A reduced model for Sustainable Operations of Water Distribution Systems

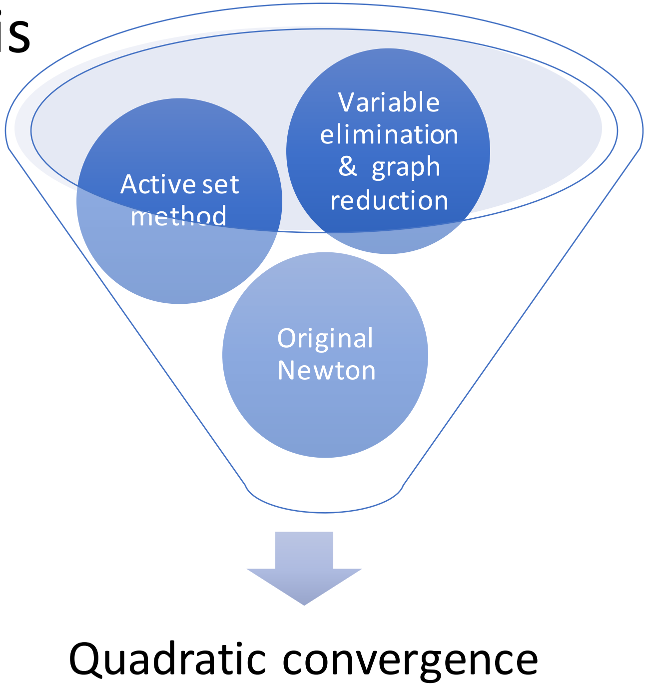
Olivier Piller, Jochen Deuerlein, Sylvan Elhay, Angus Simpson

A reduced model for Sustainable Operations of Water Distribution Systems

Olivier Piller, Jochen Deuerlein, Sylvan Elhay and Angus Simpson
INRAE/UR ETTIS, France; 3S Consult GmbH, Germany;
Univ. of Adelaide, Australia;

Overview

- Comprehensive framework that includes **linkflow and outflow min-max constraints and pressure control** for steady-state analysis



1. Background
2. Regularization
3. Reduced model
4. Illustrative example

Flow and pressure constraints for sustainable operation of WDSs

- Complex and aging infrastructures
- Operational and asset managements
- Protection
- Resilience to natural and man-made disasters

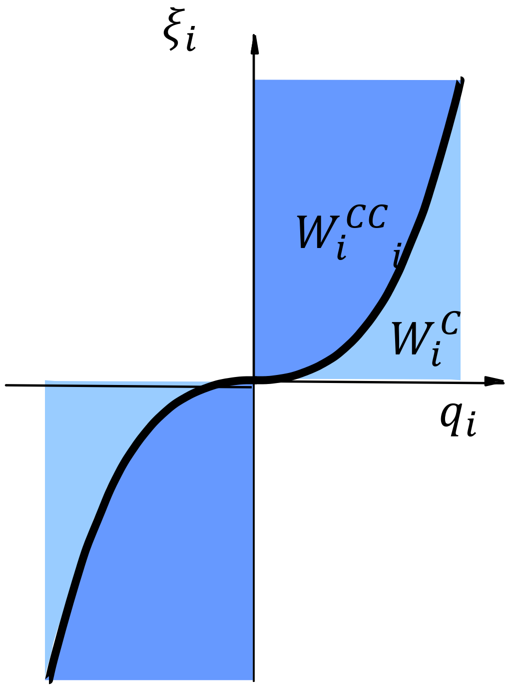
❖ Ulusoy et al. (2020) & Nerantzis et al. (2020)
  - Design-for-Control MINLP
  - 0ptimal control of VSPs &valves

❖ Abraham et al. (2017)
  - Self-cleaning networks
  - Minimum velocity & closed pipes

Control of velocity and pressure for sustainable management

The DDM Content function

Calculation of the hydraulic steady-state is equivalent to the minimization of the nonlinear, differentiable and strictly convex content function:



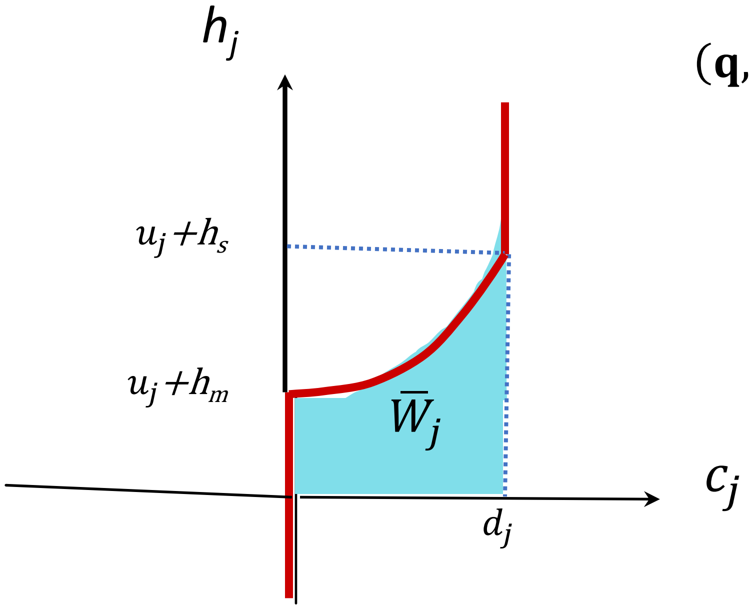
$$\min_{\mathbf{q}\in\mathbb{R}^{n_p}} C(\mathbf{q}) \stackrel{\text{def}}{=} \sum_{i=1}^{n_p} \overbrace{\int_0^{q_i} \xi_i(x)\,dx}^{W_i^C} - \boldsymbol{q}^T \mathbf{A}_0 \boldsymbol{h}_0$$

$$s.t.\quad -\mathbf{A}^{\mathrm{T}}\mathbf{q} - \mathbf{S}_d^T \mathbf{d} = \mathbf{0}_{\mathrm{n_j}}$$

Cherry (1951)
Collins et al. (1978)

The PDM Content function

Deuerlein et al. (2019)



Inverse POR

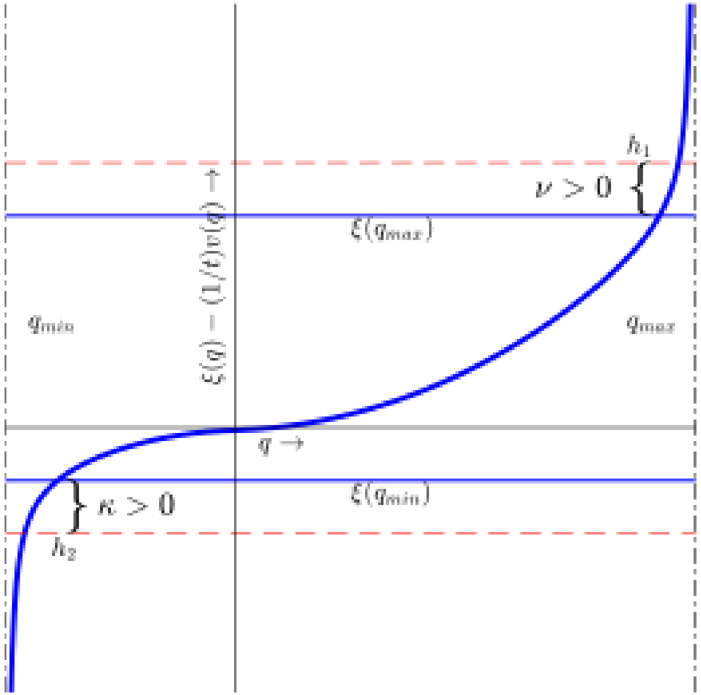
$$\min_{(\mathbf{q},\mathbf{c})\in\mathbb{R}^{n_p+n_j}} \mathrm{C}(\mathbf{q},\boldsymbol{c}) \stackrel{\text{def}}{=} C(\mathbf{q}) + \sum_{j=1}^{n_d} \overline{\boldsymbol{W}_{\boldsymbol{j}}}(\boldsymbol{c}_{\boldsymbol{j}})$$

$$s.t. -\mathbf{A}^{\mathrm{T}}\mathbf{q} - \mathbf{S}_d^T\mathbf{c} = \mathbf{0}_{\mathrm{n_j}};\ \ \mathbf{0}_{\mathrm{n_d}} \leq \mathbf{c} \leq \boldsymbol{d}$$

The heads in this problem are the equality constraints' Lagrange multipliers (LMs),
Pressure deficit & pressure surplus are the box constraint LMs

Linkflow constraints

Piller et al. (2021)
Elhay et al. (2022)



$$\min_{(\mathbf{q},\mathbf{c})\in\mathbb{R}^{n_p+n_j}} \mathrm{C}(\mathbf{q}, \boldsymbol{c})$$

$$s.t. -\mathbf{A}^{\mathrm{T}}\mathbf{q} - \mathbf{S}_{\mathbf{d}}^{\mathrm{T}}\mathbf{c} = \mathbf{0}_{\mathrm{n_j}};\ \ \mathbf{0}_{\mathrm{n_d}} \leq \mathbf{c} \leq \boldsymbol{d};\ \mathbf{q}_{\mathbf{min}} \leq \mathbf{q} \leq \mathbf{q}_{\mathbf{max}}$$

- ✓ Maximum flow: $q \leq q_{max}$ (FCV)
- ✓ Positivity: $0 \leq q$ (CV)
- ✓ No-flow: $\mathrm{q} = 0$ (Closed valve)
- ✓ Minimum flow: $0 \leq q_{min} \leq q$
- ✓ Capacities: $-q_{CAP} \leq q \leq +q_{CAP}$
- ✓ Equality constraints: $q = q_F$

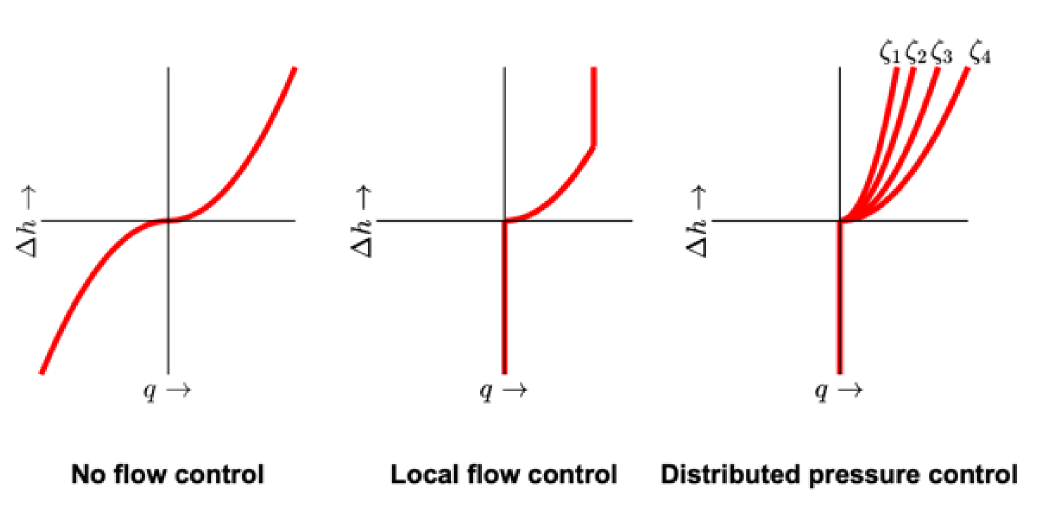
- ❑ LMs for linkflow constraints interprets as additional head loss, or gain loss

Nash equilibrium for local control with PRDs

Deuerlein et al. (2023)

$$\min_{(\mathbf{q},\mathbf{c})\in \boldsymbol{U}} \mathrm{C}_Z(\mathbf{q}, \boldsymbol{c}) \stackrel{\text{def}}{=} \mathrm{C}(\mathbf{q}, \boldsymbol{c}) + \boldsymbol{q}^T \mathbf{z}$$

$$\min_{z_k>0} \mathrm{f_k}(z_k) \stackrel{\text{def}}{=} \frac{1}{2}\left(h_{i_k} - \xi(q_k) - z_k - h_{j_k}^s\right)^2, k = 1, \dots, n_v$$



the Nash Equilibrium for a game in which the players are:

1) optimization of a system content model
2) a separate, local optimization problem for each link with a PRV treating the linkflows and outflows and heads as fixed parameters

KKT equations and Deuerlein et al. solution

$$\boldsymbol{\xi}(\mathbf{q}) - \mathbf{A}\mathbf{h} - \mathbf{a} - \mathbf{L}_{\mathbf{q}}^{T}\boldsymbol{\kappa} + \mathbf{U}_{\mathbf{q}}^{T}\boldsymbol{\nu} + \mathbf{L}_{v}^{T}\mathbf{z} = \mathbf{0}_{n_p}$$
$$\mathbf{h}_{\mathbf{d}}(\mathbf{c}) - \mathbf{S}_{d}\mathbf{h} - \mathbf{L}_{c}^{T}\boldsymbol{\lambda} + \mathbf{U}_{c}^{T}\boldsymbol{\mu} = \mathbf{0}_{n_d}$$
$$-\mathbf{A}^{T}\mathbf{q} - \mathbf{S}_{d}^{T}\mathbf{c} = \mathbf{0}_{n_j}$$
$$-\mathbf{L}_{c}\mathbf{c} = \mathbf{0}_{n_{c_l}}$$
$$\mathbf{U}_{c}(\mathbf{c} - \mathbf{d}) = \mathbf{0}_{n_{cu}}$$
$$-\mathbf{L}_{q}(\mathbf{q} - \mathbf{q}_{\min}) = \mathbf{0}_{n_{q_l}}$$
$$\mathbf{U}_{q}(\mathbf{q} - \mathbf{q}_{max}) = \mathbf{0}_{n_{qu}}$$
$$\mathbf{S}_{s}\mathbf{h} - \mathbf{L}_{v}\boldsymbol{\xi}(\mathbf{q}) - \mathbf{z} + \mathbf{L}_{z_a}^{T}\boldsymbol{\chi} - \mathbf{h}^{s} = \mathbf{0}_{n_v}$$
$$-\mathbf{L}_{z_a}\mathbf{z} = \mathbf{0}_{n_{za}}$$
$$-\mathbf{c} \leq \mathbf{0}_{n_d}$$
$$\mathbf{c} - \mathbf{d} \leq \mathbf{0}_{n_d}$$
$$-\mathbf{q} + \mathbf{q}_{min} \leq \mathbf{0}_{n_p}$$
$$-\mathbf{q}_{max} + \mathbf{q} \leq \mathbf{0}_{n_p}$$
$$-\mathbf{z} \leq \mathbf{0}_{n_v}$$
$$\lambda \geq \mathbf{0}_{n_{c_l}}, \mu \geq \mathbf{0}_{n_{cu}}, \kappa \geq \mathbf{0}_{n_{q_l}}, \nu \geq \mathbf{0}_{n_{qu}}, \chi \geq \mathbf{0}_{n_{za}}$$

KKT Eqs

Active-Set Method with Newton method

$$\begin{pmatrix} \boldsymbol{F}_{q_b} & \boldsymbol{O} & -\boldsymbol{A}_{b.} & \hat{\boldsymbol{L}}_{v_b}^{T} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M}_{c_b} & -\hat{\boldsymbol{I}}_{c_b}\boldsymbol{S}_d & \boldsymbol{O} & \boldsymbol{O} \\ -\boldsymbol{A}_{b.}^{T} & -\boldsymbol{S}_d^{T}\hat{\boldsymbol{I}}_{c_b}^{T} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ -\hat{\boldsymbol{L}}_{v_b}\boldsymbol{F}_{q_b} & \boldsymbol{O} & \boldsymbol{S}_s & -\boldsymbol{I} & \boldsymbol{L}_{z_a}^{T} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{L}_{z_a} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \delta \boldsymbol{q}_b^{(m+1)} \\ \delta \boldsymbol{c}_b^{(m+1)} \\ \delta \boldsymbol{h}^{(m+1)} \\ \delta z^{(m+1)} \\ \delta \boldsymbol{\chi}_{z_a}^{(m+1)} \end{pmatrix} = - \begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \boldsymbol{r}_3 \\ \boldsymbol{r}_4 \\ \boldsymbol{r}_5 \end{pmatrix}$$

with

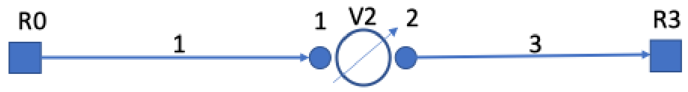
$$\begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \boldsymbol{r}_3 \\ \boldsymbol{r}_4 \\ \boldsymbol{r}_5 \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{\xi}_{q_b}^{(m)} + \boldsymbol{L}_{v_b}^{T}\boldsymbol{z}_{v_b}^{(m)} - \boldsymbol{A}_{b.}\boldsymbol{h}^{(m)} - \boldsymbol{a}_{q_b} \\ h\left(\boldsymbol{c}_b^{(m)}\right) - \boldsymbol{h}_{c_b)}^{(m)} \\ -\boldsymbol{A}^{T}\boldsymbol{q}^{(m)} - \boldsymbol{S}_d^{T}\boldsymbol{c}^{(m)} \\ \boldsymbol{S}_s\boldsymbol{h}^{(m)} - \boldsymbol{L}_v\boldsymbol{\xi}^{(m)} - \boldsymbol{z}^{(m)} + \boldsymbol{L}_{z_a}^{T}\boldsymbol{\chi}_{z_a}^{(m)} - \boldsymbol{h}^{s} \\ \boldsymbol{L}_{z_a}\boldsymbol{z}^{(m)} \end{pmatrix}$$

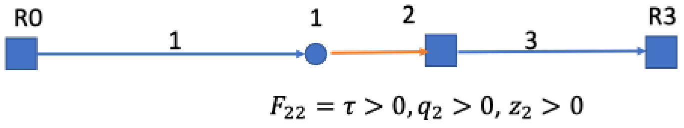
Regularization of the Schur matrix

$$\widehat{\mathbf{V}}\delta\mathbf{h}^{(\mathrm{m}+1)} = \mathbf{s}_3$$

$$\text{Where } \widehat{\mathbf{V}} = \mathbf{A}_{\mathrm{b.}}^{\mathrm{T}}\widehat{\mathbf{F}}_{\mathrm{q_b}}^{-1}\widehat{\mathbf{A}} + \hat{\mathbf{I}}_{\mathrm{c_b}}^{\mathrm{T}}\mathbf{M}_{\mathrm{c_b}}^{-1}\hat{\mathbf{I}}_{\mathrm{c_b}}$$

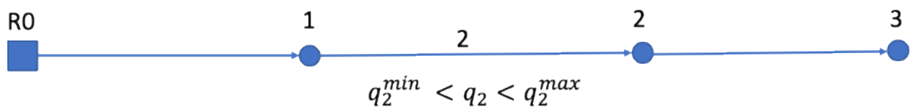
Regularizations from Deuerlein et al. (2023)

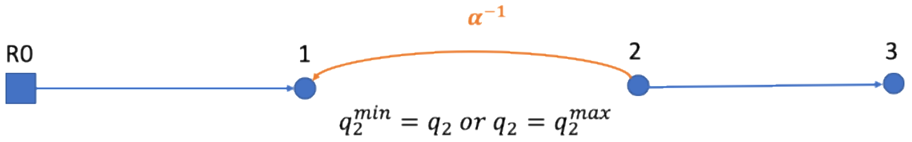




F can be singular, whenever the valve status is active (q>0 and z>0)

Or The Schur complement, $\widehat{\mathbf{V}}$, can become singular if the matrix $\widehat{\mathbf{A}}$ has less than full rank because of the particular membership of the index sets at that point





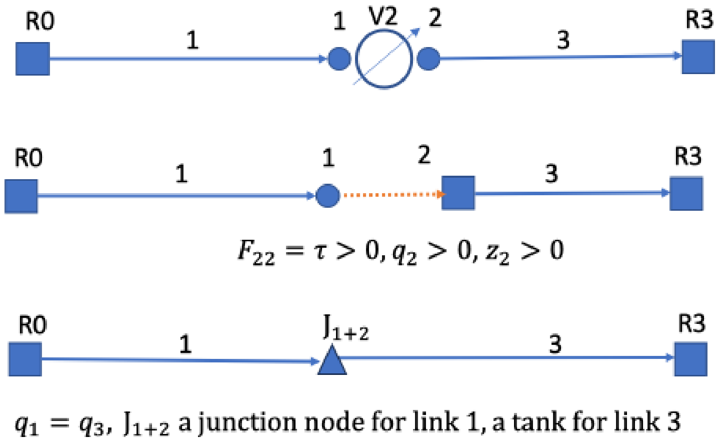
The reduced system system

- Elimination of z and χ in ASM and Newton method
- Reduction conservation energy (partitionning links)
- Conservation of mass (partitionning nodes + node merging)
- Final system Schur

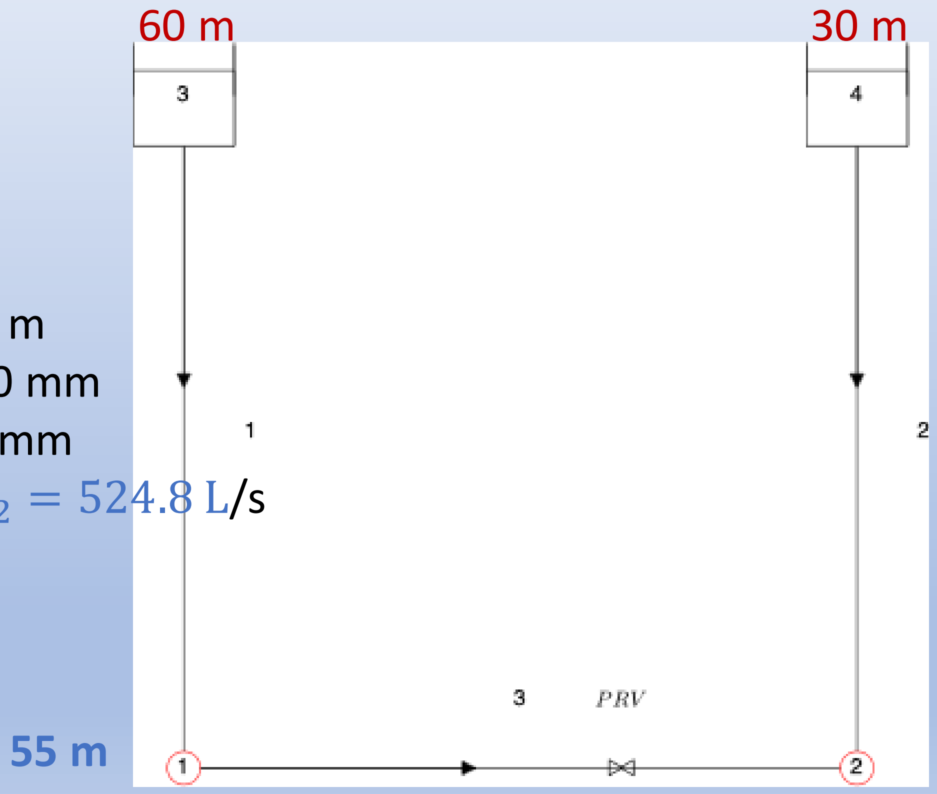
$$-\left(\overline{\boldsymbol{A}}^T_{\boldsymbol{b}.}\boldsymbol{F}^{-1}_{q_{b_r}}\widehat{\boldsymbol{A}}_{b_s} + \boldsymbol{L}_s\boldsymbol{S}^T_d\widehat{\boldsymbol{I}}^T_{c_b}\boldsymbol{M}^{-1}_{c_b}\widehat{\boldsymbol{I}}_{c_b}\boldsymbol{S}_d\boldsymbol{L}^T_s\right)\delta\boldsymbol{h}_s^{(m+1)} =$$

$$-\overline{\boldsymbol{A}}^T_{b.}\boldsymbol{F}^{-1}_{q_{b_r}}\boldsymbol{v}_1 - \boldsymbol{L}_s\boldsymbol{S}^T_d\widehat{\boldsymbol{I}}^T_{c_b}\boldsymbol{M}^{-1}_{c_b}\boldsymbol{r}_2 - \boldsymbol{L}_s\boldsymbol{r}_3$$

$$\boldsymbol{V} = \begin{pmatrix}\overline{\boldsymbol{A}}^T_{b.} & \boldsymbol{L}_s\boldsymbol{S}^T_d\hat{\boldsymbol{I}}^T_{c_b}\end{pmatrix}\begin{pmatrix}\boldsymbol{F}^{-1}_{q_{br}} & \\ & \boldsymbol{M}^{-1}_{c_b}\end{pmatrix}\begin{pmatrix}\widehat{\boldsymbol{A}}_{b.} \\ \hat{\boldsymbol{I}}_{c_b}\boldsymbol{S}_d\boldsymbol{L}^T_s\end{pmatrix}$$



An illustrative example



L = 400 m
Φ = 500 mm
k = 0.5 mm
$q_1 = q_2 = 524.8$ L/s

An active PRV example

1. a failure when no regularization is used and initial $\widehat{\mathbf{V}}$
2. a slow convergence case when the F-reg is very strict (linear CV & 15 iter.) $\tau = 2.10^0$
3. A quadratic convergence case with reduced model or small $\tau = 2.10^{-4}$

Conclusions and perspective

- **A general framework suitable for sustainable management of pressures and flow rates is presented**
  - The formulation takes into account min-max constraints for linkflows (qmin < q < qmax)) and outflows (0 < c < d)
  - No heuristics are required to determine the status of the pressure and flow control devices (Nash equilibrium + Lagrange multipliers)
- **A reduced formulation is sought to benefit from Newton's quadratic convergence**
- Future work, finding **rapid and robust algorithms**:
  - Existence & uniqueness
  - impact of graph variations during the iterations? Matrix rank or spanning trees?
  - Relaxation on inappropriate constraints with large networks and a lot of constraints

Thank you for your attention!

Any questions?

Olivier.piller@inrae.fr
deuerlein@3sconsult.de
sylvan.elhay@adelaide.edu.au
angus.simpson@adelaide.edu.au

References

- Cherry, C. (1951). "CXVII. Some general theorems for non-linear systems possessing reactance." The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 42(333), 1161-1177.
- Collins, M., Cooper, L., Helgason, R., Kennington, J., and LeBlanc, L. (1978). "Solving the Pipe Network Analysis Problem Using Optimization Techniques." Management Science, 24(7), 747-760.
- Deuerlein, J. W., Piller, O., Elhay, S., and Simpson, A. R. (2019). "Content-Based Active-Set Method for the Pressure-Dependent Model of Water Distribution Systems." Journal of Water Resources Planning and Management, 145(1), 04018082.
- Deuerlein, J. W., Elhay, S., Piller, O., and Simpson, A. R. (2023). "Modeling Flow and Pressure Control in Water Distribution Systems Using the Nash Equilibrium." Journal of Water Resources Planning and Management, 149(6), 04023019.
- Elhay, S., Piller, O., Deuerlein, J. W., and Simpson, A. R. (2022). "An Interior Point Method Applied to Flow Constraints in a Pressure-Dependent Water Distribution System." Journal of Water Resources Planning and Management, 148(1), 04021090.
- Piller, O., Elhay, S., Deuerlein, J. W., and Simpson, A. R. (2020). "A Content-Based Active-Set Method for Pressure-Dependent Models of Water Distribution Systems with Flow Controls." Journal of Water Resources Planning and Management, 146(4), 04020009.