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Marine Courtois, Kévan Rastello, Frédéric Grognard, Ludovic Mailleret, Suzanne Touzeau, et al.. How does residual fertility impact the effectiveness of the sterile insect technique in controlling *Ceratitis capitata*?. MPDEE 2023 - Mathematical Population Dynamics, Ecology and Evolution, Apr 2023, Marseille, France. hal-04144121

HAL Id: hal-04144121

<https://hal.inrae.fr/hal-04144121v1>

Submitted on 8 Aug 2023

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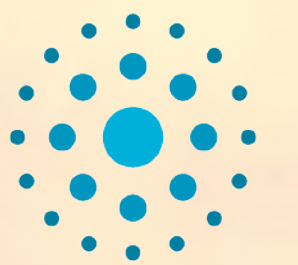
How does residual fertility impact the effectiveness of the Sterile Insect Technique (SIT) in controlling *Ceratitis capitata*?

Marine Courtois, Kévan Rastello, Frédéric Grognard,
Ludovic Mailleret, Suzanne Touzeau, Louise van Oudenhove

Mathematical Population Dynamics, Ecology and Evolution - **MPDEE 2023**
CIRM - 26 April 2023

INRAE

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UNIVERSITÉ
CÔTE D'AZUR





Pest Insects



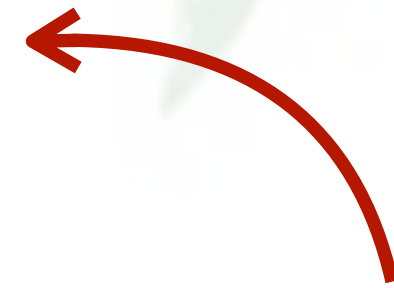
Pest Insects

Ceratitidis capitata



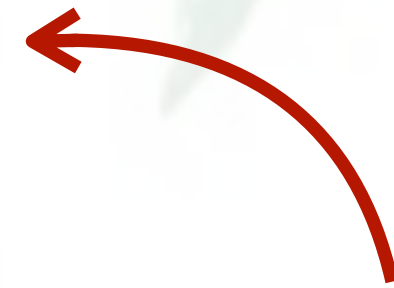
Damages

Ceratitidis capitata



Damages

Ceratitits capitata



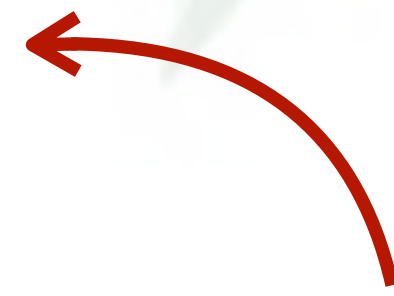
Damages



Ceratitits capitata



 **Crop yields**

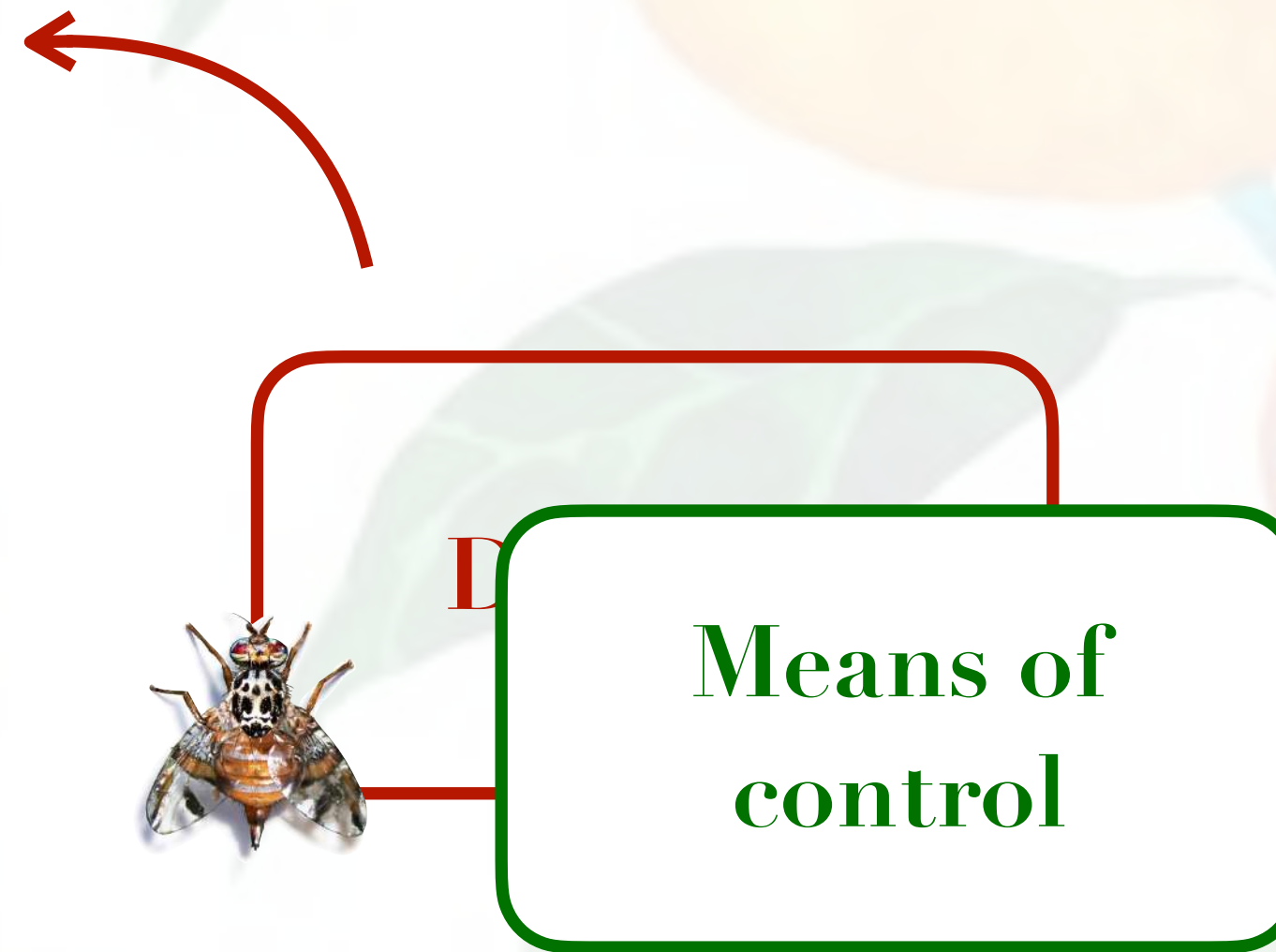


Ceratitidis capitata



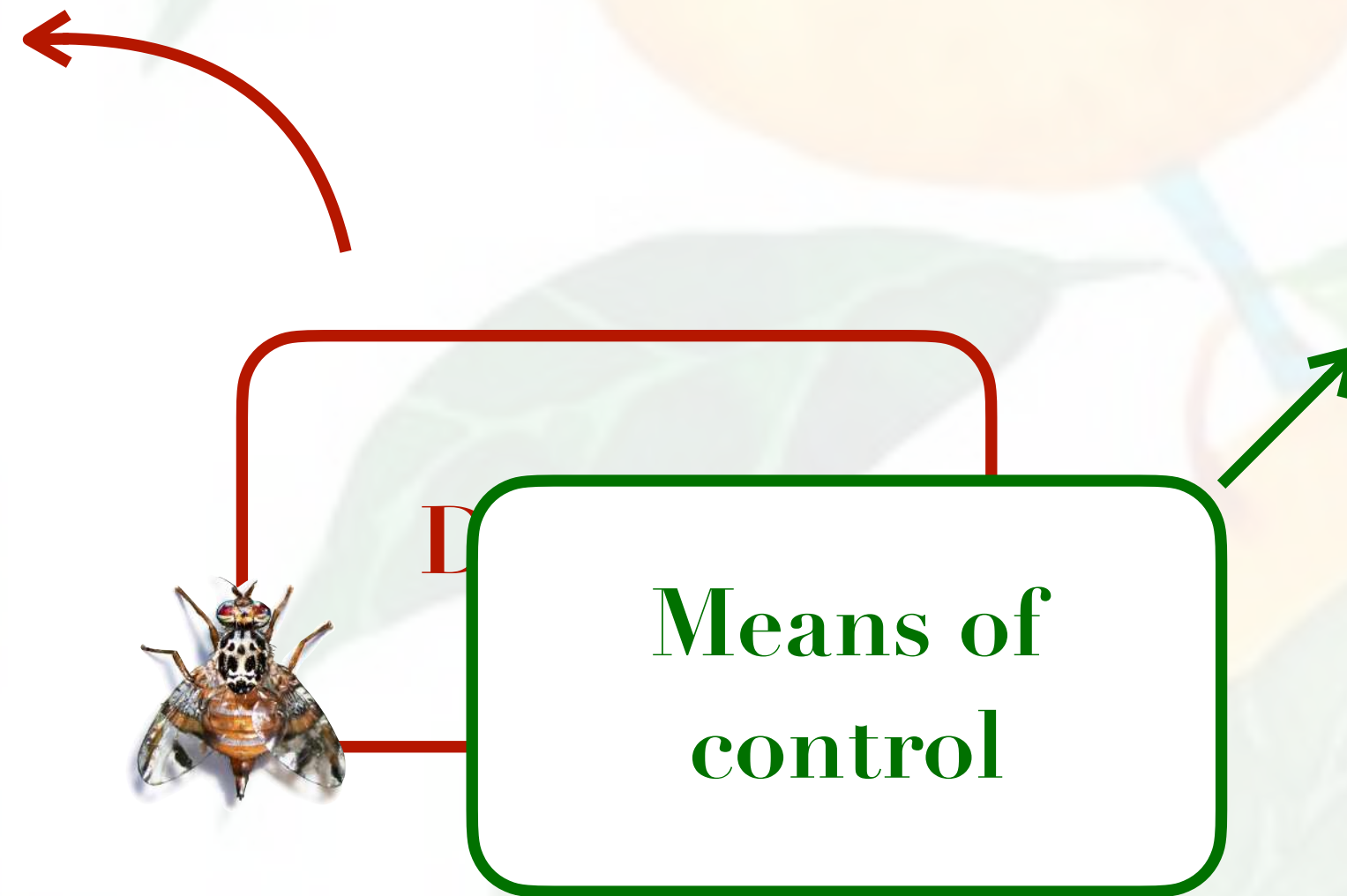
 **Crop yields**

Losses up to 100% of the production (Jerraya 2003; Rycckewaert et al. 2010)



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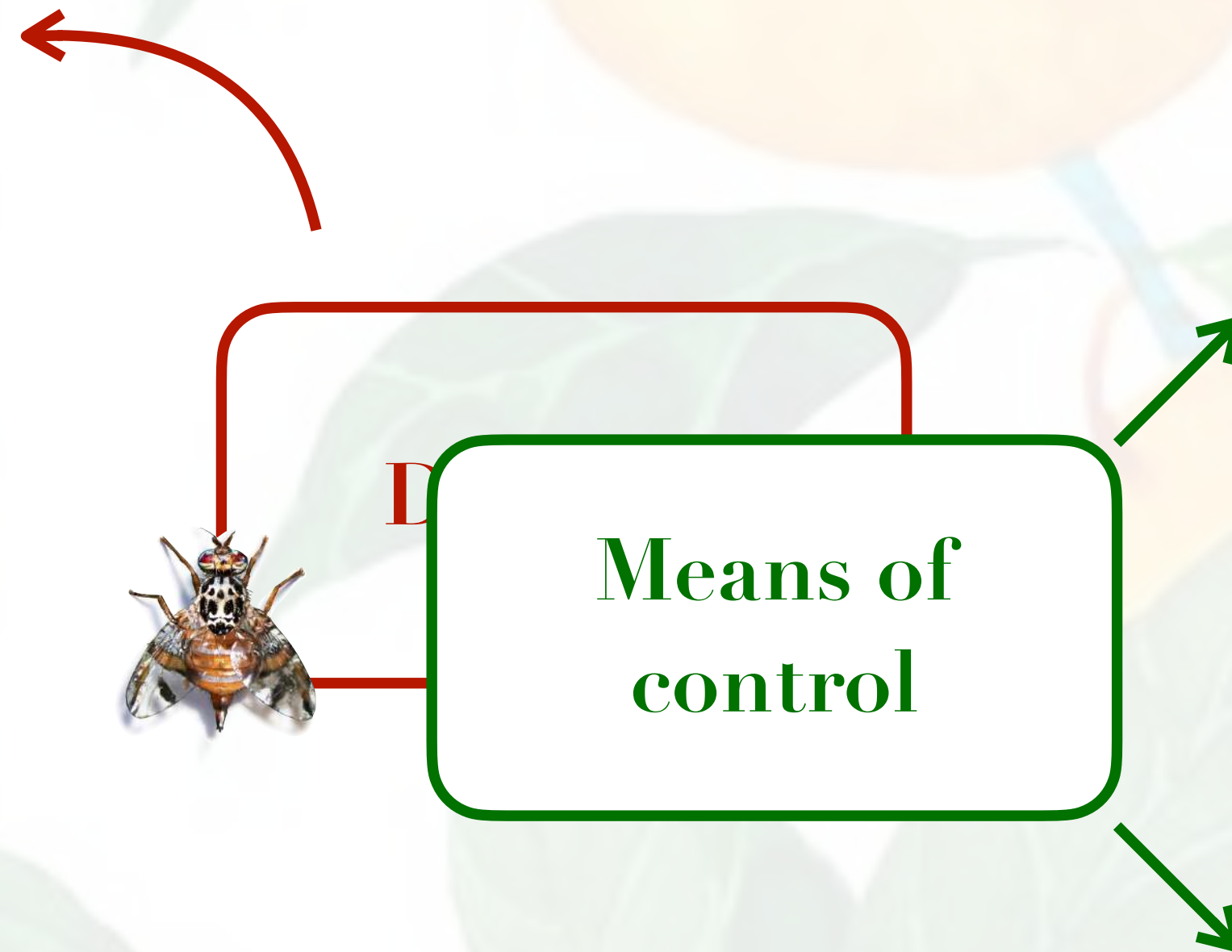
Preventive control

- Picking up and crushing fallen fruit
- Shallow tillage during winter



📈 Crop yields

Losses up to 100% of the production (Jerraya 2003; Ryckewaert et al. 2010)



Preventive control

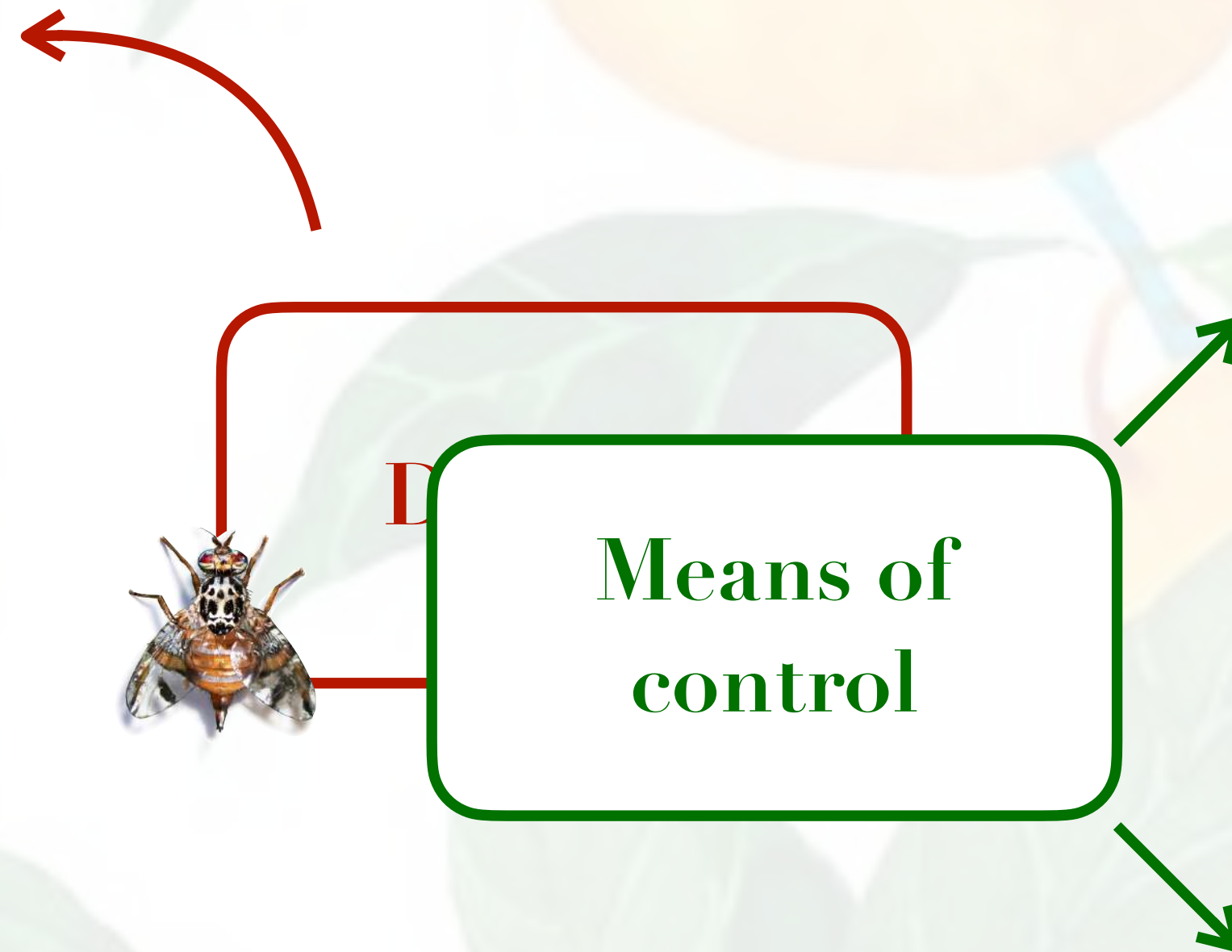
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Alternative control

- Mass trapping
- « Attract and kill »
- Parasitoids
- SIT: Sterile Insect Technique

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↓
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Sterile Insect Technique

- Ecofriendly technique specific to the targeted insect
- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

Sterile Insect Technique

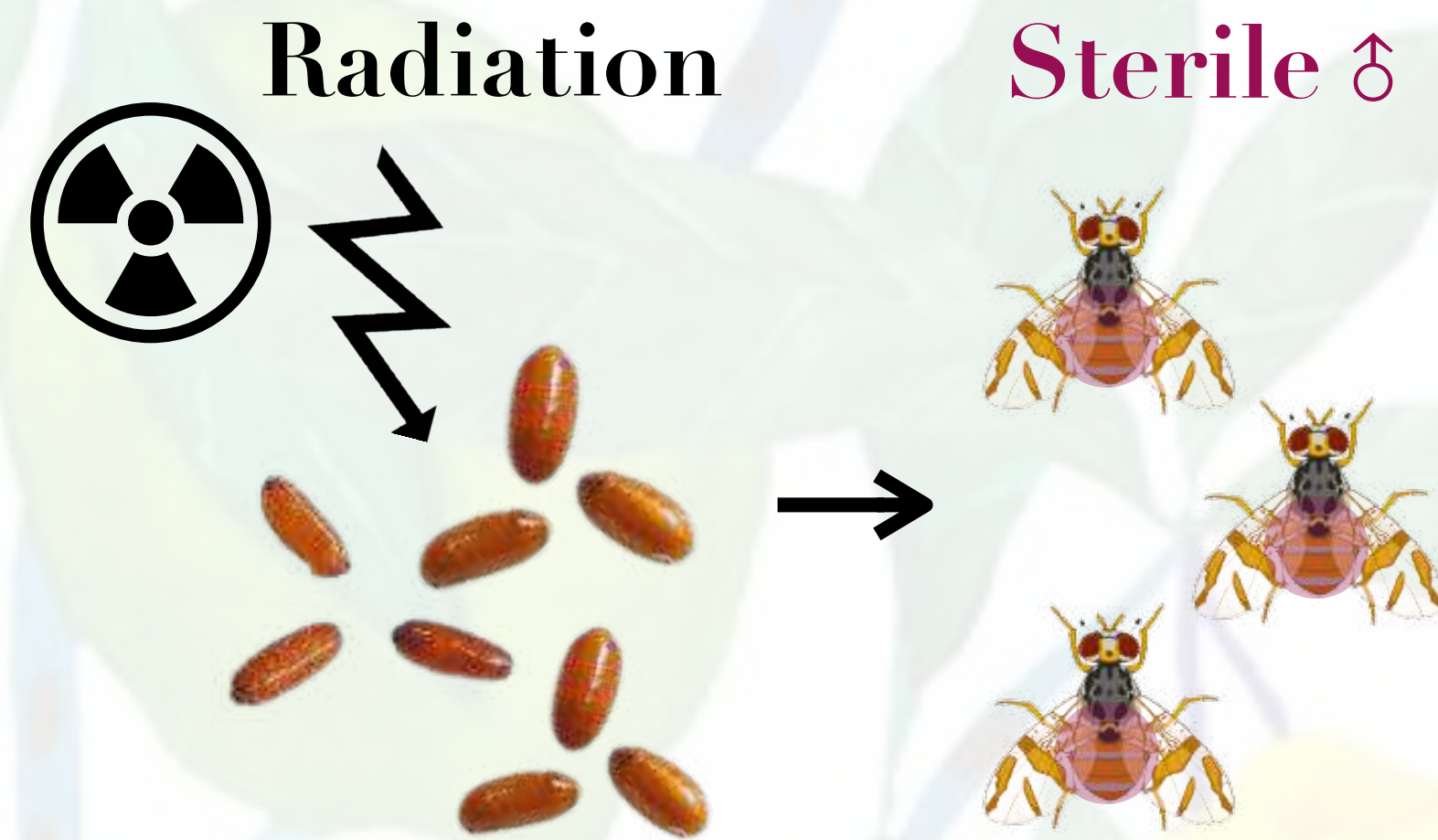
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Radiation



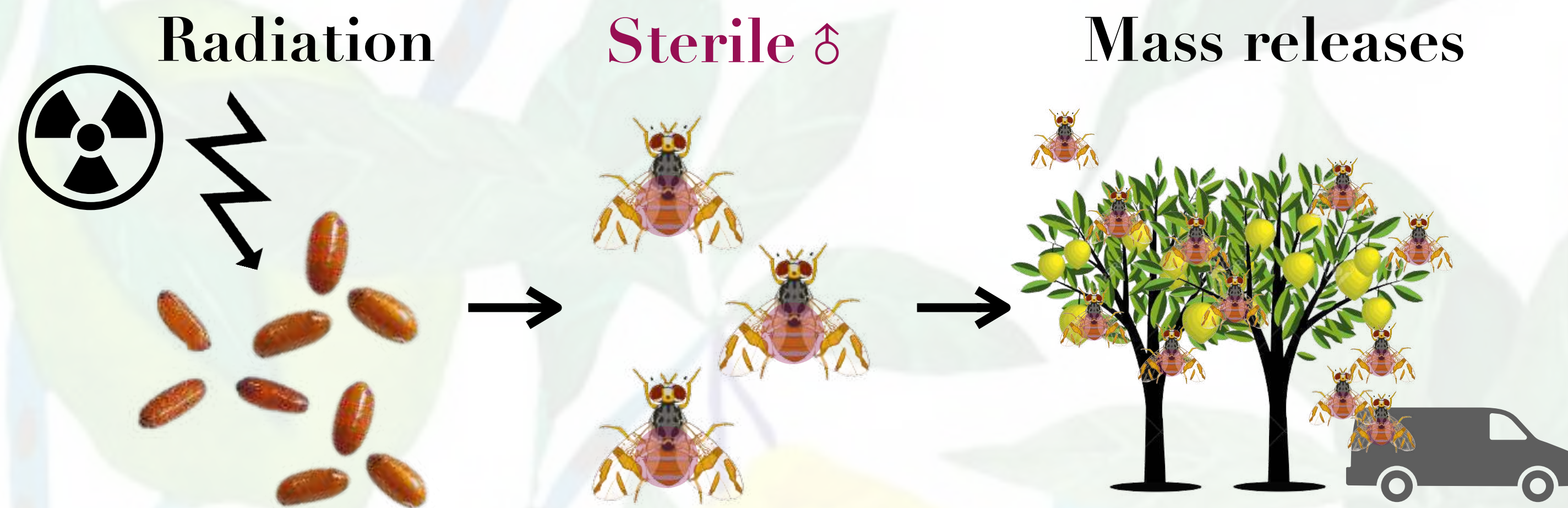
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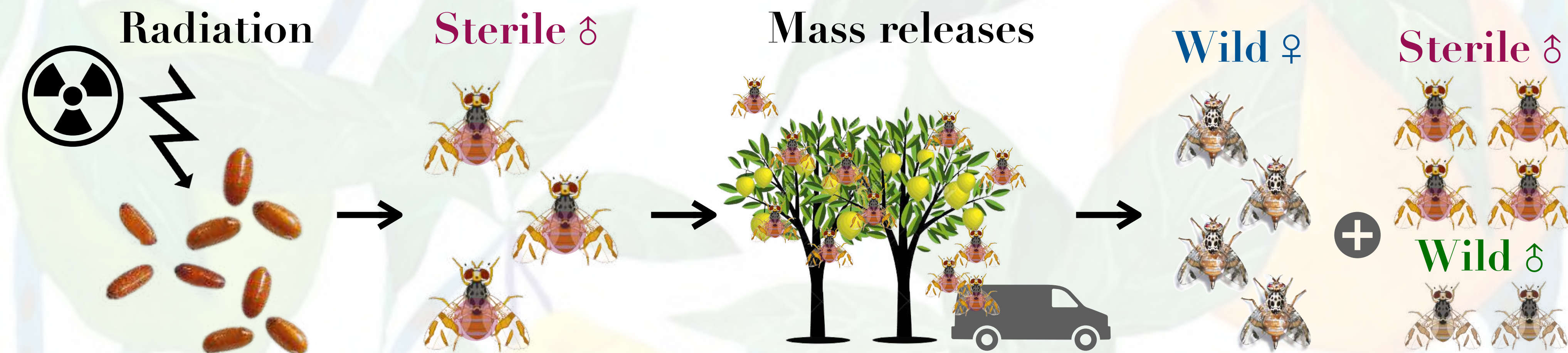
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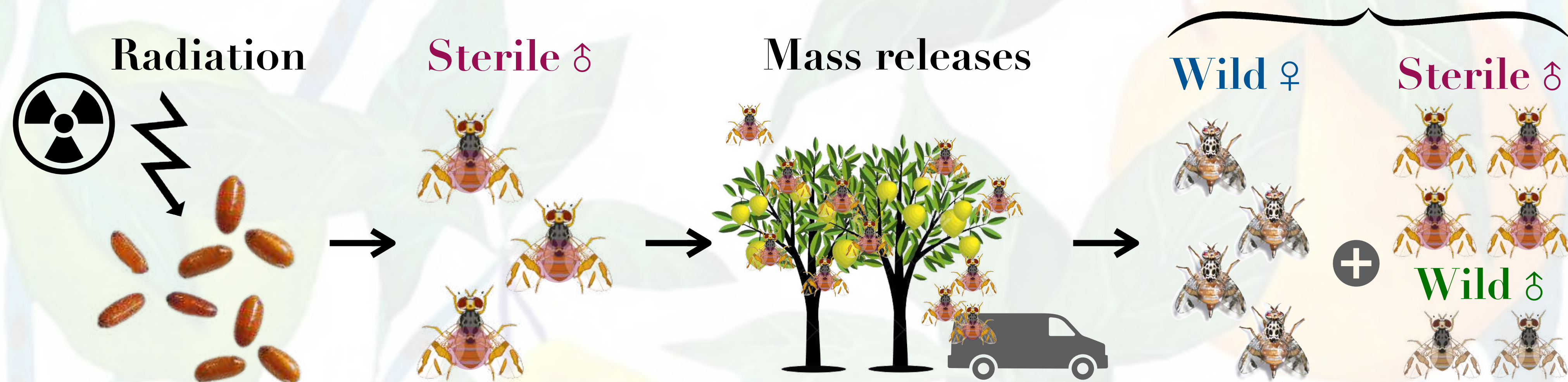
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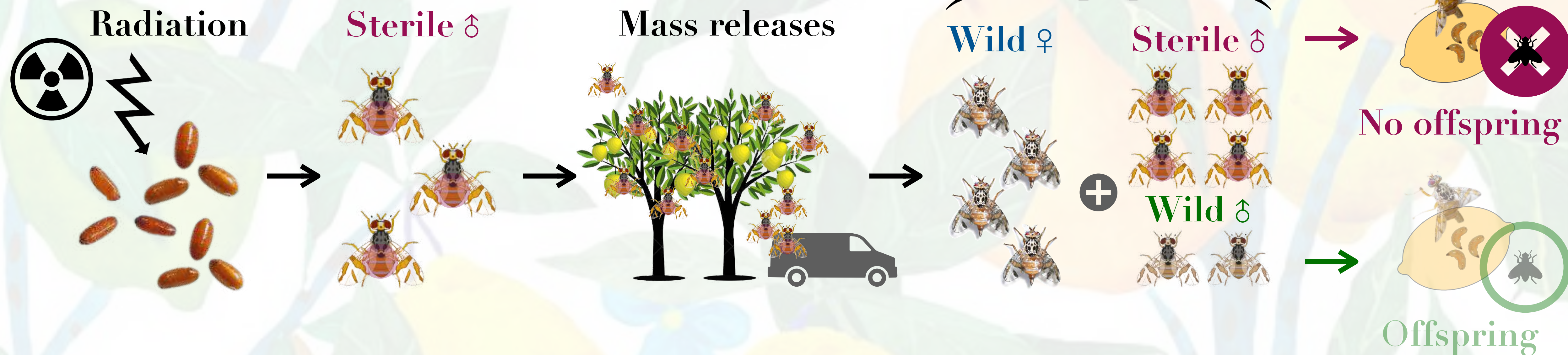
High mating between
females and **sterile males**



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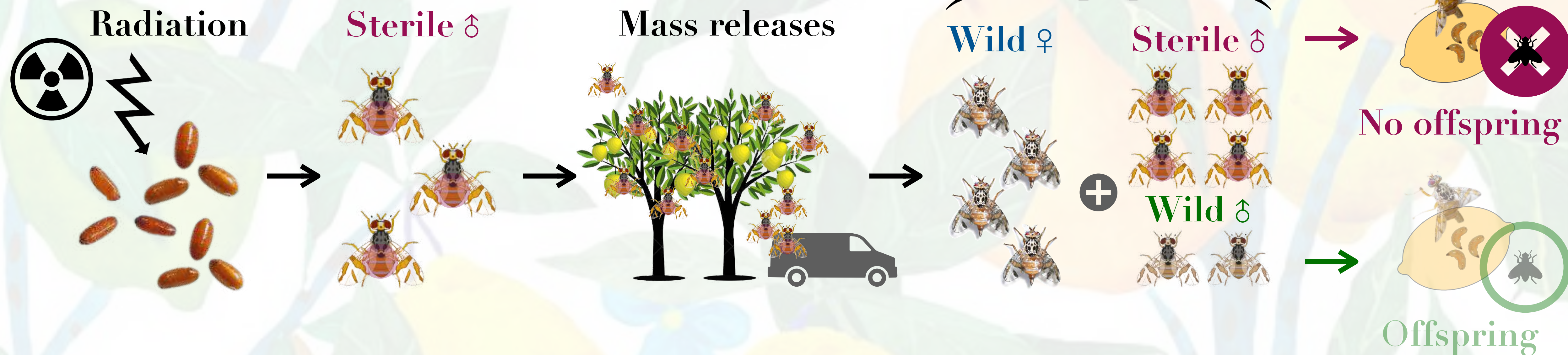
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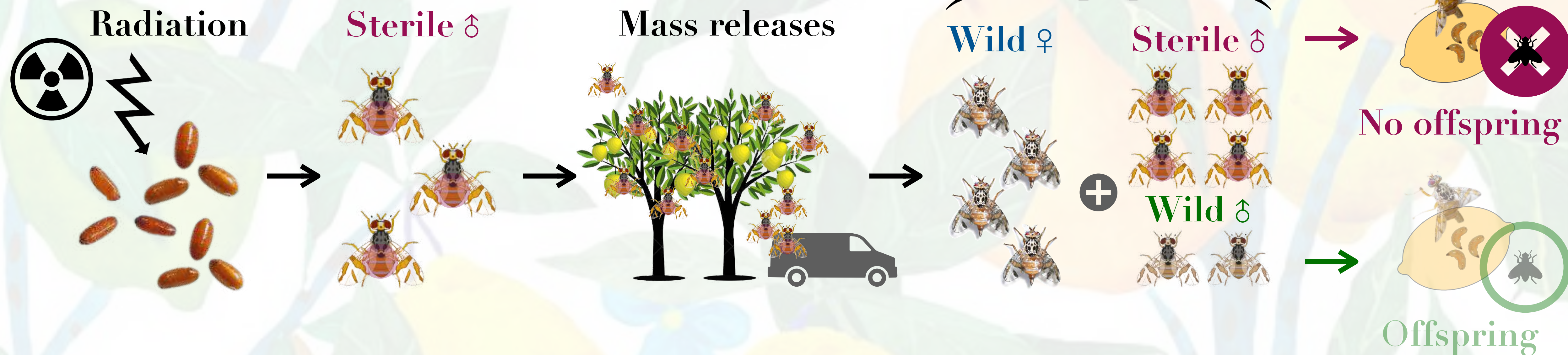


How many insects to release? How often?

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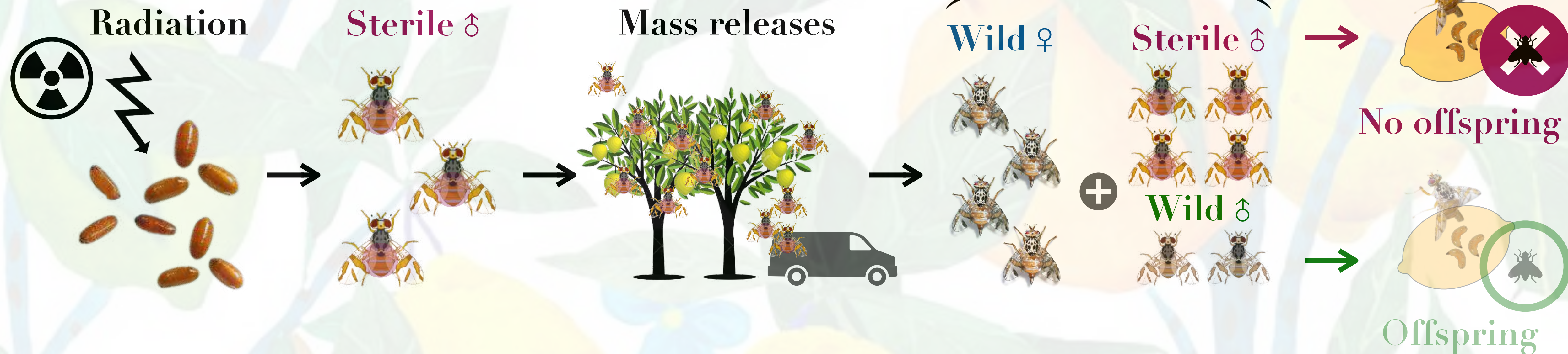
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How many insects to release? How often?

Population decrease?

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- Objective : to decrease the number of offspring in the next generation (Dyck et al., 2021)

High mating between



- Main challenge: to determine **how high the sterility rate should be** to ensure pest control in the field
- **Modeling** represents an essential and efficient tool to tackle this issue (limitation of economic and temporal costs)

Are all released insects sterile?

How many insects to release? How often?

Population decrease?

♂ Sterile

S

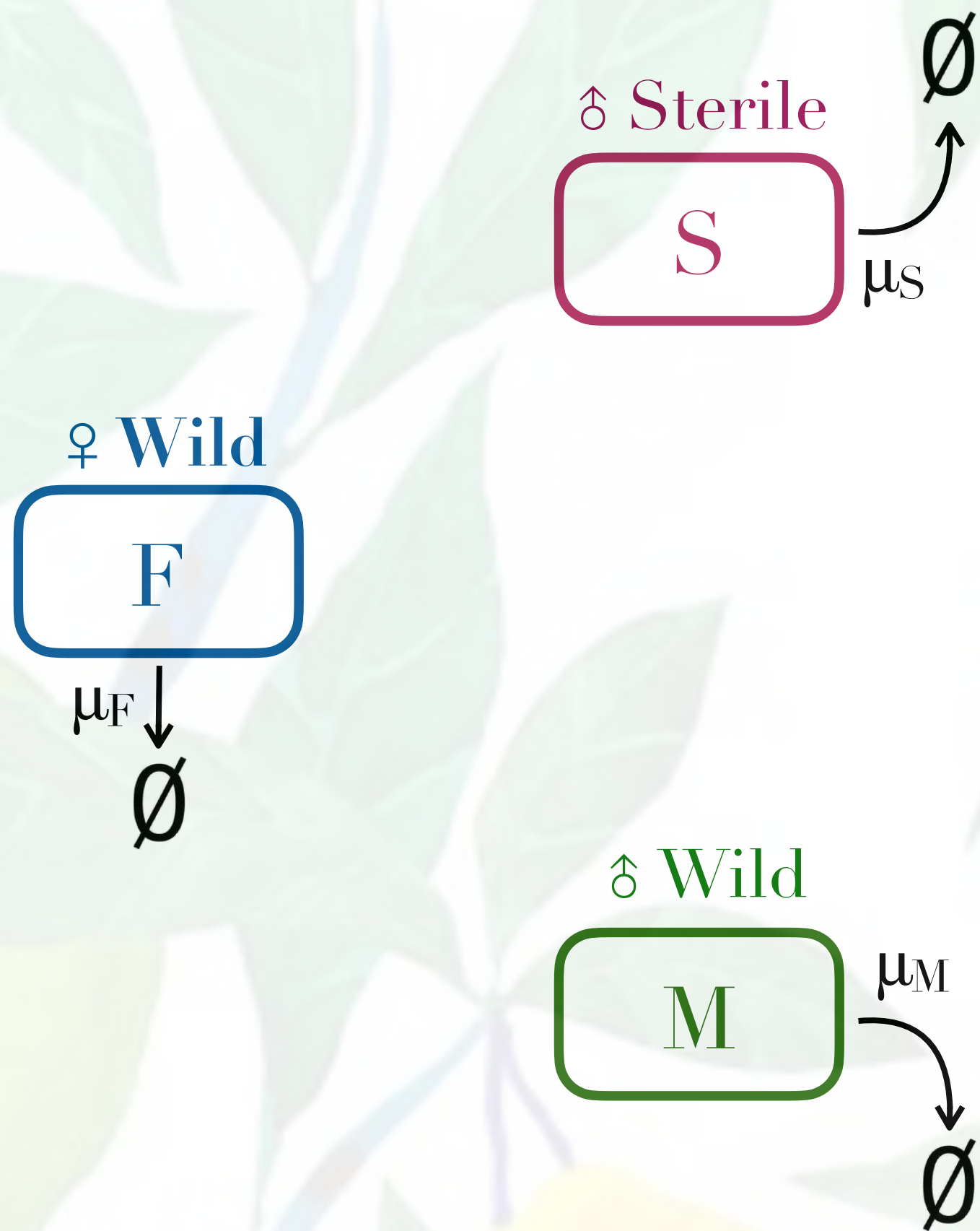
♀ Wild

F

♂ Wild

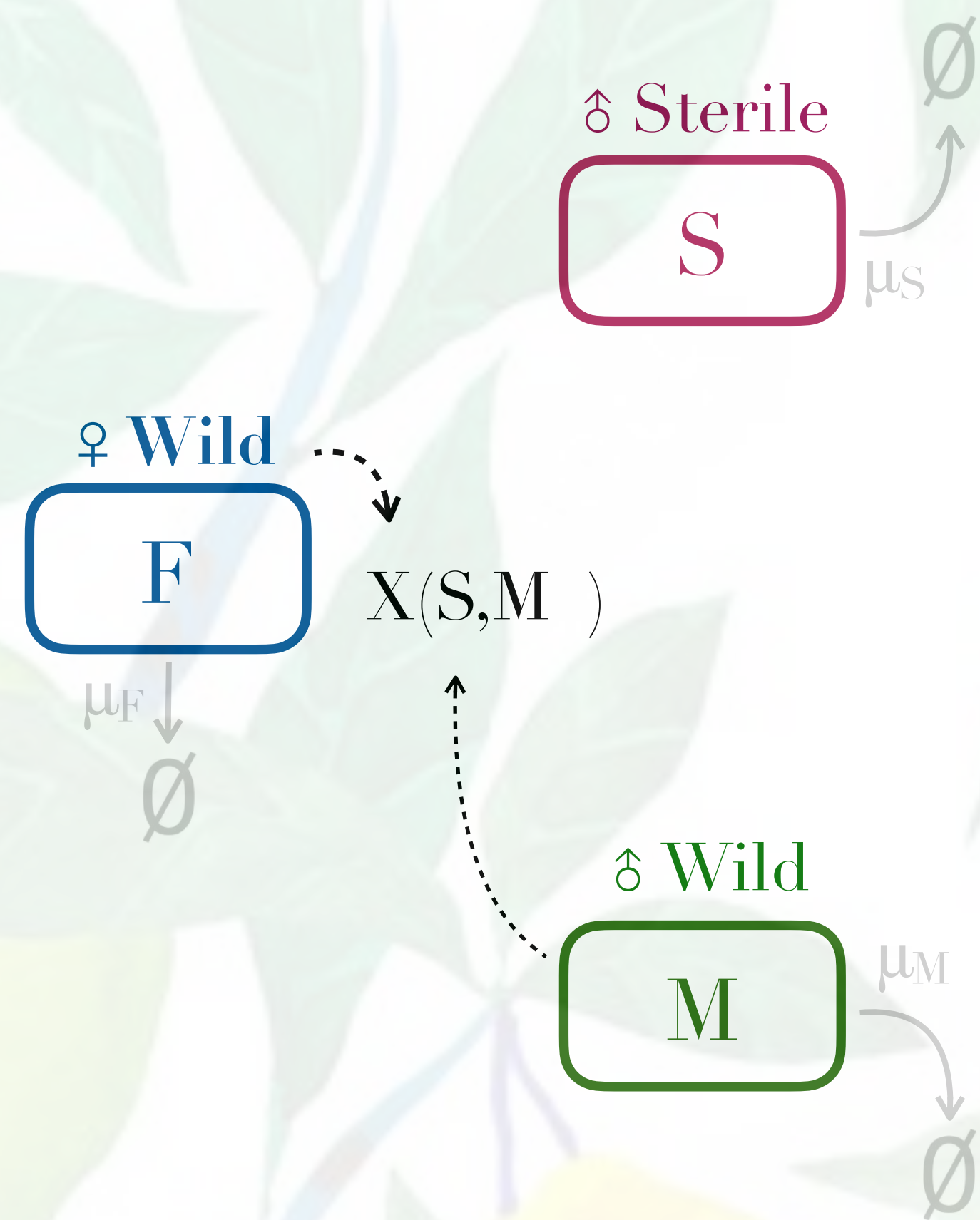
M

$$\begin{cases} \dot{S} = \\ \dot{F} = \\ \dot{M} = \end{cases}$$



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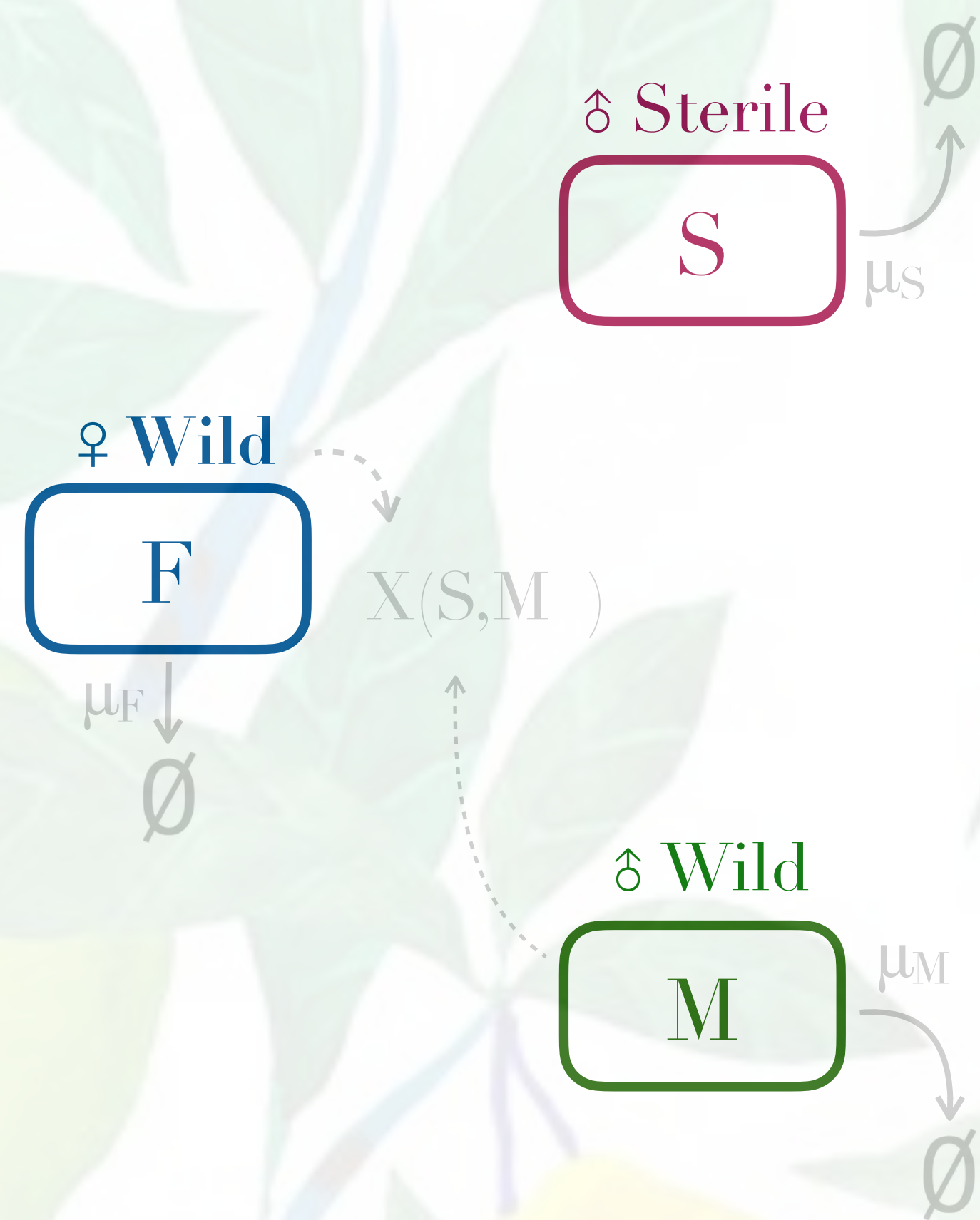
μ : mortality rate



$$\begin{cases} \dot{S} = -\mu_S S \\ \dot{F} = -\mu_F F + r(1-p) X(S, M) C(F) F \\ \dot{M} = -\mu_M M + r p X(S, M) C(F) F \end{cases}$$

μ : mortality rate

$X(S, M)$: mating probability $\frac{M}{k+M+S}$

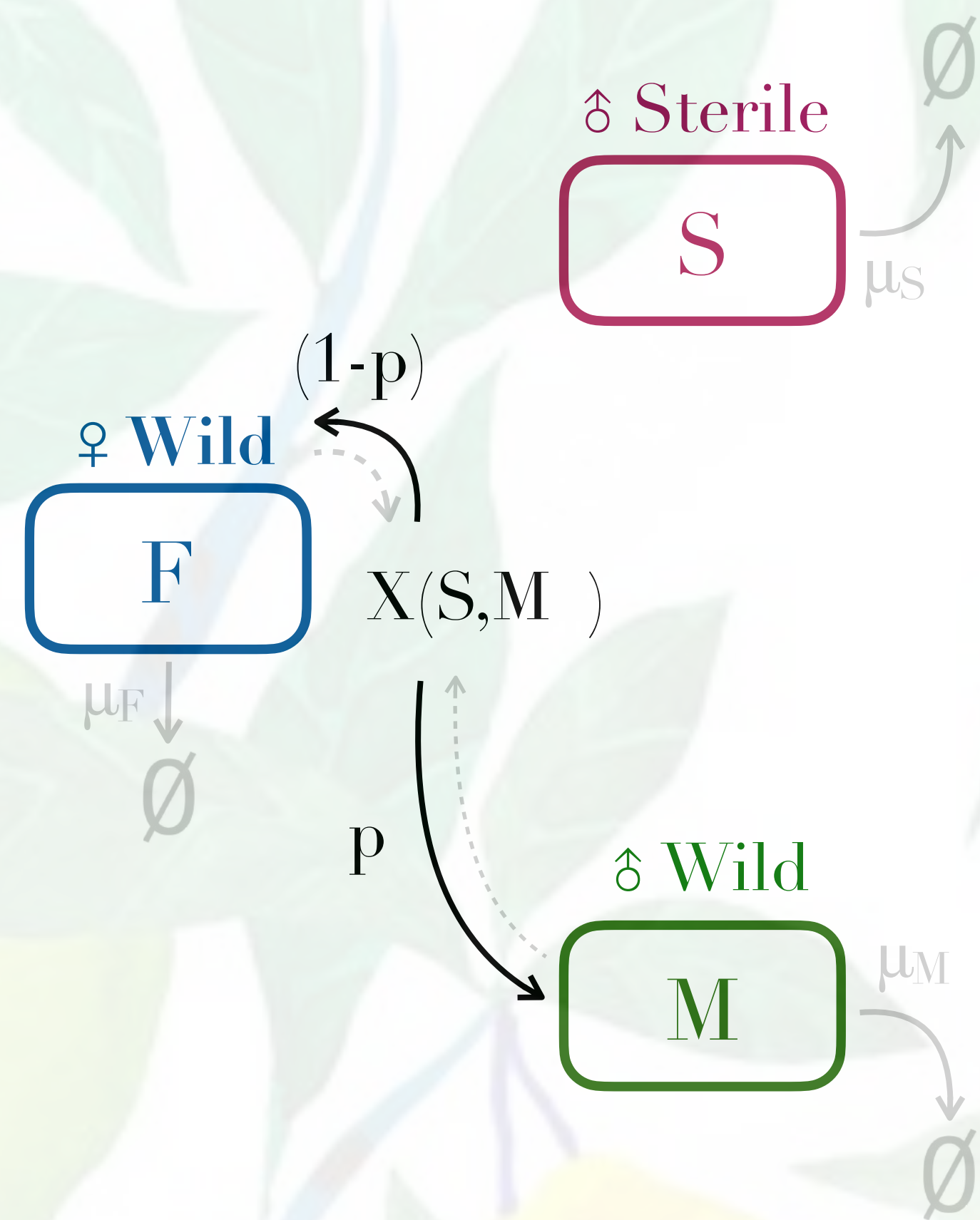


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μ : mortality rate

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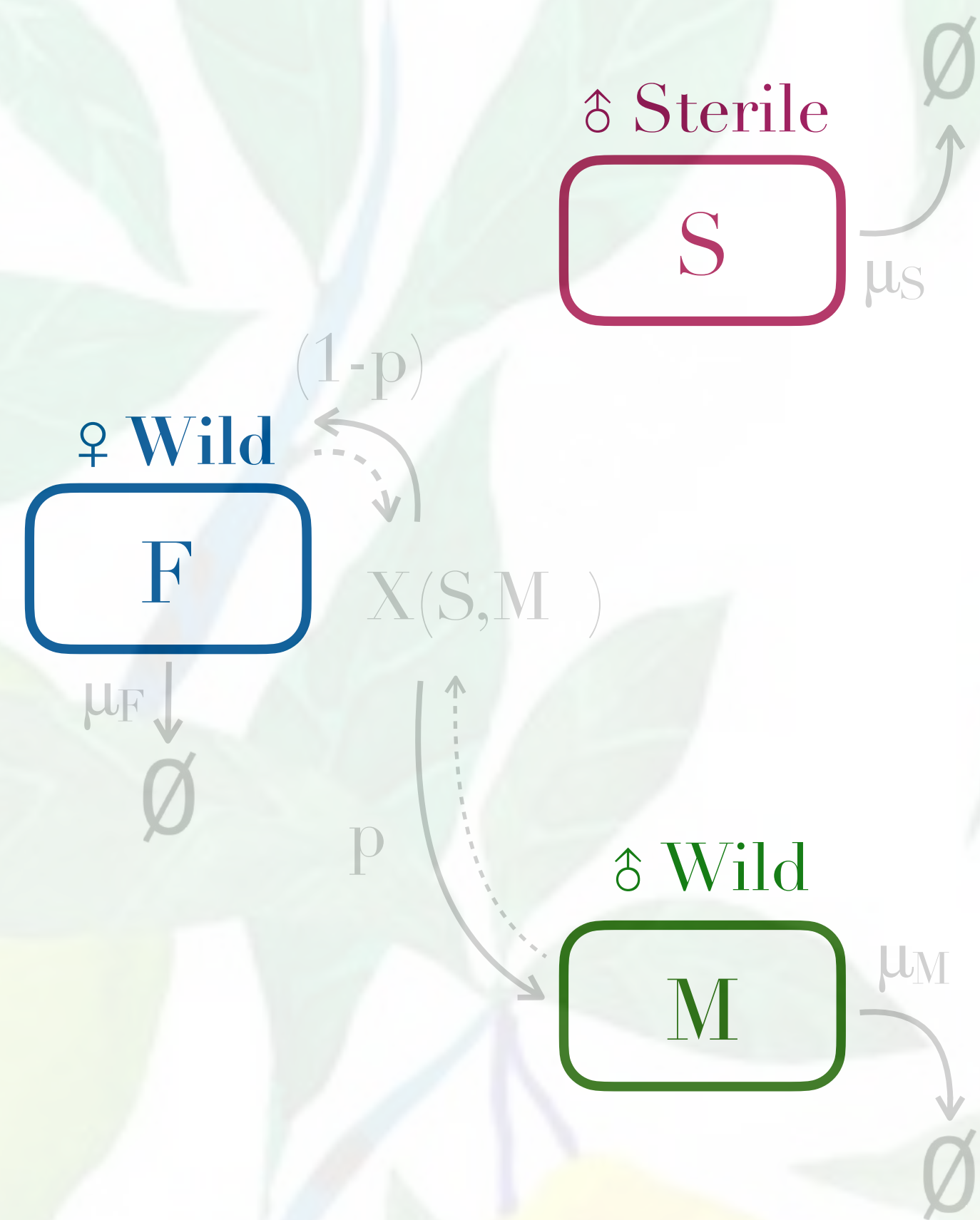
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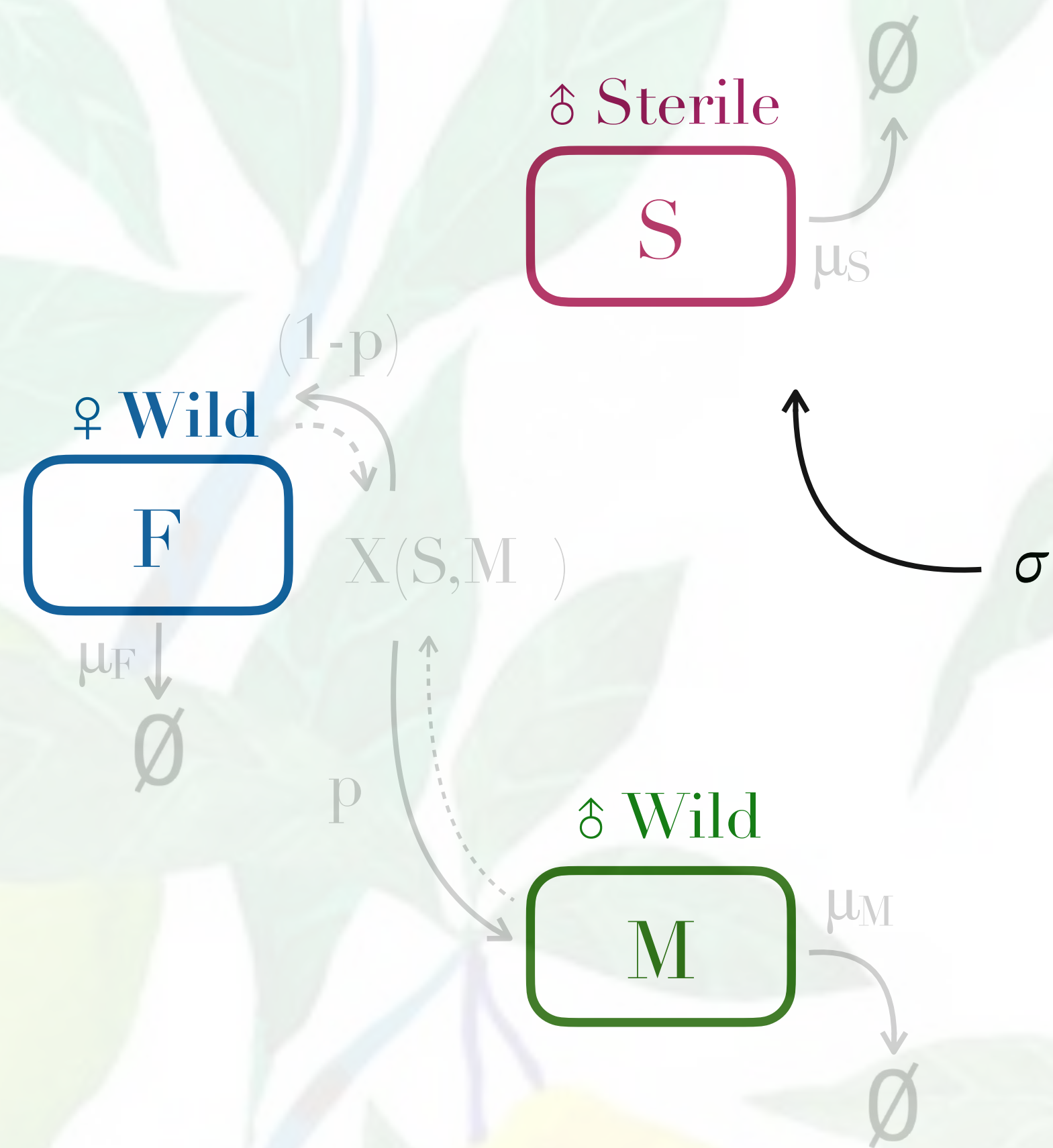
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$C(F)$: competition $\frac{1}{1 + \beta F}$

$$\frac{M}{k+M+S}$$

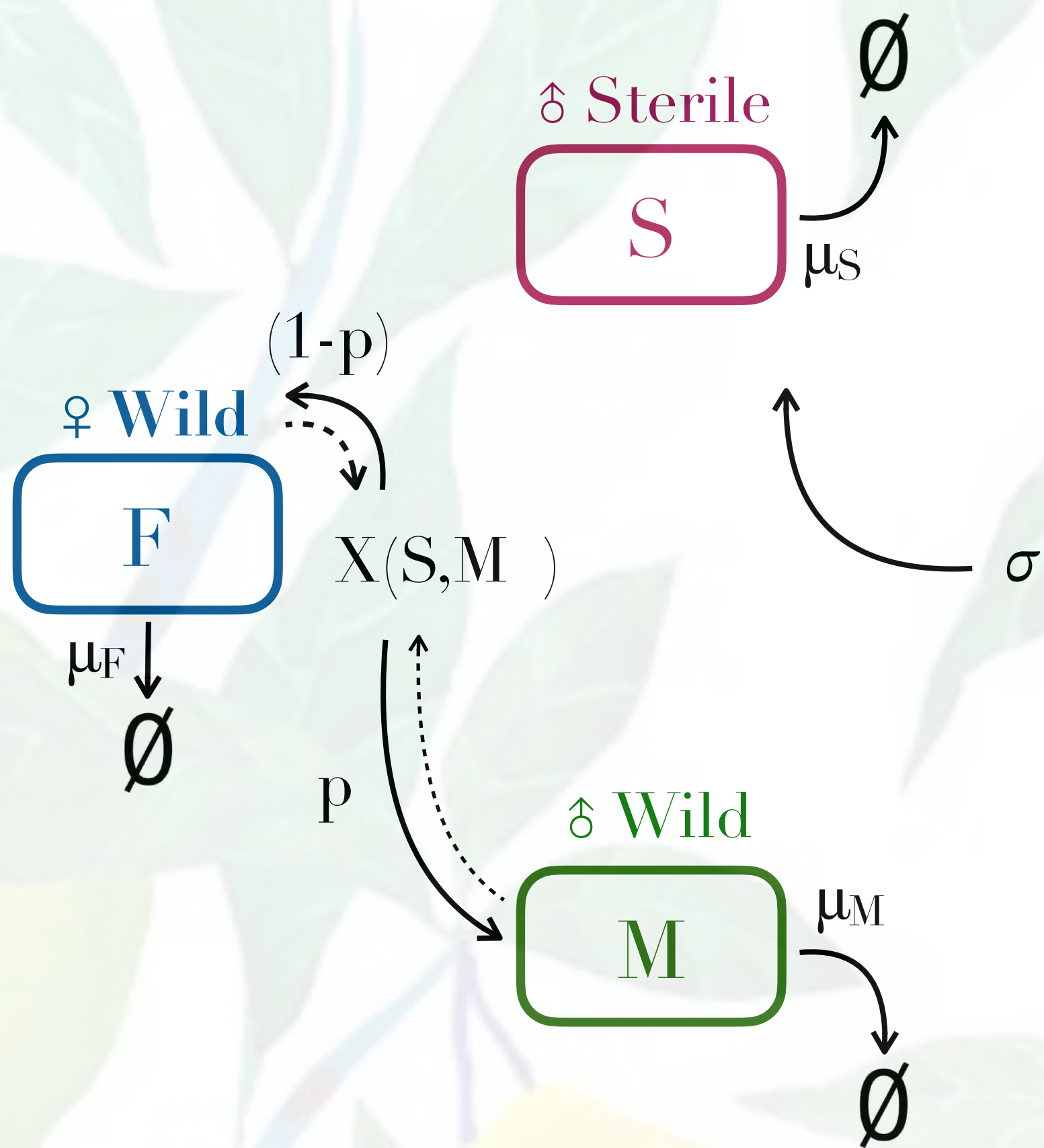


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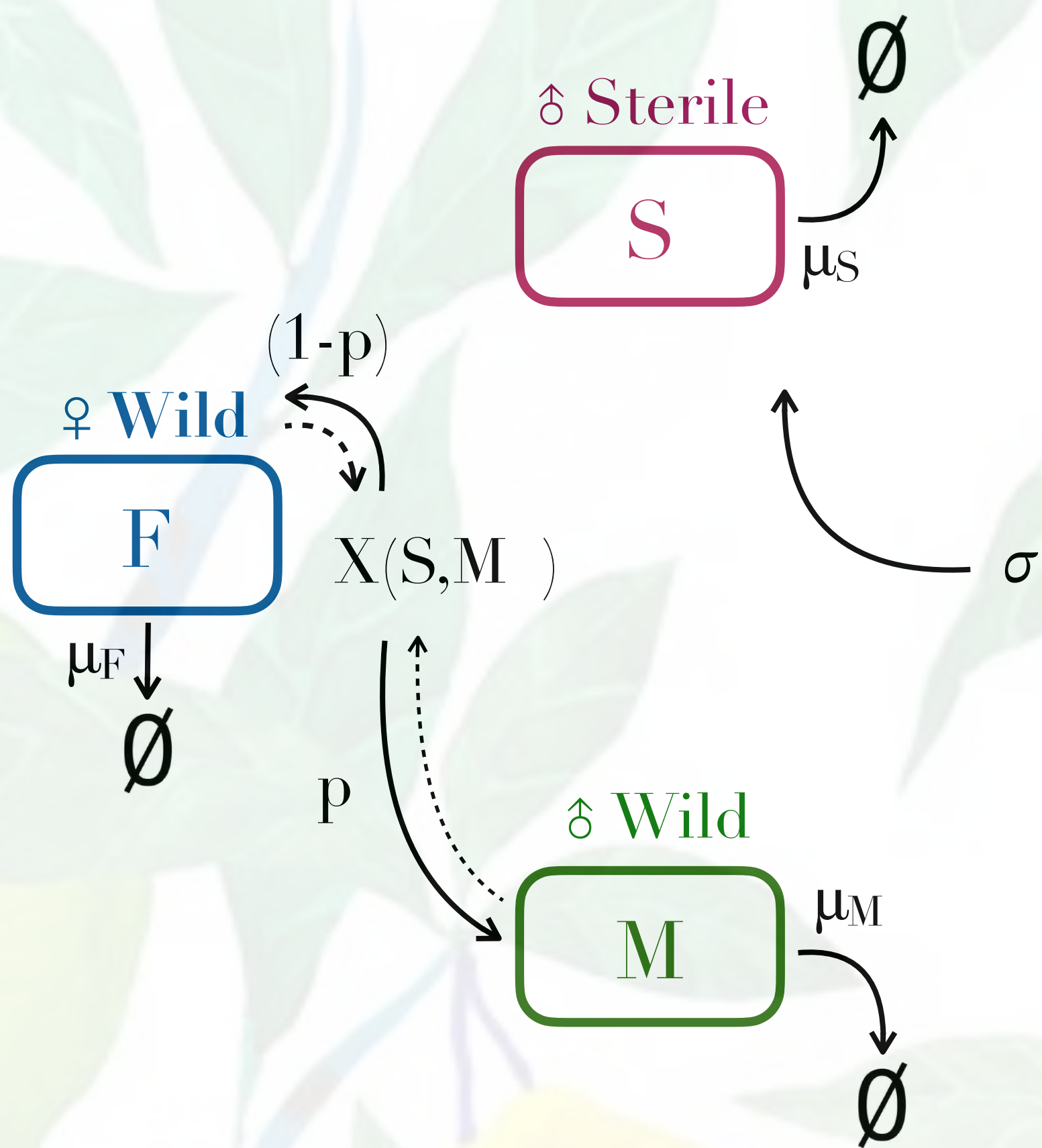
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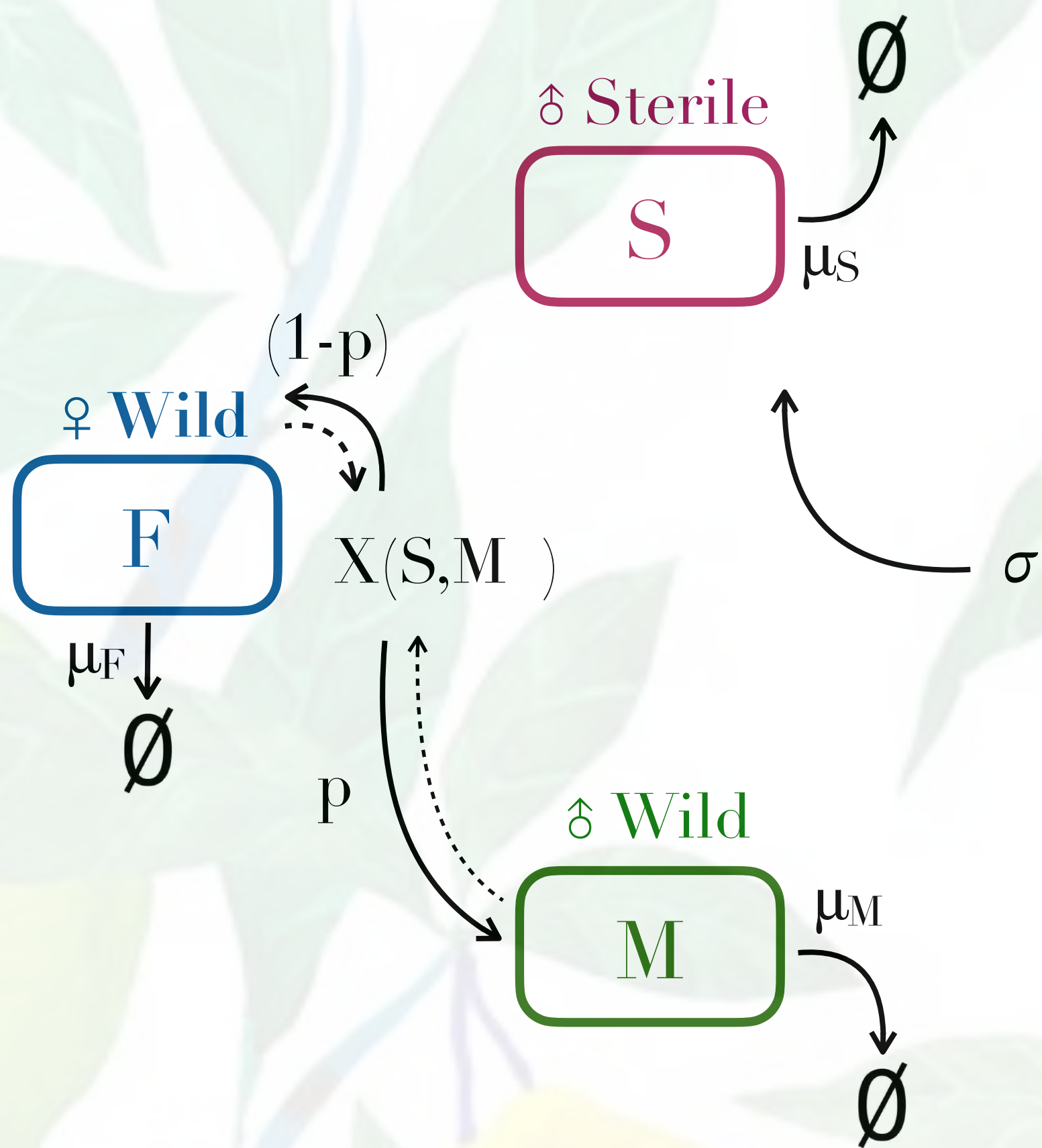


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Residual fertility

δ, ϵ : proportion of non-sterile males among the releases

$X(S, M)$: mating probability $\frac{M}{k+M+S}$



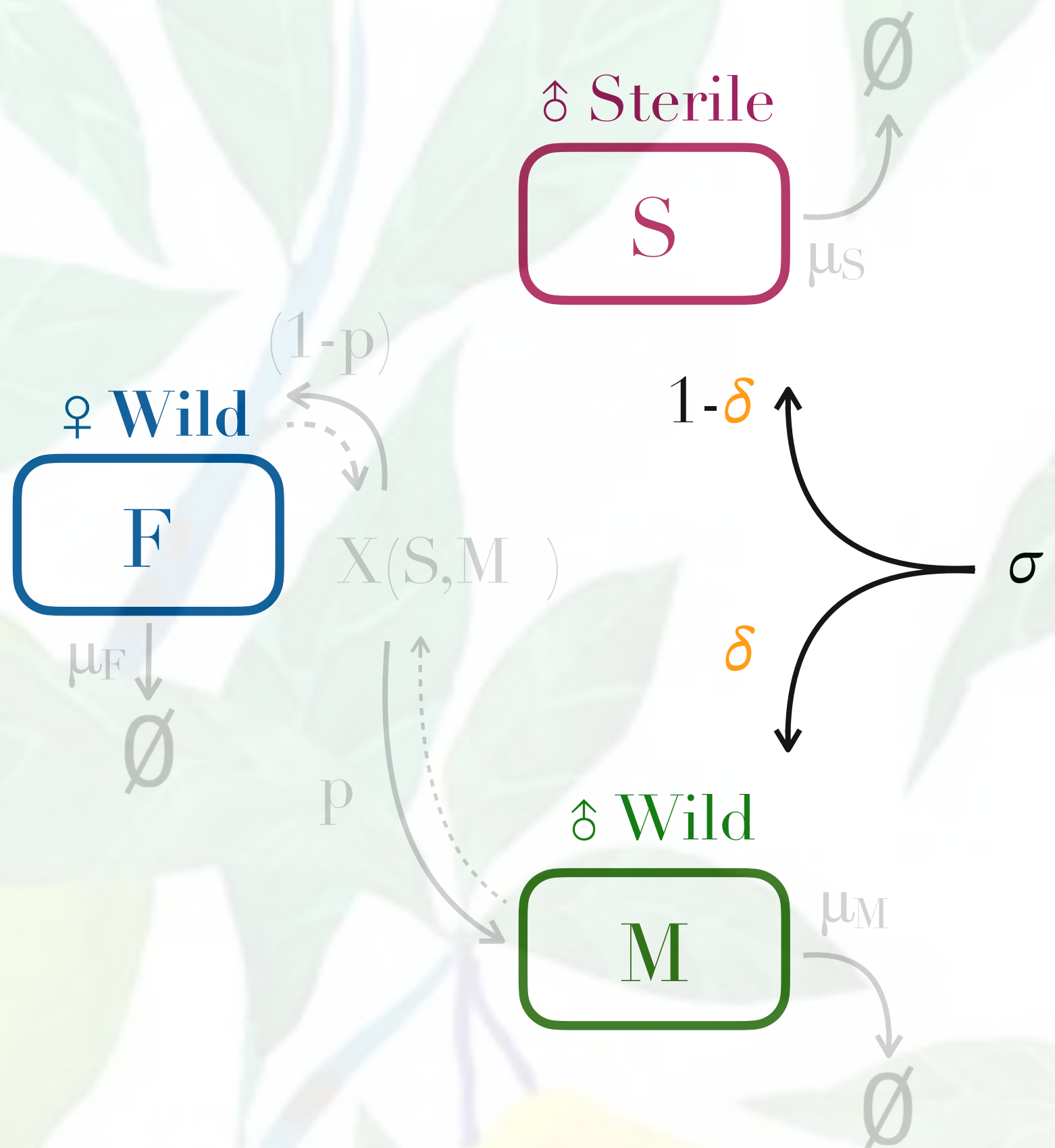
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Residual fertility

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(0) **No Residual fertility**
 $\delta = 0, \epsilon = 0$

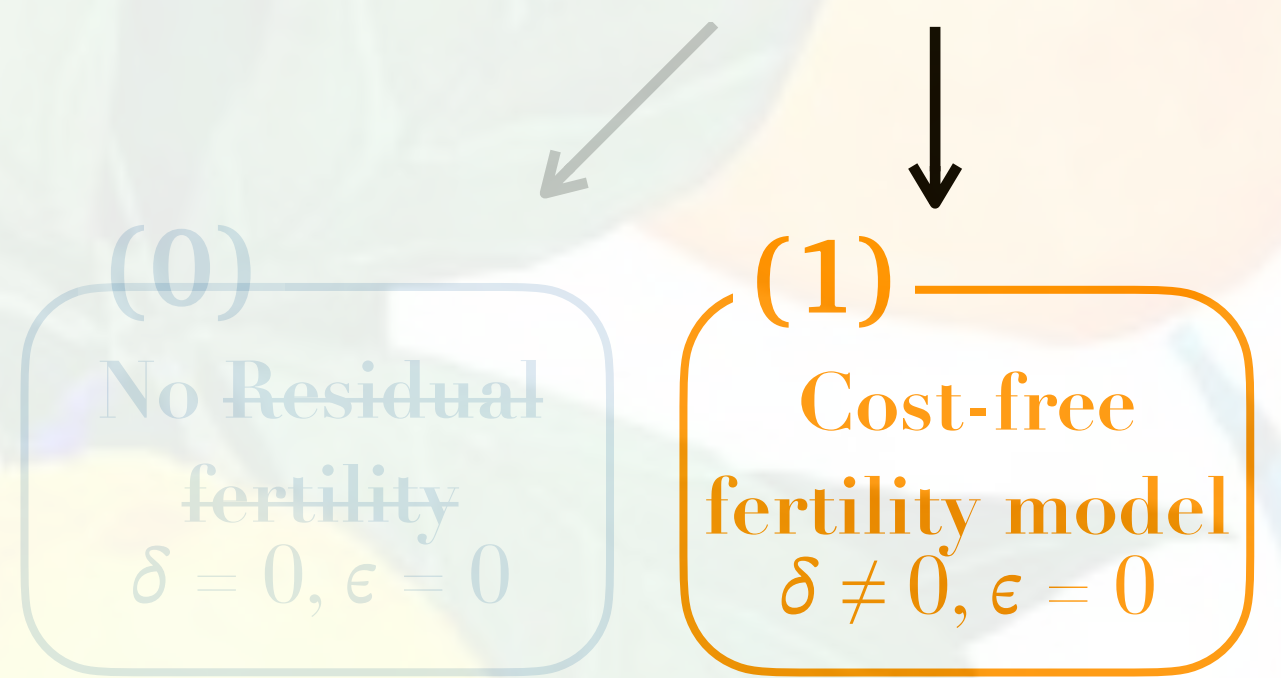
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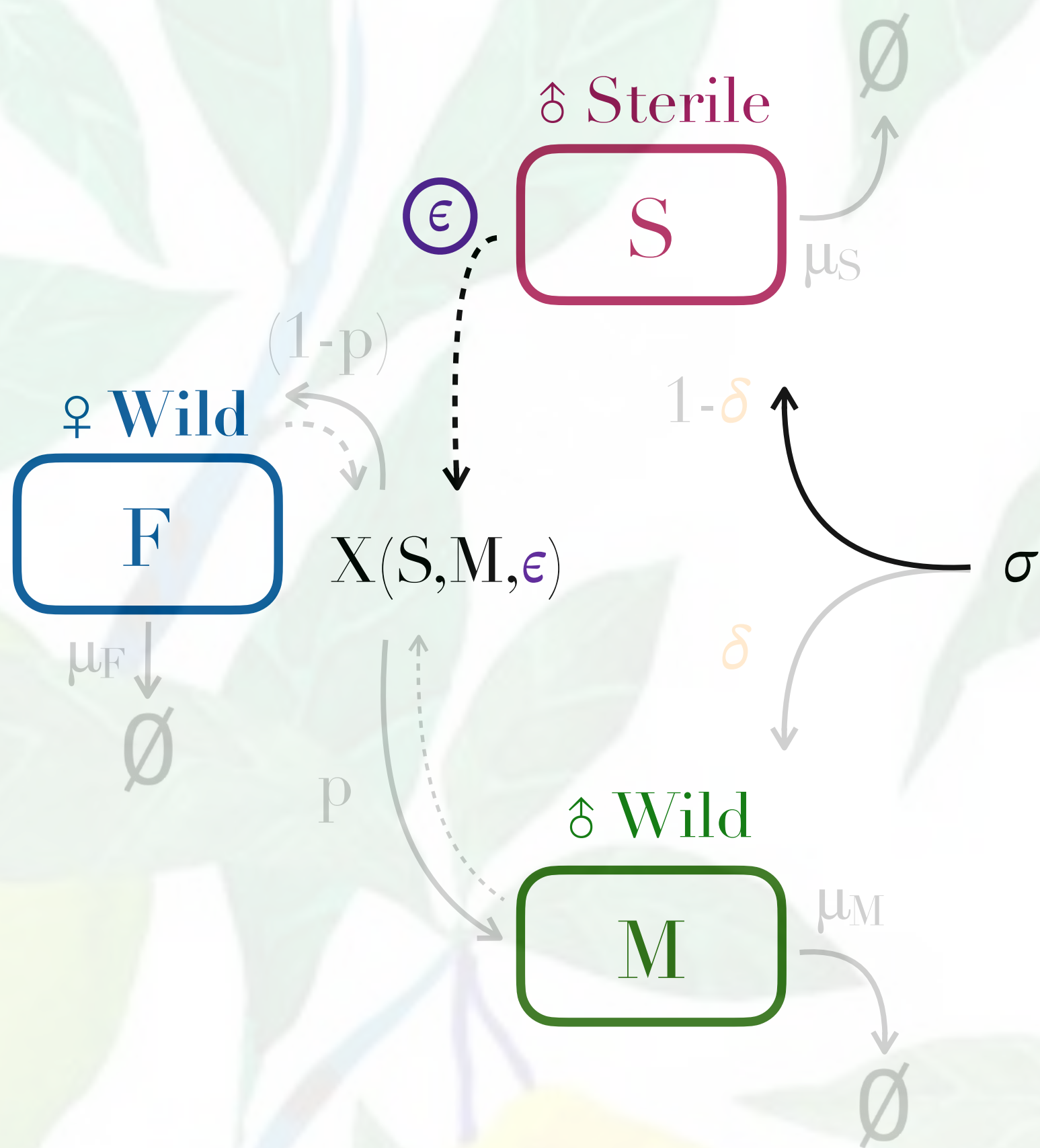
$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p)X(S, M) - C(F)F \\ \dot{M} = -\mu_M M + rpX(S, M) + C(F)F + \delta\sigma \end{cases}$$

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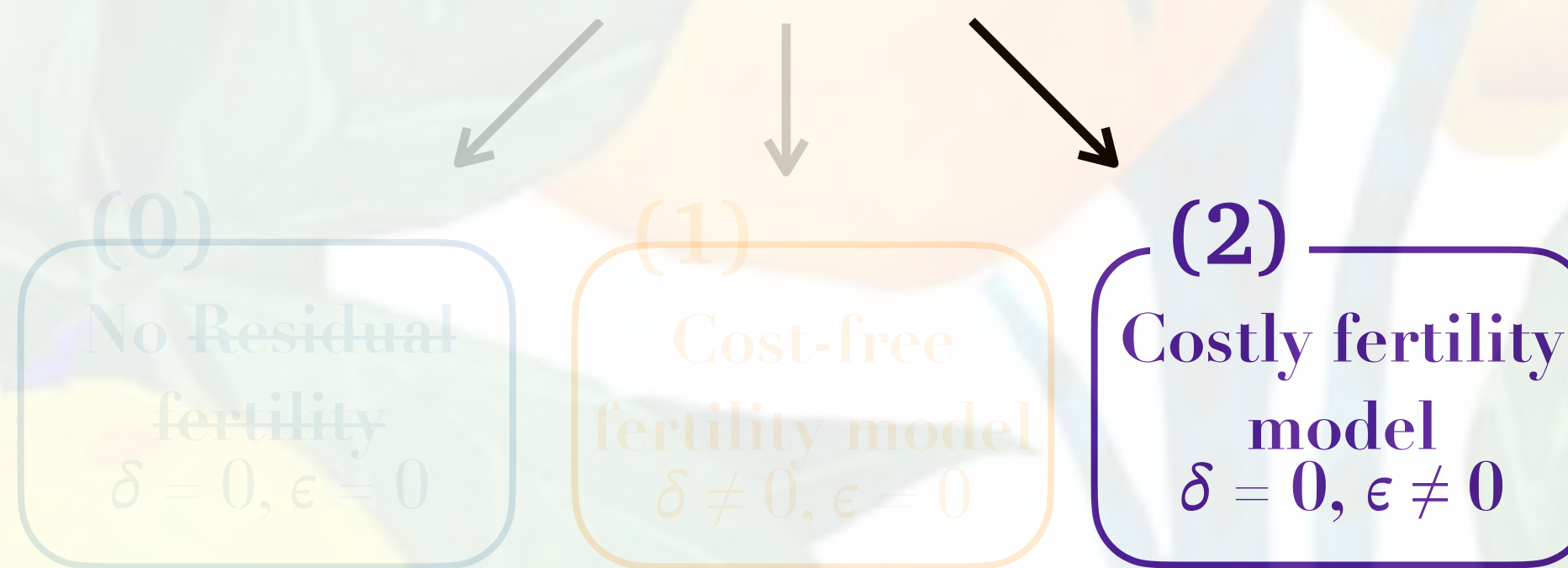
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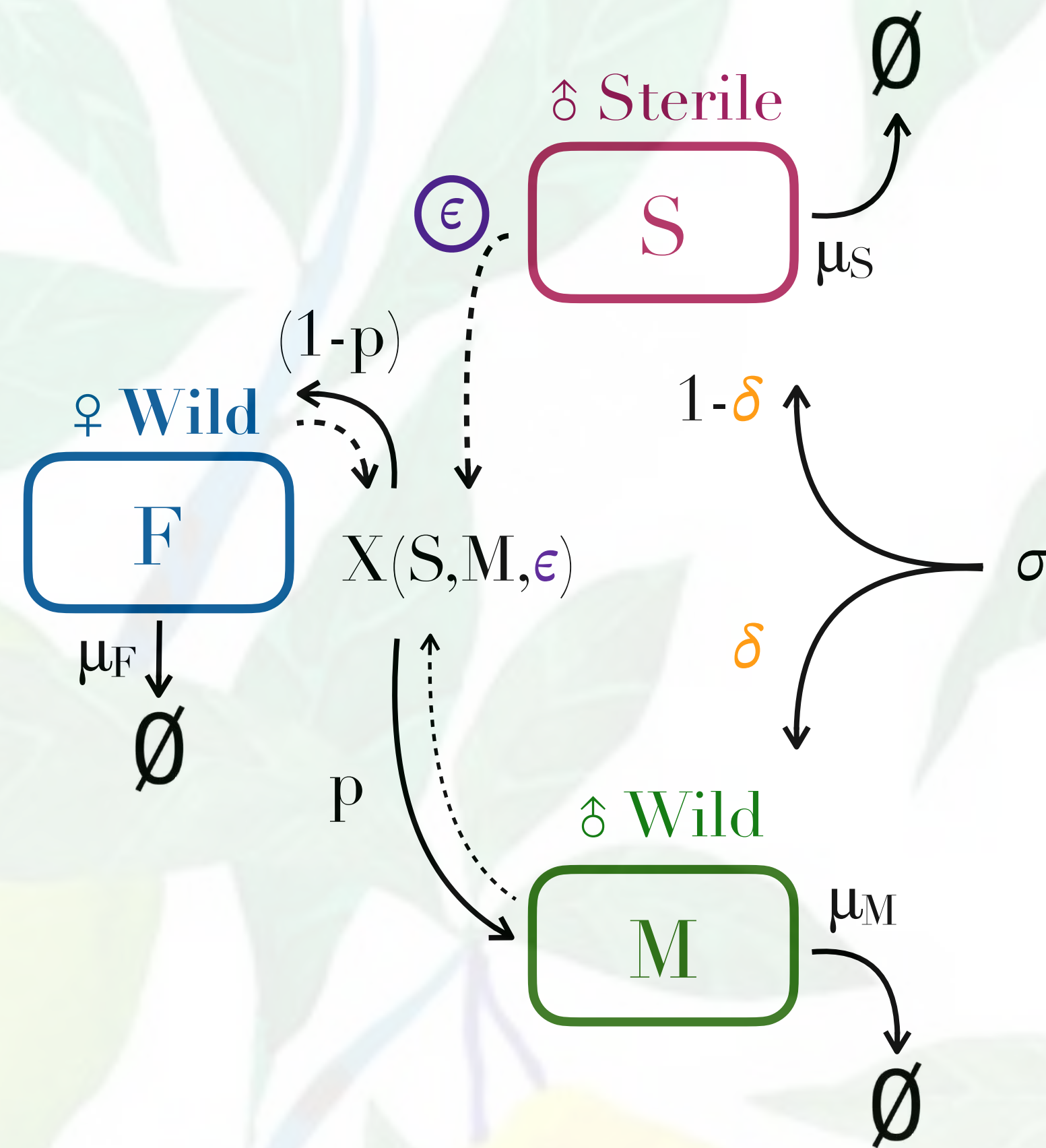
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$X(S, M, \epsilon)$: mating probability

$$\frac{M + \epsilon \eta S}{k + M + \eta S}$$

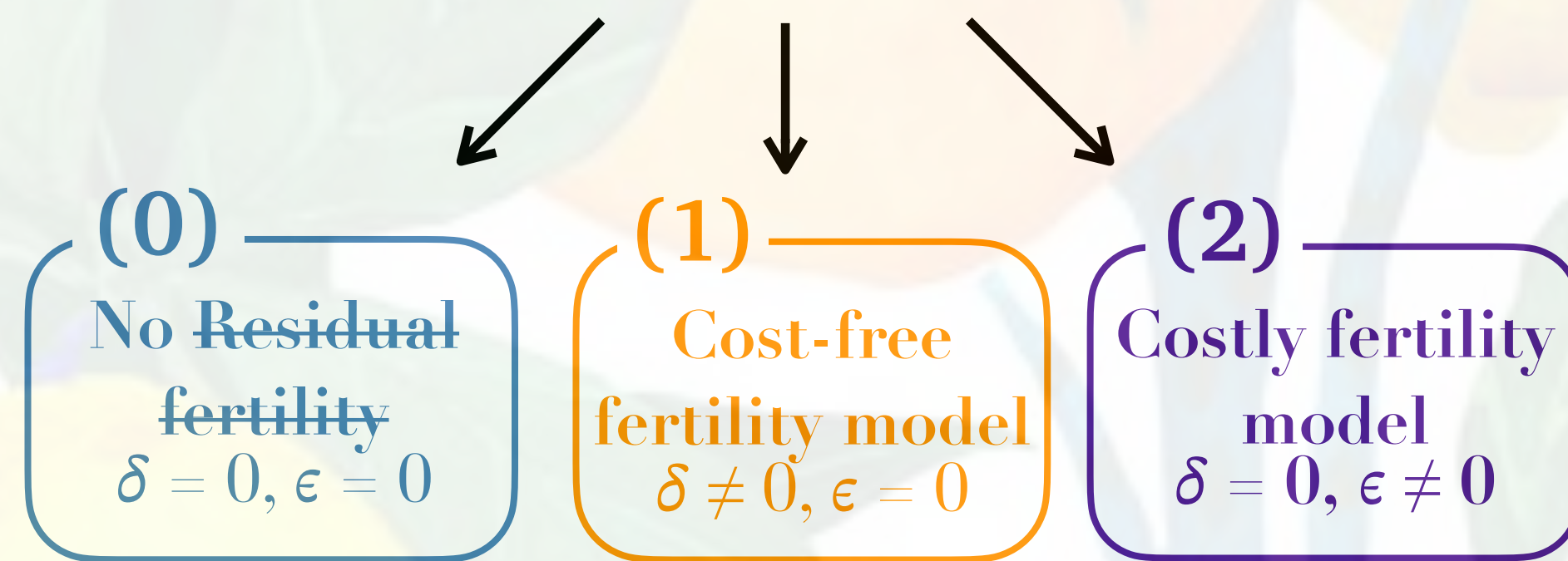
η : Fitness cost



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Table 1: Model parameters

Parameters	Descriptions	Values	Units	References
μ_F	Female mortality rate	0.050	day ⁻¹	Vargas <i>et al.</i> (2000) Pieterse <i>et al.</i> (2020)
μ_M	Male mortality rate	0.036	day ⁻¹	Vargas <i>et al.</i> (2000) Pieterse <i>et al.</i> (2020)
μ_S	Sterile male mortality rate	0.057	day ⁻¹	Calibrated value
p	Sex ratio	0.50	-	Pieterse <i>et al.</i> (2020)
r	Emergence rate (mean number of eggs leading to the adult stage per female)	1.19	eggs.♀ ⁻¹ .day ⁻¹	Shoukry and Hafez (1979) Carey (1982, 1984) Vargas <i>et al.</i> (1984, 2000) Krainacker <i>et al.</i> (1987) Duyck <i>et al.</i> (2002) Papadopoulos <i>et al.</i> (2002) Diamantidis <i>et al.</i> (2011)
k	Coupling half-saturation constant	1	♂ density	Calibrated value
β	Oviposition competition between females	0.85	(♀ density) ⁻¹	Calibrated value
σ	Sterile male release rate	Variable	♂ density.day ⁻¹	
$1 - \eta$	Sterilization cost	0.8	-	Calibrated value
δ	Proportion of non-sterile males among the releases (cost-free fertility)	Variable	-	Studied value
ϵ	Proportion of non-sterile males among the releases (costly fertility)	Variable	-	Studied value

The values noted as "Calibrated values" were determined from laboratory data.

➔ Search for equilibria to see when the population can't settle.

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$$\dot{S} = -\mu_S S + (1 - \delta)\sigma \quad \Rightarrow \quad \dot{S} = 0$$

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$$M^* = \frac{\delta\sigma}{\mu_M} = M_0^*$$

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$$M^* = \frac{\delta\sigma}{\mu_M} = M_0^* \quad M^* = M(F^*) = \frac{p\mu_F}{(1 - p)\mu_M} F^* + \frac{\delta\sigma}{\mu_M}$$

➔ Search for equilibria to see when the population can't settle.

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$$\dot{F} = -\mu_F F + r(1 - p)X(S, M, \epsilon)C(F)F \quad \Rightarrow \quad \dot{F} = 0 \iff F^*(-\mu_F + r(1 - p)X(S^*, M^*, \epsilon)C(F^*)) = 0$$

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Pest-free equilibrium

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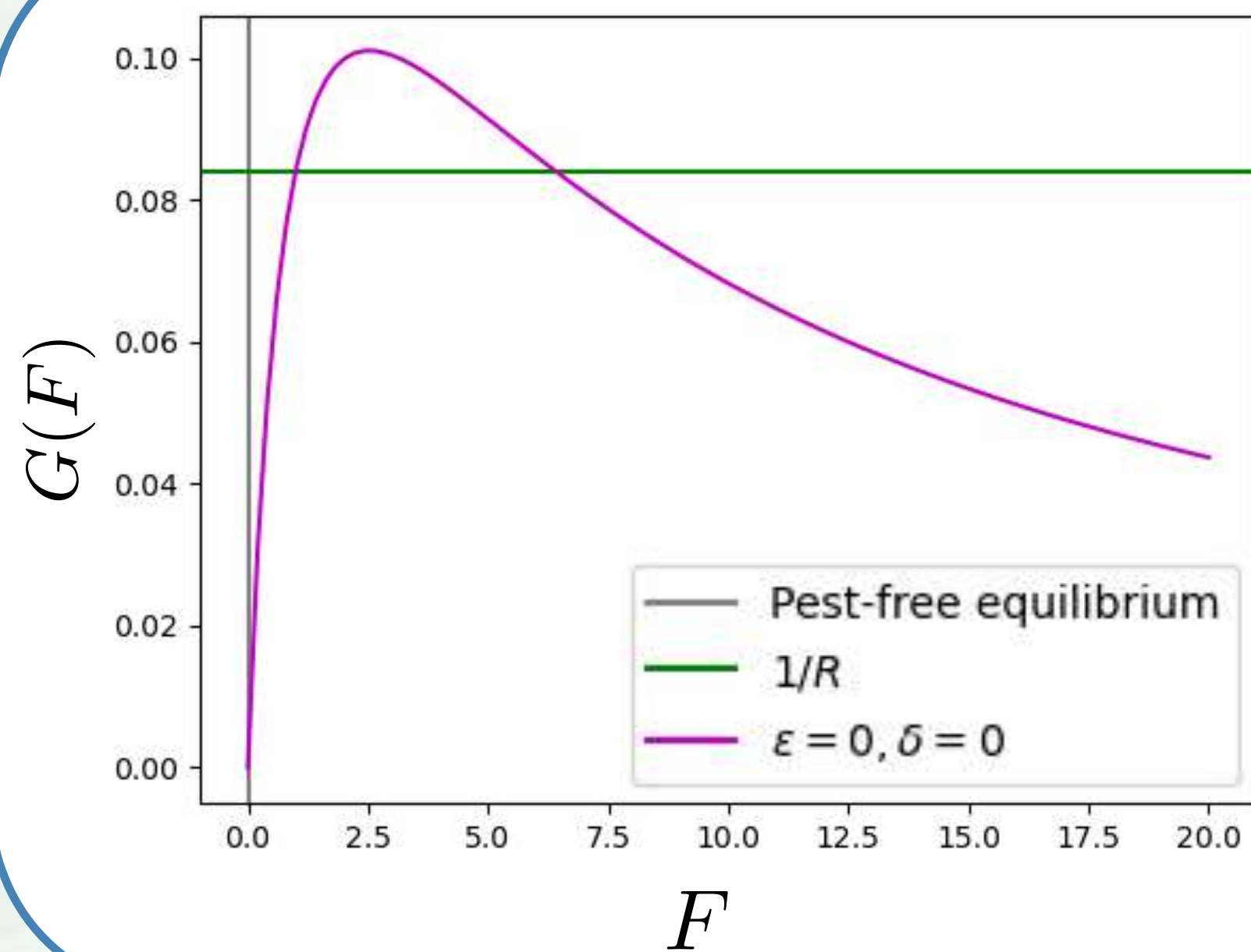
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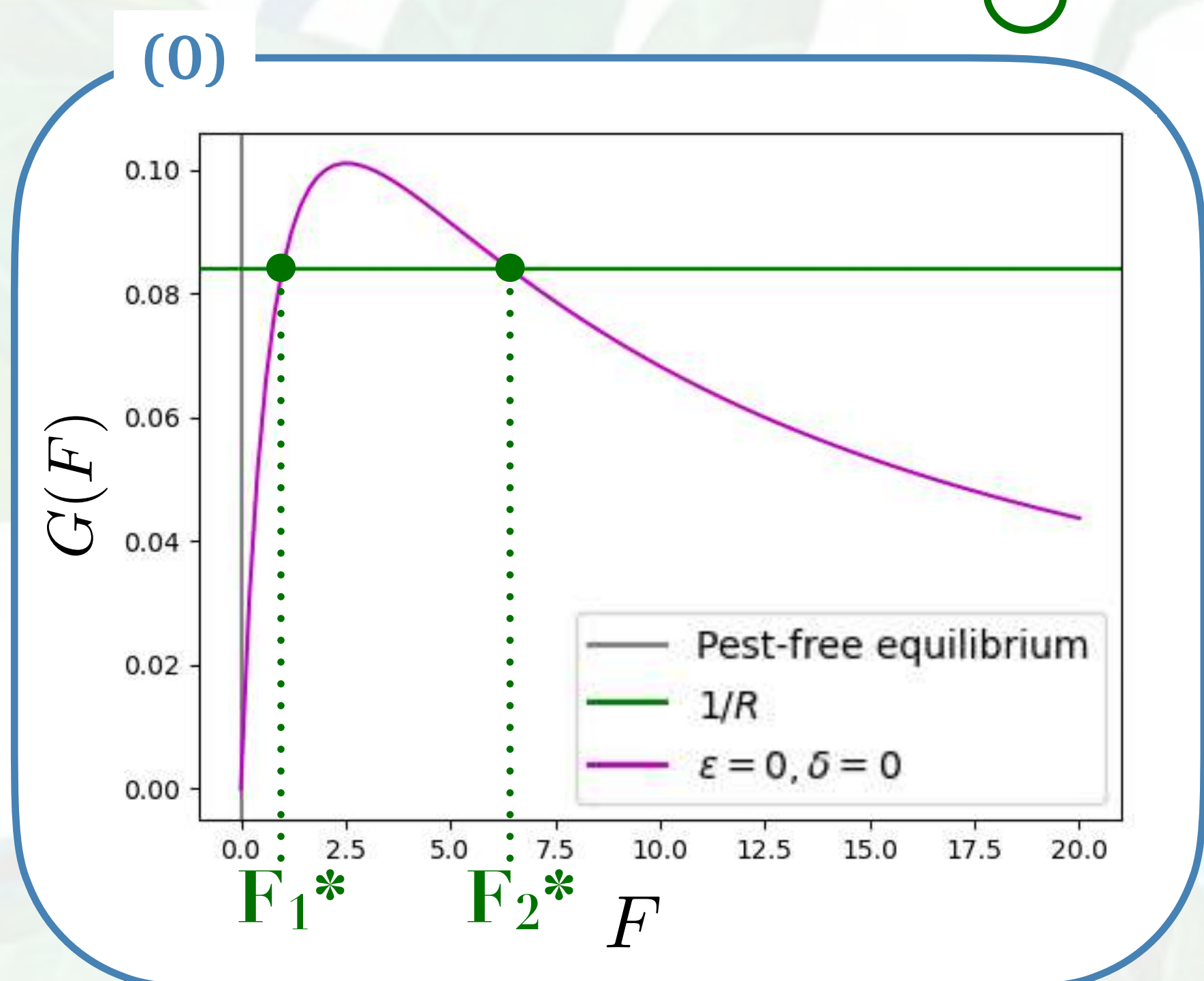
(0)



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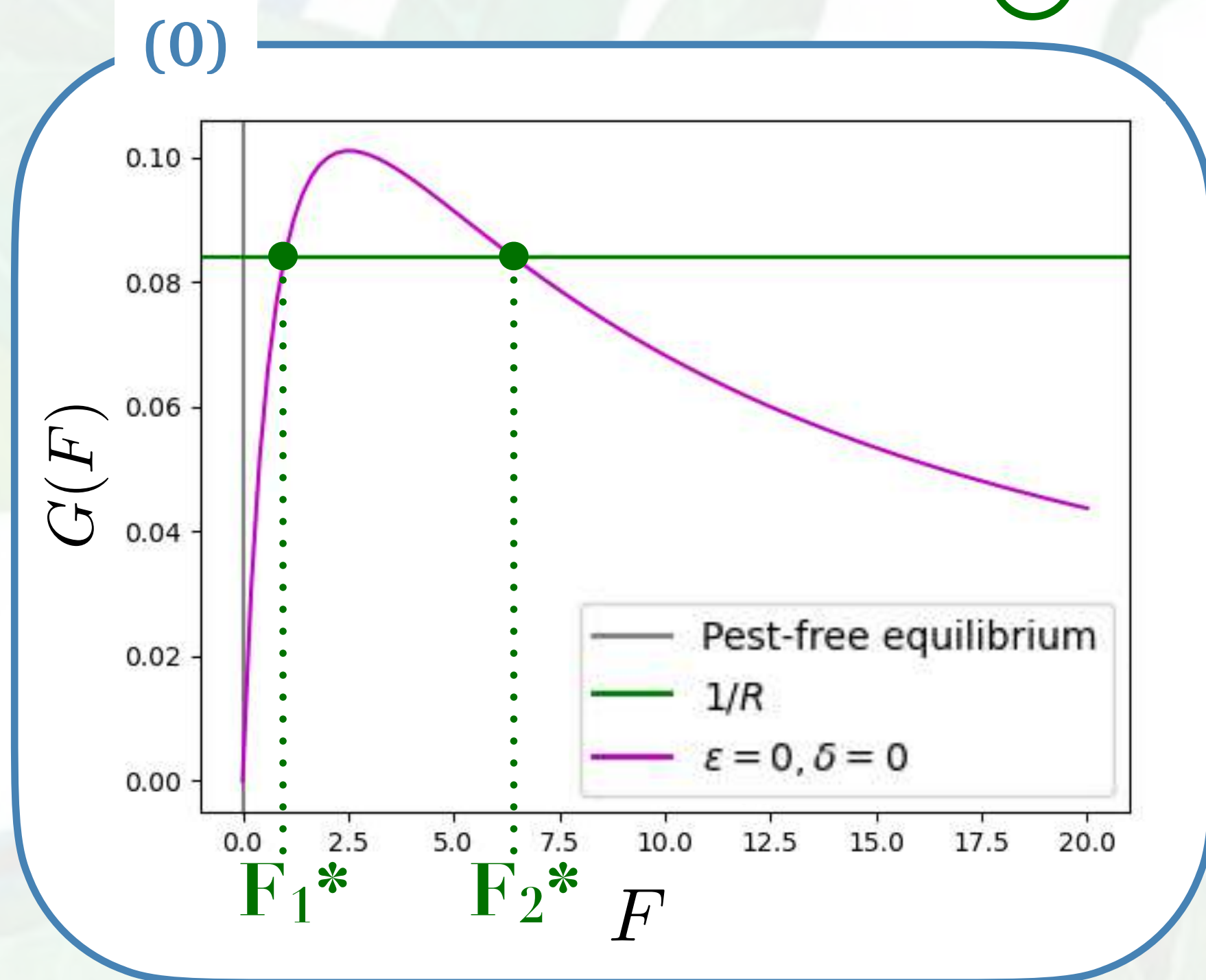


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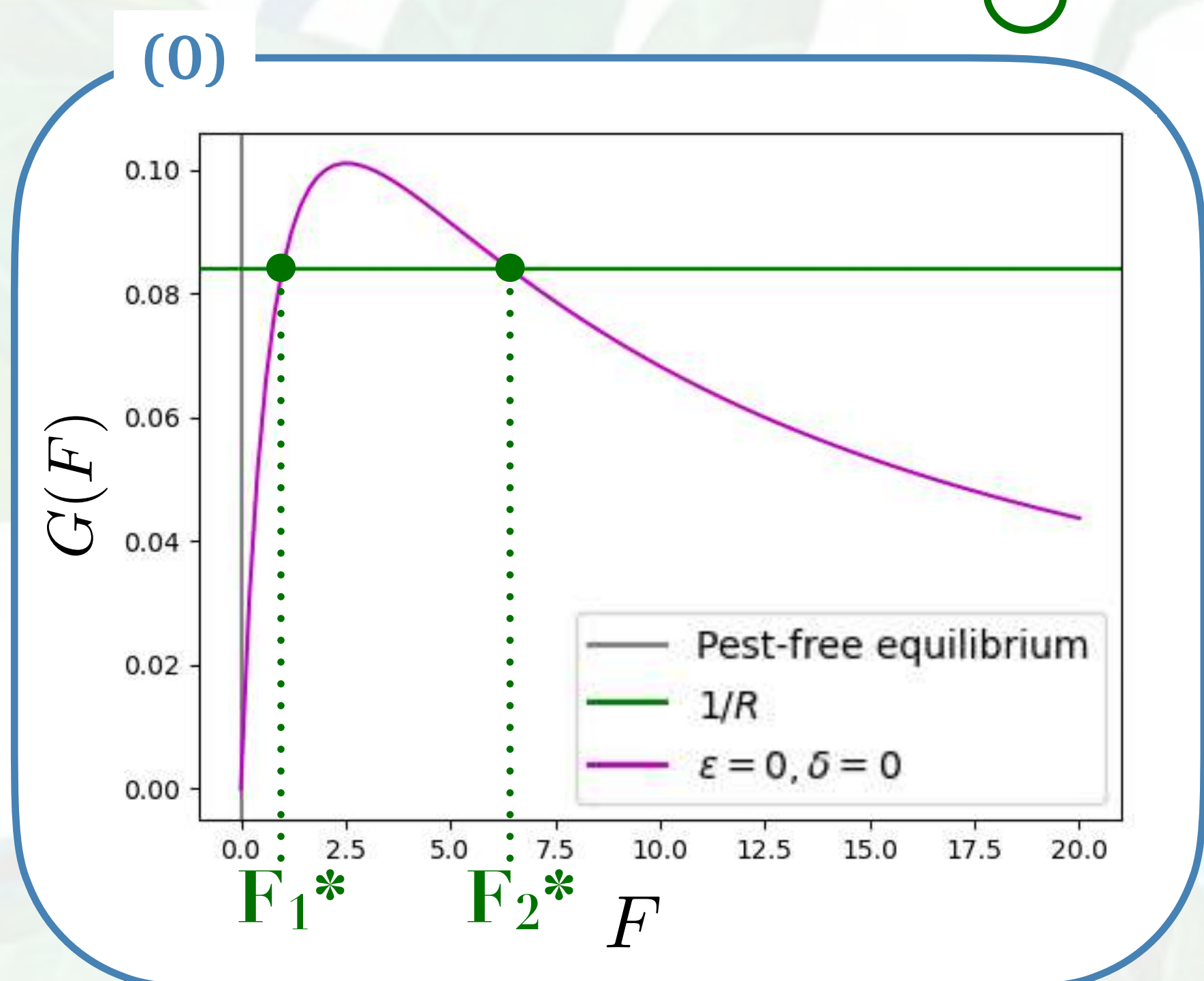
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The equilibria are: $(S^*, F_1^*, M_1^*), (S^*, F_2^*, M_2^*)$.

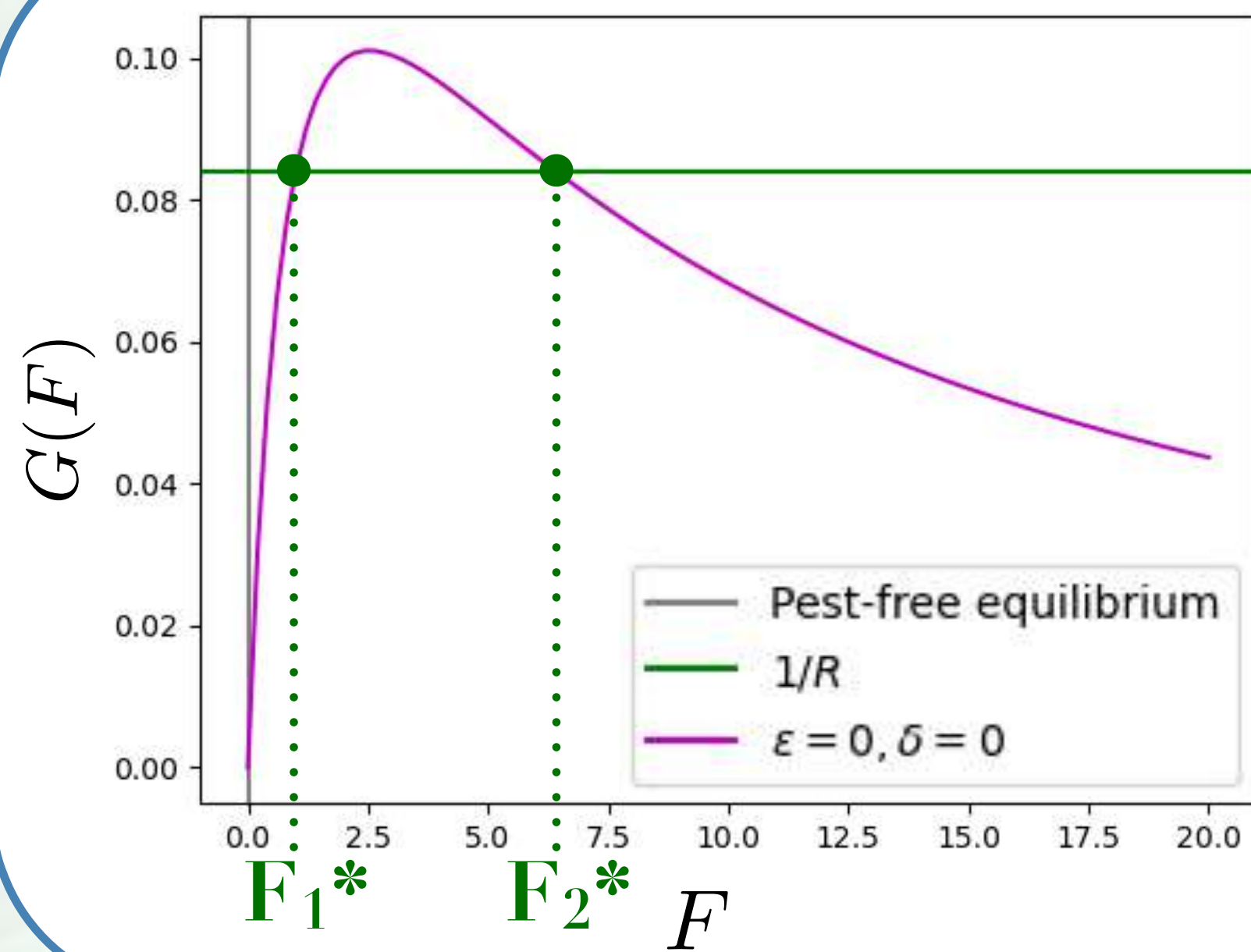
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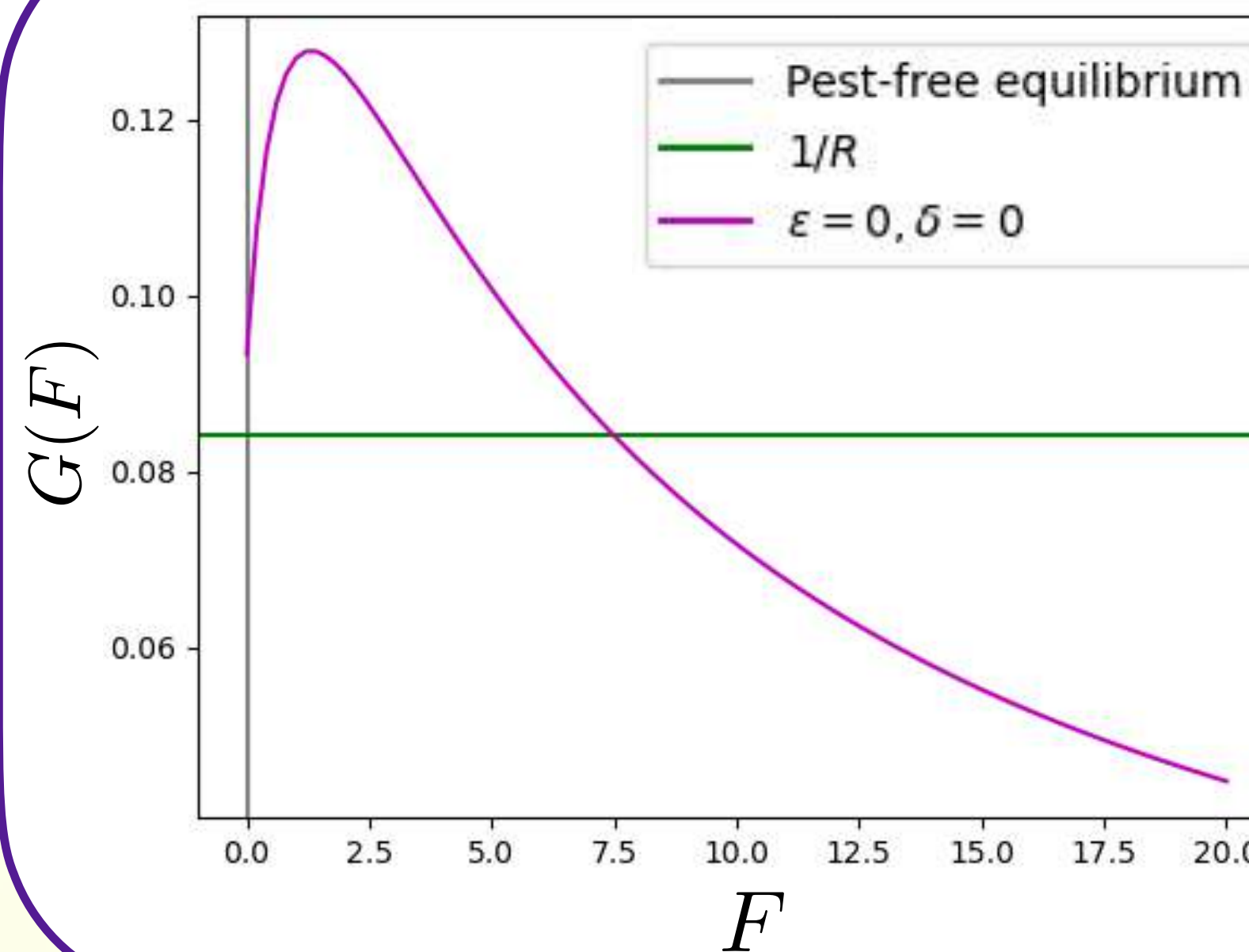
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(0)



(2)



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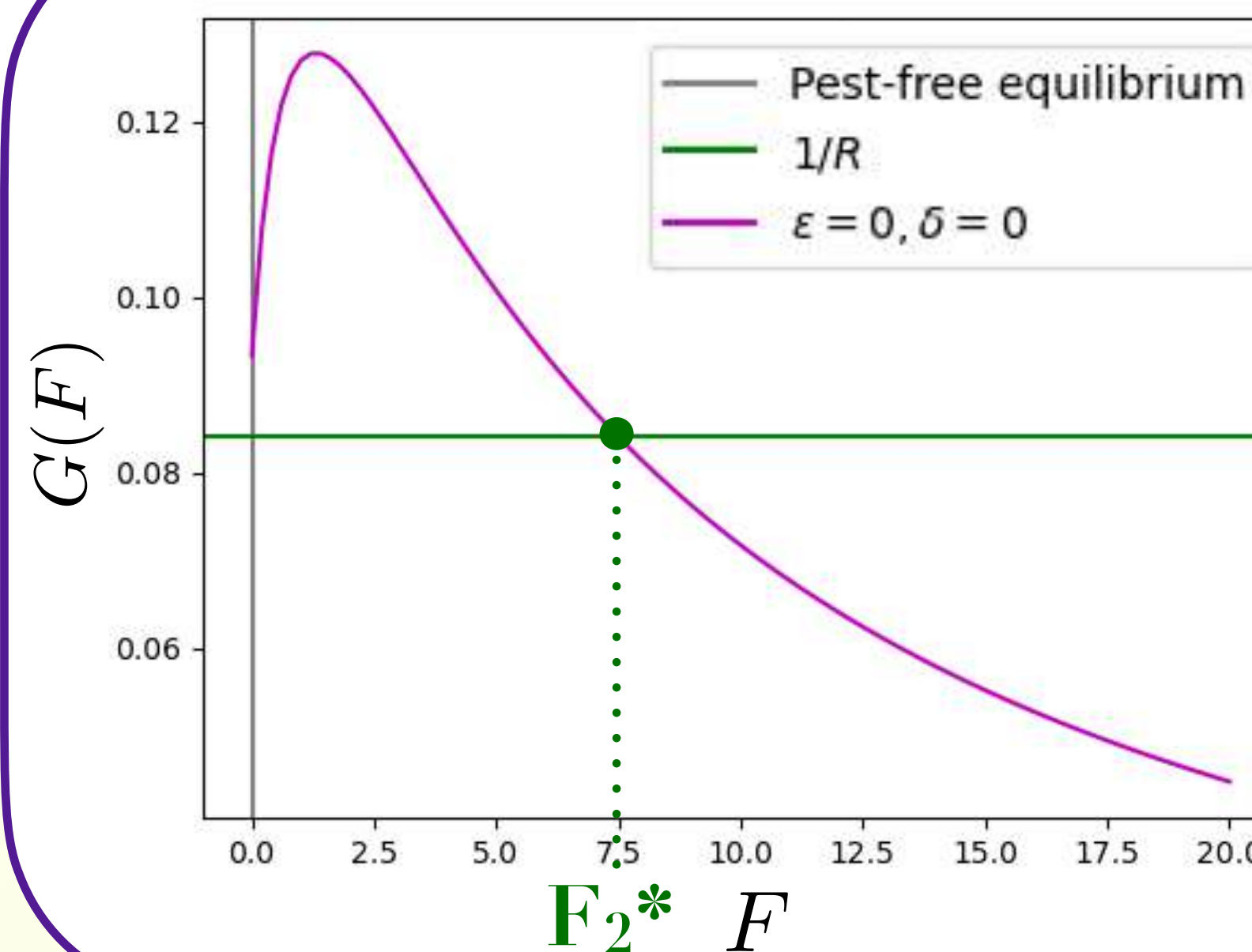
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At least 1 solution F_2^* such that:

$$\frac{dG}{dF}(F_2^*) < 0$$

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(2)



Stability of equilibria

Study of the Jacobian matrix J of the (F, M) subsystem (at $S = S^*$)

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$$J = \begin{pmatrix} -\mu_F + (1-p)X(M_0^*)r & 0 \\ pX(M_0^*)r & -\mu_M \end{pmatrix}$$

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➡ calculation of the trace and determinant

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In summary

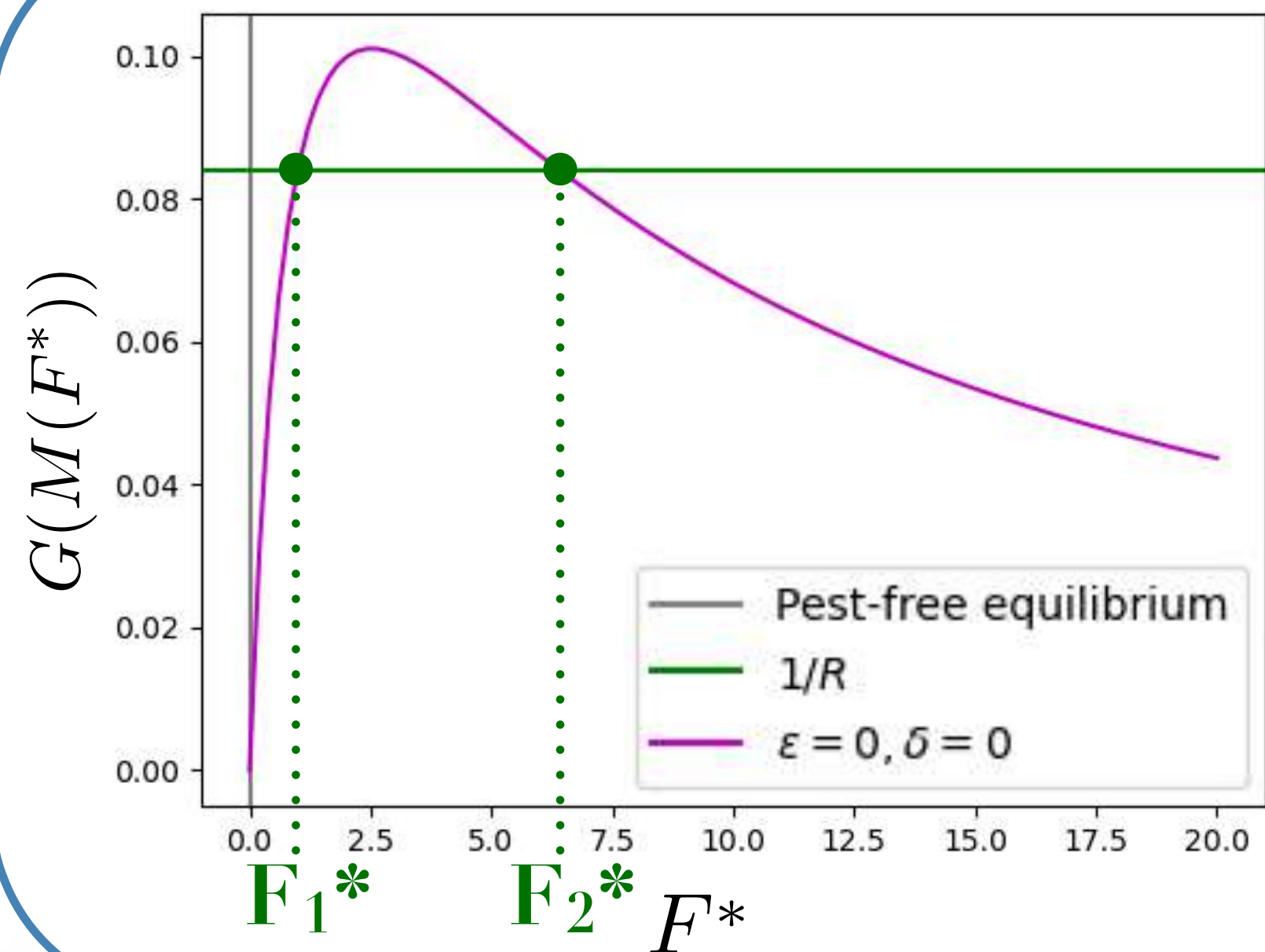
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(0)



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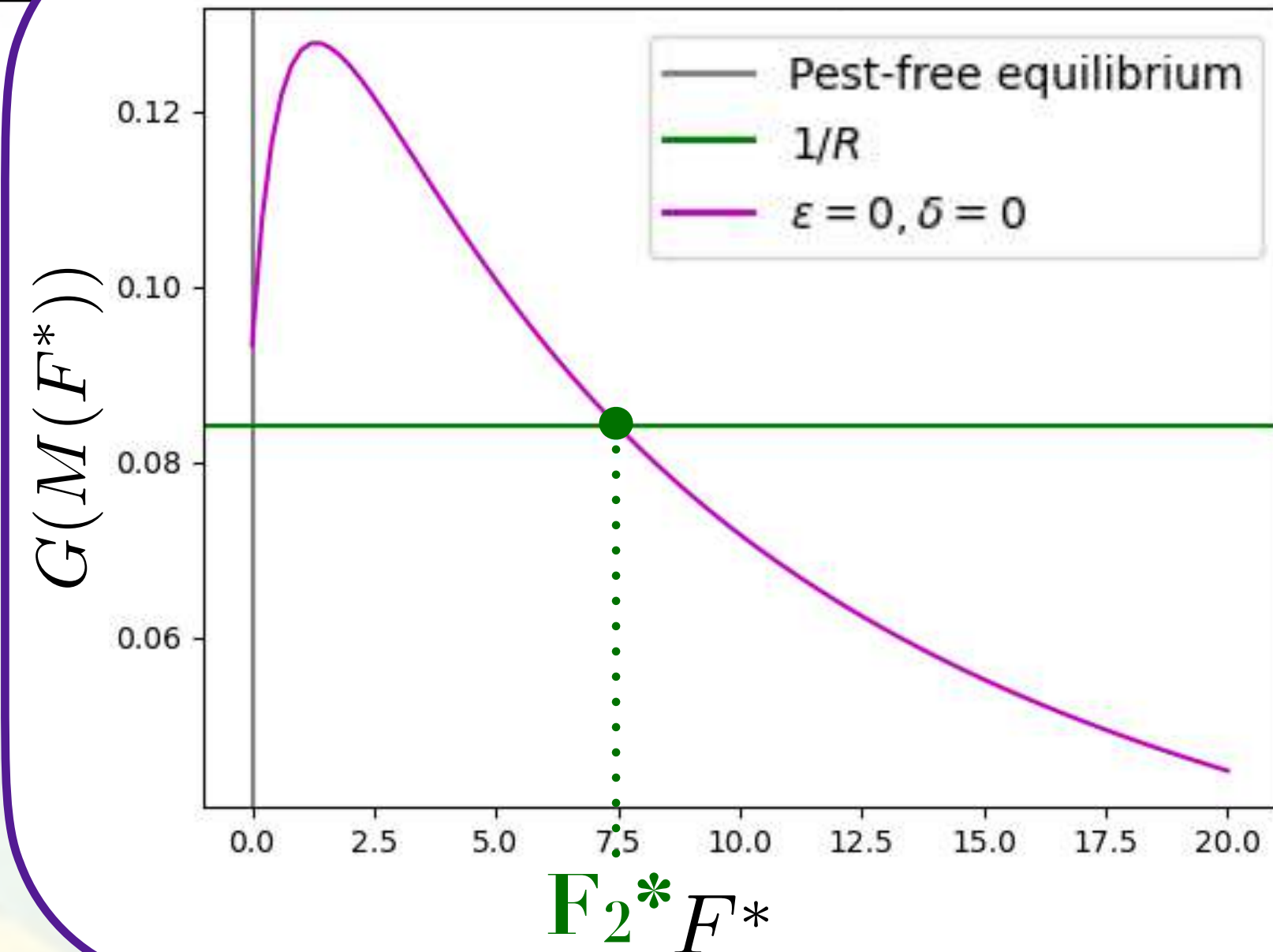
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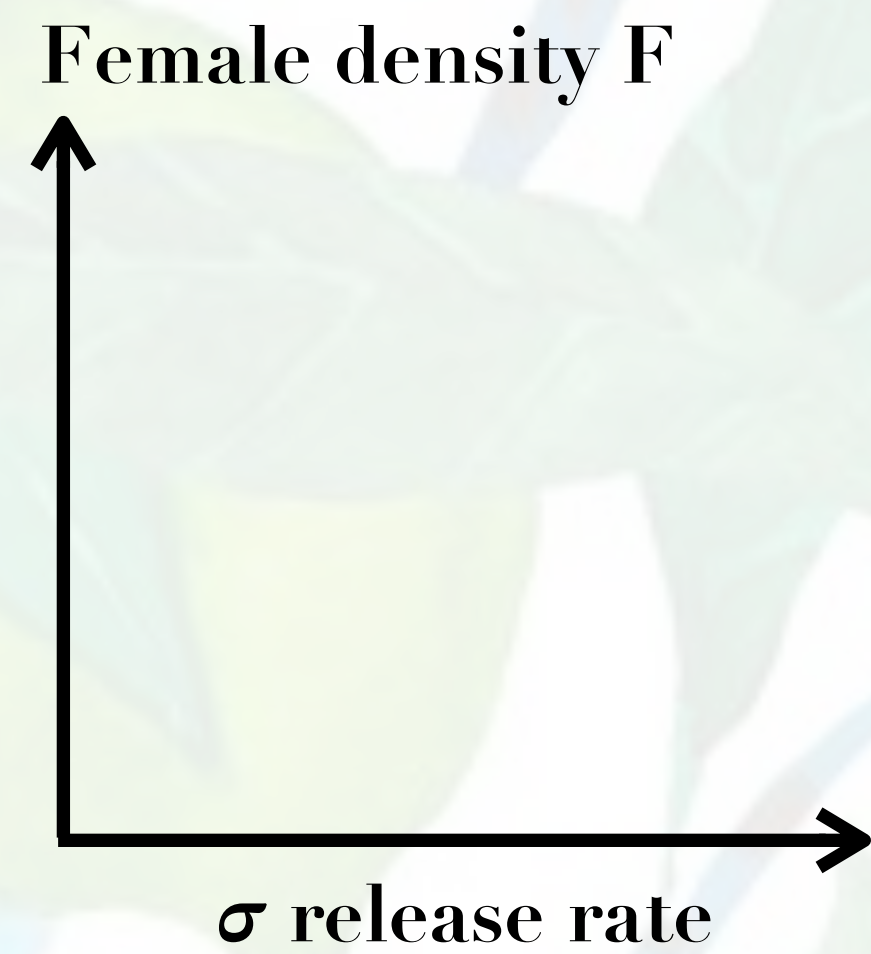
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Infestation equilibrium

(S^*, F_2^*, M_2^*) **STABLE**

Study of the bifurcation diagram in σ

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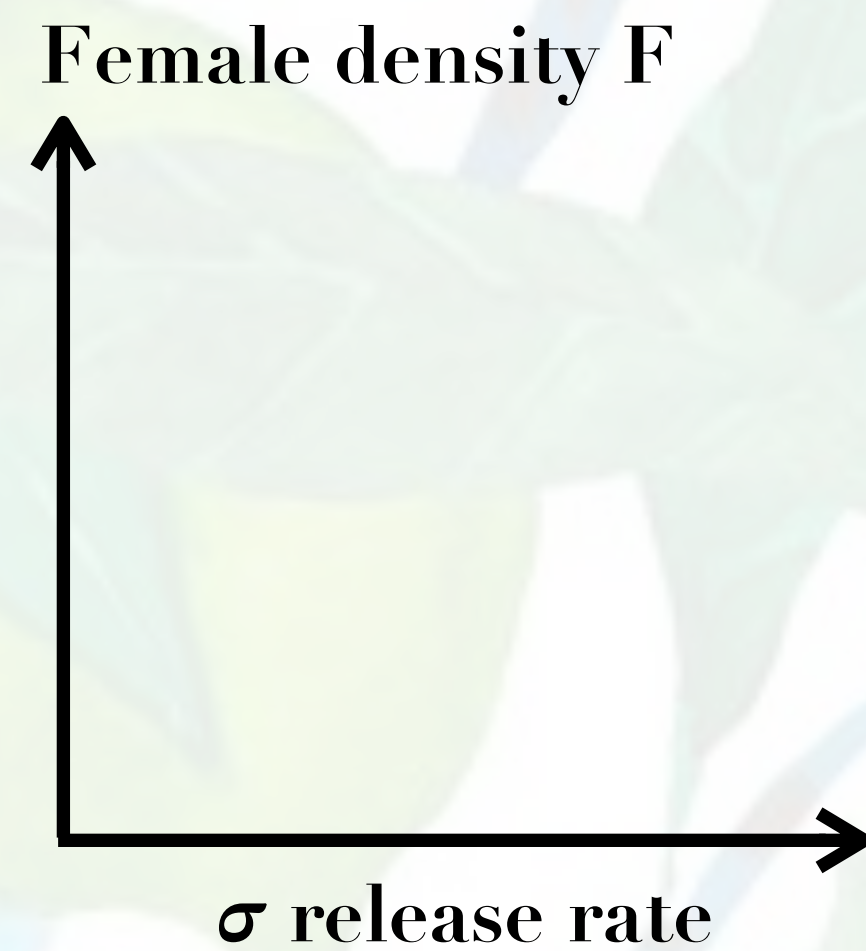


σ bifurcation diagram

Study of the bifurcation diagram in σ

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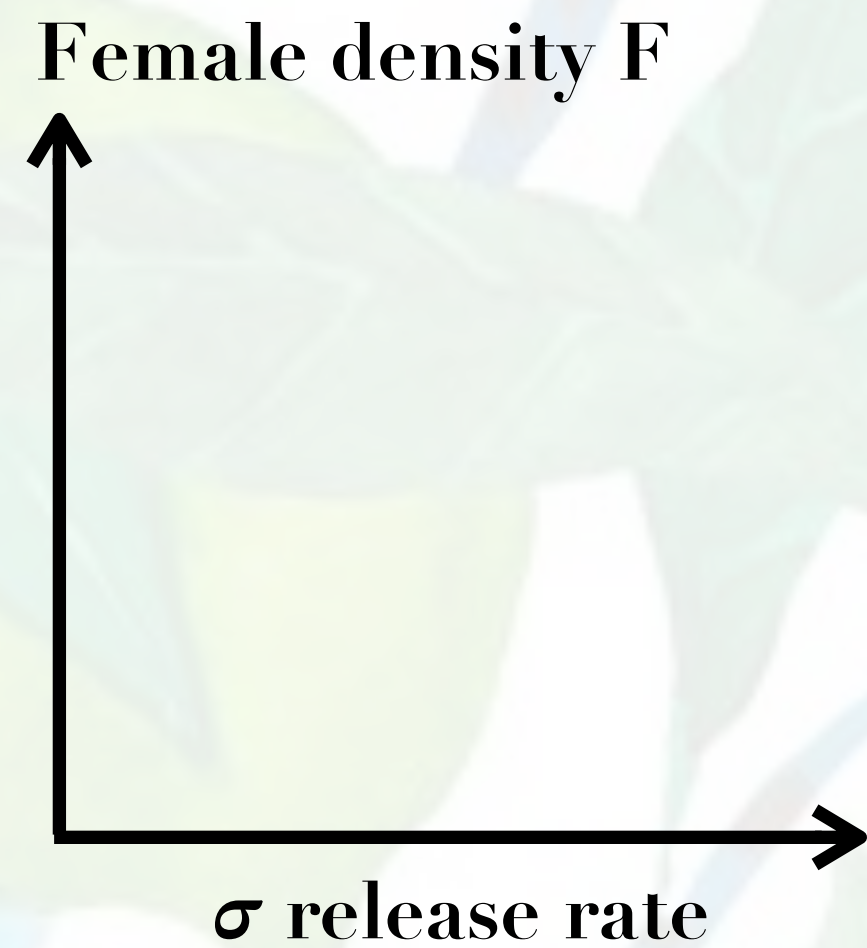


↓
 σ bifurcation diagram

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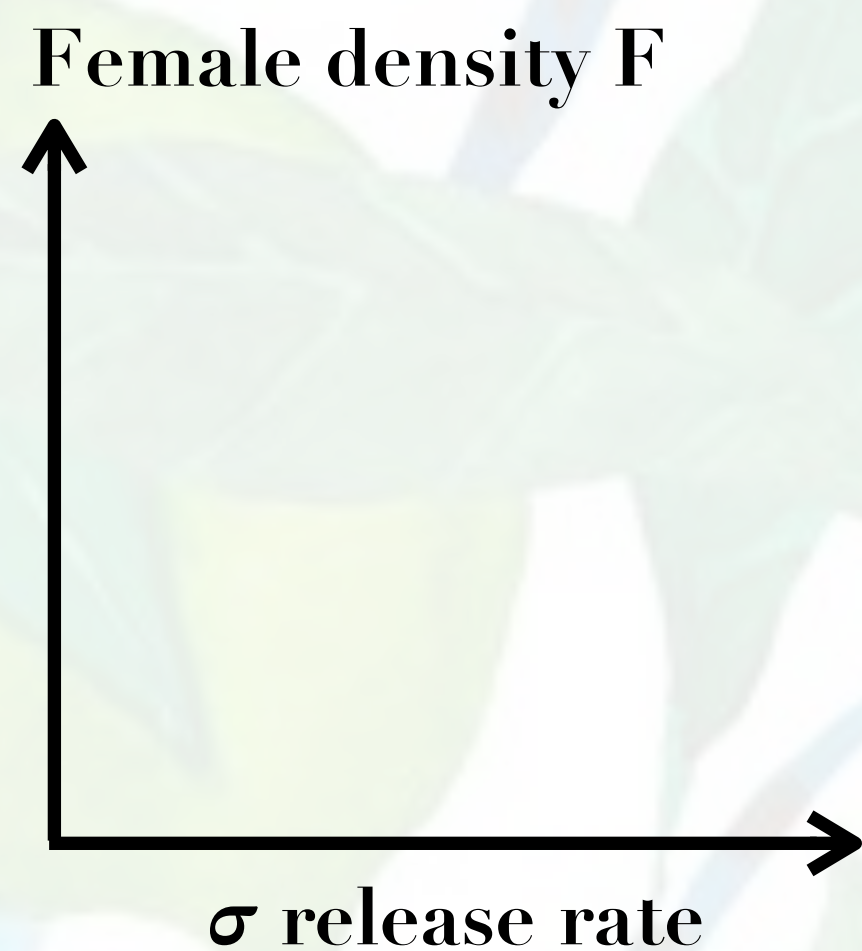
⊙ bifurcation diagram

$$\sigma = \frac{(1 + \beta F)(k + \frac{p\mu_F}{(1-p)\mu_M}F) - \frac{Rp\mu_F}{(1-p)\mu_M}F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F)\frac{\delta}{\mu_M} - (1 + \beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Study of the bifurcation diagram in σ

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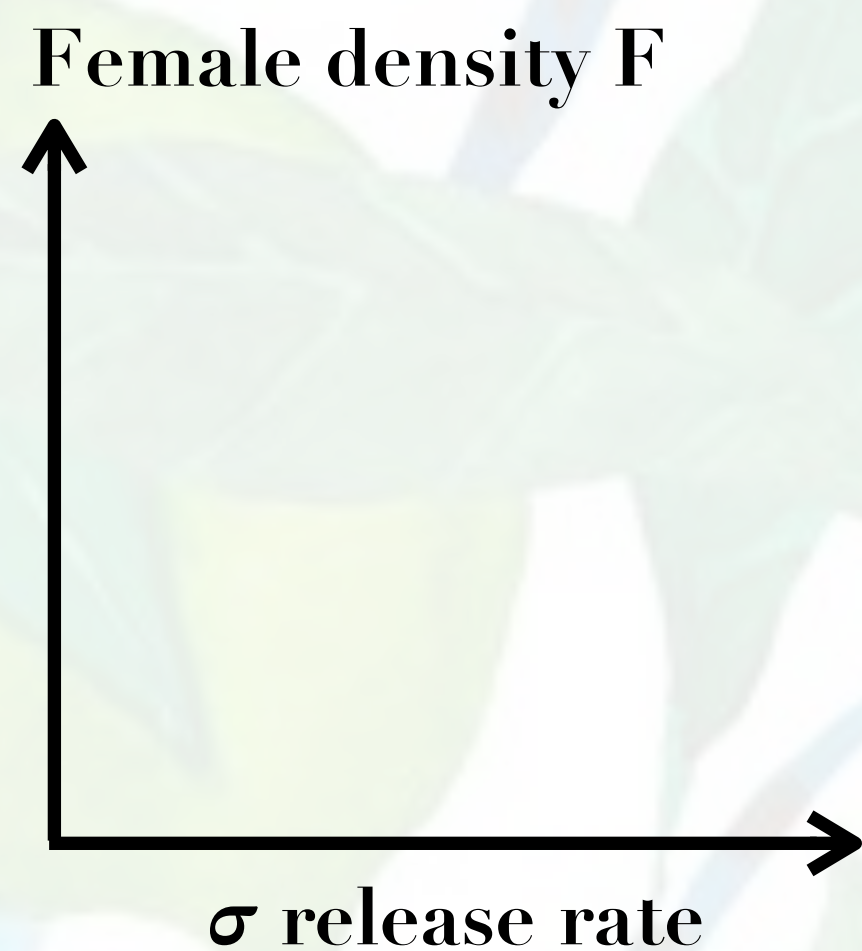
Polynomial of degree 2 not depending on residual fertility rate (δ, ϵ)

$$\sigma = \frac{\left((1 + \beta F) \left(k + \frac{p\mu_F}{(1-p)\mu_M} F \right) - \frac{Rp\mu_F}{(1-p)\mu_M} F \right)}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F) \frac{\delta}{\mu_M} - (1 + \beta F) \left(\eta \frac{(1-\delta)}{\mu_S} \right)}$$

Study of the bifurcation diagram in σ

Infestation equilibria are the F values solutions of:

$$G(F) = X(S, M)C(F) = \frac{\mu_F}{r(1-p)} = \frac{1}{R} \iff \frac{\frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \frac{\epsilon\eta(1-\delta)\sigma}{\mu_S}}{k + \frac{p\mu_F}{(1-p)\mu_M}F + \frac{\delta\sigma}{\mu_M} + \eta\frac{(1-\delta)\sigma}{\mu_S}} \cdot \frac{1}{1 + \beta F} = \frac{\mu_F}{r(1-p)} = \frac{1}{R}$$



⊙ bifurcation diagram

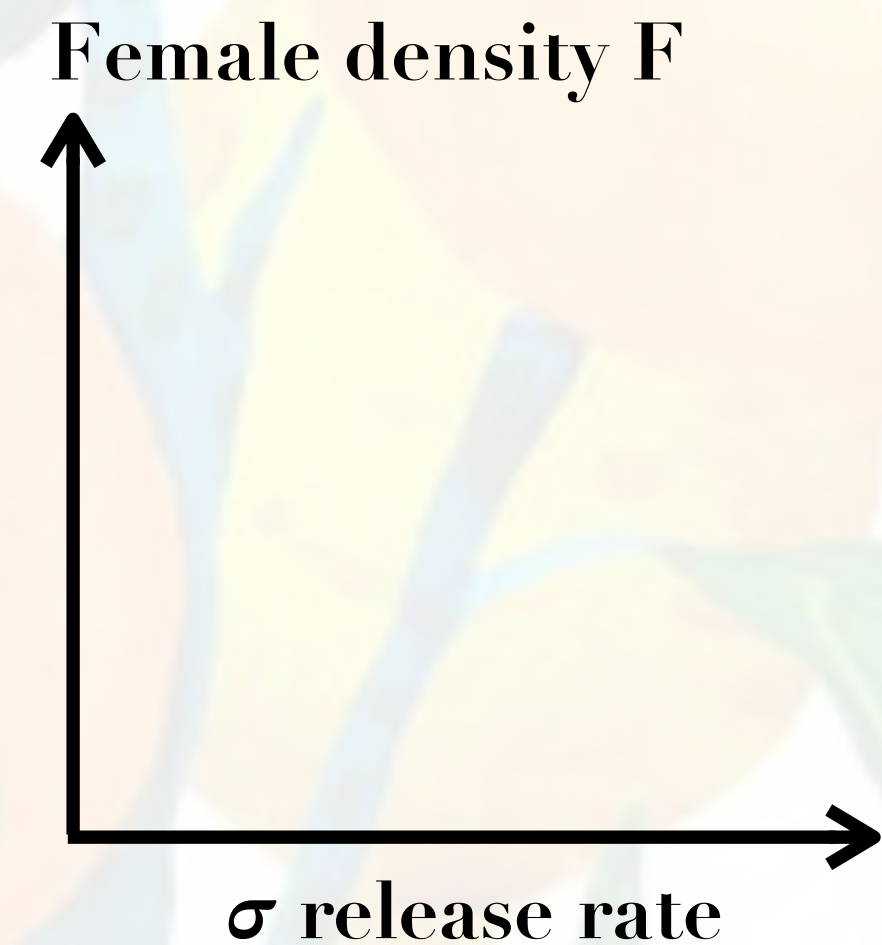
Polynomial of degree 2 not depending on residual fertility rate (δ, ϵ)

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Existence of an asymptote when the denominator cancels

Polynomial of degree 2 not depending on residual fertility rate (δ, ϵ)

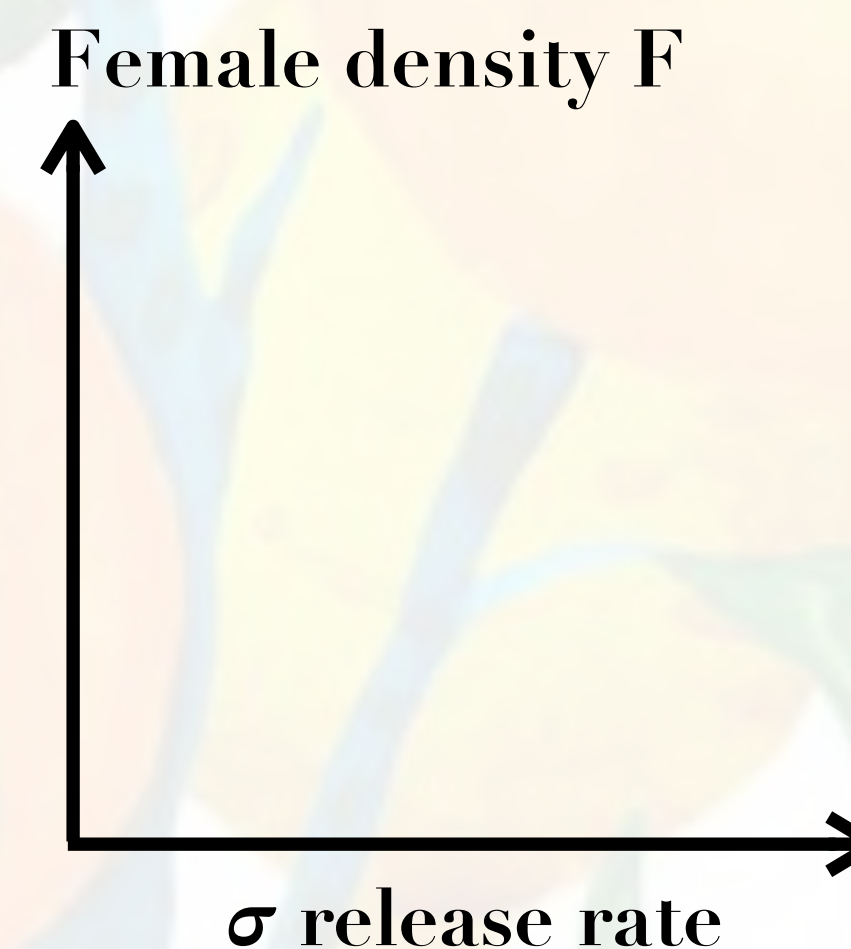
$$\sigma = \frac{\left((1 + \beta F) \left(k + \frac{p\mu_F}{(1-p)\mu_M} F \right) - \frac{Rp\mu_F}{(1-p)\mu_M} F \right)}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F) \frac{\delta}{\mu_M} - (1 + \beta F) \left(\eta \frac{(1-\delta)}{\mu_S} \right)}$$



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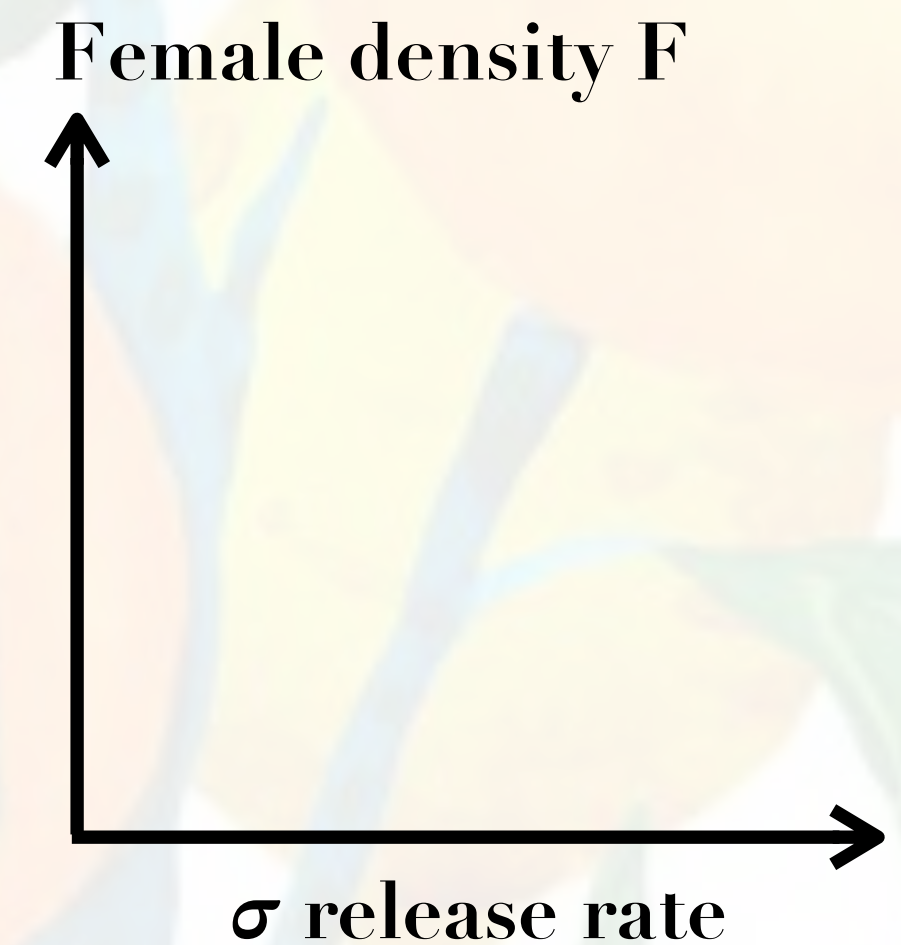
$$\iff \frac{\beta p \mu_F}{(1-p)\mu_M} F^2 + \left(\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k \right) F + k = 0$$



Polynomial of degree 2 not depending
on residual fertility rate (δ, ϵ)

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$$\Leftrightarrow \frac{\beta p \mu_F}{(1-p)\mu_M} F^2 + \left(\frac{p\mu_F(1-R)}{(1-p)\mu_M} + \beta k \right) F + k = 0$$

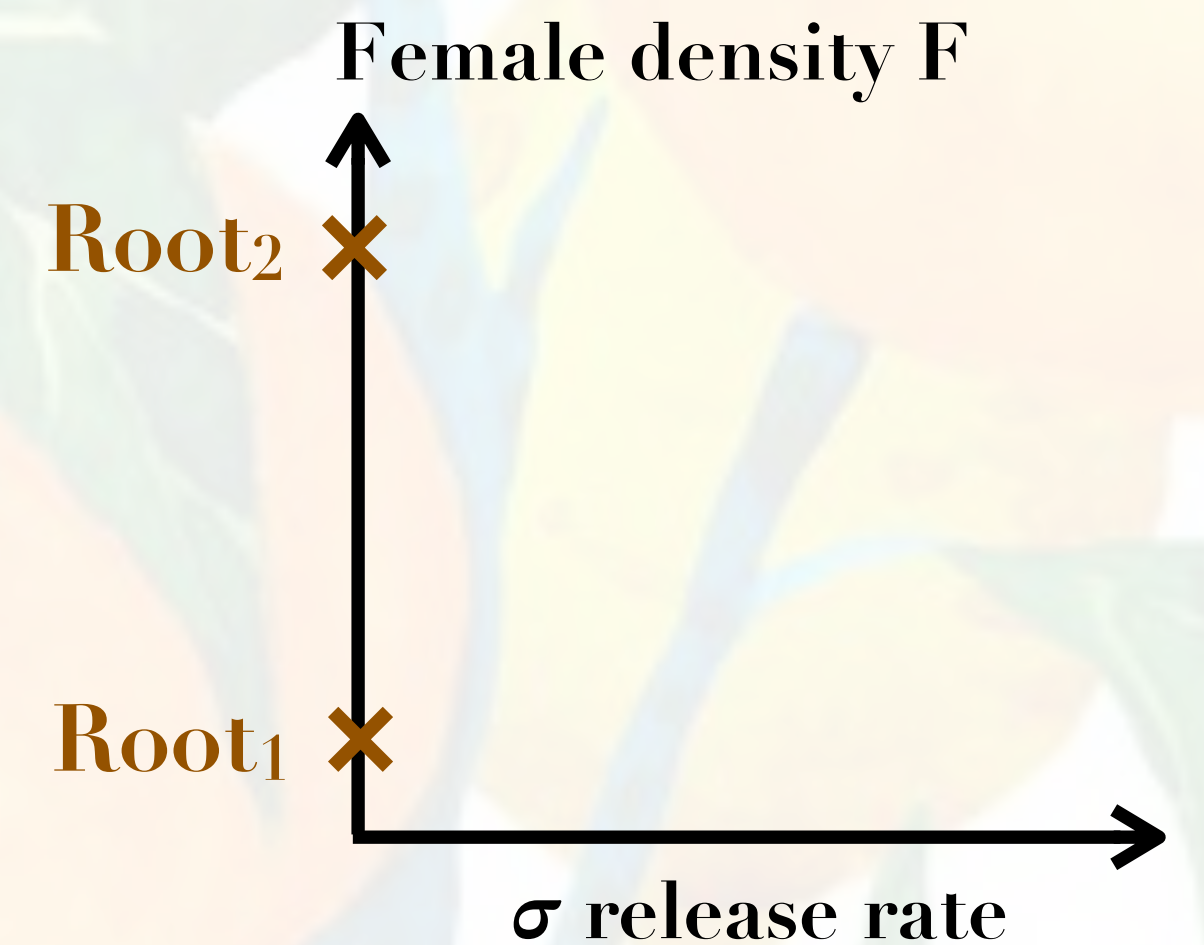


Sum of the roots > 0 and therefore the 2 roots **Root₁** and **Root₂** are > 0 if: $\frac{p\mu_F(R-1)}{\beta k(1-p)\mu_M} > 1$

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Existence of an **asymptote** when the denominator cancels

$$\sigma = \frac{(1 + \beta F)(k + \frac{p\mu_F}{(1-p)\mu_M} F) - \frac{Rp\mu_F}{(1-p)\mu_M} F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F)\frac{\delta}{\mu_M} - (1 + \beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

Existence of an asymptote when the denominator cancels

General case

$\delta \neq 0, \epsilon \neq 0$

The denominator cancels for:

$$F = \frac{1}{\beta} \left(\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} - 1 \right)$$

But $F > 0$ so existence when:

$$\frac{R\delta\mu_S + \mu_M R\epsilon\eta(1-\delta)}{\delta\mu_S + \mu_M\eta(1-\delta)} > 1$$

$$\sigma = \frac{(1 + \beta F)(k + \frac{p\mu_F}{(1-p)\mu_M} F) - \frac{Rp\mu_F}{(1-p)\mu_M} F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F)\frac{\delta}{\mu_M} - (1 + \beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

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(1)

Cost-free fertility model

$$\delta \neq 0, \epsilon = 0$$

The denominator cancels for:

$$F = \frac{1}{\beta} \left(\frac{R\delta\mu_S}{\delta\mu_S + \eta(1-\delta)\mu_M} - 1 \right)$$

But $F > 0$ so existence when:

$$\delta > \frac{\eta\mu_M}{R\mu_S - \mu_S + \eta\mu_M}$$

$$\sigma = \frac{(1 + \beta F)(k + \frac{p\mu_F}{(1-p)\mu_M} F) - \frac{Rp\mu_F}{(1-p)\mu_M} F}{\frac{R\delta}{\mu_M} + \frac{R\epsilon\eta(1-\delta)}{\mu_S} - (1 + \beta F)\frac{\delta}{\mu_M} - (1 + \beta F)(\eta\frac{(1-\delta)}{\mu_S})}$$

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(2)

Costly fertility model

$$\delta = 0, \epsilon \neq 0$$

The denominator cancels for:

$$F = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

$$\epsilon > \frac{1}{R}$$

(2)

Costly fertility
model

$$\delta = 0, \epsilon \neq 0$$

The denominator cancels for:

$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

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$$\epsilon > 0.084$$

(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

The denominator cancels for:

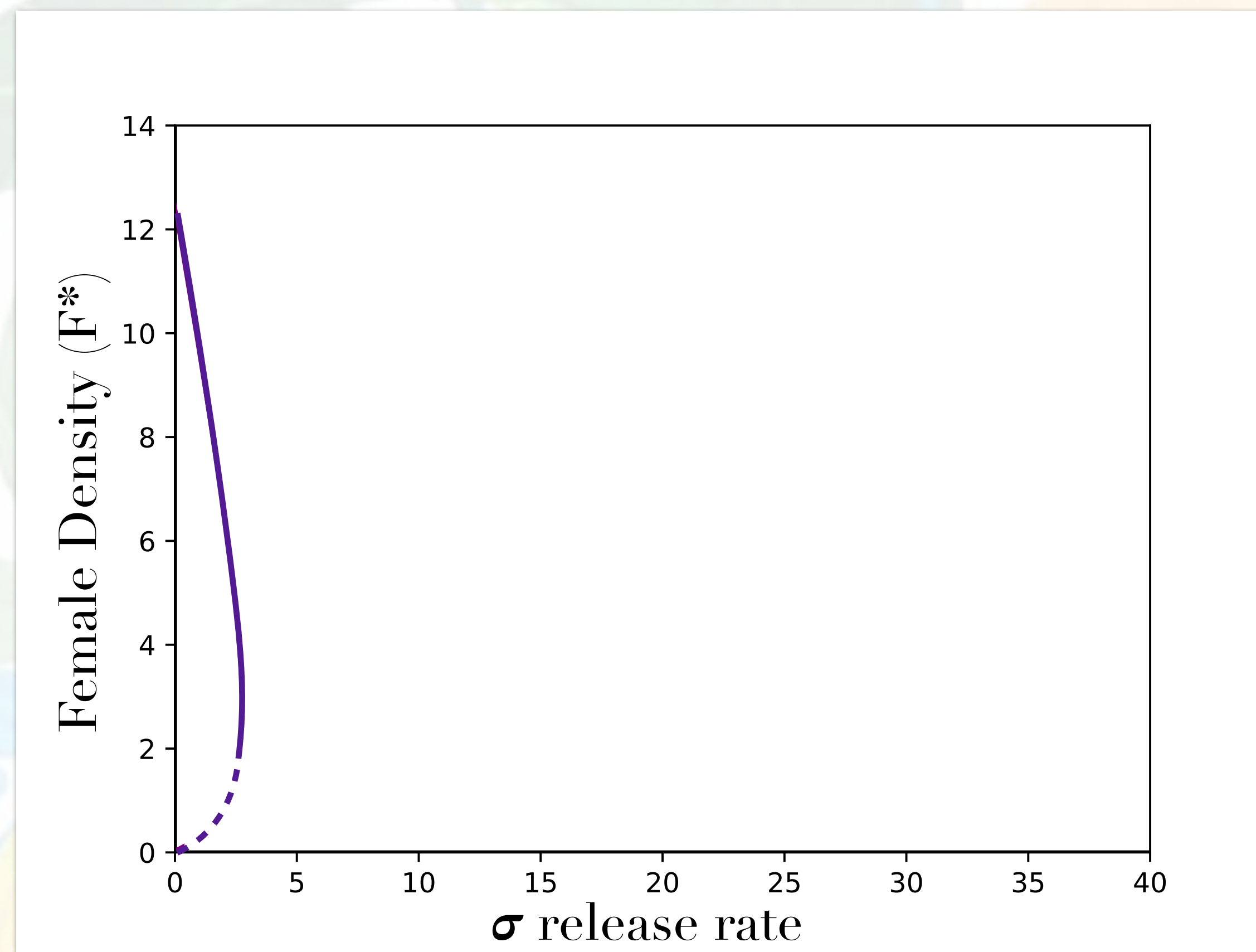
$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

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$$\epsilon > 0.084$$

- Stable
- - - Unstable



(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

The denominator cancels for:

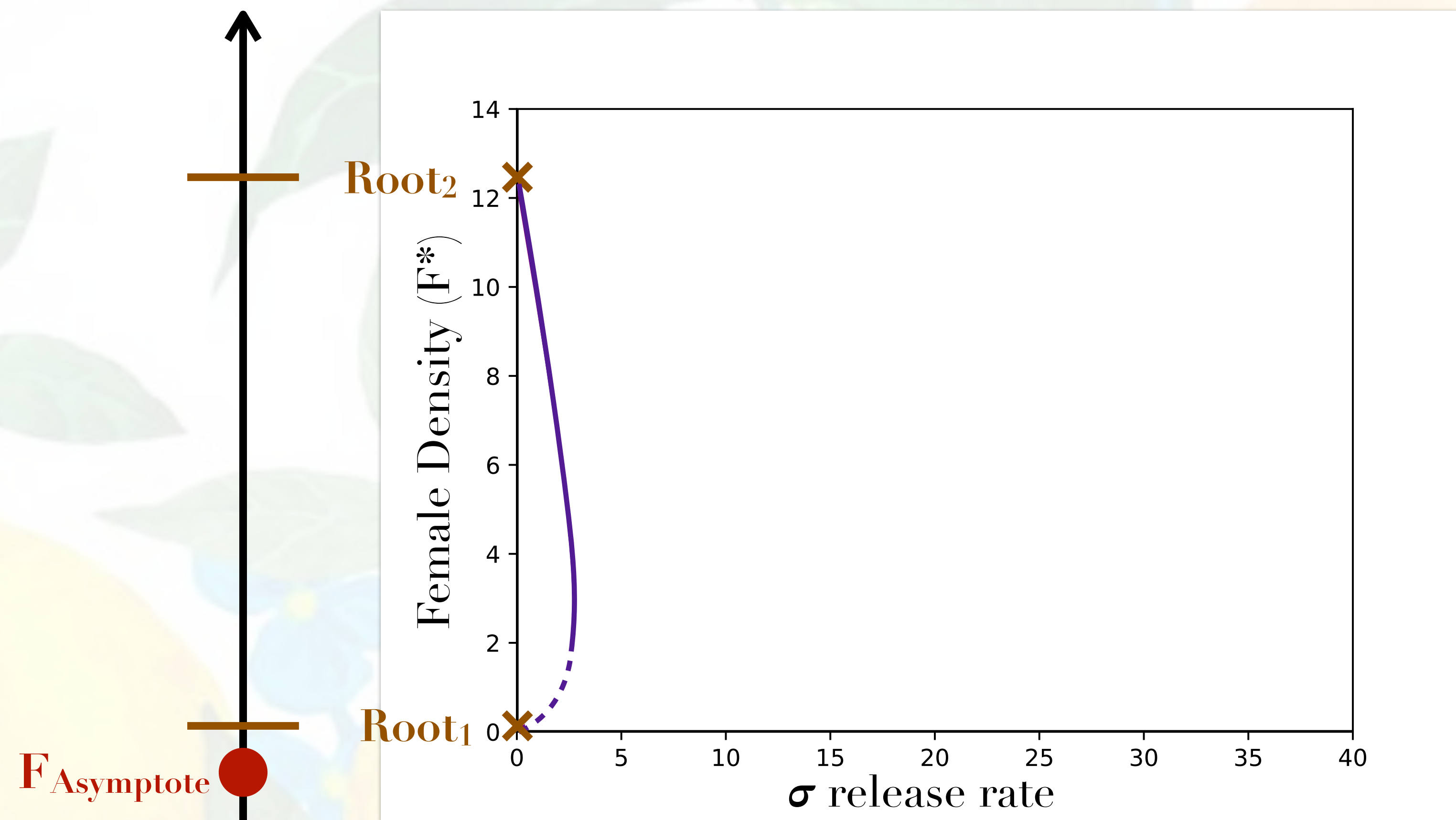
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- - - Unstable



(2)
 Costly fertility
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 $\delta = 0, \epsilon \neq 0$

The denominator cancels for:

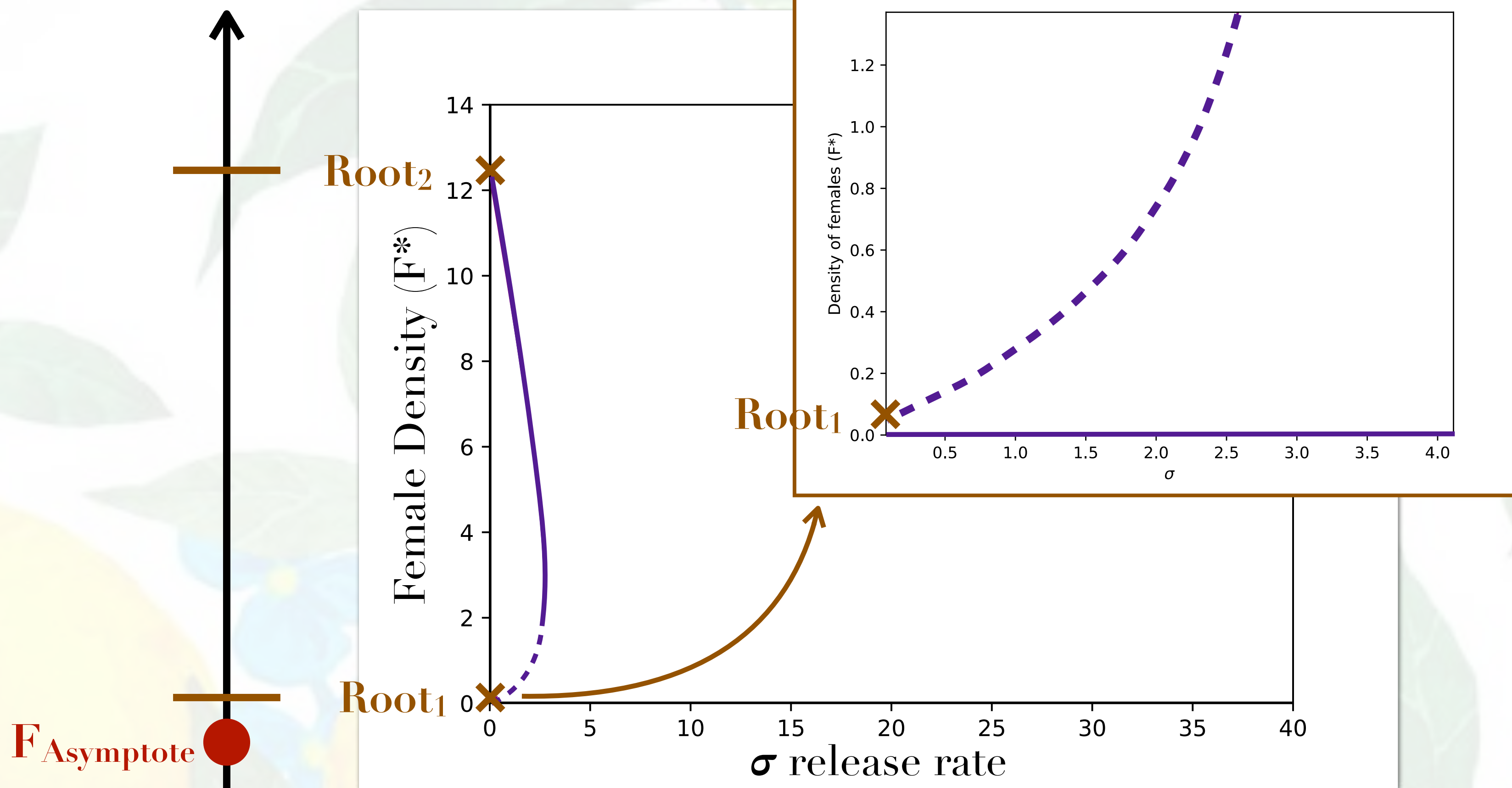
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— Stable
 - - - Unstable



(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

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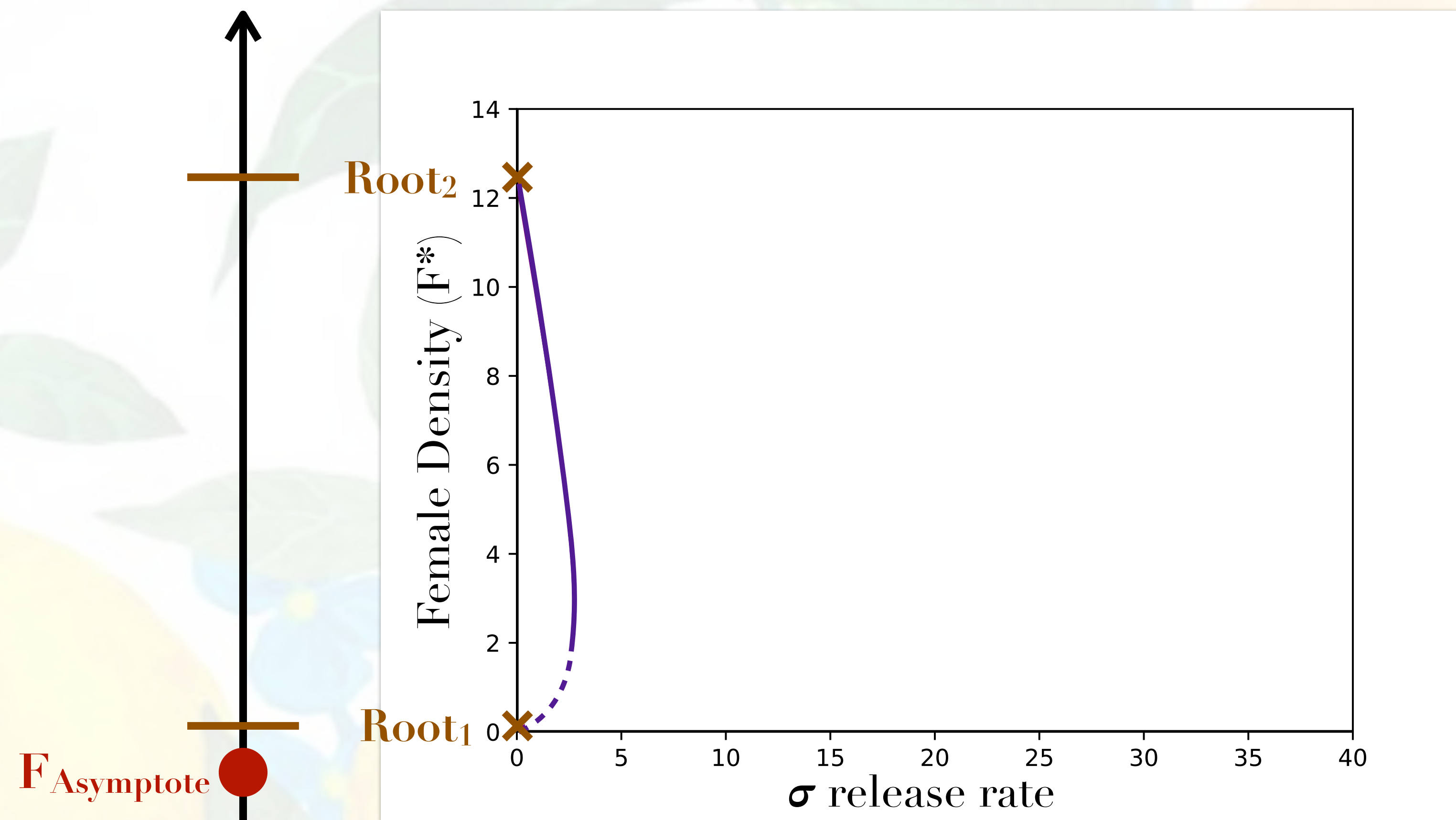
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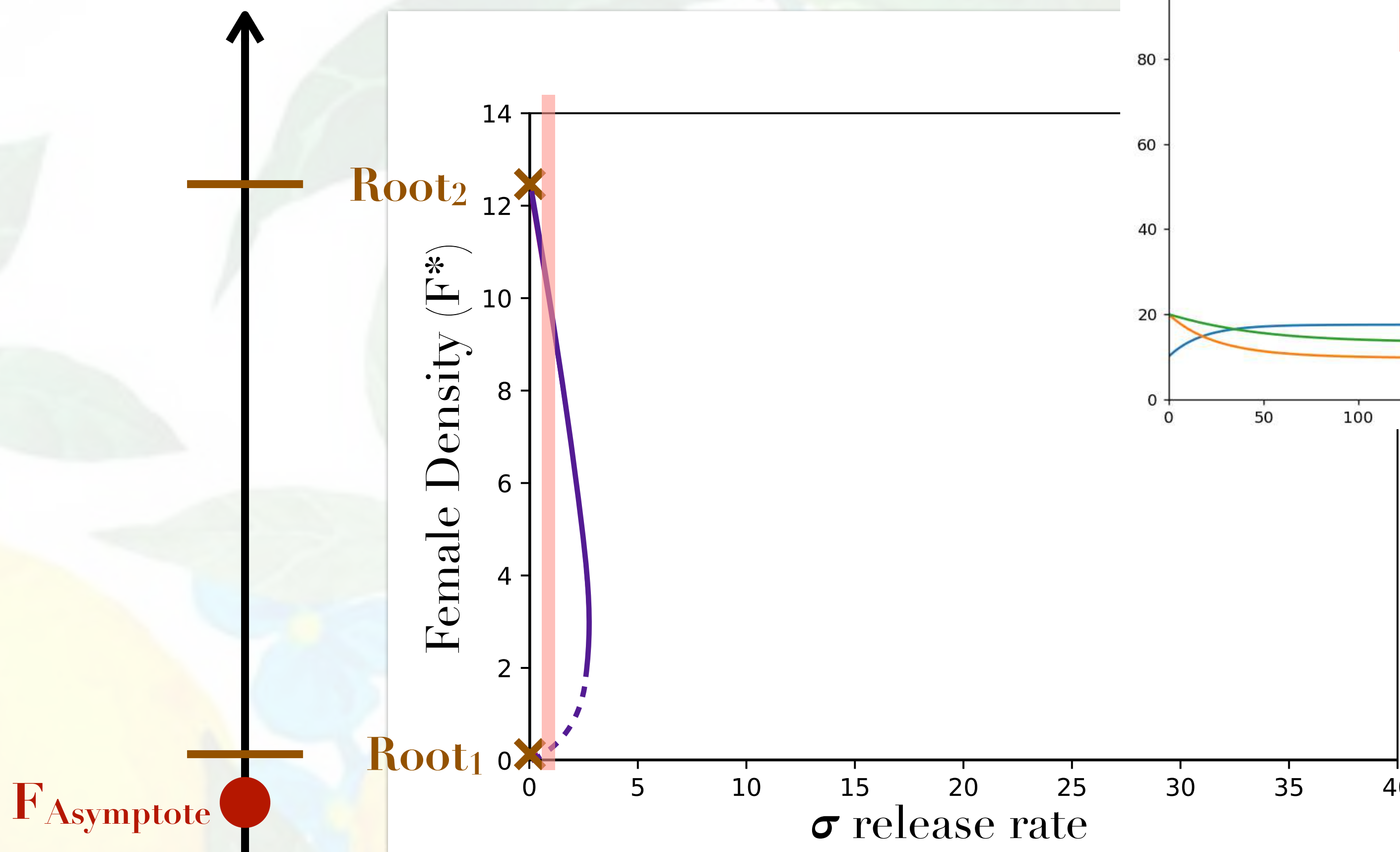
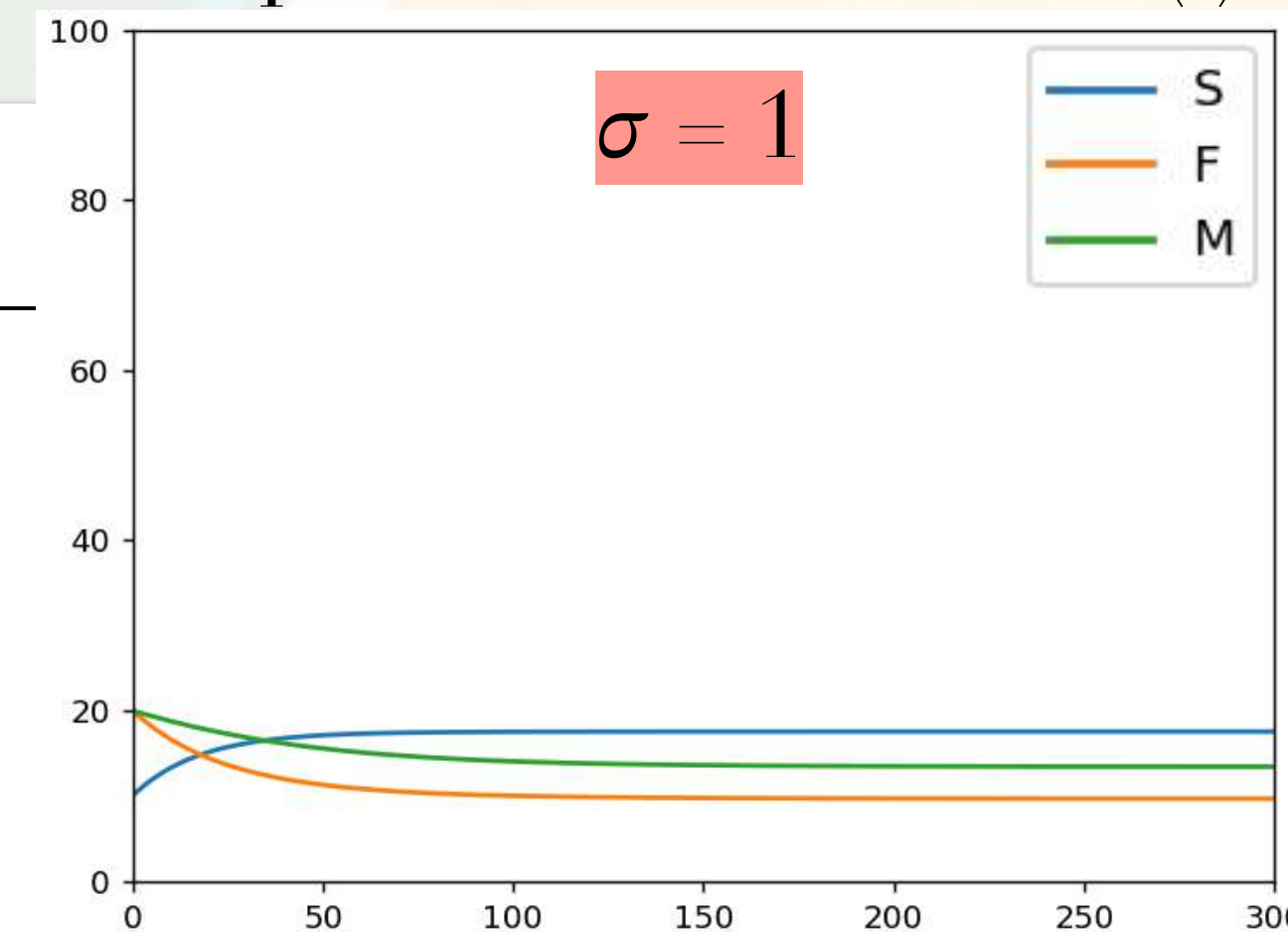
$$\epsilon > \frac{1}{R}$$

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— Stable
 - - - Unstable



Population Densities = $f(t)$



(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

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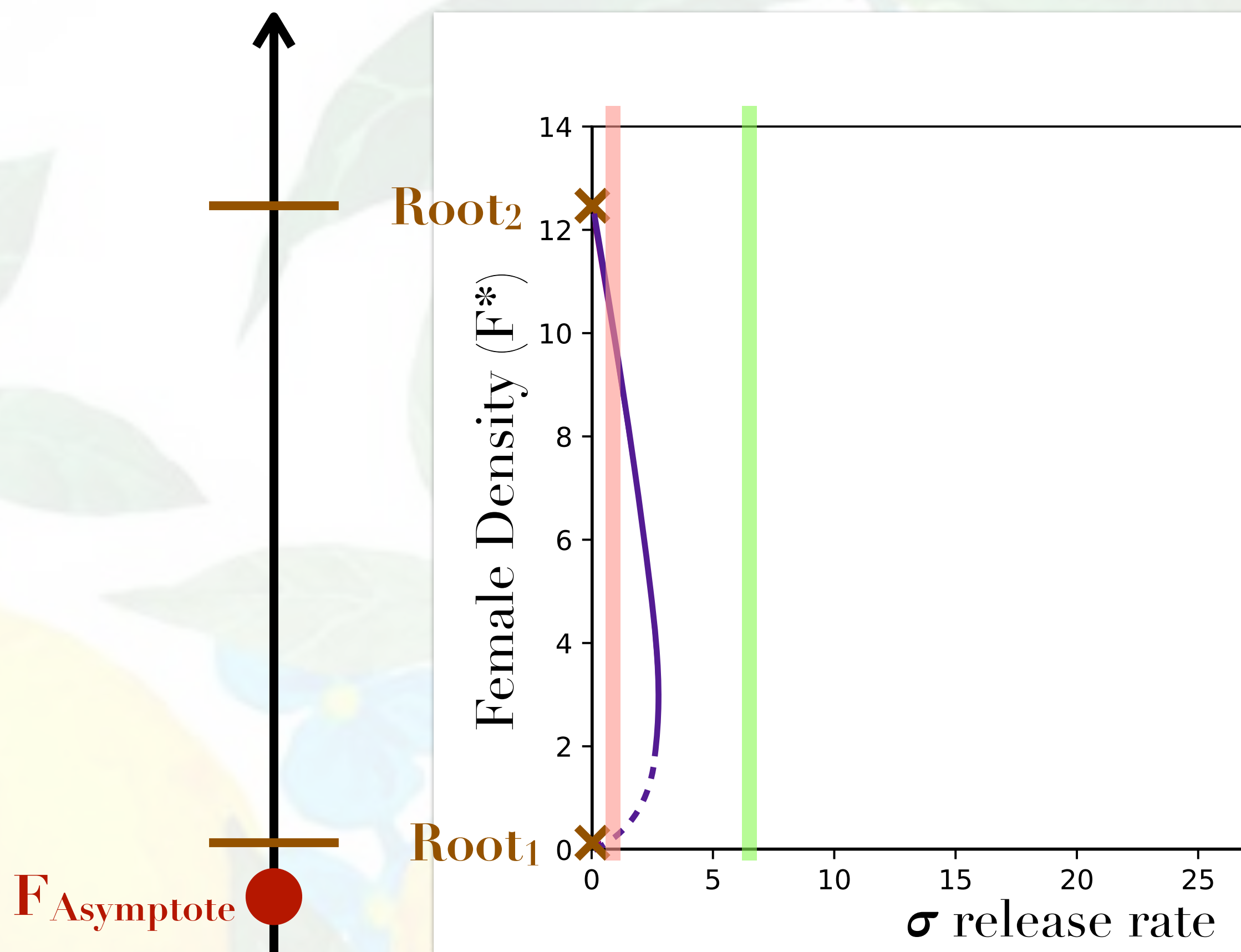
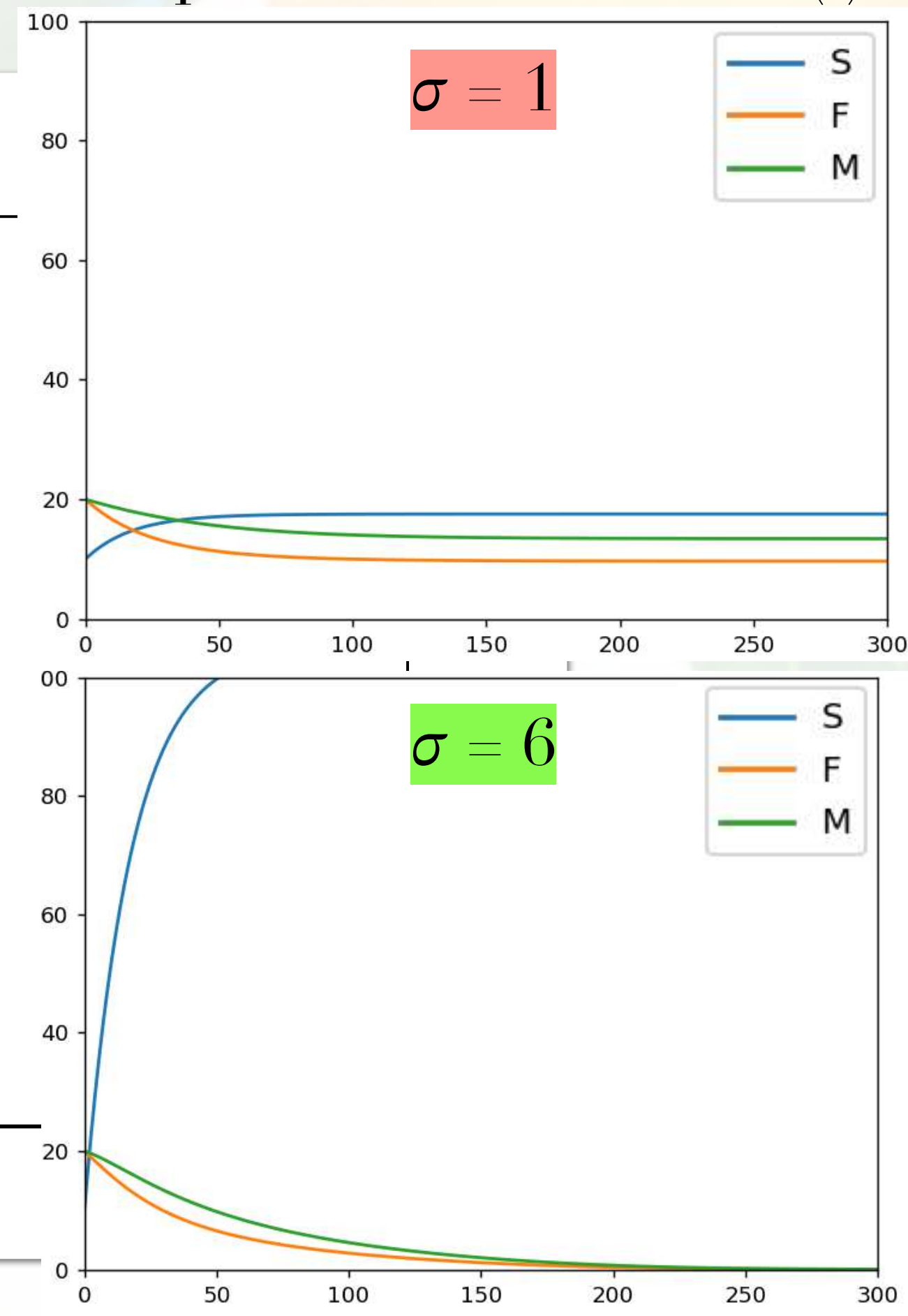
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— Stable
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(2)
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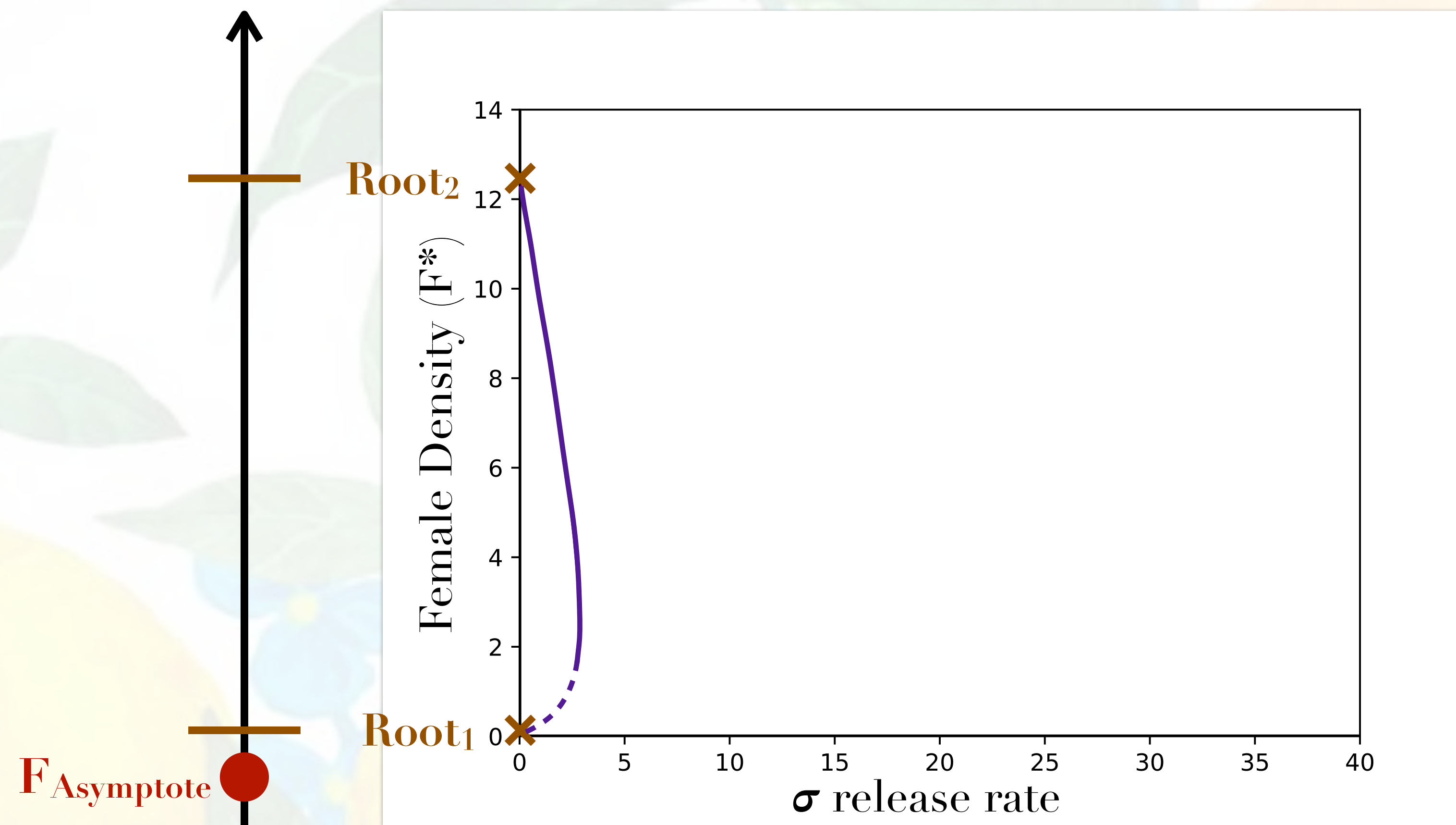
$$\epsilon > 0.084$$

- Stable
- - - Unstable

0.020

0.084

Residual
Fertility rate ϵ



(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

The denominator cancels for:

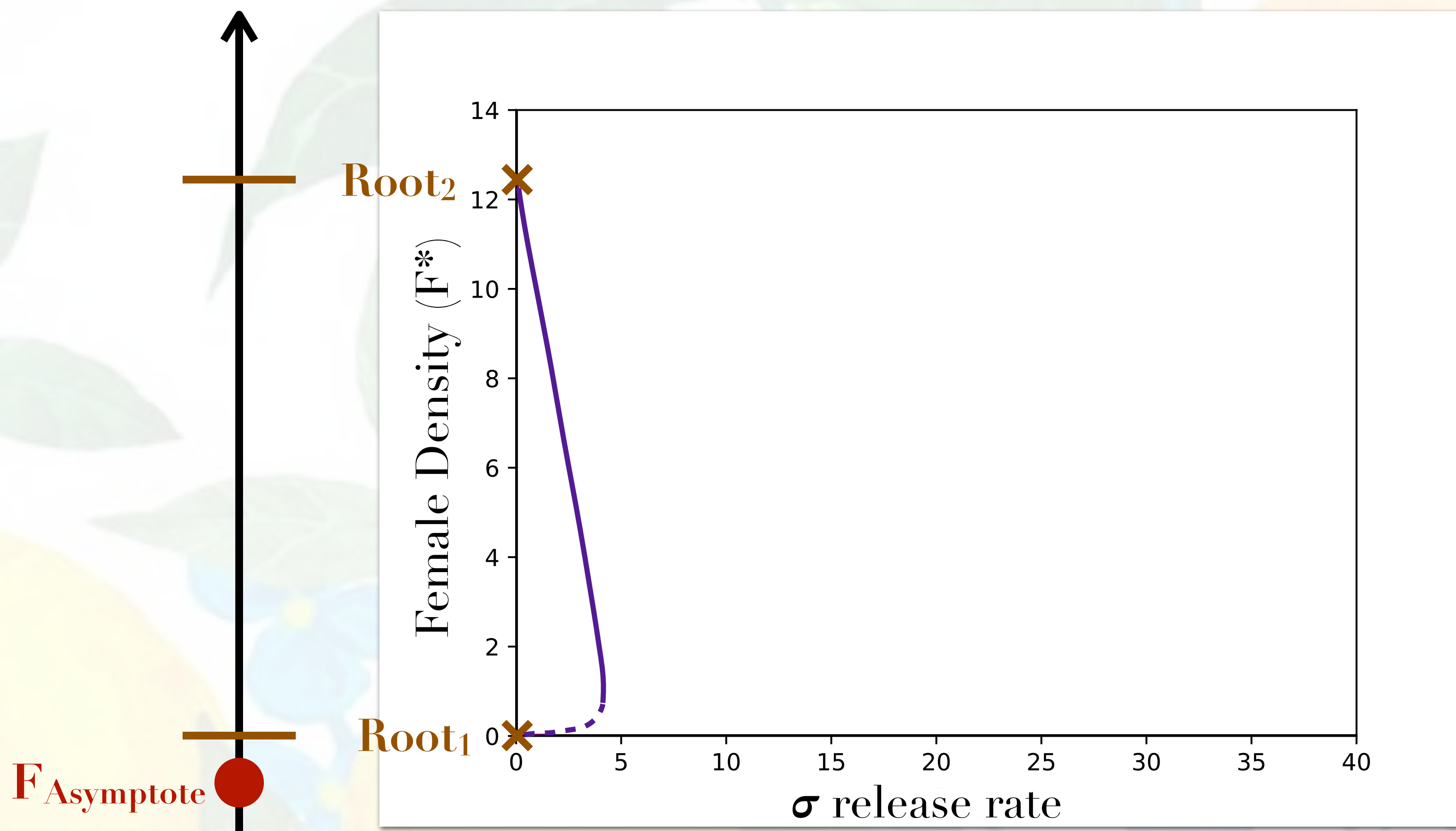
$$F_{\text{Asymptote}} = \frac{R\epsilon - 1}{\beta}$$

But $F > 0$ so existence when:

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- Stable
- - - Unstable



(2)
 Costly fertility
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 $\delta = 0, \epsilon \neq 0$

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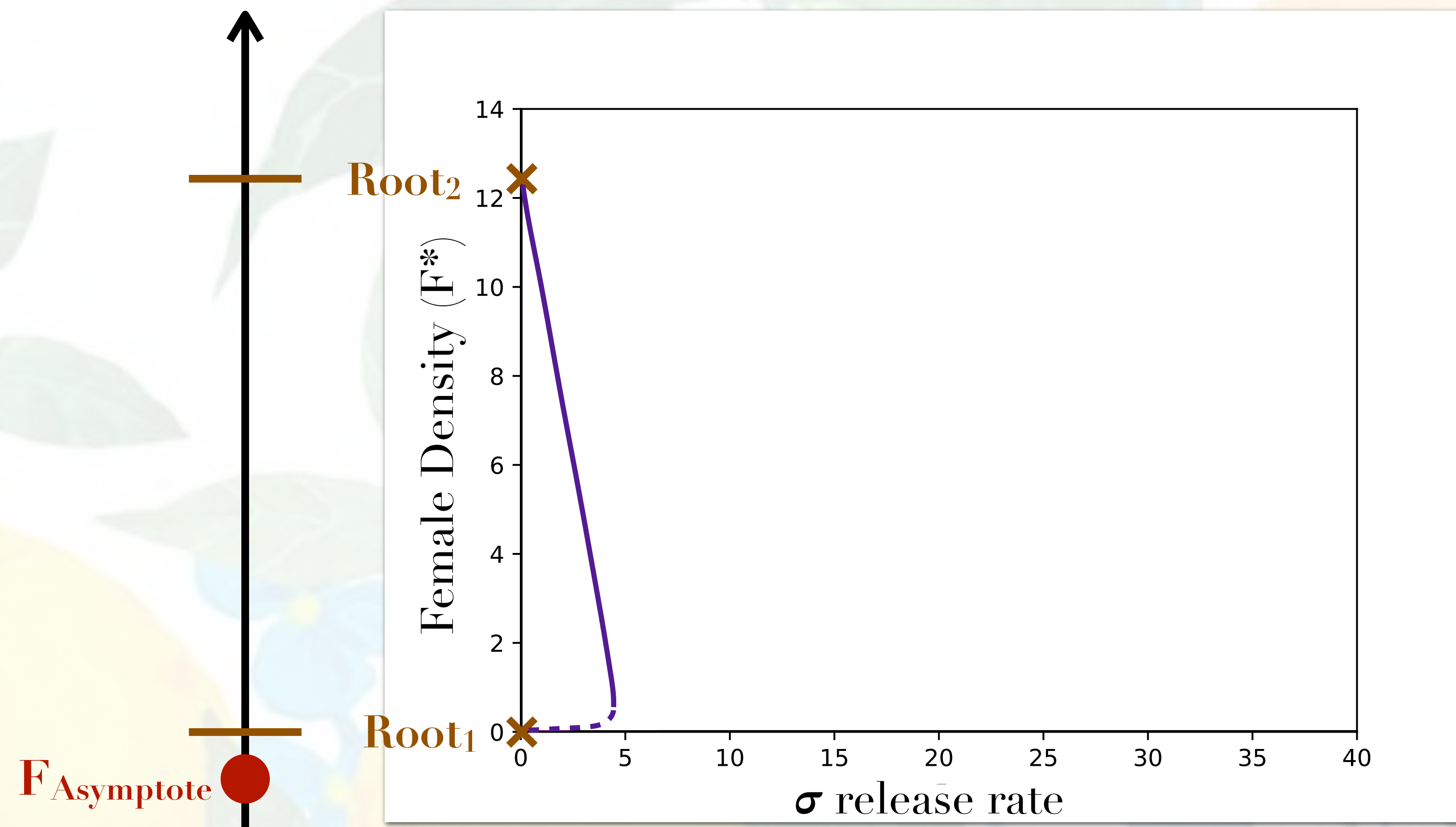
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 Costly fertility
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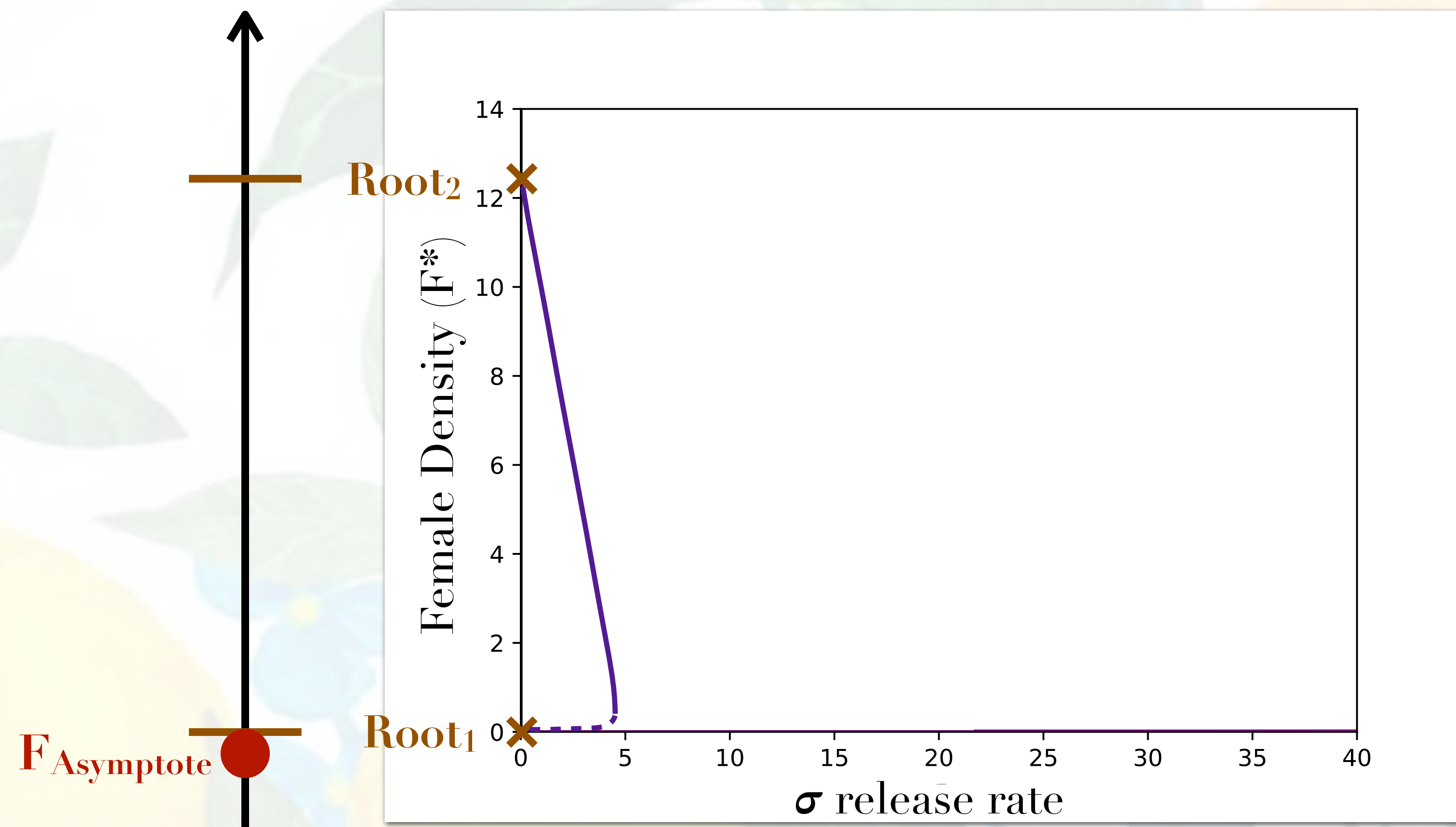
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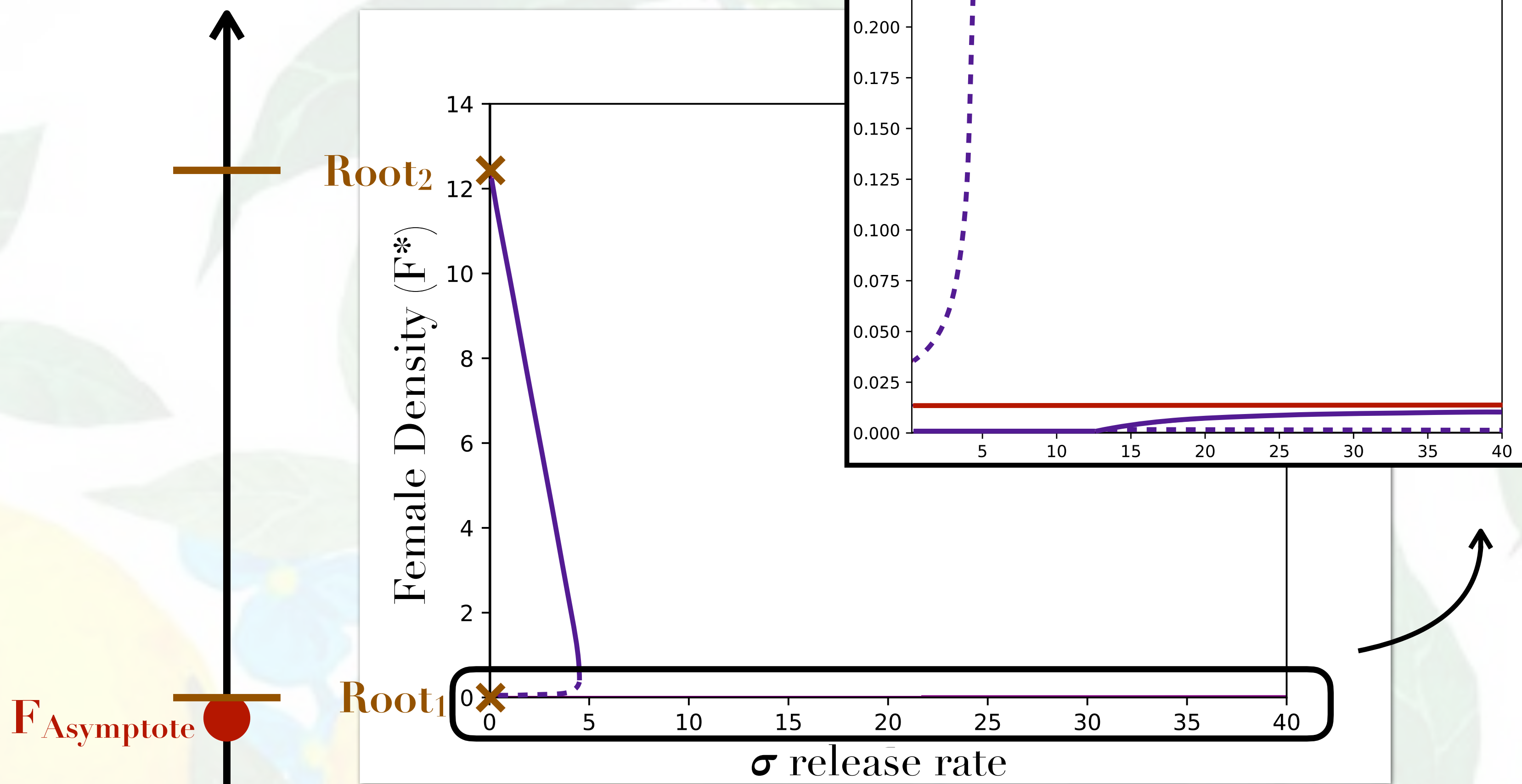
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 Costly fertility
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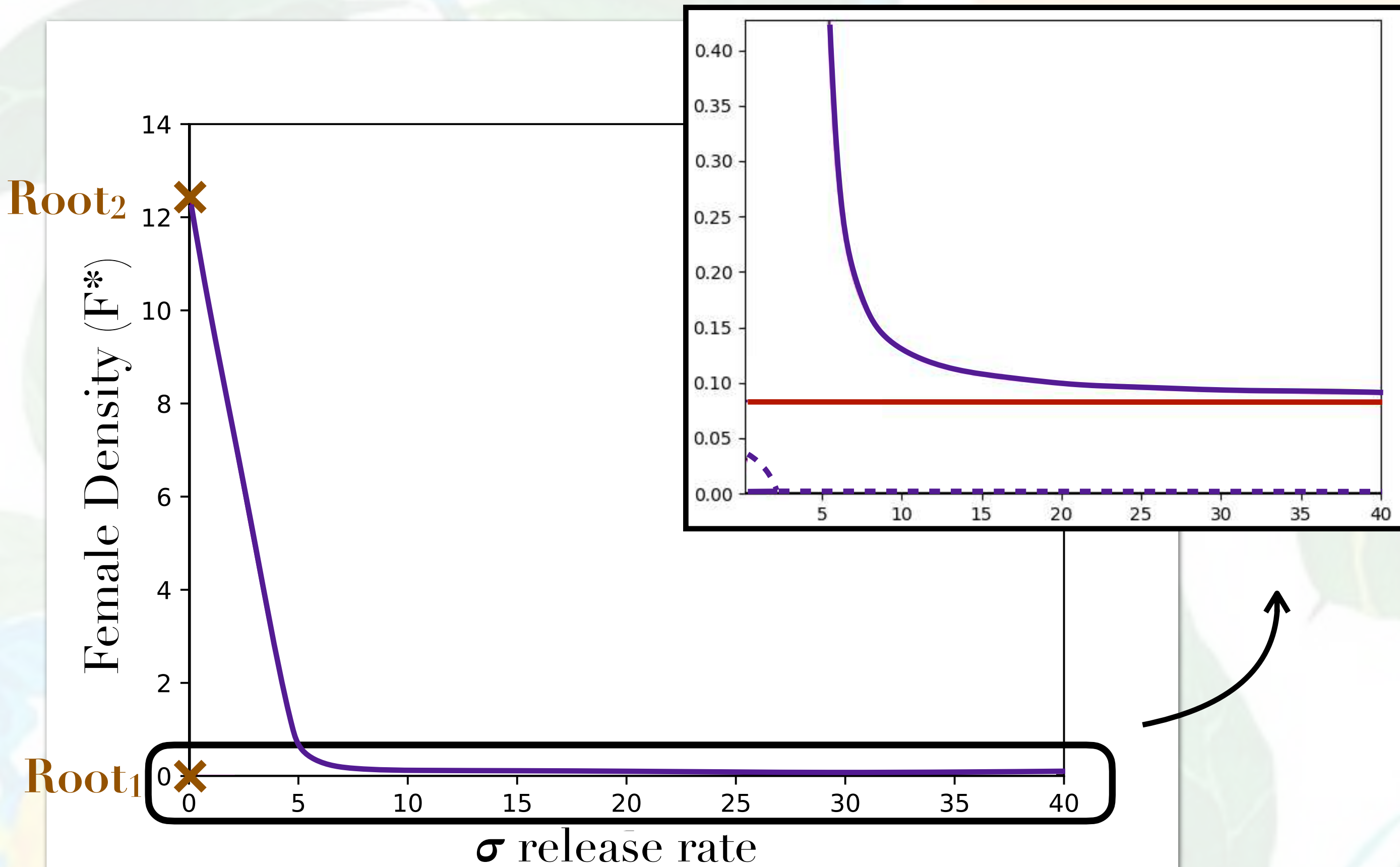
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- - - Unstable



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 Costly fertility
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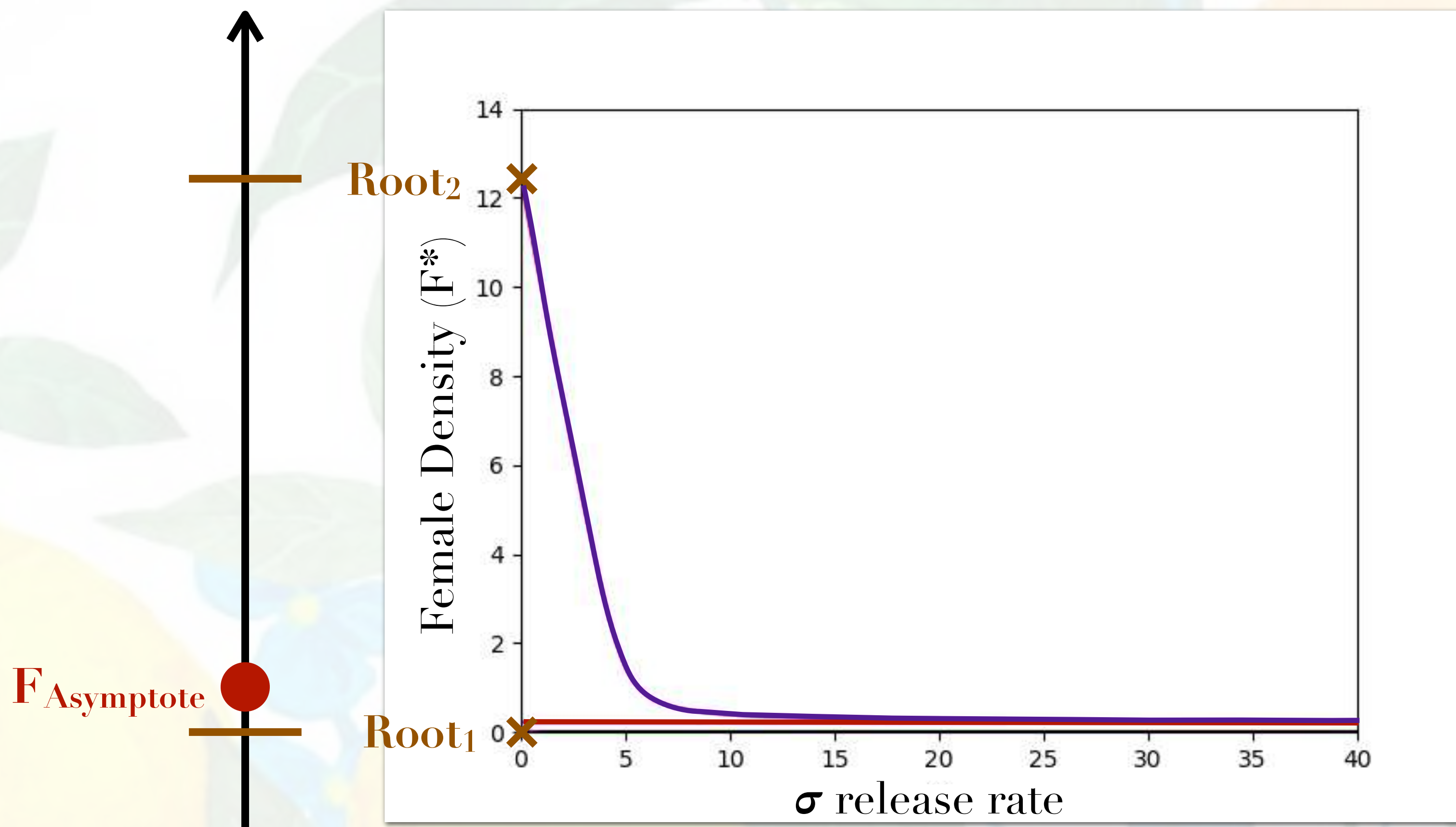
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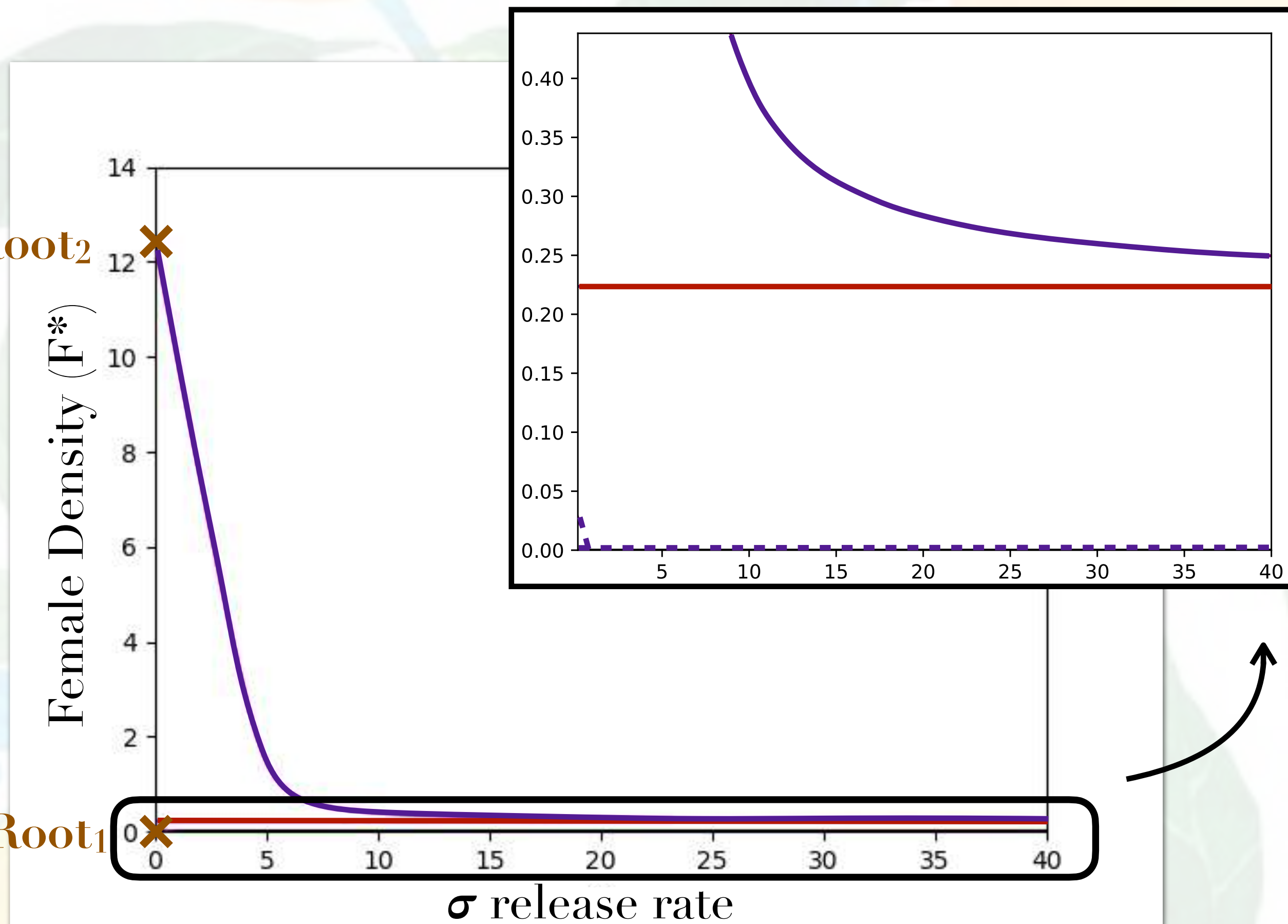
$$\epsilon > 0.084$$

- Stable
- - - Unstable



Root₂

Root₁



(2)
 Costly fertility
 model
 $\delta = 0, \epsilon \neq 0$

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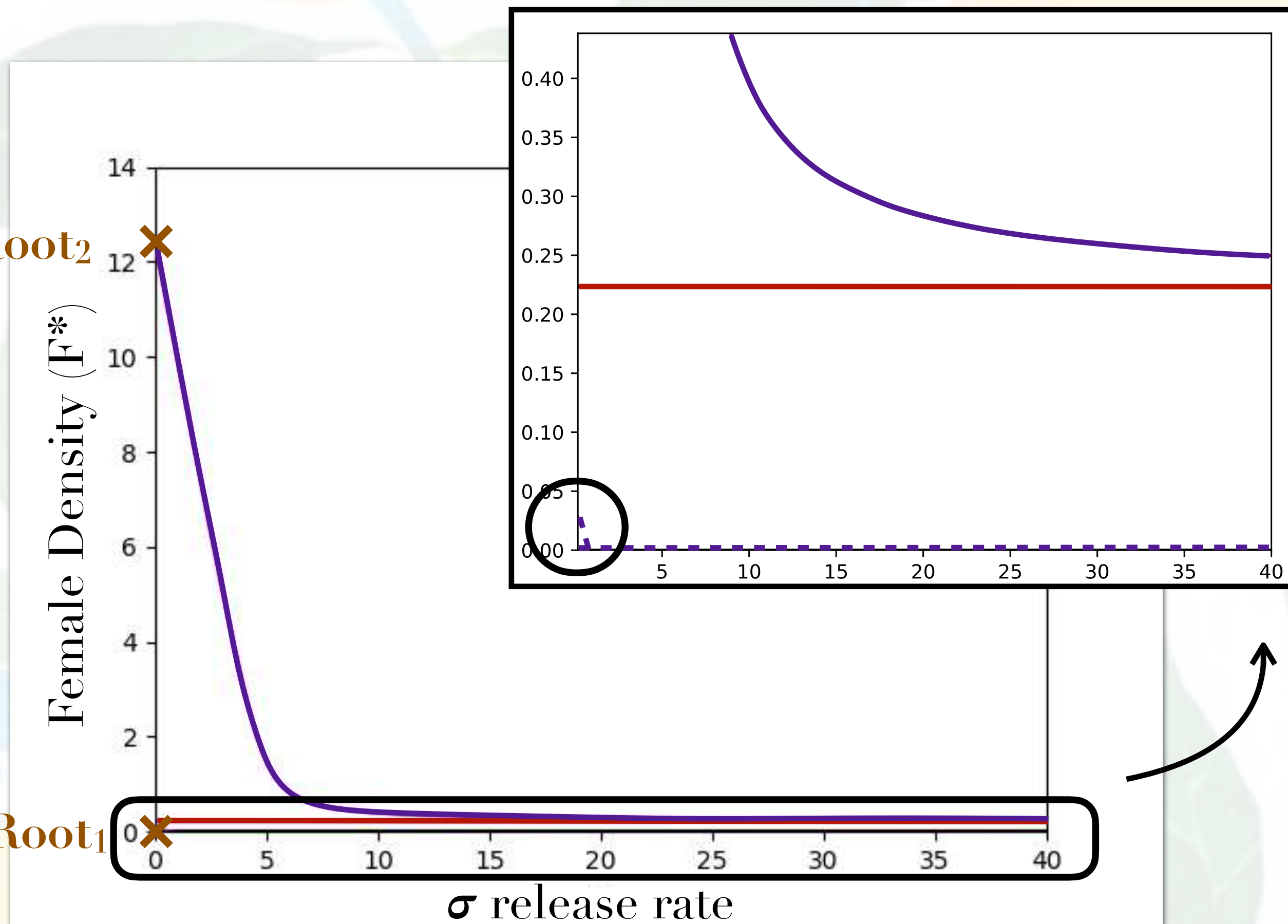
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- Stable
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Root₂

Root₁



(2)
 Costly fertility
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 $\delta = 0, \epsilon \neq 0$

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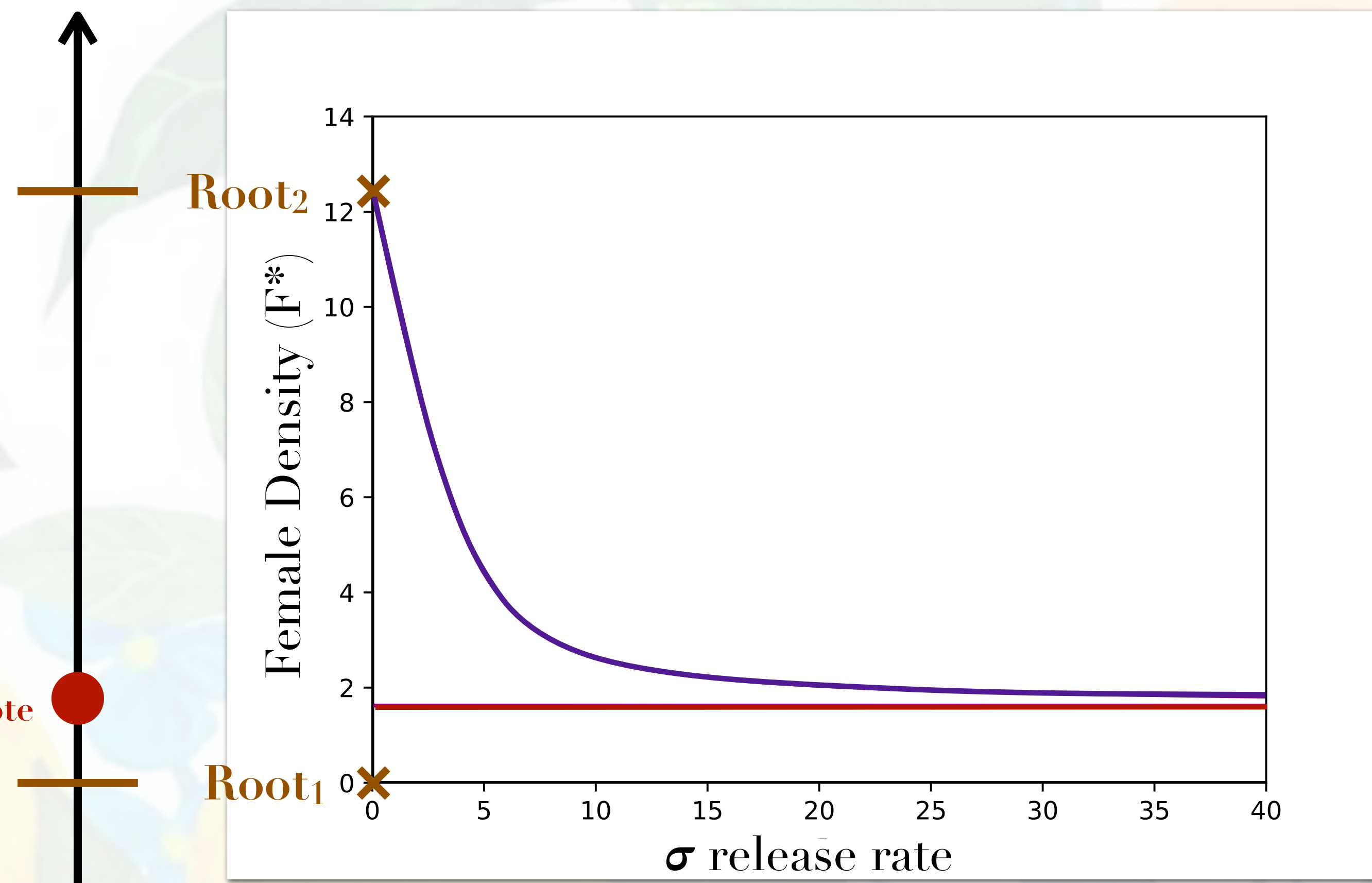
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 Costly fertility
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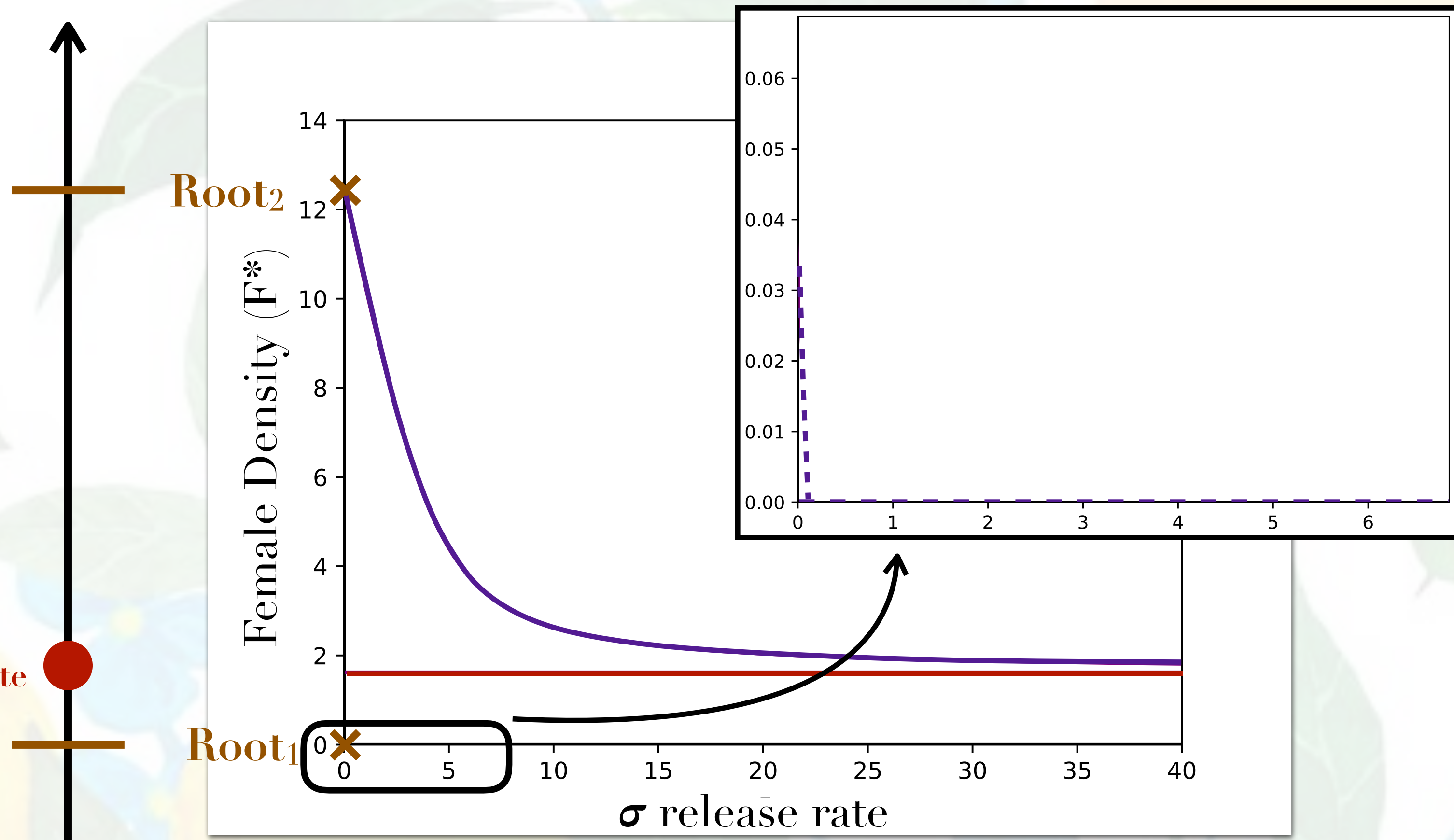
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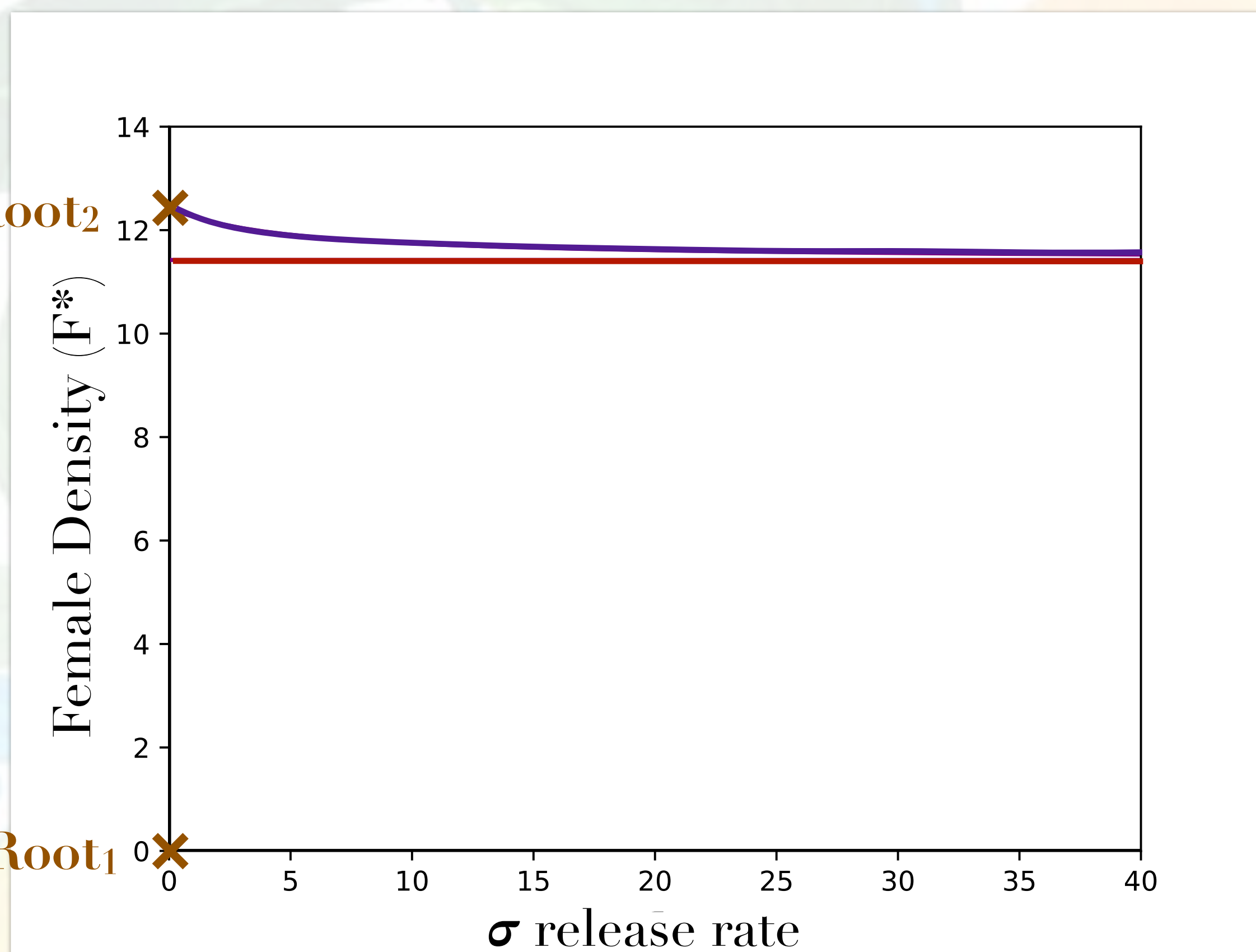
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(2)
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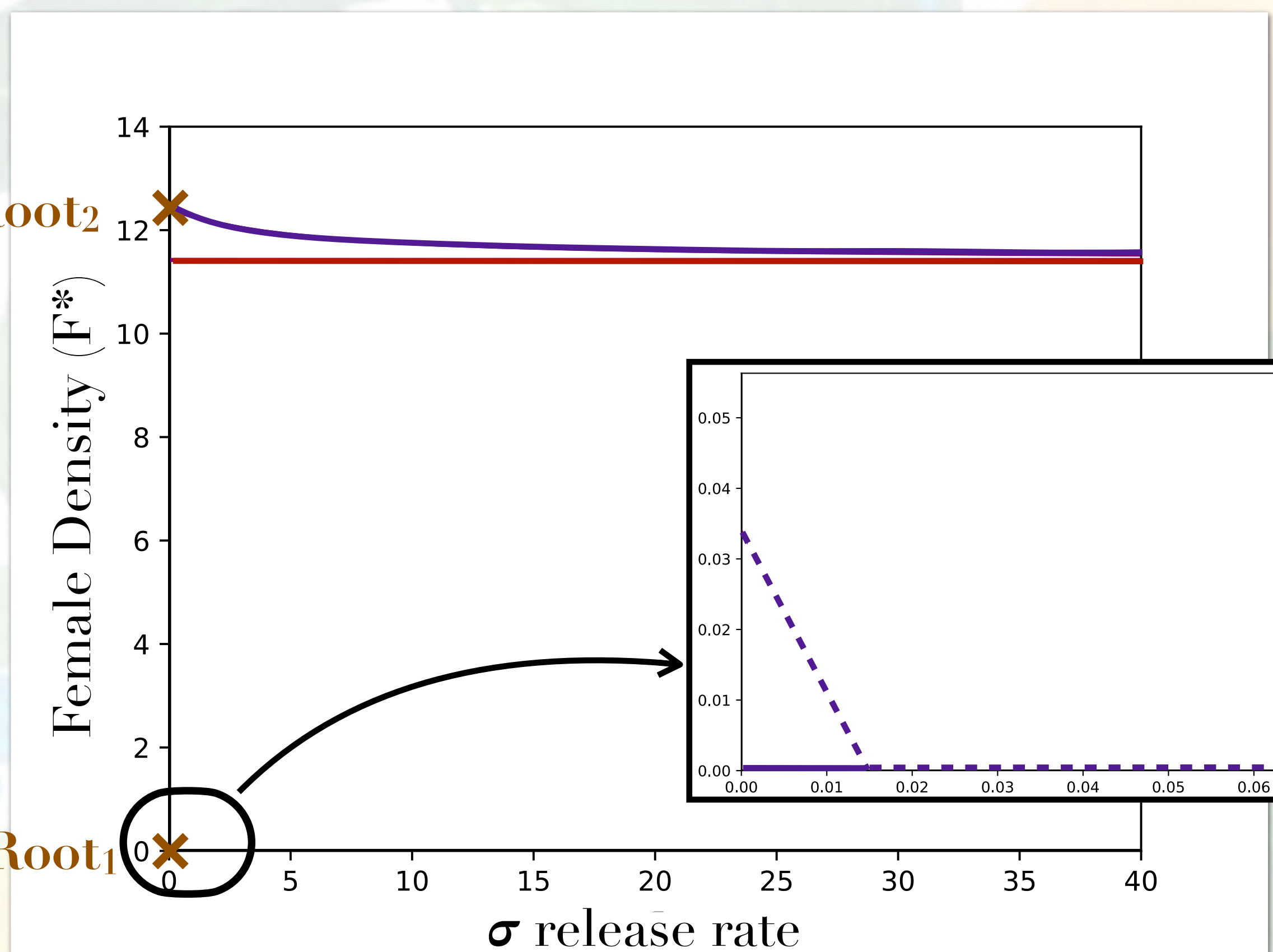
$$\epsilon > 0.084$$

- Stable
- - - Unstable



Root₂

Root₁



(2)
 Costly fertility
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 $\delta = 0, \epsilon \neq 0$

The denominator cancels for:

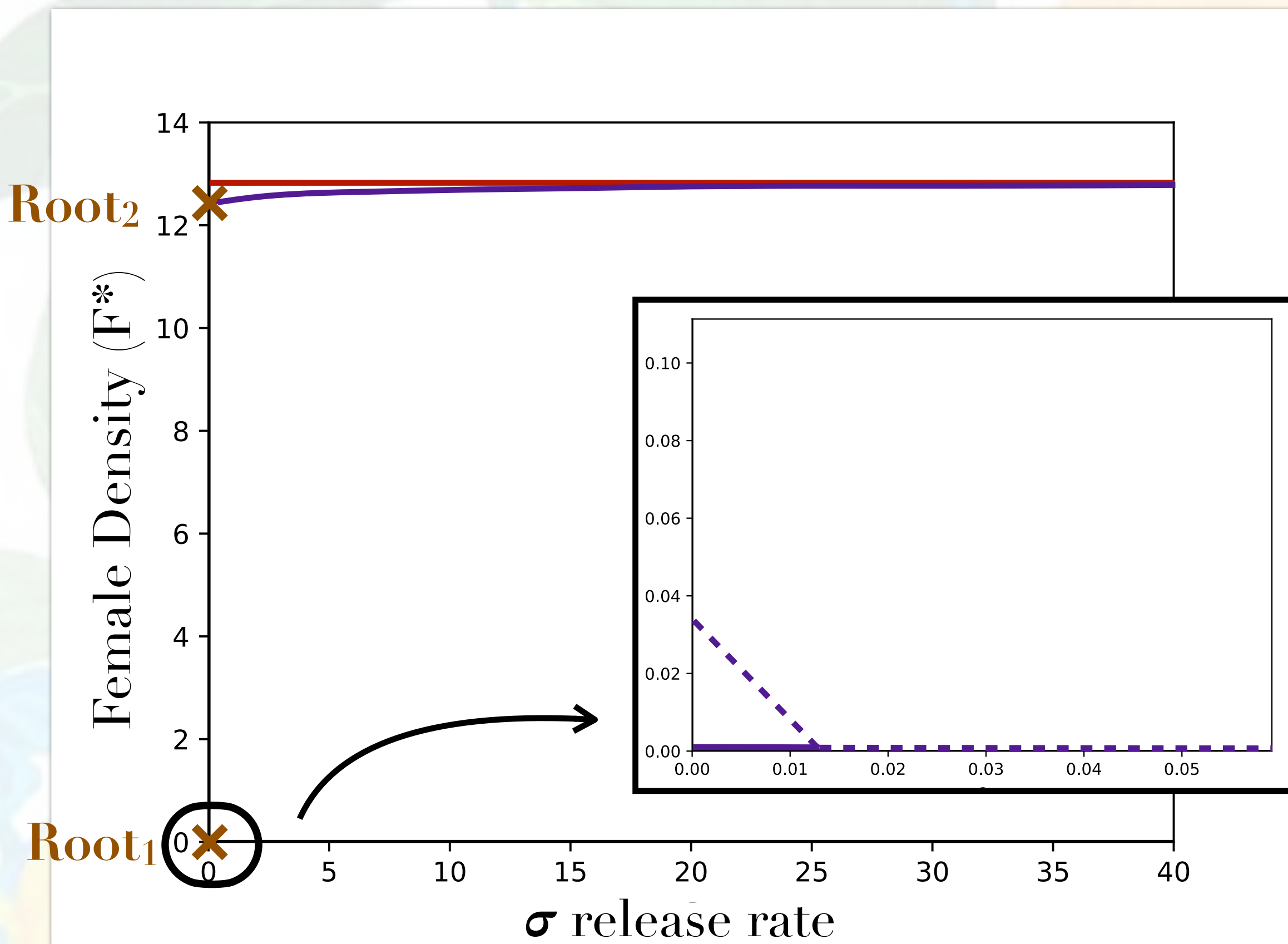
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- Stable
- - - Unstable



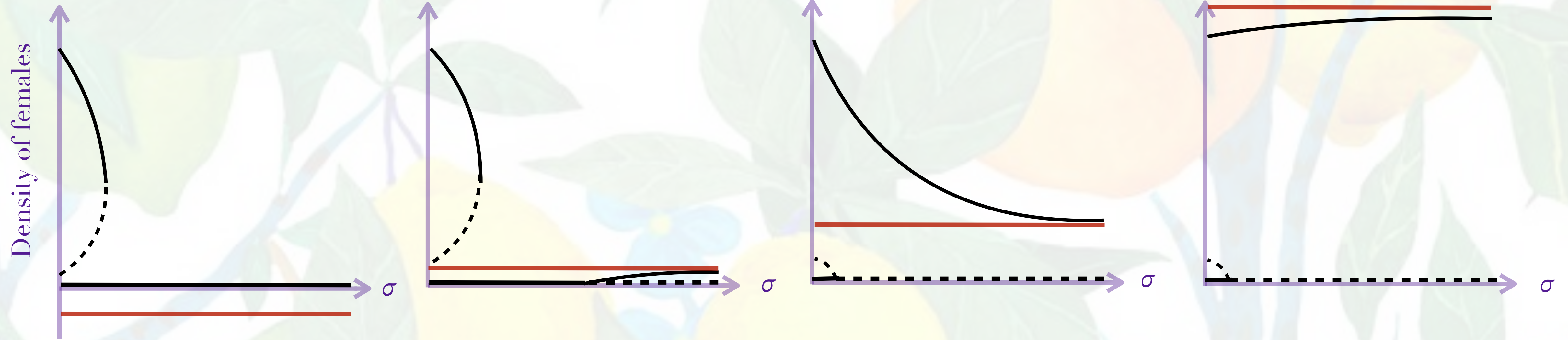
(2)

Costly fertility
model

$\delta = 0, \epsilon \neq 0$



— Stable
- - - Unstable



(2)

Costly fertility
model

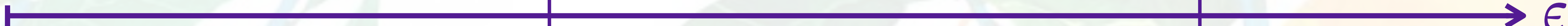
$\delta = 0, \epsilon \neq 0$

1

1'

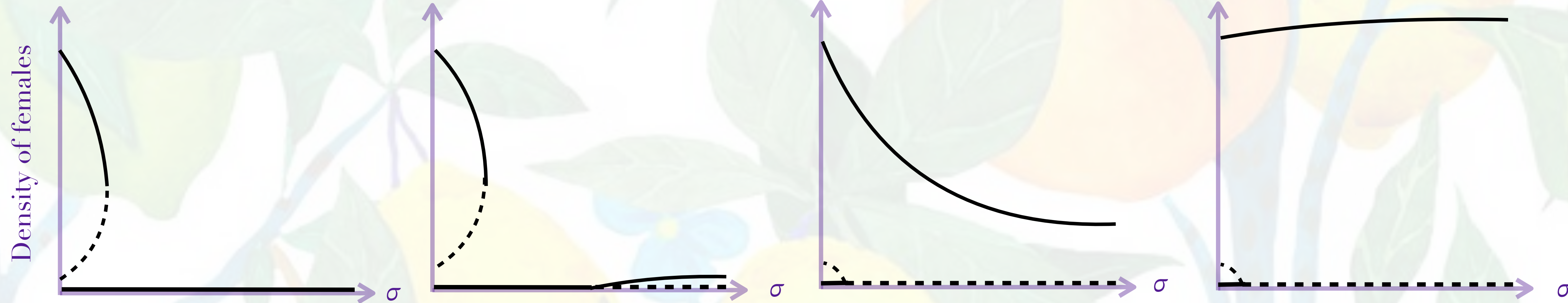
2

3



— Stable

- - - Unstable

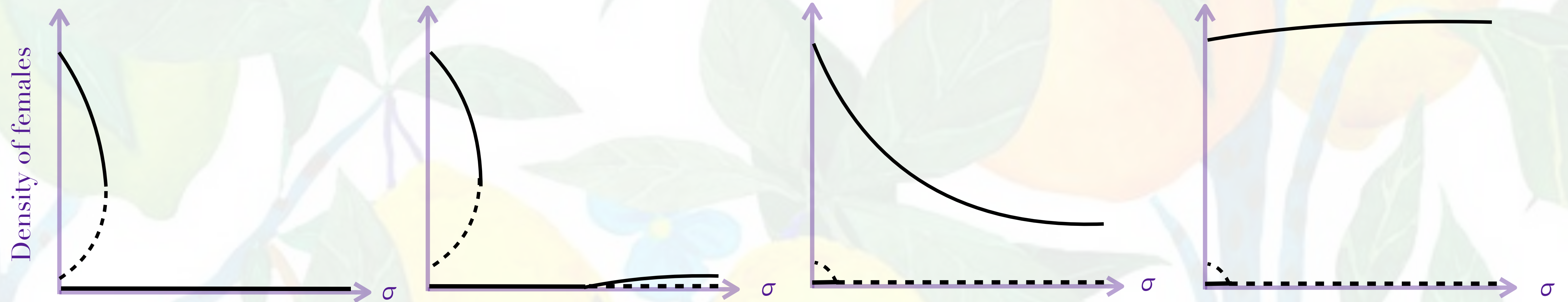


(2)

Costly fertility model
 $\delta = 0, \epsilon \neq 0$



— Stable
- - - Unstable



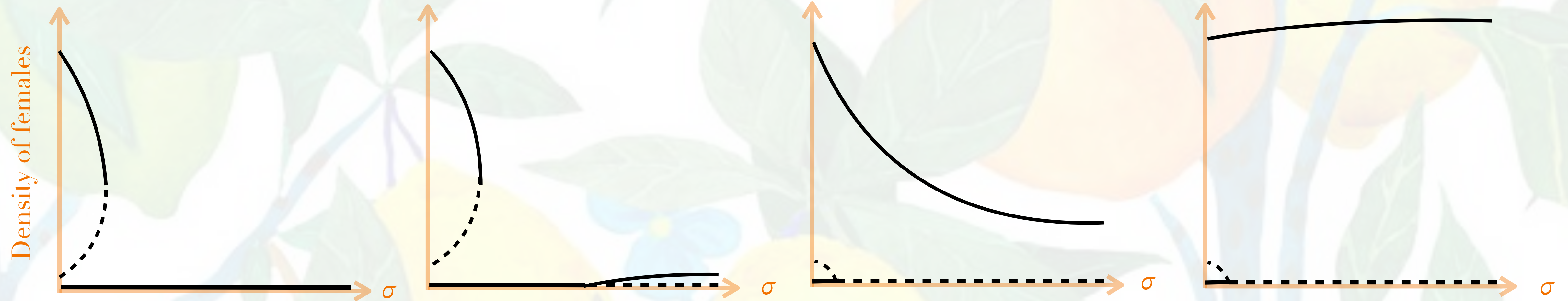
➡ Different control capabilities depending on the shape of the bifurcation diagram

(1)

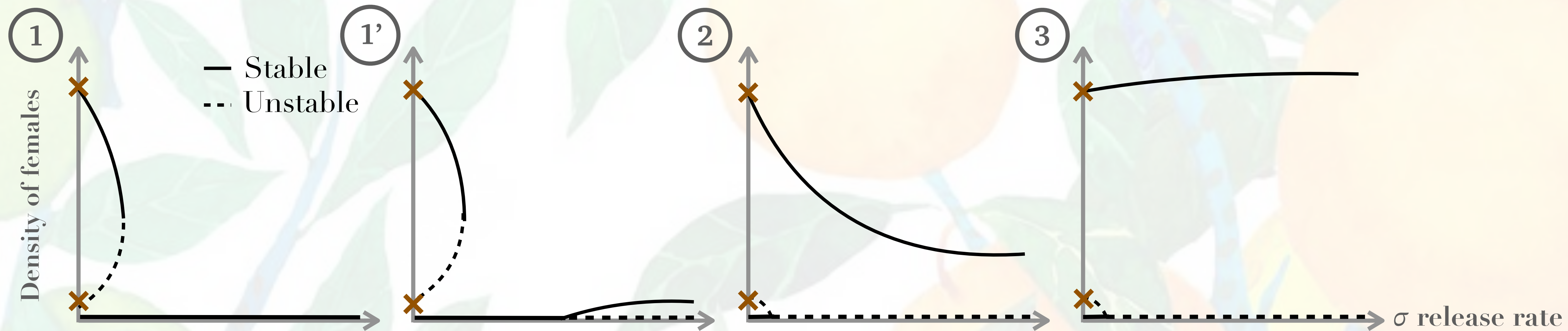
Cost-free
fertility model
 $\delta \neq 0, \epsilon = 0$

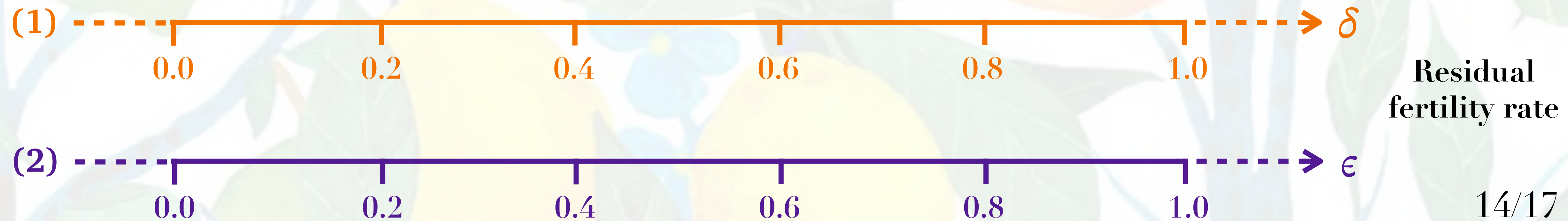
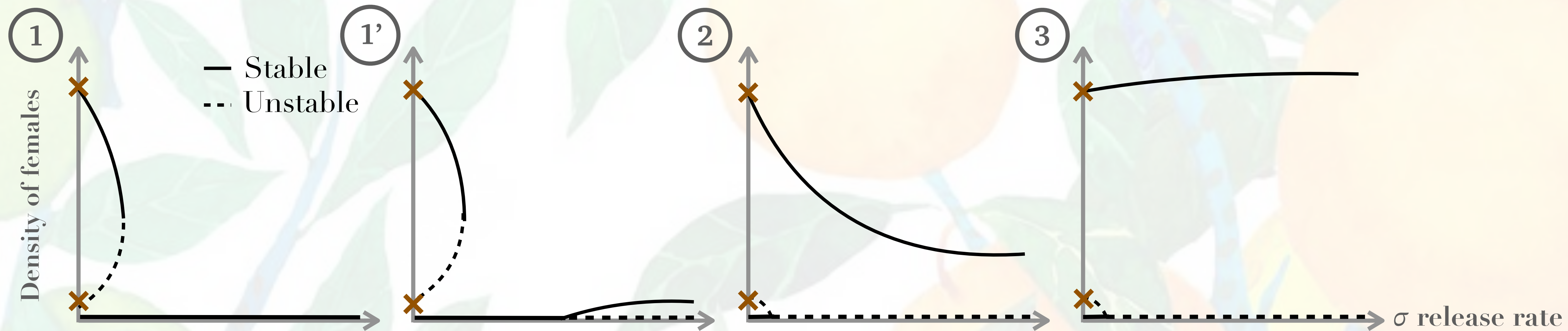


— Stable
- - - Unstable



➔ Different control capabilities depending on the shape of the bifurcation diagram





Introduction

Model

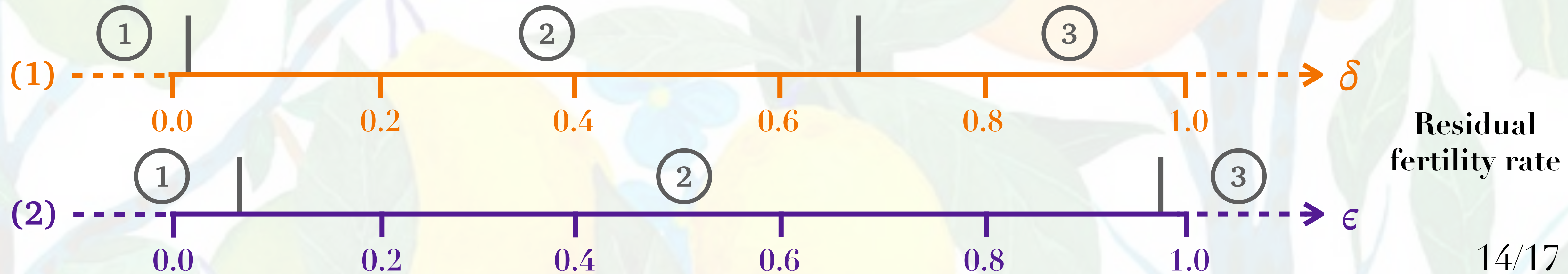
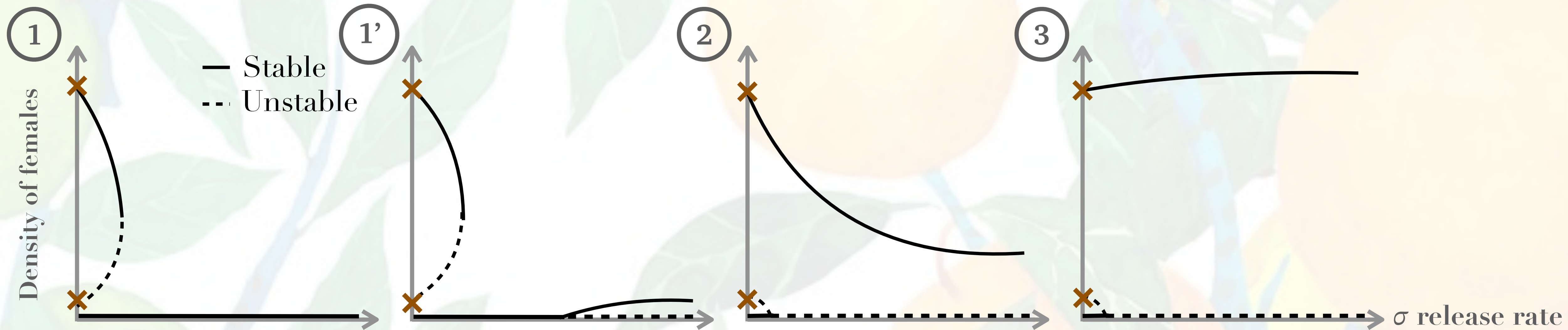
Parameters

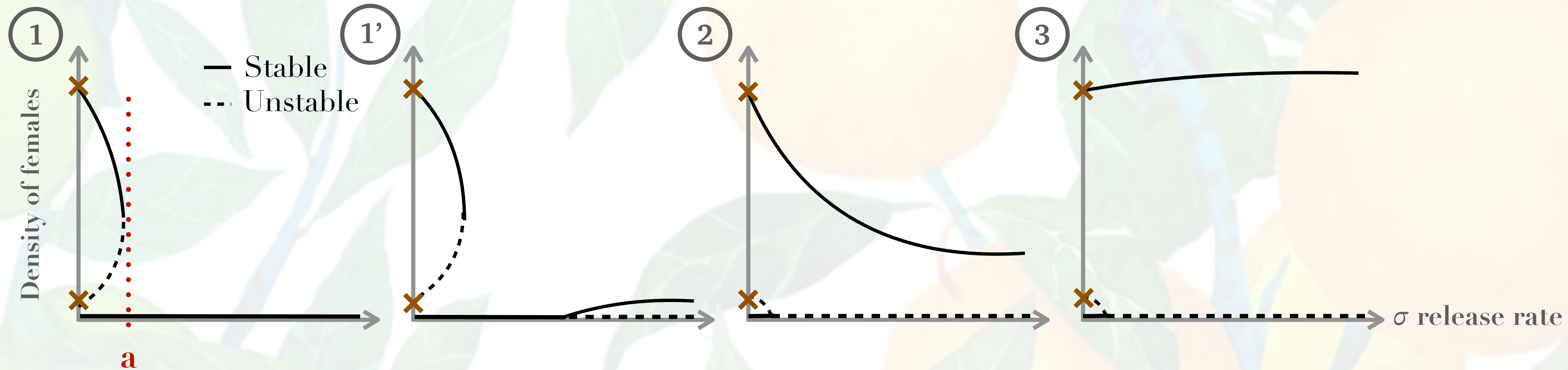
Equilibria

Bifurcation diagrams

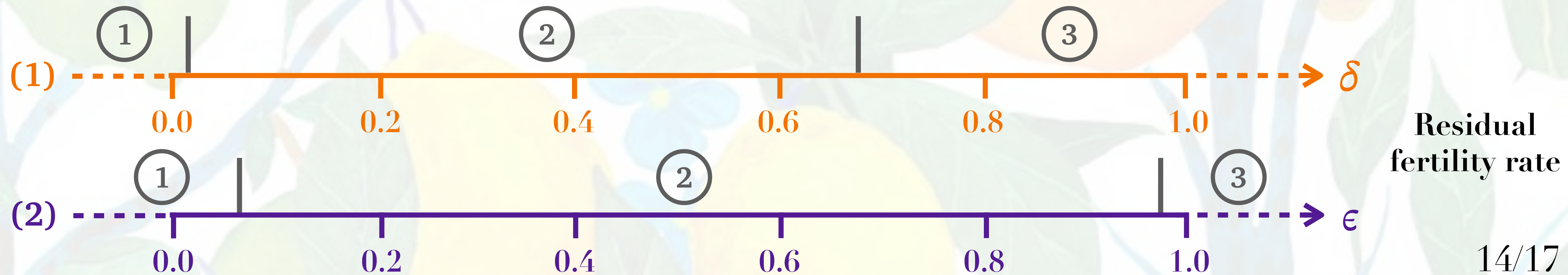
Dynamics

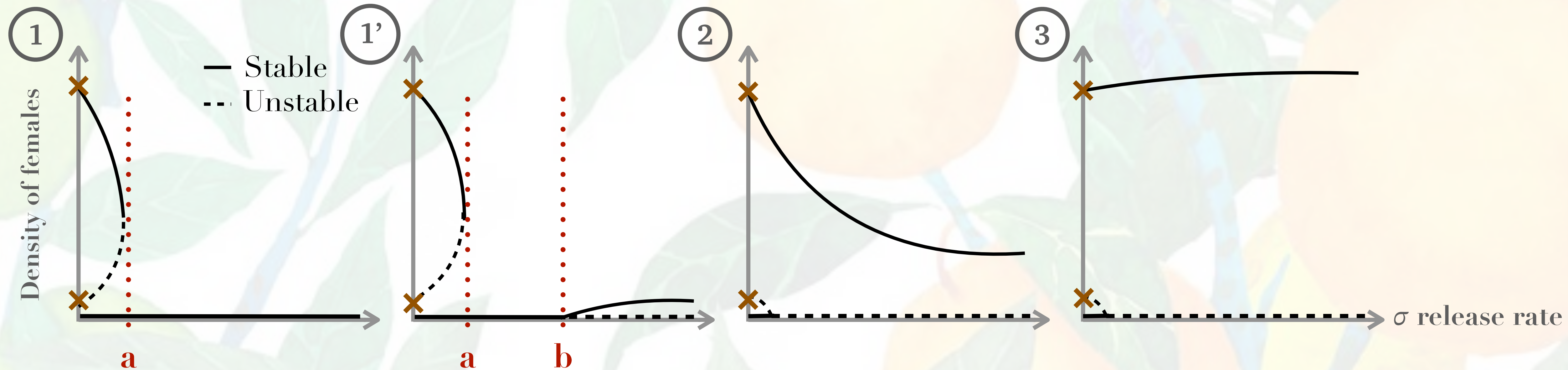
Discussion





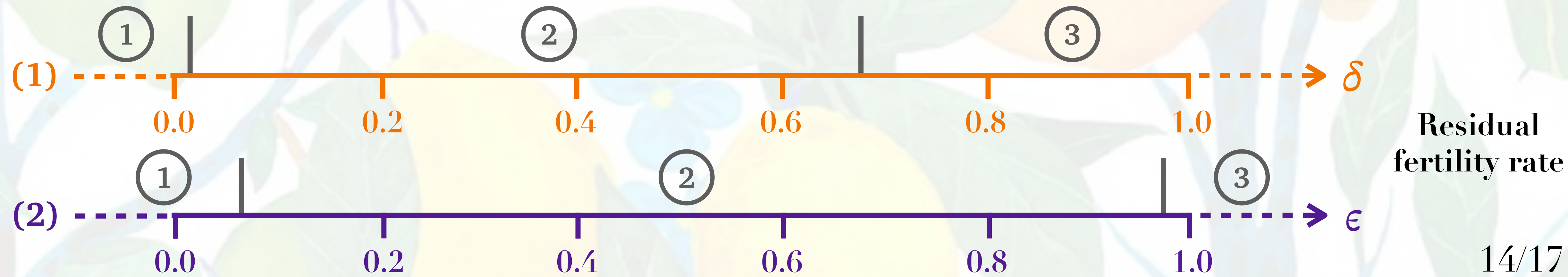
**For $\sigma > a$:
eradication**

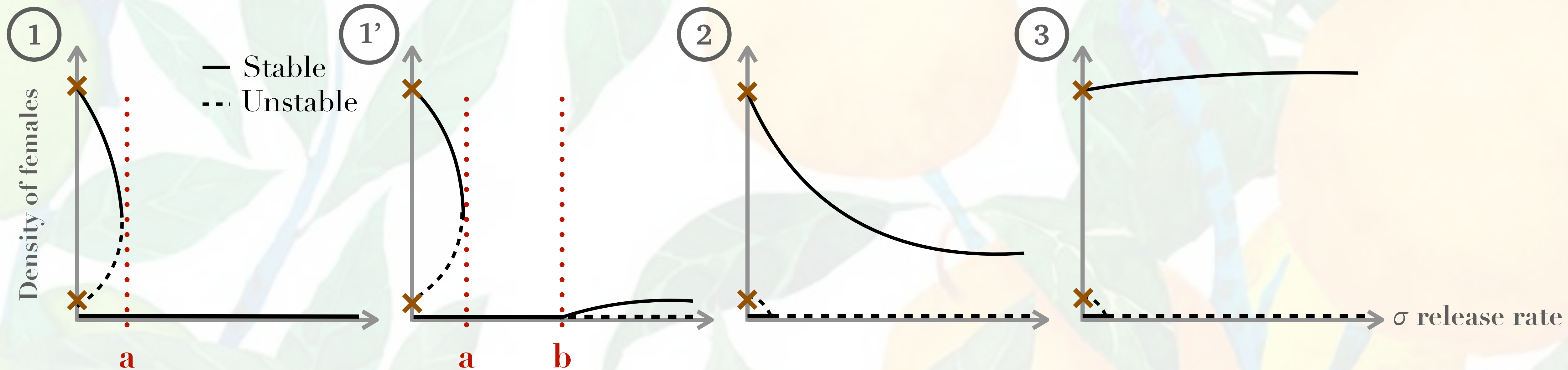




For $\sigma > a$:
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For $a < \sigma < b$: eradication
For $\sigma > b$: quasi eradication

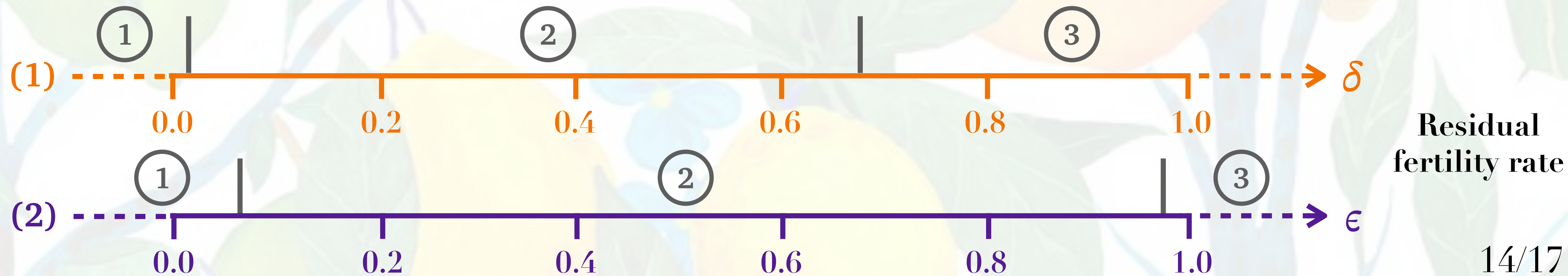


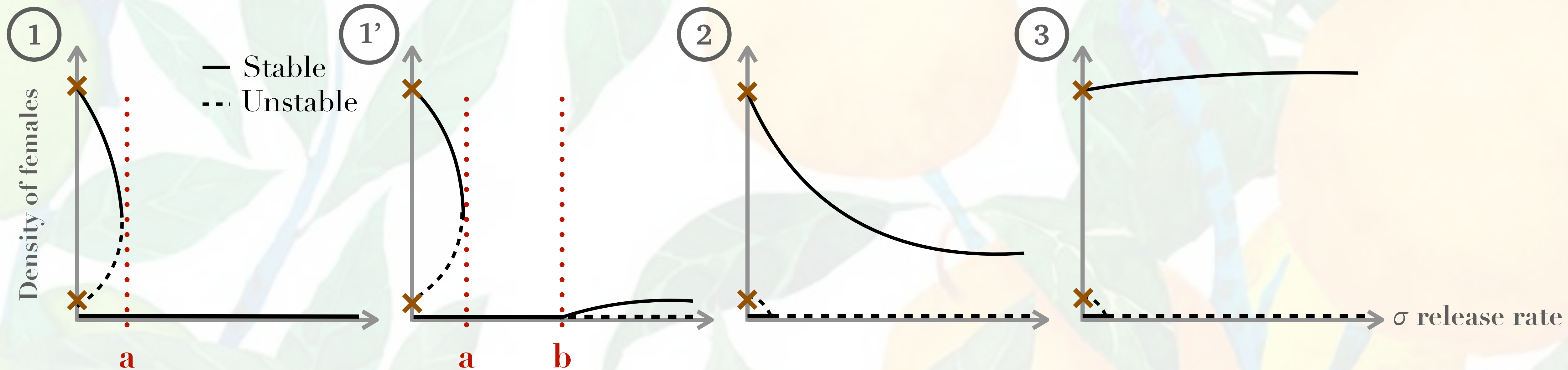


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Control only for
large σ



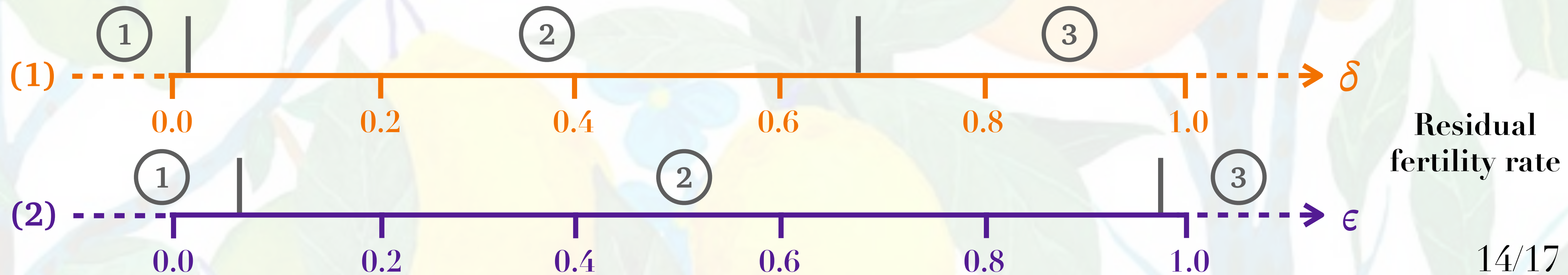


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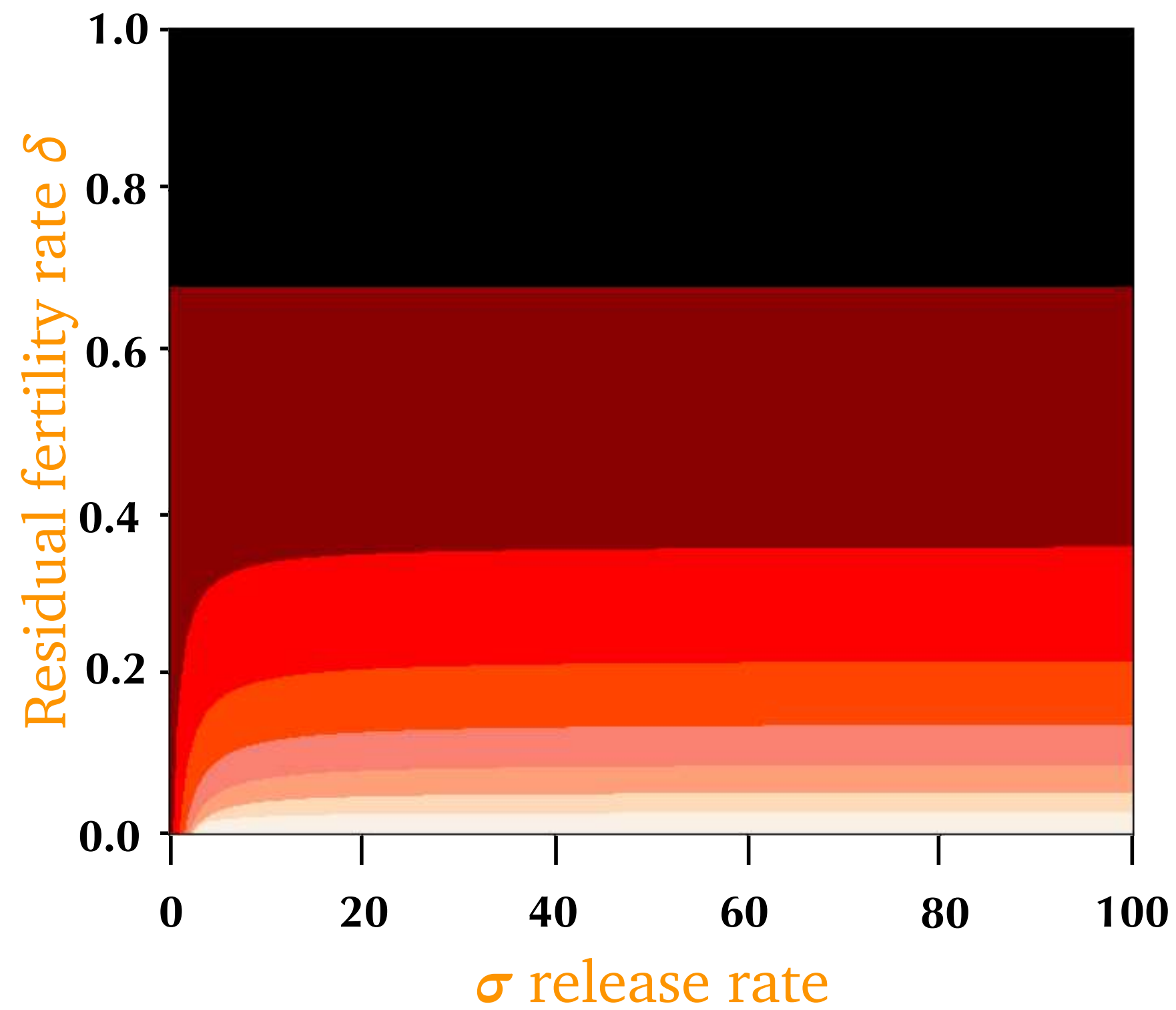
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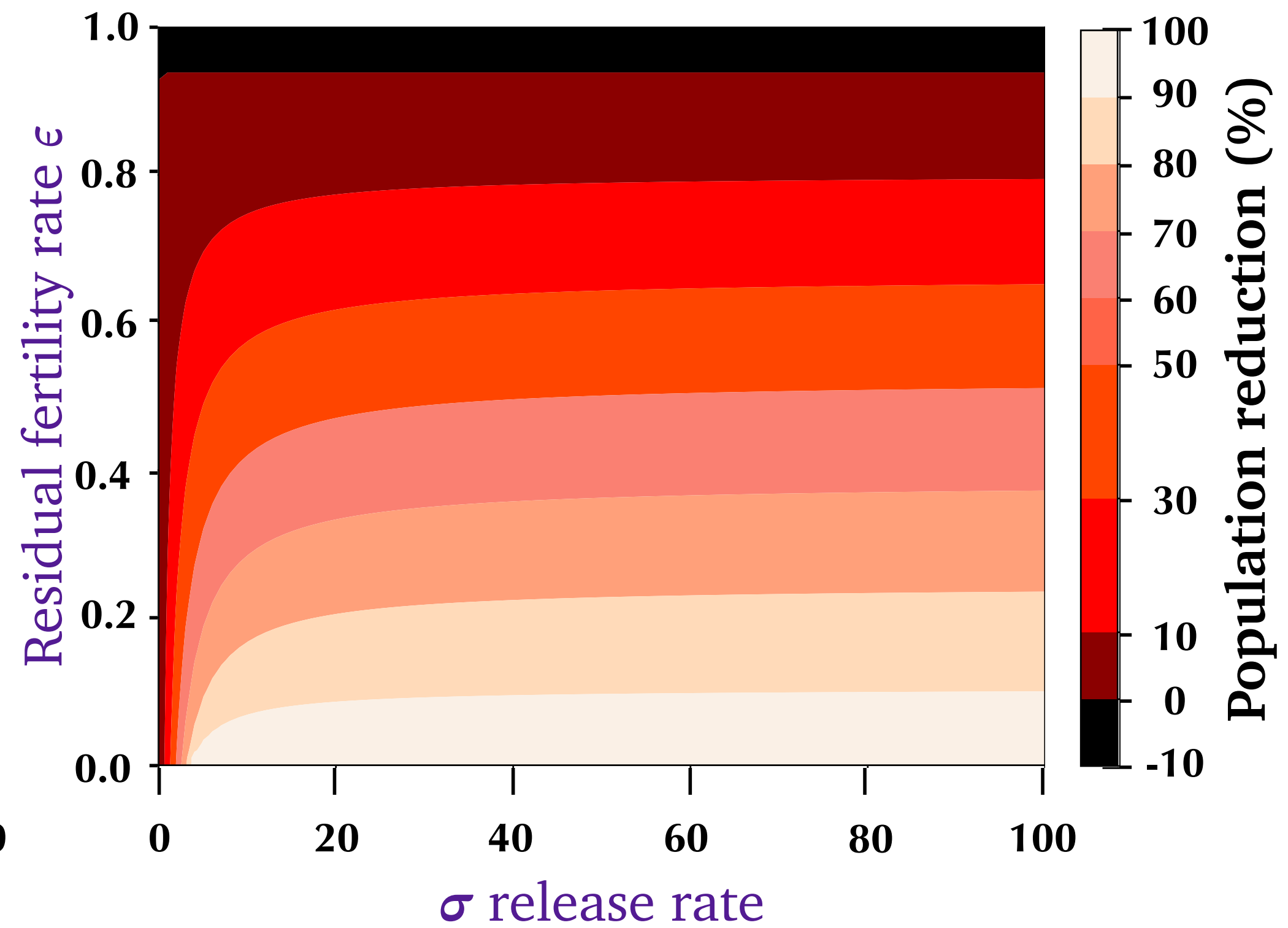
SIT is inefficient



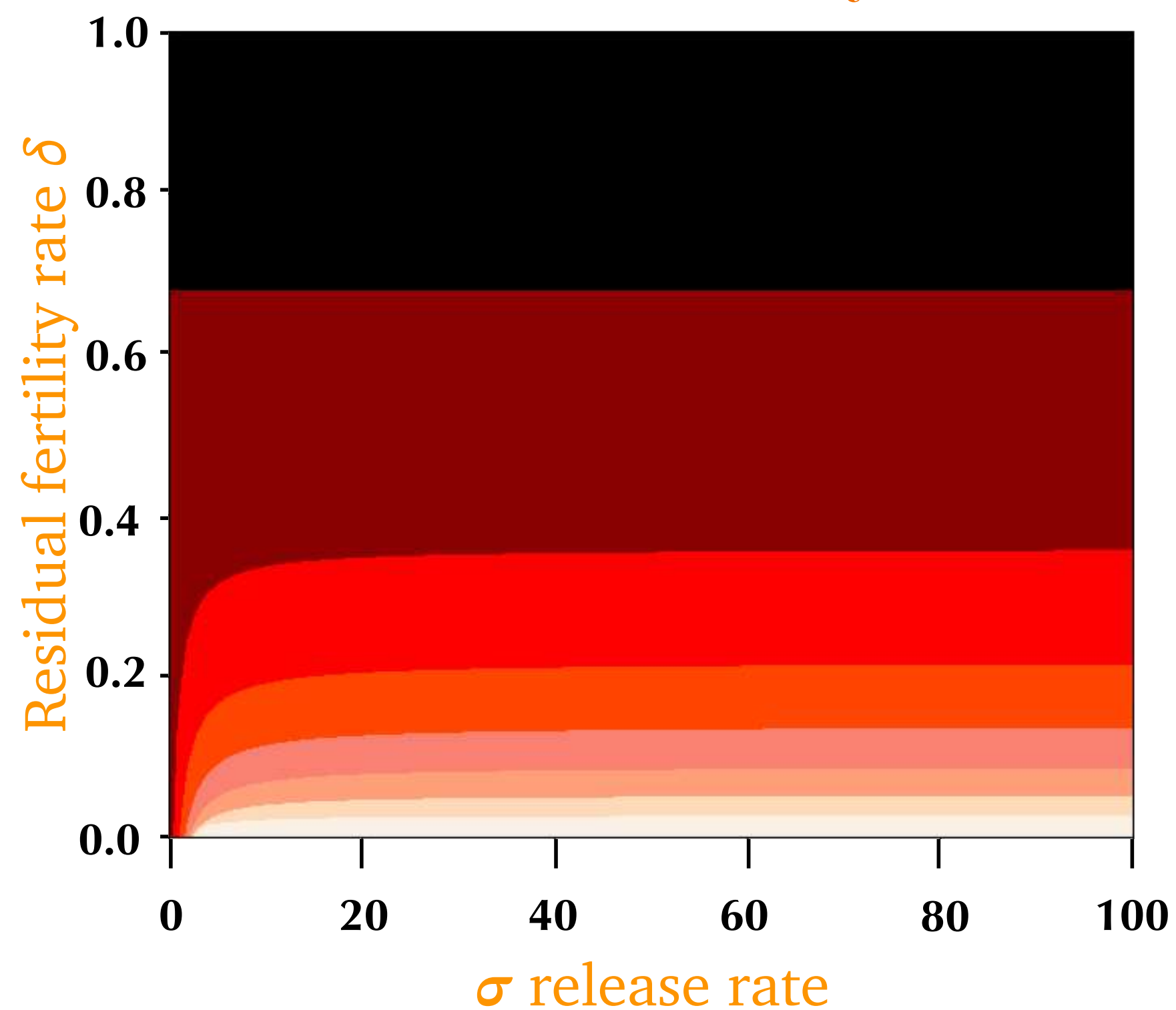
(1) Cost-free fertility model



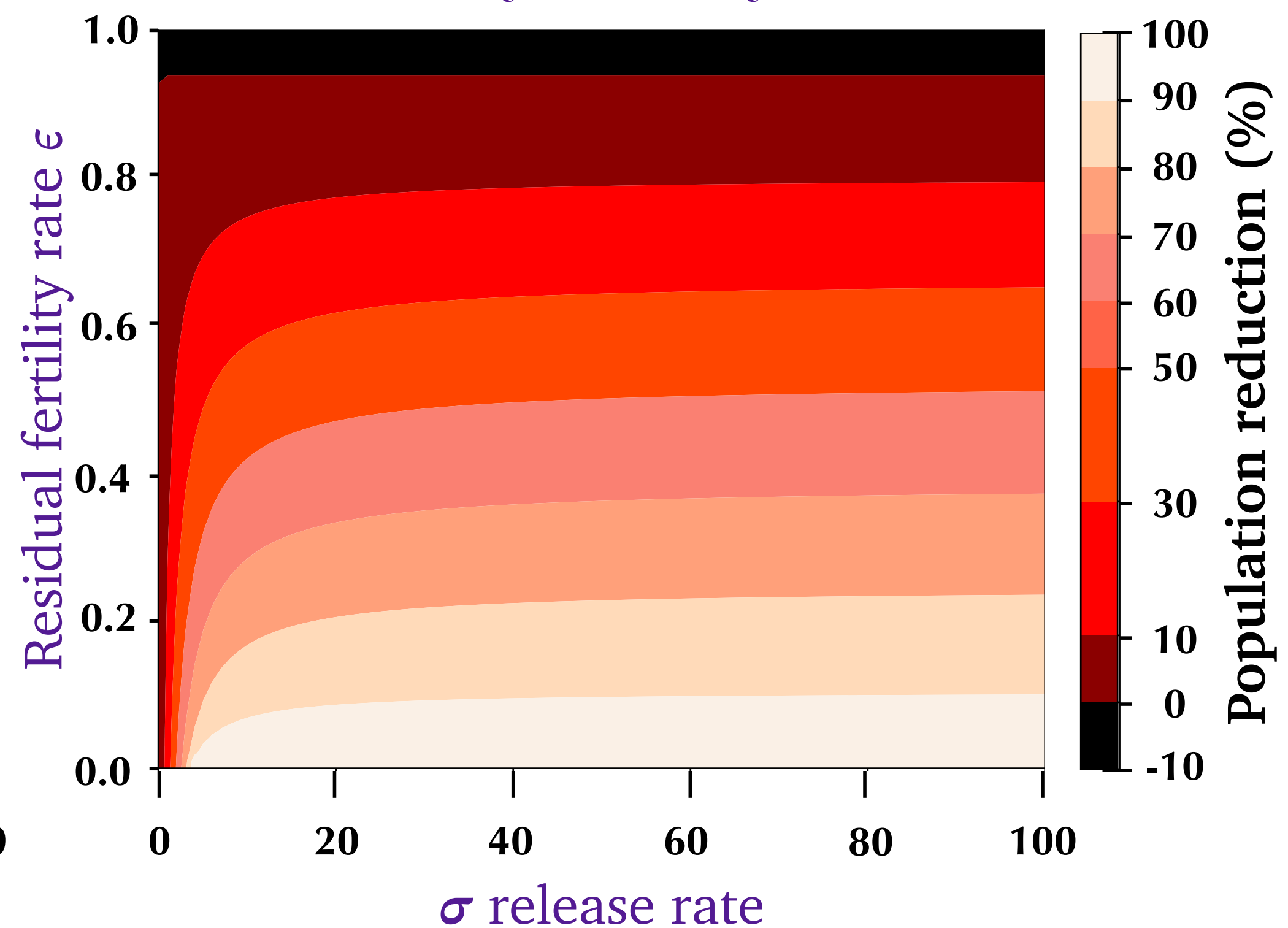
(2) Costly fertility model



(1) Cost-free fertility model

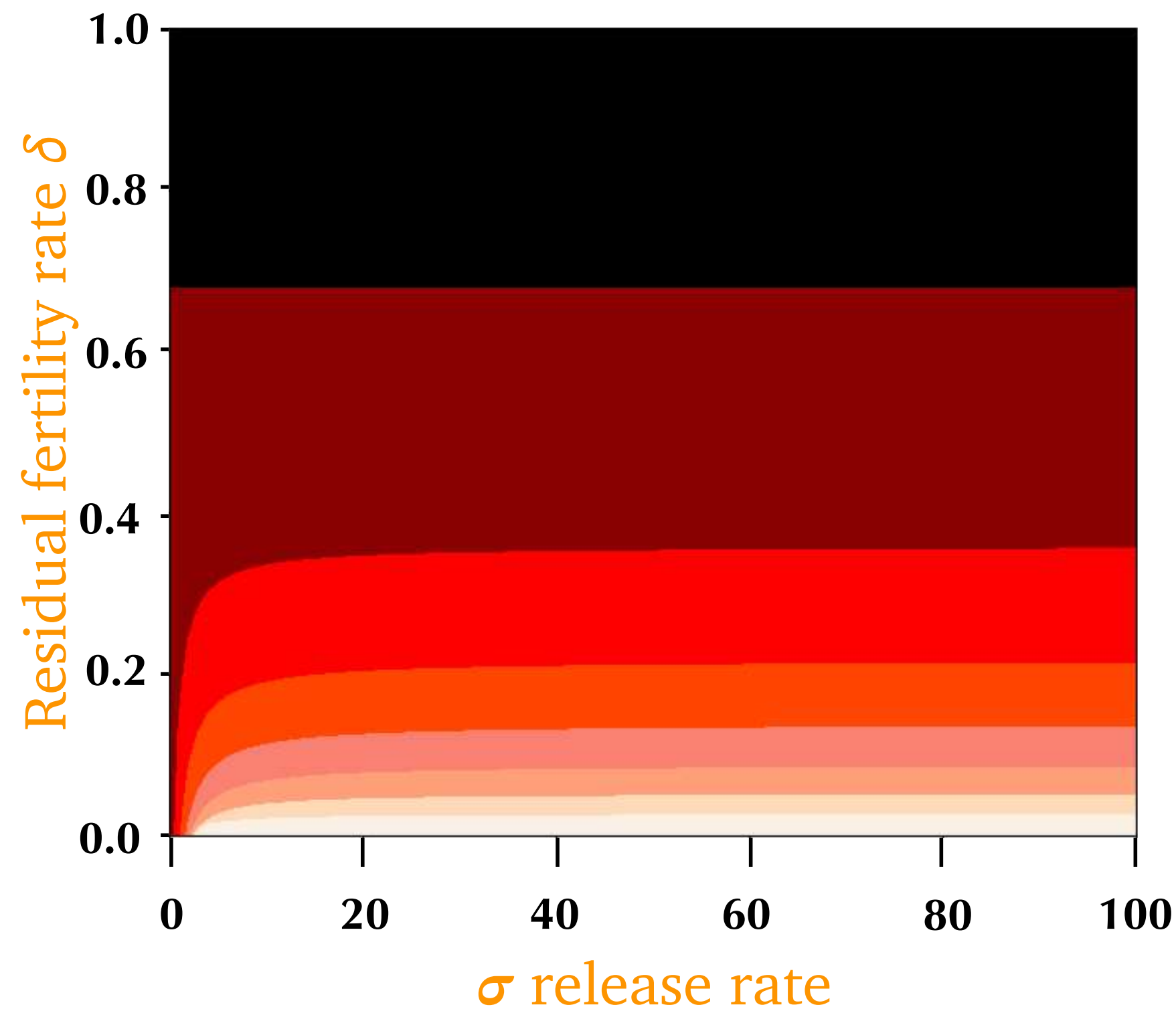


(2) Costly fertility model

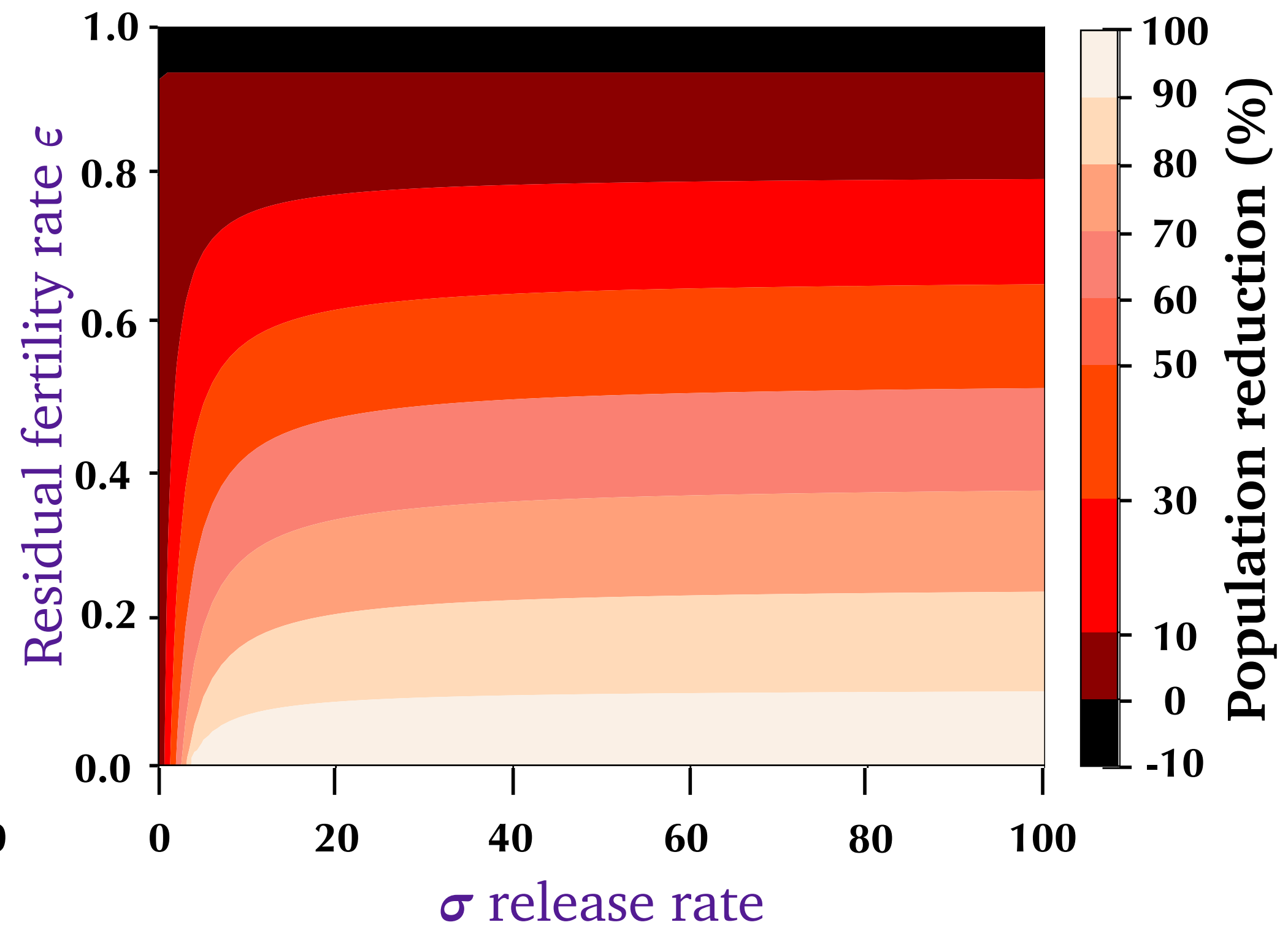


- Strong difference between the two sub models (1) and (2)

(1) Cost-free fertility model

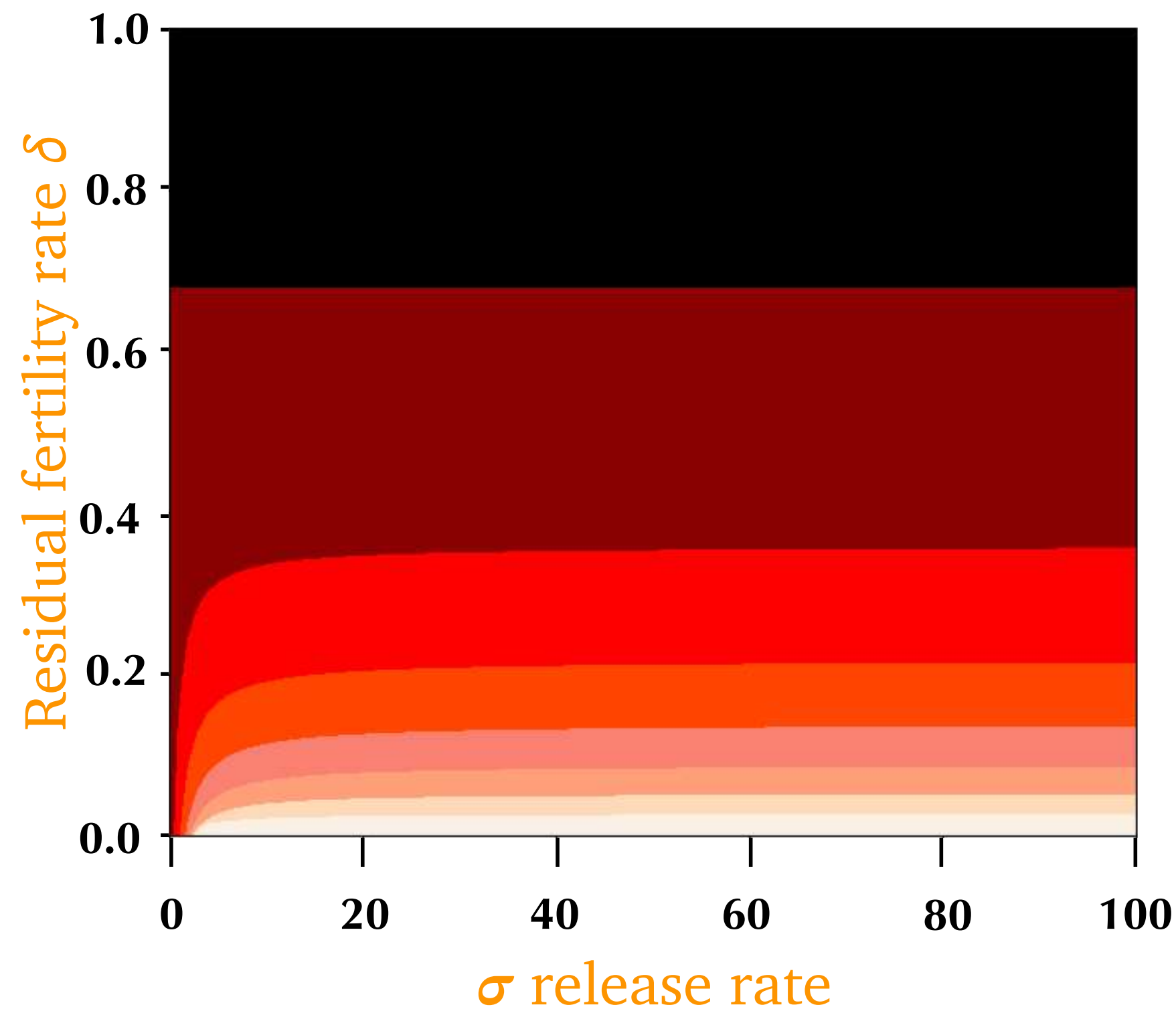


(2) Costly fertility model

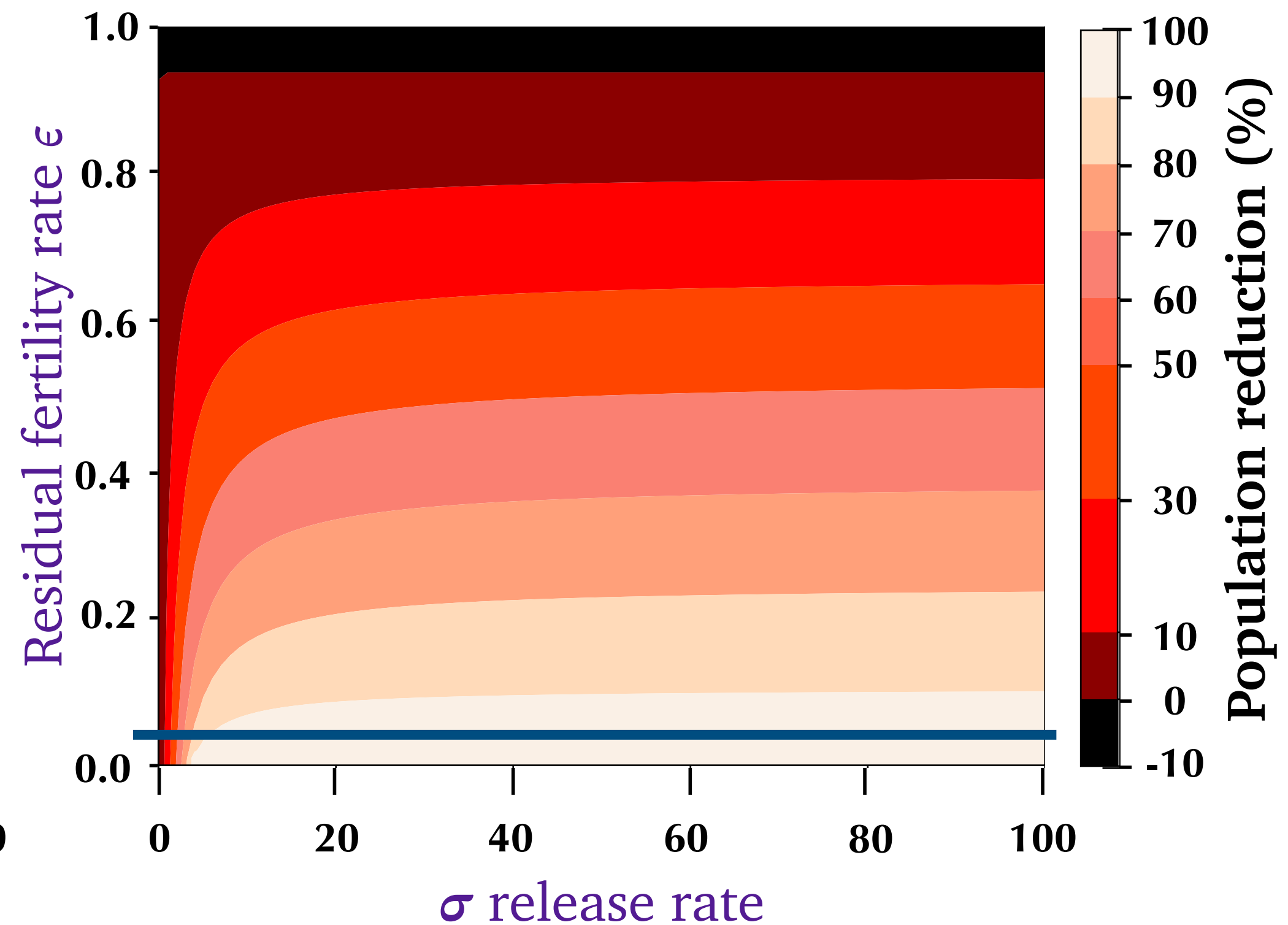


- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates

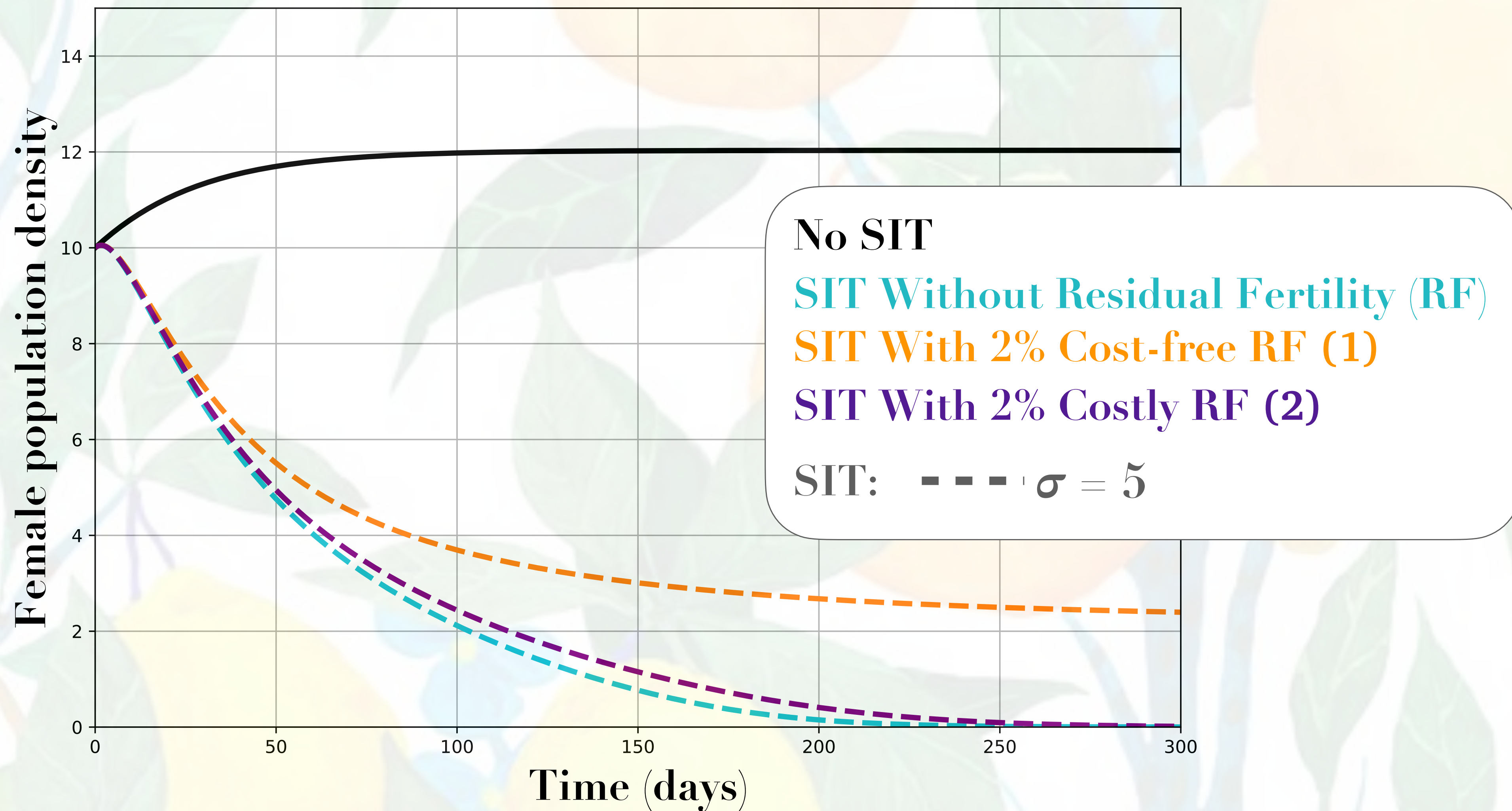
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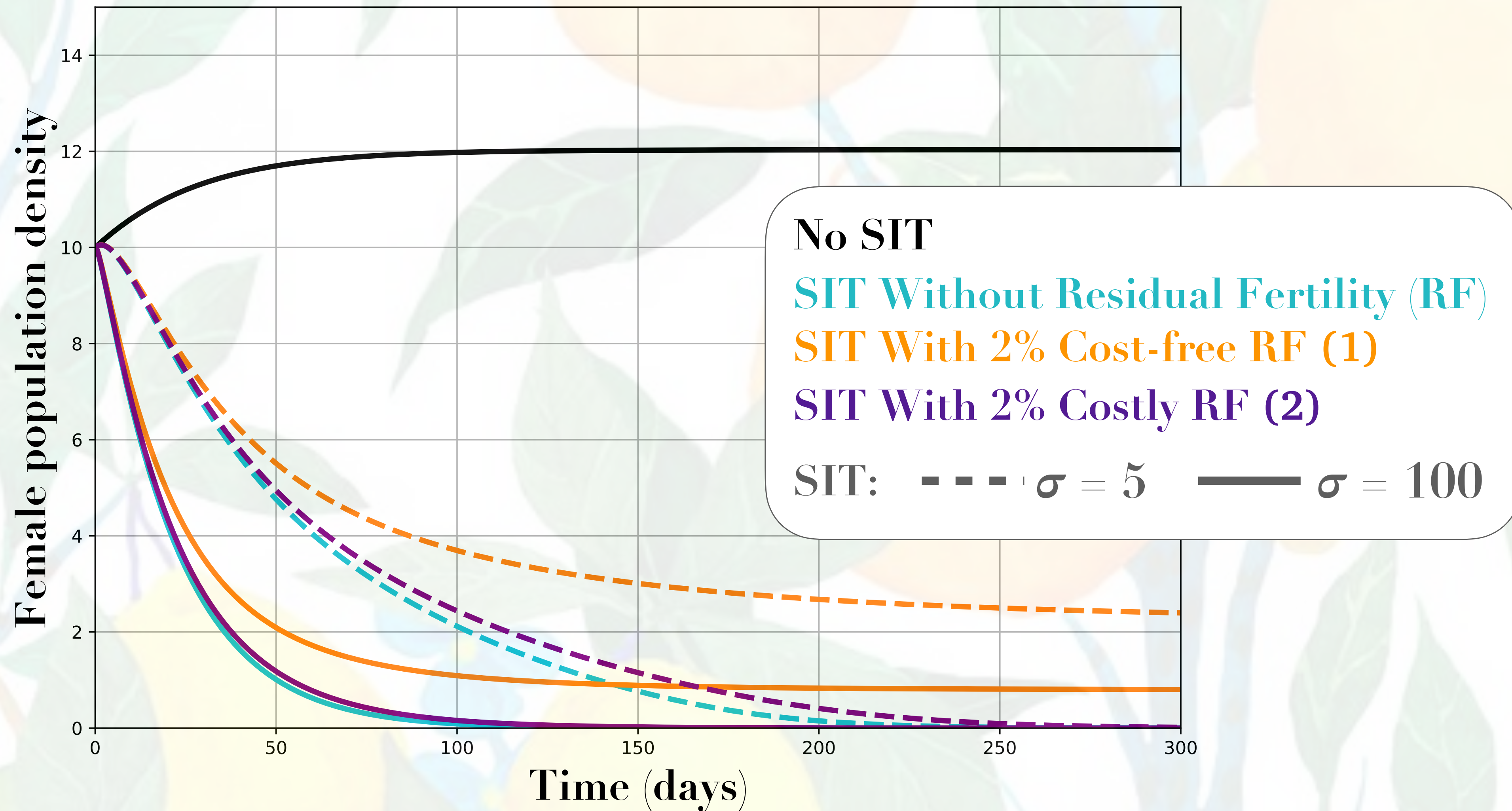


(2) Costly fertility model



- Strong difference between the two sub models (1) and (2)
- If there is a **fitness cost**: SIT effective for higher residual fertility rates
- Minimum σ ? If we increase it there is no more consequence ?





- Strong impact of residual fertility on SIT efficiency
- For **costly residual fertility**, SIT is effective at higher residual fertility rates

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 - Releases of at least **500 sterile males per day per hectare** ($\sigma = 5$)

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- **Ethics**

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• **Ethics**

•

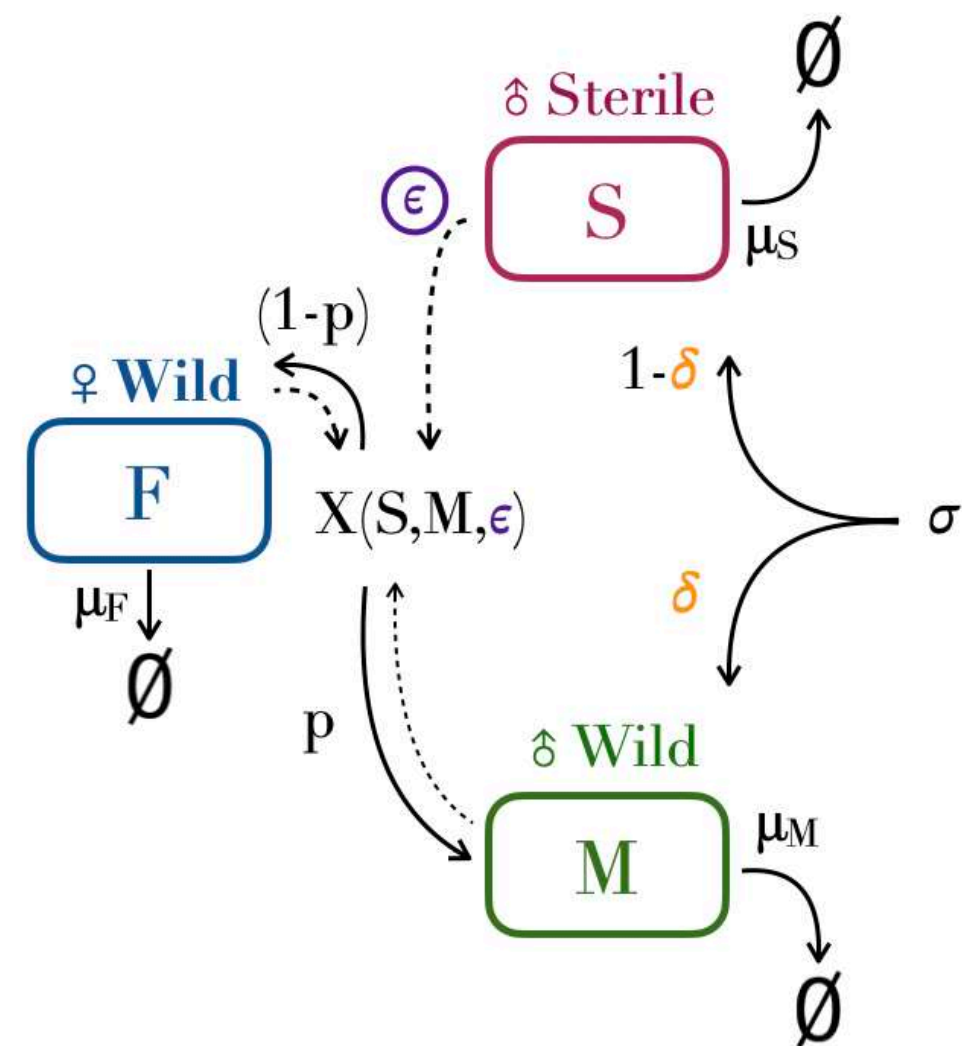
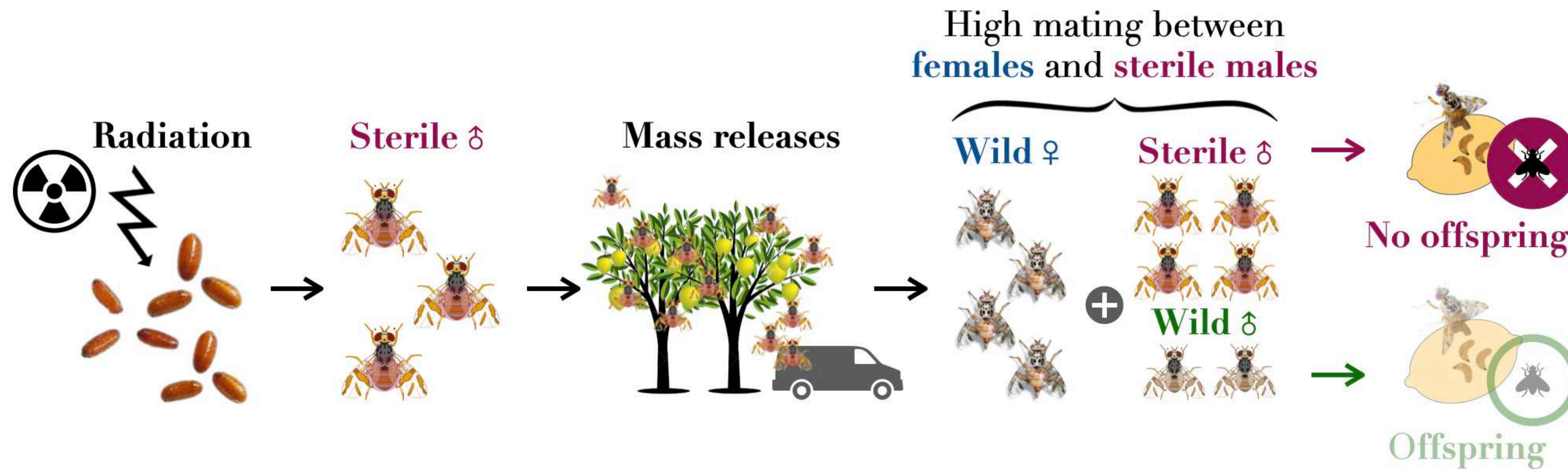
Control vs eradication

Thresholds:

$$\epsilon = \frac{1}{R} + q\left(1 - \frac{1}{R}\right)$$

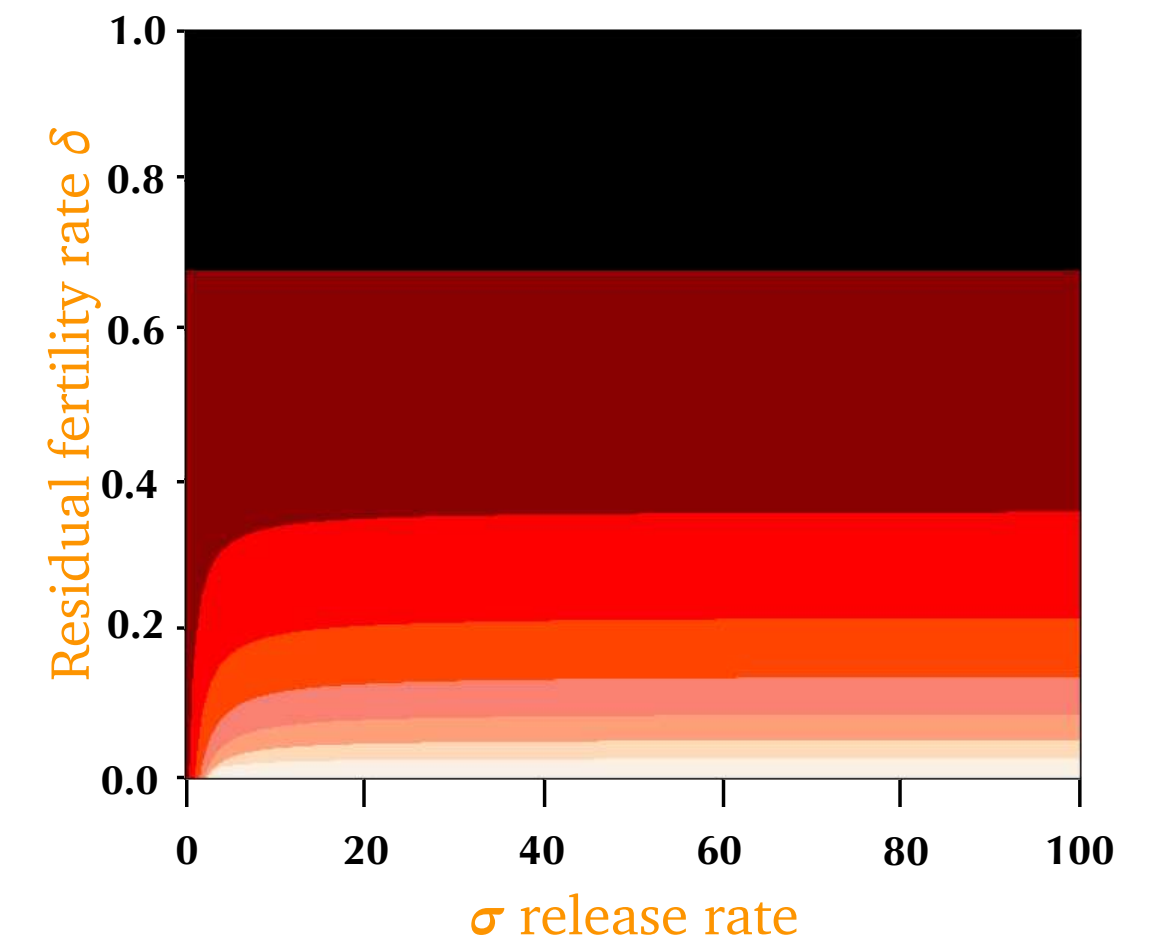
$$\epsilon = \frac{1}{R}$$

Thank you for your attention !

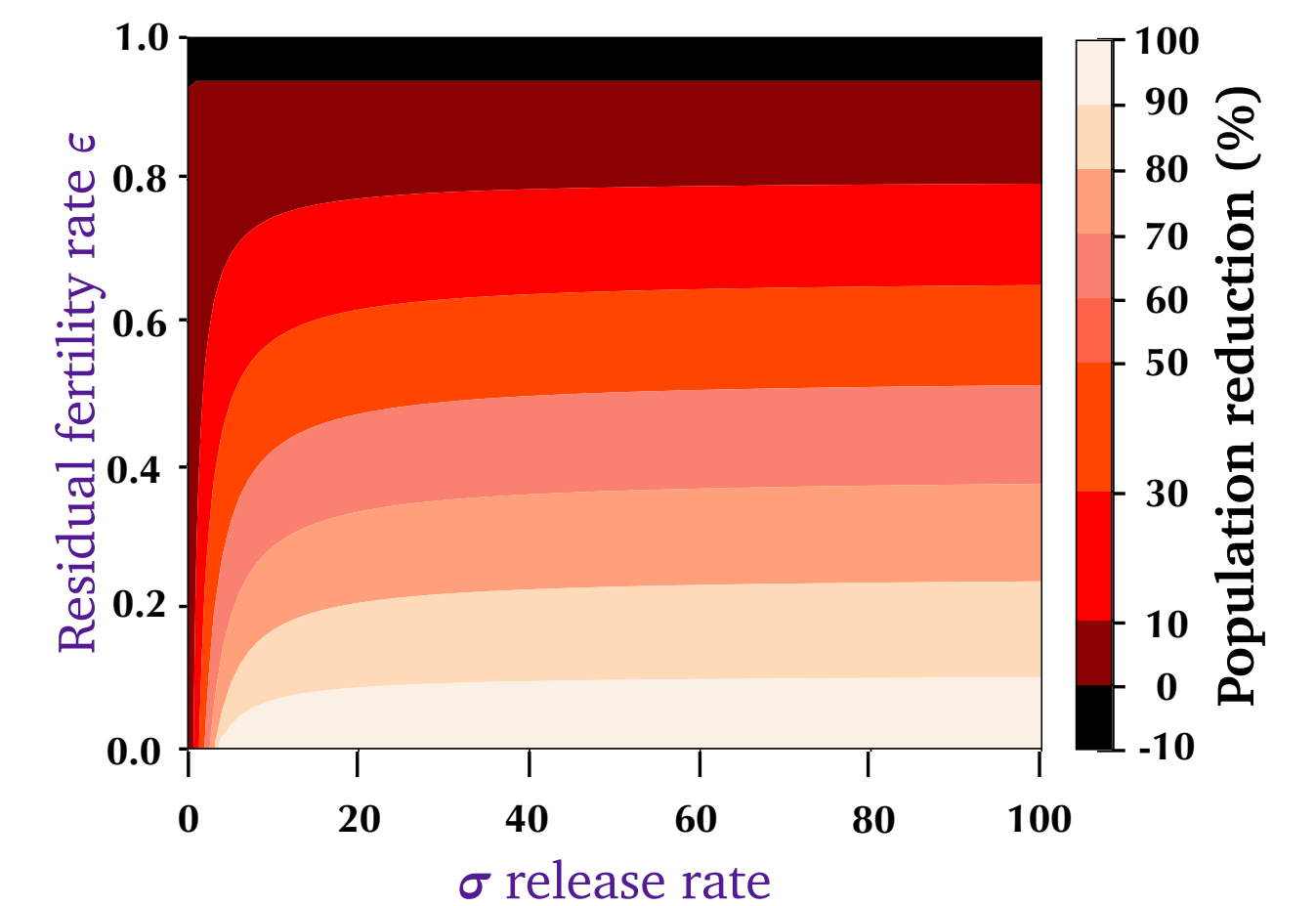


$$\begin{cases} \dot{S} = -\mu_S S + (1-\delta)\sigma \\ \dot{F} = -\mu_F F + r(1-p) X(S, M, \epsilon) C(F) F \\ \dot{M} = -\mu_M M + r p X(S, M, \epsilon) C(F) F + \delta\sigma \end{cases}$$

(1) Cost-free fertility



(2) Costly fertility model



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➔ Parameters have been estimated from the literature

✓ Easy parameters to estimate

μ : mortality rate

r : emergence rate

p : proportion of males

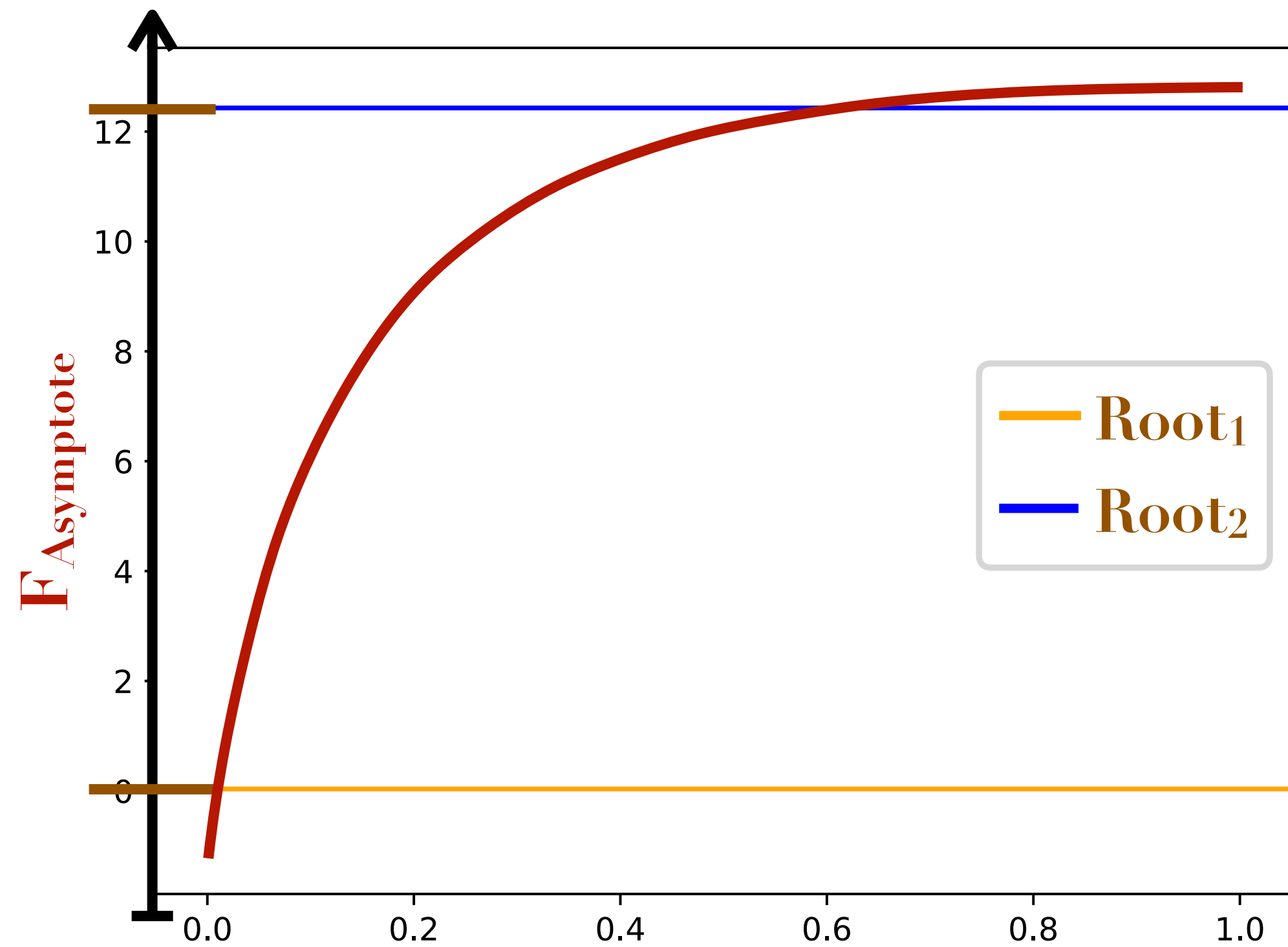
✗ Not easy parameters to estimate

$C(F)$: competition $\frac{1}{1 + \beta F}$

η : Sterilisation cost

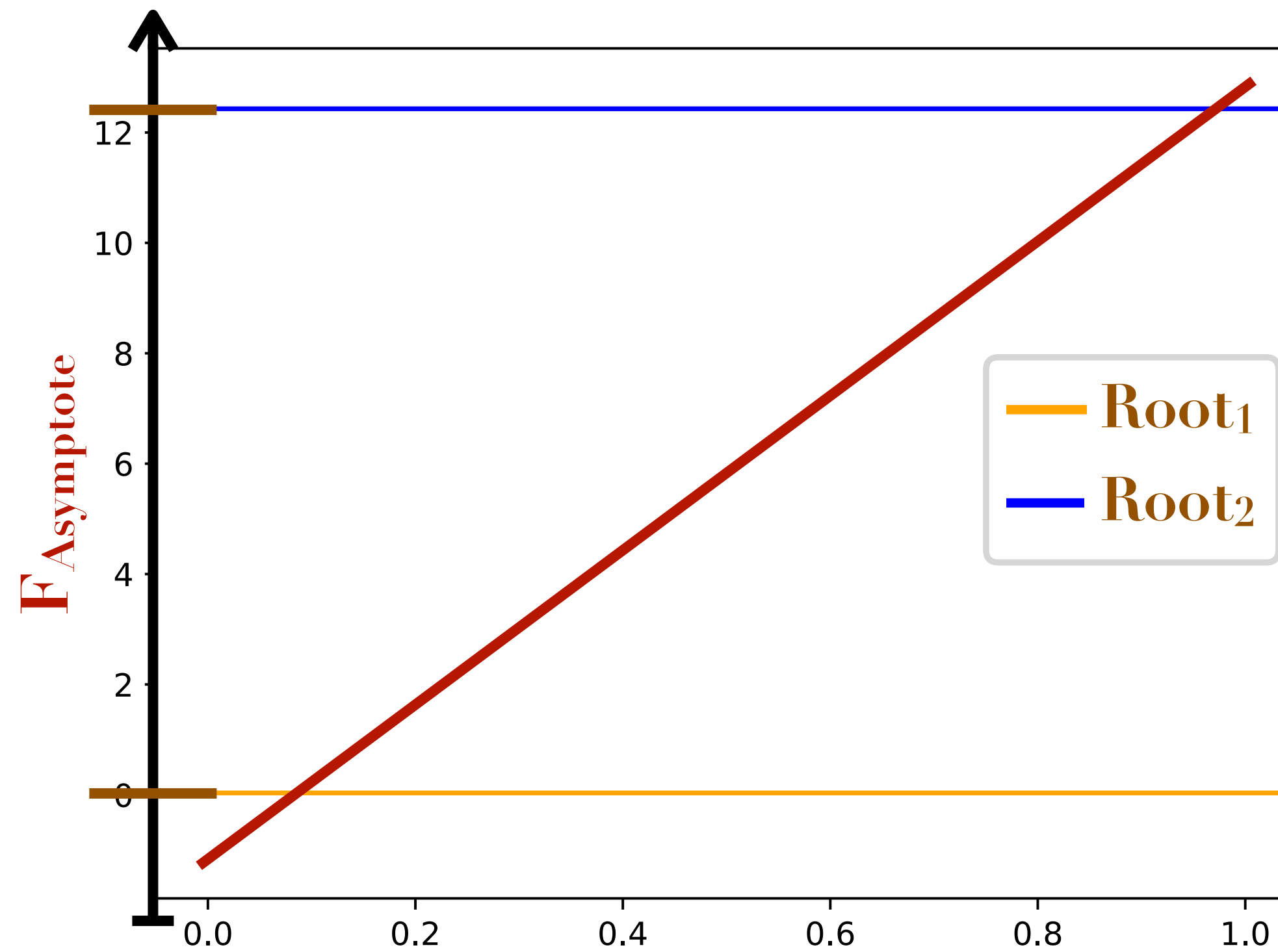
β : oviposition competition between females

(1) Cost-free fertility model
 $\delta \neq 0, \epsilon = 0$



Residual Fertility Rate δ

(2) Costly fertility model
 $\delta = 0, \epsilon \neq 0$



Residual Fertility Rate ϵ

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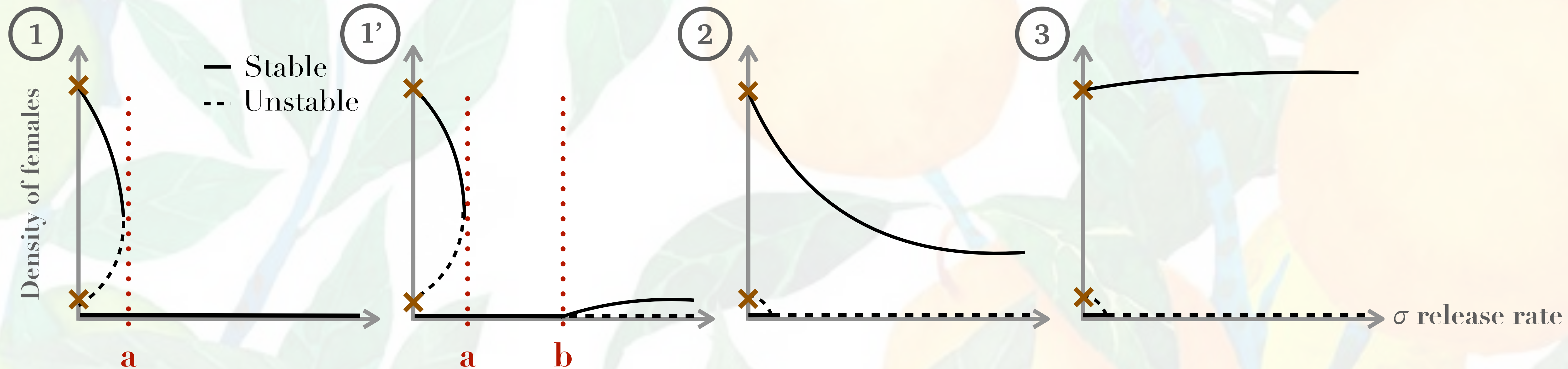
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