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Time scale separation in life-long ovarian follicles population dynamics model

Romain Yvinec

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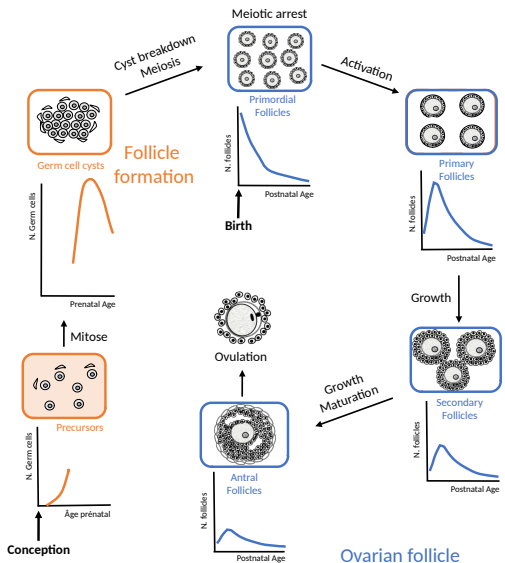
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Time scale separation in life-long ovarian follicles population dynamics model

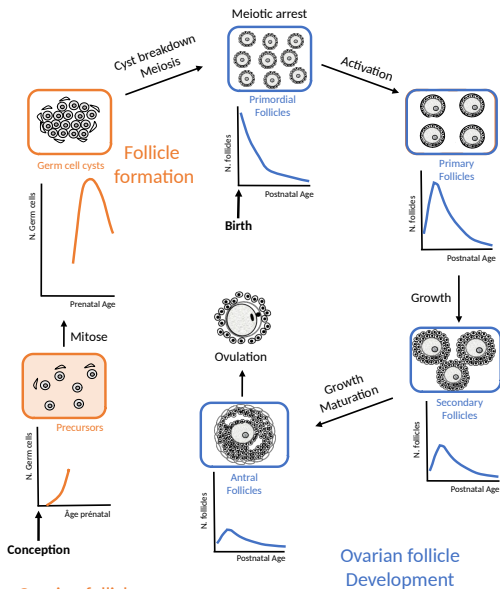
Romain Yvinec

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Ovarian follicle Development

Ovarian follicle Formation

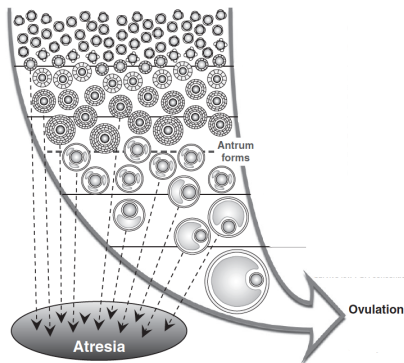


- Pool of Quiescent follicles **static reserve** (perinatal in most mammals)
Slow activation
- Basal growth **Dynamic reserve** (starting at birth) Spanning over several ovarian cycles
- Terminal growth
After puberty : **ovulation** within an ovarian cycle

Order of magnitude (in Women)

- Quiescent follicles

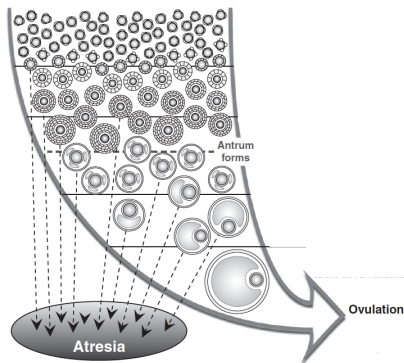
peri-natal	$\approx 5 \cdot 10^6$
At birth	$\approx 1 \cdot 10^6$
At puberty	$10^4 - 10^6$
At menopause	$< 10^3$
Activation rate	"A few per days"



Scaramuzzi et al., *Reprod.Fert. Dev.* 2011

Order of magnitude (in Women)

- Quiescent follicles
 - peri-natal $\approx 5 \cdot 10^6$
 - At birth $\approx 1 \cdot 10^6$
 - At puberty $10^4 - 10^6$
 - At menopause $< 10^3$
 - Activation rate "A few per days"
- Growing follicles
 - Maturation time 120 – 180j
 - Basal follicles $10^3 - 10^4$
 - Terminal follicles 10^2
 - Pre-Ovulatory follicles a few
 - Atresia Most of them



Scaramuzzi et al., *Reprod.Fert. Dev.* 2011

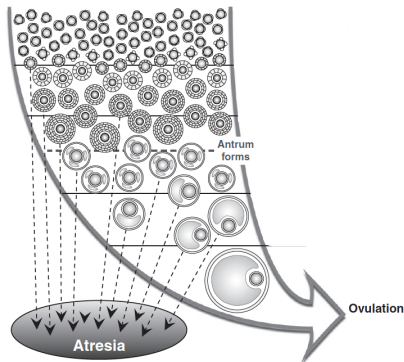
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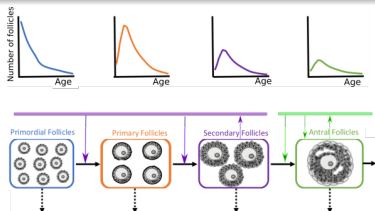
> **Only 400 follicles will ever reach the pre-ovulatory stage**



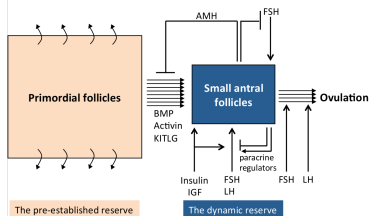
Scaramuzzi et al., *Reprod.Fert. Dev.* 2011

Population dynamics in female gametogenesis

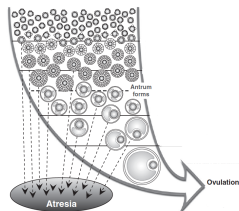
- Asynchronous growth
- Several timescales : Ten of years / Months / Weeks
- Interactions between subpopulations



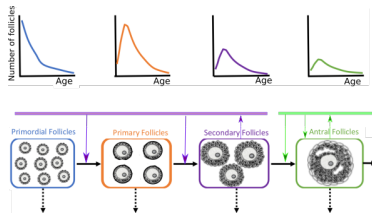
Ovarian reserves of follicles and their regulations



Monniaux, *Theriogenology* 2016

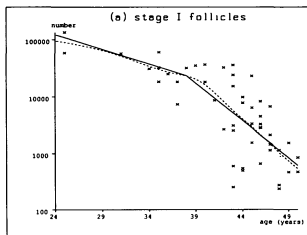


Scaramuzzi et al., *Reprod.Fert. Dev.* 2011

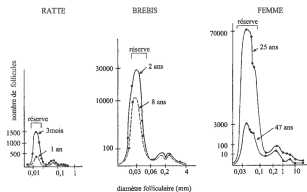


⇒ Irreversible (slow) decay of an initial pool of quiescent follicle

⇒ "Stable" repartition of growing follicle



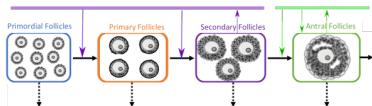
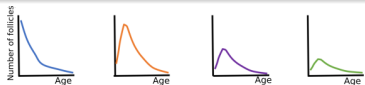
Faddy and Gosden 1995



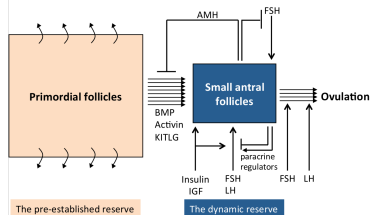
Thibault and Levesseur, 2001

Societal challenges : to preserve the reproductive ability

- Iatrogenic or physiological alterations
- Sensibility to environmental conditions
- Biodiversity preservation



Ovarian reserves of follicles and their regulations

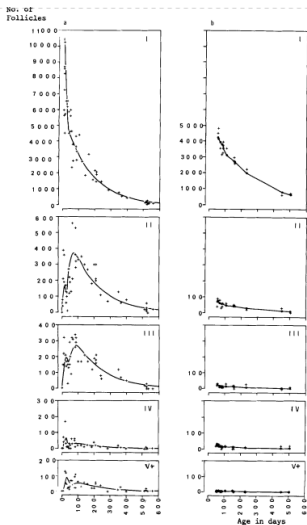


Monniaux, Theriogenology 2016

Economic and environmental challenges

- Biotechnology of reproduction
- Endocrine disruptors
- Non-hormonal contraception

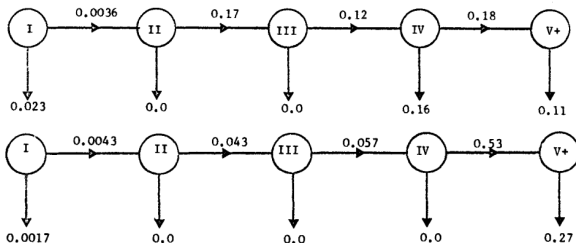
Aim : revisit compartmental model with nonlinear interaction and timescale analysis



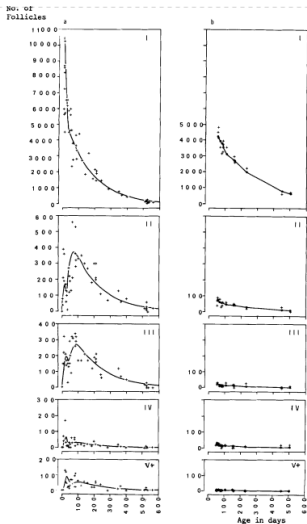
An Analytical Model for Ovarian Follicle Dynamics

M. J. FADDY,¹ ESTHER C. JONES² AND R. G. EDWARDS³
¹Department of Mathematical Statistics, University of Birmingham, Birmingham B15 2TT, U.K.; ²Department of Anatomy, University of Birmingham, Birmingham B15 2TJ, U.K., and ³Physiological Laboratory, University of Cambridge, Cambridge CB2 3EG, U. K.

- "Migration-death" model
- Linear Model



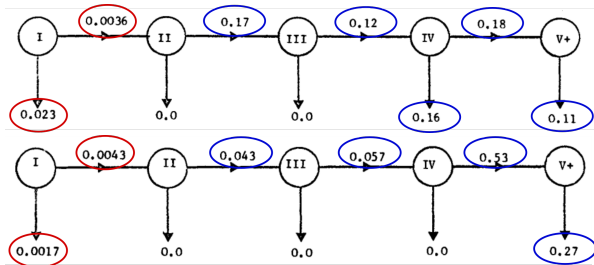
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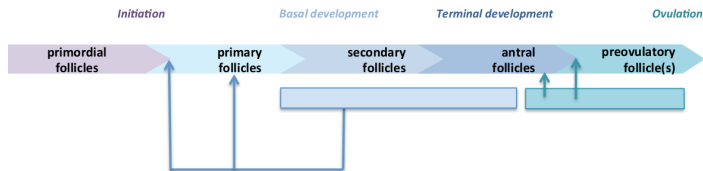
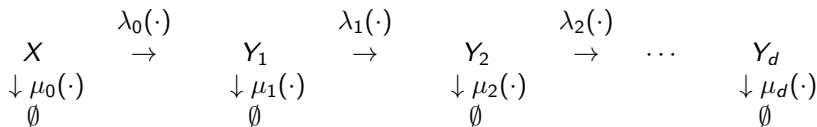
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Lifespan follicle population model

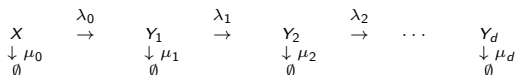
- Structured Population in compartments
- Non linear interaction between follicles *via* λ' 's and μ' 's.



Bonnet et al. *Multiscale population dynamics in reproductive biology : singular perturbation reduction in deterministic and stochastic models*, ESAIM : PROCEEDINGS AND SURVEYS, 2020.

Lifespan follicle population model

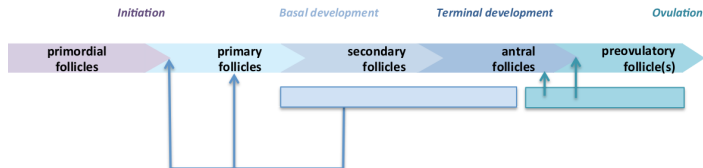
- Structured Population in compartments
- Non linear interaction between follicles *via* λ 's and μ 's.



Typical choice

$$\lambda_i(Y) = m_i + \frac{f_i}{1 + K_{1,i} \sum_{j=1}^d \omega_{1,j} Y_j},$$

$$\mu_i(Y) = g_i \left(1 + K_{2,i} \sum_{j=0}^d \omega_{2,j} Y_j \right)$$

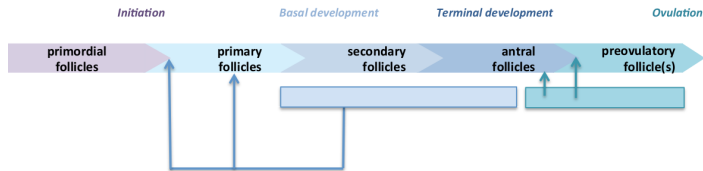


Bonnet et al. *Multiscale population dynamics in reproductive biology : singular perturbation reduction in deterministic and stochastic models*, ESAIM : PROCEEDINGS AND SURVEYS, 2020.

Lifespan follicle population model

- Structured Population in compartments
- Non linear interaction between follicles *via* λ 's and μ 's.
- Two time and abundance scales
- Quiescent Pool \gg Growing Follicles
- Slow Activation \ll Fast growth

$$\begin{array}{ccccccc}
 \frac{1}{\varepsilon} X & \xrightarrow{\varepsilon \lambda_0} & Y_1 & \xrightarrow{\lambda_1} & Y_2 & \xrightarrow{\lambda_2} & \dots & Y_d \\
 \downarrow \varepsilon \mu_0 & & \downarrow \mu_1 & & \downarrow \mu_2 & & & \downarrow \mu_d \\
 \emptyset & & \emptyset & & \emptyset & & & \emptyset
 \end{array}$$



Bonnet et al. *Multiscale population dynamics in reproductive biology : singular perturbation reduction in deterministic*

and stochastic models, ESAIM: PROCEEDINGS AND SURVEYS, 2020

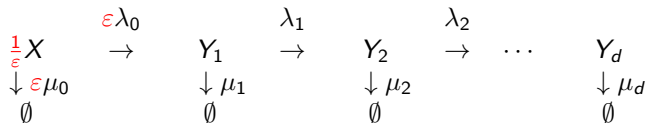
Singular Perturbation Theory (deterministic ODEs)

$$\begin{array}{ccccccc}
 \frac{1}{\varepsilon} X & \xrightarrow{\varepsilon \lambda_0} & Y_1 & \xrightarrow{\lambda_1} & Y_2 & \xrightarrow{\lambda_2} & \dots & Y_d \\
 \downarrow \varepsilon \mu_0 & & \downarrow \mu_1 & & \downarrow \mu_2 & & & \downarrow \mu_d \\
 \emptyset & & \emptyset & & \emptyset & & & \emptyset
 \end{array}$$

In the limit $\varepsilon \rightarrow 0$ We expect X and $Y = (Y_1, \dots, Y_d)$ to converge to a differential-algebraic equation :

$$\begin{cases} \frac{dx}{dt}(t) = F(x(t), y(t)), & x(0) = x^{\text{in}}, \\ 0 = G(x(t), y(t)), & t > 0 \end{cases}$$

Singular Perturbation Theory (stochastic CTMC)



Re-scaled Continuous Time Markov Chain :

$(X^\varepsilon(t) = \varepsilon X_0(t/\varepsilon), Y^\varepsilon(t) = (X_1(t/\varepsilon), \dots, X_d(t/\varepsilon))) :$

	Events	Rate
self-renew :	$(X, Y) \rightarrow (X + \varepsilon, Y),$	$\frac{1}{\varepsilon} r_0(Y)X,$
activation :	$(X, Y) \rightarrow (X - \varepsilon, Y + e_1),$	$\frac{1}{\varepsilon} \lambda_0(Y)X,$
atresia :	$(X, Y) \rightarrow (X - \varepsilon, Y),$	$\frac{1}{\varepsilon} \mu_0(Y)X,$
growth :	$(X, Y) \rightarrow (X, Y + e_{i+1} - e_i),$	$\frac{1}{\varepsilon} \lambda_i(Y)Y_i, i = 1..d - 1,$
atresia :	$(X, Y) \rightarrow (X, Y - e_i),$	$\frac{1}{\varepsilon} \mu_i(Y)Y_i, i = 1..d,$

$$\begin{array}{ccccccc}
 & \varepsilon \lambda_0 & & \lambda_1 & & \lambda_2 & & \dots & & \lambda_d \\
 \frac{1}{\varepsilon} X & \rightarrow & Y_1 & \rightarrow & Y_2 & \rightarrow & \dots & & & Y_d \\
 \downarrow \varepsilon \mu_0 & & \downarrow \mu_1 & & \downarrow \mu_2 & & & & & \downarrow \mu_d \\
 \emptyset & & \emptyset & & \emptyset & & & & & \emptyset
 \end{array}$$

Theorem (G. Ballif, F. Clément, R.Y. SIAP 2022)

(...) $(X^\varepsilon, Y^\varepsilon)$ converges in $\mathcal{D}_{\mathbb{R}}[0, \infty[\times \mathcal{L}_m(\mathbb{N}^d)$ to the unique solution of

$$\begin{cases} \frac{dx}{dt}(t) & = \Lambda_0(x(t))x(t), & x(0) = x^{\text{in}}, \\ \Lambda_0(x(t)) & = - \sum_{y \in \mathbb{N}^d} (\lambda_0(y) + \mu_0(y)) \pi_{x(t)}(y), \end{cases}$$

$$\sum_{y \in \mathbb{N}^d} L_x \psi(y) \pi_x(y) = 0, \quad \forall \psi \text{ bounded on } \mathbb{N}^d,$$

$$\begin{aligned}
 L_x \psi(y) = & \lambda_0(y) x \left[\psi(y + e_1) - \psi(y) \right] + \sum_{i=1}^{d-1} \lambda_i(y) y_i \left[\psi(y + e_{i+1} - e_i) - \psi(y) \right] \\
 & + \sum_{i=1}^d \mu_i(y) y_i \left[\psi(y - e_i) - \psi(y) \right].
 \end{aligned}$$

Schéma de preuve

- Compacité / Estimée sur les moments :

$$\forall p \geq 1, \sup_{\varepsilon} \mathbb{E} \left(\sup_{t \geq 0} \left| X^\varepsilon(t) + \sum_{i=1}^d Y_i^\varepsilon(t) \right|^p \right) < \infty$$

- Identification de la martingale "lente" :

$$M_f^\varepsilon(t) = f(X^\varepsilon(t)) - \int_0^t Af(X^\varepsilon(s), Y^\varepsilon(s)) ds + R_f^\varepsilon(t) \text{ où}$$

$$Af(x, y) = \left(r_0(y) - \lambda_0(y) - \mu_0(y) \right) x f'(x)$$

- Identification de la martingale "rapide" :

$$M_g^\varepsilon(t) := \varepsilon \left[g(Y^\varepsilon(t)) - g(Y^\varepsilon(0)) \right] - \int_0^t \int_{\mathbb{N}^d} L_{X^\varepsilon(s)} g(Y^\varepsilon(s)) ds$$

	Events	Rate
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atresia :	$(X, Y) \rightarrow (X, Y - e_i),$	$\frac{1}{\varepsilon} \mu_i(Y) Y_i, i = 1..d,$

Hypothèses

- ★ $r_0(y) < R_0, \forall y$
- ★ $\lambda_0(y) \leq B_0, \forall y$
- ★ $\lambda_i(y) > 0,$
 $i \in \llbracket 0, d - 1 \rrbracket, \forall y$
- ★ $\mu_d(y) > 0, \forall y$
- ★ $\alpha_i > 0$ tel que
 $\lambda_i(y) + \mu_i(y) \geq \alpha_i,$
 $\forall i \in \llbracket 0, d \rrbracket, \forall y$

Éléments de preuves

- Processus majorant linéaire :

$$\begin{array}{ll}
 (U, V) \rightarrow (U + \varepsilon, V), & \frac{1}{\varepsilon} R_0 U, \\
 (U, V) \rightarrow (U, V + e_1), & B_0 U, \\
 (U, V) \rightarrow (U - \varepsilon, V + 1), & \alpha_0 U, \\
 (U, V) \rightarrow (U, V + e_{i+1} - e_i), & \alpha_i V_i.
 \end{array}$$

- Par couplage : $X \leq U$ et $\sum_{j=1}^i Y_j \leq \sum_{j=1}^i V_j.$
- Lyapounov $F(y) = \sum_{i=1}^d \left(\sum_{j=1}^i y_j \right)^{p_i}$ pour $p_i \searrow.$

Pour l'unicité :

$$\frac{dx}{dt}(t) = \langle r_0 - \lambda_0 - \mu_0, \pi_{x(t)} \rangle x(t)$$

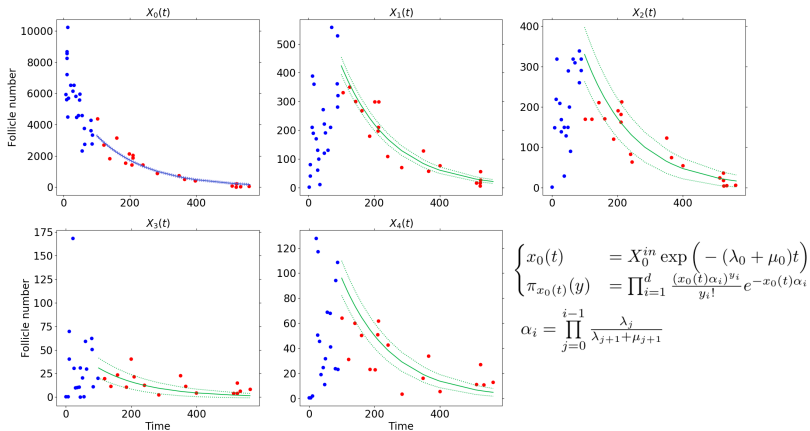
- Lyapounov $F : \langle \pi_x, F \rangle < \infty$.

- Pour toute fonction f tel que $|f| \leq F$

$$\langle f, \pi_x - \pi_{x'} \rangle = (x - x') \langle (g_x(\cdot + 1) - g_x(\cdot)) \lambda_0, \pi_{x'} \rangle$$

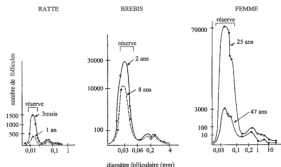
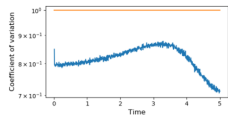
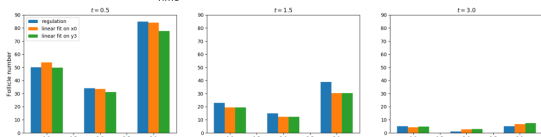
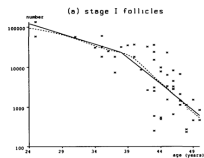
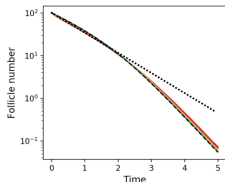
où g_x est solution de l'équation de Poisson : $L_x g_x = \langle f, \pi_x \rangle - f$ et vérifie $|g_x| \leq F$

Does it works in practice ?



- Timescale separation is coherent with published data on follicle counts in mice at the lifespan time scale, *away from a transient period*.

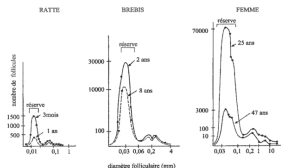
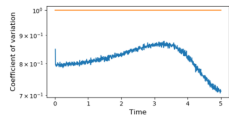
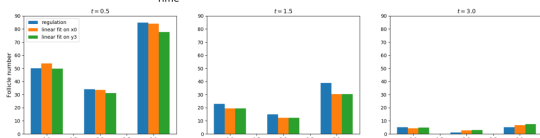
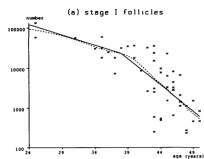
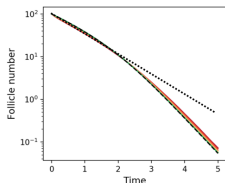
What is it useful for?



- ✓ acceleration of reserve decay with age :

$$\frac{dx}{dt} = -\left(a + \frac{b}{1+cx}\right)x$$
 -> Mechanistic explanation of previously published statistical regression model (Coxworth and Hawkes 2010)

What is it useful for?



- ✓ acceleration of reserve decay with age :

$$\frac{dx}{dt} = -\left(a + \frac{b}{1+cx}\right)x$$
 -> Mechanistic explanation of previously published statistical regression model (Coxworth and Hawkes 2010)
- ✓ "stable" evolution of growing follicles Y
 -> Antral Follicle Count for fertility test and primary ovarian insufficiency detection

Going further

- What about fluctuations ?
- Can we infer the regulation mechanism that control follicle activation ?
- Can we refine the model to model the transient phase (reserve establishment / early post-natal dynamics) ?

Fluctuations

At leading order, fluctuations are Gaussian of order $\sqrt{\varepsilon}$: $U^\varepsilon = \frac{X^\varepsilon - x}{\sqrt{\varepsilon}}$

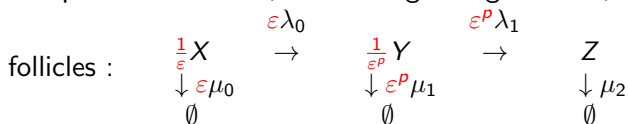
CLT

... U^ε converges towards U that satisfies

$$U(t) = U^{in} + \int_0^t \left[\lambda'_0(x(s))x(s) + \lambda(x(s)) \right] U(s) ds + \int_0^t \sqrt{\langle G(x(s), \cdot), \pi_{x(s)} \rangle} W_s$$

Fluctuations with more than two timescales

X =quiescent follicles; Y =small growing follicles; Z large terminal

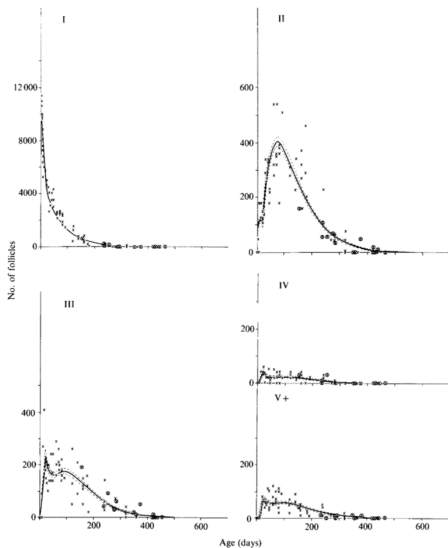


For $p > 1/2$, fluctuations on X are still on order $\sqrt{\varepsilon}$, with Y and Z contributing *equally*.

For $p \leq 1/2$: undergoing work !

"Time" course

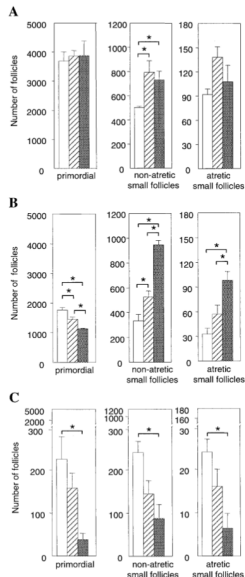
- Follicle count in mice from birth until 500 days.
- Reserve + 4 compartments (Faddy's classification)
- (Recovery of points by hand)



Faddy, Gosden and Edwards, J. Endo., 1983
 Faddy, Telfer and Gosden, Cell Proliferation, 1987

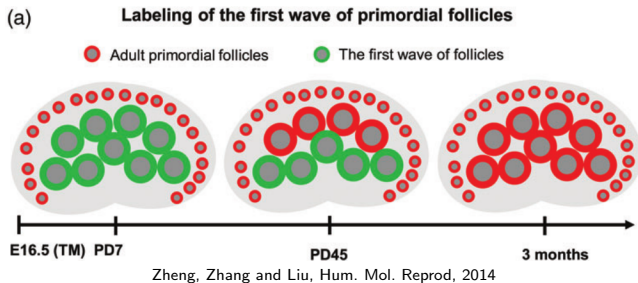
Perturbation data : KO AMH

- AMH Inhibition *in vivo* on mice
- 3 genotypes : control group (+/+), heterozygous mice KO AMH (+/-) homozygous mice KO AMH (-/-).
- Follicle counts at 3 ages :
 - 25 days (A)
 - 120 days (B)
 - 390 days (C)



Ovarian Reserve build-up

- Two distinct population of follicles are present initially
- Labelling of each population

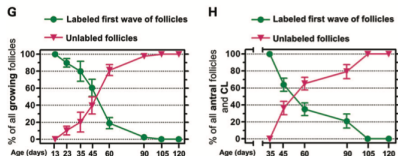
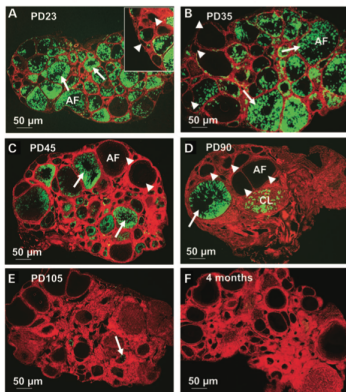


Ovarian Reserve build-up

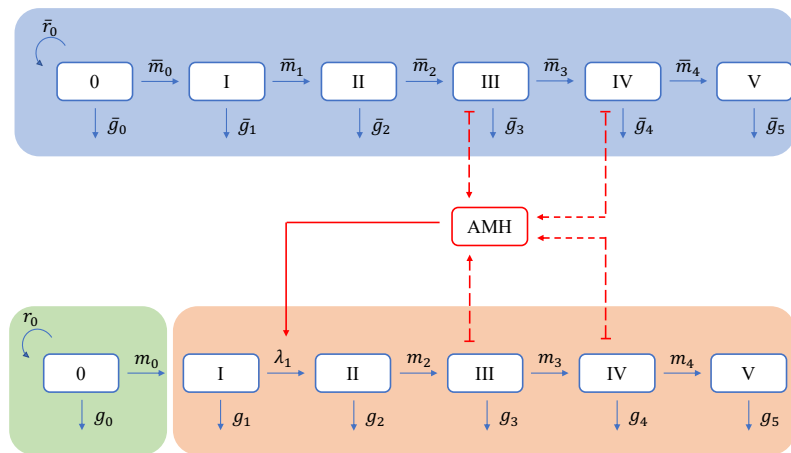
- Tracing follicles of the first wave of activated follicles.
- Proportion of first wave activated follicles among growing follicles.

$$p(t) = \frac{\sum_{i=1}^4 X_i^1(t)}{\sum_{i=1}^4 X_i^{tot}(t)}$$

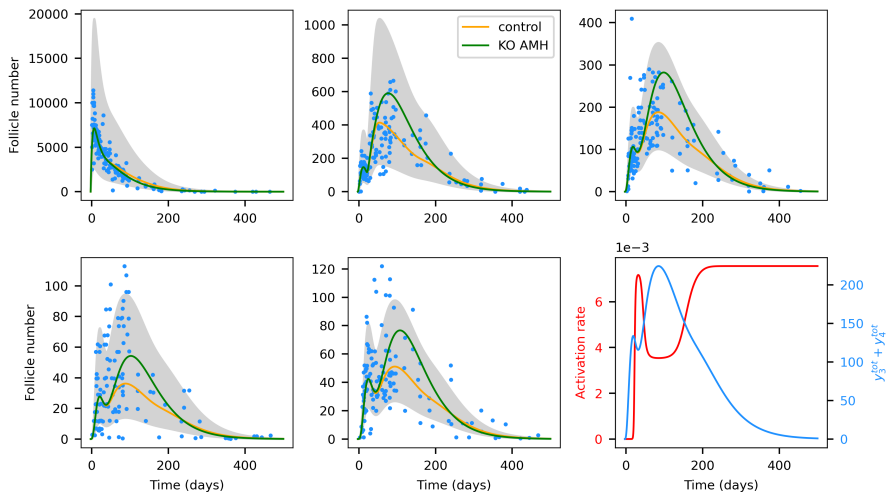
Tamoxifen was given at E16.5 and ovaries were analyzed at various ages



ODE model

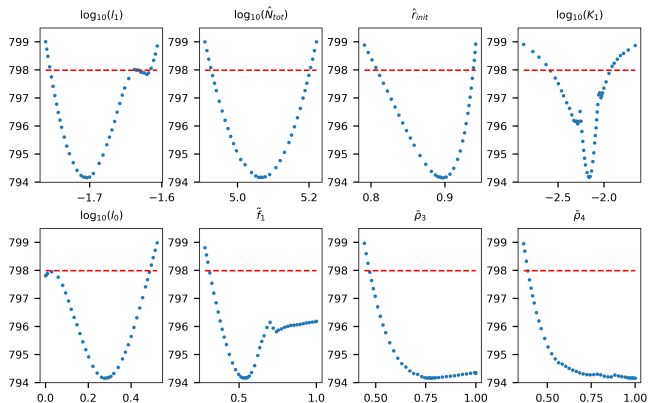


Data fitting



Identifiability

Theoretical (*Structural Identifiability Julia package*) and practical identifiability (*Data2Dynamics*)

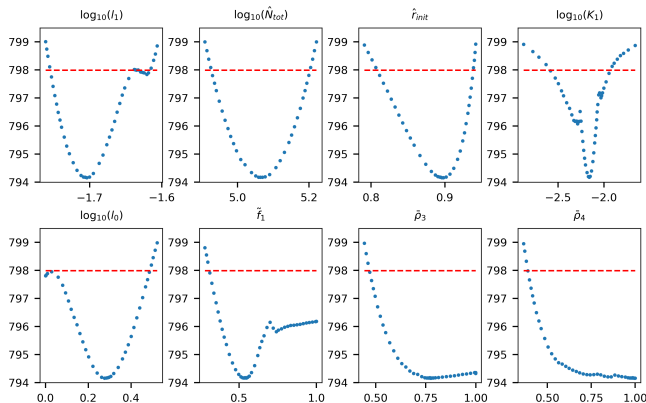


Matlab p. 14

- 20 of 31 parameters are practically identifiable.
- Mean activation time is around 200 days, while growing time is around 50 days.

Identifiability

Theoretical (*Structural Identifiability Julia package*) and practical identifiability (*Data2Dynamics*)



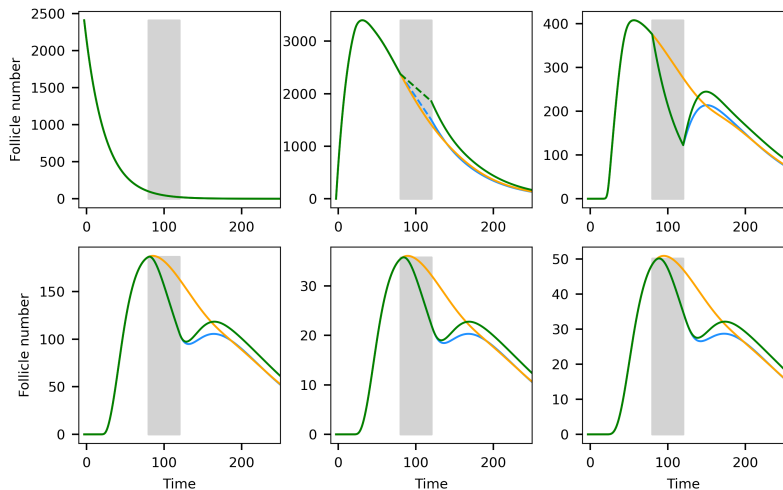
Matlab p. 21

- 20 of 31 parameters are practically identifiable.
- Additional Data on germ cell dynamics would greatly improve identifiability

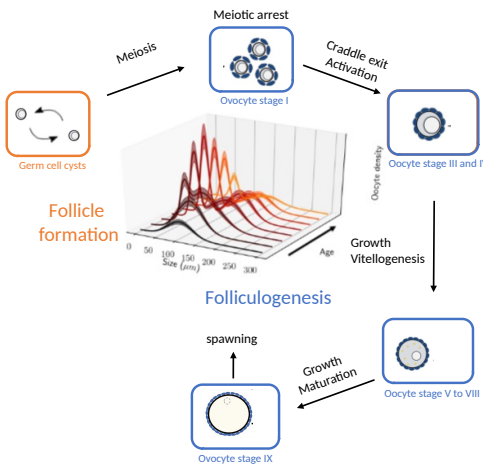
Parameter values interpretation

- Mean activation time is around 200 days, while growing time is around 50 days.
- The first-wave follicles is approx. 5 times faster than the second wave for compartments 0,1,2
- Follicles atresia is negligible in compartment 2,3,4

Prediction of AMH administration



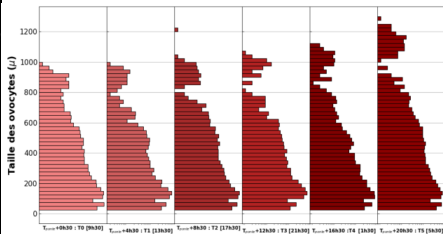
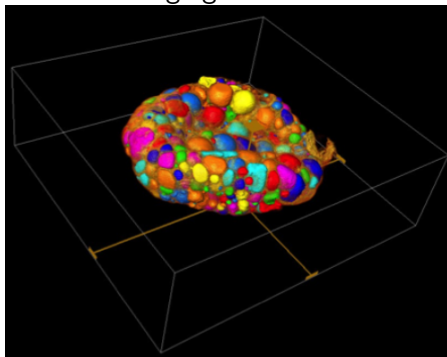
Comparison with fish oogenesis



- Study of Medaka oogenesis
- Renewed asynchronous oogenesis
- Useful for Eco-Toxicology studies

3D imaging of whole ovary in Medaka

3D imaging data : whole Follicle count and size measurement



Modèle structuré en taille

$$\left\{ \begin{array}{l} \frac{d\rho_0(t)}{dt} \\ \partial_t \rho(t, x) \\ \lim_{x \rightarrow 0} \lambda(\rho(t, \cdot), x) \rho(t, x) \end{array} \right. = \begin{array}{l} \left(r_0(\rho(t, \cdot)) - \lambda_0(\rho(t, \cdot)) - \mu_0(\rho(t, \cdot)) \right) \rho_0(t), \\ -\partial_x (\lambda(\rho(t, \cdot), x) \rho(t, x)) - \mu(\rho(t, \cdot), x) \rho(t, x) \\ \lambda_0(\rho(t, \cdot)) \rho_0(t), \end{array}$$

Dépendance du cycle ovarien ?

Distribution de la taille des ovocytes à chaque temps de l'expérience

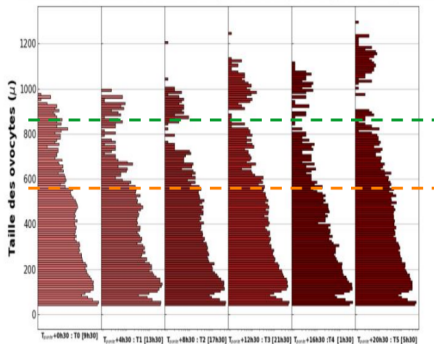
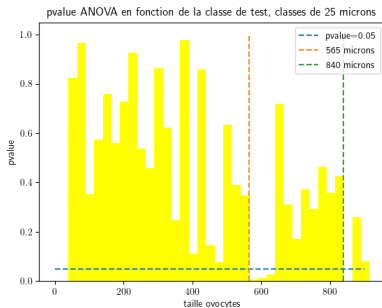


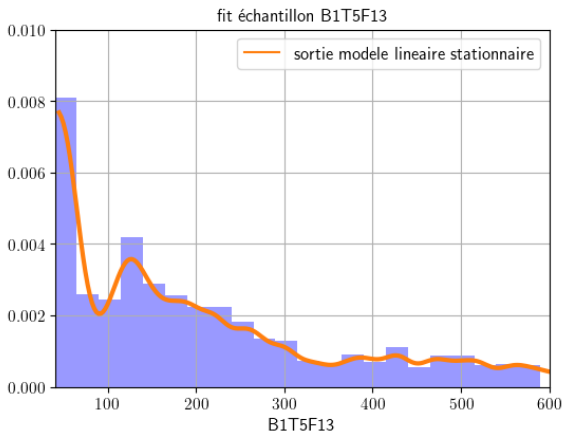
FIGURE A.1 – Présentation de l'évolution des distributions de la taille des ovocytes de 10 en 10 μm en échelle logarithmique



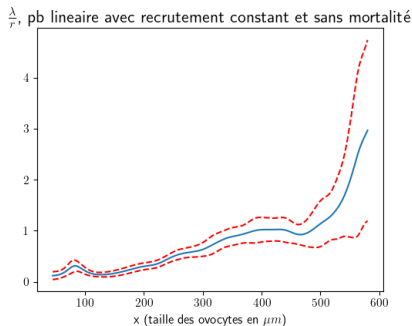
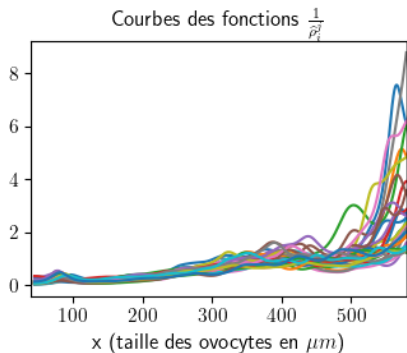
Problème stationnaire pour la partie indépendante du cycle

$$\begin{cases} 0 &= -\partial_x(\lambda(\rho, x)\rho(x)) - \mu(\rho, x)\rho(x), \quad \text{for } x \in (0, 1), \\ r &= \lim_{x \rightarrow 0} \lambda(\rho, x)\rho(x), \end{cases}$$

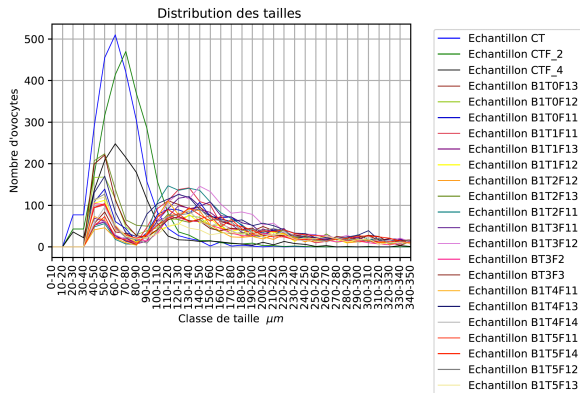
Résultat préliminaire de problème inverse (modèle linéaire, $\mu = 0$)



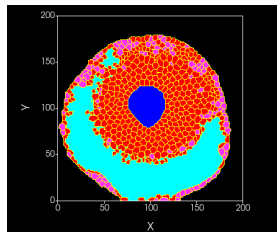
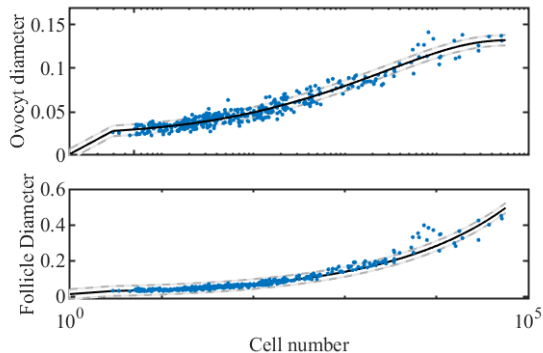
Résultat préliminaire de problème inverse (modèle linéaire, $\mu = 0$)



Transition Juvénile -> adulte



More than one structuring variable



Conclusion and perspective

- ✓ Lifespan ovarian follicle population dynamics model
- ✓ Separation of time scale explains slow decay of the reserve and quasi stable growing follicle repartition
- ✓ Follicle count data and perturbation experiments may reveal feedback mechanisms
 - Extension to three timescale (reserve, basal growth and terminal growth)
 - Extension to (several) continuous structuring variable
 - Comparative physiology approach

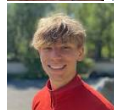
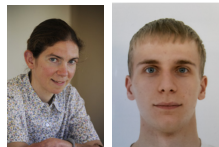
Conclusion and perspective

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Open Post-doc position available in 2023 !

Thanks for your attention !

- ★ INRIA Saclay : Frédérique Clément, Louis Fostier, Guillaume Ballif, Frédérique Robin
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