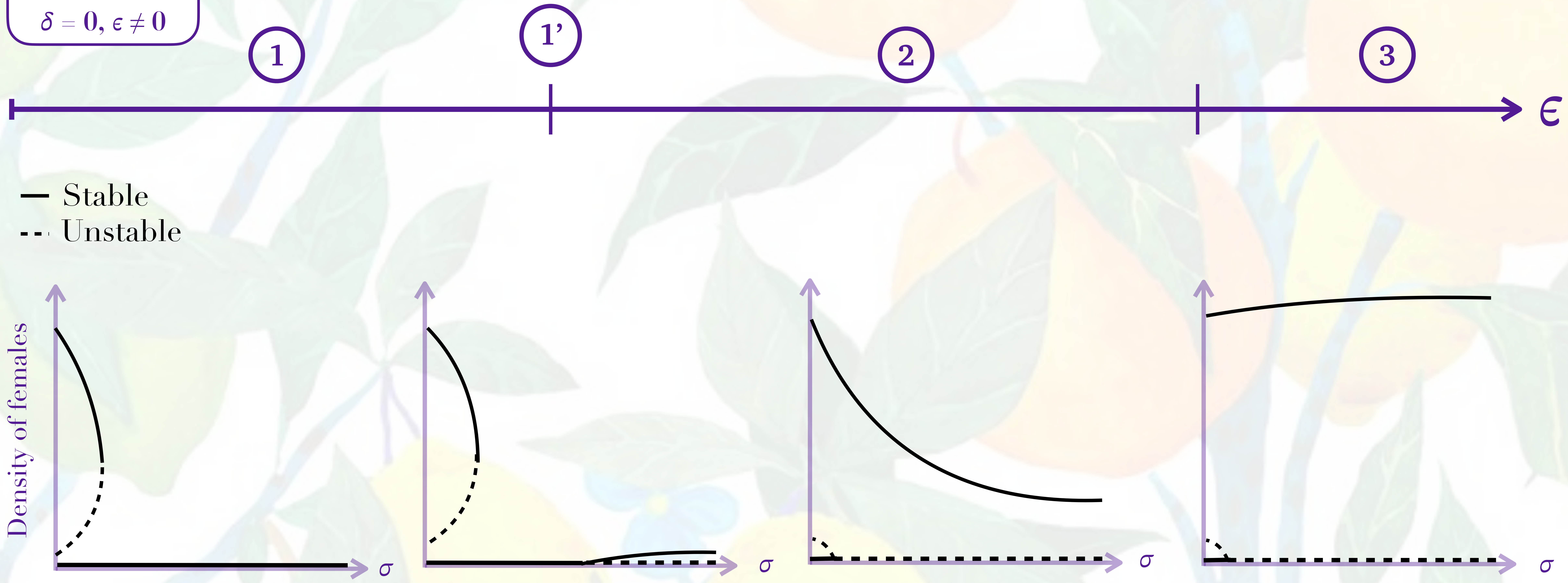


— Stable
- - Unstable

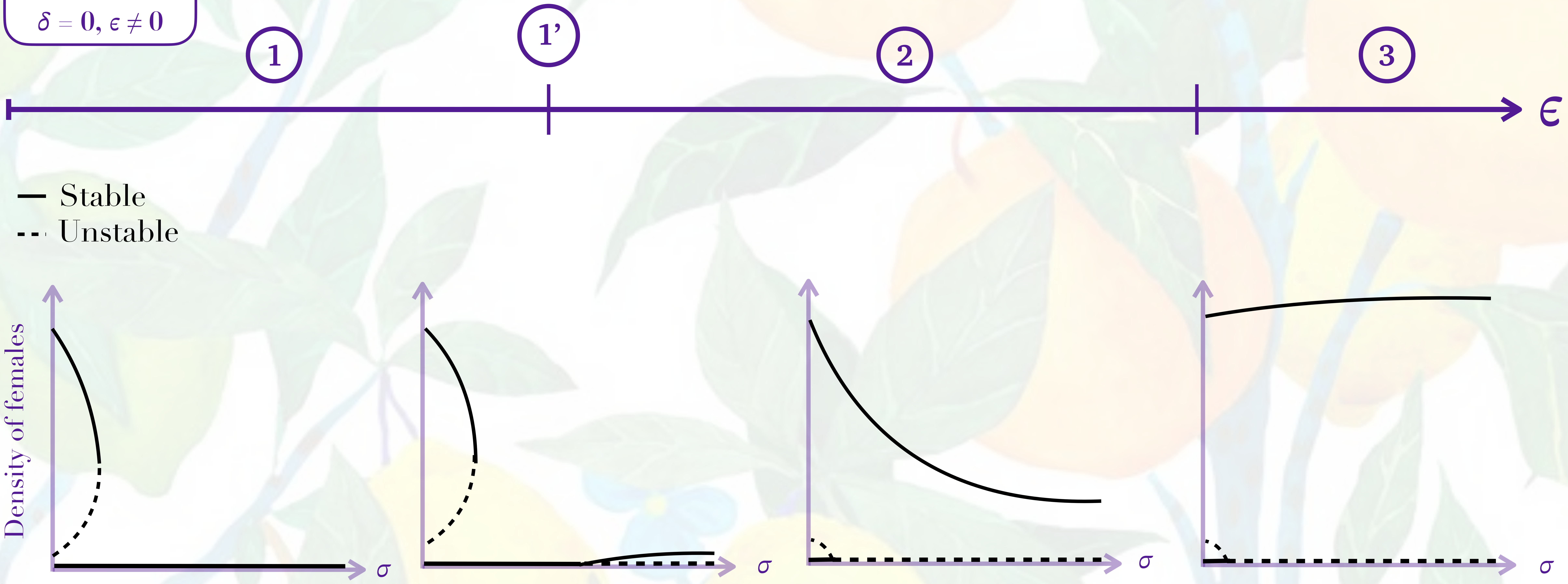
(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$



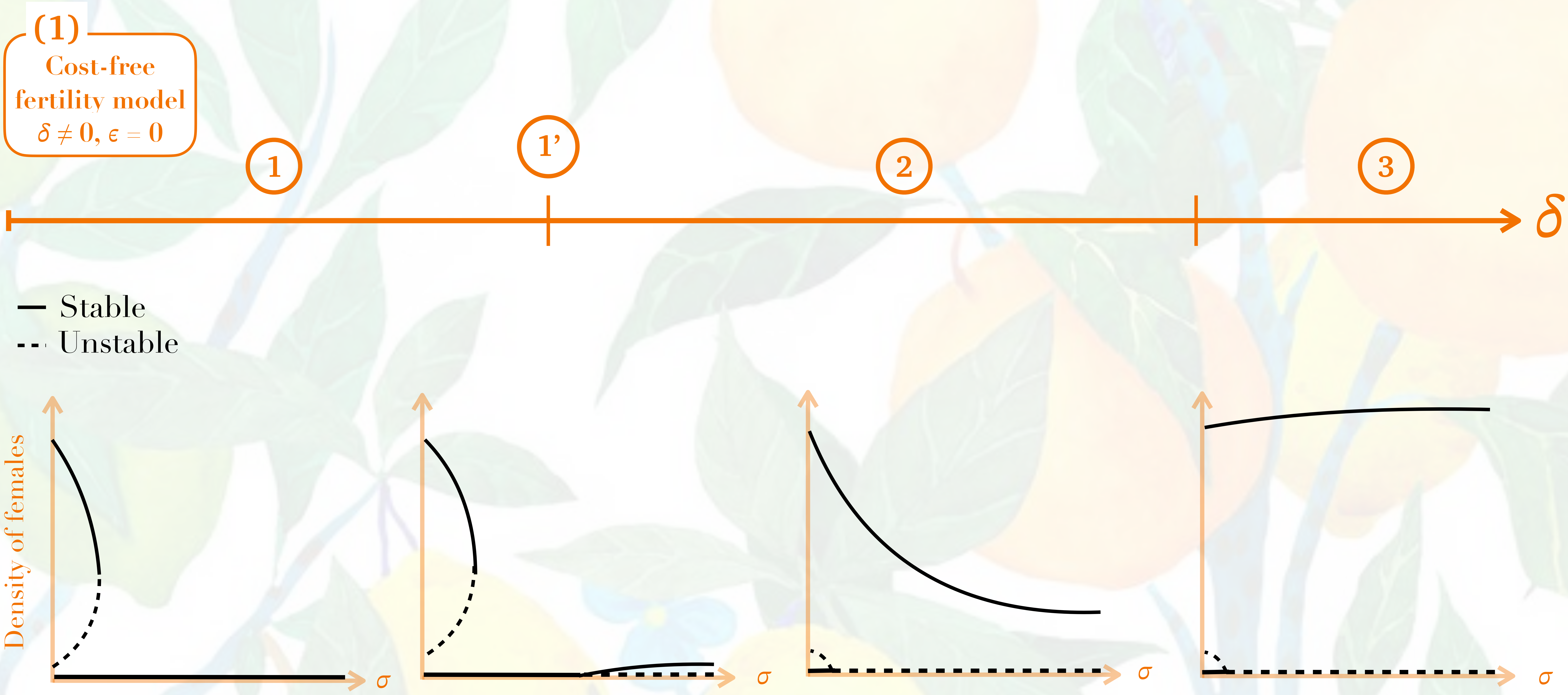
(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$



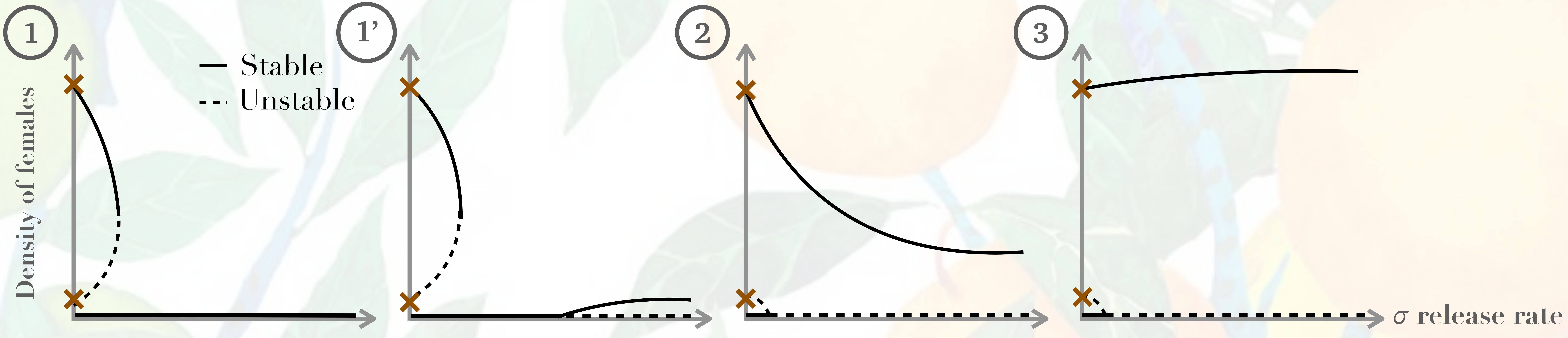
(2) Costly fertility model
$\delta = 0, \epsilon \neq 0$

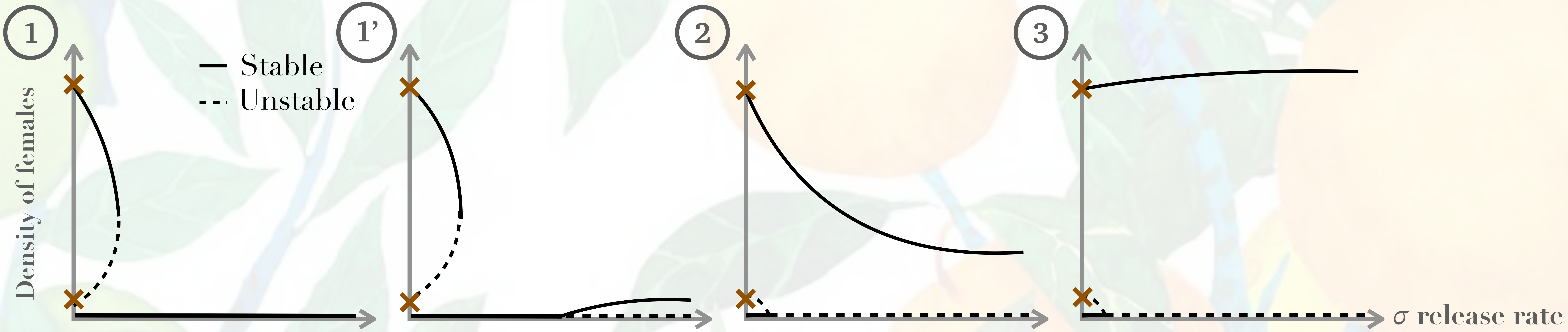


➡️ Different control capabilities depending on the shape of the bifurcation diagram



➡️ Different control capabilities depending on the shape of the bifurcation diagram

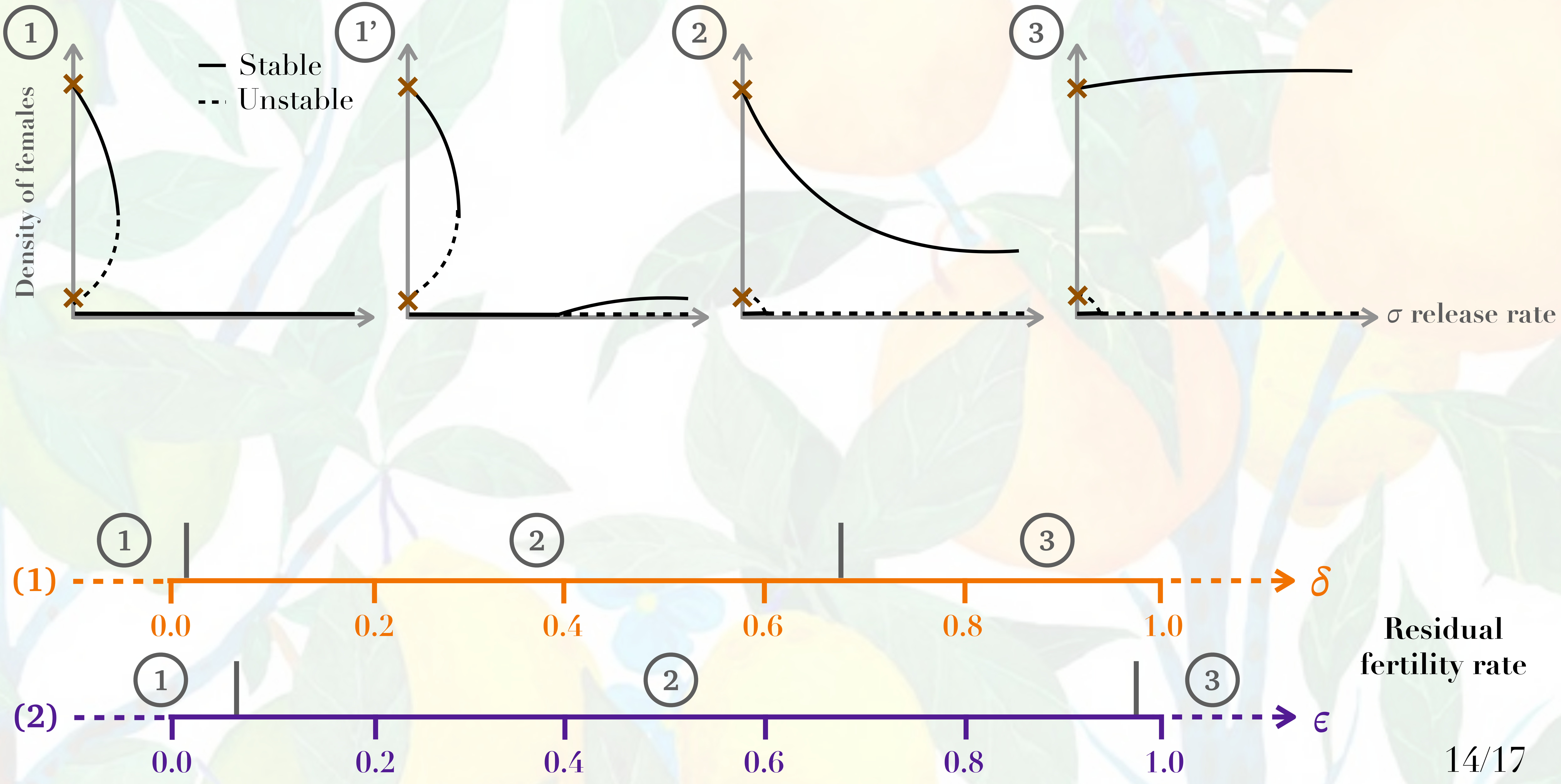

1
1'
2
3
Density of females
Stable
Unstable
σ release rate

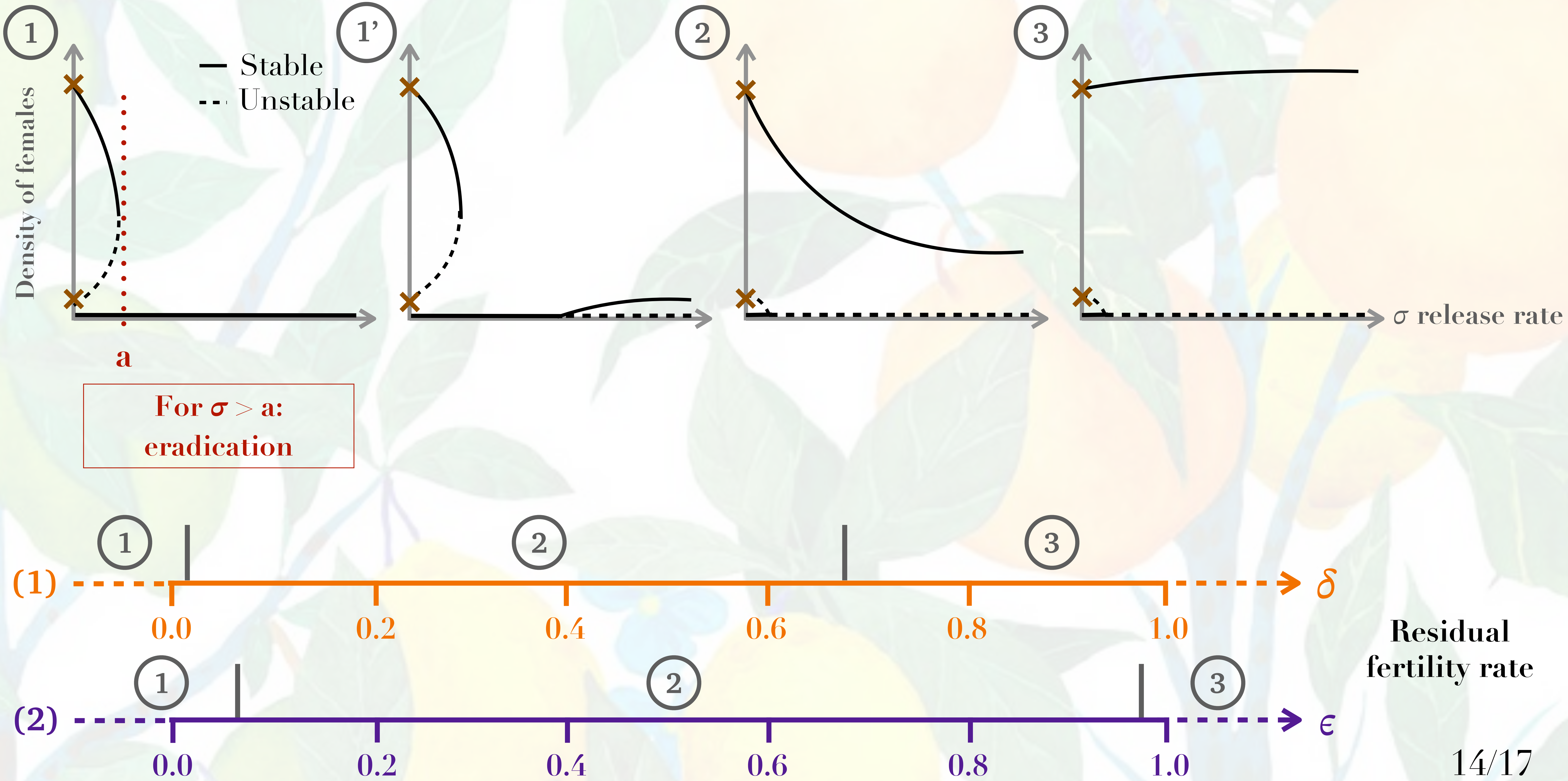


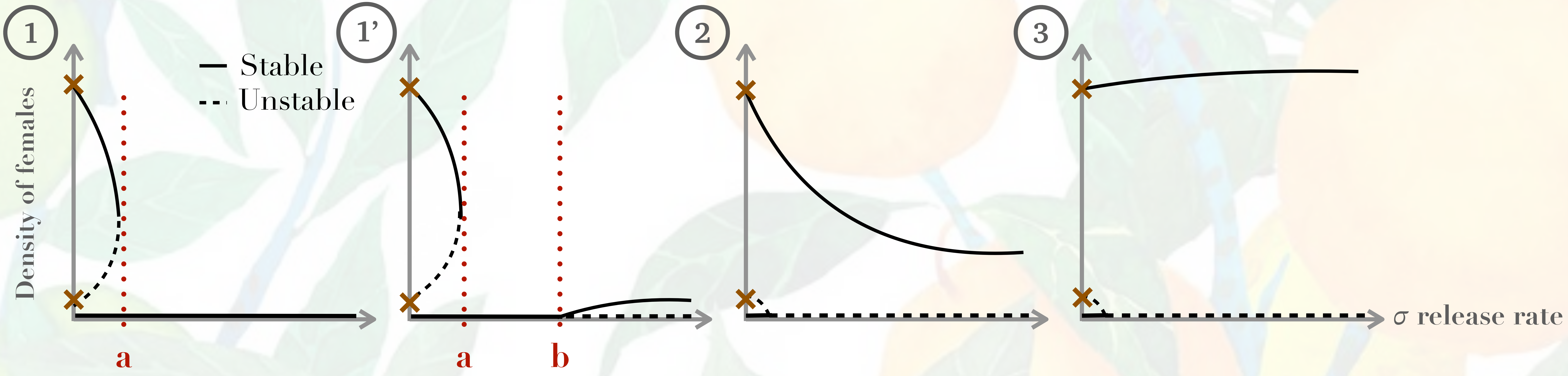
(1) 0.0 0.2 0.4 0.6 0.8 1.0 δ

Residual fertility rate

(2) 0.0 0.2 0.4 0.6 0.8 1.0 ϵ

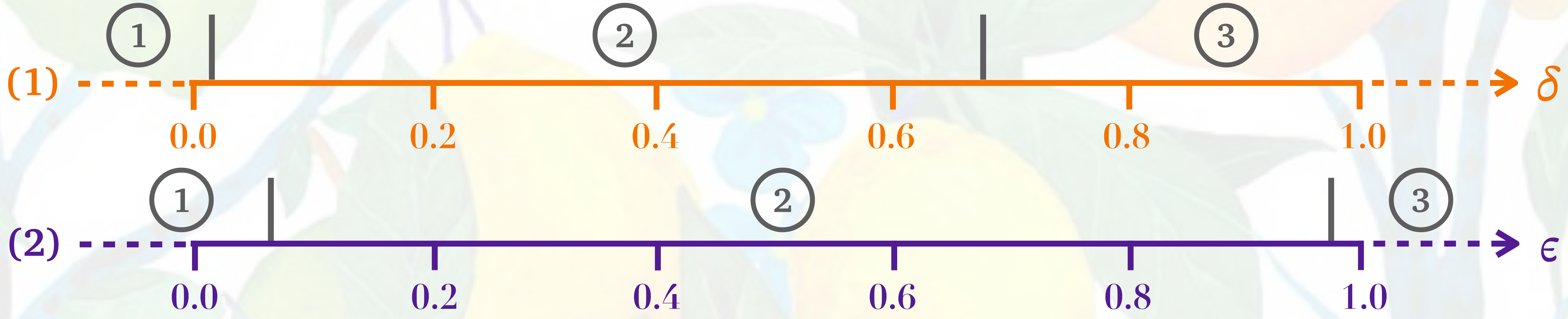

1
1'
2
3
Density of females
Stable
Unstable
σ release rate
1
2
3
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
Residual fertility rate
1
2
3
(2)
ϵ
0.0
0.2
0.4
0.6
0.8
1.0


1
1'
2
3
Stable
Unstable
Density of females
a
σ release rate
For σ > a:
eradication
1
2
3
(1)
0.0
0.2
0.4
0.6
0.8
1.0
δ
Residual
fertility rate
1
2
3
(2)
0.0
0.2
0.4
0.6
0.8
1.0
ϵ

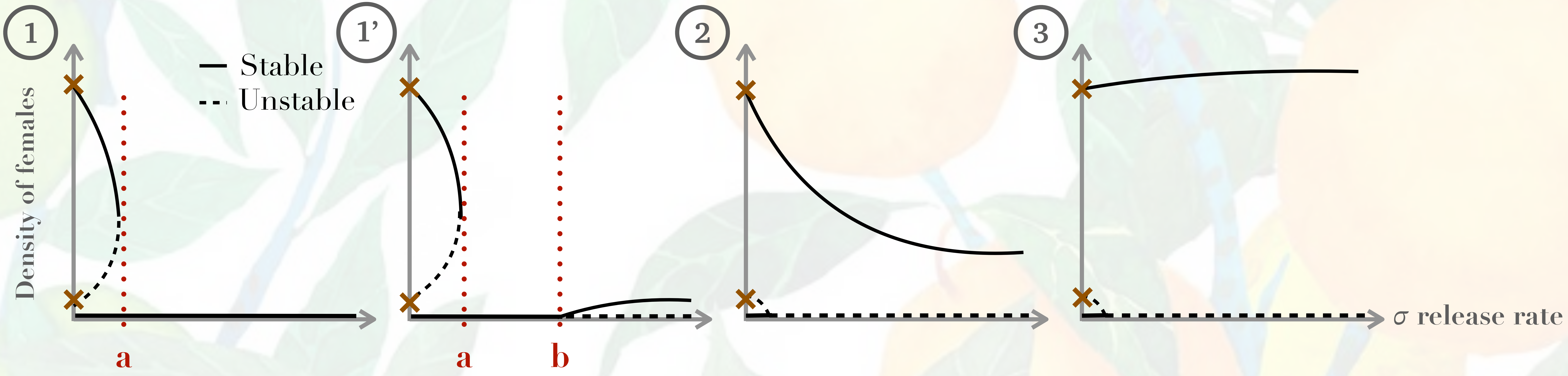


For σ > a:
eradication

For a < σ < b: eradication
For σ >b: quasi eradication



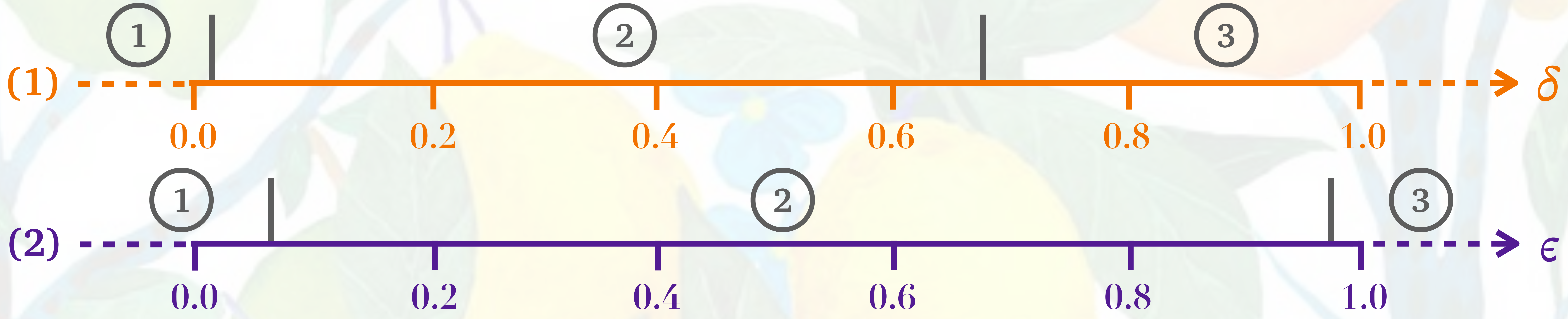
Residual fertility rate



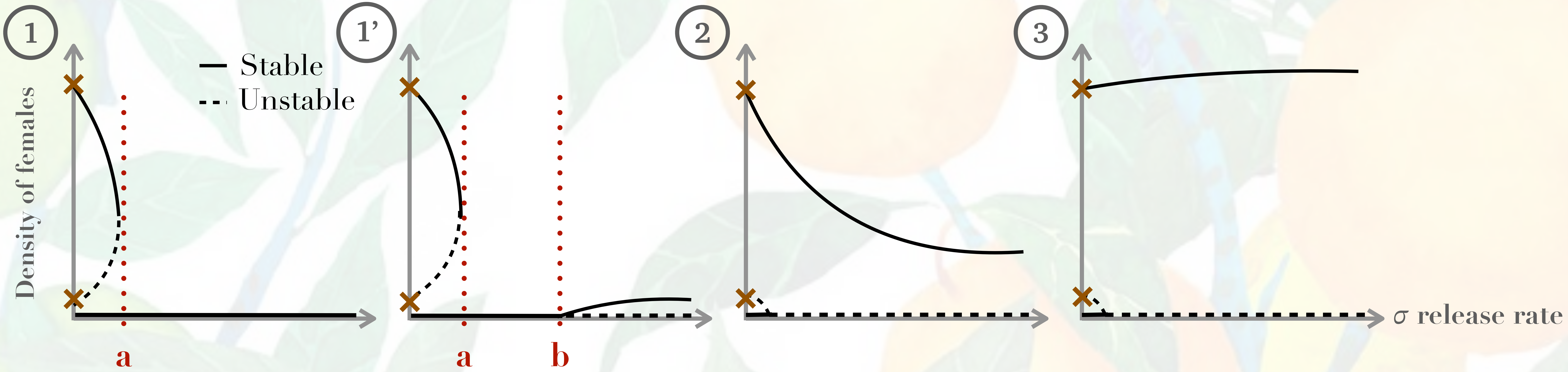
For σ > a:
eradication

For a < σ < b: eradication
For σ >b: quasi eradication

Control only for
large σ



Residual fertility rate

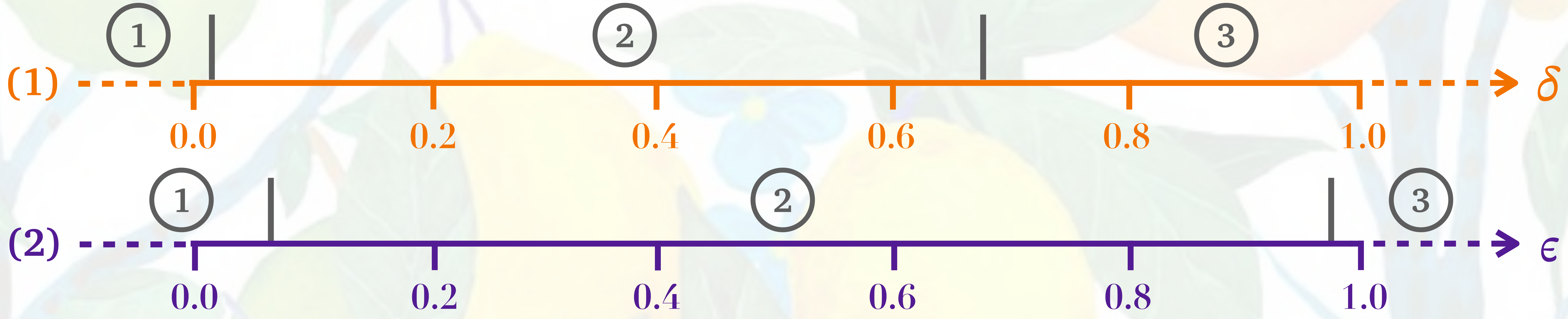


For σ > a: eradication

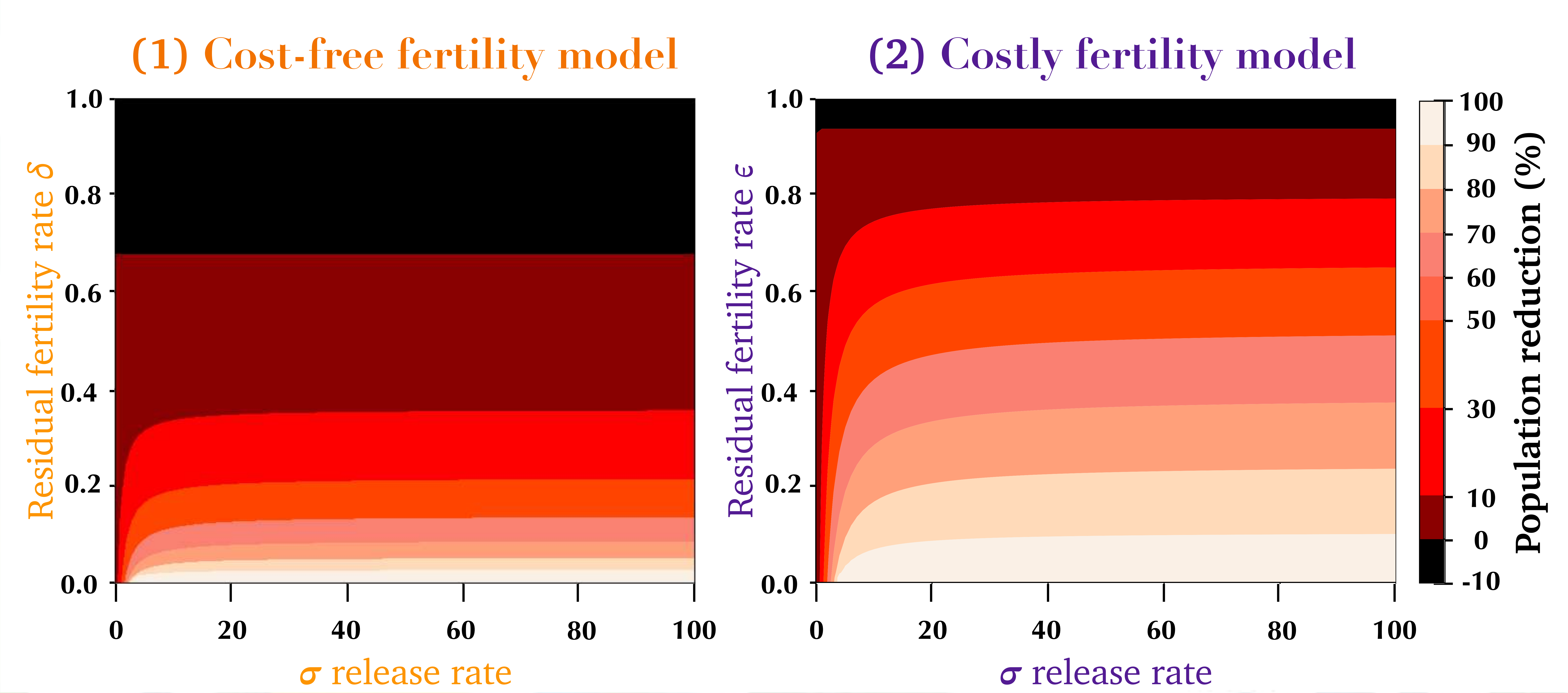
For a < σ < b: eradication
For σ >b: quasi eradication

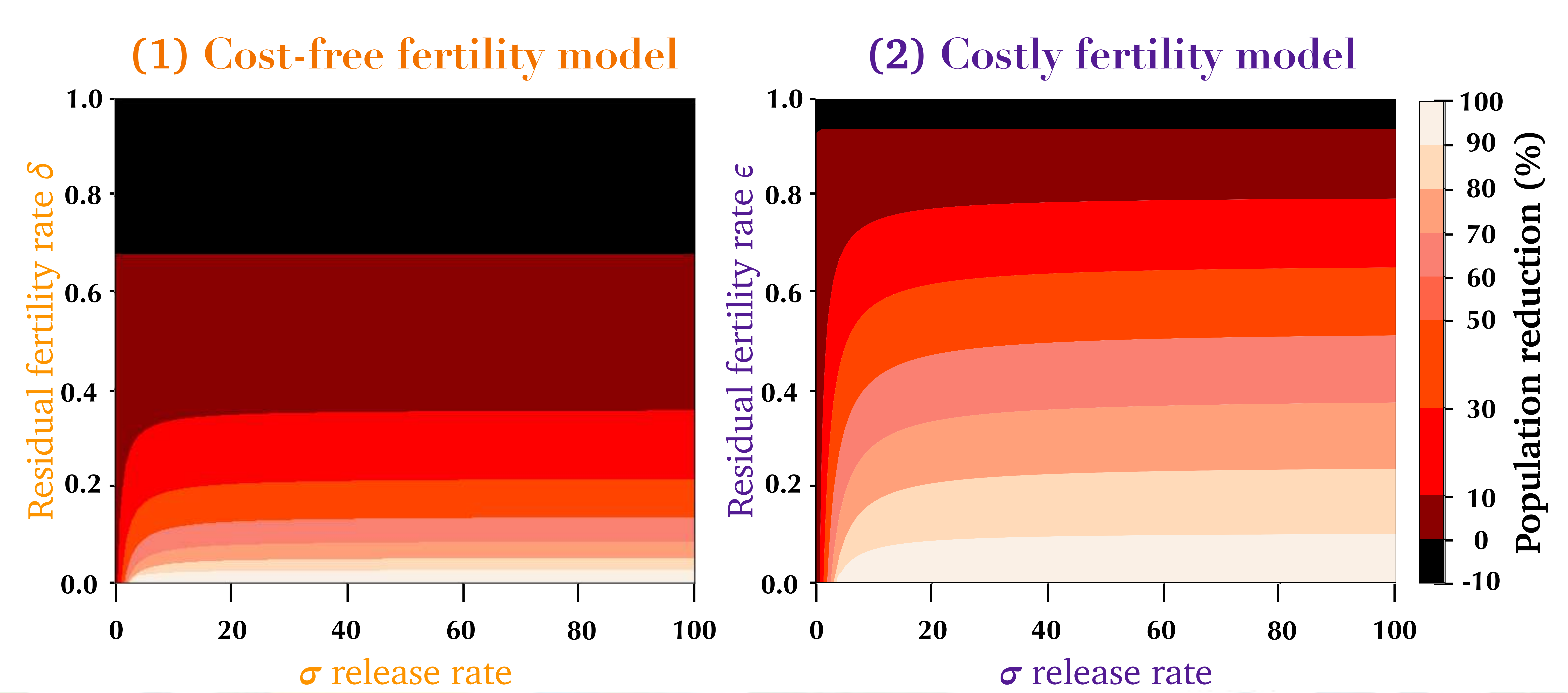
Control only for large σ

SIT is inefficient

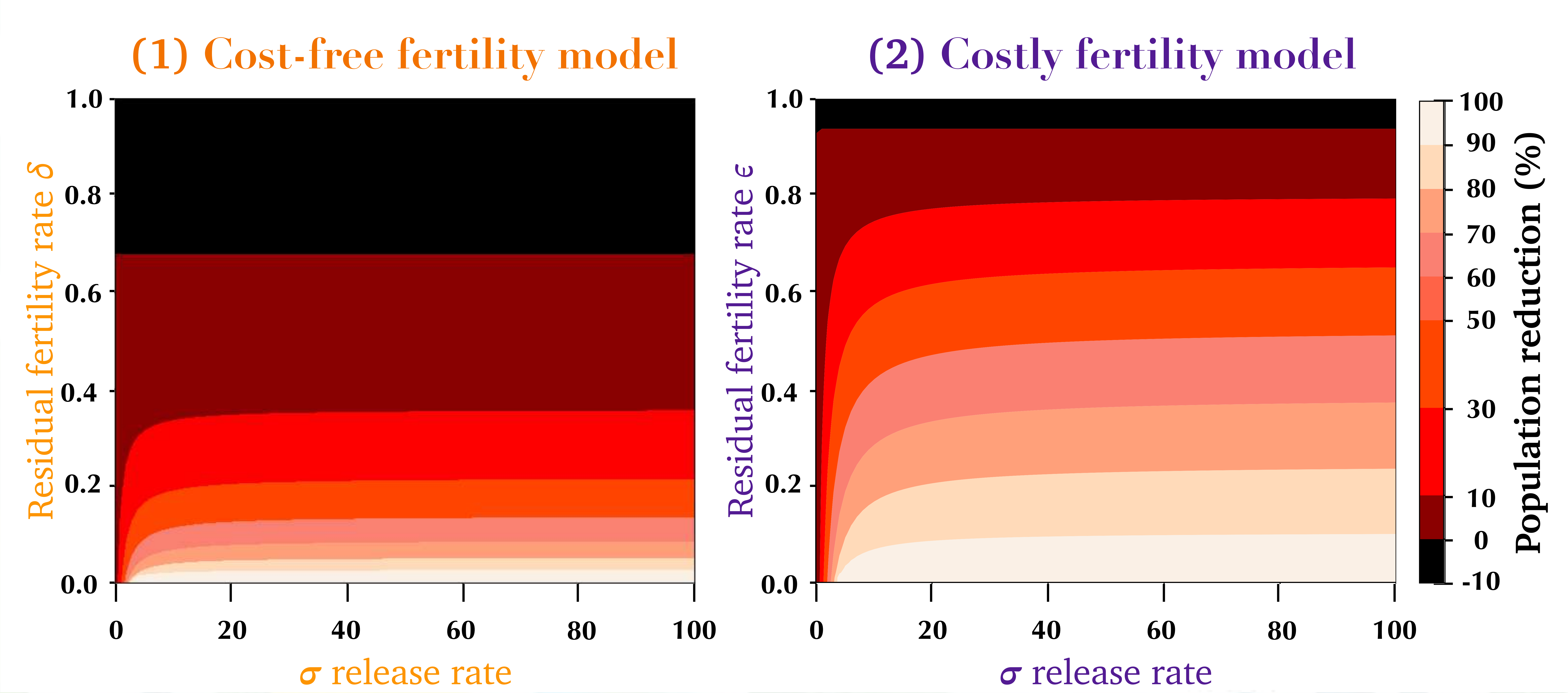


Residual fertility rate

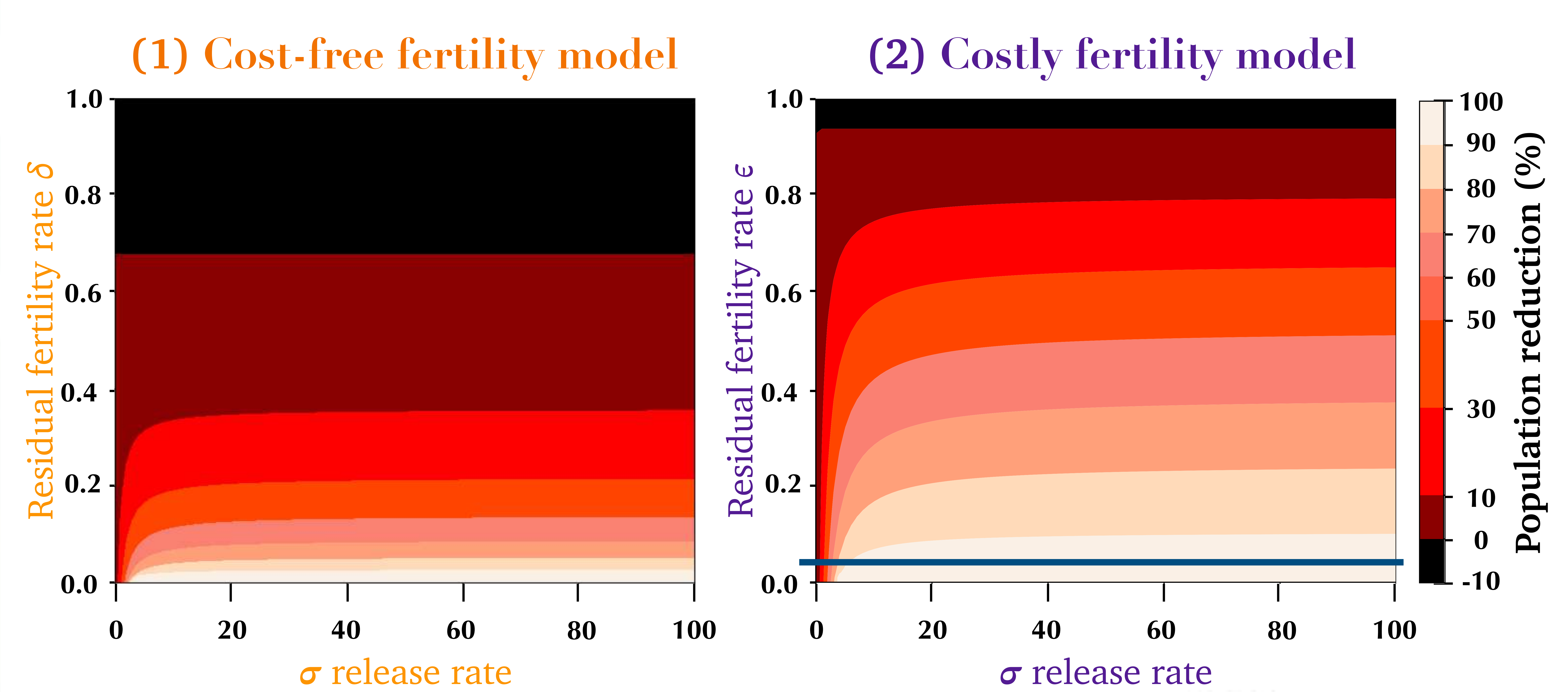

(1) Cost-free fertility model
(2) Costly fertility model
Residual fertility rate δ
Residual fertility rate ϵ
σ release rate
σ release rate
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Population reduction (%)
100
90
80
70
60
50
30
10
0
-10



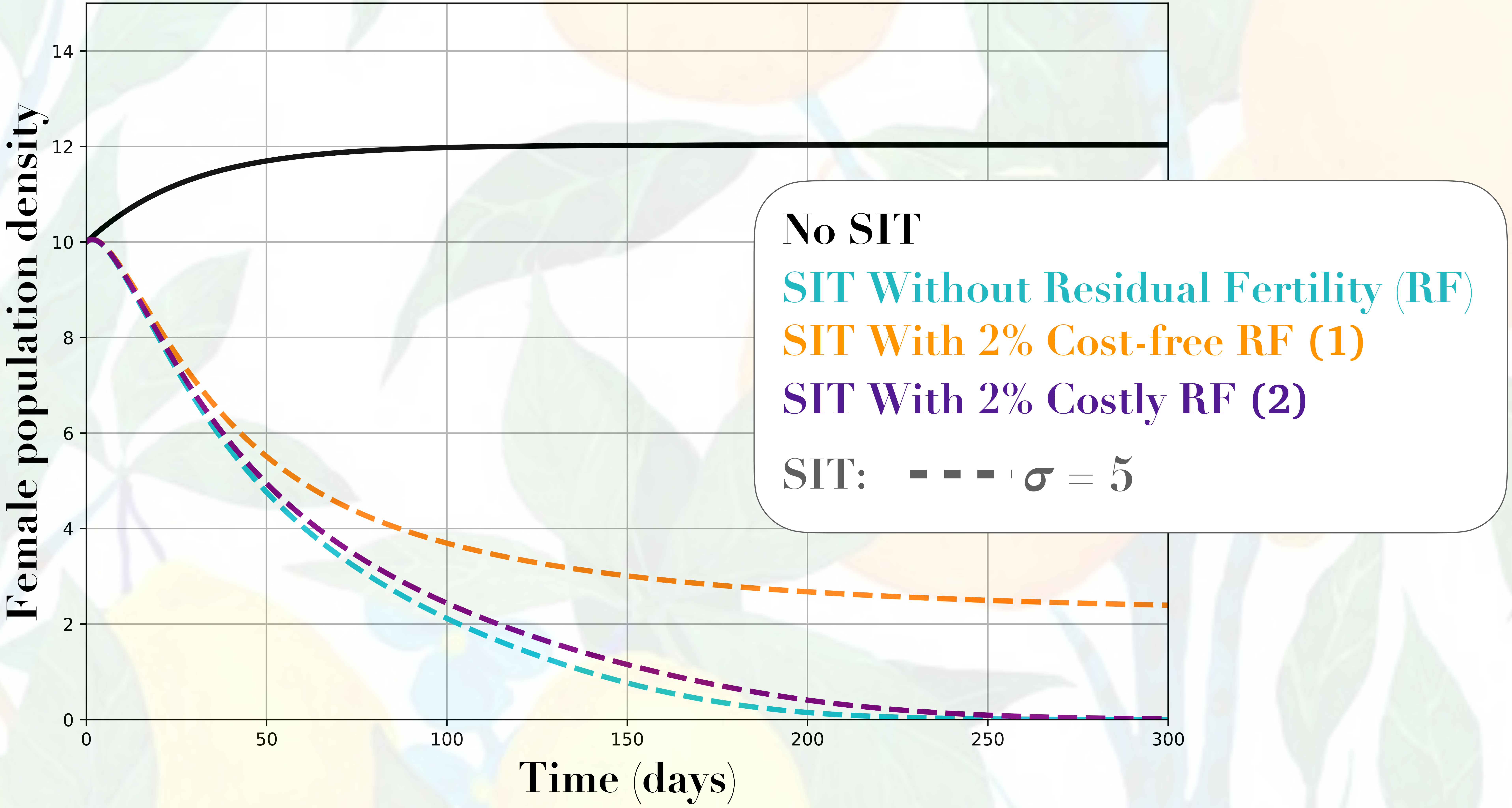
- Strong difference between the two sub models (1) and (2)



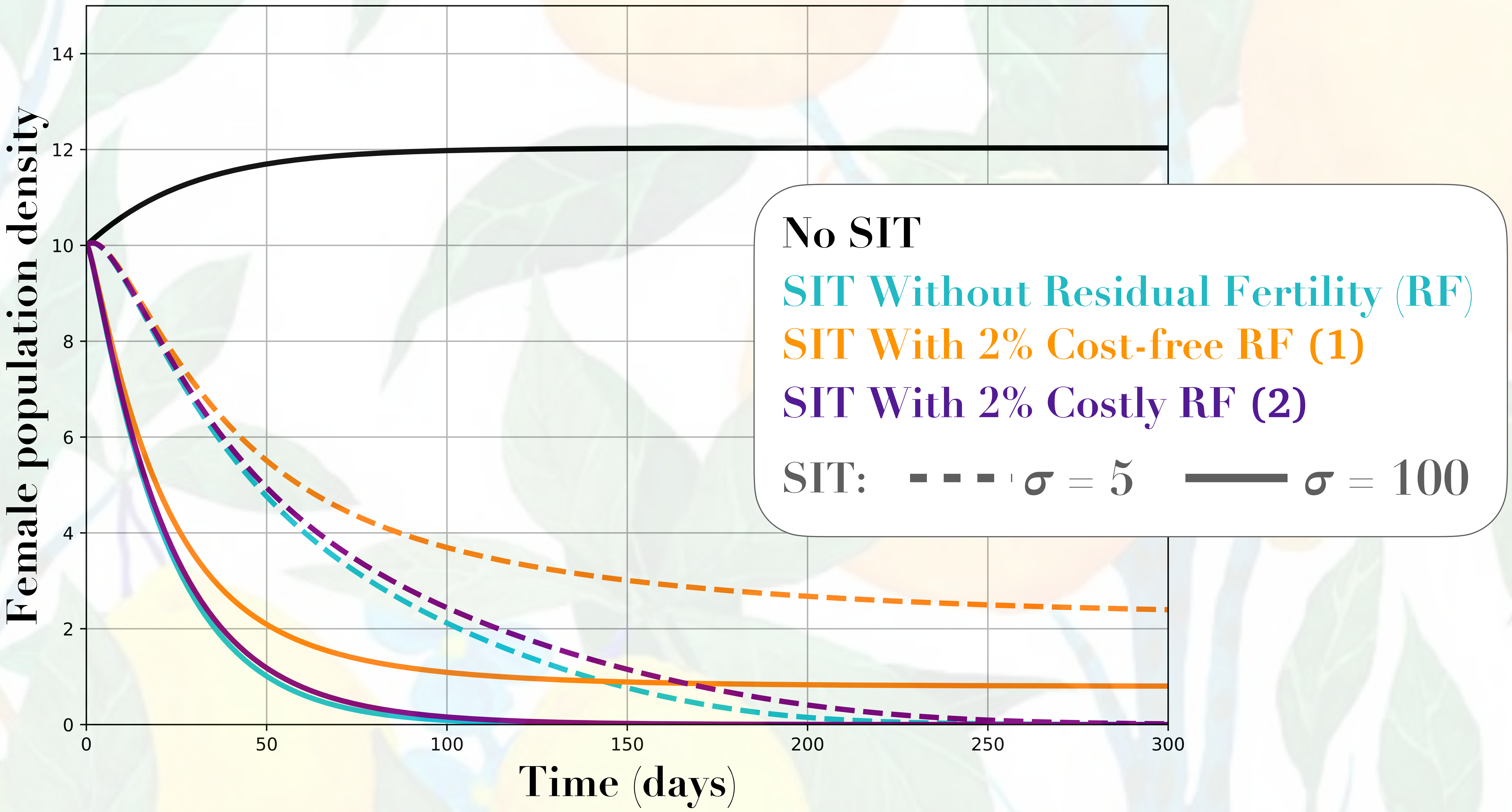
- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates



- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates
- Minimum **σ**? If we increase it there is no more consequence ?



Female population density
Time (days)
0
2
4
6
8
10
12
14
0
50
100
150
200
250
300
No SIT
SIT Without Residual Fertility (RF)
SIT With 2% Cost-free RF (1)
SIT With 2% Costly RF (2)
SIT: $\sigma = 5$


Female population density
Time (days)
0
2
4
6
8
10
12
14
0
50
100
150
200
250
300
No SIT
SIT Without Residual Fertility (RF)
SIT With 2% Cost-free RF (1)
SIT With 2% Costly RF (2)
SIT: $\sigma = 5$ $\sigma = 100$

- Strong **impact of residual fertility** on SIT efficiency
- For **costly residual fertility**, SIT is effective at higher residual fertility rates

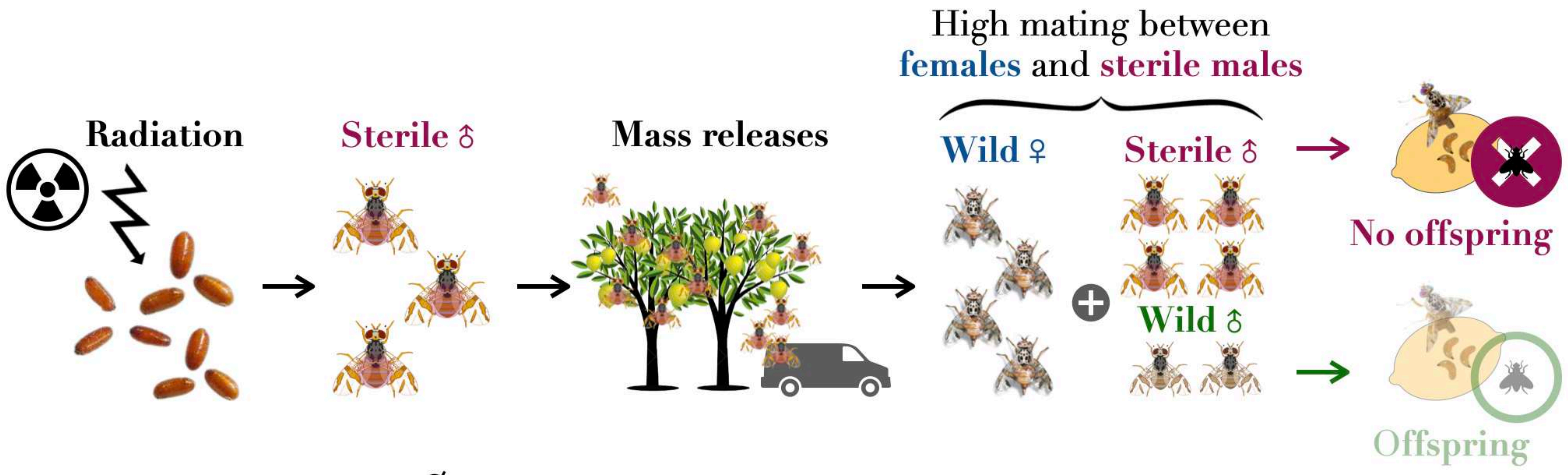
- Strong **impact of residual fertility** on SIT efficiency
- For **costly residual fertility**, SIT is effective at higher residual fertility rates
- **Applicable** in an agricultural context?
  - $\boldsymbol{\sigma}$: number of individuals released per day per 100 m^2
  - Example: For SIT effectiveness with **2% of residual fertility**
    - Releases of **at least 500 sterile males per day per hectare** ($\boldsymbol{\sigma} = 5$)

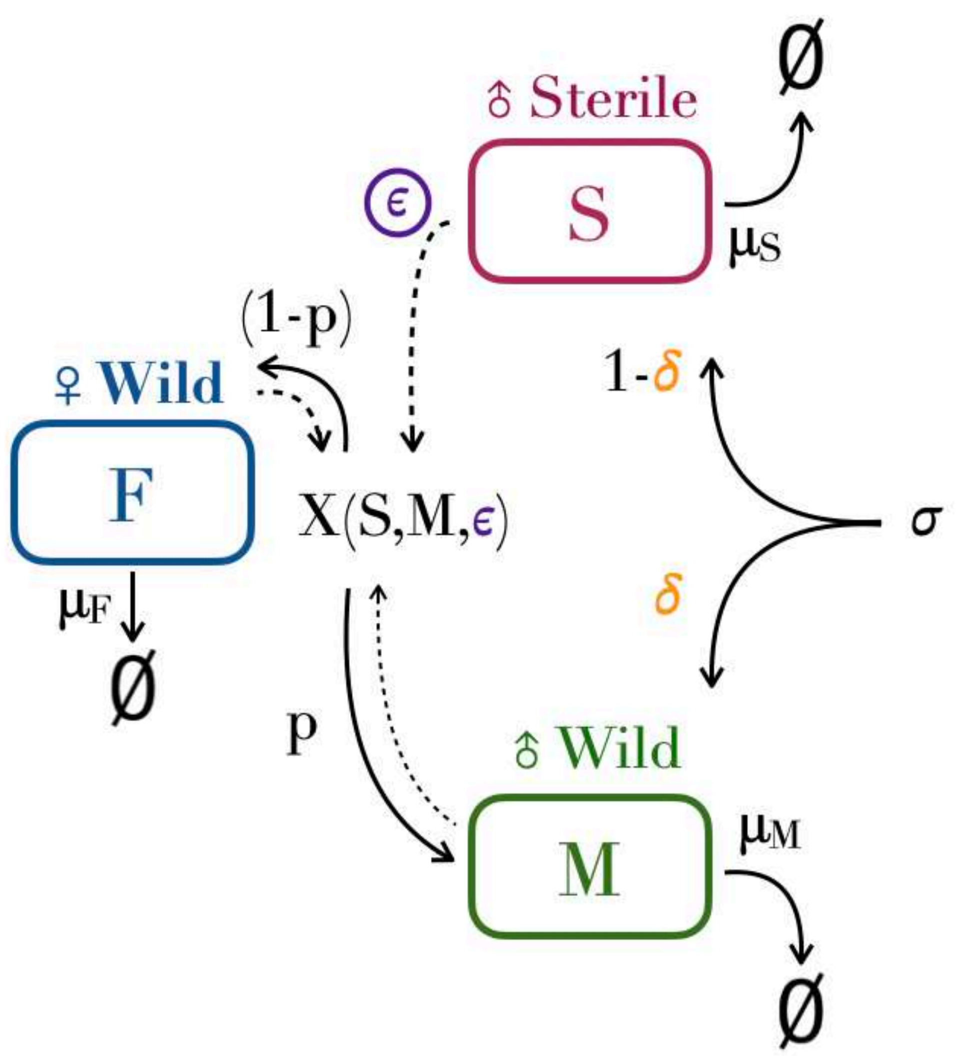
- Strong **impact of residual fertility** on SIT efficiency
- For **costly residual fertility**, SIT is effective at higher residual fertility rates
- **Applicable** in an agricultural context?
  - $\boldsymbol{\sigma}$: number of individuals released per day per 100 m^2
  - Example: For SIT effectiveness with **2% of residual fertility**
    - Releases of **at least 500 sterile males per day per hectare** ($\boldsymbol{\sigma} = 5$)
- **Ethics**

- Strong **impact of residual fertility** on SIT efficiency
- For **costly residual fertility**, SIT is effective at higher residual fertility rates
- **Applicable** in an agricultural context?
  - $\boldsymbol{\sigma}$: number of individuals released per day per 100 m^2
  - Example: For SIT effectiveness with **2% of residual fertility**
    - Releases of **at least 500 sterile males per day per hectare** ($\boldsymbol{\sigma} = 5$)
- **Ethics**
- **Control vs eradication**

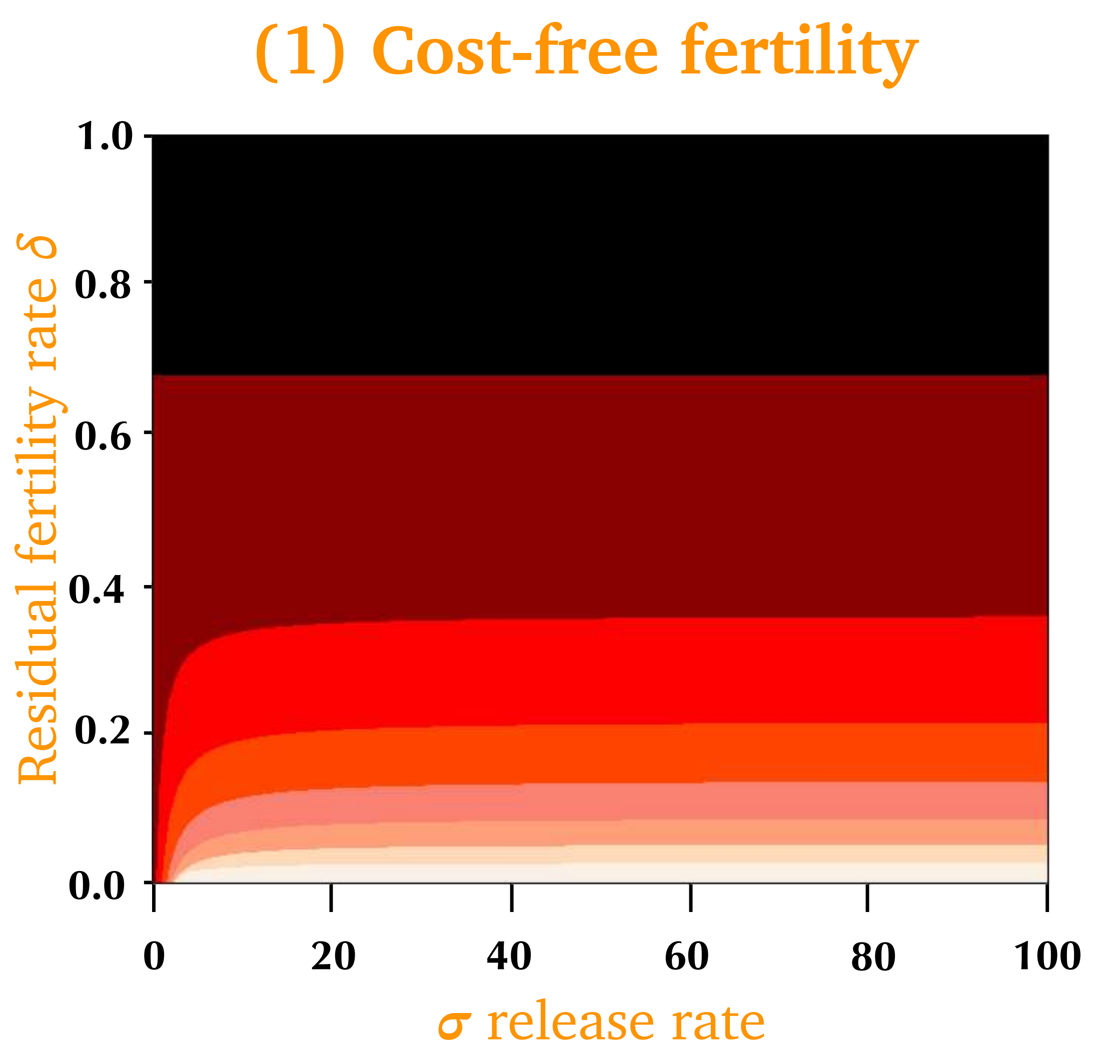
Thresholds: $\epsilon = \frac{1}{R} + q(1 - \frac{1}{R})$ | $\epsilon = \frac{1}{R}$

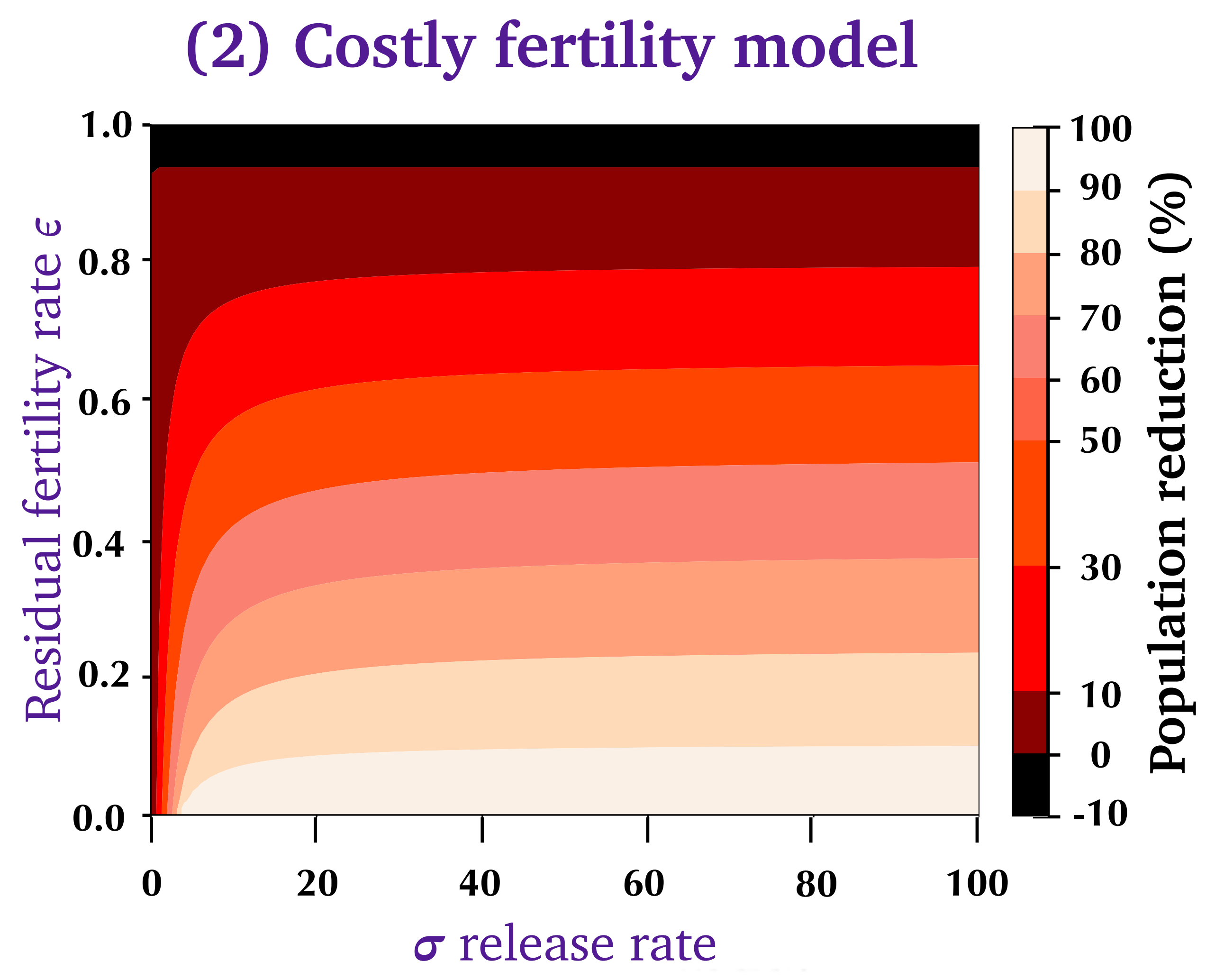
Thank you for your attention !





$$
\begin{cases}
\dot{S} = -\mu_S S + (1-\delta)\sigma \\
\dot{F} = -\mu_F F + r(1-p)\,X(S,M,\epsilon)\,C(F)\,F \\
\dot{M} = -\mu_M M + r\,p\,X(S,M,\epsilon)\,C(F)\,F + \delta\sigma
\end{cases}
$$





$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S,M,\epsilon)C(F)F \\ \dot{M} = -\mu_M M + rpX(S,M,\epsilon)C(F)F + \delta\sigma \end{cases}$$

➡️ **Parameters** have been estimated **from the literature**

✅ **Easy parameters to estimate**

μ: mortality rate

r: emergence rate

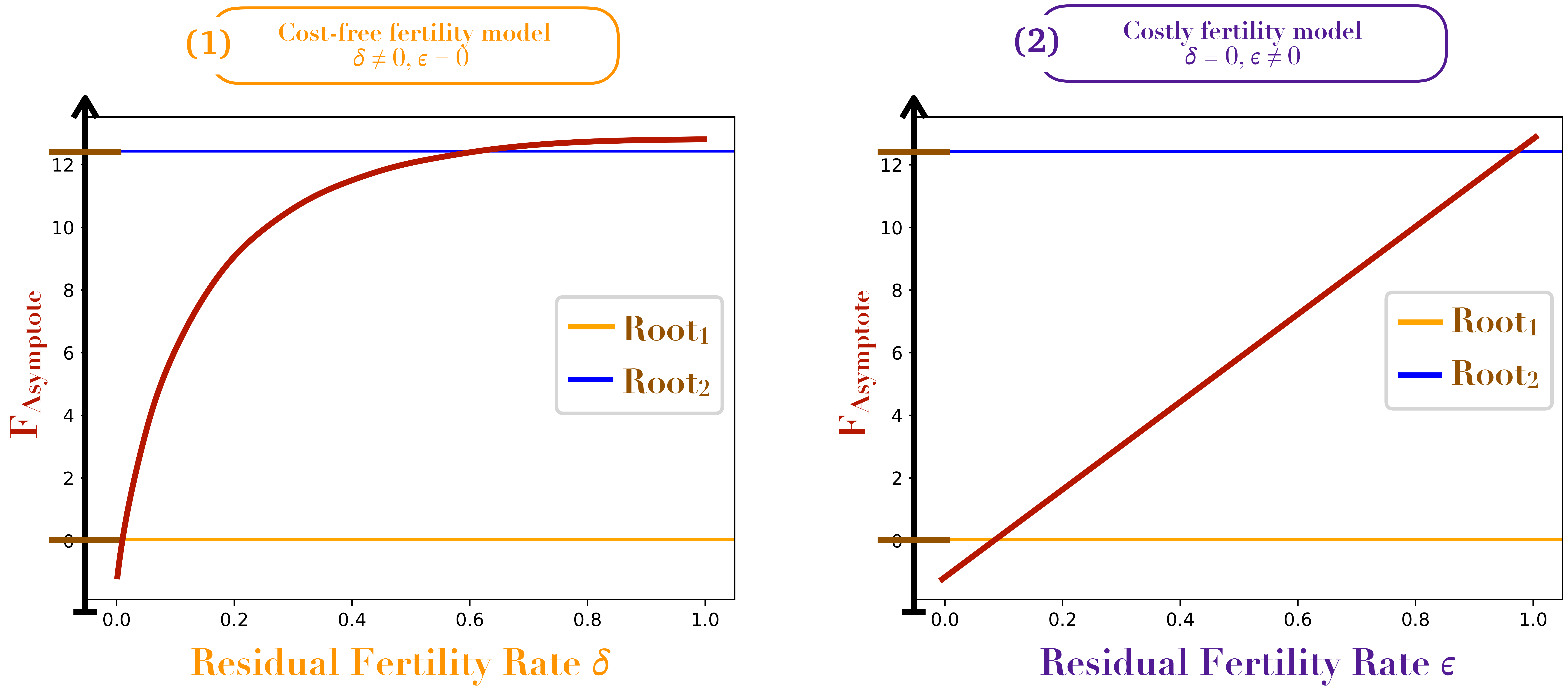
p: proportion of males

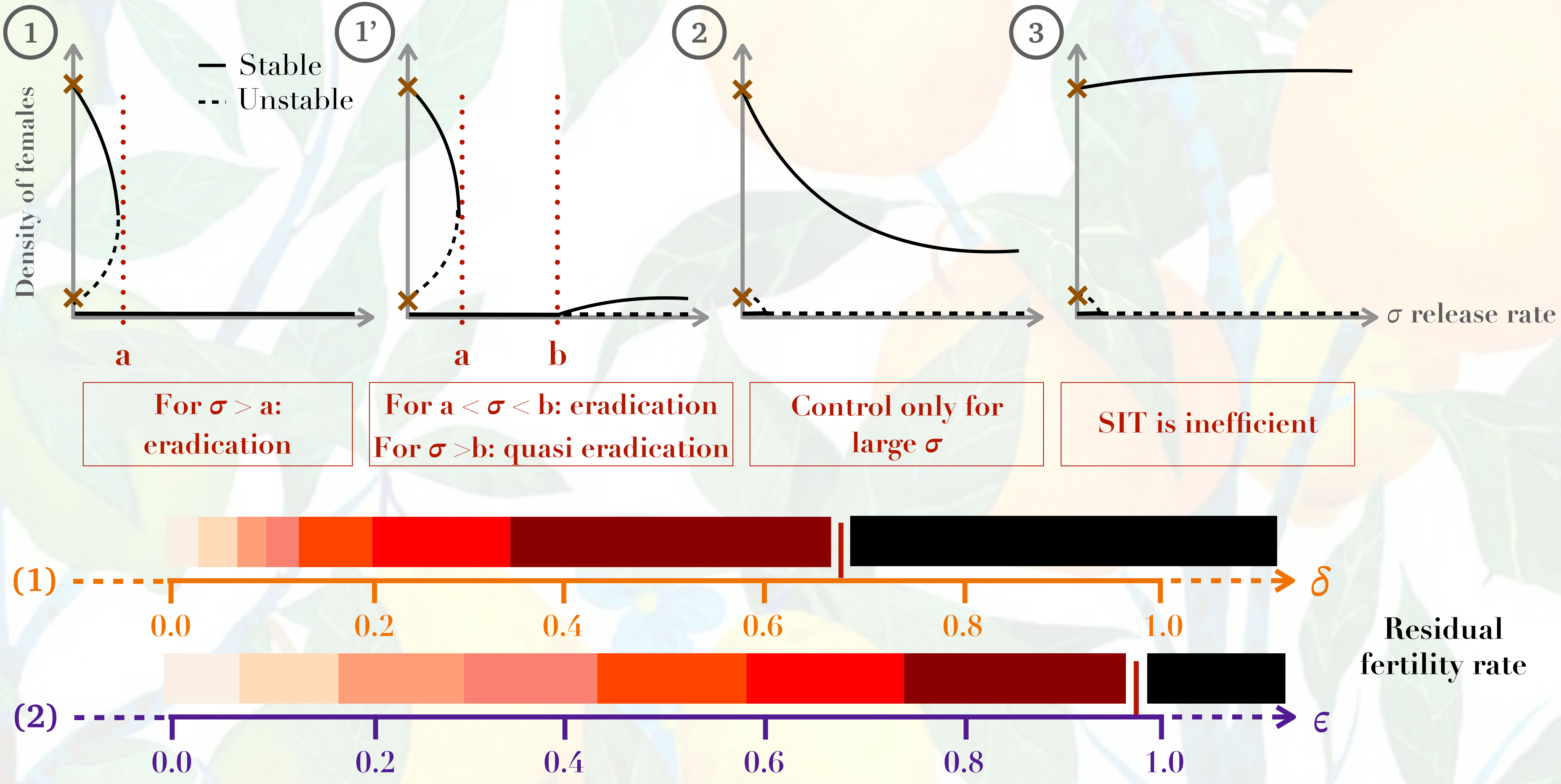
❌ **Not easy parameters to estimate**

C(F): competition $\frac{1}{1 + \beta F}$

β: oviposition competition between females

η: Sterilisation cost


(1)
Cost-free fertility model
$\delta \neq 0, \epsilon = 0$
$F_{Asymptote}$
12
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
$Root_1$
$Root_2$
Residual Fertility Rate δ
(2)
Costly fertility model
$\delta = 0, \epsilon \neq 0$
$F_{Asymptote}$
12
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
$Root_1$
$Root_2$
Residual Fertility Rate ϵ



1
1'
2
3
Stable
Unstable
Density of females
a
a
b
σ release rate
For σ > a: eradication
For a < σ < b: eradication
For σ >b: quasi eradication
Control only for large σ
SIT is inefficient
(1)
δ
0.0
0.2
0.4
0.6
0.8
1.0
(2)
ϵ
0.0
0.2
0.4
0.6
0.8
1.0
Residual fertility rate