Towards enhanced regionalization of Hydrologic-hydraulic river networks models with assimilation of multi-source data and SWOT hydraulic visibility

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Kevin Larnier[1], Pierre-André Garambois[2], Léo Pujol[3], Jérome Monnier[4], Charlotte Emery[1], Thomas Ledauphin [5], Hervé Yesou [5], Stéphane Calmant [6]

[1] CS-GROUP FRANCE, Toulouse,

[2]INRAE, Aix-Marseille Université, RECOVER, 3275 Route Cézanne, 13182 Aix-en-Provence, France

[3]Institut de Mathématiques de Toulouse (IMFT), Université de Toulouse, CNRS, 31400 Toulouse, France

[4]Hydrosciences Montpellier, France

[5]SERTIT - Icube, Strasbourg, France

[6]IRD, Cayenne

`pierre-andre.Garambois@inrae.fr ; kevin.larnier@csgroup.eu` (corresponding authors)

Abstract: In context of increasing observation of the earth by satellites and airborne sensors, this contribution investigates information extraction from those data combined with in situ data for hydrological-hydraulic modeling of river networks. This work is based on recent multi-satellite datasets over two relatively large catchment river networks with contrasted and complex hydrological variabilities and flow features (anabranching reaches, confluences, tidal effects) : the Maroni River basin in French Guyana (SWOT Cal/val site) and the Adour River basin in metropolitan France. The dataset contains Multi-mission altimetry, optical and radar image of rivers in addition to in situ data and recent SWOT data. The forward modeling approach consists in dynamic shallow water flow models of river networks, in 1D following or multi-D inflowed by catchment scale hydrological models. The inverse modeling approach uses variational data assimilation and enables to optimize spatially distributed bathymetry-friction patterns to reduce the misfit to in-situ and satellite observables as well as to optimize simultaneously inflow hydrographs.

Keywords: River networks, hydrological model, hydraulic model, coupling, data assimilation, multi-source observations

Vers une meilleure régionalisation des modèles hydrologiques et hydrauliques des réseaux fluviaux grâce à l'assimilation de données multi-sources et à la visibilité hydraulique SWOT

Résumé : Dans le contexte de l'observation croissante de la terre par des satellites et des capteurs aéroportés, cette contribution étudie l'extraction d'informations à partir de ces données combinées avec des données in situ pour la modélisation hydrologique-hydraulique des réseaux fluviaux. Ce travail est basé sur des ensembles de données multi-satellites récentes sur deux réseaux fluviaux relativement importants avec des variabilités hydrologiques et des caractéristiques d'écoulement contrastées et complexes (tronçons anabranchés, confluences, effets de marée) : le bassin du fleuve Maroni en Guyane française (site SWOT Cal/val) et le bassin de l'Adour en France métropolitaine. Le jeu de données contient des images altimétriques, optiques et radar multi-missions des fleuves ainsi que des données in situ et des données SWOT récentes. L'approche de modélisation consiste en des modèles ydraulique de réseaux fluviaux, en 1D ou multi-D, alimentés par des modèles hydrologiques à l'échelle du bassin versant. L'approche de modélisation inverse utilise l'assimilation variationnelle des données et permet d'optimiser les la bathymétrie-friction distribués dans l'espace afin de réduire l'inadéquation aux observables in-situ et satellitaires ainsi que d'optimiser simultanément les hydrogrammes amonts et latéraux.

Mots-clefs : réseaux hydrographiques, modélisation hydrologique-hydraulqiue, assimilation de données, observations multi-sources

1. Introduction

A variety of satellites and sensors allows to observe, with increasing spatio-temporal resolution, the variability of continental water surfaces and hydrological components over catchments. In particular, satellite altimetry and images bring interesting hydraulic visibility (Garambois et al., 2017), i.e. "the potential to depict a hydrological response and hydraulic variabilities within a river section or network via remote sensing". Observations at an unprecedented spatial resolution with interesting temporal revisits of worldwide rivers surface properties started to be collected by the SWOT wide swath and nadir altimetry mission launched in December 2022 (cf. (Rodriguez, 2012; Rodríguez et al., 2020)).

Satellite observations, combined with in situ measurements, represent a very interesting source of information for hydrological and hydraulic modeling. Nevertheless, the exploitation of such data requires integrated models of adapted complexity as well as adequate data assimilation methods. These forward-inverse approaches must be capable of ingesting multi-source heterogeneous data while solving high dimensional ill-posed inverse problems as encountered with non-linear and dynamic flow models involving spatio-temporal state-fluxes and parameters. We focus here on the use of flow observables over river networks and floodplains.

The estimation of uncertain or unknown spatialized bathymetry-friction and inflows from heterogeneous satellite observations of rivers surface deformations remains a difficult inverse problem. The main issues encountered in hydraulic inverse problems aiming to retrieve unknown or uncertain discharge-bathymetry-friction from WS (Water Surface) observables are:

- Despite WS observables are relatively direct observations of hydraulic states, they contain variable measurement errors and do not give acces to friction and river bottom bathymetry - except for penetrating Lidar over shallow and non turbid flows. WS variabilities are flow signatures produced by hydraulic controls variabilities and the flow that acts as a low pass filter on them in fluvial regime (Montazem et al., 2019).
- Local and spatial equifinality: different hydraulic models parameters combinations leading locally to equivalent WS properties (Garambois & Monnier, 2015; Larnier et al., 2021) and different parameters patterns leading to equivalent flow lines (Larnier et al., 2021; Garambois et al., 2020), also with the issue of spatialized lateral flow exchanges (Pujol et al., 2020); both local and spatial equifinality issues, in addition to larger modeling uncertainty, are also encountered in spatially distributed hydrological model optimization from downstream integrative discharge (Huynh et al., 2023) or even from hydraulic observables within couplings (Pujol et al., 2022).
- The spatio-temporal sparsity of altimetric observations regarding real flow controls spatialization and dynamics. This is analyzed in (Brisset et al., 2018) for inferable inflow hydrograph frequencies with the introduction of identifiability maps, also in (Pujol et al., 2020) with multiple inflows. Friction patterns scales, larger than the scale of bathymetric

variability, are analyzed in (Garambois et al., 2020) and adapted physicaly-derived covariance operators are proposed in (Malou & Monnier, 2022).

Local equifinality translates into sensitivity to prior even in local inversion of the Low Froude algebraic model from WS observations. This issue can be partially overcome if sufficient physical constraints are provided from ancillary database and system knowledge for instance (cf. (Garambois & Monnier, 2015; Larnier et al., 2021; Larnier & Monnier, 2020)).

Putting in coherence catchment river-network models and flow observables, somehow pertains to a double regionalization problem regarding the estimation of (1) hydraulic channels parameters influencing flow dynamics and of (2) the hydrological parameters influencing discharge signals production and mass inflows into the river network hydraulic model.

The combined use of dynamic flow models of river systems and variational data assimilation methods enables to solve hydrologic-hydraulic inverse problems involving high dimensional spatio-temporal unknown parameters and heterogeneous in situ and satellite observations. This contribution presents multi-dimensional river networks hydrologic-hydraulic modeling approaches based on recent multi-satellite data cocktails, used both for model geometry construction and for spatially distributed calibration performed by variational data assimilation with the DassFlow platform (2D (Monnier et al., 2016; Pujol et al., 2022), 1D hydraulic model (Brisset et al., 2018; Larnier et al., 2021; Pujol et al., 2020; Malou et al., 2021), multi-D with hydrology (Pujol et al., 2022)) (cf. GitHub : https://github.com/orgs/DassHydro-dev/repositories) .

2. Hydrological-Hydraulic network model

We consider a 2D river basin domain Ω_{rr}, on which is applied a distributed hydrological model $\mathcal{M}_{rr}$, that contains a sub-domain Ω_{hy} on which is applied 1D $\mathcal{M}_{hy}^{1D}$ or 1Dlike-2D $\mathcal{M}_{hy}^{2D}$ hydraulic model of the river network, inflowed by the hydrological. The inflow points are defined by preprocessing and correspond to upstream and lateral tributaries draining sub-catchmetns.

We only detail here for brievity our DassFlow 1D model solving the 1D Saint-Venant equations over River networks. The 1D-like-2D model, based on the resolution of the 2D shallow water equations with a single solver, and including semi lumped hydrological model in DassFlow 2D is detailled in (Pujol et al., 2022).

For 1D modeling, the hydraulic domain Ω_{hy}^{1D}, $\Omega_{hy}^{1D} \subset \Omega_{rr} \subset \mathbb{R}^2$, is a portion of a hydrographic network plus its floodplains, described by connected segments; $t \in]0, T]$ denotes the physical time and $x \in \Omega_{hy}^{1D}$ the curvilinear abscissa. Let $A\,(x,t)\,[\mathrm{m}^2]$ be the cross-sectional area of flow and $Q\,(x,t)\,[\mathrm{m}^3/\mathrm{s}]$ the flow rate such that $Q = UA$ with $U\,(x,t)$ the mean velocity $[\mathrm{m/s}]$ over a cross-sectional area of flow. The Froude number for any cross-section is defined as $\mathrm{Fr} = Uc = \sqrt{Q^2WgA^3}$, where W is the top width, and compares the flow velocity U with the wave velocity c; Fr^2 compares the kinetic energy of the moving fluid with the potential energy of gravity.

The 1D Saint-Venant equations taking into account a variable cross-section A with lateral contributions q_l, write as follows:

$$\mathcal{M}_{hy}^{1D}: \quad \partial_t \mathbf{U} + \partial_x \mathbf{F}(\mathbf{U}) \;=\; \mathbf{S}(\mathbf{U})$$

$$\mathbf{U} = \begin{bmatrix} A \\ Q \end{bmatrix}, \ \mathbf{F}(\mathbf{U}) = \begin{bmatrix} Q \\ \beta \frac{Q^2}{A} \end{bmatrix}, \ \mathbf{S}(\mathbf{U}) = \begin{bmatrix} q_l \\ -gA\left(\frac{\partial Z}{\partial x} - S_f\right) + U\delta_l q_l \end{bmatrix} \tag{1}$$

where $Z(x,t)$ is the WSE $[m]$ and $Z = (z_b + h)$ with $z_b(x)$ the river bed level $[m]$ and $h(x,t)$ the water depth $[m]$, $R_h(x,t) = A/P_h$ the hydraulic radius $[m]$, $P_h(x,t)$ the wetted perimeter $[m]$, g is the gravity magnitude $[m.s^{-2}]$, $q_{lat}(x,t)$ is the lineic lateral discharge $[m^2.s^{-1}]$ and k_{lat} is a lateral discharge coefficient chosen equal to one here since we consider inflows only. Let us recall the Froude number definition $\mathrm{Fr} = U/c$ comparing the average flow velocity U to pressure wave celerity $c = \sqrt{\frac{gA}{W}}$ where W is the flow top width $[m]$. β is a dimensionless coefficient accounting for velocity non-uniformity and set to 1 by default.
The friction term S_f is classically parameterized with the empirical Manning-Strickler law established for uniform flows $S_f = \dfrac{|Q|Q}{K^2 A^2 R_h^{4/3}}$ where K $\left[\mathrm{m}^{1/3}.\mathrm{s}^{-1}\right]$ is the Strickler coefficient. Richer formulations are also available: $K(h) = \gamma h^{\delta}$, or the classical two-bed formulation (Nicollet & Uan, 1979).

The Saint-Venant equations are solved on each segment of the river network and the continuity of the flow between segments is ensured by applying an equality constrain on water levels and mass conservation at the confluence between two segments.
Boundary conditions are classically imposed (subcritical flows here) at boundary nodes (main hydrological inflows here) with inflow discharges $Q(t)$ at upstream nodes and WSE $Z(t)$ at the downstream node; lateral hydrographs $q_l at(t)$ at in/outflow nodes. The initial condition is set as the steady state backwater curve profile $Z_0(x) = Z\left(Q_{in}(t_0), q_{lat,1..L}(t_0)\right)$ for hot-start. This 1D Saint-Venant model is discretized using the classical implicit Preissmann scheme (see e.g. (Cunge et al., 1980)) on a regular grid of spacing Δx using a double sweep method enabling to deal with flow regimes changes; hourly time step Δt here. This is implemented into the computational software DassFlow (see DassFLow documentation; accurate finite volume scheme are also available (Brisset et al., 2018)).

3. Inverse algorithms

We denote by $\boldsymbol{Y}^*_{hy}$ the set of multisource observations of hydraulic responses over the river network domain Ω_{hy} that we aim to assimilate.

Given a spatio-temporal flow model, either 1D or 2D shallow water models above, given flow observables provided by in situ and airborne sensors, a VDA algorithm aims at estimating the unknown-uncertain input parameters of the model by minimizing the misfit between the observables $\boldsymbol{Y}^*_{hy}$ and model outputs $\mathbf{U}(\boldsymbol{\theta})$. We denote by $\boldsymbol{\theta}$ the unknown or uncertain parameter of the hydraulic model, that can contain spatialized bathymetry b and friction K, spatio-temporal boundary conditions Q_{in} and source terms q_l such that:

$$\boldsymbol{\theta} = (b(x), \alpha(x), \beta(x), Q_{in}(x_{in}, t), q_l(x_l, t))^T \tag{2}$$

Where $x_i n$ denotes the spatial coordinates of the BC inflow points and x_l the lateral inflow points.
The VDA method can be applied to one of the model only as in 1D (Brisset et al., 2018; Larnier et al., 2021; Garambois et al., 2020; Pujol et al., 2020) or in 1Dlike-2D (Monnier et al., 2016; Pujol et al., 2022). The considered unknown-uncertain parameter $\boldsymbol{\theta}$ is high-dimensional and the cost gradient computation relies on the adjoint model derived by automatic differenciation using the Tapenade software (Hascoet & Pascual, 2013).
We consider the cost function:

$$J(\boldsymbol{\theta}) = J_{obs}(\mathbf{U}(\boldsymbol{\theta})) + \alpha_{reg} J_{reg}(\boldsymbol{\theta}) \tag{3}$$

where $J_{obs}(.)$ and $J_{reg}(.)$ are differentiable, convex functions and α_{reg} is the regularization weight. The observation term J_{obs} is defined to account for multi-source (depth, discharge, surface velocity, ...) as in e.g. (Pujol et al., 2022), through scaled and potentially weighted terms accounting for data-model misfit.
A variable change based on a background covariance matrix $\boldsymbol{B}$, acting as a preconditioning of the optimization problem in context of equifinality, is used (Larnier et al., 2021). We define a diagonal matrix $\boldsymbol{B} = diag\,(B_b, B_K, b_Q)$ with 2 decreasing exponential kernels $(B_Q)_{i,j} = (\sigma_Q)^2\, exp\left(-\frac{|t_j-t_i|}{L_Q}\right)$, $(B_b)_{i,j} = (\sigma_b)^2\, exp\left(-\frac{|x_j-x_i|}{L_b}\right)$ and $B_K = diag\left(\sigma_\alpha^2, \sigma_\beta^2\right)$, where L_Q and L_b act as correlation length and scalar values $\sigma_\Box$ can be seen as variances.
In this case, the new control variable is $k = B^{-1/2}\left(\boldsymbol{\theta} - \boldsymbol{\theta}^*\right)$ and the optimization problem reads:

$$\hat{k} = \underset{k}{\operatorname{argmin}} J\,(k) \tag{4}$$

Where $j(k) = J(\theta)$ and the new optimality condition reads $B^{1/2}\nabla J\,(\theta) = 0$.
The optimization problem is numerically solved using the quasi-Newtion descent algorithm L-BFGS-B (Zhu et al., 1997). For more details on the formulations and algorithms, see e.g. (Monnier, 2020). The optimization is started from a background value denoted $\boldsymbol{\theta}^*$, that can contain a priori physical knowledge of the system, and/or be given by a global search in lower dimensional control spaces as done with the Low Froude model in (Larnier et al., 2021).

4. results

This section presents the construction from multi-source datasets of 2 modeling cases of various sophistication, first of the Maroni and next of the Adour River networks. The Maroni is representative of large and sparsely gauged rivers and a priori "SWOT compatible", while the Adour is more densely gauged and known while being a priori in the limit of visibility of SWOT. Both rivers networks present physical complexities in terms of tidal effect propagation from downstrea, and complex geomorphological facies. Spatio-temporal model parameters are estimateed by VDA on synthetic (real ones showable in November) SWOT data at the end.

4.1. Multisatellite dataset and construction of a 1D model of the Maroni river network

The river network portion we consider has been defined from a fixed threshold on drained area. The inflow points simply correspond to the upstream subcatchments of those main rivers - we here neglect some small lateral tibutaries which drain only a few percent of the total basin area.

A rich multi-source dataset has been collected and processed over the Maroni River network that is discretized into 24 main reaches between main inflows. It is illustrated in Fig. 1 and is composed of:

- Water masks from Landsat optical images (Pekel water mask) and from Sentinel 1/2 radar images (ExtractEO algorithm).
- Nadir altimetric WS elevations from Sentinel and Jason satellites, but also from the drifting orbit ICESat-2 satellite.
- In situ limnimetric and discharge time series at sparse gauging stations (VigiCrues network)
- In situ longitudinal water surface profiles gained from boats equiped with GPS.
- Merit digital elevation model (DEM).

An algorithm has been set up (not presented) for ICESat-2 altimetric data processing based on an high resolution average water mask that is used to crop ground tracks as depicted on the top left of Fig. 1. The processed altimetric WS elevations compare fairly well to in situ WS GPS profiles as illustrated on one segment among 24.

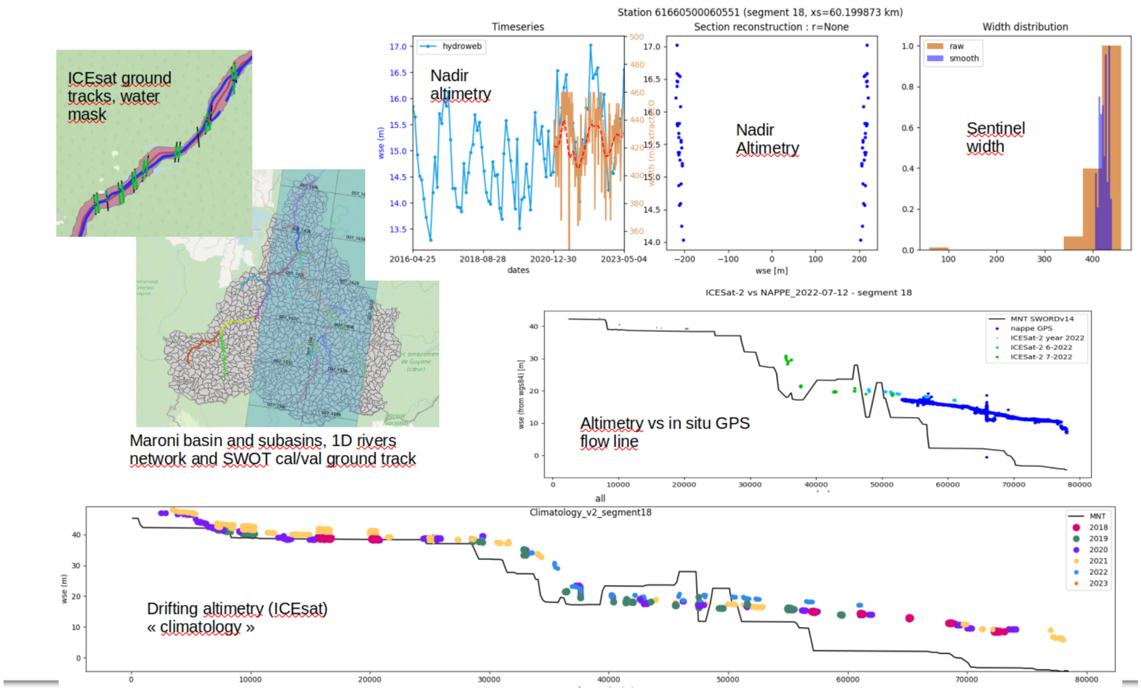

Figure 1. Maroni catchment and mains rivers network with SWOT cal-val groundtrack: Multi-source (Sentinel, ICEsat, in situ, hydroSHED) dataset over the Maroni river network for model construction; hydrological sub-basins in grey.

A spatial filtering approach is applied to the temporal water masks since we first want to capture main contraction-enlargements of river channels and main hydraulic controls for our 1D model.

The hydraulic model mesh and geometry is built as follows. A "fishbone" mesh is automatically defined on top of the water mask using rivers centerline, at a fine spatial spacing of few hundred meters (to make the most of data while ensuring accurate resolution of the hydraulic model). A simple rectangular cross section is considered here which is reasonnable for such rivers morphology (cf. discussion in (Pujol et al., 2020) for the "nearby" Negro River) and its width is given by the average water mask. The a priori channel bottom elevation b^* is obtained by inverting the low Froude model (cf. (Larnier et al., 2021; Pujol et al., 2020) given GPS flow lines and assuming a constant friction of $K = 30$.

Inflows are provided by the regionalized hydrological model MGB. Downstream hydraulic boundary condition is imposed either as a Neumann BC or with water surface elevation from altimetry.

4.2. Estimation of river network model parameter by assimilation of multisatellite data

A first data assimilation experiment is presented here, it is based on the hydraulic model built above. It consists in solving inverse problem 4 by assimilating all available altimetric data over the river domain in space and time. The control vector 3 is composed of spatially distributed bathymetry, friction coefficients and inflow discharge time series and is of high dimension. A simultaneous optimization of these hydraulic controls is performed using the data assimilation algorithm presented above.

The estimates obtained after satellite altimetry (nadir) data assimilation are shown in Figure 2 for spatially distributed bathymetry over the 23 reaches (spatially distributed friction not shown) and in Fig. 3 for the 12 inflow hydrographs. The model is also in good agreement with

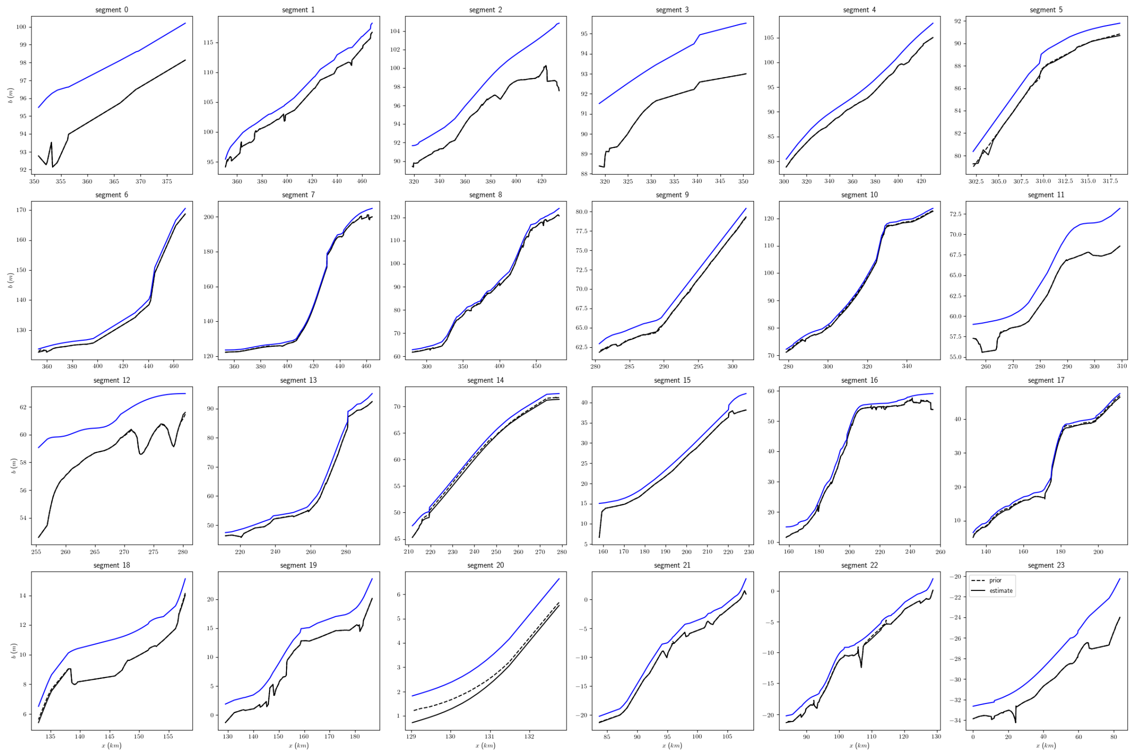

Figure 2. Estimated spatially distributed bathymetry coefficients for the 23 segments of the Maroni River network. ité14 Theta triplet.

in situ flow depth time series at two upstream gauging stations at Papaichton and Maripasoula (Fig. 3, right).

SWOT data assimilation could be presented at the conference.

4.3. 1Dlike and 1D2D modeling from LiDAR data, the Adour river network

A 1D-like - 2D model has been built in (Pujol et al., 2022) over the Adour River network. It is composed of a 2D zone over the city of Bayonne, based on an operational hydraulic model, and of a 1D-like network. Here the effective 1D-like channel geometry has been built from high resolution Lidar DEM (Fig.4).

One particular feature of the area is the tidal control, which is set around 6km from the 2D area and impacts most the the modeled area, reaching more than 50km inland. Calibration of the bathymetry, using limnigraph data and starting with a prior value based on the DEM, allows to accurately model this signature propagation up to the upstream parts of the model (not shown).

In an academic twin experiment setup, upstream hydrographs were inferred from the observation of their mixed signatures on water depth (not shown). Fair results were obtained starting from unbiased priors. This opens the way to investigate the estimation of ungauged lateral tributaries from multi-source data as well as the estimation of effective channel parameters from multi-source flow observations. Forthcoming SWOT data (see swathes in Fig.4) will be tested on this relatively narrow river network. A coupling with a regionalized hydrological model model will be studied.

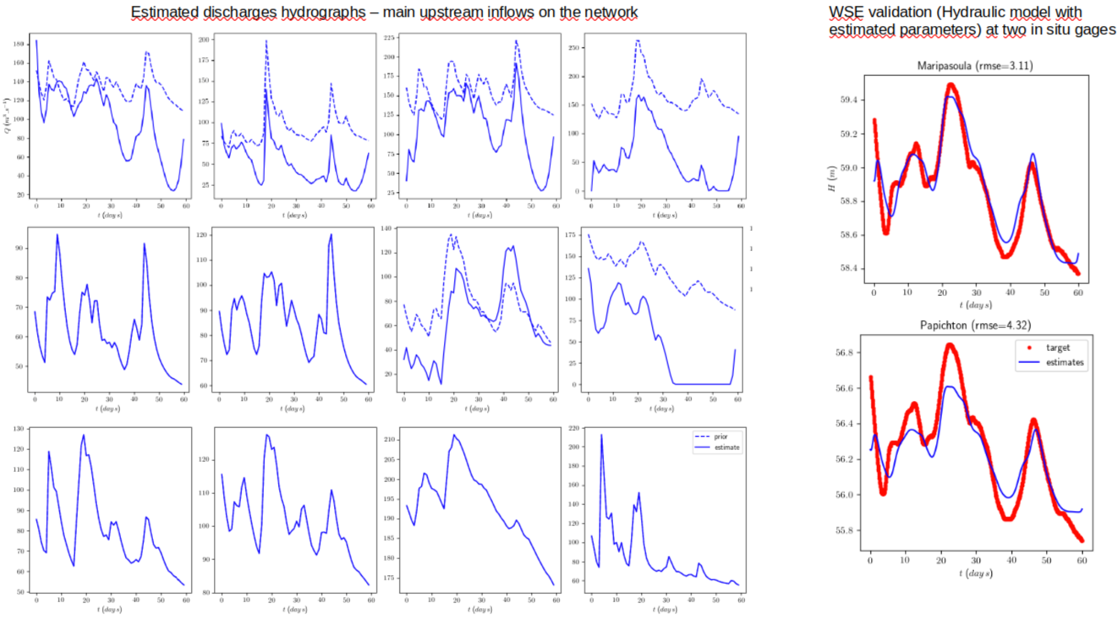

Figure 3. Estimated spatially distributed hydrographs for the 12 main inflow points over the Maroni River network. ité14 Theta triplet.

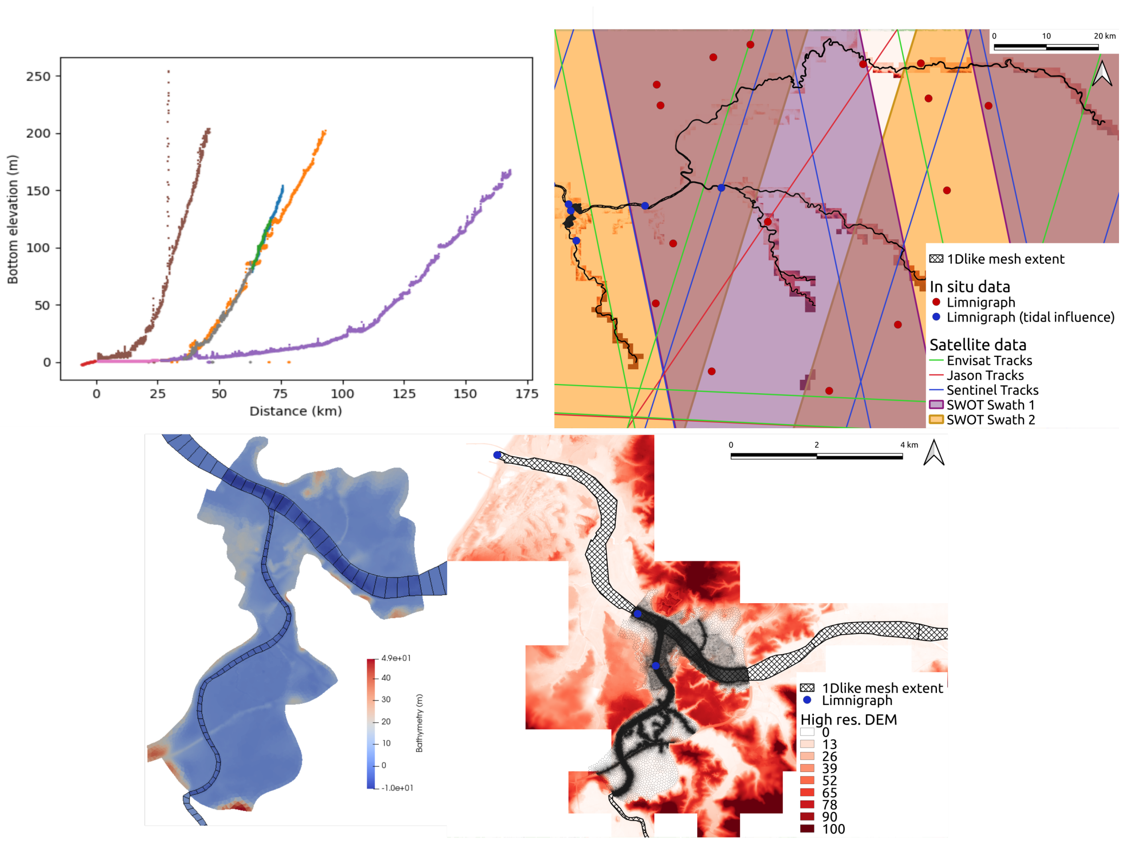

Figure 4. Adour river network upstream from Bayonne (tidal boundary), with observability from in situ stations (Banque HYDRO network) and satellite missions (altimetry from SWOT, Sentinel 3A-B, Envisat and Jason). High resolution DEM from LiDAR used to build 1Dlike network model and 1D2D model with urban flood at Bayonne.

5. Conclusion and further work

This contribution presented the construction of river network flow models, in coherence with multi-source data, over the Maroni and Adour River basins. Several preprocessing tools have been developped to pre-process multi-source data and prepare the model inputs including mesh and coupling with regional hydrology. Two river network hydraulic models inflowed by hydrological models were built. The capabilities of the data assimilation algorithm are illustrated on a simultaneous optimization of spatially distributed bathymetry-riction and inflows over the Maroni network using existing nadir altimetry missions. Further work consists in studying the assimilation of SWOT satellite spatio-temporal observations, starting with the data collected this spring on the Cal-Val track over the Maroni. This work opens perspectives for regionalization of spatially distributed river channels parameters from multi-source data as well as model hydrological regionalization through information feedback in the hydrological-hydraulic coupling (cf. (Pujol et al., 2022) also another proceeding presented). The proposed methods are transposable and implemented in open source softwares.

6. Authors contributions

KL and LP performed numerical results respectively on the Maroni and Adour cases. PAG supervised this research and wrote the article. The design and development of the inverse method was supervised by JM. All authors participated to data processing, discussions and manuscript revision.

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