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## International Conference of Numerical Analysis and Optimization Days (JANO'14)

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### Mathematical model of anaerobic digestion with hydrolytic process by means of leachate recirculation in landfills

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#### Abstract :

Controlled landfilling, also known as Technical Landfill Center (TLC), has become one of the major solid waste treatment or disposal systems in developing countries. A TLC is a highly dynamic environment [3]. During the waste degradation processes, the water contained in the waste and the percolation water [5, 6, 7] (mainly from precipitation) mix with organic and mineral matter to form a juice, called leachate. This leachate is contaminated and can, if not controlled, pollute the soil and groundwater. Mathematical modeling is of great interest for decision making in the field of waste treatment and recovery. It allows to analyze the underlying complex systems and to predict the behavior of the different components. Many new technologies have been developed to optimize energy yields [4, 8], in particular the anaerobic digestion process. The mathematical model proposed in this work is inspired by [1] where the authors introduce a landfill model with biomass recirculation in which the organic fraction is decomposed during an anaerobic digestion process. We consider a model that describes the process taking into account an additional phenomenon which is the solubilization of the insoluble or slowly biodegradable fraction [2] through a recirculation of the leachate. We also consider that a part of the dead bacteria is a new soluble substrate while the other part is transformed into carbon dioxide [9]. The dynamic system obtained allowed us to predict the evolution of the quantities of methane and carbon dioxide over time. The system admits an infinity of non-hyperbolic equilibria but a qualitative analysis characterized its global attractor whose connectivity is impacted by considering an inhibition in growth. The simulations carried out confirm these theoretical results, and also make it possible to highlight the influence of the leachate recirculation on the performance of the system.

**Keywords :** leachate recirculation, mathematical modeling, biogas, non hyperbolic equilibria.

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