Game theory. From theory to applications.

Mabel Tidball

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GTM2023

Plan of the talk

- Theory
  - Discretization of Hamiton-Jacobi-Bellman-Isaacs equations.
  - Conjectural equilibrium.
- Theoretical applications
  - Water management problems.
  - Experimental economics.
- Operational applications
  - Non linear pricing system for water management.
  - Reverse auctions to allocate payments for environmental services.

Presentation of myself

- PhD (Mathematics): 1987-1991 Universidad Nacional de Rosario, Argentina. Discovery of zero-sum games and viscosity solutions.

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- Researcher at INRAe (1998) "National Research Institute for Agriculture, Food and the Environment". Montpellier, France.

Progress

1. Theory
2. Theoretical applications
3. Operational applications

Discretization HJB equations

PhD: On the numerical resolution of Hamilton-Jacobi-Bellman and Isaacs equations

$$V(x_0) = \max_{u(.)} \int_0^{\infty} e^{-\rho t} \pi(x,u) dt, \quad \dot{x} = f(x,u), \quad x(0) = x_0$$

$$\max_u (\rho V - \pi - \nabla V * f) = 0$$

Viscosity solution when V is not a regular function.

Discretization of HJB equations

Euler discretization in time (h time discretization)

$$\max_{u^h}(V^h(x) - (1-\rho h)V^h(x + hf(x,u)) - h\pi(x,u) = 0$$

Discretization in space (using finite elements): replace $x + hf(x,u))$ by a convex combination of the vertices of the simplex.

Discretization in time and space transforms a continuous-time deterministic control (game) problem in a stochastic Markov chain (game)

HJB equations: convergence

When discretizing in time (h) and space (k) the order of convergence is

$$k + \sqrt{h}.$$

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Fast solution of general non-linear fixed point problems.
Algorithms mixing value iteration and policy iteration with an example of zero-sum game where the convergence is as bad as using value iteration.

Approximating games and convergence

Unifying approach for approximating a zero-sum game by a sequence of approximating games. Discounted payoff and average payoff

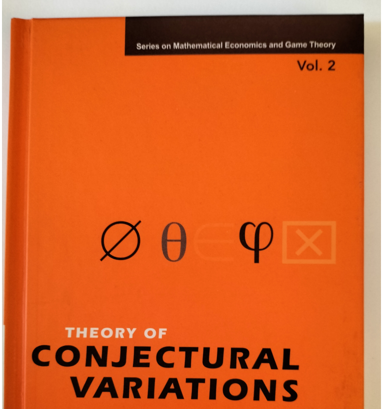
- Convergence of the values and of optimal strategies of the approximating games to the "limit" game.
- Conversely, based on optimal policies for the "limit" game, we construct policies which are almost optimal for the approximating games.
- Applications to state approximation of stochastic games, convergence of finite horizon problems to infinite horizon problems, convergence in the discount factor ....

Some papers

M. Tidball, (1991) « Numerical Approach to the Infinite Horizon Problem of Deterministic Control Theory ». *Appl. Math. Optim.*.
M. Tidball, R. González, (1993) « Zero Sum Differential games with Stopping Time. Some Results about their Numerical Resolution ». *Annals of Dynamics Games*.
M. Tidball, (1995) « Undiscounted Zero Sum Differential Games with Stopping Times ». *New Trends in Dynamic Games and Applications*.
M. Tidball, E. Altman, (1996)« Approximations in Dynamic Zero-Sum Games, I ». *SIAM J. Control and Optimization*.
M. Tidball, O. Pourtallier, E. Altman, (1997)« Approximations in Dynamic Zero-Sum Games, II ». *SIAM J. Control and Optimization*.

The conjectural equilibrium

Ch. Figuières, A. Jean-Marie, N. Quérou, M. Tidball, (2004) « Theory of Conjectural Variations ». In Monograph series in Mathematical Economics and Game Theory, World Scientific Publishing.



Conjectural equilibrium

What are Conjectures ?

A game-theoretical concept in which players have a conjecture about the behaviour of their opponents: they think the others will play in function of their own decision.

Conjectural equilibrium

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A game-theoretical concept in which players have a conjecture about the behaviour of their opponents: they think the others will play in function of their own decision.

What is it useful for?

- A shorthand for dynamic interactions
- As a possible alternative to Nash Equilibria... specially when information is incomplete
- Or to explain implicit cooperation when agents behave non cooperatively

Conjectures in Dynamic Games

- n players, time horizon T
- $x(t) = (x_1(t), ...x_m(t)) \in R^m$ state variable
- $e_i(t)$ control variable of i in $[t, t+1]$, $e(t)$

Dynamics

$$x(t+1) = f(x(t), e(t)), \quad x(0) = x_0$$

Payoff

$$\sum_{t=0}^{T} \rho^t \pi^i(x(t), e(t))$$

Conjectures

Conjecture of i

$$e_j^c(t) = \phi_t^{ij}(x(t)) \quad \rightarrow \quad x(t+1) = \tilde{f}_i(x(t), e_i(t)) \ .$$

optimal control problem

Conjectures replace the necessity of knowing the profit function of the others players
$\rightarrow$ optimal policy $e_i^{i*}(t)$ that we suppose unique.
Player i can compute his estimation of the state $x^{i*}(t+1)$ and $e_j^{i*}(t)$
via ϕ_t^{ij}.

A Dynamic Game with consistent conjectures

- $\phi_t^1, ...\phi_t^n$ is a (weak) control-consistent conjectural equilibrium $\iff$

$$e_j^{i*}(t) = e^{j*}(t), \quad \forall i \neq j,\ t,\ x(0) = x_0 \ \ (\textit{with } x(0) \textit{ given})$$

- Optimization problem: $\rightarrow e_i^{i*}(t) = \psi_t^i(x(t))$
  $\phi_t^1, ...\phi_t^n$ is a feedback-consistent conjectural equilibrium $\iff$

$$\psi_t^i = \phi_t^{ji}, \quad \forall i \neq j,\ t,\ x(0) = x_0$$

Consistency in dynamic games

Fershtman and Kamien (1985) in differential games,
Jean-Marie and Tidball (2005) in discrete-time games, prove:

Open-loop Nash equilibria coincides with weak control-consistent conjectural equilibria

Feedback Nash equilibria coincides with feedback-consistent conjectural equilibria

Consistency in dynamic games

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Feedback Nash equilibria coincides with feedback-consistent conjectural equilibria

Fershtman and Kamien (1985) propose to consider other kinds of conjectures (state and strategy based conjectures) of the form

$$e_j^c = \phi^{ij}(x(t), e_i(t)).$$

What about state and strategy based conjectures. Quérou and Tidball (2014)

x is a non renewable resource and c_i is consumption of player i. The problem of player i is

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}.$$

Player i conjectures that player j's consumption decision at period t is given by:

$$c_{j,t}^c = a_i x_t + b_i c_{i,t-1},$$

where a_i and b_i model the player's beliefs. In other words, agent i assumes that the consumption policy of agent j at period t is a function of the state of the resource at period t and his own consumption strategy at period $t-1$.

What about state and strategy based conjectures.

Consistent Conjectural problem

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \; x_{t+1} = x_t - (a_i x_t + b_i y_{i,t}) - c_{i,t}, \; y_{i,t+1} = c_{i,t}, c_{i,0}, x_0, \text{given}$$

What about state and strategy based conjectures.

Consistent Conjectural problem

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \; x_{t+1} = x_t - (a_i x_t + b_i y_{i,t}) - c_{i,t}, \; y_{i,t+1} = c_{i,t}, c_{i,0}, x_0, \text{given}$$

Benchmarks

- Non cooperative problem (Nash feedback)

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}, \; x_0, \; \text{given}.$$

- Cooperative solution (Pareto solution)

$$\max_{\{c_t^i, c_t^j\}} \sum_{t=0}^{\infty} \rho^t \sum_{i=1}^{2} log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}, \; x_0, \; \text{given}.$$

What about state and strategy based conjectures. Results

- In the conjectured model the optimal consumption policies depend on the initial consumption level. This implies that one might influence the consumption path that will be chosen by focusing on the initial level of consumption.

What about state and strategy based conjectures. Results

- In the conjectured model the optimal consumption policies depend on the initial consumption level. This implies that one might influence the consumption path that will be chosen by focusing on the initial level of consumption.
- The feedback consistent conjectural equilibrium coincides with the cooperative solution under complete information provided that the initial level of consumption is cooperate in both cases. If agents cooperate initially then asking for consistency ensures that cooperation will be sustained in the long run.

What about state and strategy based conjectures. Results

- If the initial consumption is too high, then the conjectural procedure leads to a more aggressive pattern than even in the non-cooperative case under full information. The effects of strategic behaviours are reinforced by incomplete information.

What about state and strategy based conjectures. Results

- If the initial consumption is too high, then the conjectural procedure leads to a more aggressive pattern than even in the non-cooperative case under full information. The effects of strategic behaviours are reinforced by incomplete information.
- When initial consumption is sufficiently low, we obtain that the procedure leads to an under-exploitation of the resource compared to the full information cooperative benchmark.

The learning model

Each player (two players) maximises at each period t (myopic behaviour)

$$\max_{e_i^t} \pi_i(e_i^t, e_j^t, x^t), \quad x^{t+1} = f(e_i^t, e_j^t, x^t).$$

Each player at each period makes a conjecture about the behaviour of the other player

$$e_j^{t,c} = \phi_i(e_i^t, x^t).$$

At each period player i solves the following optimisation problem

$$\max_{e_i^t} \pi_i(e_i^t, \phi_i(e_i^t, x^t), x^t).$$

The general learning model

Call the optimal solution for player i ($i = 1, 2$), e_i^* and the corresponding conjectured solution of the other player $e_j^{c*} = \phi_i(e_i^*, x)$.

This conjecture is in general different of e_j^* (at time t). Then player i updates his conjecture

Adapting conjectures

This update process can take one of the general forms:

$$\phi_i^{t+1} = \mathcal{U}_i(\phi_i^t, e_i^{t*}, e_j^{t*}, x^t) \tag{1}$$

$$\phi_i^{t+1} = \mathcal{S}_i(e_j^{0*}, x^0, \dots, e_j^{t*}, x^t)\ . \tag{2}$$

In the first form (1), the "functions" are updated (hence "$\mathcal{U}$") based on the most recent observation of the opponent. In the second form (2), some "statistic" (hence "$\mathcal{S}$") is performed on the whole history of observations.

Note that ϕ_i^t is a sequence of functions. In practical situations we are going to consider particular functional forms for the conjecture. This functional form will depend on a certain numbers of parameters. These parameters are going to be learned in the learning procedure.

A groundwater exploitation problem

- Water extraction is the only input in the production process of the farmers and gives a profit $P_i(e_i^t)$,
- Unitary cost increases when the level of the water table is low,
- Players can also take into account the state of the resource and have an extra (subjective) profit of maintaining the resource, in this case ρ is the discount factor and γ_i is his resource preference,
- The dynamics is given by the evolution of the level of the water table.

$$\pi_i(e_i^t, e_j^t, x^t, x^{t+1}) = P_i(e_i^t) - c(x^t)e_i^t + \rho\gamma_i x^{t+1},$$

such that

$$x^{t+1} = x^t + R - e_i^t - e_j^t.$$

Near-sighted or short-sighted procedures.

Three kinds of conjectures

- Linear conjecture in e : $e_j^c = \beta_i e_i, \quad i \neq j.$
- Copy e (imitation): $e_j^c = \beta_i + e_i, \quad i \neq j.$
- Affine conjecture in x: $e_j^c = \beta_i (x_t + R), \quad i \neq j.$

$$\beta \quad \text{such that} \quad \phi_i(\beta, e_i) = e_j^* \quad (or \ \ \phi_i(\beta, x) = e_j^*) \quad \to \tilde{\beta},$$

$$\beta_i^{t+1} = \mu \beta_i^t + (1 - \mu) \tilde{\beta}^t.$$

μ speed of adjustment.

Simulations

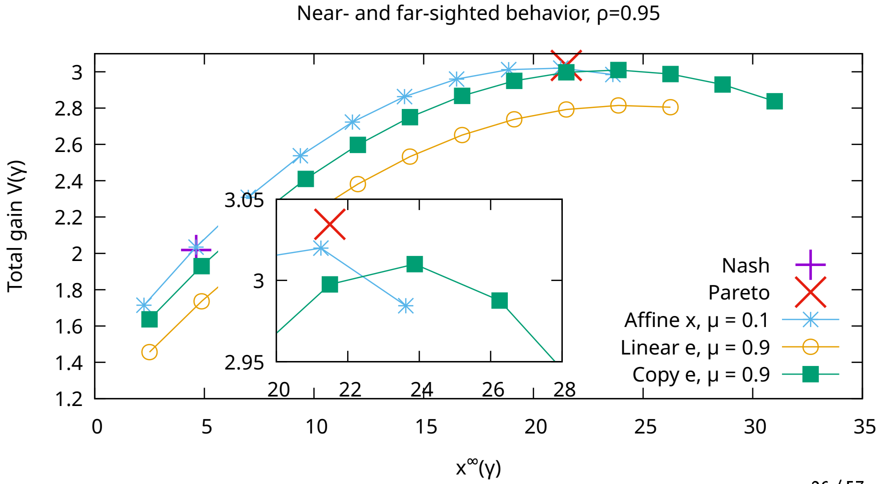
The "performance" of a learning scheme can be evaluated using many criteria. Taking into account environmental and economic concerns, we select for our comparisons:

a) the limiting/steady state stock of water x^∞, representing the environment, and
b) the total discounted sum of profits for both players, representing the welfare of the society, V.

Experiments:

- The graphs represent the result of simulations in the plane (x^∞, V).
- Near-sighted solution (x^∞, V) are functions of γ.

Simulations



Conjectures: some papers

A. Jean-Marie, M. Tidball, (2005) « Consistent conjectures, equilibria and dynamic games ». *Chapter of book: Dynamic games: Theory and applications*.

A. Jean-Marie, M. Tidball, (2006) « Adapting behaviors through a learning process ». *Journal of Economic Behavior and Organization*.

N. Quérou, M. Tidball, (2010) « Incomplete information, learning, and natural resource management ». *European Journal of Operational Research*.

N. Quérou, M. Tidball, (2014) « Consistent conjectures in a dynamic model of non-renewable resource management ». *Annals of Operations Research*.

A. Jean-Marie, T. Jimenez, M. Tidball, (2021) « Nearsighted, farsighted behaviors and learning. Application to a water management problem ». [Research Report] RR-9406, Inria.

Progress

1. Theory
2. Theoretical applications
3. Operational applications

Water management. A simple model

$$\pi_i(e_i, e_j, x) = P_i(e_i) - c(x)e_i = p_i(ae_i - be_i^2 + f) - (c - zx)e_i,$$

- e_i water extraction
- p_i price
- $ae_i - be_i^2 + f$ production function
- $c - zx$ unitary cost of pumping

such that

$$x^{t+1} = x^t + R - \sum_i e_i^t, \quad \dot{x} = R - \sum_i e_i.$$

Water management: shocks on the resource

Thesis Julia de Frutos Cachorro (2014). Continuous time infinite horizon model with several players. L-Q game problem.

- How can drought risk be incorporated into a model of groundwater management for irrigation and what is the impact of drought (exogenous shock on the resource) on the behaviour of farmers

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J. de Frutos, G. Martín Herrán and M. Tidball. Stackelberg competition in groundwater resources with multiple uses. Endogenous shock caused by the other user.

Water management. Taking into account cultivated areas

Isabelle Alvarez, Katrin Erdlenburch, Sophie Martin (work in progress): Water and surface choice in a dynamic setting.
The problem is no longer a linear- quadratic model. We use discretization tools.

- Water management problem where strategies are the water extraction and the cultivated area.
- Dynamic game and Viability approach.
- Application to the Beauce aquifer.

Water management: inducing cooperation through the pumping cost

- Dynamic model, discrete time, infinite horizon.
- A water agency can act on the marginal extraction cost of water users (Leader). The water users (farmers) are selfish and myopic (Nash).
- The goal of the agency is to give incentives to improve total welfare. She has two control instruments: one for tuning the degree of strategic interaction between agents, and the other that discourages or encourages extraction.

$$(c - zx_t) \quad \longrightarrow \quad c - z(x_t + mR - n\sum_i e_i), \quad m, n \in [0, 1].$$

Water management: inducing cooperation through the pumping cost. Results

- It is always optimal to encourage water consumption ($m = 1$).
- When the water agency is patient (large discount factor), it is optimal to maximize strategic interactions between agents ($n = 1$). For small discounting ($n = 0$).
- When m and n can be functions of t, we obtain conditions on the parameters where the optimal solutions are constant over time.
- When these conditions are not verified we exhibit through simulations, that non-constant, e.g. periodic or threshold-based policies are improving on the constant ones, and could be optimal. The optimal control problem for the water agency is no longer a L-Q model.

Water management: some papers

J. De Frutos, K. Erdlenbruch, M. Tidball, (2014) « Optimal adaptation strategies to face shocks on groundwater resources ». *Journal of Economic Dynamics and Control*.

J. De Frutos, K. Erdlenbruch, M. Tidball, (2017) « A dynamic model of irrigation and land-use choice: an application to the Beauce aquifer in France ». *European review of agricultural economics*.

J. De Frutos, K. Erdlenbruch, M. Tidball, (2017) « A dynamic model of irrigation and land-use choice: an application to the Beauce aquifer in France ». *European review of agricultural economics*.

A. Jean-Marie, M. Tidball, V. Bucarey, (2022) « The Stackelberg games of water extraction with myopic agents ». *IGTR Special Issue on Game Theory and Applications*.

Experimental economics

Experimental economics is a branch of economics that studies human behaviour in a controlled laboratory setting or out in the field, rather than just as mathematical models.

It is used

- to test economic theories,
- to test what choices people make in specific circumstances,
- to give some elements of behaviour when theory is missing
- to study alternative market or non market mechanisms
- ....

Experimental economics

Thesis Anmina Dulcie Murielle Djiguemde (2021). Dynamic Games and Renewable Common Pool Resources: Modelling and Experiments

- Lab protocols of continuous/discrete time infinite horizon control and game water management problem.
  - How does the nature of time affect the nature of strategic interactions?
  - Can continuous time foster cooperation?
- How some kind of information can help cooperation.

Experimental economics: continuous time versus discrete time. One player

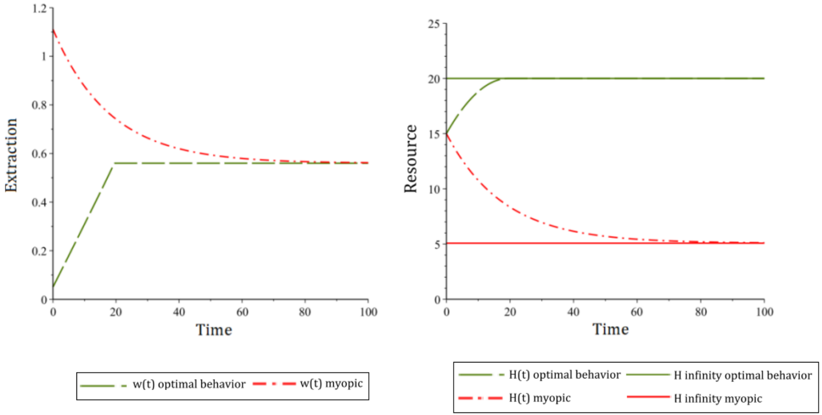

Figure: ***Extraction behaviors and resource levels in sole-agent continuous time***

Experimental economics: continuous time versus discrete time. Two player

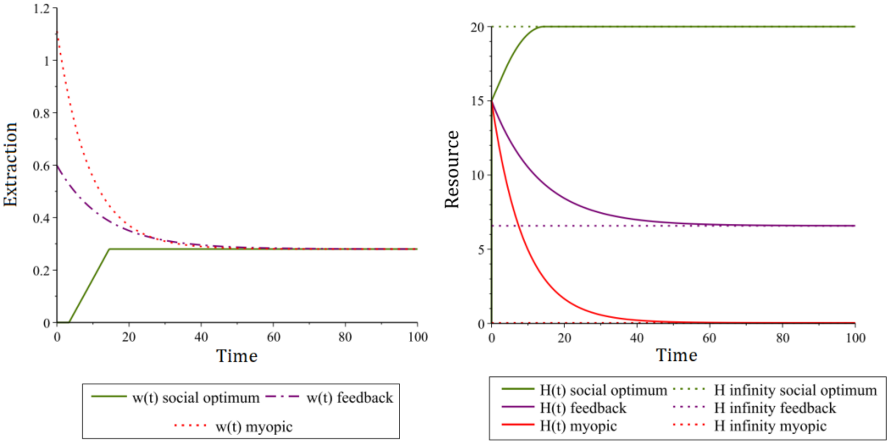

Figure: ***Extraction behaviors and resource levels in multiple-agent continuous time***

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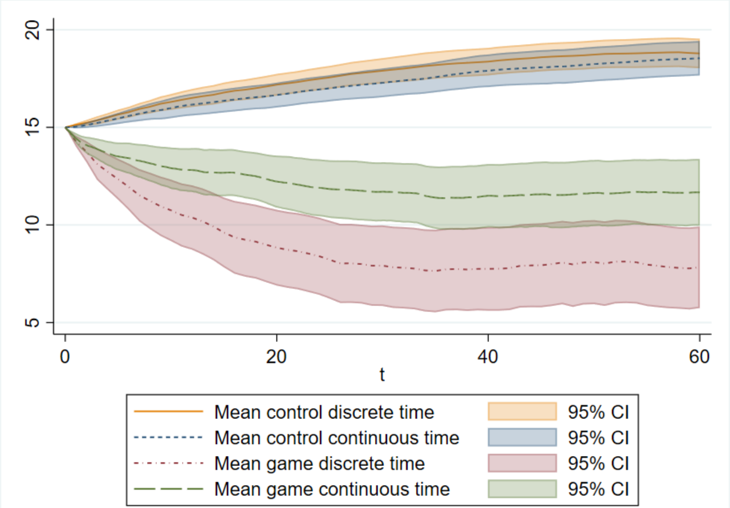


Experimental economics: continuous time versus discrete time.Results

Continuous time offers more opportunities to change one's extraction level.

This possibility can be used to test the reaction of the other players and to try to induce a change in their behaviour. For example, one player can temporarily lower his extraction level to see if the other player will do the same. This type of test is less expensive in continuous time than in discrete time.

Can nudges help cooperation? Injunctive norm

Injunctive Social Norms are rules or behaviour that individuals feel they are supposed to follow in a given context.

The theoretical paths serve as a normative guide or standard that indicates the "ideal" or "approved" behaviour in the given context. By showing the subjects what the optimal extraction path looks like, we are essentially communicating a normative message about what they "should" do to achieve the best possible outcome.

Theoretical results: Two player

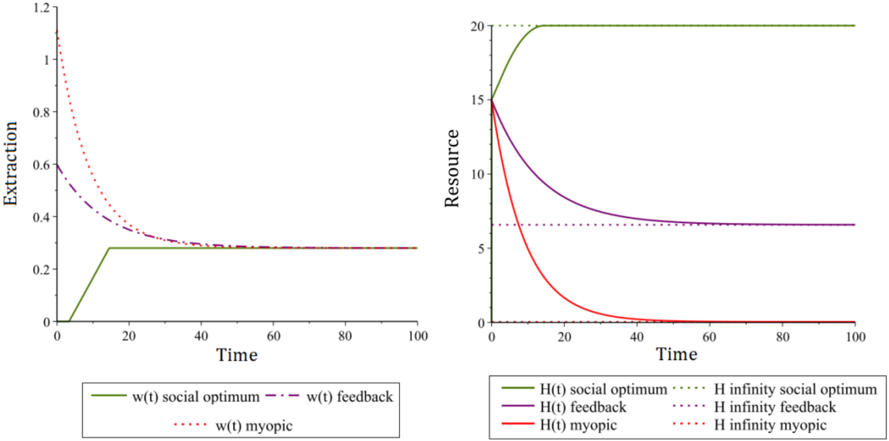


Figure: ***Extraction behaviours and resource levels in multiple-agent continuous time***

Can nudges help cooperation? Injunctive norm

In the theoretical analysis of this game without communication, three typical behaviours were identified. The resource evolution curves for these behaviours are shown in the figure. For each curve, you also have information on the individual payoff.

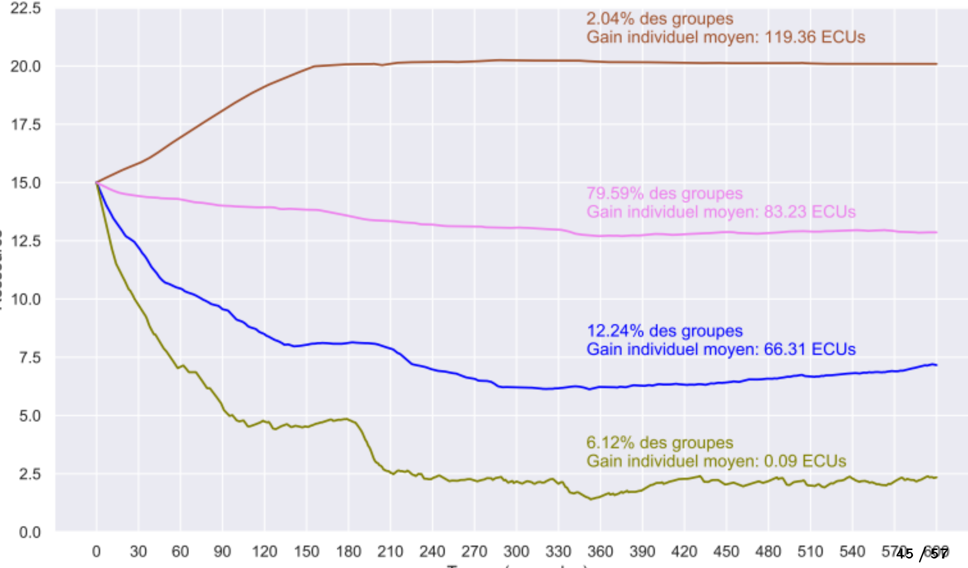
- The green curve results from the extraction choices of two perfectly symmetrical players who jointly maximize the group's payoff over the long term
- The blue curve results from the extraction choices of two perfectly symmetrical players who maximize their individual payoff over the long term
- The red curve results from the extraction choices of two perfectly symmetrical players who maximize their individual payoff over the short term

Can nudges help cooperation? Descriptive norm

Descriptive norms refer to beliefs about what others do.

Descriptive norms will drive a behaviour or practice when a person engages in a particular behavior because they think that others in their community and social circle do the same.

Can nudges help cooperation? Descriptive norm



Can nudges help cooperation? Descriptive norm

In previous sessions four typical behaviours were identified. The average resource evolution curves for these behaviours, observed frequency and payoff are shown in the figure.

- The brown curve results from the extraction choices of two players who, according to the interpretation suggested by the theory, corresponds to a joint maximization of the group's payoff over the long term
- The blue curve results from the extraction choices of two players who, ... corresponds to a maximization of their individual payoff over the long term
- The green curve results ..., corresponds to a maximization of their individual payoff over the short term
- The purple curve results from the extraction choices of two players with atypical behaviours whose interpretation escapes the theory

Experimental economics: some papers

A. Djiguemde, D. Dubois, A. Sauquet, M. Tidball, (2022) « Continuous versus Discrete Time in Dynamic Common Pool Resource Game Experiments ». *Environmental and Resource Economics*.

A. Djiguemde, D. Dubois, A. Sauquet, M. Tidball, (2022) « Individual and strategic behaviors in a dynamic extraction problem: results from a within-subject experiment in continuous time ». *Applied Economics incorporating Applied Finantial Economics*.

A. Djiguemde, D. Dubois, A. Sauquet, M. Tidball, (2023) « Nudging Behaviors in a Dynamic Common Pool Renewable Resource Experiment ». Working in progress.

Progress

1. Theory
2. Theoretical applications
3. Operational applications

Operational applications. Water bill

Project Noviwam (2010-2015): Novel Integrated Water Management Systems for Southern European Regions. Economic performance implications of a linear water pricing system and a non-linear pricing system. The non-linear pricing was really applied in the Deux-Sèvres region. The linear pricing was in vigour in Marseilles.

$$F(C,S) = \lambda B \left(aS + (1-a)\frac{\max(C,bS)C}{S} \right)$$

S: amount of water subscribed by the farmer (reserved water).
C: amount of water actually used during the farming season (actual consumption).
B: manager's budget
a b and λ are adjustment parameters.

Operational applications. Water Bill

$$F(C,S) = \lambda B \left(aS + (1-a)\frac{\max(C,bS)C}{S} \right)$$

- this pricing system is linear in C if consumption is below a certain proportion of the quantity reserved (bS),
- But, if consumption exceeds this fraction of the reservation, the bill is quadratic in C which has the effect of discouraging excessive consumption.

Operational applications. Water bill

$$F(C,S) = p_1 S + \max(0, p_2(C - S))$$

- p_1 is the price of the quantity of water reserved.
- p_2 is the price of the consumption beyond the volume reserved.

Operational applications. Water Bill. Results

The linear and non-linear pricing systems are compared according to various economic criteria including the profit of farmers, the income of the manager, and the total value of agricultural production.

An empirical application is made with a crop growth model and data collected from the Midi-Pyrenees region.

Non-linear pricing systems allows irrigation water managers to reduce the impact of drought on production.

It depends on the characteristics and capabilities of the water company.

It can be used to anticipate agricultural water demand in order to avoid imbalance with water availability.

Operational applications: Reverse auctions

Project FAST, (2021–2027):Facilitate public Action to exit from peSTicides.
Design of innovative agro-environmental policies based on reverse auctions

- Lack of theoretical results for reverse auctions when the budget is announced.
- Use experimental economics to compare the performance of target-constrained and budget-constrained reverse auctions.
- Choice experiments/survey. To analyse the acceptability of farmers to this kind of mechanism as a tool to allocate agri-environmental payments.

Operational applications: some papers

Y. Sidibé, J.P. Terreaux, M. Tidball, A. Reynaud, (2012) « Coping with drought with innovative pricing systems : The case of two irrigation water management companies in France ». *Agricultural Economics*.

J.P. Terreaux, Y. Sidibe, M. Tidball, (2012) « Préservation des étiages par des méthodes originales de tarification de l'eau d'irrigation ». *La Houille Blanche*.

Y. Sidibé, J.P. Terreaux, M. Tidball, J.M. Berland, N. Jacquin, (2016) « Le projet euroéen Noviwam et la gestion intégrée de l'eau ». *Revue des Sciences de l'Eau*.

Adrien Coiffard, Raphaele Préget, Mabel Tidball, (2023). « An Experimental Comparison of Target and Budget Constraints in Conservation Auctions ». Working paper.

Adrien Coiffard, Raphaele Préget, Mabel Tidball, (2023). « Conservation auctions: an online double constraint reverse auction experiment ». Working paper.

Challenges

For me

- Considering constraints in dynamic games
- Impulsional games
- Reverse auctions, and in particular discrete reverse auctions

Challenges

For me

- Considering constraints in dynamic games
- Impulsional games
- Reverse auctions, and in particular discrete reverse auctions

For the game community

- Cooperative games (time consistency)
- Stochastic games
- Mean field games
- Evolutionary games
- Learning... numerical procedures...
- How to chose the “correct” model in applications
- ...

Thanks to my coauthors

Thanks you