ISDG and Mabel: Importance of being strategic and dynamic. Lucky to be lucky

Mabel Tidball

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Importance of being strategic and dynamic
Lucky to be lucky

Mabel Tidball
INRAe, CEEM, Montpellier, France
Argentinian ... French   Mathematics ... Economy ... OR

ISDG 2022. July 2022

Plan of the talk

- Some important dates for ISDG and me.
- The evolution of being dynamic and strategic: from my PhD in Mathematics to working in an applied institute. From proofs of existence and uniqueness of equilibria, discretization and convergence to the construction of bio-hydro-economics models and decision-making tools.
- Lucky to be lucky: my coauthors and my students!

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- Rosario sometime between 1967 and 1970. Elementary school, the meaning of infinity and my aunt Irma.

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- First ISDG: 1992. Switzerland. Tutorial of Martino Bardi about viscosity solutions.
- ISDG Adelaide, South Australia, 2000. I meet Guiomar Martín Herrán and Francisco Cabo.

ISDG Segovia 2005

Some important dates

- INRA Montpellier, France, 1998. Application of dynamic games to environmental management
- Habilitation à Diriger des Recherches, University of Montpellier 1, 2005. Environmental economics and strategic behavior

Fist steps in dynamic games: PhD

PhD: On the numerical resolution of Hamilton-Jacobi-Bellman and Isaacs equations

Discretization in time and space transforms a continuous time deterministic control (game) problem in a stochastic Markov chain (game)

Fist steps in dynamic games: PhD

- Zero-sum differential games with stopping time. « Some results about their numerical resolution ».
  Annals of Dynamics Games, 1993.

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- Fast solution of general nonlinear fixed point problems. Algorithms mixing value iteration and policy iteration with and an example of zero-sum game where the convergence is as bad as using value iteration.

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When discretizing in time (h) and space (k) the order of convergence is

$$k + \sqrt{h}.$$

First steps in France: INRIA Sophia Antipolis

M. Tidball, E. Altman, « Approximations in Dynamic Zero-Sum Games, I ». *SIAM J. Control and Optimization*, 1996.

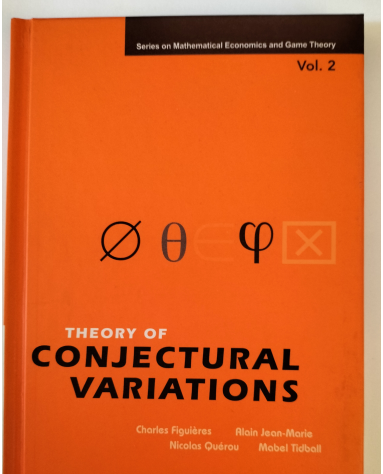
M. Tidball, O. Pourtallier, E. Altman, « Approximations in Dynamic Zero-Sum Games, II ». *SIAM J. Control and Optimization*, 1997.

First steps in France: INRIA Sophia Antipolis

Unifying approach for approximating a zero-sum game by a sequence of approximating games. Discounted payoff and average payoff

- Convergence of the values and of optimal (or almost optimal) strategies of the approximating games to the "limit" game.
- Conversely, based on optimal policies for the "limit" game, we construct policies which are almost optimal for the approximating games.
- Applications to state approximation of stochastic games, convergence of finite horizon problems to infinite horizon problems, convergence in the discount factor ....

Ch. Figuières, A. Jean-Marie, N. Quérou, M. Tidball, (2004)
« Theory of Conjectural Variations ».
In Monograph series in Mathematical Economics and Game Theory, World Scientific Publishing.



Conjectural equilibrium

What are Conjectures ?

A game-theoretical concept in which players have a conjecture about the behaviour of their opponents: they think the others will play in function of their own decision.

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What is it useful for?

- A shorthand for dynamic interactions
- As a possible alternative to Nash Equilibria... specially when information is incomplete
- Or to explain implicit cooperation when agents behave non cooperatively

Conjectures in Dynamic Games

- n players, time horizon T
- $x(t) = (x_1(t), ...x_m(t)) \in R^m$ state variable
- $e_i(t)$ control variable of i in $[t, t+1]$, $e(t)$

Dynamics

$$x(t+1) = f(x(t), e(t)), \quad x(0) = x_0$$

Payoff

$$\sum_{t=0}^{T} \rho^t \pi^i(x(t), e(t))$$

Conjectures

Conjecture of i

$$e_j^c(t) = \phi_t^{ij}(x(t)) \quad \rightarrow \quad x(t+1) = \tilde{f}_i(x(t), e_i(t)) \ .$$

optimal control problem

$\rightarrow$ optimal policy $e_i^{i*}(t)$ that we suppose unique.
Player i can compute $e_j^{i*}(t)$ and his estimation of the state $x^{i*}(t+1)$ via ϕ_t^{ij}.

A Dynamic Game with consistent conjectures

- $\phi_t^1, ...\phi_t^n$ is a (weak) control-consistent conjectural equilibrium $\iff$

$$e_j^{i*}(t) = e^{j*}(t), \quad \forall i \neq j,\ t,\ x(0) = x_0 \ \ \textit{(with } x(0) \textit{ given)}$$

- Optimization problem: $\to e_i^{i*}(t) = \psi_t^i(x(t))$
  $\phi_t^1, ...\phi_t^n$ is a feedback-consistent conjectural equilibrium $\iff$

$$\psi_t^i = \phi_t^{ji}, \quad \forall i \neq j,\ t,\ x(0) = x_0$$

Consistency in dynamic games

Fershtman and Kamien (1985) in differential games,
Jean-Marie and Tidball (2005) in discrete-time games, prove:

- Open-loop Nash equilibria coincides with weak control-consistent conjectural equilibria
- Feedback Nash equilibria coincides with feedback-consistent conjectural equilibria

Fershtman and Kamien (1985) propose to consider other kinds of conjectures (state and strategy based conjectures) of the form

$$e_j^c = \phi^{ij}(x(t), e_i(t)).$$

What about state and strategy based conjectures. Quérou and Tidball (2014)

x is a non renewable resource and c_i is consumption of player i. The problem of player i is

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}.$$

Player i conjectures that player j's consumption decision at period t is given by:

$$c_{j,t}^c = a_i x_t + b_i c_{i,t-1},$$

where a_i and b_i model the player's beliefs. In other words, agent i assumes that the consumption policy of agent j at period t is a function of the state of the resource at period t and his own consumption strategy at period $t-1$.

What about state and strategy based conjectures.

Consistent Conjectural problem

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \quad x_{t+1} = x_t - (a_i x_t + b_i y_t) - c_{i,t}, \; y_{t+1} = c_{i,t}, \; c_0, x_0, \text{ given}$$

Benchmarks

- Non cooperative problem (Nash feedback)

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \rho^t log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}, \; x_0, \text{ given.}$$

- Cooperative solution (Pareto solution)

$$\max_{\{c_t^i, c_t^j\}} \sum_{t=0}^{\infty} \rho^t \sum_{i=1}^{2} log c_{i,t}, \quad x_{t+1} = x_t - c_{i,t} - c_{j,t}, \; x_0, \text{ given.}$$

What about state and strategy based conjectures. Results

- In the conjectured model the optimal consumption policies depend on the initial consumption level. This implies that one might influence the consumption path that will be chosen by focusing on the initial level of consumption.

What about state and strategy based conjectures. Results

- In the conjectured model the optimal consumption policies depend on the initial consumption level. This implies that one might influence the consumption path that will be chosen by focusing on the initial level of consumption.
- The feedback consistent conjectural equilibrium coincides with the cooperative solution under complete information provided that the initial level of consumption is cooperate in both cases. If agents cooperate initially then asking for consistency ensures that cooperation will be sustained in the long run.

What about state and strategy based conjectures. Results

- If the initial consumption is too high, then the conjectural procedure leads to a more aggressive pattern than even in the non-cooperative case under full information. The effects of strategic behaviours are reinforced by incomplete information.

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- If the initial consumption is too high, then the conjectural procedure leads to a more aggressive pattern than even in the non-cooperative case under full information. The effects of strategic behaviours are reinforced by incomplete information.
- When initial consumption is sufficiently low, we obtain that the procedure leads to an under-exploitation of the resource compared to the full information cooperative benchmark.

The learning model

Each player (two players) maximises at each period t (myopic behaviour)

$$\max_{e_i^t} \pi_i(e_i^t, e_j^t, x^t), \quad x^{t+1} = f(e_i^t, e_j^t, x^t).$$

Each player at each period makes a conjecture about the behaviour of the other player

$$e_j^{t,c} = \phi_i(e_i^t, x^t).$$

At each period player i solves the following optimisation problem

$$\max_{e_i^t} \pi_i(e_i^t, \phi_i(e_i^t, x^t), x^t).$$

The general learning model

Call the optimal solution for player i $(i = 1, 2)$, e_i^* and the corresponding conjectured solution of the other player $e_j^{c*} = \phi_i(e_i^*, x)$.

This conjecture is in general different of e_j^* (at time t). Then player i updates his conjecture

Adapting conjectures

This update process can take one of the general forms:

$$\phi_i^{t+1} = \mathcal{U}_i(\phi_i^t, e_i^{t*}, e_j^{t*}, x^t) \tag{1}$$

$$\phi_i^{t+1} = \mathcal{S}_i(e_j^{0*}, x^0, \ldots, e_j^{t*}, x^t) \ . \tag{2}$$

In the first form (1), the “functions” are updated (hence “$\mathcal{U}$”) based on the most recent observation of the opponent. In the second form (2), some “statistic” (hence “$\mathcal{S}$”) is performed on the whole history of observations.

Note that ϕ_i^t is a sequence of functions. In practical situations we are going to consider particular functional forms for the conjecture. This functional form will depend on a certain numbers of parameters. These parameters are going to be learned in the learning procedure.

A groundwater exploitation problem

- Water extraction is the only input in the production process of the farmers and gives a profit $P_i(e_i^t)$,
- Unitary cost increases when the level of the water table is low,
- Players can also take into account the state of the resource and have an extra (subjective) profit of maintaining the resource, in this case ρ is the discount factor and γ_i is his resource preference,
- The dynamics is given by the evolution of the level of the water table.

$$\pi_i(e_i^t, e_j^t, x^t, x^{t+1}) = P_i(e_i^t) - c(x^t)e_i^t + \rho\gamma_i x^{t+1},$$

such that

$$x^{t+1} = x^t + R - e_i^t - e_j^t.$$

Near-sighted or short-sighted procedures.

Three kinds of conjectures

- Linear conjecture in e : $e_j^c = \beta_i e_i, \quad i \neq j.$
- Copy e (imitation): $e_j^c = \beta_i + e_i, \quad i \neq j.$
- Affine conjecture in x: $e_j^c = \beta_i (x_t + R), \quad i \neq j.$

$$\beta \quad \text{such that} \quad \phi_i(\beta, e_i) = e_j^* \quad \to \tilde{\beta},$$
$$\beta_i^{t+1} = \mu \beta_i^t + (1-\mu)\tilde{\beta}^t.$$

Simulations

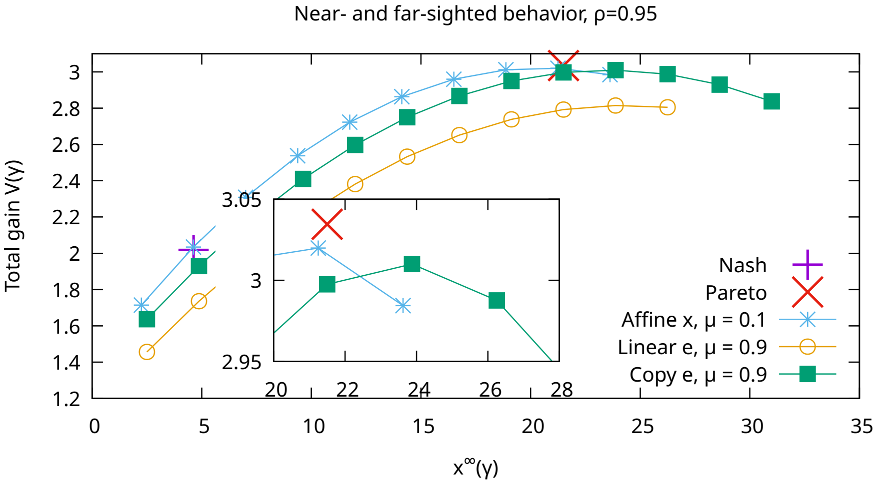
The "performance" of a learning scheme can be assessed using many criteria. Taking into account environmental and economic concerns, we select for our comparisons:

a) the limiting/steady state stock of water x^∞, representing the environment, and
b) the total discounted sum of profits for both players, representing the welfare of the society, V.

Experiments:

- The graphs represent the result of simulations in the plane (x^∞, V).
- Near-sighted solution (x^∞, V) are functions of γ.

Simulations



Conjectures: some papers

A. Jean-Marie, M. Tidball, (2005) « Consistent conjectures, equilibria and dynamic games ». *Chapter of book: Dynamic games: Theory and applications*.

A. Jean-Marie, M. Tidball, (2006) « Adapting behaviors through a learning process ». *Journal of Economic Behavior and Organization*.

N. Quérou, M. Tidball, (2010) « Incomplete information, learning, and natural resource management ». *European Journal of Operational Research*.

N. Quérou, M. Tidball, (2014) « Consistent conjectures in a dynamic model of non-renewable resource management ». *Annals of Operations Research*.

A. Jean-Marie, T. Jimenez, M. Tidball, (2021) « Nearsighted, farsighted behaviors and learning. Application to a water management problem ». [Research Report] RR-9406, Inria.

Biodiversity and agriculture

I. Brunetti, M. Tidball, D. Couvet, « Relationship between biodiversity and agricultural production ». *Natural Resource Modeling*, 2019.

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Spatial model. Consider the negative impact of agriculture on biodiversity and the fact that biodiversity can increase production.

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Spatial model. Consider the negative impact of agriculture on biodiversity and the fact that biodiversity can increase production.

- a tax on chemical inputs can have a positive effect on yield since it can be considered as a social signal helping farmers to avoid myopic behavior concerning the positive effect of biodiversity on yield
- increasing biodiversity productivity affects negatively the level of biodiversity, a counter-intuitive result; due to the fact that when biodiversity is more productive, farmers can maintain lower biodiversity to get the same yield.

When carrying capacity is taken into account

Two state variables: natural resource and carrying capacity.

Ngo Van Long, Georges Zaccour, Mabel Tidball
« Optimal harvesting and taxation when accounting for the marine environmental quality of the fishery » got the Rollie Lamberson award for best paper, 2021.
RMA Newsletter, fall, 2021.

PhD students

I only cite some of them... but all of them are really very intelligent!

Nicolas Quérou (2003). Contributions to the modeling and mathematical study of negotiation processes.
Katrin Erdlenbruch (2005). Essays in Renewable Resource Economics.
Fabien Prieur (2006). Growth and environment in the overlapping generations model: dynamic analysis and evaluations of public policies.

They are at CEEM laboratory, Nicolas was deputy director (2014 -2019) then Fabien (2020-2025). They work in dynamic games! I have my succession assured.

PhD students

Julia de Frutos Cachorro (2014). Groundwater resource management subject to droughts: analysis of adaptation strategies.
Supervisors: Mabel and Katrin

- How can drought risk be incorporated into a model of groundwater management for irrigation and what is the impact of drought (exogenous shock on the resource) on the behavior of farmers

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J. de Frutos, G. Martín Herrán and M. Tidball. Stackelberg competition in groundwater resources with multiple uses. Endogenous shock caused by the other user. Presented in this ISDG.

PhD students

Anmina Dulcie Murielle Djiguemde (2021). Dynamic Games and Renewable Common Pool Resources: Modeling and Experiments

- Lab protocol of the continuous time game versus lab protocol of an approximation in discrete time of the game. (paper published in Environmental Resource Economics 2022).
- How some kind of information can help cooperation.

Behavioral Economics: Positional and conformist consumers in a public good game

Positional consumer enjoys if his/her relative contribution to the public good is higher than the average contribution by others. Conformist consumers feel better if their behavior fits the average behavior in society, i.e., near the average contribution by others.

Alain Jean-Marie, Francisco Cabo, Mabel Tidball
Positional effects in public good provision. Strategic interaction and inertia.
Positional and conformist effects in public good provision. Strategic interaction and inertia.
Presented in this ISDG.

Operational applications

- Project Noviwam (2010-2015): Novel Integrated Water Management Systems for Southern European Regions. Thesis Yoro Sidibé (2012): Economic performance implications of a linear water pricing system and a nonlinear pricing system. The nonlinear pricing was really applied in the Deux-Sèvres region and compared with the linear pricing in vigor in Marseilles.

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- Project FAST, (2021–2027): Facilitate public Action to exit from peSTicides.
  Thesis Adrien Coiffard: Design of innovative agro-environmental policies based on reverse auctions

Challenges

For me

- Considering constraints in dynamic games (see for example papers of Agnieszka Wiszniewska-Matyszkiel)
- Impulsional games
- Discrete reverse auctions

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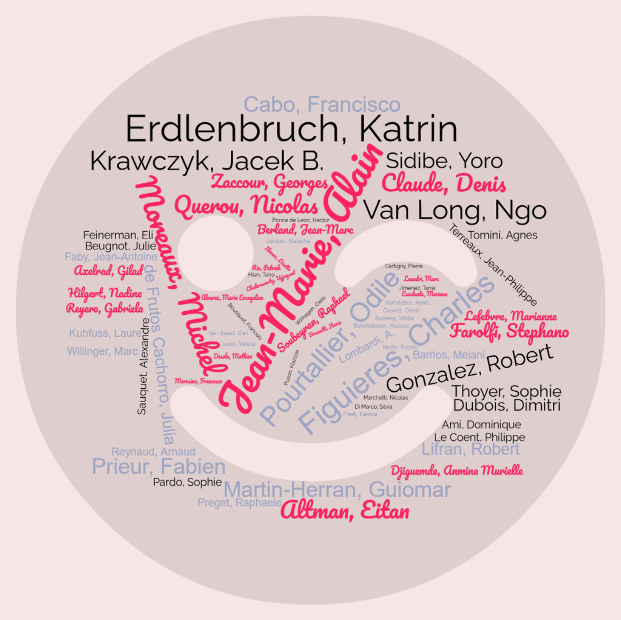
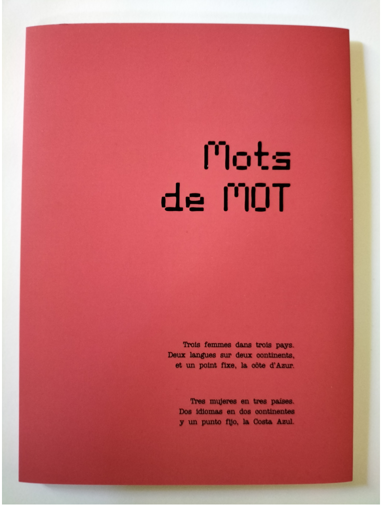
For the community of ISDG

- Stochastic games
- Mean field games
- Evolutionary games
- Learning... numerical procedures...
- How to chose the “correct” model in applications
- ...

Thanks to my coauthors



Mabel - Odile - Tania. Mots de MOT.



3 women from 3 countries (and from ISDG!!). 2 languages (French and Spanish) and one fixed point the French riviera.

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Thanks you – Merci – Gracias

One day I am going to live in Theory because in Theory things go well

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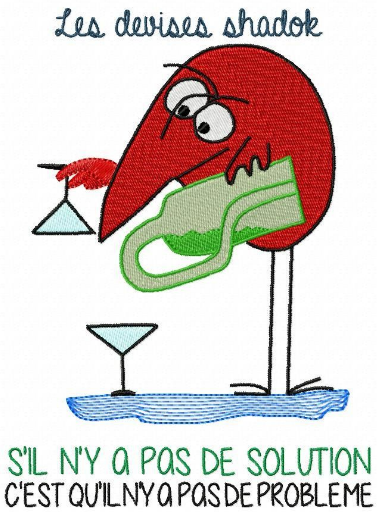

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By trying continuously one finally succeeds. So, the more it fails, the more likely it is to work.

Thanks you – Merci – Gracias