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CEE-M Working Paper 2023-07

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June 27, 2023

Abstract

Social context gives rise, in some individuals, to a desire to conform with other people. Conversely, other people desire to be exclusive or different from the "common herd". We study the interaction between two different agents: the positional player, characterized by the snob effect, and the conformist player, characterized by the bandwagon effect. Both agents engage in a public good game. For a static game we prove that some contribution is feasible only if the status concern of the positional player is sufficiently large. Moreover, contribution by both players requires also that the conformist player sees private contributions as complements. We also analyze how status concerns influence social welfare. In the second part of the paper, we extent the game to a dynamic setting, considering individuals who are change-adverse and who compare their current action against the opponent's past action. Convergence to the static Nash equilibrium can be monotone, oscillating, or spiral. Numerical simulations show that some properties of contributions and utilities along the transition path can be different than those in the long run; specifically, non-monotone convergence can induce overshooting in contributions allowing for over/undershooting in welfare.

Keywords: Public good, positional concerns, inertia, static and dynamic game.
JEL code: H41, D91, C72, C73.

*IMUVa, Universidad de Valladolid, Spain.
Francisco Cabo received funding from the Spanish Government (project PID2020-112509GB-I00), and from Junta de Castilla y León (project VA105G18), co-financed by FEDER funds.
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1 Introduction

Status-seeking behavior or the influence of social context on people's behavior has already been highlighted at the early stages of the economic discipline. Adam Smith noted the propensity of individuals to expend effort in order to attain status. Due to social context, people's preferences are not exclusively determined from absolute consumption. Many classic economic thinkers, from Thorstein Veblen to John Kenneth Galbraith, acknowledged that relative consumption also matters. The idea that preferences are interdependent is clearly stated in Duesenberry's relative income hypothesis (Duesenberry, 1949). Leibenstein (1950) states that an individual, in her search for status, seeks to signal wealth through her consumption decisions. He distinguishes three situations when social influences are considered. The bandwagon effect - or conformism - occurs when the agent seeks to follow the consumption behavior of others; the snob effect, contrarily, refers to the agent's desire for exclusiveness; finally, the Veblen effect is the desire to buy expensive goods to signal wealth (conspicuous consumption).

According to Corneo and Jeanne (1997) the snob effect arises when the desire to be identified with the rich is stronger than the desire not to be identified with the poor, while a bandwagon effect takes place when the former is weaker than the latter. We will define conformity as the act of changing behavior to match the behavior of others; see, for example, Ding (2017). On the other hand, snob agents, denoted here simply as positional agents, wish to behave not just differently, but better than their peers. In the words of Blanton and Christie (2003), "people try to stick out from others in good ways but not in bad ways".

The dichotomy between the wish for uniqueness and the desire to conform with others is present in the social psychology and the social influence literature. Kim and Markus (1999) analyze whether uniqueness is more common in Western culture, while conformity is more typical in East Asian culture, acknowledging that the norm to conform is as real as the norm to be unique. From the two behavioral alternatives, normative and counternormative, Blanton and Christie (2003) predict that people attend to the second, focusing on how they differ from others. On the other hand, Cialdini and Goldstein (2004) highlight that conformity can be based on three motivations: to form an accurate interpretation of reality and behave correctly, to obtain social approval from others, and to maintain one's self-concept. Although we will typically refer to social approval, our analysis encompasses the three motivations.

This dual vision can also be found in the theory of fashion, which by definition is consumed conspicuously and satisfies two social needs: the need to be similar to their social counterparts and the need for differentiation or uniqueness (see Simmel (1904) and Brewer (1991)). Similarly, Yoganarasimhan (2012) presents a model of social interaction, as a signaling game, where fashion has two roles: a way of conforming with others (emulation) and a signal of taste (distinction).

Ever since Becker (1974) and his theory of social interaction, it has been known that the existence of status concerns is potentially welfare reducing because agents may devote too much effort to acquire status. If people consume to signal wealth in order to climb the social ladder, then conspicuous consumption by one agent generates a negative externality in all other agents, which can reduce welfare (see, for example, Frank (2005)). However, this is not necessarily the case when the conspicuous good is a private contribution to a public good. In that case, positive contributions to signal wealth also have an additional associated externality (in this case positive): the utility from public good consumption. It is then possible that conspicuous generosity enhances social welfare, see Bougherara et al.

(2019) and Cabo et al. (2022).

Numerous recent experimental studies show that status concern/social approval is important in the public good provision and, in particular, in charitable giving (see reference within Wendner and Goulder (2008) or Ellingsen and Johannesson (2011)). The main idea behind these experimental studies is that making private donations public tends to raise contributions. It is nonetheless difficult to disentangle whether people increase their donations to contribute more than or the same as others - that is, to outrun or to conform with what others contribute.

From a theoretical perspective, the effect of status concern on the private provision of a public good[1] was analyzed by Muñoz-García (2011), who compared the contribution in the case of a simultaneous versus a sequential mode of play. Its effect on social welfare was explored by Bougherara et al. (2019) and extended by Cabo et al. (2022), who studied a one-shot game and also considered a dynamic mode of play. All these works assume snob agents, who get a positive positional payoff for contributions above their peers. They contribute more than their neighbors to signal wealth and to gain social status. The conclusion is that positionality, or the desire to overrun others, can lead to positive private contributions in a public good game overcoming the zero-contribution hypothesis predicted by Ostrom (2000). Cabo et al. (2022) noted that this result would not be attained if agents are exclusively guided by conformism.

In the present paper, we analyze private contribution to a public good when some agents are governed by a wish for uniqueness and others by a desire for conformity. Within the framework of a public good game, we study the conditions under which a virtuous sequence of private provisions can be opened by snob agents and followed by the conformist players. The initial contributions of the positional agents would induce some contributions on the conformist agents. Subsequently, by closing the gap, the conformist agents would oblige further contributions on the positional agents.

With that aim, we define a static game of private contribution to a public good between a positional player and a conformist player. In the standard formulation of the game, players enjoy public good consumption but dislike contributing to the public good (the utility associated with the absolute quantities contributed by the two players is denoted here as intrinsic utility). Moreover, when social context is considered, players can show status concerns. Hence, the preferences of a player are also affected by the comparison of her contribution against her opponent's contribution. This comparison represents a gain or a loss for the positional player, depending on whether she contributes above or below the conformist player. However, the conformist player can only lose from this comparison, and the loss is higher the wider the gap between contributions, regardless of whether she contributes more or less than her opponent.

Status concerns of the players determine whether no one, only the positional player, or both players contribute - and in the latter case, whether no one, only one or both players contribute their total endowment. Contributions are only possible if the status concern of the positional player is sufficiently large. Only if the positional player contributes, and only if the conformist player sees private contributions as strong complements can this latter start contributing. Nevertheless, the conformist player will never contribute more than the positional player.

[1] Wendner and Goulder (2008) also study the public good provision (and the excess burden) when utility depends on the consumption or income of others. However, they put the focus on the social optimum and not its private provision.

Positionality raises both players' contributions as well as the gap between them. Conversely, conformism reduces this gap and does not modify the global contribution. In the interior case, where both players contribute part of but not all of their wealth, we study the effect of positionality and the degree of conformism on individual and global contributions, and on welfare.

Positionality raises global contribution which, considering a concave utility from public good consumption, also raises global (jointly for the two players) intrinsic utility, if positionality is initially low and vice versa. Its effect on the global payoff linked to status concerns, and on social welfare, is undoubtedly positive. Conversely, the degree of conformism has no effect on global contribution and hence on the intrinsic utility. The payoff linked to status concerns rises for the society only if the conformist player values the public good less than the positional player, and if her status concern is not too large. Otherwise, the degree of conformism reduces the global payoff from the status concerns, and with it social welfare.

The second half of the paper considers a dynamic version of the game in a discrete time setting. Status concerned players compare their contribution against what they observe their opponent did the previous period. Moreover, players have inertia, i.e. they are reluctant to change previous actions. From these two assumptions, the strategic interaction between the two agents gives rise to a dynamic process which collects each player contribution across time. We show that this process converges to the static Nash equilibrium of the game, except when the inertia of players vanishes. We find that, depending on change-aversion parameters, convergence can be monotone, oscillatory, or spiral. This is at odds with the findings of Cabo et al. (2022) where both players are positional and convergence is always monotone.

Numerical simulations illustrate the main results obtained in the static and the dynamic versions of the game. Higher positionality raises both players' contribution and higher conformism makes them more egalitarian, both in the long run and across the transition. However, some properties of contributions and utilities along the transition path can be different from those in the long run (or, equivalently, at the static equilibrium). Specifically, overshooting contributions can temporarily lower intrinsic and global utility below their long-run values or even below their value under zero contribution. Likewise, the contribution of the positional is larger than the contribution of the conformist in the long run, but not necessarily at every time through the transition.

The remainder of the paper is organized as follows. In Section 2, we present the public good game and distinguish between the intrinsic utility and the payoffs associated with status concerns. Section 3 characterizes the different Nash equilibria of the game, depending on the status concerns of the players. Section 4 analyzes how these concerns affect social welfare for the interior solution. Section 5 generalizes the model to a dynamic setting and characterizes the behavior of the dynamic solution. The main results are numerically illustrated in Section 6. Conclusions are presented in Section 7. Technical details are explained in the Appendix.

2 Public good game

This section describes the strategic interaction in a static public good game between two different individuals. Agents get inner or intrinsic utility from public good consumption, at the cost of private contributions. However, social context gives rise to additional gains/losses from the comparison of one player's contribution in relative terms to the other.

Depending on how they are influenced by social context we distinguish two types of players.

They both get the same inner satisfaction from public good consumption, but they differ in the satisfaction they get when comparing their contribution against other people's contribution; that is, the payoff associated with status concerns, henceforth denoted as SC payoff or SCP. We distinguish between the positional player, P, who seeks for exclusiveness, and the conformist player, C, who "follows the herd" and tries to mimic other people's behavior.

Player $i \in \{P, C\}$ is endowed with w_i, contributes $g_i \in [0, w_i]$ to a public good, and consumes $w_i - g_i$ in other private goods. The total utility of this player is the addition of the intrinsic utility plus the SC payoff:

$$U_i(g_i, g_{-i}) = u_i(g_i, g_{-i}) + V_i(g_i, g_{-i}), \tag{1}$$

where the index "$-i$" denotes, as usual, the opponent of player i. The global utility or social welfare is defined as the addition: $U(g_i, g_{-i}) = U_i(g_i, g_{-i}) + U_{-i}(g_i, g_{-i})$.

Each player $i \in \{P, C\}$ solves a maximization problem:

$$\max_{g_i} U_i(g_i, g_{-i}), \tag{2}$$

$$\text{s.t.: } 0 \leq g_i \leq w_i. \tag{3}$$

2.1 Intrinsic utility

Both players obtain intrinsic utility from absolute consumption, defined as a quasi-linear function (see, for example, Varian (1994)):

$$u_i(g_i, g_{-i}) = w_i - g_i + b_i(G), \qquad G = g_{\mathrm{P}} + g_{\mathrm{C}}. \tag{4}$$

The utility from public good consumption, $b_i(G)$, strictly increases at a decreasing rate with the total amount contributed. Conversely, private consumption is assumed to provide constant marginal utility. The assumption of linear utility from private consumption is based on the idea that the noncontributed amount can be used for many different purposes. Since $b_i(G)$ is an increasing function, player i has an incentive to free-ride on other agents' contribution, $\partial u_i / \partial g_{-i}(g_i, g_{-i}) > 0$. Moreover since $b_i(G)$ marginally decreases, private contributions are substitutes,[2] $\partial^2 u_i / (\partial g_i \partial g_{-i})(g_i, g_{-i}) < 0$. Global intrinsic utility for society is $u(g_{\mathrm{P}}, g_{\mathrm{C}}) = u_{\mathrm{P}}(g_{\mathrm{P}}, g_{\mathrm{C}}) + u_{\mathrm{C}}(g_{\mathrm{C}}, g_{\mathrm{P}})$.

The public good game is defined by an intrinsic utility satisfying two additional properties.

C1 No one wants to contribute:

$$\frac{\partial u_i}{\partial g_i}(g_i, g_{-i}) < 0 \Leftrightarrow b_i'(G) < 1, \ \forall G \in [0, W];$$

with $W = w_{\mathrm{P}} + w_{\mathrm{C}}$.

C2 Some private contribution to the public good but also some private consumption are socially desirable:

$$\frac{\partial u}{\partial g_i}(0,0) > 0, \quad \frac{\partial u}{\partial g_i}(w_{\mathrm{P}}, w_{\mathrm{C}}) < 0, \quad \Leftrightarrow \quad b_i'(W) + b_{-i}'(W) < 1 < b_i'(0) + b_{-i}'(0).$$

[2]Functions b_{P} and b_{C} are assumed to be twice continuously differentiable.

By condition C1 in a Nash equilibrium, no player would individually contribute, even if by Condition C2 positive contributions are socially desirable.

From (4), the global intrinsic utility for the two players only depends on the global amount of public good:[3]

$$u(G) = W - G + b_{\mathrm{P}}(G) + b_{\mathrm{C}}(G).$$

From condition C2, the maximum global intrinsic utility is reached at an interior contributed amount, $0 < G^{SO} < W$. Moreover, although $b_i'(G) > 0$, given the assumption of diminishing marginal utility, nothing prevents the possibility of a large contribution, G^{E0}, above which intrinsic utility falls below the utility at zero contribution, $U(G^{E0}) = W$ and $U(G) < W, \forall G > G^{E0}$. From the point of view of the intrinsic utility, contributions can be in shortage $G < G^{SO}$, in excess $G > G^{SO}$, or at its efficient level, $G = G^{SO}$. Under shortage, additional contributions improve the intrinsic utility. However, under excess, contributions reduce intrinsic utility and indeed, if contributions are very high, intrinsic utility would fall below W. Thus, contributions are intrinsic welfare improving when $G < G^{E0}$ (IW-I), or intrinsic welfare reducing when $G > G^{E0}$ (IW-R).

2.2 Positional player

A positional or snob agent, P, seeks social status by contributing more than others. This agent gains (loses) an SCP, defined as a constant fraction of her contribution above (below) that of the other player:

$$V_{\mathrm{P}}(g_{\mathrm{P}}, g_{\mathrm{C}}) = v_{\mathrm{P}}(g_{\mathrm{P}} - g_{\mathrm{C}}). \tag{5}$$

The SCP grows at rate $v_{\mathrm{P}} \geq 0$ with the contributions of the positional player and decays at this same rate with the contributions of the other player. Parameter v_{P} will be called the player's positional concern.

The marginal utility of a positional player is:

$$\frac{\partial U_{\mathrm{P}}}{\partial g_{\mathrm{P}}} = -1 + v_{\mathrm{P}} + b_{\mathrm{P}}'(G). \tag{6}$$

Since $b_i(G)$ is a strictly increasing function, an agent with a positional concern greater or equal to one would always contribute her total endowment, regardless of what the other player might do. Hence, to avoid this extreme situation, we assume:

Assumption 1 $0 \leq v_P < 1$.

From the FOC of problem (2)-(3) for the P player, the wished "uncoordinated" amount of public good for the positional reads:

Definition 1 (Wished amount) *The amount of PG wished by palyer P, is defined as:*

$$\begin{cases} 0 & \text{if } v_P \leq 1 - b_P'(0), \\ A = (b_P')^{-1}(1 - v_P) & \text{if } v_P \in (1 - b_P'(0), 1 - b_P'(W)), \\ w_P & \text{if } v_P \geq 1 - b_P'(W). \end{cases} \tag{7}$$

[3]Notice here a slight abuse of notation, because we have defined u as a function of (g_i, g_{-i}).

Note that since b_P is concave, the wished amount increases with positionality, strictly in the interior case $A = (b'_\mathrm{P})^{-1}(1 - v_\mathrm{P})$ since $\partial A / \partial v_\mathrm{P} > 0$. If $0 \le v_\mathrm{P} < 1 - b'_\mathrm{P}(0)$, the marginal utility of the public good never surpasses the marginal cost from its private provision, and hence, no public good would be privately provided. However, if $v_\mathrm{P} > 1 - b'_\mathrm{P}(0)$, then the marginal utility from the public good surpasses, at least initially, the marginal cost from private provision, and agents are willing to privately provide some public good.

2.3 Conformist player

The second type of player is also influenced by social context, although in a different way than the positional player. Conformist agents feel better if their behavior fits the average behavior in society. In our formulation, the well-being of the conformist player decreases if her contribution deviates (above or below) from the contribution level of the other player. For the conformist player, the SCP always represents a loss which enlarges with the contribution gap, regardless of whether she contributes above or below her opponent:

$$V_\mathrm{C}(g_\mathrm{C}, g_\mathrm{P}) = -\frac{v_\mathrm{C}}{2}(g_\mathrm{C} - g_\mathrm{P})^2. \tag{8}$$

Parameter $v_\mathrm{C} \ge 0$ denotes this agent's degree of conformism. Introducing conformism to intrinsic utility, the FOC from the maximization problem (2)-(3) for the C player now reads:

$$\frac{\partial U_\mathrm{C}}{\partial g_\mathrm{C}} = -1 + b'_\mathrm{C}(G) - v_\mathrm{C}(g_\mathrm{C} - g_\mathrm{P}). \tag{9}$$

The marginal utility from the private provision can be lower or greater than in the case without conformism, depending on whether the conformist player contributes more or less than the positional. If the other player provides more public good than the C player, this latter has an incentive to increase contributions, which increases the marginal utility from the private provision. However, opposite reasoning applies when the P player provides less than the C player.

3 Nash equilibria

This section characterizes the Nash equilibria of a game between a positional and a conformist player.[4] The analysis is carried out considering a particular functional form for the intrinsic utility.

3.1 First-order conditions and best-response functions

In what follows we assume $b_i(G) = \alpha_i\left(G - \varepsilon G^2/2\right)$, and hence, the intrinsic utility in (4) particularizes to the following linear quadratic function: A quadratic utility from public good consumption can be found in Laury et al. (1999) or Bougherara et al. (2019).

$$u_i(g_i, g_{-i}) = w_i - g_i + b_i(G) = w_i - g_i + \alpha_i\left(G - \frac{\varepsilon}{2}G^2\right), \quad i \in \{P, C\}.$$

[4]The case where one player is positional and conformist at the same time is an schizophrenic behavior not considered here. In the case where both players are conformist, it is easy to prove that the equilibrium is characterized by no contribution. The case of two positional players is studied in Cabo et al. (2022).

Given this particular expression, $G^{SO} = (\alpha_{\rm P} + \alpha_{\rm C} - 1)/(\varepsilon(\alpha_{\rm P} + \alpha_{\rm C}))$ and $G^{EO} = 2G^{SO}$. For this specification, $b_i(G)$ is concave. To guarantee that $b_i'(G) > 0 \; \forall i \in \{P, C\} \; G \in [0, W]$, as well as Conditions C1 and C2, we must further assume:

Assumption 2 $0 < \alpha_i < 1$, $i \in \{P, C\}$.

Assumption 3 $0 < \varepsilon < \frac{1}{W}$.

Assumption 4 $\alpha_P + \alpha_C > 1$.

Assumption 5 $W > \dfrac{\alpha_P + \alpha_C - 1}{\varepsilon(\alpha_P + \alpha_C)}$.

Note that from Assumption 3 it follows that $0 \leq G \leq W < 1/\varepsilon$. Moreover, since $1 < \alpha_{\rm P} + \alpha_{\rm C} < 2$, then $0 < G^{SO} < G^{E0} < 1/\varepsilon$. As a result, the global contribution can be IW-I or IW-R.

Considering social context, the utility function does not necessarily satisfy the properties assumed for the intrinsic utility.

Condition C1 can be reversed if:

$$\frac{\partial U_{\rm P}}{\partial g_{\rm P}}(g_{\rm P}, g_{\rm C}) > 0 \iff v_{\rm P} > 1 - \alpha_{\rm P} + \alpha_{\rm P}\varepsilon G \equiv \theta_{\rm P}(G),$$
$$\frac{\partial U_{\rm C}}{\partial g_{\rm C}}(g_{\rm C}, g_{\rm P}) > 0 \iff v_{\rm C}(g_{\rm P} - g_{\rm C}) > 1 - \alpha_{\rm C} + \alpha_{\rm C}\varepsilon G \equiv \theta_{\rm C}(G).$$

For $v_{\rm P} > 1 - \alpha_{\rm P}$, then $\partial U_{\rm P}/(\partial g_{\rm P})(0,0) = -1 + \alpha_{\rm P} + v_{\rm P} > 0$. Thus, C1 does not hold for a sufficiently strong positional concern, at least for small contributions. Likewise, if the positional player contributes above the conformist, $g_{\rm P} > g_{\rm C}$, and if her degree of conformism is strong enough, then the conformist player would also be willing to contribute. Therefore, status concerns can make some contribution individually desirable for both types of agents.

Derivatives in condition C2 can be written as:

$$\frac{\partial U}{\partial g_{\rm P}}(0,0) = -1 + (\alpha_{\rm P} + \alpha_{\rm C}) + v_{\rm P}, \quad \frac{\partial U}{\partial g_{\rm C}}(0,0) = -1 + (\alpha_{\rm P} + \alpha_{\rm C}),$$
$$\frac{\partial U}{\partial g_{\rm P}}(w_{\rm P}, w_{\rm C}) = -1 + (\alpha_{\rm P} + \alpha_{\rm C})[1 - \varepsilon W] + v_{\rm P} - v_{\rm C}(w_{\rm P} - w_{\rm C}),$$
$$\frac{\partial U}{\partial g_{\rm C}}(w_{\rm P}, w_{\rm C}) = -1 + (\alpha_{\rm P} + \alpha_{\rm C})[1 - \varepsilon W] - v_{\rm P} + v_{\rm C}(w_{\rm P} - w_{\rm C}).$$

Under Assumption 4, $\partial U/(\partial g_i)(0,0) > 0$, and some contribution by either agent is socially desirable. However, our Assumptions does not guarantee $\partial U/(\partial g_i)(w_{\rm P}, w_{\rm C}) < 0$. Full contribution by one or both types of agents could be socially desirable.

The cross partial derivative for each player reads:

$$\frac{\partial^2 U_{\rm P}}{\partial g_{\rm P}\partial g_{\rm C}}(g_{\rm P}, g_{\rm C}) = -\alpha_{\rm P}\varepsilon < 0, \quad \frac{\partial^2 U_{\rm C}}{\partial g_{\rm C}\partial g_{\rm P}}(g_{\rm C}, g_{\rm P}) = -\alpha_{\rm C}\varepsilon + v_{\rm C}.$$

The positional player regards contributions as substitutes. However, for the conformist player two situations are possible. If $v_{\rm C} < \alpha_{\rm C}\varepsilon$ contributions are subtitute goods. However, if $v_{\rm C} > \alpha_{\rm C}\varepsilon$, conformism becomes so strong that contributions from the positional agent raises the conformist's willingness to contribute; i.e. g_{P} and g_{C} become complements for the conformist agent.

From (7) and (9), the players' FOC for an interior solution are:

$$-1 + \alpha_{\mathrm{P}}(1 - \varepsilon G) + v_{\mathrm{P}} = 0,$$
$$-1 + \alpha_{\mathrm{C}}(1 - \varepsilon G) + v_{\mathrm{C}}(g_{\mathrm{P}} - g_{\mathrm{C}}) = 0.$$

And therefore, the interior best-response functions of the positional and the conformist players read:

$$g_{\mathrm{P}}^{b}(g_{\mathrm{C}}) = A - g_{\mathrm{C}}, \qquad A = \frac{v_{\mathrm{P}} - (1 - \alpha_{\mathrm{P}})}{\alpha_{\mathrm{P}}\varepsilon}. \tag{10}$$
$$g_{\mathrm{C}}^{b}(g_{\mathrm{P}}) = (-B + g_{\mathrm{P}})r, \qquad B = \frac{1 - \alpha_{\mathrm{C}}}{v_{\mathrm{C}} - \alpha_{\mathrm{C}}\varepsilon}, \quad r = \frac{v_{\mathrm{C}} - \alpha_{\mathrm{C}}\varepsilon}{v_{\mathrm{C}} + \alpha_{\mathrm{C}}\varepsilon}. \tag{11}$$

The intercept of the downward-sloping best response by the positional player is positive for $v_{\mathrm{P}} > 1 - \alpha_{\mathrm{P}}$. The best-response function of the conformist player (11) is downward/upward sloping under substitutability/complementarity. In either case, the intercept, given by $-rB$, takes a negative value. Note also that B, which represents the contribution of the positional player at which $g_{\mathrm{C}}^{b}(B) = 0$, takes a positive value only under complementarity.

Positive contribution by the positional player requires $A > 0$, i.e. the necessary condition:

$$v_{\mathrm{P}} > 1 - \alpha_{\mathrm{P}}. \tag{12}$$

This opens up the possibility for positive contributions for the conformist player as well. But, assuming (12) is satisfied, under what circumstances would the conformist player imitate the positional player and contribute? For a moderate degree of conformism, $v_{\mathrm{C}} < \alpha_{\mathrm{C}}\varepsilon$, this player regards g_{P} and g_{C} as substitutes, and would contribute nothing, no matter how much the positional player contributes.

Interestingly, if the degree of conformism is above $\alpha_{\mathrm{C}}\varepsilon$, complementarity can induce the conformist player to contribute, following the behavior of the positional. However, the conformist player imitates the positional only if the contribution of this latter is large enough; in particular, greater than B, which marks the minimum contribution by the positional player to induce positive contributions from the conformist (positive marginal utility of the first unit contributed by this player). Therefore, $w_{\mathrm{P}} > B$ is a first necessary condition, otherwise the positional player would never contribute enough to induce the conformist player to contribute. Moreover, since the positional player never contributes above A, then $A > B$ is also a necessary condition. That is, $0 < B < \min\{A, w_{\mathrm{P}}\}$, or equivalently,

$$v_{\mathrm{C}} > \alpha_{\mathrm{C}}\varepsilon + \max\left\{\frac{1 - \alpha_{\mathrm{C}}}{w_{\mathrm{P}}}, \frac{1 - \alpha_{\mathrm{C}}}{A}\right\} \equiv \underline{v}_{\mathrm{C}}(v_{\mathrm{P}}). \tag{13}$$

Given that this threshold is larger than $\alpha_{\mathrm{C}}\varepsilon$, condition (13) can be interpreted as a strong complementarity requirement for the conformist player to contribute a positive amount. Conversely, weak complementarity is defined as $v_{\mathrm{C}} \in (\alpha_{\mathrm{C}}\varepsilon, \underline{v}_{\mathrm{C}})$. In this case, complementarity is not enough to induce the conformist agent to contribute.

3.2 Existence and uniqueness of the Nash equilibrium

Condition (12) is necessary and sufficient for positive contributions by the positional player. It is also a necessary condition for positive contributions by the conformist player. Moreover, the latter contributes only under the strong complementarity condition (13). Under substitutability or weak complementarity, only the positional player can contribute. In this case, she would build the amount of public good which maximizes her welfare, A, unless this amount surpasses her endowment, w_{P}.

For each pair $(v_{\mathrm{P}}, v_{\mathrm{C}}) \in \mathbb{R}^{+} \times \mathbb{R}^{+}$ a unique equilibrium exists. The equilibrium in the first quadrant is characterized in the following proposition. Note that the equilibrium is a continuous function of $(v_{\mathrm{P}}, v_{\mathrm{C}})$.

Proposition 1 (Nash equilibrium) *Under Assumptions 2 and 3 there is a unique Nash equilibrium:*

a) If $v_P \leq 1-\alpha_P$,

$$(g_P^N, g_C^N) = (0,0).$$

b) If $v_P > 1-\alpha_P$ and $v_C \leq \underline{v}_C$, then

$$(g_P^N, g_C^N) = \begin{cases} a_{int} = (A, 0) & \text{if} \quad v_P < \theta_P(w_P), \\ a_P = (w_P, 0) & \text{if} \quad v_P \geq \theta_P(w_P). \end{cases}$$

c) If $v_P > 1-\alpha_P$ and $v_C > \underline{v}_C$, then

- *an interior equilibrium $g_{int}^N = (g_{Pint}^N, g_{Cint}^N)$ exists if*

$$v_P < \theta_P(2w_P) \;\wedge\; v_C > B_P(v_P) \;\wedge\; \begin{cases} v_P < \theta_P(2w_C) \\ \text{or} \\ v_C < B_C(v_P) \end{cases} \tag{14}$$

- *equilibria where at least one of the players contributes her total endowment exist: $\big(w_P,\, g_C^b(w_P)\big)$, (w_P, w_C) and $\big(g_P^b(w_C), w_C\big)$. Moreover the two equilibria where the conformist player contributes w_C disappear if $w_P \leq w_C$.*

Proof. See Appendix A for the detailed proof of all these equilibria. ■

An interior solution arises when positionality does not push the positional player to want more than twice her endowment and the conformism is strong enough so that the positional player can rely on some contribution by the conformist player and does not need to contribute in full. Moreover, either the positional player does not want more than twice the endowment of the conformist, or if she does, the conformist does not fully contribute.

Status concerns could be so strong to induce one or both players to contribute all of their endowments. Full contribution by both agents occurs if both the positional concern and the degree of conformism are very high. Conversely, one agent would contribute her total endowment while the other would respond with an interior contribution when positionality is large and either only positionality or only conformism is very large.

Figure 1 depicts the regions for the different Nash equilibria depending on the positional concern and the degree of conformism,[5] considering the following parameters values: α_{P} =

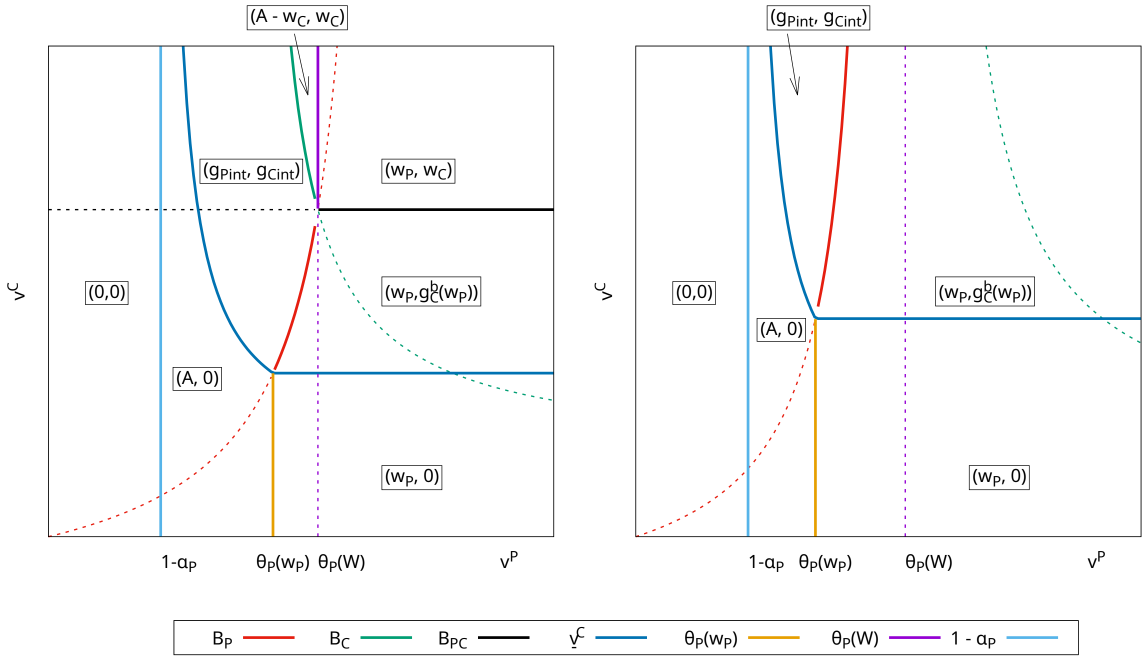

Figure 1: The zones of parameters in the $(v_{\mathrm{P}}, v_{\mathrm{C}})$-plane corresponding to the different configurations for the Nash equilibrium when $w_{\mathrm{P}} > w_{\mathrm{C}}$ (left) and $w_{\mathrm{P}} < w_{\mathrm{C}}$ (right). The zones are limited by continuous lines; dashed lines are the irrelevant parts of the boundary conditions.

$\alpha_{\mathrm{C}} = 2/3$, $\varepsilon = 1$ and $w_{\mathrm{P}} = 0.5 > w_{\mathrm{C}} = 0.2$ (left) or $w_{\mathrm{P}} = 0.3 < w_{\mathrm{C}} = 0.4$ (right). The definition of functions and explicit formulas are given in Appendix A.

Moreover we have the following properties for contributions:

Proposition 2 *Under the necessary condition (12) for positive contribution,*

i) global contribution never exceeds the amount wished by the positional player, $G^N \leq A$,

ii) the positional player always contributes above the conformist, $g_P^N > g_C^N$.

Proof. See Appendix A. ■

[5] In Figure 1 and henceforth,

- $B_i = B_i(v_{\mathrm{P}})$ represents the degree of conformism at which the interior solution of player i reaches her endowment, $g_{iint} = w_i$, provided that $v_{\mathrm{P}} < \theta_{\mathrm{P}}(2w_{\mathrm{P}})$;
- B_{PC} the minimum degree of conformism at which the best response of the conformist player to full contribution by the positional is also full contribution ($g_{\mathrm{C}}^b(w_{\mathrm{P}}) = w_{\mathrm{C}}$), provided that $w_{\mathrm{P}} > w_{\mathrm{C}}$ (if, instead, $w_{\mathrm{P}} < w_{\mathrm{C}}$ the conformist player never fully contributes);
- $\theta_{\mathrm{P}}(W)$ the minimum positional concern at which the best response of the positional player to full contribution by the conformist is also full contribution $g_{\mathrm{P}}^b(w_{\mathrm{C}}) = w_{\mathrm{P}}$;
- and $\theta_{\mathrm{P}}(w_{\mathrm{P}})$ the minimum positional concern at which the positional player wants a total amount greater than her endowment.

4 Welfare analysis in the interior solution

This section analyzes the effect of v_P and v_C on the private provision of the public good and on social welfare, assuming that positionality and conformism are strong enough to guarantee positive contributions, but not so large as to imply full contribution by one or both players. Thus, we focus the analysis on the interior solution, which we write as:

$$g_{\text{P}int} = \frac{A + \Delta g_{int}}{2}, \quad g_{\text{C}int} = \frac{A - \Delta g_{int}}{2}, \quad \Delta g_{int} = g_{\text{P}int} - g_{\text{C}int}. \tag{15}$$

From which the global contribution and the gap between players' contribution read:

$$G_{int} = A, \quad \Delta g_{int} = \frac{h(v_\text{P})}{\alpha_\text{P} v_\text{C}} := \frac{\alpha_\text{P} - \alpha_\text{C} + \alpha_\text{C} v_\text{P}}{\alpha_\text{P} v_\text{C}} = \frac{1 - \alpha_\text{C} + \alpha_\text{C}\varepsilon A}{v_\text{C}} > 0. \tag{16}$$

The Nash equilibrium belongs to the *interior* intersection of the two reaction curves under the following assumption:

Assumption 6 *Interior equilibrium: The positional player wants a positive amount of public good (Condition (12)), the conformist player regards contributions as strong complements (Condition (13)) and the static equilibrium does not lie on the boundary (Condition (14)).*

In the interior solution both players contribute a positive amount and also consume part of their endowment. As shown in (15), players' contributions can be written as dependent on two terms: the global contribution to the public good which matches the positional player's wished amount, A, (FOC for this player can be written as $G_{int} = A$); and the contribution gap wished by the conformist player (FOC for this player can be written as $\Delta g_{int} = (1 - \alpha_\text{C} + \alpha_\text{C}\varepsilon G_{int})/v_\text{C}$). The positional player would contribute half the amount of public good he wishes, plus half the gap. Correspondingly, the conformist player would contribute half the amount wished by the positional player reduced in half the gap.

To study the importance of the positional concern/conformism on social welfare, we first characterize how one player's positional concern affects her as well as her opponent's contribution.

Proposition 3 *Under Assumption 6,*

- *A greater positional concern, v_P implies:*

$$\frac{\partial g_{\text{P}int}}{\partial v_P} = \frac{v_C + \alpha_C\varepsilon}{2\alpha_P\varepsilon v_C} > 0, \quad \frac{\partial g_{\text{C}int}}{\partial v_P} = \frac{v_C - \alpha_C\varepsilon}{2\alpha_P\varepsilon v_C} > 0, \quad \frac{\partial G_{int}}{\partial v_P} = \frac{1}{\alpha_P\varepsilon} > 0, \quad \frac{\partial \Delta g_{int}}{\partial v_P} = \frac{\alpha_C}{\alpha_P v_C} > 0.$$

- *A greater degree of conformism, v_C implies:*

$$\frac{\partial g_{\text{P}int}}{\partial v_C} = -\frac{h(v_P)}{2\alpha_P(v_C)^2} < 0, \quad \frac{\partial g_{\text{C}int}}{\partial v_C} = -\frac{\partial g_{\text{P}int}}{\partial v_C} > 0, \quad \frac{\partial G_{int}}{\partial v_C} = 0, \quad \frac{\partial \Delta g_{int}}{\partial v_C} = 2\frac{\partial g_{\text{P}int}}{\partial v_C} < 0.$$

Proof. The proof is straightforward. ■

A stronger positional concern increases the contribution of the positional player, $g_{\text{P}int}$. Moreover, the contribution of the conformist player, $g_{\text{C}int}$, increases due to the strong complementarity. The contribution rise is stronger for the positional player than for the conformist one. As a result, the gap between players' contributions widens.

A higher degree of conformism will induce this player to increase her contribution in the exact same amount as the positional player reduces her contribution. Hence, the distribution of contributions between the two players becomes more equalitarian, although the global contribution remains unchanged.

Depending on the value of v_{P}, the next proposition characterizes the conditions under which global contribution is in shortage, in excess, or decreases the global intrinsic utility below its zero-contribution value:

Proposition 4 *Under Assumption 6, global contribution, G_{int}, is:*

$$\underset{1-\alpha_{\mathrm{P}}}{\vdash}\;\; G_{int} < G^{\mathrm{SO}} \text{ (shortage)} \;\; \underset{\frac{\alpha_{\mathrm{C}}}{\alpha_{\mathrm{P}}+\alpha_{\mathrm{C}}}}{+} \;\; G_{int} \in (G^{\mathrm{SO}}, G^{\mathrm{E0}}) \text{ (excess)} \;\; \underset{\frac{\alpha_{\mathrm{C}}}{\alpha_{\mathrm{P}}+\alpha_{\mathrm{C}}} + \frac{\alpha_{\mathrm{P}}+\alpha_{\mathrm{C}}-1}{\alpha_{\mathrm{P}}+\alpha_{\mathrm{C}}}\alpha_{\mathrm{P}}}{+} \;\; G_{int} > G^{\mathrm{E0}} \text{ (IW-R)} \;\; \rightarrow v_{\mathrm{P}}$$

Proof. Proof is straightforward from the definition of shortage, excess and intrinsic welfare reducing contribution, as well as the expression for the global contribution in (16). ■

Next, we analyze the effect of the positional concern, first, and the degree of conformism, later, on the global intrinsic utility, u_{int}, the global SCP, $V_{int} = V_{\mathrm{P}int} + V_{\mathrm{C}int}$, and the global social welfare, U_{int}.

Proposition 5 *Under Assumption 6, the positional concern, v_P, has an effect on:*

i) the global intrinsic utility,

$$\frac{\partial u_{int}}{\partial v_P} = \frac{\alpha_C - (\alpha_P + \alpha_C)v_P}{\alpha_P^2 \varepsilon} \gtrless 0 \Leftrightarrow v_P \lesseqgtr \frac{\alpha_C}{\alpha_P + \alpha_C},$$

ii) the global SCP,

$$\frac{\partial V_{int}}{\partial v_P} = \frac{(h(v_P))^2 + \alpha_C^2 v_P (1 - v_P)}{\alpha_P^2 v_C} > 0,$$

iii) and the social welfare,

$$\begin{aligned} v_P \leq \frac{\alpha_C}{\alpha_P + \alpha_C} \quad &\Rightarrow \quad \frac{\partial U_{int}}{\partial v_P} > 0, \\ v_P > \frac{\alpha_C}{\alpha_P + \alpha_C} \quad &\Rightarrow \quad \left(\frac{\partial U_{int}}{\partial v_P} \gtrless 0 \Leftrightarrow v_C \lesseqgtr B_{v_P} \equiv \frac{(h(v_P))^2 + \alpha_C^2 v_P (1 - v_P)}{v_P(\alpha_P + \alpha_C) - \alpha_C} \varepsilon \right). \end{aligned}$$

Proof. $\partial V_{int}/(\partial v_{\mathrm{P}}) > 0$ because $v_{\mathrm{P}} \in (0,1)$. The effect of v_{P} on social welfare can be written as:

$$\frac{\partial U_{int}}{\partial v_{\mathrm{P}}} = \frac{v_{\mathrm{C}}[\alpha_{\mathrm{C}} - v_{\mathrm{P}}(\alpha_{\mathrm{P}} + \alpha_{\mathrm{C}})] + [h^2(v_{\mathrm{P}}) + \alpha_{\mathrm{C}}^2 v_{\mathrm{P}}(1 - v_{\mathrm{P}})]\varepsilon}{v_{\mathrm{C}} \alpha_{\mathrm{P}}^2 \varepsilon}.$$

The result in Proposition 5 *iii)* straightforwardly follows. ■

Figure 2 illustrates the results of Proposition 5. The parameters values are the same as in Figure 1 (left). A higher positional concern enhances both players' contributions, and a greater contribution increases global intrinsic utility if the marginal cost of providing it is lower

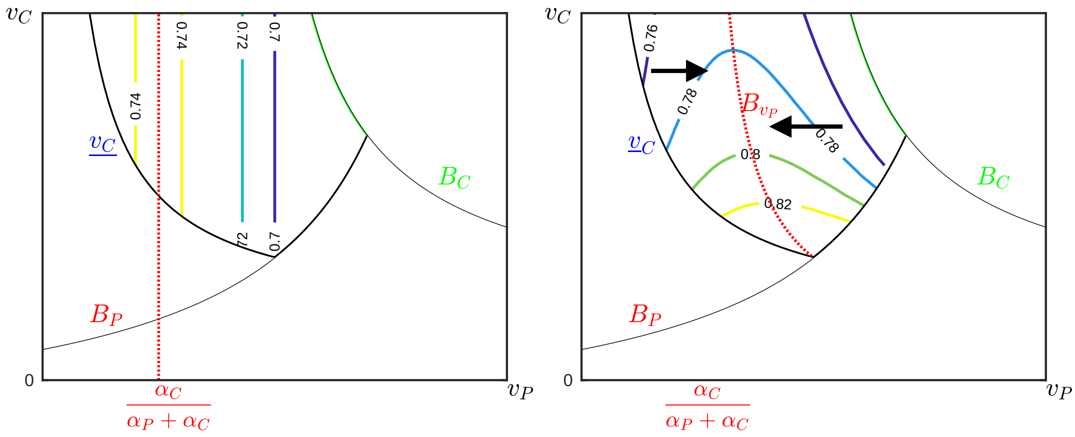

Figure 2: Level curves of u_{int} (left) and U_{int} (right), $\alpha_P = \alpha_C$

than the global marginal benefit. This occurs when the contribution is in shortage, i.e. iff $v_P < \alpha_C/(\alpha_P + \alpha_C)$. Opposite reasoning applies under excess contribution, $v_P > \alpha_C/(\alpha_P + \alpha_C)$, see Figure 2 (left).

Moreover, the SCP to both players are also influenced by a higher v_P from price and quantity effects. The price effect only affects the positional player (positively because $\Delta g_{int} > 0$). The quantity effect of a wider gap between the players' contributions enhances the SC gains for the positional player but also the SC losses for the conformist. Adding up the price and quantity effects for the two players, the global SCP rises with v_P.

Under a shortage in contributions, higher positional concern raises contributions, intrinsic utility, and global SCP, hence implying greater social welfare. Under excess contributions, a rise in v_P worsens intrinsic utility although it improves global SCP. In aggregate, the second effect is stronger and social welfare increases if the positional concern is small enough, below B_{v_P}, (see Proposition 5 *iii*)) and vice versa. As shown in Figure 2 (right), the level curves for social welfare increase with v_P to the left of curve B_{v_P} but they decrease with v_P to the right of this curve.

Proposition 6 *Under Assumption 6, the degree of conformism, v_C, does not affect intrinsic utility. Therefore its effect on social welfare matches its effect on global SCP, given by:*

$$\frac{\partial U_{int}}{\partial v_C} = \begin{cases} > 0 & \textit{if} \quad \alpha_P > \alpha_C \wedge v_P < \dfrac{\alpha_P - \alpha_C}{2\alpha_P - \alpha_C} \\ \leq 0 & \textit{otherwise.} \end{cases}$$

Proof. See Appendix A. ■

When $\alpha_P \leq \alpha_C$ a higher degree of conformism undoubtedly reduces intrinsic utility, as illustrated in Figure 2 (for the case when both players equally value public good consumption). Its effect when $\alpha_P > \alpha_C$ depends on the status concern of the positional player, as shown in Figure 3. A higher degree of conformism will induce the conformist player to increase her contribution to approach that of the positional one, and the latter will reduce hers by the same amount. Because the amount of public good does not change, neither does the intrinsic utility for the two players altogether. However, a higher v_C also influences the SCP. The "price" effect of a rise in the degree of conformism affects the conformist player negatively. Moreover, a higher degree of conformism narrows the gap between the players' contributions, which reduces the SC gains for the positional player and the SC losses for the conformist player. Thus, the total effect for society will be positive (negative) if the gain from a narrower gap for the conformist player is larger (smaller) than the addition of two negative effects: the negative "price effect" for this player from a more costly status concern plus the negative effect of a narrower gap on the positional player. The net effect is positive if the positional player values the public good more than the conformist (α_P relatively large) and her positional concern is sufficiently small. Conversely, if her positional concern is large a higher degree of conformism reduces social welfare. Conformism also reduces social welfare if the conformist player values the public good more than the positional player (α_C relatively large), no matter how large this latter's positional concern. Figure 3 depicts the case with $\alpha_P = 4/5 > \alpha_C = 1/3$. For $v_P < (\alpha_P - \alpha_C)/(2\alpha_P - \alpha_C)$, level curves increase with v_C (up-movements). And conversely, for $v_P > (\alpha_P - \alpha_C)/(2\alpha_P - \alpha_C)$, the level curves increase with reductions in v_C (down-movements).

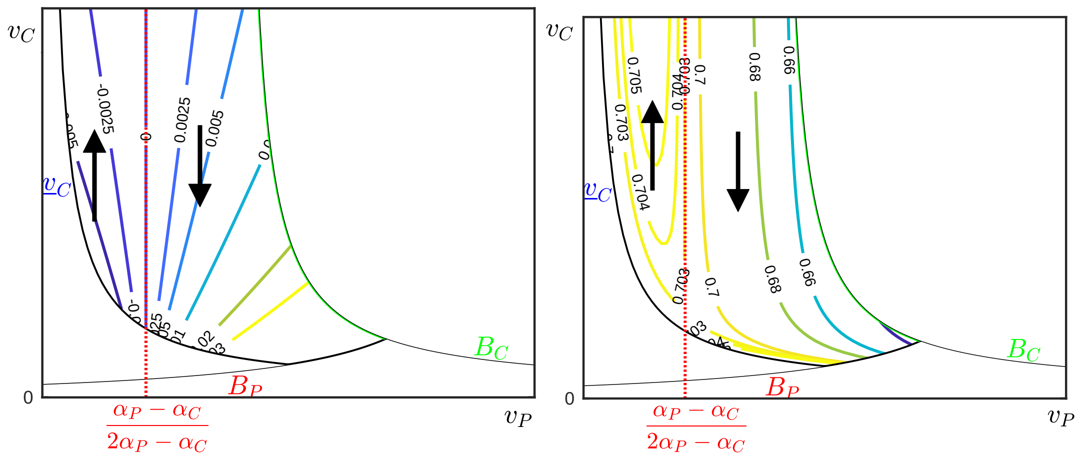

Figure 3: Level curves of V_{int} (left) and U_{int} (right), $\alpha_P > \alpha_C$

5 The dynamic model with myopic players

This section extends the static model previously described to a dynamic context. The dynamics comes from two facts. First, the social context is based on the comparison of one player's contribution against the other players' contribution observed in the previous period. Second, players are reluctant to deviate from previous actions: inertia with respect to past contributions.

In the present section, we define the model and present its main properties. In the subsequent Section 6, we numerically illustrate the main results. The dynamic version of the game is presented considering discrete time. The intrinsic utility at time step t results from current contributions g_{Pt} and g_{Ct}. The positional agent gets joy from contributing above the other player's contribution at the previous step: $+v_P(g_{Pt} - g_{Ct-1})$. On the other hand, the conformist agent gets a disutility from deviations from the other player's previous action: $-v_C(g_{Ct} - g_{Pt-1})^2/2$. Coefficients v_i, $i \in \{P, C\}$ are nonnegative.

Additionally, it is assumed that players show inertia: they are reluctant to change their action from a time step to the next. They get a disutility from deviations from the previous action: $-v_i^I(g_{it} - g_{it-1})^2/2$, $i \in \{P, C\}$. Coefficients v_i^I, $i \in \{P, C\}$ are nonnegative. As a result, the utilities of both players at time step t now read:

$$U_{\mathrm{P}}(\bullet) = w_{\mathrm{P}} - g_{\mathrm{P}t} + \alpha_{\mathrm{P}}\left[G_t - \frac{\varepsilon}{2}G_t^2\right] - \frac{v_{\mathrm{P}}^{\mathrm{I}}}{2}(g_{\mathrm{P}t} - g_{\mathrm{P}t-1})^2 + v_{\mathrm{P}}(g_{\mathrm{P}t} - g_{\mathrm{C}t-1}) \tag{17}$$
$$U_{\mathrm{C}}(\bullet) = w_{\mathrm{C}} - g_{\mathrm{C}t} + \alpha_{\mathrm{C}}\left[G_t - \frac{\varepsilon}{2}G_t^2\right] - \frac{v_{\mathrm{C}}^{\mathrm{I}}}{2}(g_{\mathrm{C}t} - g_{\mathrm{C}t-1})^2 - \frac{v_{\mathrm{C}}}{2}(g_{\mathrm{C}t} - g_{\mathrm{P}t-1})^2.$$

The problem of a myopic agent $i \in \{P, C\}$ is:

$$\max_{g_{it}} U_i(g_{it}, g_{-it}, g_{it-1}, g_{-it-1}), \tag{18}$$

the values g_{it-1} being given. The first-order conditions for an interior equilibrium are:

$$-1 + \alpha_{\mathrm{P}}[1 - \varepsilon(g_{\mathrm{P}t} + g_{\mathrm{C}t})] - v_{\mathrm{P}}^{\mathrm{I}}(g_{\mathrm{P}t} - g_{\mathrm{P}t-1}) + v_{\mathrm{P}} = 0 \tag{19}$$
$$-1 + \alpha_{\mathrm{C}}[1 - \varepsilon(g_{\mathrm{P}t} + g_{\mathrm{C}t})] - v_{\mathrm{C}}^{\mathrm{I}}(g_{\mathrm{C}t} - g_{\mathrm{C}t-1}) - v_{\mathrm{C}}(g_{\mathrm{C}t} - g_{\mathrm{P}t-1}) = 0. \tag{20}$$

Solving these linear equations, we obtain the unconstrained optimum as a function of the opponent's current action and both players' actions at the previous step:

$$r_{\mathrm{P}}(g_{\mathrm{C}t}; g_{\mathrm{P}t-1}, g_{\mathrm{C}t-1}) = \frac{A\alpha_{\mathrm{P}}\varepsilon + v_{\mathrm{P}}^{\mathrm{I}}g_{\mathrm{P}t-1} - \alpha_{\mathrm{P}}\varepsilon g_{\mathrm{C}t}}{v_{\mathrm{P}}^{\mathrm{I}} + \alpha_{\mathrm{P}}\varepsilon} \tag{21}$$
$$r_{\mathrm{C}}(g_{\mathrm{P}t}; g_{\mathrm{P}t-1}, g_{\mathrm{C}t-1}) = \frac{(-1 + \alpha_{\mathrm{C}})\alpha_{\mathrm{C}}\varepsilon + v_{\mathrm{C}}^{\mathrm{I}}g_{\mathrm{C}t-1} + v_{\mathrm{C}}g_{\mathrm{P}t-1} - \alpha_{\mathrm{C}}\varepsilon g_{\mathrm{P}t}}{v_{\mathrm{C}}^{\mathrm{I}} + v_{\mathrm{C}} + \alpha_{\mathrm{C}}\varepsilon}. \tag{22}$$

Taking the constraints into account, we deduce the dynamic reaction function as:

$$r_i^D(g_{-it}; g_{it-1}, g_{-it-1}) = \begin{cases} w_i & \text{if } r_i(g_{-it}; g_{it-1}, g_{-it-1}) \geq w_i \\ r_i(g_{-it}; g_{it-1}, g_{-it-1}) & \text{if } 0 \leq r_i(g_{-it}; g_{it-1}, g_{-it-1}) \leq w_i \\ 0 & \text{if } r_i(g_{-it}; g_{it-1}, g_{-it-1}) \leq 0. \end{cases} \tag{23}$$

It readily follows that a Nash equilibrium at step t must satisfy $g_{it} = r_i^D(g_{-it}; g_{it-1}, g_{-it-1})$ for $i \in \{P, C\}$. Provided the Nash equilibrium exists and is unique, this procedure defines a dynamical process $\{(g_{\mathrm{P}t}, g_{\mathrm{C}t}); t = 0, 1, \ldots\}$. First, we connect the fixed points of this dynamic system to the static Nash equilibrium.

Proposition 7 *The static Nash equilibrium, g^N, is a fixed point of the dynamical system, $g_{it} = r_i^D(g_{-it}; g_{it-1}, g_{-it-1})$ for $i \in \{P, C\}$.*

Proof. See Appendix B. ■

We are interested in trajectories that lie in the interior of the domain of constraints, for all t. Accordingly, under Assumption 6, we are certain that the static Nash equilibrium itself is interior. Then, the dynamic Nash equilibrium $g_{it} = r_i^D(g_{-it}; g_{it-1}, g_{-it-1})$, $i \in \{P, C\}$ is actually the unconstrained equilibrium $g_{it} = r_i(g_{-it}; g_{it-1}, g_{-it-1})$, at least when $(g_{\mathrm{P}t-1}, g_{\mathrm{C}t-1})$ is close enough to the static Nash equilibrium (see Proposition 8 *i*) below). As detailed in Appendix C.1, the resulting dynamical system is linear and can be written in vector and matrix form as:

$$g_t \;=\; \mathbf{M}g_{t-1} + \Gamma^0. \tag{24}$$

In what follows, we are interested in: a) whether this sequence does or does not converge to the fixed point $g^{\mathrm{N}} = (\mathbf{I} - \mathbf{M})^{-1}\Gamma^0$, and b) when it does, *how* it converges. Convergence can be *monotone*, *oscillating*, or *spiral*. In the last two cases, it is possible to observe *overshooting*: a situation in which one or both players contribute, albeit temporarily, more than in the long-run equilibrium.

Proposition 8 states that the trajectory stemming from the dynamic system (24), g_t, converges towards the static Nash equilibrium, provided that at least one of the players shows some inertia.

Proposition 8 *Under Assumption 6,*

i) If the initial condition lies in a neighborhood of the static Nash equilibrium, g_{int}, then the trajectory of the dynamical system remains an interior solution at every time step.

ii) When $v_i^I \neq 0$ at least for one player, the solution to the dynamical system (24) converges to a unique steady state, given by the point g_{int}.

iii) When $v_i^I = v_j^I = 0$, the solution to the dynamical system (24) does not converge in general. Its trajectory reaches, in finite time, a cycle of order 2, which oscillates around the point g_{int} and which depends on the initial condition.

Proof. See Appendix C.4. ■

While Proposition 8 addresses the convergence of the sequence g_t, it is not specific about the type of convergence: monotone, oscillating, or spiral. The following proposition provides this information.

Proposition 9 *Assume 6, and let $\chi(\lambda)$ be the quadratic polynomial defined as:*

$$\chi(\lambda) = (\lambda-1)v_P^I(\lambda(v_C+v_C^I)-v_C^I)+\lambda\varepsilon\left(\alpha_P(\lambda(v_C+v_C^I)-v_C^I+v_C)+\alpha_C v_P^I(\lambda-1)\right), \tag{25}$$

whose discriminant with respect to λ is:

$$\Delta_\lambda = (\varepsilon\alpha_C+v_C)^2(v_P^I)^2+(\varepsilon\alpha_P)^2(v_C^I)^2+2\varepsilon\alpha_P(\varepsilon\alpha_C-3v_C)v_P^Iv_C^I-2\varepsilon\alpha_P[v_C((\varepsilon\alpha_C+v_C)v_P^I+\varepsilon\alpha_P v_C^I)]+(\varepsilon\alpha_P v_C)^2 .$$

The convergence of the dynamical system (24) is: a) monotone when $\Delta_\lambda \geq 0$ and $\varepsilon\alpha_P v_C - \varepsilon\alpha_P v_C^I - (\varepsilon\alpha_C+v_C)v_P^I - 2v_P^Iv_C^I \leq 0$; b) oscillating when $\Delta_\lambda \geq 0$ and $\varepsilon\alpha_P v_C - \varepsilon\alpha_P v_C^I - (\varepsilon\alpha_C+v_C)v_P^I - 2v_P^Iv_C^I \geq 0$; c) spiral when $\Delta_\lambda < 0$. In this last case, d) the speed of convergence decreases with v_i^I and increases with v_C.

Proof. See Appendix C, including the proof of Proposition 9 in Appendix C.4. ■

A graphical illustration of how the type of convergence depends on inertia parameters, is presented in Figure 4. The parameters chosen for this figure are those used in the simulations of Section 6.1. The diagram shows the curve of points where the discriminant of polynomial $\chi(\lambda)$ vanishes.[6]

Roughly speaking, convergence is monotone when one of the player's inertia is relatively large[7] with respect to the other player's inertia. It is oscillating when inertia is relatively small for both players. Finally, convergence is spiral in the remaining cases - that is, when the sum of inertias is large enough, and neither one is "very large" with respect to the other.

[6] It also depicts the data points used in Section 6.1.

[7] The concept of relatively large is not symmetric between the two players. Convergence is monotone when the positional's inertia is much larger than the conformist's inertia, or when the conformist's inertia is very much larger than the positional's inertia.

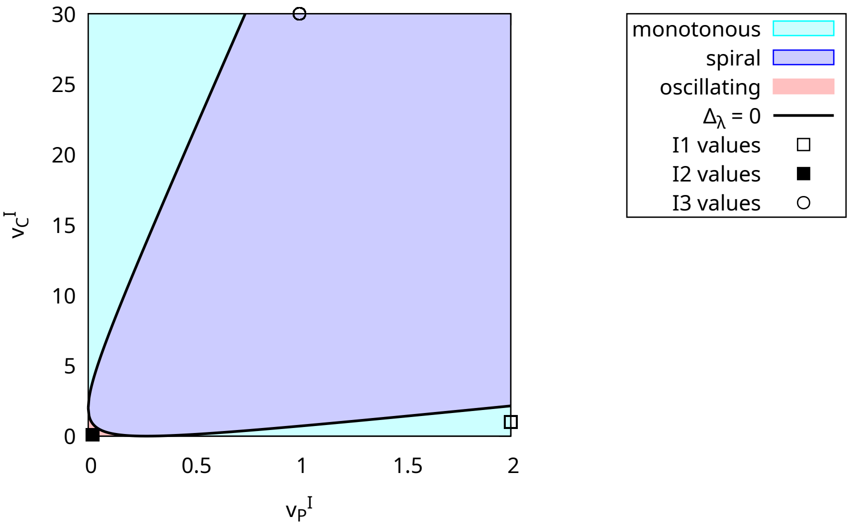

Figure 4: Type of convergence as a function of inertia parameters

We conclude this section with an observation on the relative sensitivity of players to their respective inertia parameter. Looking back at Figure 4 we identify two particular points where $\Delta_\lambda = 0$ and either $v_{\mathrm{P}}^{\mathrm{I}} = 0$ or $v_{\mathrm{C}}^{\mathrm{I}} = 0$. Using Proposition 9, it is easily seen that these points have coordinates:

$$\left(\frac{\varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}}}{\varepsilon\alpha_{\mathrm{C}} + v_{\mathrm{C}}}, 0\right) \quad \text{and} \quad (0, v_{\mathrm{C}})\,.$$

This leads to the idea to normalize the function $\chi(\cdot)$ by performing the following change of parameters:

$$v_{\mathrm{P}}^{\mathrm{I}} = \theta\,\frac{\varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}}}{\varepsilon\alpha_{\mathrm{C}} + v_{\mathrm{C}}} \qquad v_{\mathrm{C}}^{\mathrm{I}} = \varphi\, v_{\mathrm{C}}.$$

The new parameters θ and φ are dimensionless. The interpretation of φ is simply the relative intensity of the conformist player's inertia with respect to her conformism. The interpretation of θ is not as straightforward: it involves a relative inertia term $v_{\mathrm{P}}^{\mathrm{I}}/(\varepsilon\alpha_{\mathrm{P}})$ proper to the positional player, but also the modifying factor $(\varepsilon\alpha_{\mathrm{C}} + v_{\mathrm{C}})/v_{\mathrm{C}}$ relative to the conformist player.

After the change of parameters, the function χ becomes symmetric in θ and φ. Concerning the roots of χ, which determine the type of convergence, this implies two phenomena. First, swapping the values of θ and φ lead to the same roots. Second, a variation of θ is equivalent to the same variation of φ. This allows us to deduce how much the inertia parameters $v_{\mathrm{P}}^{\mathrm{I}}$ and $v_{\mathrm{C}}^{\mathrm{I}}$ must vary, in order to produce the same effect on convergence.

6 Simulations

We present in this section simulations corresponding to different cases under the assumption of an interior static Nash equilibrium. Our purpose is twofold. On the one hand, we illustrate the dependence of the static Nash equilibrium on the parameters. On the other, we illustrate different types of transition. In all the examples, the players' endowments are $w_{\mathrm{P}} = w_{\mathrm{C}} = 1$ and the initial contribution is set to $(g_{\mathrm{P}0}, g_{\mathrm{C}0}) = (0, 0)$.

In a first example, we let inertia, positionality, and conformism to vary, in order to illustrate different features of the dynamics. In the second example, we focus on utilities, in particular in the case where the players' utility, individually as well as globally, can be temporarily reduced with respect to the situation of no contribution (which corresponds to the Nash equilibrium without status concerns). In the third example, we study a situation where the contribution of the conformist player can exceed that of the positional in the transition path.

6.1 First example

The common parameters in all experiments are:

$$\varepsilon = \frac{4}{10}, \alpha_{\mathrm{P}} = \alpha_{\mathrm{C}} = \frac{4}{5}.$$

Consequently, the critical values determining shortage, excess and intrinsic welfare reduction (see Section 2.1) are: $G^{SO} = 15/16$ and $G^{E0} = 15/8$.

Varying inertia. Assume that the degree of positionalism and conformism are fixed at $v_{\mathrm{P}} = 1/3$ and $v_{\mathrm{C}} = 2$, while inertia is determined by four different sets of parameters:

$$\text{Case I1: } v_{\mathrm{P}}^{\mathrm{I}} = 2, v_{\mathrm{C}}^{\mathrm{I}} = 1, \quad \text{Case I2: } v_{\mathrm{P}}^{\mathrm{I}} = \frac{2}{100}, v_{\mathrm{C}}^{\mathrm{I}} = \frac{1}{10},$$

$$\text{Case I3: } v_{\mathrm{P}}^{\mathrm{I}} = 1, v_{\mathrm{C}}^{\mathrm{I}} = 30, \quad \text{Case I4: } v_{\mathrm{P}}^{\mathrm{I}} = v_{\mathrm{C}}^{\mathrm{I}} = 0.$$

The static Nash equilibrium for all four experiments (which is also the steady state of the dynamical system) is $(g_{\mathrm{P}}^{\mathrm{N}}, g_{\mathrm{C}}^{\mathrm{N}}) = (7/24, 1/8) \simeq (0.2917, 0.125)$. At the long-run equilibrium contributions are in shortage, $g_{\mathrm{P}}^{\mathrm{N}} + g_{\mathrm{C}}^{\mathrm{N}} = A = 5/12 < G^{SO}$. The first three cases are placed in the three different regions in Figure 4: monotone (I1), oscillating (I2) and spiral (I3).

The phase diagram for these three cases is displayed in Figure 5 (left) and the time paths are represented in Figure 5 (right). The trajectory of Case I4 is periodic (see below) and is omitted for the sake of clarity.

In Case I1, the convergence to the steady state is monotone: both players' contributions increase toward that of the static Nash equilibrium (the eigenvalues of the linear dynamic system are real and positive: $\lambda_1 \simeq 0.36$ and $\lambda_2 \simeq 0.74$).

In Case I2, the convergence is oscillating: contributions successively overshoot and undershoot the Nash equilibrium[8] (the eigenvalues are real and negative: $\lambda_1 \simeq -0.77$ and $\lambda_2 \simeq -0.36 \times 10^{-2}$).

In Case I3, the convergence is spiral (the eigenvalues are complex: $\lambda_i \simeq 0.84 \pm 0.06i$).

The speed of convergence is determined by the modulus of the largest eigenvalue: 0.74, 0.77, and 0.84, respectively.

Case I4 is the case without inertia: the trajectory does not converge to the static Nash equilibrium. Instead, it perpetually oscillates between contributions (5/12,0) and (1/6,1/4). The middle point between them is the Nash equilibrium (7/24,1/8). This behavior is predicted by Proposition 8 *iii*).

[8] Because the second eigenvalue is very small, the successive contributions seem to be lying on the same straight line, which slope is determined by the eigenvector of the linear recurrence.

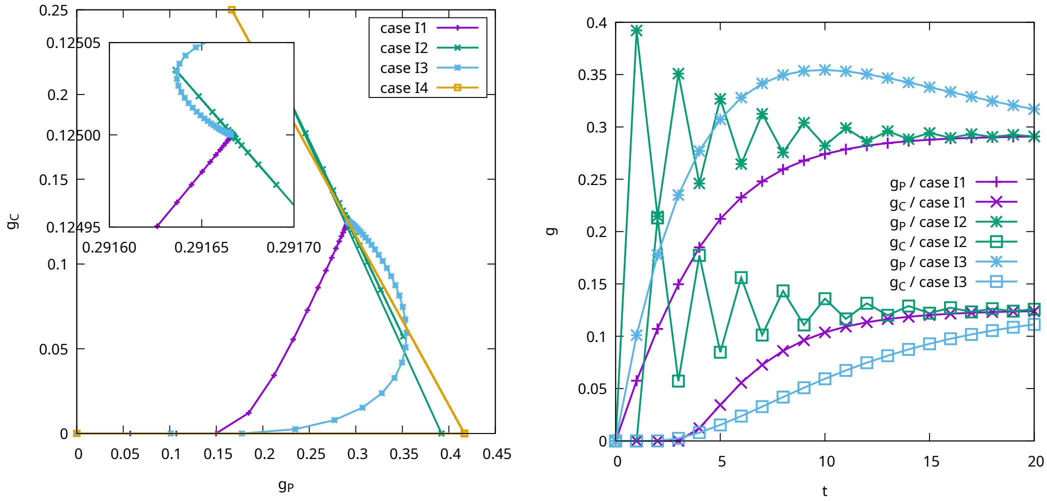

Figure 5: Phase diagram (left) and trajectories (right); varying inertia

In Case I3, with spiral convergence, both players temporarily contribute more than at the steady-state equilibrium. Likewise, in Case I2 with oscillating convergence, and Case I4 with fluctuation and non-convergence, the two players alternate contributions above and below the long-run Nash equilibrium. However, global contribution never surpasses G^{SO} and the trajectory remains under shortage across the transition.

Varying positionalism. Assume that the degree of conformism and inertia are fixed at $v_{\mathrm{C}} = 2$, $v^{\mathrm{I}}_{\mathrm{P}} = 1$ and $v^{\mathrm{I}}_{\mathrm{C}} = 10$, while the degree of positionalism takes three different values:

$$\text{Case P1: } v_{\mathrm{P}} = 1/3, \qquad \text{Case P2: } v_{\mathrm{P}} = 0.4373, \qquad \text{Case P3: } v_{\mathrm{P}} = 0.56.$$

Being interior, the static Nash equilibria for these three experiments must satisfy $g^{\mathrm{N}}_{\mathrm{C}} = g^{b}_{\mathrm{C}}(g^{\mathrm{N}}_{\mathrm{P}}) = r(g^{\mathrm{N}}_{\mathrm{P}} - B)$. Consequently, they all lie on the line $g^{\mathrm{N}}_{\mathrm{C}} = 21g^{\mathrm{N}}_{\mathrm{P}}/29 - 5/58$. The phase diagram displayed in Figure 6 (left) shows that a greater positional concern enhances each player's contribution in the long run, as proven in Proposition 3. It appears to be so through the transition as well.

The type of convergence to the equilibrium depends on the eigenvalues of the dynamical system, which are independent of parameter v_{P}. Then, the eigenvalues are given by the complex numbers: $\lambda_i \simeq 0.77 \pm 0.16$, with modulus $|\lambda_i| \simeq 0.79$, in all three cases. Convergence to the Nash equilibrium is therefore spiral.

Varying conformism. Assume that positionalism and inertia are fixed at $v_{\mathrm{P}} = 1/3$, $v^{\mathrm{I}}_{\mathrm{P}} = 1$ and $v^{\mathrm{I}}_{\mathrm{C}} = 10$, while the degree of conformism, v_{C}, takes three different values:

$$\text{Case C1: } v_{\mathrm{C}} = 2, \qquad \text{Case C2: } v_{\mathrm{C}} = 10, \qquad \text{Case C3: } v_{\mathrm{C}} = 20.$$

The Nash equilibria for these three experiments all lie on the line $g_{\mathrm{P}} + g_{\mathrm{C}} = A = 5/12$, as defined in (10). Global contribution in the long run is the same in all three cases. However, as predicted by Proposition 3, it is more equally distributed between the two players (closer to the 45° line) the greater the degree of conformism. The phase diagram displayed in Figure 6 (right), shows that a greater conformism implies more equalitarian contributions not only in the long run, but also through the transition. In all three cases, the eigenvalues are complex: respectively $\lambda_i \simeq 0.77 \pm 0.16i$, $0.57 \pm 0.23i$, and $0.46 \pm 0.19i$, with respective modulus: 0.79, 0.61, and 0.50. Convergence is therefore spiral and faster the greater the degree of conformism, in accordance with Proposition 9 d).

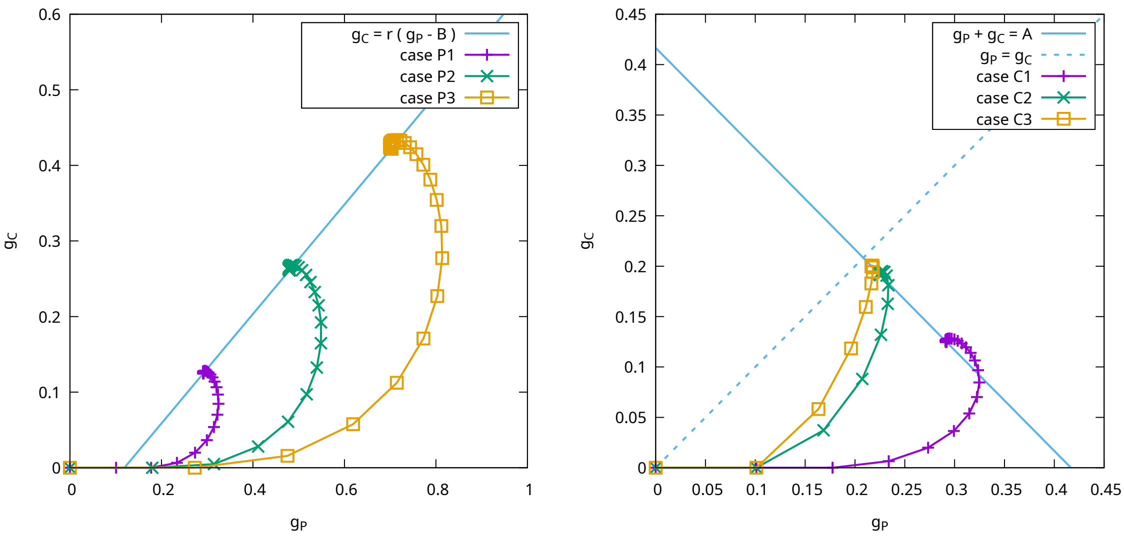

Figure 6: Phase diagram for the first example; varying v_{P} (left) and varying v_{C} (right)

6.2 Second example

We focus now on the sequence of players' "instantaneous" utilities along the dynamic trajectory. This example illustrates the possibility of a non-monotone time path for the intrinsic and the global utility. This feaure is associated with overshooting in the global contribution. Large overshooting may raise contributions so high as to be in excess and even intrinsic welfare reducing. In this latter case, intrinsic utility decreases below its level when players do not contribute at all. Moreover, global utility can also be temporarily lower than in the case of zero contribution.

The fixed parameters for this experiment are:

$$\varepsilon = \frac{4}{10}, \alpha_{\mathrm{P}} = \frac{4}{5}, v_{\mathrm{P}} = 0.56, v_{\mathrm{C}} = 2, v_{\mathrm{P}}^{\mathrm{I}} = 1.$$

And we distinguish four cases depending on the degree of conformism and on whether the conformist player values the public good equal or less than the positional one:[9]

$$\text{Case 1: } \alpha_{\mathrm{C}} = \frac{4}{5}, v_{\mathrm{C}}^{\mathrm{I}} = 10, \qquad \text{Case 2: } \alpha_{\mathrm{C}} = \frac{4}{5}, v_{\mathrm{C}}^{\mathrm{I}} = 30,$$

$$\text{Case 3: } \alpha_{\mathrm{C}} = \frac{1}{2}, v_{\mathrm{C}}^{\mathrm{I}} = 10, \qquad \text{Case 4: } \alpha_{\mathrm{C}} = \frac{1}{2}, v_{\mathrm{C}}^{\mathrm{I}} = 30.$$

[9]Note that Case 1 in this example coincides with Case P3 in example 6.1.

The steady state (static Nash equilibrium) for these parameters is interior, with values $(g_{\rm P}^{\rm N}, g_{\rm C}^{\rm N}) = (281/400, 169/400)$ in the first two cases, and $(g_{\rm P}^{\rm N}, g_{\rm C}^{\rm N}) = (119/160, 61/160)$ in the last two. Both lie on the line $g_{\rm P}^{\rm N} + g_{\rm C}^{\rm N} = 9/8$.

Figure 7 (left) shows the difference between the current intrinsic utility and its value with no contribution, W. In Cases 3 and 4, the trajectory for contributions successively exhibits shortage, excess, welfare reducing, and finally excess again. Correspondingly, the instantaneous intrinsic utility in Figure 7 (left) increases first and decays later to fall even below W, to finally rise above this value. Cases 1 and 2 follow a similar pattern, but the intrinsic utility never drops below W. Figure 7 (right) depicts the difference between global utility with and without contribution. Its behavior is similar to the behavior of the intrinsic utility, although showing greater variations. In Case 3, where the conformist player has low inertia, even though intrinsic utility falls below the zero-contribution case, the global utility does not; benefits associated with status concerns compensate for the loss of intrinsic utility.

Figure 8 (left) depicts the SCP over time, for each player. Social comparison always implies utility losses for the conformist player and utility gains for the positional player (who contributes more than the conformist player). The conformist player has a relatively higher inertia in Cases 2 and 4 than in Cases 1 and 3. When the conformist player has a high change-aversion she allows a wider contribution gap to open and waits longer to close it. Within this period, when the gap is wide, the SC gains for the positional increase, although the SC losses for the conformist player increase even further. The addition of gains and losses can become negative for some periods of time, especially when the conformist player has high change-aversion, as illustrated in Figure 8 (right). This explains why when comparing Figure 7 left and right, one observes a wider variation in global than in intrinsic utility. While the change-aversion of the conformist player has a negligible effect on intrinsic utility, it considerably reduces global SCP.

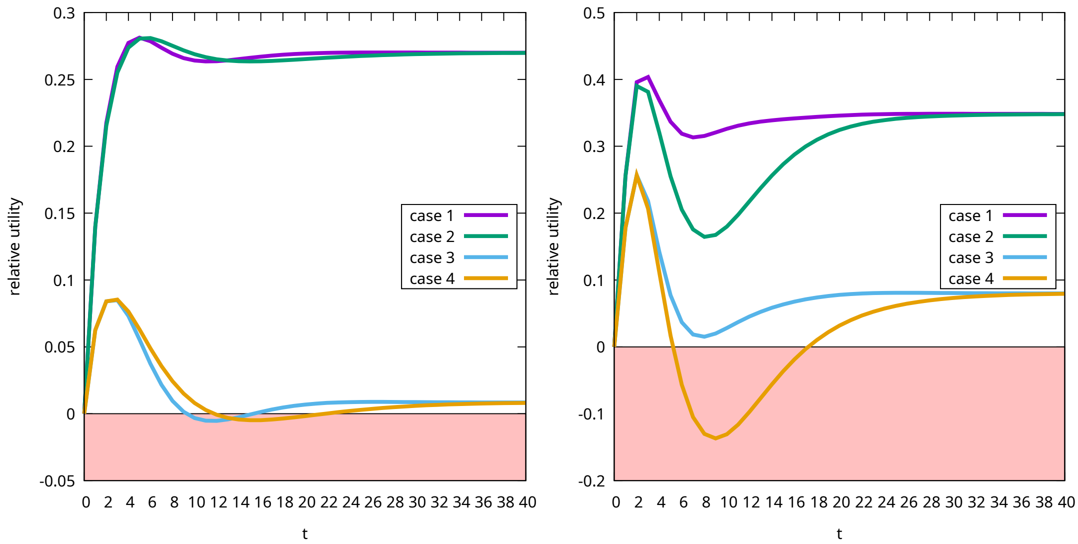

Figure 7: Global intrinsic utility (left) and global utility (right), relative to $u_0 := W$

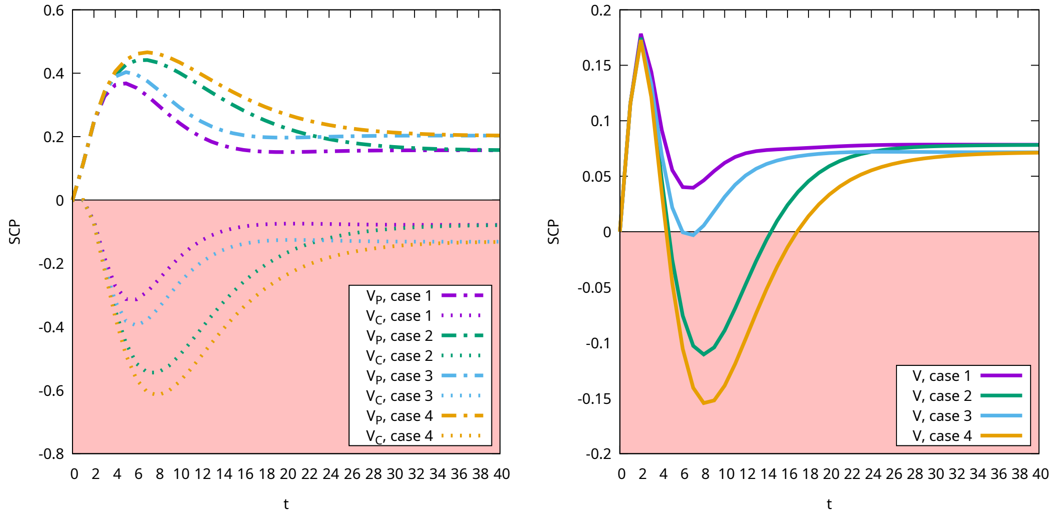

Figure 8: SCP for each player (left) and globally (right)

6.3 Third example

In the static (or the long-run) equilibrium, the positional player always contributes more than the conformist. However, this experiment illustrates that the overshooting phenomenon may cause the conformist player to temporarily contribute more than the positional. We consider two sets of parameters:

$$\text{Case 1:} \qquad \varepsilon = \frac{4}{10}, \alpha_P = \alpha_C = \frac{4}{5}, v_P = 0.56, v_C = 50, v_P^I = 1, v_C^I = 150$$
$$\text{Case 2:} \qquad \varepsilon = \frac{4}{10}, \alpha_P = \alpha_C = \frac{4}{5}, v_P = \frac{1}{3}, v_C = 50, v_P^I = \frac{2}{100}, v_C^I = \frac{1}{10}.$$

In both cases, conformism is very high and the conformist player has much larger inertia than the positional one. The trajectories of the contributions are displayed in Figure 9 (right), as time paths, and in Figure 9 (left), as a phase diagram. Convergence to the Nash equilibrium is spiral in Case 1, due to the large inertia of the conformist player. Convergence is oscillating in Case 2 because both inertia parameters are small. In both cases, the contribution of the conformist player surpasses that of the positional at several time steps. Eventually, contributions become close enough to their equilibrium and the contribution of the positional player remains larger than that of the conformist.

7 Concluding remarks

The paper analyzes the strategic interaction between agents keen on distinguishing themselves from the common herd, and other agents who are willing to imitate this common-herd behavior. Social context is introduced in a game of private provision of a public good considering two different players. The utility of the positional player rises/decreases when she contributes more/less than her opponent. The conformist player gets lower utility the more she deviates, above or below, from the contribution of her opponent.

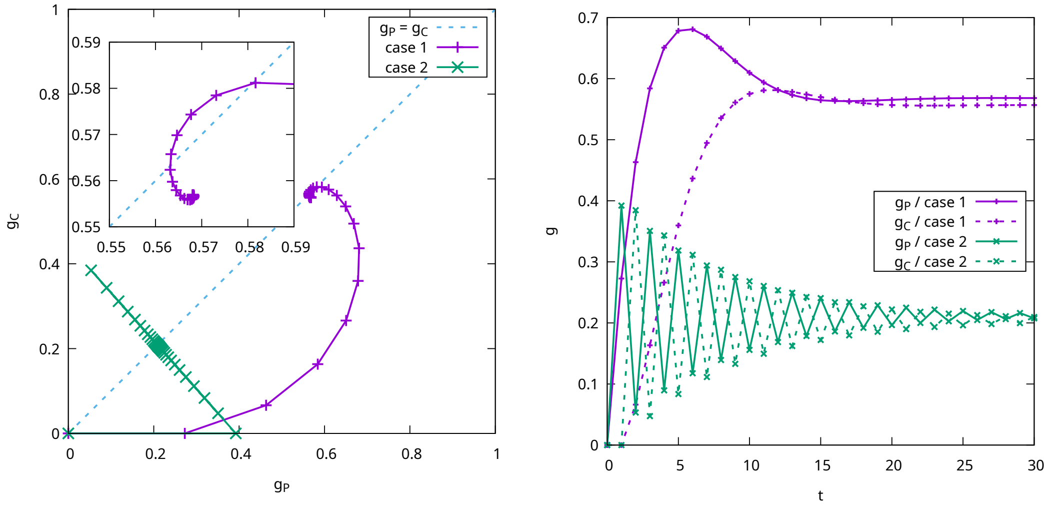

Figure 9: Contributions of players in the third example: phase diagram (left) and trajectories (right)

For a static game, we characterize the minimum positionality for the positional player to start contributing. Above this threshold, the conformist player would imitate the contributing behavior, only if she regards contributions as strong complementary goods (a degree of conformism sufficiently large). We also characterize the conditions for the contributions not to exhaust the global players' endowment. In either case, with positive contribution, the positional player always contributes more than the conformist

The status concerns of the players have different effect on contributions. Positionality enhances both players' contribution, and the increment is higher for the positional player than for the conformist one. Thus, positionality increases both intrinsic utility and the status concern payoffs (globally, for the two players). The degree of conformism increases the contribution by the conformist player in the exact same amount as it reduces the contribution by the positional, hence, leaving global contribution unchanged. Therefore, the contribution gap between the players widens with positionality and it narrows with the degree of conformism. Conformism has no effect on intrinsic utility, while the status concern payoffs increase only if the conformist player values the public good little and has a small status concern.

We extend the static game to a dynamic setting, considering that peoples' social concerns are built by comparing their current contribution to what they saw their peers contributed in the past, and assuming further that agents show inertia. The time path for the contributions which solves this dynamic problem converges to the static Nash equilibrium.

First, we analytically characterize the dynamic solution and the type of convergence: monotonous, oscillatory, or spiral. Later, numerical simulations help to highlight which properties of the static equilibrium remain valid through the whole transition and which properties do not.

In a first set of examples, we observe how the difference in players' inertia can lead to monotone, oscillating, or spiral convergence. Moreover, the simulations show that a stronger status concern for the positional player raises contributions for the two players at the long-run (the static Nash equilibrium), as well as through the transition period. Likewise, a stronger status concern for the conformist player induces more egalitarian contributions between the two players, again at the long-run equilibrium and through the transition.

A second set of examples focuses on the overshooting period, when either one or both players contribute above their long-run equilibrium. Overshooting can be so strong as to temporarily lead intrinsic utility to below its value with zero contribution. This is also true for global utility or social welfare that also takes into account the payoffs linked to status concerns. This effect is more likely the more change-averse the conformist player is, and assuming that she values public good more than the positional player.

In a final example we show that for specific parameter values, it is even possible that the conformist player temporarily contributes more than the positional player. This can happen when the positional player has a strong status concern and she is much more change-averse than the positional.

We believe that an interesting extension would be to define a truly dynamic game, in which agents decisions would be made dependent on the stock of contributions carried out by the players up until a certain time. Another line for future research could introduce a prosocial agent as a third type of player.

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A Proofs for Nash equilibria

Proof of Proposition 1. With the notation $A = (\alpha_{\rm P} - 1 + v_{\rm P})/(\alpha_{\rm P}\varepsilon)$, $h(v_{\rm P}) = \alpha_{\rm P} - \alpha_{\rm C} + \alpha_{\rm C} v_{\rm P}$, introduced in the proposition, the potential interior solution can be written as:

$$g^{\rm N}_{{\rm P}int} = \frac{1}{2}\left(A + \frac{h(v_{\rm P})}{\alpha_{\rm P} v_{\rm C}}\right), \qquad g^{\rm N}_{{\rm C}int} = \frac{1}{2}\left(A - \frac{h(v_{\rm P})}{\alpha_{\rm P} v_{\rm C}}\right). \tag{26}$$

Define function $\theta_i(x) = 1 - \alpha_i + \alpha_i \varepsilon x \geq 1 - \alpha_i \geq 0$ and remember that

$$B = \frac{1 - \alpha_{\rm C}}{v_{\rm C} - \alpha_{\rm C}\varepsilon}, \qquad r = \frac{v_{\rm C} - \alpha_{\rm C}\varepsilon}{v_{\rm C} + \alpha_{\rm C}\varepsilon}.$$

The best-response functions are, from (10), (11)

$$g^b_{\rm P}(g_{\rm C}) = \begin{cases} 0 & \text{if} \quad A \leq g_{\rm C} & \Leftrightarrow \quad v_{\rm P} \leq \theta_{\rm P}(g_{\rm C}) \\ A - g_{\rm C} & \text{if} \quad A - w_{\rm P} \leq g_{\rm C} \leq A & \Leftrightarrow \quad v_{\rm P} \in (\theta_{\rm P}(g_{\rm C}), \theta_{\rm P}(w_{\rm P} + g_{\rm C})) \\ w_{\rm P} & \text{if} \quad g_{\rm C} \leq A - w_{\rm P} & \Leftrightarrow \quad v_{\rm P} \geq \theta_{\rm P}(w_{\rm P} + g_{\rm C}). \end{cases}$$

$$g^b_{\rm C}(g_{\rm P}) = \begin{cases} 0 & v_{\rm C} g_{\rm P} \leq \theta_{\rm C}(g_{\rm P}) \\ r(g_{\rm P} - B) & v_{\rm C}(g_{\rm P} - w_{\rm C}) \leq \theta_{\rm C}(g_{\rm P} + w_{\rm C}) \ \wedge \ v_{\rm C} g_{\rm P} \geq \theta_{\rm C}(g_{\rm P}) \\ w_{\rm C} & v_{\rm C}(g_{\rm P} - w_{\rm C}) \geq \theta_{\rm C}(g_{\rm P} + w_{\rm C}). \end{cases}$$

Remember that $\alpha_i - 1 < 0$, $i = \{P, C\}$, by Assumption 2. We have three main cases:

Case a. When $v_{\rm P} \leq 1 - \alpha_{\rm P} \equiv \theta_{\rm P}(0) \leq \theta_{\rm P}(g_{\rm C})$, then $g^b_{\rm P}(g_{\rm C}) = 0$, $\forall g_{\rm C} \in [0, w_{\rm C}]$. Then, for any Nash equilibrium, $g^{\rm N}_{\rm P} = 0$ and $g^{\rm N}_{\rm C} = g^b_{\rm C}(0)$. Because $\alpha_{\rm C} - 1 < 0$, $g^b_{\rm C}(0) = 0$. In that case, the unique Nash equilibrium is $(g^{\rm N}_{\rm P}, g^{\rm N}_{\rm C}) = (0, 0)$. This proves statement c).

In the rest of this proof, we assume that $v_{\rm P} > 1 - \alpha_{\rm P}$. Then $A > 0$ and since $h(v_{\rm P}) > 0$. Consequently, from (15) and (16), $g_{{\rm P}int} > g_{{\rm C}int}$ and $g_{{\rm P}int} > 0$.

Case b. Note first that in this case $g_{\rm P}$ can not be 0 at the equilibrium. Indeed, if it were the case, then $g^{\rm N}_{\rm C} \geq A > 0$ and $g^{\rm N}_{\rm C} = 0$ is excluded. However, when computing $g^b_{\rm C}(0)$, only the first case is satisfied since $0 \leq \theta_{\rm C}(0)$. This is a contradiction, hence the impossibility.
Consider now the possibility that $g^{\rm N}_{\rm C} = 0$. We have $g^b_{\rm C}(g_{\rm P}) = 0$ iff $v_{\rm C} \leq \theta_{\rm C}(g_{\rm P})/g_{\rm P}$ or equivalently $v_{\rm C} \leq \alpha_{\rm C}\varepsilon + (1 - \alpha_{\rm C})/g_{\rm P}$. From (10) $g_{\rm P} \leq \min\{A, w_{\rm P}\}$, and then $g^b_{\rm C}(g_{\rm P}) = 0$ for all $g_{\rm P}$ satisfying the previous condition if:

$$v_{\rm C} \leq \alpha_{\rm C}\varepsilon + \max\left\{\frac{1 - \alpha_{\rm C}}{A}, \frac{1 - \alpha_{\rm C}}{w_{\rm P}}\right\} = \underline{v}_{\rm C}(v_{\rm P}).$$

Then necessarily $g^{\rm N}_{\rm C} = 0$ and $g^{\rm N}_{\rm P} = g^b_{\rm P}(0)$. Accordingly, when $v_{\rm C} \leq \underline{v}_{\rm C}(v_{\rm P})$, there is a unique Nash equilibrium which is:

$$(g^{\rm N}_{\rm P}, g^{\rm N}_{\rm C}) = \begin{cases} a_{int} \equiv (A, 0) & v_{\rm P} < \theta_{\rm P}(w_{\rm P}) \\ a_{\rm P} \equiv (w_{\rm P}, 0) & v_{\rm P} \geq \theta_{\rm P}(w_{\rm P}). \end{cases}$$

This proves statement a).

Case c. Assume finally that $v_{\mathrm{P}} > 1-\alpha_{\mathrm{P}}$ and $v_{\mathrm{C}} > \underline{v}_{\mathrm{C}}(v_{\mathrm{P}})$. Under these conditions it is easy to see that $0 < B < \min\{A, w_{\mathrm{P}}\}$. As a preliminary, we prove now that $g^{\mathrm{N}}_{\mathrm{C}int} > 0$ when $g^{\mathrm{N}}_{\mathrm{P}int} \leq w_{\mathrm{P}}$. Indeed, when $A \leq w_{\mathrm{P}}$

$$g^{\mathrm{N}}_{\mathrm{C}int} > 0 \iff v_{\mathrm{C}} > \frac{h(v_{\mathrm{P}})}{\alpha_{\mathrm{P}} A} = \frac{\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}} + \alpha_{\mathrm{C}} v_{\mathrm{P}}}{\alpha_{\mathrm{P}} A} = \frac{1 - \alpha_{\mathrm{C}} + \alpha_{\mathrm{C}} A \varepsilon}{A}.$$

This last condition is satisfied because $v_{\mathrm{C}} > \underline{v}_{\mathrm{C}}(v_{\mathrm{P}})$. When $A > w_{\mathrm{P}}$, $g^{\mathrm{N}}_{\mathrm{C}int} \leq 0$ implies $g^{\mathrm{N}}_{\mathrm{P}int} \geq A$ (because $g^{\mathrm{N}}_{\mathrm{P}int} + g^{\mathrm{N}}_{\mathrm{C}int} = A$) which is a contradiction.
In this case one interior and three boundary equilibria are feasible. To analyse the condition in v_{P} and v_{C} under which these equilibria are feasible we have (remember that $h(v_{\mathrm{P}}) > 0$)

$$g^{\mathrm{N}}_{\mathrm{P}int} \leq w_{\mathrm{P}} \iff \varepsilon h(v_{\mathrm{P}}) \leq v_{\mathrm{C}}[\theta_{\mathrm{P}}(2w_{\mathrm{P}}) - v_{\mathrm{P}}] \iff \theta_{\mathrm{P}}(2w_{\mathrm{P}}) > v_{\mathrm{P}} \wedge v_{\mathrm{C}} \geq \frac{\varepsilon h(v_{\mathrm{P}})}{\theta_{\mathrm{P}}(2w_{\mathrm{P}}) - v_{\mathrm{P}}} := B_{\mathrm{P}}(v_{\mathrm{P}}). \tag{27}$$

Note that $B_{\mathrm{P}}(v_{\mathrm{P}})$ is an increasing function when $v_{\mathrm{P}} < \theta_{\mathrm{P}}(2w_{\mathrm{P}})$, goes to infinity when v_{P} tends to $\theta_{\mathrm{P}}(2w_{\mathrm{P}})$ from below and $B_{\mathrm{P}}(v_{\mathrm{P}})$ intercepts $\underline{v}_{\mathrm{C}}(v_{\mathrm{P}})$ when $v_{\mathrm{P}} = \theta_{\mathrm{P}}(w_{\mathrm{P}})$.
Similarly,

$$g^{\mathrm{N}}_{\mathrm{C}int} > w_{\mathrm{C}} \iff \varepsilon h(v_{\mathrm{P}}) < v_{\mathrm{C}}[v_{\mathrm{P}} - \theta_{\mathrm{P}}(2w_{\mathrm{C}})] \iff \theta_{\mathrm{P}}(2w_{\mathrm{C}}) < v_{\mathrm{P}} \wedge v_{\mathrm{C}} > \frac{\varepsilon h(v_{\mathrm{P}})}{v_{\mathrm{P}} - \theta_{\mathrm{P}}(2w_{\mathrm{C}})} := B_{\mathrm{C}}(v_{\mathrm{P}}). \tag{28}$$

Note that $B_{\mathrm{C}}(v_{\mathrm{P}})$ is a decreasing function when $v_{\mathrm{P}} > \theta_{\mathrm{P}}(2w_{\mathrm{C}})$, tends to infinity when v_{P} tends to $\theta_{\mathrm{P}}(2w_{\mathrm{C}})$ from above and its limit when v_{P} goes to infinity is $\varepsilon\alpha_{\mathrm{C}}$.
Moreover, $B_{\mathrm{P}}(v_{\mathrm{P}})$ intersects $B_{\mathrm{C}}(v_{\mathrm{P}})$ when $v_{\mathrm{P}} = \theta_{\mathrm{P}}(w_{\mathrm{P}}, w_{\mathrm{C}})$ and $B_{11}(\theta_{\mathrm{P}}(W)) = \frac{\theta_{\mathrm{C}}(W)}{w1 - w_{\mathrm{C}}}$.
Now we can analyse the different equilibria. Remember that we are in the case where $v_{\mathrm{P}} > 1 - \alpha_{\mathrm{P}}$ and $v_{\mathrm{C}} > \underline{v}_{\mathrm{C}}(v_{\mathrm{P}})$. In particular we know that $g^{\mathrm{N}}_{iint} > 0, i = \{P, C\}$ and $B < w_{\mathrm{P}}$. In this region

- $(g^{\mathrm{N}}_{\mathrm{P}int}, g^{\mathrm{N}}_{\mathrm{C}int})$ is an equilibrium if and only if (27) and the negation of (28) hold:

$$\left[\theta_{\mathrm{P}}(2w_{\mathrm{P}}) > v_{\mathrm{P}} \wedge v_{\mathrm{C}} \geq \frac{\varepsilon h(v_{\mathrm{P}})}{\theta_{\mathrm{P}}(2w_{\mathrm{P}}) - v_{\mathrm{P}}}\right] \wedge \left[\theta_{\mathrm{P}}(2w_{\mathrm{C}}) \geq v_{\mathrm{P}} \ \vee \ v_{\mathrm{C}} \leq \frac{\varepsilon h(v_{\mathrm{P}})}{v_{\mathrm{P}} - \theta_{\mathrm{P}}(2w_{\mathrm{C}})}\right].$$

  Note that when $w_{\mathrm{P}} < w_{\mathrm{C}}$ the second bracket is not relevant because $\theta_{\mathrm{P}}(2w_{\mathrm{C}}) > \theta_{\mathrm{P}}(2w_{\mathrm{P}}) > v_{\mathrm{P}}$ is automatically satisfied. Indeed we knew that $g^{\mathrm{N}}_{\mathrm{P}int} < w_{\mathrm{P}}$ implies $g^{\mathrm{N}}_{\mathrm{C}int} < g^{\mathrm{NPC}}_{\mathrm{P}int} < w_{\mathrm{C}}$.

- $(w_{\mathrm{P}}, w_{\mathrm{C}})$ is an equilibrium if and only if

$$v_{\mathrm{P}} \geq \theta_{\mathrm{P}}(W) \wedge v_{\mathrm{C}} \geq \frac{\theta_{\mathrm{C}}(W)}{w_{\mathrm{P}} - w_{\mathrm{C}}} := B_{\mathrm{PC}} \wedge w_{\mathrm{P}} > w_{\mathrm{C}}.$$

- $(A - w_{\mathrm{C}}, w_{\mathrm{C}})$ is an equilibrium if and only if

$$v_{\mathrm{P}} \in (\theta_{\mathrm{P}}(w_{\mathrm{C}}), \theta_{\mathrm{P}}(W)) \wedge v_{\mathrm{C}}(A - 2w_{\mathrm{C}}) \geq \theta_{\mathrm{C}}(A).$$

  Note that the second condition requires $A - 2w_{\mathrm{C}}$ to be positive, that is $v_{\mathrm{P}} > \theta_{\mathrm{P}}(2w_{\mathrm{C}})$. Then

$$v_{\mathrm{C}}(A - 2w_{\mathrm{C}}) \geq \theta_{\mathrm{C}}(A) \iff v_{\mathrm{P}} > \theta_{\mathrm{P}}(2w_{\mathrm{C}}) \wedge v_{\mathrm{C}} \geq B_{\mathrm{C}}(v_{\mathrm{P}}) \iff g^{\mathrm{N}}_{\mathrm{C}int} \geq w_{\mathrm{C}}.$$

  Note that $\theta_{\mathrm{P}}(2w_{\mathrm{C}}) < \theta_{\mathrm{P}}(w_{\mathrm{P}} + w_{\mathrm{C}}) \iff w_{\mathrm{P}} > w_{\mathrm{C}}$. In consequence this equilibrium exists when

$$w_{\mathrm{P}} > w_{\mathrm{C}} \wedge v_{\mathrm{P}} \in (\theta_{\mathrm{P}}(2w_{\mathrm{C}}), \theta_{\mathrm{P}}(W)) \wedge v_{\mathrm{C}} \geq B_{\mathrm{C}}(v_{\mathrm{P}}).$$

- $(w_{\mathrm{P}}, r(w_{\mathrm{P}} - B))$ is an equilibrium if and only if

$$r(w_{\mathrm{P}} - B) \leq A - w_{\mathrm{P}} \wedge v_{\mathrm{C}}(w_{\mathrm{P}} - w_{\mathrm{C}}) \leq \theta_{\mathrm{C}}(W) \wedge v_{\mathrm{C}} \geq \frac{\theta_{\mathrm{P}}(w_{\mathrm{P}})}{w_{\mathrm{P}}} = \alpha_{\mathrm{C}}\varepsilon + \frac{1-\alpha_{\mathrm{C}}}{w_{\mathrm{P}}}.$$

The last inequality holds because $v_{\mathrm{C}} > \underline{v}_{\mathrm{C}}(v_{\mathrm{P}})$ and we have that

$$r(w_{\mathrm{P}} - B) \leq A - w_{\mathrm{P}} \iff \varepsilon h(v_{\mathrm{P}}) \geq v_{\mathrm{C}}[\theta_{\mathrm{P}}(2w_{\mathrm{P}}) - v_{\mathrm{P}}] \iff g^{\mathrm{N}}_{\mathrm{P}int} \geq w_{\mathrm{P}}.$$

The first equivalence follows after some manipulations and the second one from (27). Then this case is possible if and only if

$$[v_{\mathrm{C}} \leq B_{\mathrm{PC}} \ \vee \ w_{\mathrm{P}} \leq w_{\mathrm{C}}] \wedge [v_{\mathrm{C}} \leq B_{\mathrm{P}}(v_{\mathrm{P}}) \ \vee \ v_{\mathrm{P}} \geq \theta_{\mathrm{P}}(2w_{\mathrm{P}}))]$$

This concludes the proof. ■

Proof of Proposition 2.

i) The proof is straightforward in cases $(w_{\mathrm{P}}, 0)$ $(G^{\mathrm{N}} = w_{\mathrm{P}} \leq A)$, $(w_{\mathrm{P}}, w_{\mathrm{C}})$ $(G^{\mathrm{N}} = W \leq A)$, and $(A, 0)$, $\left(g^b_{\mathrm{P}}(w_{\mathrm{C}}), w_{\mathrm{C}}\right)$ and $(g_{\mathrm{P}int}, g_{\mathrm{C}int})$ $(G^{\mathrm{N}} = G_{int} = A)$. Case $\left(w_{\mathrm{P}}, g^b_{\mathrm{C}}(w_{\mathrm{P}})\right)$ is characterized by:

$$w_{\mathrm{P}} \leq g_{\mathrm{P}int} \overset{r>0}{\Longrightarrow} g^b_{\mathrm{C}}(w_{\mathrm{P}}) \leq g^b_{\mathrm{C}}(g_{\mathrm{P}int}) \Longrightarrow w_{\mathrm{P}} + g^b_{\mathrm{C}}(w_{\mathrm{P}}) \leq g_{\mathrm{P}int} + g^b_{\mathrm{C}}(g_{\mathrm{P}int}) = A.$$

ii) In case b) the conformist does not contribute and the positional does. In case c) four equilibria are possible. For the interior equilibrium, $(g_{\mathrm{P}int}, g_{\mathrm{C}int})$, $g_{\mathrm{P}int} > g_{\mathrm{C}int} \iff \alpha_{\mathrm{P}} - \alpha_{\mathrm{C}} + \alpha_{\mathrm{C}} v_{\mathrm{P}} > 0$, which holds because $v_{\mathrm{P}} > 1 - \alpha_{\mathrm{P}}$ and $\alpha_{\mathrm{C}} < 1$.

For equilibrium $\left(w_{\mathrm{P}}, g^b_{\mathrm{C}}(w_{\mathrm{P}})\right)$, since $1 - r > 0$ and $-rB < 0$ it follows that $w_{\mathrm{P}} > g^b_{\mathrm{C}}(w_{\mathrm{P}})$.

Finally, in the equilibrium $(\left(g^b_{\mathrm{P}}(w_{\mathrm{C}}), w_{\mathrm{C}}\right)$, $A - w_{\mathrm{C}} > w_{\mathrm{C}} \iff \alpha_{\mathrm{P}}(1-\alpha_{\mathrm{C}}) + \alpha_{\mathrm{C}}[v_{\mathrm{P}} - (1 - \alpha_{\mathrm{P}})] > 0$, which again also holds true under (12).

■

Proof of Proposition 6.

The derivative of social welfare with respect to v_{C} reads:

$$\frac{\partial U_{int}}{\partial v_{\mathrm{C}}} = \frac{\partial V_{int}}{\partial v_{\mathrm{C}}} = \frac{h(v_{\mathrm{P}})[\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}} - (2\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}})v_{\mathrm{P}}]}{2\alpha_{\mathrm{P}}^2 v_{\mathrm{C}}},$$

which sign is given by the sign of $H = \alpha_{\mathrm{P}} - \alpha_{\mathrm{C}} - (2\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}})v_{\mathrm{P}}$. To study the sign of H we can distinguish 3 different situations:

i) $\alpha_{\mathrm{P}} > \alpha_{\mathrm{C}} \Rightarrow 2\alpha_{\mathrm{P}} > \alpha_{\mathrm{C}}$, then

$$H \gtrless 0 \Leftrightarrow v_{\mathrm{P}} \lessgtr \frac{\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}}{2\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}} \in (0,1)\,.$$

iia) $\alpha_{\mathrm{P}} < \alpha_{\mathrm{C}} \wedge 2\alpha_{\mathrm{P}} > \alpha_{\mathrm{C}}$, then $H < 0$ for any $v_{\mathrm{P}} \in (0,1)$.

iib) $2\alpha_{\mathrm{P}} < \alpha_{\mathrm{C}} \Rightarrow \alpha_{\mathrm{P}} < \alpha_{\mathrm{C}}$, then

$$H \gtrless 0 \Leftrightarrow v_{\mathrm{P}} \gtrless \frac{\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}}{2\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}}$$

and

$$v_{\mathrm{P}} < 1 < \frac{\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}}{2\alpha_{\mathrm{P}} - \alpha_{\mathrm{C}}} \Rightarrow H < 0.$$

■

B Proof that the static equilibrium is a fixed point of the dynamical system

We present here the proof of Proposition 7. For the sake of compactness and symmetry, we use the notation $A_i, v_i^{\mathrm{C}}, v_i^{\mathrm{P}}, i = \{P, C\}$. The correspondence with Proposition 7 is: $v_{\mathrm{C}}^{\mathrm{P}} = v_{\mathrm{P}}^{\mathrm{C}} = 0$, $v_{\mathrm{C}}^{\mathrm{C}} = v_{\mathrm{C}}$, $v_{\mathrm{P}}^{\mathrm{P}} = v_{\mathrm{P}}$, $A_{\mathrm{P}} = A$, and $\alpha_{\mathrm{P}}\varepsilon A_{\mathrm{C}} = \alpha_{\mathrm{C}} - 1$.
Proof. The best-response functions of the static case (see equations (10)–(11)) are:

$$r_i^S(g_{-i}) = \begin{cases} w_i & \text{if } r_i^b(g_{-i}) \geq w_i \\ r_i^b(g_{-it}) & \text{if } 0 \leq r_i^b(g_{-i}) \leq w_i \\ 0 & \text{if } r_i^b(g_{-i}) \leq 0, \end{cases} \quad \text{where} \quad r_i^b(g_{-i}) = \frac{A_i\alpha_i\varepsilon + (v_i^{\mathrm{C}} - \alpha_i\varepsilon)g_{-i}}{v_i^{\mathrm{C}} + \alpha_i\varepsilon} \tag{29}$$

and the Nash static equilibrium verifies $g_i = r_i^S(g_{-i})$ for $i \in \{P, C\}$.
If the dynamic process has a steady state, at the steady state the reaction function reads $r_i^{D\infty}(g_i, g_{-i}) = r_i^D(g_{-i}; g_i, g_{-i})$, with:

$$r_i^{D\infty}(g_i, g_{-i}) = \begin{cases} w_i & \text{if } r_i^\infty(g_i, g_{-i}) \geq w_i \\ r_i^\infty(g_i, g_{-i}) & \text{if } 0 \leq r_i^\infty(g_i, g_{-i}) \leq w_i \\ 0 & \text{if } r_i^\infty(g_i, g_{-i}) \leq 0, \end{cases} \text{where } r_i^\infty(g_i, g_{-i}) = \frac{A_i\alpha_i\varepsilon + v_i^{\mathrm{I}} g_i + (v_i^{\mathrm{C}} - \alpha_i\varepsilon)g_{-i}}{v_i^{\mathrm{I}} + v_i^{\mathrm{C}} + \alpha_i\varepsilon}, \tag{30}$$

and the steady state satisfies $g_i^\infty = r_i^{D\infty}(g_i^\infty, g_{-i}^\infty)$.

We are going to see that

$$g_i^\infty = r_i^{D\infty}(g_i^\infty, g_{-i}^\infty) \quad \Longleftrightarrow \quad g_i^\infty = r_i^S(g_{-i}^\infty).$$

In fact

$$0 < r_i^\infty(g_i^\infty, g_{-i}^\infty) < w_i \quad \Longleftrightarrow 0 < g_i^\infty = \frac{A_i\alpha_i\varepsilon + v_i^{\mathrm{I}} g_i^\infty + v_i^{\mathrm{C}} g_{-i}^\infty - \alpha_i\varepsilon g_{-i}^\infty}{v_i^{\mathrm{I}} + v_i^{\mathrm{C}} + \alpha_i\varepsilon} < w_i,$$

$$\Longleftrightarrow \quad w_i(v_i^{\mathrm{I}} + v_i^{\mathrm{C}} + \alpha_i\varepsilon) > \frac{A_i\alpha_i\varepsilon + v_i^{\mathrm{C}} g_{-i}^\infty - \alpha_i\varepsilon g_{-i}^\infty}{v_i^{\mathrm{C}} + \alpha_i\varepsilon} > 0 \quad \Longleftrightarrow \quad w_i > r_i^b(g_{-i}^\infty) > 0,$$

$$\Longleftrightarrow \quad 0 < g_i^\infty = \frac{A_i\alpha_i\varepsilon + v_i^{\mathrm{C}} g_{-i} - \alpha_i\varepsilon g_{-i}}{v_i^{\mathrm{C}} + \alpha_i\varepsilon} < w_i \Longleftrightarrow \quad g_i^\infty = r_i^S(g_{-i}^\infty).$$

The same reasoning applies if $r_i(g_{-i}^\infty) \leq 0$ or $r_i(g_{-i}^\infty) \geq w_i$. ∎

C Analysis of the dynamics

C.1 The linear dynamical system

We derive here the equations of the dynamics defined in Section 5 when the successive myopic Nash equilibria are interior. FOC in (19) and (20), can be rewritten in matrix form as:

$$\begin{pmatrix} \varepsilon\alpha_{\mathrm{P}} + \omega_{\mathrm{P}} & \varepsilon\alpha_{\mathrm{P}} \\ \varepsilon\alpha_{\mathrm{C}} & \varepsilon\alpha_{\mathrm{C}} + \omega_{\mathrm{C}} \end{pmatrix} \begin{pmatrix} g_{\mathrm{P}t} \\ g_{\mathrm{C}t} \end{pmatrix} = \begin{pmatrix} v_{\mathrm{P}}^{\mathrm{I}} & 0 \\ v_{\mathrm{C}} & v_{\mathrm{C}}^{\mathrm{I}} \end{pmatrix} \begin{pmatrix} g_{\mathrm{P}t-1} \\ g_{\mathrm{C}t-1} \end{pmatrix} + \begin{pmatrix} \Phi_{\mathrm{P}} \\ \Phi_{\mathrm{C}} \end{pmatrix}, \tag{31}$$

where

$$\omega_{\mathrm{P}} = v_{\mathrm{P}}^{\mathrm{I}} \qquad \omega_{\mathrm{C}} = v_{\mathrm{C}}^{\mathrm{I}} + v_{\mathrm{C}} \qquad \Phi_{\mathrm{P}} = \alpha_{\mathrm{P}} - 1 + v_{\mathrm{P}} \qquad \Phi_{\mathrm{C}} = \alpha_{\mathrm{C}} - 1.$$

Denote the matrices and vectors:

$$\mathbf{S} := \begin{pmatrix} \varepsilon\alpha_{\mathrm{P}} + \omega_{\mathrm{P}} & \varepsilon\alpha_{\mathrm{P}} \\ \varepsilon\alpha_{\mathrm{C}} & \varepsilon\alpha_{\mathrm{C}} + \omega_{\mathrm{C}} \end{pmatrix} \qquad \mathbf{B} := \begin{pmatrix} v_{\mathrm{P}}^{\mathrm{I}} & 0 \\ v_{\mathrm{C}} & v_{\mathrm{C}}^{\mathrm{I}} \end{pmatrix} \qquad \Phi := \begin{pmatrix} \Phi_{\mathrm{P}} \\ \Phi_{\mathrm{C}} \end{pmatrix} \qquad g_t := \begin{pmatrix} g_{\mathrm{P}t} \\ g_{\mathrm{C}t} \end{pmatrix}.$$

The solution to (31) depends on whether matrix $\mathbf{S}$ is singular or not. The determinant of $\mathbf{S}$ is:

$$D = \omega_{\mathrm{P}}\omega_{\mathrm{C}} + \varepsilon(\alpha_{\mathrm{C}}\omega_{\mathrm{P}} + \alpha_{\mathrm{P}}\omega_{\mathrm{C}}). \tag{32}$$

Since ε and the α_i are assumed to be strictly positive, $D = 0$ can happen only if $\omega_{\mathrm{P}} = \omega_{\mathrm{C}} = 0$, which is equivalent to $v_{\mathrm{P}}^{\mathrm{I}} = v_{\mathrm{C}}^{\mathrm{I}} = v_{\mathrm{C}} = 0$. By Assumption 6, $v_{\mathrm{C}} = 0$ is excluded. Consequently, $D > 0$ and matrix $\mathbf{S}$ is invertible.

Then, if $\mathbf{M} = \mathbf{S}^{-1}\mathbf{B}$ and $\Gamma^0 = \mathbf{S}^{-1}\Phi$, the system (31) can be written as:

$$g_t = \mathbf{M}g_{t-1} + \Gamma^0 \ . \tag{24}$$

Explicitly, the matrix $\mathbf{M}$ and the vector Γ^0 are:

$$\mathbf{M} = \frac{1}{D}\begin{pmatrix} v_{\mathrm{P}}^{\mathrm{I}}\omega_{\mathrm{C}} + \varepsilon(\alpha_{\mathrm{C}}v_{\mathrm{P}}^{\mathrm{I}} - \alpha_{\mathrm{P}}v_{\mathrm{C}}) & -\varepsilon\alpha_{\mathrm{P}}v_{\mathrm{C}}^{\mathrm{I}} \\ v_{\mathrm{C}}\omega_{\mathrm{P}} + \varepsilon(\alpha_{\mathrm{P}}v_{\mathrm{C}} - \alpha_{\mathrm{C}}v_{\mathrm{P}}^{\mathrm{I}}) & v_{\mathrm{C}}^{\mathrm{I}}(\omega_{\mathrm{P}} + \varepsilon\alpha_{\mathrm{P}}) \end{pmatrix} \quad \Gamma^0 = \frac{1}{D}\begin{pmatrix} \omega_{\mathrm{C}}\Phi_{\mathrm{P}} + \varepsilon(\alpha_{\mathrm{C}}\Phi_{\mathrm{P}} - \alpha_{\mathrm{P}}\Phi_{\mathrm{C}}) \\ \omega_{\mathrm{P}}\Phi_{\mathrm{C}} + \varepsilon(\alpha_{\mathrm{P}}\Phi_{\mathrm{C}} - \alpha_{\mathrm{C}}\Phi_{\mathrm{P}}) \end{pmatrix}. \tag{33}$$

C.2 Eigenvalues of the dynamical system

The dynamical system (24) is affine: many of its properties are characterized by the eigenvalues of $\mathbf{M} = \mathbf{S}^{-1}\mathbf{B}$. Let λ be an eigenvalue. Then, for some non-zero vector u, $\mathbf{S}^{-1}\mathbf{B}u = \lambda u$ or equivalently $\mathbf{B}u = \lambda\mathbf{S}u$. The matrix $\lambda\mathbf{S} - \mathbf{B}$ is therefore singular. Accordingly, the condition for λ to be an eigenvalue of $\mathbf{M}$ is:

$$\begin{aligned} 0 &= (\lambda\varepsilon\alpha_{\mathrm{P}} + \lambda\omega_{\mathrm{P}} - v_{\mathrm{P}}^{\mathrm{I}})(\lambda\varepsilon\alpha_{\mathrm{C}} + \lambda\omega_{\mathrm{C}} - v_{\mathrm{C}}^{\mathrm{I}}) - (\lambda\varepsilon\alpha_{\mathrm{P}})(\lambda\varepsilon\alpha_{\mathrm{C}} - v_{\mathrm{C}}) \\ 0 &= (\lambda\omega_{\mathrm{P}} - v_{\mathrm{P}}^{\mathrm{I}})(\lambda\omega_{\mathrm{C}} - v_{\mathrm{C}}^{\mathrm{I}}) + \lambda\varepsilon\left(\alpha_{\mathrm{P}}(\lambda\omega_{\mathrm{C}} - v_{\mathrm{C}}^{\mathrm{I}} + v_{\mathrm{C}}) + \alpha_{\mathrm{C}}(\lambda\omega_{\mathrm{P}} - v_{\mathrm{P}}^{\mathrm{I}})\right) \ . \end{aligned} \tag{34}$$

Let $\chi(\lambda)$ be the polynomial in the right-hand side of (34).

The following proposition summarizes the principal properties of this polynomial.

Proposition 10 *Under the assumption $\varepsilon > 0$ and $\alpha_i > 0$, $i \in \{P, C\}$, the eigenvalues λ_1, λ_2 of $\mathbf{M}$, roots of $\chi(\lambda)$, have the following properties.*

i) Because $v_C > 0$ (by Assumpion 6) no eigenvalues is equal to 1; an eigenvalue is equal to -1 if and only if $v_P^I = v_C^I = 0$, that is, no inertia; an eigenvalue is equal to 0 if and only if $v_P^I = 0$ or $v_C^I = 0$.

ii) When both eigenvalues are real, they lie in the interval $[-1, 1]$ and they have the same sign.

iii) When they are both complex, their modulus is strictly less than 1.

Proof. As preliminary, remember that the product of the roots of χ is equal to the ratio of the constant term of the polynomial to its leading term. Therefore, from (34),

$$\lambda_1\lambda_2 = \frac{v_{\mathrm{P}}^{\mathrm{I}}v_{\mathrm{C}}^{\mathrm{I}}}{\omega_{\mathrm{P}}\omega_{\mathrm{C}} + \varepsilon(\alpha_{\mathrm{P}}\omega_{\mathrm{C}} + \alpha_{\mathrm{C}}\omega_{\mathrm{P}})} = \frac{v_{\mathrm{P}}^{\mathrm{I}}v_{\mathrm{C}}^{\mathrm{I}}}{v_{\mathrm{P}}^{\mathrm{I}}(v_{\mathrm{C}}^{\mathrm{I}} + v_{\mathrm{C}}) + \varepsilon(\alpha_{\mathrm{P}}(v_{\mathrm{C}}^{\mathrm{I}} + v_{\mathrm{C}}) + \alpha_{\mathrm{C}}v_{\mathrm{P}}^{\mathrm{I}})} \ . \tag{35}$$

Next, we evaluate $\chi(1) = 2\varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}}$, $\chi(-1) = 2v^{\mathrm{I}}_{\mathrm{P}}(2v^{\mathrm{I}}_{\mathrm{C}} + v_{\mathrm{C}}) + 2\varepsilon(\alpha_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}} + \alpha_{\mathrm{C}} v^{\mathrm{I}}_{\mathrm{P}})$ and $\chi(0) = v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}}$. From these values we conclude that $i)$ holds.

We now turn to $ii)$. Using the values above, we find that $\chi(1) \geq 0$ and $\chi(-1) \geq 0$. Through elementary computations, we also find that $\chi'(1) \geq 0$ and $\chi'(-1) \leq 0$. This implies that both eigenvalues are in the interval $[-1, 1]$. They have the same sign since their product is positive.

Consider now $iii)$. If $v^{\mathrm{I}}_{\mathrm{C}} = 0$ or $v^{\mathrm{I}}_{\mathrm{P}} = 0$, then $\lambda = 0$ is an eigenvalue, so that eigenvalues are not complex. In the case where both eigenvalues are complex, they are conjugate and $|\lambda_1|^2 = |\lambda_2|^2 = |\lambda_1 \lambda_2|$. Then (35) implies:

$$|\lambda_i|^2 \ \leq \ \frac{v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}}}{\omega_{\mathrm{P}} \omega_{\mathrm{C}}} \ = \ \frac{v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}}}{v^{\mathrm{I}}_{\mathrm{P}}(v^{\mathrm{I}}_{\mathrm{C}} + v_{\mathrm{C}})} \ \leq \ 1.$$

In order to prove $iii)$, we argue that the inequality is strict. Equality $|\lambda_i| = 1$ occurs if and only if $v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}} > 0$ and

$$v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}} = \omega_{\mathrm{P}} \omega_{\mathrm{C}} + \varepsilon(\alpha_{\mathrm{C}} \omega_{\mathrm{C}} + \alpha_{\mathrm{C}} \omega_{\mathrm{P}})$$

or equivalently,

$$0 = v^{\mathrm{I}}_{\mathrm{P}} v_{\mathrm{C}} + \varepsilon(\alpha_i(v_{\mathrm{C}} + v^{\mathrm{I}}_{\mathrm{C}}) + \alpha_{\mathrm{C}} v^{\mathrm{I}}_{\mathrm{P}}),$$

and this in turns implies $v^{\mathrm{I}}_{\mathrm{P}} = v^{\mathrm{I}}_{\mathrm{C}} = v_{\mathrm{C}} = 0$. Since this contradicts $v^{\mathrm{I}}_{\mathrm{P}} v^{\mathrm{I}}_{\mathrm{C}} > 0$, the situation cannot occur and $iii)$ is proved. ■

C.3 Players without inertia

In this situation, no player has inertia: $v^{\mathrm{I}}_{\mathrm{P}} = v^{\mathrm{I}}_{\mathrm{C}} = 0$. By Assumption 6, $v_{\mathrm{C}} > 0$. According to Proposition 10 $ii)$, one eigenvalue of $\mathbf{M}$ is equal to -1. Indeed, since $\omega_{\mathrm{P}} = 0$ and $\omega_{\mathrm{C}} = v_{\mathrm{C}}$, the matrix $\mathbf{M}$ in (33) reduces to:

$$\mathbf{M} \ = \ \frac{1}{\varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}}} \begin{pmatrix} -\varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}} & 0 \\ \varepsilon\alpha_{\mathrm{P}} v_{\mathrm{C}} & 0 \end{pmatrix} \ = \ \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}.$$

Accordingly, the first steps of the dynamical system (24) are

$$g_0 = \begin{pmatrix} g_{\mathrm{P}0} \\ g_{\mathrm{C}0} \end{pmatrix} \quad g_1 = \begin{pmatrix} -g_{\mathrm{P}0} + \Gamma^0_{\mathrm{P}} \\ g_{\mathrm{P}0} + \Gamma^0_{\mathrm{C}} \end{pmatrix} \quad g_2 = \begin{pmatrix} g_{\mathrm{P}0} \\ -g_{\mathrm{P}0} + \Gamma^0_{\mathrm{P}} + \Gamma^0_{\mathrm{C}} \end{pmatrix} \quad g_3 = g_1$$

and then $g_4 = g_{2k} = g_2$, $g_{2k+1} = g_1$ for all $k \in \mathbb{N}$. Except for the initial value, the sequence solution of (24) is a cycle of order 2. It is independent on $g_{\mathrm{C}0}$ but does depend on the initial value $g_{\mathrm{P}0}$. We observe that the midpoint of the cycle is the point:

$$g^{\infty} \ = \ \begin{pmatrix} \frac{1}{2}\Gamma^0_{\mathrm{P}} \\ \frac{1}{2}\Gamma^0_{\mathrm{P}} + \Gamma^0_{\mathrm{C}} \end{pmatrix},$$

which is precisely the fixed point of (24), $(\mathbf{I} - \mathbf{M})^{-1}\Gamma^0$ and also the static Nash equilibrium g_{int} in accordance with Proposition 8.

In this case, a necessary and sufficient condition for the trajectory to lie in the set of constraints is that g_0, g_1 and g_2 be in this set. This leads to the following conditions:

$$\Gamma^0_{\mathrm{P}} - \min\{w_{\mathrm{C}} - \Gamma^0_{\mathrm{C}}, w_{\mathrm{P}}\} \leq g_{\mathrm{P}0} \leq \Gamma^0_{\mathrm{P}} + \min\{\Gamma^0_{\mathrm{C}}, 0\},$$

$$0 \leq g_{i0} \leq w_i, \quad i = P, C.$$

C.4 Convergence of the dynamical system

Convergence occurs from any initial condition if and only if both eigenvalues have absolute value less than 1. When it is the case, the convergence can be: *monotone* if both eigenvalues are real and positive, *oscillating* if at least one of them is real and negative, and *spiral* if both eigenvalues are complex. Using Proposition 10, we can now prove Proposition 8 and Proposition 9.

Proof of Proposition 8.

According to Proposition 10, the modulus of both eigenvalues of matrix $\mathbf{M}$ is smaller than 1, whether they are real (Proposition 10 *ii*)) or complex (Proposition 10 *iii*)). Moreover, 1 cannot be an eigenvalue under Assumption 6 (Proposition 10 *i*)). This implies that the matrix $\mathbf{I}-\mathbf{M}$ is invertible and $g^{\mathrm{N}} = (\mathbf{I}-\mathbf{M})^{-1}\Gamma^0$ is the fixed point of (24). Moreover, the distance between g_t and g^{N} cannot increase with t. Since g^{N} is in the interior of the rectangle of constraints, there is a neighborhood of g^{N} inside the rectangle which satisfies statement *i*).

Statement *ii*) is also a consequence of Proposition 10: under Assumption 6 and if $v_{\mathrm{P}}^{\mathrm{I}} \neq 0$ or $v_{\mathrm{IC}} \neq 0$, then by Proposition 10 *i*), 1 and −1 are not eigenvalues, and from Proposition 10 *ii*) both eigenvalues are strictly smaller than 1 in modulus. This implies that $\|g_t - g^{\mathrm{N}}\| < \rho\,\|g_{t-1} - g^{\mathrm{N}}\|$ for some $\rho < 1$. This in turn implies the convergence of g_t to g^{N}.

The proof of statement *iii*) is a consequence of Appendix C.3. ■

Proof of Proposition 9. Let Δ_λ be the discriminant of the polynomial $\chi(\lambda)$. The roots of χ are real when $\Delta_\lambda \geq 0$ and complex conjugate if $\Delta_\lambda < 0$. This justifies statement *c*). When the roots are real, they have the same sign according to Proposition 10 *ii*). This sign is the sign of their sum, which is opposite to the sign of the ratio between the linear and the quadratic coefficients of the quadratic polynomial, $\chi(\lambda)$. The leading term in (25) is $v_{\mathrm{P}}^{\mathrm{I}}(v_{\mathrm{C}} + v_{\mathrm{C}}^{\mathrm{I}}) + \varepsilon\,(\alpha_{\mathrm{P}}(v_{\mathrm{C}} + v_{\mathrm{C}}^{\mathrm{I}}) + \alpha_{\mathrm{C}} v_{\mathrm{P}}^{\mathrm{I}}) > 0$. The sign of eigenvalues is therefore the opposite of that of the linear coefficient in χ, which leads to the conditions in *a*) and *b*).
For statement *d*), the speed of convergence is faster the smaller the spectral radius of $\mathbf{M}$, that is, the modulus of its largest eigenvalue. When $\Delta_\lambda < 0$ and eigenvalues are complex conjugate, their modulus is given by $|\lambda_i| = |\lambda_1\lambda_2|^{1/2}$, with $|\lambda_1\lambda_2|$ given in (35). It is easy to see that this is an increasing function of $v_{\mathrm{P}}^{\mathrm{I}}$ and $v_{\mathrm{C}}^{\mathrm{I}}$, and a decreasing function of v_{C}. ■

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