Multidimensional Performance Analysis of the Forest Ecosystems: Economic-Environmental Performance Nexus

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Abstract

This paper aims to propose an economic instrument evaluating the multidimensional performance of the forest ecosystems. The production theoretic approach to index numbers is considered as theoretical foundation for the performance assessment. Specifically, disaggregated index number is introduced highlighting multidimensional performance measure of the forest ecosystems, by considering economic and environmental dimensions alike. As a result, this paper provides environmentally-adjusted index number permitting to evaluate the economic health of the forest ecosystems. Interestingly, the approach introduced in this paper highlights the Economic-Environmental (EE) performance nexus of the forest ecosystems. Moreover, an econometric model is proposed allowing to estimate the multidimensional performance index of the forest ecosystems. Specifically, a non parametric analytical framework is provided highlighting the practicability of the approach introduced in this paper.

Keywords: Data Envelopment Analysis, Distance Functions, Environmentally-adjusted Performance Index, Forest Ecosystems.

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JEL: D21, D24, Q50

1 Introduction

Forests represent 31% of the earth's land surface -*i.e.*, 4.06 billion ha - and further, they are vital resources for human health and well-being (FAO, 2022). Indeed, forests are ecosystems rich in biodiversity that offer a wide range of economic goods and Ecosystem Services (ES) such as carbon sequestration, recreation, soil and water protection. According to de Groot *et al.* (2002), the multiple goods and services provided by the forest ecosystems may be partitioned into three main categories, namely economic goods (*e.g.*, timber and non-timber based products, such as cork and mushroom), environmental services (*e.g.*, carbon sequestration, soil and water protection) and socio-cultural services (*e.g.*, recreational and tourism based activities). As a result, forest ecosystems simultaneously/jointly provide private and public goods/services, which may have positive (and/or negative) external effects affecting economic systems (Amacher et al., 2014). Whilst private goods and services have market values, it remains difficult to precisely assess the economic impact of the environmental services provided by the forest ecosystems, though these ones are of major importance for human health and well-being. Therefore, the forest ecosystems contribution on the economic systems are often neglected. In this line, designing economic instrument which permits to evaluate the impact of the forest ecosystems on the economic systems must be considered as prominent issue.

This paper introduces a multidimensional performance measure of the forest ecosystems through the production theoretic approach to index numbers (Briec and Kerstens, 2004; Chambers, 2002; Bjurek, 1996; Caves *et al.*, 1982)[1]. Specifically, micro-economic approach is considered to model timber, non-timber and other forest ecosystems services (including biodiversity) production jointness (Murty and Russell, 2020; Abad and Briec, 2019). Astonishing, Forest Joint Production (FJP) processes studies are scarce in production theory and index number theory alike, although such analyses provide information about the trade-offs

[1] Throughout the paper the terms "multidimensional", "Economic-Environmental" and "Environmentally-adjusted" are considered to name the forest ecosystems performance measure.

and synergies between wood (as well as non-wood) goods, biodiversity and Forest Ecosystem Services (FES) provision. To do so, multidimensional distance function is considered as functional representation of the FJP processes (Abad and Ravelojaona, 2021, 2022). Interestingly, the distance function based approach is more general than the traditional production and cost functions ones, by considering multiple output production processes without price information (Briec, 1997; Chambers *et al.*, 1996; Färe and Primont, 1995). This fundamental property of the distance function is of particular interest to deal with ecosystem services, whose prices are not well defined and often unavailable (Chavas, 2009). Moreover, the multidimensional distance function permits to focus on either economic or environmental objectives in order to identify the forest ecosystems transformation processes. As a result, the distance function is considered as the main instrument for the multidimensional evaluation of the forest ecosystems by taking into account economic and environmental dimensions.

Even though the combination of production theory and index number theory provides useful theoretical background evaluating the performance of economic systems (OECD, 2001; Färe et al., 1994), few modelling has been provided to appraise the environmentally-adjusted performance of the forest ecosystems (Gutiérrez and Lozano, 2020). Interestingly, according to the value of the environmentally-adjusted index number, multidimensional forest ecosystems performance variation is highlighted. Moreover, the main components of the multidimensional forest ecosystems performance variation are laid out, indicating economic and environmental sources respectively. Remarkably, the forest ecosystems performance index incorporates spatial and temporal dimensions to analyse the multidimensional performance variation of the forest ecosystems. Indeed, spatial heterogeneity of the forest ecosystems affects the FES provision (including timber and non-timber based products) and biodiversity. As well, temporal variation due to climate change have impacts on the quantity and quality of the multiple goods and services provided by the forest ecosystems. The practicability of the approach laid out in this paper is highlighted through a micro-econometric model providing estimation rules to set up empirical analysis. Specifically, a non-parametric approach is introduced. In this line, the multidimensional performance measure of the forest ecosystems presented in this paper provides an economic tool helping decision-making for sustainable forest management (Zhou et al., 2021).

To sum up the rationale and the specific objectives of this paper are twofold. First, (**i**) the trade-offs and synergies between the timber (as well as non-timber) production, biodiversity conservation and FES provision are analysed by considering a micro-economic approach for the identification of the FJP processes. Second, (**ii**) a multidimensional performance measure of the forest ecosystems is introduced through the production theoretic approach to index numbers and further, it is estimated based upon non parametric econometric techniques.

The remainder of this paper is structured as follows. The definition and identification rule of the FJP processes are highlighted in section 2. The section 3 introduces environmentally-adjusted performance index of the forest ecosystems, by displaying economic and environmental sources of the performance variation. A micro-econometric model based upon non-parametric approach is provided in section 4, highlighting the practicability of the approach proposed in this paper.

2 Preliminaries

This section lays out the notations and basic concepts which are employed throughout the paper. Besides, the definition and properties of the FJP processes are introduced and further, functional representation allowing to identify the FJP processes is presented.

2.1 Notations and material

Let T and S be the sets of observed time periods and spatial units, respectively. Moreover, assume that $\mathrm{T} := \{0, ..., \tau\}$ and $\mathrm{S} := \{0, ..., \zeta\}$, where $\tau, \zeta \in \mathbb{N}^*$. In such case, $\mathrm{U} := \{0, ..., \tau \times \zeta\}$ defined the total number of observed spatial units over all time periods. Additionally, assume that the set of economic and environmental factors is defined as follows, $\mathrm{I} := \{\mathbf{x} = (x^{\mathsf{ec}}, x^{\mathsf{en}}) \in \mathbb{R}_+^{n^{\mathsf{ec}}+n^{\mathsf{en}}} : n^{\mathsf{ec}}, n^{\mathsf{en}} \in \mathbb{N}^*\}$. In the same vein, consider that the production of goods/services is separated into economic and environmental components such that the output set is defined as follows, $\mathrm{O} := \{\mathbf{y} = (y^{\mathsf{ec}}, y^{\mathsf{en}}) \in \mathbb{R}_+^{m^{\mathsf{ec}}+m^{\mathsf{en}}} : m^{\mathsf{ec}}, m^{\mathsf{en}} \in \mathbb{N}^*\}$. As a result, economic and environmental commodities vector is defined as $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+m}$, where $n = n^{\mathsf{ec}} + n^{\mathsf{en}}$ and $m = m^{\mathsf{ec}} + m^{\mathsf{en}}$.

Let F be a multidimensional forest transformation process encompassing technologically and environmentally feasible economic and environmental commodities. Precisely, $\mathrm{F} \subseteq \mathbb{R}_+^{n+m}$ is defined as follows[2]

$$\mathrm{F} := \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+m} : \mathbf{x} \text{ can produce } \mathbf{y}\}. \tag{2.1}$$

Remark that spatial- and temporal-based multidimensional forest transformation processes may be defined by considering $t \in \mathrm{T}$ and $l \in \mathrm{S}$, respectively.[3]

Economic and environmental restrictions of the multidimensional forest transformation process is defined through the economic set, $\mathrm{Y}(x^{\mathtt{en}}, y^{\mathtt{en}}) : \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}} \mapsto \mathbb{R}_+^{n^{\mathtt{ec}}+m^{\mathtt{ec}}}$, and the environmental set, $\mathrm{E}(x^{\mathtt{ec}}, y^{\mathtt{ec}}) : \mathbb{R}_+^{n^{\mathtt{ec}}+m^{\mathtt{ec}}} \mapsto \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}}$, respectively (Abad *et al.*, 2023). Specifically, $\mathrm{Y}(x^{\mathtt{en}}, y^{\mathtt{en}}) \subseteq \mathbb{R}_+^{n^{\mathtt{ec}}+m^{\mathtt{ec}}}$ and $\mathrm{E}(x^{\mathtt{ec}}, y^{\mathtt{ec}}) \subseteq \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}}$ are defined as follows:

$$\mathrm{Y}(x^{\mathtt{en}}, y^{\mathtt{en}}) := \{(x^{\mathtt{ec}}, y^{\mathtt{ec}}) \in \mathbb{R}_+^{n^{\mathtt{ec}}+m^{\mathtt{ec}}} : (\mathbf{x}, \mathbf{y}) \in \mathrm{F}\} \tag{2.2}$$

and

$$\mathrm{E}(x^{\mathtt{ec}}, y^{\mathtt{ec}}) := \{(x^{\mathtt{en}}, y^{\mathtt{en}}) \in \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}} : (\mathbf{x}, \mathbf{y}) \in \mathrm{F}\}. \tag{2.3}$$

Note that the economic and environmental sets permit to characterise the multidimensional forest transformation process such that $(x^{\mathtt{ec}}, y^{\mathtt{ec}}) \in \mathrm{Y}(x^{\mathtt{en}}, y^{\mathtt{en}}) \Leftrightarrow (\mathbf{x}, \mathbf{y}) \in \mathrm{F} \Leftrightarrow (x^{\mathtt{en}}, y^{\mathtt{en}}) \in \mathrm{E}(x^{\mathtt{ec}}, y^{\mathtt{ec}})$.

The reciprocal formulation of the economic and environmental sets (2.2)-(2.3) highlights the forest ecosystems economic-environmental nexus. Precisely, $\mathrm{Y}^{-}(x^{\mathtt{ec}}, y^{\mathtt{ec}}) \subseteq \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}}$ and $\mathrm{E}^{-}(x^{\mathtt{en}}, y^{\mathtt{en}}) \subseteq \mathbb{R}_+^{n^{\mathtt{ec}}+m^{\mathtt{ec}}}$ are defined as follows:

$$\mathrm{Y}^{-}(x^{\mathtt{ec}}, y^{\mathtt{ec}}) := \{(x^{\mathtt{en}}, y^{\mathtt{en}}) \in \mathbb{R}_+^{n^{\mathtt{en}}+m^{\mathtt{en}}} : (x^{\mathtt{ec}}, y^{\mathtt{ec}}) \in \mathrm{Y}(x^{\mathtt{en}}, y^{\mathtt{en}})\} \tag{2.4}$$

[2] In this paper, set-theoretic representation of the forest transformation processes is considered. However, classical characterisation of the forest transformation processes through the production function approach can be immediately provided. Assuming $\mathbf{y}$ is restricted to be scalar such that $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+1}$ then, the forest production function is defined as follows: $\mathrm{f}(\mathbf{x}) := \{\max(\mathbf{y}) : (\mathbf{x}, \mathbf{y}) \in \mathrm{F}\}$.

[3] Temporal-based multidimensional forest transformation process is defined as $\mathrm{F}_t := \{(\mathbf{x}_t, \mathbf{y}_t) \in \mathbb{R}_+^{n+m} : \mathbf{x}_t \text{ can produce } \mathbf{y}_t\}$, where $t \in \mathrm{T}$. As well, spatial-based multidimensional forest transformation process is defined as $\mathrm{F}_l := \{(\mathbf{x}_l, \mathbf{y}_l) \in \mathbb{R}_+^{n+m} : \mathbf{x}_l \text{ can produce } \mathbf{y}_l\}$, with $l \in \mathrm{S}$.

and

$$\mathrm{E}^{-}(x^{\mathtt{en}}, y^{\mathtt{en}}) := \{(x^{\mathtt{ec}}, y^{\mathtt{ec}}) \in \mathbb{R}_{+}^{n^{\mathtt{ec}}+m^{\mathtt{ec}}} : (x^{\mathtt{en}}, y^{\mathtt{en}}) \in \mathrm{E}(x^{\mathtt{ec}}, y^{\mathtt{ec}})\}. \tag{2.5}$$

The reciprocal economic set (2.4) displays all the forest ecosystems environmental goods/services which are needed in order given economic commodities happen and further, the reciprocal environmental set (2.5) maps all the forest economic components that are caused from given environmental commodities.

2.2 Forest joint production processes: axiomatic approach

F (2.1) is the more general representation of multidimensional forest transformation processes however, some restrictions (*i.e.*, properties) on F are needed providing practical representation of the forest transformation activities. In this regard, axiomatic characterisation of the multidimensional forest transformation processes is laid out in the upcoming statements.

Consider that multidimensional forest transformation processes satisfy the following regular properties (Färe *et al.*, 1985):

F1. *Inaction and No free lunch*: $\forall \mathbf{x} \in \mathbb{R}_{+}^{n}$, $(\mathbf{x}, 0) \in \mathrm{F} \wedge (0, \mathbf{y}) \in \mathrm{F} \Rightarrow \mathbf{y} = 0$.
F2. *Boundedness*: $\mathrm{F}_{\mathbf{y}} := \{(\mathbf{x}, \mathbf{v}) \in \mathrm{F} : \mathbf{v} \leq \mathbf{y}\}$ is bounded for all $\mathbf{y} \in \mathbb{R}_{+}^{m}$.
F3. *Closedness*: F is closed.

The first axiom has two parts. The inaction condition ensures that it is always possible to produce nothing whilst, the no free lunch principle implies that positive production cannot come from null production factors. The two parts of F1 implies that $(0, 0) \in \mathrm{F}$. Interestingly, these conditions are fairly weak and therefore, they may be strengthened notably for the environmental components. For illustrative purposes, considering $(\mathbf{x}, \mathbf{y}) = (0, 0)$ might appear as strong assumption and as a result, F1 can be refined by assuming $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_{++}^{n+m}$. F2 and F3 are mathematical properties imposing the production set is compact, which ensure the feasibility of the micro-economic analysis that follows (Debreu, 1951).

In the literature, environmental commodities are usually understood as joint production resulting into negative externalities (Färe *et al.*, 1989). However, in forestry the environmental goods/services can also cause positive externalities (*e.g.*, soil protection and carbon sequestration), which can be considered as desirable commodities arising through forest joint production processes. In either case, the environmental components causing positive and/or negative externalities are not freely disposable[4]. Let K and $\mathcal{K}$ be convex cones which are defined as $\mathrm{K} = \mathbb{R}^n_+ \times -\mathbb{R}^m_+$ and $\mathcal{K} = \mathbb{R}^{n^{\mathrm{ec}}+m^{\mathrm{en}}}_+ \times -\mathbb{R}^{n^{\mathrm{en}}+m^{\mathrm{ec}}}_+$, respectively. Consider that the forest joint production processes satisfies the following restricted disposal property (Abad and Briec, 2019):

F4. *Generalised B-disposability*: $\mathrm{F} := \big((\mathrm{F} + \mathrm{K}) \cap (\mathrm{F} + \mathcal{K}) \big) \cap \mathbb{R}^n_+ \times \mathbb{R}^m_+$.

According to the axioms $\mathrm{F1} - \mathrm{F4}$ the forest joint production processes are defined as a conjunction of economic and environmental sub-processes (Abad and Briec, 2019; Murty *et al.*, 2012). Precisely, intended commodities of the economic activities satisfy the traditional free disposal property -*i.e.,* $(\mathrm{F} + \mathrm{K}) \cap \mathbb{R}^n_+ \times \mathbb{R}^m_+$- and further, environmental goods/services causing externalities fulfil limited free disposal property; *i.e.*, $(\mathrm{F} + \mathcal{K}) \cap \mathbb{R}^n_+ \times \mathbb{R}^m_+$. Notice that the classical convexity property of the joint production processes is not necessary through the axiomatic framework $\mathrm{F1} - \mathrm{F4}$ allowing to consider convex neutral forest joint production processes. As a result, the axiomatic approach $\mathrm{F1} - \mathrm{F4}$ permits to analyse the trade-offs and synergies between the economic and environmental forest ecosystems commodities through a general framework admitting non linearities; see Figures 1-2.

2.3 Multidimensional distance function

Traditionally, distance functions are considered as functional representations of theoretic production set (Chambers and Färe, 2020). In this paper, a multidimensional distance function is introduced as functional form of the forest joint production processes.

The next result lays out the multidimensional distance function separating economic and environmental dimensions of the forest ecosystems (Abad and Ravelojaona, 2022).

[4]Relaxing the classical Free Disposal (FD) axiom implies that environmental outputs disposal (or environmental factors stock) is costly. As a result, it is not possible to scale down environmental goods/services causing negative external effects (*e.g.*, pollution) without reducing economic components. As well, scaling up environmental factors which induce positive externalities (*e.g.*, soil protection) implies to increase the economic components.

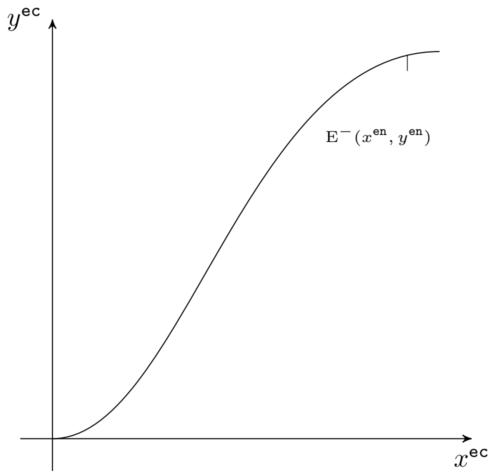

Figure 1: Reciprocal environmental set

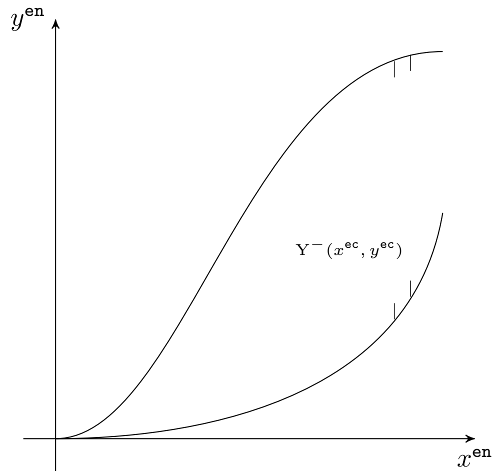

Figure 2: Reciprocal economic set

Definition 2.1 *Let* F *be a forest joint production process which satisfies properties* F1 − F4. *For any* $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+m}$*, the multidimensional distance function is defined as follows:*

$$D^{\alpha;\beta}(\mathbf{x}, \mathbf{y}) := \sup_{\theta} \left\{ \theta \geq 1 : \left(\theta^{-\alpha^{\mathsf{ec}}} x^{\mathsf{ec}}, \theta^{-\alpha^{\mathsf{en}}} x^{\mathsf{en}}, \theta^{\beta^{\mathsf{ec}}} y^{\mathsf{ec}}, \theta^{\beta^{\mathsf{en}}} y^{\mathsf{en}} \right) \in \mathrm{F} \right\} \tag{2.6}$$

where $\alpha = (\alpha^{\mathsf{ec}}, \alpha^{\mathsf{en}}) \in \mathbb{R}_+^{n^{\mathsf{ec}}} \times \mathbb{R}^{n^{\mathsf{en}}}$ *and* $\beta = (\beta^{\mathsf{ec}}, \beta^{\mathsf{en}}) \in \mathbb{R}_+^{m^{\mathsf{ec}}} \times \mathbb{R}^{m^{\mathsf{en}}}$.

Interestingly, the general shape of the multidimensional distance function (2.6) permits to focus on economic- and environmental-oriented distance functions.

The next result presents the economic- and environmental-oriented multidimensional distance functions:

Proposition 2.2 *For any* $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+m}$,

i. *The economic-oriented multidimensional distance function is defined as follows,*

$$D^{\alpha^{\mathsf{ec}},\beta^{\mathsf{ec}}}(\mathbf{x}, \mathbf{y}) \equiv D^{\mathrm{I}^{\mathsf{ec}},\mathrm{O}^{\mathsf{ec}}}(\mathbf{x}, \mathbf{y}), \tag{2.7}$$

where $(\alpha^{\mathsf{ec}}, \beta^{\mathsf{ec}}) \in \mathbb{R}_+^{n^{\mathsf{ec}}} \times \mathbb{R}_+^{m^{\mathsf{ec}}}$ *and* $\alpha^{\mathsf{en}} = \beta^{\mathsf{en}} = 0$.

ii. *The environmental-oriented multidimensional distance function is defined as follows,*

$$D^{\alpha^{\mathsf{en}},\beta^{\mathsf{en}}}(\mathbf{x}, \mathbf{y}) \equiv D^{\mathrm{I}^{\mathsf{en}},\mathrm{O}^{\mathsf{en}}}(\mathbf{x}, \mathbf{y}), \tag{2.8}$$

where $(\alpha^{\mathsf{en}}, \beta^{\mathsf{en}}) \in \mathbb{R}^{n^{\mathsf{en}}} \times \mathbb{R}^{m^{\mathsf{en}}}$ *and* $\alpha^{\mathsf{ec}} = \beta^{\mathsf{ec}} = 0$.

Assuming that $(\alpha^i, \beta^i) \in [0,1] \times [0,1]$ and $\alpha^j = \beta^j = 0$ with $i, j = \texttt{ec}, \texttt{en}$ and $i \neq j$ then, the economic- and environmental-oriented multidimensional distance functions inherit the structure of the traditional Shephard distance function (Färe *et al.*, 1985)[5].

The parametrisation of the environmental-oriented multidimensional distance function permits to consider separately environmental commodities resulting into either positive external effects or negative externalities. Let ε^+ and ε^- be the positive and negative externalities cones. For any $(x^{\texttt{en}}, y^{\texttt{en}}) \in \mathrm{Y}^-(x^{\texttt{ec}}, y^{\texttt{ec}})$, positive and negative externalities cones are defined as $\varepsilon^+ := \{(x^{\texttt{en}}, y^{\texttt{en}}) : (x^{\texttt{en}}, y^{\texttt{en}}) + \mathbb{R}_+^{n^{\texttt{en}}+m^{\texttt{en}}} \in \mathrm{Y}^-(x^{\texttt{ec}}, y^{\texttt{ec}})\}$ and $\varepsilon^- := \{(x^{\texttt{en}}, y^{\texttt{en}}) : (x^{\texttt{en}}, y^{\texttt{en}}) + \mathbb{R}_-^{n^{\texttt{en}}+m^{\texttt{en}}} \in \mathrm{Y}^-(x^{\texttt{ec}}, y^{\texttt{ec}})\}$, respectively. The next results define the environmental-oriented multidimensional distance function by considering positive and negative externalities cones.

$$D_{\varepsilon^+}^{\alpha^{\texttt{en}}, \beta^{\texttt{en}}}(\mathbf{x}, \mathbf{y}) \equiv D_{\varepsilon^+}^{\mathrm{I}^{\texttt{en}}, \mathrm{O}^{\texttt{en}}}(\mathbf{x}, \mathbf{y}), \tag{2.9}$$

where $(\alpha^{\texttt{en}}, \beta^{\texttt{en}}) \in [-1, 0] \times [0, 1]$ and $\alpha^{\texttt{ec}} = \beta^{\texttt{ec}} = 0$.

And

$$D_{\varepsilon^-}^{\alpha^{\texttt{en}}, \beta^{\texttt{en}}}(\mathbf{x}, \mathbf{y}) \equiv D_{\varepsilon^-}^{\mathrm{I}^{\texttt{en}}, \mathrm{O}^{\texttt{en}}}(\mathbf{x}, \mathbf{y}), \tag{2.10}$$

where $(\alpha^{\texttt{en}}, \beta^{\texttt{en}}) \in [0, 1] \times [-1, 0]$ and $\alpha^{\texttt{ec}} = \beta^{\texttt{ec}} = 0$.

3 Methodology

In this section environmentally-adjusted forest ecosystems performance measure is laid out based upon combination of economic- and environmental-oriented multidimensional distance functions. Specifically, the proposed multidimensional forest ecosystems performance measure inherits the structure of the Färe-Primont index, that is the fixed base version

[5]Note that if $\alpha^{\texttt{i}} = 1$ and $\beta^{\texttt{i}} = \alpha^j = \beta^j = 0$ with $i, j = \texttt{ec}, \texttt{en}$ and $i \neq j$ then, the economic- and environmental-oriented multidimensional distance functions take the form of the sub-vector Shephard distance function (Färe *et al.*, 2004). In such case, $D^{\alpha^{\texttt{i}}, \beta^{\texttt{i}}}(\mathbf{x}, \mathbf{y}) \equiv D^{\mathrm{I}^{\texttt{i}}}(x_t, y_t)$ where $i = \texttt{ec}, \texttt{en}$. In the same way, when $\beta^{\texttt{i}} = 1$ and $\alpha^{\texttt{i}} = \alpha^j = \beta^j = 0$ with $i, j = \texttt{ec}, \texttt{en}$ and $i \neq j$ then, $D^{\alpha^{\texttt{i}}, \beta^{\texttt{i}}}(\mathbf{x}, \mathbf{y}) \equiv D^{\mathrm{O}^{\texttt{i}}}(x_t, y_t)$ with $i = \texttt{ec}, \texttt{en}$.

of the Hicks-Moorsteen index (O'Donnell, 2014; Bjurek, 1996). Moreover, the economic-environmental performance nexus is highlighted through a disaggregation of the environmentally-adjusted forest ecosystems performance index.

3.1 Environmentally-adjusted forest ecosystems performance index: definition

Let $\mathrm{U}^{\mathbf{x},\mathbf{y}} := \{(\mathbf{x}_u, \mathbf{y}_u) \in \mathbb{R}_+^{n+m} : u \in \mathrm{U}\}$ be the overall set of observations encompassing the total number of spatial commodities over all time periods. The next result defines the environmentally-adjusted forest ecosystems performance index by considering both positive and negative externalities cones.

Definition 3.1 *Assume that* F *is a forest joint production process satisfying properties* F1 − F4. *For any* $(\mathbf{x}_{q,s}, \mathbf{y}_{q,s}) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}$ *and a representative observation* $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+m}$*, the Environmentally-adjusted Forest ecosystems Performance (EFP) index considering both positive and negative externalities cones is defined as follows:*

$$\mathrm{EFP}_{\varepsilon^i}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}) = \frac{g_i\big(D^{\mathrm{O}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)}{h_i\big(D^{\mathrm{I}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)}, \quad i = +, - \tag{3.1}$$

such that $g_i(\cdot)$ *and* $h_i(\cdot)$ *display output- and input-based quantity indices taking into account positive and negative environmental externalities, respectively.*

To be consistent with the production theoretic approach to index numbers, the environmentally-adjusted forest ecosystems performance measure is defined as a ratio of output quantity index over input quantity index by considering economic and environmental components of the forest ecosystems. Specifically, these quantity indices are defined by means of economic- and environmental-oriented multidimensional distance functions[6].

The next results lay out the output- and input-based quantity indices by considering both positive and negative externalities cones.

[6] In this contribution, the economic- and environmental-oriented multidimensional distance functions are considered as aggregator functions defining input and output quantity indices. Nonetheless, different aggregator functions may be considered affecting the shape of the performance measure; see O'Donnell (2014).

$$
\begin{cases}
g_+\big(D^{\mathrm{O}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z},\mathbf{y}_{q,s,z})\big) = \dfrac{D^{\mathrm{O}^{\mathrm{ec}}}(\mathbf{x}_z, y_q^{\mathbf{ec}}, y_z^{\mathbf{en}})}{D^{\mathrm{O}^{\mathrm{ec}}}(\mathbf{x}_z, y_s^{\mathbf{ec}}, y_z^{\mathbf{en}})} \times \dfrac{D_{\varepsilon^+}^{\mathrm{O}^{\mathrm{en}}}(\mathbf{x}_z, y_z^{\mathbf{ec}}, y_q^{\mathbf{en}})}{D_{\varepsilon^+}^{\mathrm{O}^{\mathrm{en}}}(\mathbf{x}_z, y_z^{\mathbf{ec}}, y_s^{\mathbf{en}})} \\
g_-\big(D^{\mathrm{O}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z},\mathbf{y}_{q,s,z})\big) = \dfrac{D^{\mathrm{O}^{\mathrm{ec}}}(\mathbf{x}_z, y_q^{\mathbf{ec}}, y_z^{\mathbf{en}})}{D^{\mathrm{O}^{\mathrm{ec}}}(\mathbf{x}_z, y_s^{\mathbf{ec}}, y_z^{\mathbf{en}})} \times \dfrac{D_{\varepsilon^-}^{\mathrm{O}^{\mathrm{en}}}(\mathbf{x}_z, y_z^{\mathbf{ec}}, y_s^{\mathbf{en}})}{D_{\varepsilon^-}^{\mathrm{O}^{\mathrm{en}}}(\mathbf{x}_z, y_z^{\mathbf{ec}}, y_q^{\mathbf{en}})}
\end{cases} \tag{3.2}
$$

and

$$
\begin{cases}
h_+\big(D^{\mathrm{I}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z},\mathbf{y}_{q,s,z})\big) = \dfrac{D^{\mathrm{I}^{\mathrm{ec}}}(x_s^{\mathbf{ec}}, x_z^{\mathbf{en}}, \mathbf{y}_z)}{D^{\mathrm{I}^{\mathrm{ec}}}(x_q^{\mathbf{ec}}, x_z^{\mathbf{en}}, \mathbf{y}_z)} \times \dfrac{D_{\varepsilon^+}^{\mathrm{I}^{\mathrm{en}}}(x_z^{\mathbf{ec}}, x_q^{\mathbf{en}}, \mathbf{y}_z)}{D_{\varepsilon^+}^{\mathrm{I}^{\mathrm{en}}}(x_z^{\mathbf{ec}}, x_s^{\mathbf{en}}, \mathbf{y}_z)} \\
h_-\big(D^{\mathrm{I}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z},\mathbf{y}_{q,s,z})\big) = \dfrac{D^{\mathrm{I}^{\mathrm{ec}}}(x_s^{\mathbf{ec}}, x_z^{\mathbf{en}}, \mathbf{y}_z)}{D^{\mathrm{I}^{\mathrm{ec}}}(x_q^{\mathbf{ec}}, x_z^{\mathbf{en}}, \mathbf{y}_z)} \times \dfrac{D_{\varepsilon^-}^{\mathrm{I}^{\mathrm{en}}}(x_z^{\mathbf{ec}}, x_s^{\mathbf{en}}, \mathbf{y}_z)}{D_{\varepsilon^-}^{\mathrm{I}^{\mathrm{en}}}(x_z^{\mathbf{ec}}, x_q^{\mathbf{en}}, \mathbf{y}_z)}.
\end{cases} \tag{3.3}
$$

Assuming that environmental commodities cause positive externalities, $g_+(\cdot) > 1$ implies that more economic and environmental outputs are produced by the observation q than the observation s for given environmental and economic factors $\mathbf{x}_z$. Moreover, if $h_+(\cdot) < 1$ then, less economic inputs and more environmental factors are used by the observation s relatively to the observation q for given economic and environmental goods/services $\mathbf{y}_z$. In such case, $\mathrm{EFP}_{\varepsilon^+}(\cdot) > 1$ indicates environmentally-adjusted forest ecosystems performance improvement. Reversely, when $g_+(\cdot) < 1$ and $h_+(\cdot) > 1$ then, opposite outcomes arise exhibiting multidimensional forest ecosystems performance loss, that is $\mathrm{EFP}_{\varepsilon^+}(\cdot) < 1$.[7]

According to the combination of the quantity indices $g_i(\cdot)$ and $h_i(\cdot)$ values, where $i = +, -$, remark that different environmentally-adjusted forest ecosystems performance characterisation may be identified; see Table 1.

Interestingly enough, the EFP index can be disaggregated highlighting economic and environmental sources of the multidimensional forest ecosystems performance variation. The disaggregation rule of the EFP index is provided in the next statement.

Proposition 3.2 *For any* $(\mathbf{x}_{q,s}, \mathbf{y}_{q,s}) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}$ *and a representative observation* $(\mathbf{x}_z, \mathbf{y}_z) \in$

[7] In the same vein, considering that environmental commodities cause negative external effects then, $g_-(\cdot) > 1$ indicates that more economic outputs and less environmental goods/services are produced by the observation q than the observation s for given environmental and economic factors $\mathbf{x}_z$. As well, $h_-(\cdot) < 1$ implies that less economic and environmental factors are employed by the observation s relatively to the observation q for given economic and environmental goods/services $\mathbf{y}_z$. As a result, $\mathrm{EFP}_{\varepsilon^-}(\cdot) > 1$ reveals environmentally-adjusted forest ecosystems performance increase. If $\mathrm{EFP}_{\varepsilon^-}(\cdot) < 1$ then, the reverse reasoning holds.

	$h_i(\cdot) > 1$	$h_i(\cdot) < 1$
$g_i(\cdot) > 1$	**i.** $g_i(\cdot) > h_i(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) > 1$ **ii.** $g_i(\cdot) < h_i(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) < 1$	$\text{EFP}_{\varepsilon^i}(\cdot) > 1$
$g_i(\cdot) < 1$	$\text{EFP}_{\varepsilon^i}(\cdot) < 1$	**i.** $g_i(\cdot) > h_i(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) > 1$ **ii.** $g_i(\cdot) < h_i(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) < 1$

Table 1: $\text{EFP}_{\varepsilon^i}(\cdot)$ characterization, where $i = +, -$

$\mathbb{R}_+^{n+m}$*, the EFP index disaggregation taking into account positive and negative externalities cones is defined as follows:*

$$\begin{aligned}\text{EFP}_{\varepsilon^i}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}) &= \frac{g^{\mathsf{ec}}\big(D^{\text{O}^{\text{ec}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)}{h^{\mathsf{ec}}\big(D^{\text{I}^{\text{ec}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)} \times \frac{g_i^{\mathsf{en}}\big(D_{\varepsilon^i}^{\text{O}^{\text{en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)}{h_i^{\mathsf{en}}\big(D_{\varepsilon^i}^{\text{I}^{\text{en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z})\big)}, \quad i = +, - \\ &\equiv \text{EFP}^{\mathsf{ec}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}) \times \text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}), \quad i = +, - \end{aligned} \tag{3.4}$$

where $\text{EFP}^{\mathsf{ec}}(\cdot)$ *and* $\text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot)$ *highlight economic- and environmental-based forest ecosystems performance indices considering positive and negative environmental externalities, respectively.*

Disaggregating the EFP index permits to display the economic-environmental nexus of the forest ecosystems performance variation. Specifically, the next table identifies the main schemes driving the foret ecosystems performance change.

	$\text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot) > 1$	$\text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot) < 1$
$\text{EFP}^{\mathsf{ec}}(\cdot) > 1$	$\text{EFP}_{\varepsilon^i}(\cdot) > 1$	**i.** $\text{EFP}^{\mathsf{ec}}(\cdot) > \left[\text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot)\right]^{-1}$ then, $\text{EFP}_{\varepsilon^i}(\cdot) > 1$ **ii.** $\text{EFP}^{\mathsf{ec}}(\cdot) < \left[\text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot)\right]^{-1}$ then, $\text{EFP}_{\varepsilon^i}(\cdot) < 1$
$\text{EFP}^{\mathsf{ec}}(\cdot) < 1$	**i.** $[\text{EFP}^{\mathsf{ec}}(\cdot)]^{-1} < \text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) > 1$ **ii.** $[\text{EFP}^{\mathsf{ec}}(\cdot)]^{-1} > \text{EFP}_{\varepsilon^i}^{\mathsf{en}}(\cdot)$ then, $\text{EFP}_{\varepsilon^i}(\cdot) < 1$	$\text{EFP}_{\varepsilon^i}(\cdot) < 1$

Table 2: $\text{EFP}_{\varepsilon^i}(\cdot)$ disaggregation, where $i = +, -$

3.2 Spatial- and temporal-based EFP indices

The upcoming statement highlights spatial- and temporal-based multidimensional forest ecosystems performance measures.

Proposition 3.3 *Let* T *and* S *be the sets of observed time periods and spatial units. For any* $(\mathbf{x}_{q,s}, \mathbf{y}_{q,s}) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}$ *and a representative observation* $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+m}$*, the spatial- and temporal-based EFP indices taking into account positive and negative externalities cones respectively are defined as follows:*

$$\mathrm{EFP}_{\varepsilon^i}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; t) = \frac{g_i\big(D^{\mathrm{O}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; t)\big)}{h_i\big(D^{\mathrm{I}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; t)\big)}, \quad i = +,- \ \wedge \ t \in \mathrm{T}. \tag{3.5}$$

and

$$\mathrm{EFP}_{\varepsilon^i}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; l) = \frac{g_i\big(D^{\mathrm{O}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; l)\big)}{h_i\big(D^{\mathrm{I}^{\mathrm{ec,en}}}(\mathbf{x}_{q,s,z}, \mathbf{y}_{q,s,z}; l)\big)}, \quad i = +,- \ \wedge \ l \in \mathrm{S} \tag{3.6}$$

Spatial version of the EFP index (3.5) permits to compare two different spatial units, namely q and s, for a given time period $t \in \mathrm{T}$ and a representative bundle of commodities $(\mathbf{x}_z, \mathbf{y}_z)$. In the same way, temporal formulation of the EFP index (3.6) compares temporal observations q and s for a given spatial unit $l \in \mathrm{S}$ and a representative observation $(\mathbf{x}_z, \mathbf{y}_z)$. Interestingly, combining spatial- and temporal-based EFP formulations allows to compare different spatial units (e.g., $a, b \in \mathrm{S}$) in different periods (e.g., $e, c \in \mathrm{T}$), namely $q \equiv \{a, e\}$ and $s \equiv \{b, c\}$, for a given commodities $(\mathbf{x}_z, \mathbf{y}_z)$.[8]

Remark that spatial and temporal formulations of the EFP index are based upon the identification of a base observation allowing to compare $q, s \in \mathrm{U}$. Specifically, the choice of the base observation for comparisons is affected by the identification of remarkable spatial and/or temporal units. Regarding the selection of the representative bundle of commodities $(\mathbf{x}_z, \mathbf{y}_z)$, the average input and output of the set $\mathrm{U}^{\mathbf{x},\mathbf{y}}$, which encompasses the total number of observed spatial and temporal commodities, is considered (O'Donnell, 2014)[9].

[8]The results provided in the Tables 1-2 about the characterisation and the disaggregation of the EFP index remains valid through spatial- and temporal-based formulations of the EFP measure, namely (3.5) and (3.6).

[9]Note that different fixed reference points are used in the literature (Briec *et al.*, 2018). In this paper, the average input and output of the set $\mathrm{U}^{\mathbf{x},\mathbf{y}}$ is selected to be consistent with the original formulation of the FP index (O'Donnell, 2014).

4 Econometric estimation: non parametric approach

In this section, non parametric specification of the EFP index is introduced through a DEA analytical framework. Specifically, the EFP index is evaluated based upon multiplicative approximation of the FJP processes which permits the estimation of classical S-shaped production frontier (Kao, 1986; Banker and Maindiratta, 1986). Interestingly, the S-shaped production frontier results into non convex production set allowing to consider increasing marginal products.

4.1 Multiplicative FJP processes

The next definition presents non parametric approximation of the multiplicative FJP processes through the DEA method. Precisely, multiplicative formulation of the by-production model is considered which satisfies the axiomatic approach $\mathrm{F1}-\mathrm{F4}$ (Abad and Briec, 2019; Murty *et al.*, 2012).

Definition 4.1 *Assuming that the forest joint production process* F *satisfies properties* $\mathrm{F1}-\mathrm{F4}$*, for any* $(\mathbf{x},\mathbf{y}) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}$ *the non parametric approximation of the multiplicative FJP process is defined as follows:*

$$\mathrm{F}^{\mathtt{DEA}}(\mathbf{x},\mathbf{y}) := \Bigg\{(\mathbf{x},\mathbf{y}) \in \mathbb{R}^{n+m}_{+} : \mathbf{x} \geq \prod_{u\in\mathrm{U}} (\mathbf{x}_u)^{\eta_u}, y^{\mathtt{ec}} \leq \prod_{u\in\mathrm{U}} (y^{\mathtt{ec}}_u)^{\eta_u}, x^{\mathtt{en}} \leq \prod_{u\in\mathrm{U}} (x^{\mathtt{en}}_u)^{\lambda_u}, \\ y^{\mathtt{en}} \geq \prod_{u\in\mathrm{U}} (y^{\mathtt{en}}_u)^{\lambda_u}; \lambda,\eta \geq 0; \sum_{u\in\mathrm{U}} \lambda_u = \sum_{u\in\mathrm{U}} \eta_u = 1\Bigg\}. \quad (4.1)$$

Remark that the aforementioned result (4.1) provides non parametric approximation of the overall forest transformation process including all observed spatial units over time.

Let $\mathrm{U}^{\mathbf{x},\mathbf{y}}_{++} := \{(\mathbf{x}_u,\mathbf{y}_u) \in \mathbb{R}^{n+m}_{++} : u \in \mathrm{U}\}$ be the strictly positive overall set of observations. Considering natural logarithmic transformation of (4.1) provides log-linear production set which inherits the structure of the classical BCC piecewise linear model (Banker *et al.*, 1984). Precisely, the next result lays out the natural logarithmic transformation of (4.1).

Proposition 4.2 *For any* $(\mathbf{x},\mathbf{y}) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}_{++}$*, non parametric natural logarithmic transformation* *of the multiplicative FJP process is defined as follows:*

$$\ln\left(\mathrm{F}^{\mathtt{DEA}}(\mathbf{x},\mathbf{y})\right) := \Bigg\{(\mathbf{x},\mathbf{y}) \in \mathbb{R}^{n+m}_{++} : \ln(\mathbf{x}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(\mathbf{x}_u\right), \ln(y^{\mathtt{ec}}) \leq \sum_{u\in \mathrm{U}} \eta_u \ln\left(y^{\mathtt{ec}}_u\right), \ln(x^{\mathtt{en}}) \leq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(x^{\mathtt{en}}_u\right), \ln(y^{\mathtt{en}}) \geq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(y^{\mathtt{en}}_u\right); \lambda,\eta \geq 0; \sum_{u\in \mathrm{U}} \lambda_u = \sum_{u\in \mathrm{U}} \eta_u = 1\Bigg\}. \quad (4.2)$$

4.2 Non parametric EFP index

In this contribution, the multidimensional distance functions permitting to estimate the EFP index are evaluated through the same reference technology, namely the non parametric multiplicative FJP process (4.1).[10]

For any $(\mathbf{x}_k, \mathbf{y}_k) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}$, the non parametric multidimensional distance function of the observation $(\mathbf{x}_k, \mathbf{y}_k)$ is defined as follows:

$$D^{\alpha;\beta}(\mathbf{x},\mathbf{y})|_{DEA} := \sup_{\theta}\Big\{\theta \geq 1 : \theta^{-\alpha^{\mathtt{ec}}} x^{\mathtt{ec}}_{k,i} \geq \prod_{u\in \mathrm{U}} \left(x^{\mathtt{ec}}_{u,i}\right)^{\eta_u}, \theta^{-\alpha^{\mathtt{en}}} x^{\mathtt{en}}_{k,i} \geq \prod_{u\in \mathrm{U}} \left(x^{\mathtt{en}}_{u,i}\right)^{\eta_u}, \theta^{\beta^{\mathtt{ec}}} y^{\mathtt{ec}}_{k,j} \leq \prod_{u\in \mathrm{U}} \left(y^{\mathtt{ec}}_{u,j}\right)^{\eta_u}, \quad (4.3)$$

$$\theta^{-\alpha^{\mathtt{en}}} x^{\mathtt{en}}_{k,i} \leq \prod_{u\in \mathrm{U}} \left(x^{\mathtt{en}}_{u,i}\right)^{\lambda_u}, \theta^{\beta^{\mathtt{en}}} y^{\mathtt{en}}_{k,j} \geq \prod_{u\in \mathrm{U}} \left(y^{\mathtt{en}}_{u,j}\right)^{\lambda_u}, \lambda,\eta \geq 0,$$

$$\sum_{u\in \mathrm{U}} \lambda_u = \sum_{u\in \mathrm{U}} \eta_u = 1, i \in [n], j \in [m]\Big\}$$

where $\alpha = (\alpha^{\mathtt{ec}}, \alpha^{\mathtt{en}}) \in \mathbb{R}^{n^{\mathtt{ec}}}_{+} \times \mathbb{R}^{n^{\mathtt{en}}}$ and $\beta = (\beta^{\mathtt{ec}}, \beta^{\mathtt{en}}) \in \mathbb{R}^{m^{\mathtt{ec}}}_{+} \times \mathbb{R}^{m^{\mathtt{en}}}$.

Considering a natural logarithmic transformation of (4.3), the non parametric multidimensional distance function of the observation $(\mathbf{x}_k, \mathbf{y}_k)$ is defined through a log-linear model. For any $(\mathbf{x}_k, \mathbf{y}_k) \in \mathrm{U}^{\mathbf{x},\mathbf{y}}_{++}$, the natural logarithmic transformation of the non parametric multidimensional distance function of the observation $(\mathbf{x}_k, \mathbf{y}_k)$ is defined as follows:

[10] Considering a single reference technology for the the multidimensional distance functions assessment ensures that the EFP index satisfies the transitivity property which permits multilateral and multi-temporal comparisons (O'Donnell, 2014).

$$\ln\left[D^{\alpha;\beta}(\mathbf{x},\mathbf{y})|_{DEA}\right] := \sup_{\ln(\theta)}\Big\{\ln(\theta)\geq 0 : \ln(x^{\mathtt{ec}}_{k,i}) - \alpha^{\mathtt{ec}}\ln(\theta) \geq \sum_{u\in\mathrm{U}}\eta_u \ln\left(x^{\mathtt{ec}}_{u,i}\right), \ln(x^{\mathtt{en}}_{k,i}) - \alpha^{\mathtt{en}}\ln(\theta) \geq \sum_{u\in\mathrm{U}}\eta_u\ln\left(x^{\mathtt{en}}_{u,i}\right),$$
$$\ln(y^{\mathtt{ec}}_{k,j}) + \beta^{\mathtt{ec}}\ln(\theta) \leq \sum_{u\in\mathrm{U}}\eta_u\ln\left(y^{\mathtt{ec}}_{u,j}\right), \ln(x^{\mathtt{en}}_{k,i}) - \alpha^{\mathtt{en}}\ln(\theta) \leq \sum_{u\in\mathrm{U}}\lambda_u\ln\left(x^{\mathtt{en}}_{u,i}\right),$$
$$\ln(y^{\mathtt{en}}_{k,j}) + \beta^{\mathtt{en}}\ln(\theta) \geq \sum_{u\in\mathrm{U}}\lambda_u\ln\left(y^{\mathtt{en}}_{u,j}\right), \lambda,\eta\geq 0, \sum_{u\in\mathrm{U}}\lambda_u = \sum_{u\in\mathrm{U}}\eta_u = 1,$$
$$i\in[n], j\in[m]\Big\} \tag{4.4}$$

where $\alpha = (\alpha^{\mathtt{ec}}, \alpha^{\mathtt{en}}) \in \mathbb{R}^{n^{\mathtt{ec}}}_{+} \times \mathbb{R}^{n^{\mathtt{en}}}$ and $\beta = (\beta^{\mathtt{ec}}, \beta^{\mathtt{en}}) \in \mathbb{R}^{m^{\mathtt{ec}}}_{+} \times \mathbb{R}^{m^{\mathtt{en}}}$.

The computation of the EFP index requires to implement eight distance functions, namely four economic-oriented distance functions and four environmental-oriented distance functions. For any two observations $k = q, s$, the economic-based EFP index is defined by resolving the next linear programs:

$$\begin{aligned}
\ln\left[D^{\mathrm{I}^{\mathtt{ec}}}(x^{\mathtt{ec}}_k, x^{\mathtt{en}}_z, \mathbf{y}_z)|_{DEA}\right] = \quad & \max \quad \ln(\theta) \\
& \text{s.t.} \quad \ln(x^{\mathtt{ec}}_{k,i}) - \ln(\theta) \geq \sum_{u\in\mathrm{U}}\eta_u\ln\left(x^{\mathtt{ec}}_{u,i}\right), i = 1,...,n^{\mathtt{ec}} \\
& \qquad \ln(x^{\mathtt{en}}_{z,i}) \geq \sum_{u\in\mathrm{U}}\eta_u\ln\left(x^{\mathtt{en}}_{u,i}\right), i = 1,...,n^{\mathtt{en}} \\
& \qquad \ln(y^{\mathtt{ec}}_{z,j}) \leq \sum_{u\in\mathrm{U}}\eta_u\ln\left(y^{\mathtt{ec}}_{u,j}\right), j = 1,...,m^{\mathtt{ec}} \\
& \qquad \ln(x^{\mathtt{en}}_{z,i}) \leq \sum_{u\in\mathrm{U}}\lambda_u\ln\left(x^{\mathtt{en}}_{u,i}\right), i = 1,...,n^{\mathtt{en}} \\
& \qquad \ln(y^{\mathtt{en}}_{z,j}) \geq \sum_{u\in\mathrm{U}}\lambda_u\ln\left(y^{\mathtt{en}}_{u,j}\right), j = 1,...,m^{\mathtt{en}} \\
& \qquad \sum_{u\in\mathrm{U}}\lambda_u = 1 \\
& \qquad \sum_{u\in\mathrm{U}}\eta_u = 1 \\
& \qquad \lambda,\eta \geq 0
\end{aligned} \tag{4.5}$$

and

$$
\begin{aligned}
\ln\left[D^{\mathrm{O}^{\mathrm{ec}}}(\mathbf{x}_z, y_k^{\mathtt{ec}}, y_z^{\mathtt{en}})|_{DEA}\right] = \quad & \max \quad \ln(\theta) \\
& \text{s.t.} \quad \ln(x_{z,i}^{\mathtt{ec}}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x_{u,i}^{\mathtt{ec}}\right), i = 1, ..., n^{\mathtt{ec}} \\
& \qquad \ln(x_{z,i}^{\mathtt{en}}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x_{u,i}^{\mathtt{en}}\right), i = 1, ..., n^{\mathtt{en}} \\
& \qquad \ln(y_{k,j}^{\mathtt{ec}}) + \ln(\theta) \leq \sum_{u\in \mathrm{U}} \eta_u \ln\left(y_{u,j}^{\mathtt{ec}}\right), j = 1, ..., m^{\mathtt{ec}} \\
& \qquad \ln(x_{z,i}^{\mathtt{en}}) \leq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(x_{u,i}^{\mathtt{en}}\right), i = 1, ..., n^{\mathtt{en}} \\
& \qquad \ln(y_{z,j}^{\mathtt{en}}) \geq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(y_{u,j}^{\mathtt{en}}\right), j = 1, ..., m^{\mathtt{en}} \\
& \qquad \sum_{u\in \mathrm{U}} \lambda_u = 1 \\
& \qquad \sum_{u\in \mathrm{U}} \eta_u = 1 \\
& \qquad \lambda, \eta \geq 0.
\end{aligned} \tag{4.6}
$$

Regarding the environmental-based EFP index computation, the next environmental-oriented distance functions need to be implement.

$$
\begin{aligned}
\ln\left[D_{\varepsilon^+}^{\mathrm{I}^{\mathrm{en}}}(x_z^{\mathtt{ec}}, x_k^{\mathtt{en}}, \mathbf{y}_z)|_{DEA}\right] = \quad & \max \quad \ln(\theta) \\
& \text{s.t.} \quad \ln(x_{z,i}^{\mathtt{ec}}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x_{u,i}^{\mathtt{ec}}\right), i = 1, ..., n^{\mathtt{ec}} \\
& \qquad \ln(x_{k,i}^{\mathtt{en}}) + \ln(\theta) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x_{u,i}^{\mathtt{en}}\right), i = 1, ..., n^{\mathtt{en}} \\
& \qquad \ln(y_{z,j}^{\mathtt{ec}}) \leq \sum_{u\in \mathrm{U}} \eta_u \ln\left(y_{u,j}^{\mathtt{ec}}\right), j = 1, ..., m^{\mathtt{ec}} \\
& \qquad \ln(x_{k,i}^{\mathtt{en}}) + \ln(\theta) \leq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(x_{u,i}^{\mathtt{en}}\right), i = 1, ..., n^{\mathtt{en}} \\
& \qquad \ln(y_{z,j}^{\mathtt{en}}) \geq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(y_{u,j}^{\mathtt{en}}\right), j = 1, ..., m^{\mathtt{en}} \\
& \qquad \sum_{u\in \mathrm{U}} \lambda_u = 1 \\
& \qquad \sum_{u\in \mathrm{U}} \eta_u = 1 \\
& \qquad \lambda, \eta \geq 0
\end{aligned} \tag{4.7}
$$

and

$$\ln\left[D^{\mathrm{O^{en}}}_{\varepsilon^+}(\mathbf{x}_z, y^{\mathrm{ec}}_z, y^{\mathrm{en}}_k)|_{DEA}\right] = \begin{array}{ll} \max & \ln(\theta) \\ \text{s.t.} & \ln(x^{\mathrm{ec}}_{z,i}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x^{\mathrm{ec}}_{u,i}\right), i = 1, ..., n^{\mathrm{ec}} \\ & \ln(x^{\mathrm{en}}_{z,i}) \geq \sum_{u\in \mathrm{U}} \eta_u \ln\left(x^{\mathrm{en}}_{u,i}\right), i = 1, ..., n^{\mathrm{en}} \\ & \ln(y^{\mathrm{ec}}_{z,j}) \leq \sum_{u\in \mathrm{U}} \eta_u \ln\left(y^{\mathrm{ec}}_{u,j}\right), j = 1, ..., m^{\mathrm{ec}} \\ & \ln(x^{\mathrm{en}}_{z,i}) \leq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(x^{\mathrm{en}}_{u,i}\right), i = 1, ..., n^{\mathrm{en}} \\ & \ln(y^{\mathrm{en}}_{k,j}) + \ln(\theta) \geq \sum_{u\in \mathrm{U}} \lambda_u \ln\left(y^{\mathrm{en}}_{u,j}\right), j = 1, ..., m^{\mathrm{en}} \\ & \sum_{u\in \mathrm{U}} \lambda_u = 1 \\ & \sum_{u\in \mathrm{U}} \eta_u = 1 \\ & \lambda, \eta \geq 0. \end{array} \tag{4.8}$$

Remark that the aforementioned environmental-oriented distance functions (4.7)-(4.8) are defined by assuming that the environmental commodities result into positive externalities. In the case where the environmental commodities result into negative external effects, the parametrisation of the environmental-oriented distance functions needs to be modified as stated in (2.10).

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