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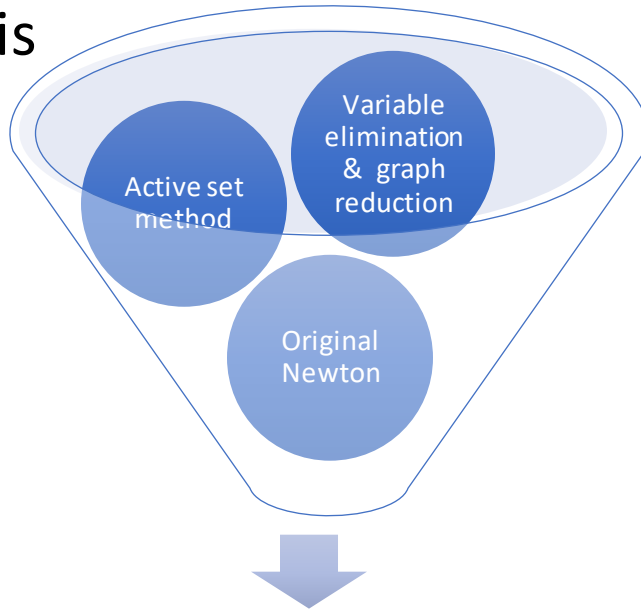
A reduced model for Sustainable Operations of Water Distribution Systems

Olivier Piller, Jochen Deuerlein, Sylvan Elhay and Angus Simpson
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Overview

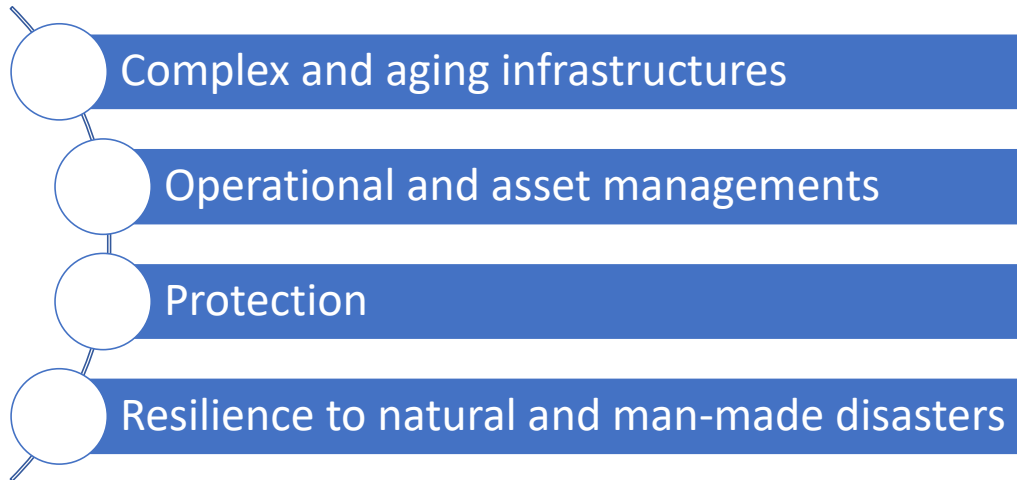
- Comprehensive framework that includes **linkflow and outflow min-max constraints and pressure control** for steady-state analysis



Quadratic convergence

1. Background
2. Regularization
3. Reduced model
4. Illustrative example

Flow and pressure constraints for sustainable operation of WDSs

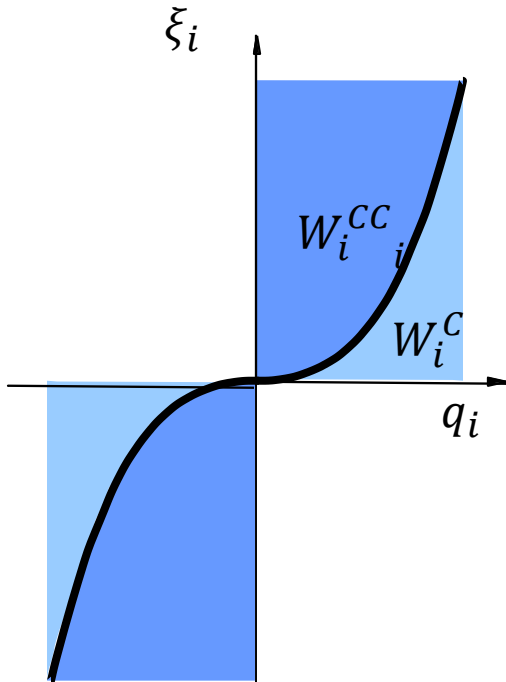


- ❖ Ulusoy et al. (2020) & Nerantzis et al. (2020)
 - Design-for-Control MINLP
 - Optimal control of VSPs & valves
- ❖ Abraham et al. (2017)
 - Self-cleaning networks
 - Minimum velocity & closed pipes

Control of velocity and pressure for sustainable management

The DDM Content function

Calculation of the hydraulic steady-state is equivalent to the minimization of the nonlinear, differentiable and strictly convex content function:



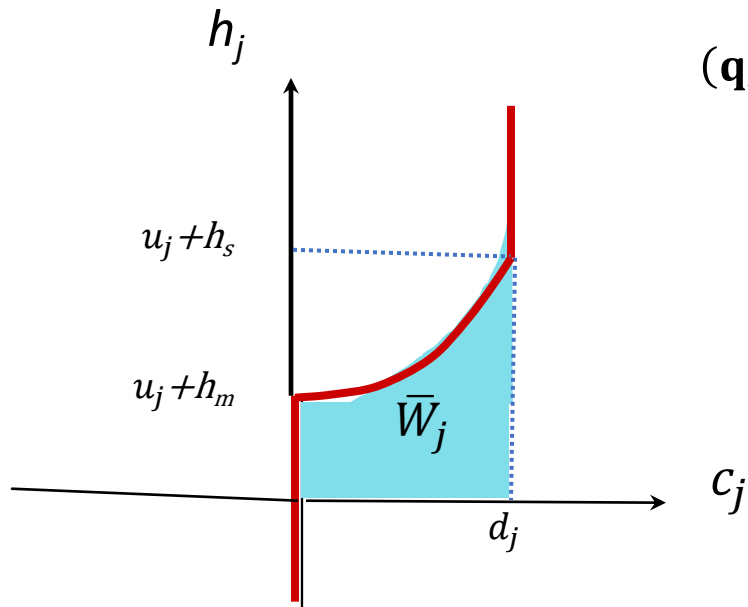
$$\min_{\mathbf{q} \in \mathbb{R}^{n_p}} C(\mathbf{q}) \stackrel{\text{def}}{=} \sum_{i=1}^{n_p} \overbrace{\int_0^{q_i} \xi_i(x) dx}^{W_i^C} - \mathbf{q}^T \mathbf{A}_0 \mathbf{h}_0$$

$$\text{s. t. } -\mathbf{A}^T \mathbf{q} - \mathbf{S}_d^T \mathbf{d} = \mathbf{0}_{n_j}$$

Cherry (1951)
Collins et al. (1978)

The PDM Content function

Deuerlein et al. (2019)



Inverse POR

$$\min_{(\mathbf{q}, \mathbf{c}) \in \mathbb{R}^{n_p + n_j}} C(\mathbf{q}, \mathbf{c}) \stackrel{\text{def}}{=} C(\mathbf{q}) + \sum_{j=1}^{n_d} \bar{W}_j(\mathbf{c}_j)$$

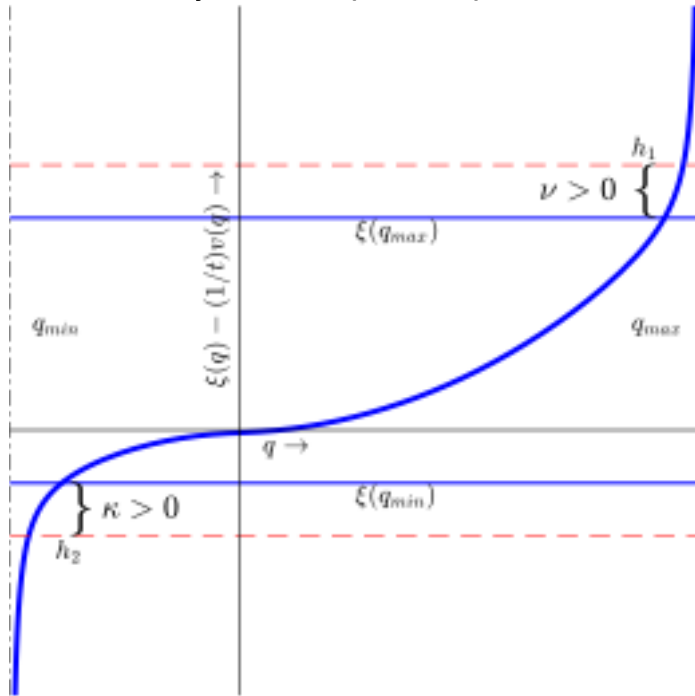
$$s. t. -\mathbf{A}^T \mathbf{q} - \mathbf{S}_d^T \mathbf{c} = \mathbf{0}_{n_j}; \quad \mathbf{0}_{n_d} \leq \mathbf{c} \leq \mathbf{d}$$

The heads in this problem are the equality constraints' Lagrange multipliers (LMs),
Pressure deficit & pressure surplus are the box constraint LMs

Linkflow constraints

Piller et al. (2021)

Elhay et al. (2022)



$$\min_{(\mathbf{q}, \mathbf{c}) \in \mathbb{R}^{n_p + n_j}} C(\mathbf{q}, \mathbf{c})$$

$$s.t. -\mathbf{A}^T \mathbf{q} - \mathbf{S}_d^T \mathbf{c} = \mathbf{0}_{n_j}; \quad \mathbf{0}_{n_d} \leq \mathbf{c} \leq \mathbf{d}; \quad \mathbf{q}_{min} \leq \mathbf{q} \leq \mathbf{q}_{max}$$

- ✓ Maximum flow: $q \leq q_{max}$ (FCV)
- ✓ Positivity: $0 \leq q$ (CV)
- ✓ No-flow: $q = 0$ (Closed valve)
- ✓ Minimum flow: $0 \leq q_{min} \leq q$
- ✓ Capacities: $-q_{CAP} \leq q \leq +q_{CAP}$
- ✓ Equality constraints: $q = q_F$

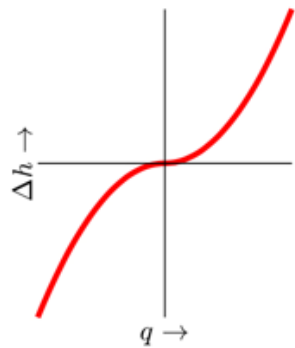
□ LMs for linkflow constraints interprets as additional head loss, or gain loss

Nash equilibrium for local control with PRDs

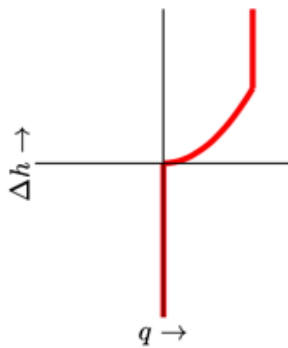
Deuerlein et al. (2023)

$$\min_{(\mathbf{q}, \mathbf{c}) \in \mathcal{U}} C_Z(\mathbf{q}, \mathbf{c}) \stackrel{\text{def}}{=} C(\mathbf{q}, \mathbf{c}) + \mathbf{q}^T \mathbf{z}$$

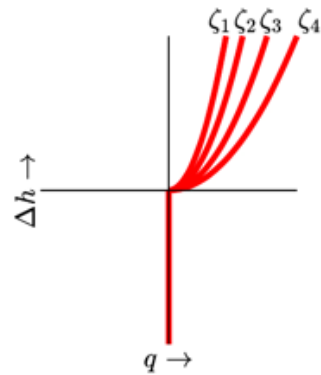
$$\min_{z_k > 0} f_k(z_k) \stackrel{\text{def}}{=} \frac{1}{2} (h_{i_k} - \xi(q_k) - z_k - h_{j_k}^s)^2, k = 1, \dots, n_v$$



No flow control



Local flow control



Distributed pressure control

the Nash Equilibrium for a game in which the players are:

- 1) optimization of a system content model
- 2) a separate, local optimization problem for each link with a PRV treating the linkflows and outflows and heads as fixed parameters

KKT equations and Deuerlein et al. solution

$$\xi(\mathbf{q}) - \mathbf{A}\mathbf{h} - \mathbf{a} - \mathbf{L}_q^T \boldsymbol{\kappa} + \mathbf{U}_q^T \boldsymbol{\nu} + \mathbf{L}_v^T \mathbf{z} = \mathbf{0}_{n_p}$$

$$\mathbf{h}_d(\mathbf{c}) - \mathbf{S}_d \mathbf{h} - \mathbf{L}_c^T \boldsymbol{\lambda} + \mathbf{U}_c^T \boldsymbol{\mu} = \mathbf{0}_{n_d}$$

$$-\mathbf{A}^T \mathbf{q} - \mathbf{S}_d^T \mathbf{c} = \mathbf{0}_{n_j}$$

$$-\mathbf{L}_c \mathbf{c} = \mathbf{0}_{n_{c_l}}$$

$$\mathbf{U}_c(\mathbf{c} - \mathbf{d}) = \mathbf{0}_{n_{c_u}}$$

$$-\mathbf{L}_q(\mathbf{q} - \mathbf{q}_{\min}) = \mathbf{0}_{n_{q_l}}$$

$$\mathbf{U}_q(\mathbf{q} - \mathbf{q}_{\max}) = \mathbf{0}_{n_{q_u}}$$

$$\mathbf{S}_s \mathbf{h} - \mathbf{L}_v \xi(\mathbf{q}) - \mathbf{z} + \mathbf{L}_{z_a}^T \boldsymbol{\chi} - \mathbf{h}^s = \mathbf{0}_{n_v}$$

$$-\mathbf{L}_{z_a} \mathbf{z} = \mathbf{0}_{n_{z_a}}$$

$$-\mathbf{c} \leq \mathbf{0}_{n_d}$$

$$\mathbf{c} - \mathbf{d} \leq \mathbf{0}_{n_d}$$

$$-\mathbf{q} + \mathbf{q}_{\min} \leq \mathbf{0}_{n_p}$$

$$-\mathbf{q}_{\max} + \mathbf{q} \leq \mathbf{0}_{n_p}$$

$$-\mathbf{z} \leq \mathbf{0}_{n_v}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}_{n_{c_l}}, \boldsymbol{\mu} \geq \mathbf{0}_{n_{c_u}}, \boldsymbol{\kappa} \geq \mathbf{0}_{n_{q_l}}, \boldsymbol{\nu} \geq \mathbf{0}_{n_{q_u}}, \boldsymbol{\chi} \geq \mathbf{0}_{n_{z_a}}$$

KKT Eqs

Active-Set Method with Newton method

$$\begin{pmatrix} \mathbf{F}_{q_b} & \mathbf{0} & -\mathbf{A}_b & \hat{\mathbf{L}}_{v_b}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{c_b} & -\hat{\mathbf{I}}_{c_b} \mathbf{S}_d & \mathbf{0} & \mathbf{0} \\ -\mathbf{A}_b^T & -\mathbf{S}_d^T \hat{\mathbf{I}}_{c_b}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\hat{\mathbf{L}}_{v_b} \mathbf{F}_{q_b} & \mathbf{0} & \mathbf{S}_s & -\mathbf{I} & \mathbf{L}_{z_a}^T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{z_a} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \delta \mathbf{q}_b^{(m+1)} \\ \delta \mathbf{c}_b^{(m+1)} \\ \delta \mathbf{h}^{(m+1)} \\ \delta \mathbf{z}^{(m+1)} \\ \delta \boldsymbol{\chi}_{z_a}^{(m+1)} \end{pmatrix} = - \begin{pmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \mathbf{r}_4 \\ \mathbf{r}_5 \end{pmatrix}$$

with

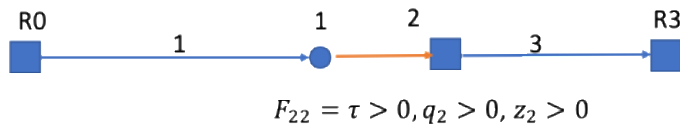
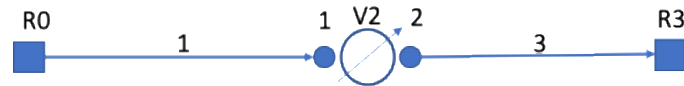
$$\begin{pmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \mathbf{r}_4 \\ \mathbf{r}_5 \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} \xi_{q_b}^{(m)} + \mathbf{L}_{v_b}^T \mathbf{z}^{(m)} - \mathbf{A}_b \mathbf{h}^{(m)} - \mathbf{a}_{q_b} \\ \mathbf{h}(\mathbf{c}_b^{(m)}) - \mathbf{h}_{c_b}^{(m)} \\ -\mathbf{A}^T \mathbf{q}^{(m)} - \mathbf{S}_d^T \mathbf{c}^{(m)} \\ \mathbf{S}_s \mathbf{h}^{(m)} - \mathbf{L}_v \xi^{(m)} - \mathbf{z}^{(m)} + \mathbf{L}_{z_a}^T \boldsymbol{\chi}_{z_a}^{(m)} - \mathbf{h}^s \\ \mathbf{L}_{z_a} \mathbf{z}^{(m)} \end{pmatrix}$$

Regularization of the Schur matrix

$$\widehat{\mathbf{V}}\delta\mathbf{h}^{(m+1)} = \mathbf{s}_3$$

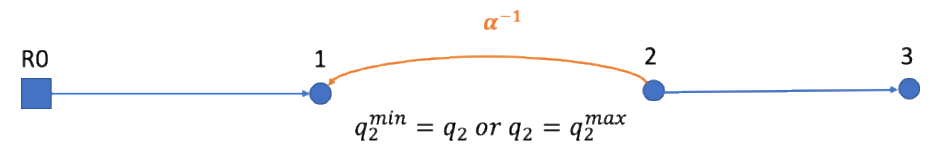
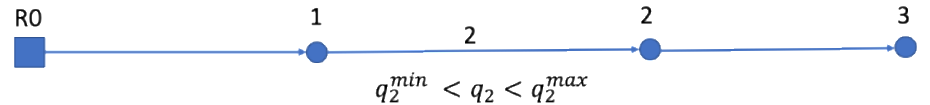
$$\text{Where } \widehat{\mathbf{V}} = \mathbf{A}_{\text{b.}}^T \widehat{\mathbf{F}}_{\text{qb}}^{-1} \widehat{\mathbf{A}} + \widehat{\mathbf{I}}_{\text{Cb}}^T \mathbf{M}_{\text{Cb}}^{-1} \widehat{\mathbf{I}}_{\text{Cb}}$$

Regularizations from Deuerlein et al. (2023)



\mathbf{F} can be singular, whenever the valve status is active ($q > 0$ and $z > 0$)

Or The Schur complement, $\widehat{\mathbf{V}}$, can become singular if the matrix $\widehat{\mathbf{A}}$ has less than full rank because of the particular membership of the index sets at that point



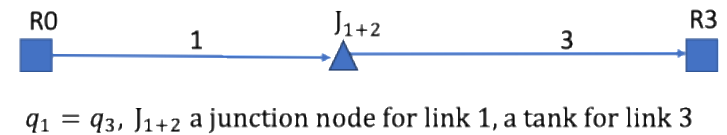
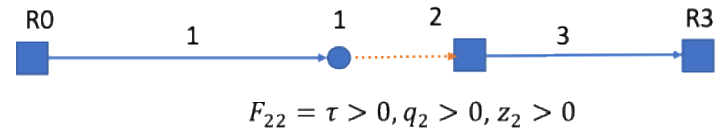
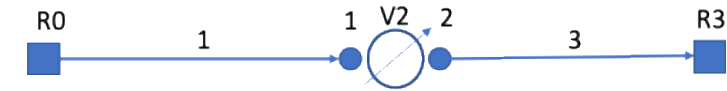
The reduced system system

- Elimination of z and χ in ASM and Newton method
- Reduction conservation energy (partitioning links)
- Conservation of mass (partitioning nodes + node merging)
- Final system Schur

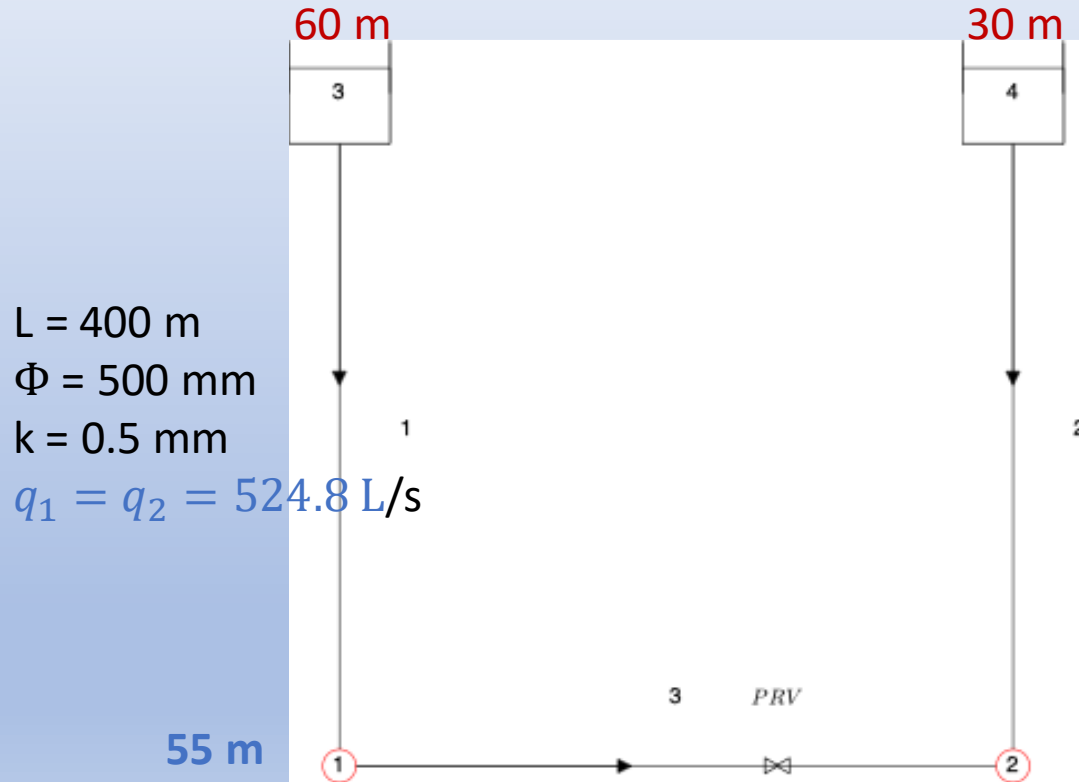
$$-\left(\bar{\mathbf{A}}_b^T \mathbf{F}_{q_{br}}^{-1} \hat{\mathbf{A}}_{b_s} + \mathbf{L}_s \mathbf{S}_d^T \hat{\mathbf{I}}_{c_b}^T \mathbf{M}_{c_b}^{-1} \hat{\mathbf{I}}_{c_b} \mathbf{S}_d \mathbf{L}_s^T\right) \delta \mathbf{h}_s^{(m+1)} =$$

$$-\bar{\mathbf{A}}_b^T \mathbf{F}_{q_{br}}^{-1} \mathbf{v}_1 - \mathbf{L}_s \mathbf{S}_d^T \hat{\mathbf{I}}_{c_b}^T \mathbf{M}_{c_b}^{-1} \mathbf{r}_2 - \mathbf{L}_s \mathbf{r}_3$$

$$\mathbf{V} = \begin{pmatrix} \bar{\mathbf{A}}_b^T & \mathbf{L}_s \mathbf{S}_d^T \hat{\mathbf{I}}_{c_b}^T \\ \mathbf{F}_{q_{br}}^{-1} & \mathbf{M}_{c_b}^{-1} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{A}}_{b_s} \\ \hat{\mathbf{I}}_{c_b} \mathbf{S}_d \mathbf{L}_s^T \end{pmatrix}$$



An illustrative example



An active PRV example

1. a failure when no regularization is used and initial \hat{V}
2. a slow convergence case when the F-reg is very strict (linear CV & 15 iter.) $\tau = 2 \cdot 10^0$
3. A quadratic convergence case with reduced model or small $\tau = 2 \cdot 10^{-4}$

Conclusions and perspective

- **A general framework suitable for sustainable management of pressures and flow rates is presented**
 - The formulation takes into account min-max constraints for linkflows ($q_{\min} < q < q_{\max}$) and outflows ($0 < c < d$)
 - No heuristics are required to determine the status of the pressure and flow control devices (Nash equilibrium + Lagrange multipliers)
- **A reduced formulation is sought to benefit from Newton's quadratic convergence**
- Future work, finding **rapid and robust algorithms**:
 - Existence & uniqueness
 - impact of graph variations during the iterations? Matrix rank or spanning trees?
 - Relaxation on inappropriate constraints with large networks and a lot of constraints

Thank you for your attention!
Any questions?

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