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A Luenberger-Hicks-Moorsteen Agricultural Soil Performance Indicator

Abad A., Hien T.

Abstract

This paper introduces an agricultural soil performance measure which inherits the structure of the Luenberger-Hicks-Moorsteen productivity indicator. The proposed agricultural soil performance measure is relevant in the context of quality changes in input and output dimensions. Specifically, the method provided in this paper permits to disaggregate the agricultural performance variation by highlighting the soil-based agricultural productivity component aside with the economic-based one. Moreover, an econometric model is provided illustrating the practicability of the Luenberger-Hicks-Moorsteen agricultural soil performance indicator.

Keywords: Agricultural soil performance, Luenberger-Hicks-Moorsteen indicator, Non Convexity, Total Factor Productivity.

JEL: C43, D21, D24

1 Introduction

Faced with galloping population growth, overconsumption of resources and the resulting level of pollution, several authors have pointed out that the earth's resources would be insufficient to feed this population. One of the first solutions proposed was that of Malthus (1872). This idea was taken up in the first report of the Club of Rome, entitled "The limits to Growth" (Meadows et al., 2013). It consisted in reducing the population. In 1990, the concept of sustainable development appeared, which expressed the need to produce while taking into account the impact on the environment: to satisfy the needs of today's populations without compromising the ability of future generations to satisfy theirs. Making the best use of the earth's resources would allow us to feed humanity and conserve other resources for future generations. Achieving this goal implies improving productivity to replace expansionist agricultural practices: moving towards ecologically sustainable development. To this end, the production frontier analysis allows to estimate the level of effort required to reduce resource wastage (Färe et al., 1994); *i.e.*, to position the production units on the best-practice frontier. This approach can be used to identify the main sources of Total Factor Productivity (TFP) change (Hulten, 2001).

As a major economic measure, TFP productivity change is usually defined as an index number allowing to evaluate the performance of production units (Prasada Rao, 2020). The change in Total Factor Productivity (TFP) has for many years been measured using the traditional Solow residual model (Solow, 1957). This approach attributes productivity growth solely to technical progress, thus ignoring the role of better use of production factors. There is a growing awareness in recent decades that ignoring inefficiency in input use or output production gives a biased measure of productivity growth (Boussemart et al., 2003). Nishimizu and Page (1982) were the first to propose a methodology that decomposes changes in total factor productivity into technological progress and changes in technical efficiency, by considering distance functions as general representation of multi input-output production process. Two approaches allowing to measure TFP change have emerged and become popular: (i) the multiplicative productivity measures which are defined as ratios of multiplicative distance functions (Bjurek, 1996; Färe et al., 1994; Caves et al., 1982); and (ii) the additive productivity measures which are defined as difference-based indicators of directional distance functions (Briec and Kerstens, 2004; Chambers, 2002; Luenberger, 1992)[1].

In contrary of the widely applied Malmquist and Luenberger productivity measures (Boussemart et al., 2003; Färe et al., 1994), the Hicks-Moorsteen and the Luenberger-Hicks-Moorsteen indices inherit multiplicative and additive complete structures, respectively (Briec and Kerstens, 2004;

[1]The main differences between the additive and the multiplicative approaches are presented in Briec and Kerstens (2004), Chambers (1998, 2002) and Diewert (1998), among others.

Bjurek, 1996). As a result, the Hicks-Moorsteen and the Luenberger-Hicks-Moorsteen productivity measures can be defined as TFP indices, permitting TFP comparisons over consecutive periods (O'Donnell, 2012). O'Donnel (2014) extends the classical ratio-based TFP framework by considering fixed base Hicks-Moorsteen index, which allows multi-lateral and multi-temporal TFP comparisons. Based upon the works of Färe and Primont (1995), the fixed base version of the Hicks-Moorsteen index was named the Färe-Primont (FP) TFP measure. Interestingly, the FP index satisfies the classical Hicks-Moorsteen properties and further the transitivity axiom, permitting spatial TFP comparisons.

In this paper, an Agricultural Soil Performance (ASP) indicator is introduced based upon combination of directional distance functions (Briec, 1997; Chambers et al., 1996, 1998; Luenberger, 1992). The ASP indicator inherits the structure of the fixed base Luenberger-Hicks-Moorsteen TFP measure. Specifically, this contribution introduces an additive version of the FP TFP index by integrating the natural capital of soils. To the best of our knowledge, no study ever considers TFP variation in agriculture by taking into account soil characteristics through the LHM methodology. In fact, existing approaches are principally based upon Luenberger and Malmquist productivity measures (Pieralli, 2017; Hailu and Chambers, 2012; Jaenicke and Lengnick, 1999). Besides, the difference-based ASP measure introduced in this paper is defined based upon disaggregation of inputs and outputs through economic- and soil-based components. This difference-based performance indicator is notably relevant when the impact of input and output quality changes on TFP growth needs to be separated. In this line, the additive complete TFP indicator which is proposed in this paper can be disaggregated by displaying the main sources of TFP change (Abad and Ravelojaona, 2022). Specifically, the disaggregation of the ASP measure allows to partition the agricultural performance change based upon quality attributes; *ie.*, soil- and economic-based components.

The production theoretic approach lays out suitable theoretical background to analyse agricultural TFP growth by considering distance function as functional representation of multiple input-output production technology (Chambers and Färe, 2020; Shen et al., 2019). In this paper, the difference-based ASP indicator is defined through the generalised B-disposal scheme (Abad and Briec, 2019)[2]. Specifically, the generalised B-disposal property allows to relax the usual free disposal assumption of the soil-quality inputs and outputs, displaying both complementarity and substitutability between soil- and economic-based components. In such case, the agricultural transformation process is defined as conjunction of soil and economic sub-technologies. Interestingly enough, the classical property of convexity is not required to characterize the agricultural transformation process and therefore, to define

[2]The generalised B-disposal approach is an axiomatic representation of environmental production technology in input and output dimensions.

the ASP indicator, through the generalised B-disposal framework. In the context of agricultural activities, relaxing the convexity property presents some theoretical and empirical advantages (Ruijs et al., 2013). In fact, agricultural transformation processes imply complex and multiple interactions between ecological elements and human activities that may induce non linear relationship between economic and natural commodities such that the production technology is non convex (Abad and Ravelojaona, 2022; Brown et al., 2011; Chavas, 2009).

The remaining of the paper unfolds as follows. Section 2 introduces basic concepts permitting to define and characterise the production process. The methodological framework considered in this paper is laid out in Section 3. Particularly, the agricultural soil performance indicator is defined and further, disaggregation procedure is provided. Section 4 illustrates the practicability of the approach highlighted in this paper by providing an econometric model. Specifically, the agricultural soil performance measure is estimated through a non parametric analytical framework.

2 Preliminaries

This section presents the basic notations considered in throughout the paper. In particular, the definition and axioms of the production process are displayed. Moreover, disaggregated additive distance function is laid out as functional representation of the production technology.

2.1 Production technology: definition and properties

Consider the input vector $\mathbf{x} \in \mathbb{R}^n_+$ allowing to produce the output vector $\mathbf{y} \in \mathbb{R}^q_+$. In addition, assume that the input and output vectors are separated in soil- and economic-based components, such that $\mathbf{x} := (x^s, x^e) \in \mathbb{R}^{n^s+n^e}_+$ and $\mathbf{y} := (y^s, y^e) \in \mathbb{R}^{q^s+q^e}_+$ where $n = n^s + n^e$ and $q = q^s + q^e$.[3]

Let T be the production process that transforms inputs $\mathbf{x} \in \mathbb{R}^n_+$ into outputs $\mathbf{y} \in \mathbb{R}^q_+$. The production set is defined as follows,

$$T := \big\{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{n+q}_+ : \mathbf{x} \text{ can produce } \mathbf{y}\big\}. \tag{2.1}$$

Traditionally, the production process (2.1) is characterized based upon the input set, $L(\cdot) : \mathbb{R}^q_+ \mapsto 2^{\mathbb{R}^n_+}$, and the output set, $P(\cdot) : \mathbb{R}^l_+ \mapsto 2^{\mathbb{R}^q_+}$.

$$P(\mathbf{x}) := \{\mathbf{y} \in \mathbb{R}^q_+ : (\mathbf{x}, \mathbf{y}) \in T\} \tag{2.2}$$

and

[3]The superscripts s and e are considered to display the soil- and economic-based components in the remainder of the paper.

$$L(\mathbf{y}) := \left\{\mathbf{x} \in \mathbb{R}^n_+ : (\mathbf{x}, \mathbf{y}) \in T\right\}. \tag{2.3}$$

Let us assume that the production technology satisfies the following regularity properties (Färe et al., 1985):

T1. - *No free lunch and inaction* - $(0, 0) \in T$, $(0, \mathbf{y}) \in T \Rightarrow \mathbf{y} = 0$;

T2. - *Boundedness* - $T(\mathbf{y}) := \{(\mathbf{u}, \mathbf{v}) \in T : \mathbf{v} \leq \mathbf{y}\}$ is bounded for all $\mathbf{y} \in \mathbb{R}^q_+$;

T3. - *Closedness* - T is closed.

Suppose that $\mathcal{K}$ is a convex cone defined as follows, $\mathcal{K} := \{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{n+q} : x^s \leq 0, x^e \geq 0, y^s \geq 0 \text{ and } y^e \leq 0\}$. Besides the axioms T1-T3, assume that the production technology satisfies the following restricted inputs and outputs disposal assumption (Abad and Briec, 2019):

T4. - *Generalised B-disposability* - $T := \Bigg(\big(T + (\mathbb{R}^n_+ \times -\mathbb{R}^q_+)\big) \cap \big(T + \mathcal{K}\big)\Bigg) \cap \big(\mathbb{R}^n_+ \times \mathbb{R}^q_+\big)$.

The axiomatic framework T1-T4 permits to define the production process as an intersection of sub-technologies (Abad and Briec, 2019; Murty et al., 2012). Specifically, the economic production set satisfies the usual inputs and outputs strong disposability; *ie.*, $\Bigg(T + (\mathbb{R}^n_+ \times -\mathbb{R}^q_+)\Bigg) \cap \big(\mathbb{R}^n_+ \times \mathbb{R}^q_+\big)$. Moreover, the soil-based production activities respect unusual disposal property by restricting inputs rise and outputs decrease for the soil components; *ie.*, $\big(T + \mathcal{K}\big) \cap \big(\mathbb{R}^n_+ \times \mathbb{R}^q_+\big)$. Remarkably, the classical convexity property of the production technology is not imposed through the axiomatic approach T1-T4. In this way, the coupled economic- and soil-based production activities may be analysed based upon a convex-neutral framework. Figures 1 and 2 illustrate the production process by displaying non convex input and output sets, respectively.

Interestingly enough, the production theoretic model T1-T4 allows to highlight both complementarity and substitutability between soil- and economic-based components. Specifically, complementarity happens along the congested frontiers of the the input and output sets; see, the curved line connecting points A and B in the Figures 1 and 2. Regarding the output side, the Figure 2 shows that along the congested frontier the soil-based output can not be discarded freely. As a result, complementarity among economic- and soil-based arises when the soil-based by-products are detrimental for the development of the economic activities. Turning to the input side, the Figure 1 shows that along the congested frontier wasting soil-based input is not permitted. In this case, the maximal level of the natural capital of soil is reached such that decreasing economic inputs can not be compensated by the rise of the natural capital of soil.

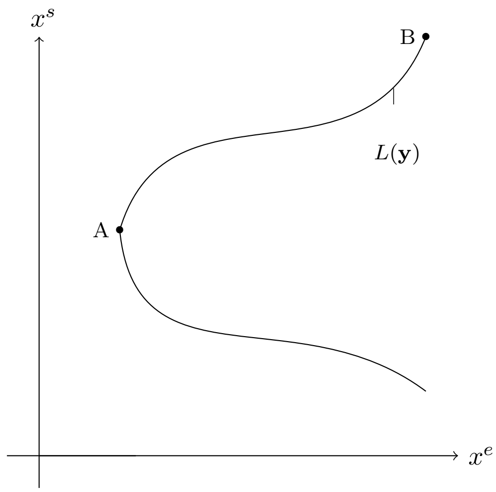

Figure 1: Non convex input set

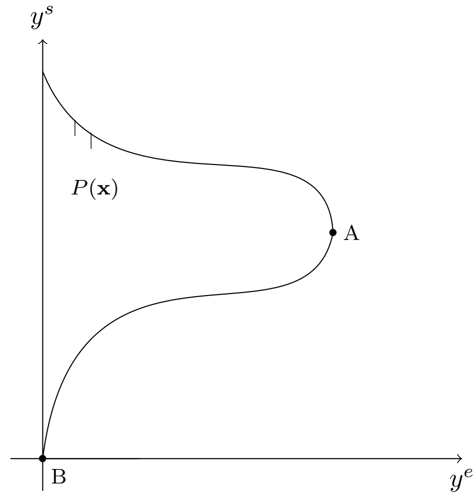

Figure 2: Non convex output set

2.2 Production technology: functional representation

In this section, an additive distance function is introduced. This distance function can be understood as functional representation of the production technology (Chambers and Färe, 2020). Specifically, an additive disaggregated distance function is presented, allowing to consider separately economic and soil-based components (Abad and Ravelojaona, 2022).

The next statement lays out the definition of the additive distance function.

Definition 2.1 *Let T be a production process that satisfies properties T1-T4. For any $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{l+q}$, the additive disaggregated distance function is defined as follows:*

$$\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) := \sup_{\delta} \left\{ \delta \in \mathbb{R} : \Big((1-\delta\gamma^s)x^s, (1-\delta\gamma^e)x^e, (1+\delta\beta^s)y^s, (1+\delta\beta^e)y^e \Big) \in T \right\}, \tag{2.4}$$

such that $\gamma = (\gamma^s, \gamma^e)$ and $\beta = (\beta^s, \beta^e)$ with $\gamma^s = \{0, -1\}$, $\gamma^e = \{0, 1\}$ and $\beta^s = \beta^e = \{0, 1\}$.

The functional form (2.4) fully characterises the production technology, such that:

$$\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) \geq 0 \Leftrightarrow (\mathbf{x}, \mathbf{y}) \in T. \tag{2.5}$$

According to the specification of the parameters γ and β, Economic- and Soil-based sub-vectors Distance Functions (respectively, EDF and SDF) are introduced. The input and output oriented EDF and SDF are laid out in the next statement.

Proposition 2.2 *Let* $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^{n+q}$ *be the input and output vectors, such that* $\mathbf{x} := (x^s, x^e) \in \mathbb{R}_+^{n^s+n^e}$ *and* $\mathbf{y} := (y^s, y^e) \in \mathbb{R}^{q^s+q^e}$. *For any* $\gamma = (\gamma^s, \gamma^e) \in \{0, -1\} \times \{0, 1\}$ *and* $\beta = (\beta^s, \beta^e) \in \{0, 1\} \times \{0, 1\}$*:*

a) $\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) \equiv \overrightarrow{D}^{i^s}(\mathbf{x}, \mathbf{y})$, *if* $\gamma^s = -1$ *and* $\gamma^e = \beta = 0$.

b) $\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) \equiv \overrightarrow{D}^{i^e}(\mathbf{x}, \mathbf{y})$, *if* $\gamma^e = 1$ *and* $\gamma^s = \beta = 0$.

c) $\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) \equiv \overrightarrow{D}^{o^s}(\mathbf{x}, \mathbf{y})$, *if* $\beta^s = 1$ *and* $\beta^e = \gamma = 0$.

d) $\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}, \mathbf{y}) \equiv \overrightarrow{D}^{o^e}(\mathbf{x}, \mathbf{y})$, *if* $\beta^e = 1$ *and* $\beta^s = \gamma = 0$.

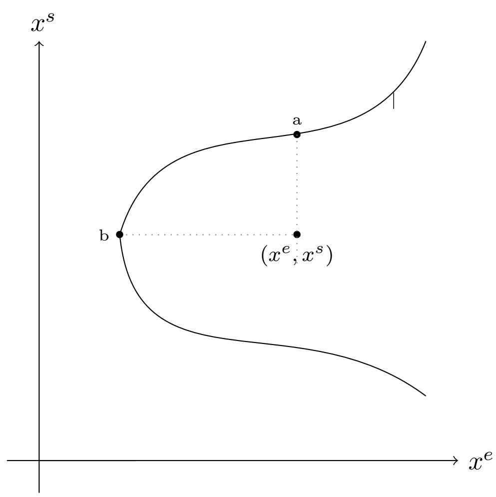

Figure 3: Economic- and soil-based input sub-vector distance functions

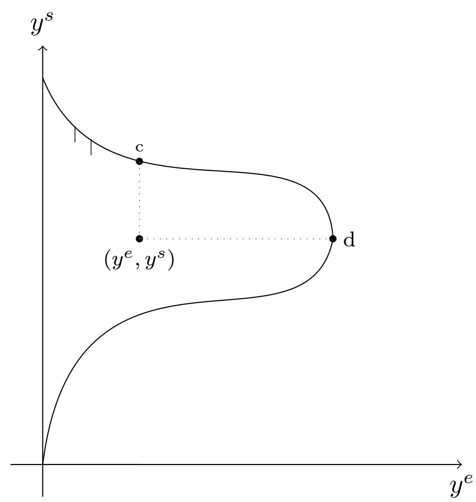

Figure 4: Economic- and soil-based output sub-vector distance functions

The Figure 3 shows that the input oriented EDF scales down the economic inputs towards the production frontier (*ie.*, point b), for a given level of outputs and soil-based inputs. Reversely, the input oriented SDF increases the natural capital of soil in direction of the production boundary (*ie.*, point a), for a given amount of outputs and economic inputs. In such case, the outputs are produced through the maximal natural capital of soil.

Regarding the output side, the Figure 4 shows that the output oriented EDF scales up the economic output towards the production frontier (*ie.*, point d), for a given level of inputs and soil-based outputs. In the same way, the output oriented SDF increases the soil-based output arising from agricultural activities in direction of the production boundary (*ie.*, point c), for a fixed amount of inputs and economic outputs.

Obviously, if the aforementioned sub-vector distance functions are equal to 0 then, the production unit belongs to the boundary of the production set.

3 Methodology

This section lays out the agricultural soil performance indicator. Moreover, multi-lateral and multi-temporal versions of the agricultural soil measure are proposed.

3.1 Agricultural soil performance indicator: definition

The ASP measure inherits the structure of the fixed base Luenberger-Hicks-Moorsteen productivity indicator and therefore, it corresponds to an additive version of the FP TFP measure.

Definition 3.1 *Assume that the production technology T satisfies the assumptions T1-T4. For any $(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \in \mathbb{R}+^{n+p}$, such that $\mathbf{x} := (x^s_{z,k,l}, x^e_{z,k,l}) \in \mathbb{R}_+^{n^s+n^e}$ and $\mathbf{y} := (y^s_{z,k,l}, y^e_{z,k,l}) \in \mathbb{R}^{q^s+q^e}$ where $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+q}$ refers to fixed observation, the agricultural soil performance indicator is defined as follows,*

$$ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) = ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) - ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \quad (3.1)$$

with $ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ and $ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ are the output and input quantity indicators, respectively.

The output and input quantity indicators mentioned in (3.1) are respectively defined as follows,

$$\begin{aligned} ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) &= ASO^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) + ASO^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \\ &= \left(\overrightarrow{D}^{o^e}(\mathbf{x}_z, y^s_z, y^e_k) - \overrightarrow{D}^{o^e}(\mathbf{x}_z, y^s_z, y^e_l)\right) + \\ &\quad \left(\overrightarrow{D}^{o^s}(\mathbf{x}_z, y^e_z, y^s_k) - \overrightarrow{D}^{o^s}(\mathbf{x}_z, y^e_z, y^s_l)\right) \end{aligned} \quad (3.2)$$

and

$$\begin{aligned} ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) &= ASI^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) + ASI^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \\ &= \left(\overrightarrow{D}^{i^e}(x^e_l, x^s_z, \mathbf{y}_z) - \overrightarrow{D}^{i^e}(x^e_k, x^s_z, \mathbf{y}_z)\right) + \\ &\quad \left(\overrightarrow{D}^{i^s}(x^s_l, x^e_z, \mathbf{y}_z) - \overrightarrow{D}^{i^s}(x^s_k, x^e_z, \mathbf{y}_z)\right) \end{aligned} \quad (3.3)$$

The economic output quantity indicator displays the ability of the observation l to operate more efficiently than the production unit k in the economic dimension, for given inputs $\mathbf{x}_z$ and soil-based outputs y^s_z. In such case, $ASO^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ is greater than 0. A similar reasoning applies for the soil output quantity indicator. As a result, if $ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) > 0$

then, the production unit l is more efficient than the observation k in both economic and soil outputs directions.

The input quantity indicator $ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ sets the economic and soil inputs inefficiencies of the observation l against ones of the observation k, for given output $\mathbf{y}_z$. If $ASI^{\{e\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) < 0$ then, the observation l operates more efficiently than the observation k for the economic inputs dimension. A similar outcome occurs in the soil inputs direction when $ASI^{\{s\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) < 0$. Consequently, $ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) < 0$ reveals that the production unit l is more efficient than the observation k in economic and soil inputs directions alike.

The agricultural soil performance measure $ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$, as the difference between output and input quantity indicators, highlights the productivity of the observation l comparatively to the observation k, for given fixed observation $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+p}$. As a result, if $ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ is greater (lesser) than 0 then, productivity gain (loss) arises.

3.2 Agricultural soil performance indicator: disaggregation

Disaggregating the ASP indicator permits to separate the economic- and the soil-based sources of the agricultural productivity variation. The next statement proposes to disaggregate the ASP indicator by displaying the economic- and the soil-based components of the agricultural soil performance measure.

Proposition 3.2 *Let T be a production technology that verifies axioms T1-T4. For any $(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \in \mathbb{R}+^{l+p}$, where $\mathbf{x} := (x^s_{z,k,l}, x^e_{z,k,l}) \in \mathbb{R}_+^{n^s+n^e}$ and $\mathbf{y} := (y^s_{z,k,l}, y^e_{z,k,l}) \in \mathbb{R}^{q^s+q^e}$ with $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+p}$ refers to fixed observation, the agricultural soil performance indicator is disaggregated as follows:*

$$ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) = ASPI^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) + ASPI^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) \quad (3.4)$$

where $ASPI^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ and $ASPI^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ correspond to the economic- and the soil-soil based ASP indicators, respectively.

The preceding economic- and soil-soil based ASP indicators are respectively defined as follows,

$$\begin{aligned} ASPI^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) = & \left(\overrightarrow{D}^{o^e}(\mathbf{x}_z, y^s_z, y^e_k) - \overrightarrow{D}^{o^e}(\mathbf{x}_z, y^s_z, y^e_l)\right) - \\ & \left(\overrightarrow{D}^{i^e}(x^e_l, x^s_z, \mathbf{y}_z) - \overrightarrow{D}^{i^e}(x^e_k, x^s_z, \mathbf{y}_z)\right) \end{aligned} \quad (3.5)$$

and

$$\begin{aligned} ASPI^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) = & \left(\overrightarrow{D}^{o^s}(\mathbf{x}_z, y^e_z, y^s_k) - \overrightarrow{D}^{o^s}(\mathbf{x}_z, y^e_z, y^s_l)\right) - \\ & \left(\overrightarrow{D}^{i^s}(x^s_l, x^e_z, \mathbf{y}_z) - \overrightarrow{D}^{i^s}(x^s_k, x^e_z, \mathbf{y}_z)\right) \end{aligned} \quad (3.6)$$

Economic-based agricultural performance progress happens when $ASPI^e(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l})$ is greater than 0. In such case, the economic performance of the production unit l relatively to that one of the production unit k is better, for given level of input and output $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+p}$. Likewise, if $ASPI^s(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}) > 0$ then, soil-based agricultural performance advance arises.

The Table 1 summarizes the conditions of the agricultural soil performance variation.

	$ASPI^s > 0$	$ASPI^s < 0$
$ASPI^e > 0$	$ASPI^{\{\gamma,\beta\}} > 0$	**i.** $\lvert ASPI^e\rvert > \lvert ASPI^s\rvert$ then $ASPI^{\{\gamma,\beta\}} > 0$, **ii.** $\lvert ASPI^e\rvert < \lvert ASPI^s\rvert$ then $ASPI^{\{\gamma,\beta\}} < 0$,
$ASPI^e < 0$	**i.** $\lvert ASPI^e\rvert < \lvert ASPI^s\rvert$ then $ASPI^{\{\gamma,\beta\}} > 0$, **ii.** $\lvert ASPI^e\rvert > \lvert ASPI^s\rvert$ then $ASPI^{\{\gamma,\beta\}} < 0$,	$ASPI^{\{\gamma,\beta\}} < 0$,

Table 1: Agricultural soil performance characterization

3.3 Agricultural soil performance indicator: multi-lateral and multi-temporal versions

The upcoming statement presents multi-lateral and multi-temporal versions of the agricultural soil performance indicator.

Proposition 3.3 *Let $\mathcal{T}$ and $\mathcal{U}$ be the index sets of the observed periods and production units, respectively. Moreover, assume that $\mathcal{T} = \{0, ..., \tau\}$ and $\mathcal{U} = \{0, ..., \nu\}$, where $\tau, \nu \in \mathbb{N}$. For any fixed observation $(\mathbf{x}_z, \mathbf{y}_z) \in \mathbb{R}_+^{n+q}$, multi-lateral and multi-temporal versions of the agricultural soil performance indicator are defined as follows,*

i. $ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; t) = ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; t) - ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; t)$, *where $t \in \mathcal{T}$, corresponds to the multi-lateral agricultural soil performance indicator.*

ii. $ASPI^{\{\gamma,\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; \nu) = ASO^{\{\beta\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; \nu) - ASI^{\{\gamma\}}(\mathbf{x}_{z,k,l}, \mathbf{y}_{z,k,l}; \nu)$, *where $\nu \in \mathcal{U}$, denotes the multi-temporal agricultural soil performance indicator.*

According to the first statement **i.**, the agricultural soil performance indicator permits to compare two different observations over the same period $t \in \mathcal{T}$. The second specification of the agricultural performance indicator **ii.** considers two different periods for the same production unit $\nu \in \mathcal{U}$. Obviously, the combination of the specifications **i.** and **ii.** allows to compare two different observations at two different periods[4].

[4] Note that the specifications of the ASP performance indicator quoted in the proposition 3.3 remains valid through the disaggregation of the agricultural soil performance measure (3.4).

4 Econometric model

This section illustrates the practicability of the methodology defined in the preceding sections. In particular, non parametric estimation of the agricultural soil performance indicator is laid out.

4.1 Non parametric specification

The non parametric estimation of the ASP measure is based upon the Data Envelopment Analysis (DEA) approach. Specifically, non convex Free Disposal Hull (FDH) production model is considered as reference approximation of the technology.

4.1.1 Production process: non parametric estimation

The FDH production model of Tulkens (1993) is considered to model non convex production set. Precisely, approximation of the coupled economic- and soil-based production technology is defined through the FDH non parametric analytical framework (Abad et al., 2023).

Let $\mathcal{A} := \{(\mathbf{x}_g, \mathbf{y}_g) : g \in \mathcal{G}\}$ be the set of all production units over all periods, where $\mathcal{G}$ is an index of natural number. The next statement lays out non parametric approximation of the FDH agricultural production process.

Definition 4.1 *Assume that T satisfies the properties T1-T4. For any $(\mathbf{x}_g, \mathbf{y}_g) \in \mathcal{A}$, non parametric approximation of the FDH production technology is defined as follows,*

$$T^{\texttt{FDH}} := \Bigg\{ (\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{n+q}_{+} : \mathbf{x}_i \geq \sum_{g \in \mathcal{G}} \theta_g \mathbf{x}_{g,i}, y_j \leq \sum_{g \in \mathcal{G}} \theta_g y_{g,j}, x^s_p \leq \sum_{g \in \mathcal{G}} \lambda_g x^s_{g,p}, x^e_r \geq \sum_{g \in \mathcal{G}} \lambda_g x^e_{g,r}, y^s_w \geq \sum_{g \in \mathcal{G}} \lambda_g y^s_{g,w}, y_m \leq \sum_{g \in \mathcal{G}} \lambda_g y_{g,m}, i \in [n], j \in [q], p \in [n^s], r \in [n^e], w \in [q^s], m \in [q^e], \lambda, \theta \in \{0,1\}, \sum_{j \in \mathcal{J}} \lambda_j = \theta_j = 1 \Bigg\}. \quad (4.1)$$

4.1.2 ASP indicator on non parametric technology

The next result defines additive disaggregated distance function with respect to the FDH production set (4.1).

Definition 4.2 *Let T be a production set satisfying the properties T1-T4. For any $(\mathbf{x}_g, \mathbf{y}_g) \in \mathcal{A}$, $\gamma = (\gamma^s, \gamma^e) \in \{0, -1\} \times \{0, 1\}$ and $\beta = (\beta^s, \beta^e) \in \{0, 1\} \times \{0, 1\}$, the additive disaggregated distance function is defined as fol-*

lows:

$$\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}_1,\mathbf{y}_1) = \begin{array}{ll} \max & \delta \\ s.t. & (1-\delta\gamma^e)x^e_{1,r} \geq \sum\limits_{g\in\mathcal{G}} \theta_g x^e_{g,r}, r\in[n^e] \\ & (1-\delta\gamma^s)x^s_{1,p} \geq \sum\limits_{g\in\mathcal{G}} \theta_g x^s_{g,p}, p\in[n^s] \\ & (1+\delta\beta^e)y^e_{1,m} \leq \sum\limits_{g\in\mathcal{G}} \theta_g y^e_{g,m}, m\in[q^e] \\ & (1+\delta\beta^s)y^s_{1,w} \leq \sum\limits_{g\in\mathcal{G}} \theta_g y^s_{g,w}, w\in[q^s] \\ & (1-\delta\gamma^e)x^e_{1,r} \geq \sum\limits_{g\in\mathcal{G}} \lambda_g x^e_{g,r}, r\in[n^e] \\ & (1-\delta\gamma^s)x^s_{1,p} \leq \sum\limits_{g\in\mathcal{G}} \lambda_g x^s_{g,p}, p\in[n^s] \\ & (1+\delta\beta^e)y^e_{1,m} \leq \sum\limits_{g\in\mathcal{G}} \lambda_g y^e_{g,m}, m\in[q^e] \\ & (1+\delta\beta^s)y^s_{1,w} \geq \sum\limits_{g\in\mathcal{G}} \lambda_g y^s_{g,w}, w\in[q^s] \\ & \sum\limits_{j\in\mathcal{J}} \lambda_j = \sum\limits_{j\in\mathcal{J}} \theta_j = 1 \\ & \lambda,\theta\in\{0,1\}. \end{array} \tag{4.2}$$

Notice that non parametric approximation of the additive disaggregated distance function (4.2) is related to a non linear optimisation problem. Following Abad and Ravelojaona (2022), the next statement provides an enumeration process allowing to estimate the additive disaggregated distance function.

$$\overrightarrow{D}^{\{\gamma,\beta\}}(\mathbf{x}_1,\mathbf{y}_1) \equiv \begin{cases} \overrightarrow{D}^{i^e}(\mathbf{x}_1,\mathbf{y}_1) = \min\limits_{g\in\mathcal{G}}\left(\max\left\{\min\limits_{r\in[n^e]}\left(1-\frac{x_{g,r}}{x_{1,r}}\right)\right\}\right) \\ \overrightarrow{D}^{i^s}(\mathbf{x}_1,\mathbf{y}_1) = \min\limits_{g\in\mathcal{G}}\left(\max\left\{\max\limits_{p\in[n^s]}\left(\frac{x_{g,p}}{x_{1,p}}-1\right)\Big|_{y^s_{1,w}\leq y^s_{g,w}} ; \min\limits_{p\in[n^s]}\left(\frac{x_{g,p}}{x_{1,p}}-1\right)\Big|_{y^s_{1,w}\geq y^s_{g,w}}\right\}\right) \\ \overrightarrow{D}^{o^e}(\mathbf{x}_1,\mathbf{y}_1) = \min\limits_{g\in\mathcal{G}}\left(\max\left\{\min\limits_{m\in[q^e]}\left(\frac{y_{g,m}}{y_{1,m}}-1\right)\right\}\right) \\ \overrightarrow{D}^{o^s}(\mathbf{x}_1,\mathbf{y}_1) = \min\limits_{g\in\mathcal{G}}\left(\max\left\{\min\limits_{w\in[q^s]}\left(\frac{y_{g,w}}{y_{1,w}}-1\right)\Big|_{x^s_{1,p}\geq x^s_{g,p}} ; \max\limits_{w\in[q^s]}\left(\frac{y_{g,w}}{y_{1,w}}-1\right)\Big|_{x^s_{1,p}\leq x^s_{g,p}}\right\}\right) \end{cases} \tag{4.3}$$

References

[1] Abad, A., Briec, W., P. Ravelojaona (2023) forthcoming, Energy-Economic Environmental Productivity Growth Nexus: A Micro-economic Approach,in Gomes M. and Arouri M. (eds) The Handbook of Energy, Sustainability and Economic Growth, Edward Elgar Publishing Ltd.

[2] Abad, A., P. Ravelojoana (2022) A Generalization of Environmental Productivity Analysis, *Journal of Productivity Analysis*, 57, 61-78.

[3] Abad, A., W. Briec (2019) On the Axiomatic of Pollution-generating Technologies: a Non-Parametric Approach, *European Journal of Operational Research*, 277(1), 377-390.

[4] Bjurek, H. (1996) The Malmquist Total Factor Productivity Index, *Scandinavian Journal of Economics*, 98, 303-313.

[5] Boussemart, J.P., W. Briec, K. Kerstens, J.-C. Poutineau (2003) Luenberger and Malmquist Productivity Indices: Theoretical Comparisons and Empirical Illustration, *Bulletin of Economic Research*, 55(4), 391-405.

[6] Brown, G., Patterson, T., N. Cain (2011) The devil in the details: Non-convexities in ecosystem service provision, *Resource and Energy Economics*, 33, 355-365.

[7] Briec, W. (1997) A Graph-Type Extension of Farrell Technical Efficiency Measure, *Journal of Productivity Analysis*, 8, 95-110.

[8] Briec, W., K. Kerstens (2004) A Luenberger-Hicks-Moorsteen Productivity Indicator: Its Relation to the Hicks-Moorsteen Productivity Index and the Luenberger Productivity Indicator, *Economic Theory*, 23(4), 925-939.

[9] Caves, D.W., L.R. Christensen, W.E. Diewert (1982) The Economic Theory of Index Numbers and the Measurement of Inputs, Outputs and Productivity, *Econometrica*, 50, 1393-1414.

[10] Chambers, R.G. (2002) Exact Nonradial Input, Output, and Productivity Measurement, *Economic Theory*, 20, 751-765.

[11] Chambers, R.G. (1998) Input and output indicators, dans : Färe, R., Grosskopf, S., Russell, R. (Eds.) *Index Numbers: Essays in Honour of Sten Malmquist*, Kluwer Academic Publishers.

[12] Chambers, R.G., R. Färe (2020) Distance Functions in Production Economics, in Ray S.C., Chambers R., Kumbhakar S. (eds) *Handbook of Production Economics*, Springer, Singapore.

[13] Chambers, R.G., Chung, Y., R. Färe (1998) Profit, Directional Distance Functions, and Nerlovian Efficiency, *Journal of Optimization Theory and Applications*, 98, 351-364.

[14] Chambers, R.G., Chung, Y., R. Färe (1996) Benefit and Distance Functions, *Journal of Economic Theory*, 70, 407-419.

[15] Chavas, J.P. (2009) On the productive value of biodiversity, *Environmental and Resource Economics*, 42, 109-131.

[16] Diewert, W.E. (1998) *Index Number Theory Using Differences rather than Ratios*, Vancouver, University of British Columbia (Department of Economics: DP 98-10).

[17] Färe, R., Grosskopf, S., Norris, M., Z. Zhang (1994) Productivity growth, technical progress, and efficiency change in industrialized countries, *The American Economic Review*, 84, 66-83.

[18] Färe, R., D. Primont (1995) *Multi Output Production and Duality: Theory and Applications*, Kluwer Academic Publishers, Boston.

[19] Hailu, A., R.G. Chambers (2012) A Luenberger soil-quality indicator, *Journal of Productivity Analysis*, 38, 145-154.

[20] Hulten, C.R. (2001) Total Factor Productivity: A Short Biography, in *New Developments in Productivity Analysis* C.R. Hulten, E.R. Dean and M. Harper (eds.), National Bureau of Economic Research Books, University of Chicago Press, 1-54.

[21] Jaenicke, E.C., L.L. Lengnick (1999) A Soil-Quality Index and its Relationship to Efficiency and Productivity Growth Measures: Two Decompositions, *American Journal of Agricultural Economics*, 81, 881-893.

[22] Luenberger, D.G. (1992) Benefit Function and Duality, *Journal of Mathematical Economics*, 21, 461-481.

[23] Murty, S., R. R. Russell, S. B. Levkoff (2012) On Modeling Pollution-Generating Technologies, Journal of Environmental Economics and Management, 64, 117-135.

[24] Nishimizu, M., J. Page (1982) Total Factor Productivity Growth, Technological Progress and Technical Efficiency Change: Dimensions of Productivity Change in Yugoslavia, 1965-78, *Economic Journal*, 92, 920-936.

[25] O'Donnell, C.J (2012) An Aggregate Quantity-Price Framework for Measuring and Decomposing Productivity and Profitability Change, *Journal of Productivity Analysis*, 38(3), 255-272.

[26] O'Donnell, C.J. (2014) Econometric Estimation of Distance Functions and Associated Measures of Productivity and Efficiency Change, *Journal of Productivity Analysis*, 41(2), 187-200.

[27] Prasada Rao, D.S. (2020) *Index Numbers and Productivity Measurement*, in Ray S.C., Chambers R., Kumbhakar S. (eds) Handbook of Production Economics, Springer, Singapore.

[28] Pierallii, S. (2017) Introducing a new non-monotonic economic measure of soil quality, *Soil & Tillage Research*, 169, 92-98.

[29] Solow, R. (1957) Technical Change and the Aggregate Production Function, *The Review of Economics and Statistics*, 39,312-320.

[30] Shen, Z., Balazentis, T., G. D. Ferrier (2019) Agricultural productivity evolution in China: A generalized decomposition of the Luenberger-Hicks-Moorsteen productivity indicator, *China Economic Review*, 57, 101315.