Spatial embeddings for cattle trade networks

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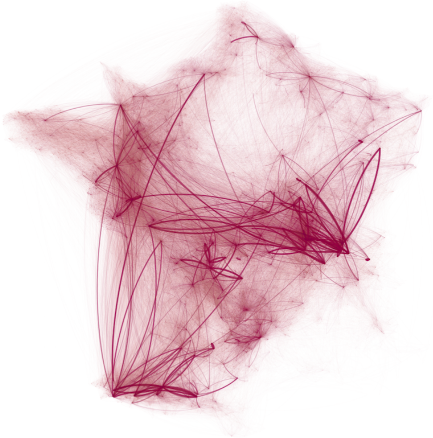
French cattle trade network

- $\sim 250,000$ nodes (holdings)
- $\sim 18,000,000$ animals at any given time
- $\sim 5,000,000$ time-stamped edges (exchanges) per year

Movements between holdings logged on a daily basis since 2005 (Base de Données Nationale d'Identification, BDNI)



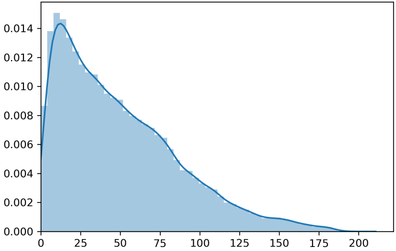
(Figures by Gaël Beaunée)

Spatial structure

Cattle trade network in Brittany, 2015 :

- 17,245 holdings spread over ~27,000 sq km
- Most movements on short scale (<100 km)
- Long edges mostly between high-volume nodes (markets/assembly centres)

Goal : formulate a generic model for cattle trade taking spatial and commercial inhomogeneity into account



Scale-free percolation

Given a set of vertices $V \subset \mathbf{R}^2$, draw ***weights*** $(W_x,\ x \in V)$ independently, according to a Pareto distribution with parameter $\tau - 1 > 0$:

$$\mathbb{P}(W \in dx) = \frac{\tau - 1}{x^\tau} dx.$$

Conditionally on the weights, connect vertices $x, y \in V$ with probability :

$$p_{x,y} = \mathbb{P}(x \leftrightarrow y) = 1 - \exp\left(-\lambda \frac{W_x W_y}{\|x - y\|^\alpha}\right).$$

- First introduced by [Deijfen et al. 2013] for $V = \mathbf{Z}^d$
- Studied by [Deprez and Wüthrich 2019] and [Dalmau and Salvi 2021] when V is Poisson-distributed on $\mathbf{R}^d$

Properties of SFP

If G is distributed as SFP(τ, λ, α) on the vertex set V :

- If $\alpha > 2$ and $\gamma = \alpha(\tau - 1)/2 > 1$, all vertices have finite degree a.s.
- G has scale-free degree distribution with exponent γ :

  $$\mathbb{P}(D_x > s) = \ell(s)s^{-\gamma},$$

  where ℓ is slowly varying at infinity.
- If $\gamma \in (1, 2)$, the graph distance between two points x, y scales like $\log\log \|x - y\|$ (***ultra-small world***)
- If $\gamma \geq 2$ and $\alpha \in (2, 4)$, graph distance scales like $\log \|x - y\|$ (***small world***)
- G has positive clustering (if $x \leftrightarrow y$ and $x \leftrightarrow z$, chances are that $y \leftrightarrow z$ also)

Simulation of SFP graphs

$$p_{x,y} = 1 - \exp\left(-\lambda \frac{W_x W_y}{\|x - y\|^\alpha}\right).$$

Naive sampling strategy inefficient for large networks :

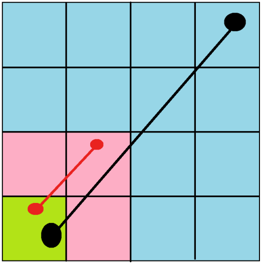
- Requires $\mathcal{O}(|V|^2)$ computations of $p_{x,y} = \mathbb{P}(x \leftrightarrow y)$
- For typical parameter values, only a small fraction of edges present (sparsity)
- Empirically, simulation time ∼20s for $|V| \sim 1000$
- Makes simulation-based inference methods impossible to use

Simulation of SFP graphs

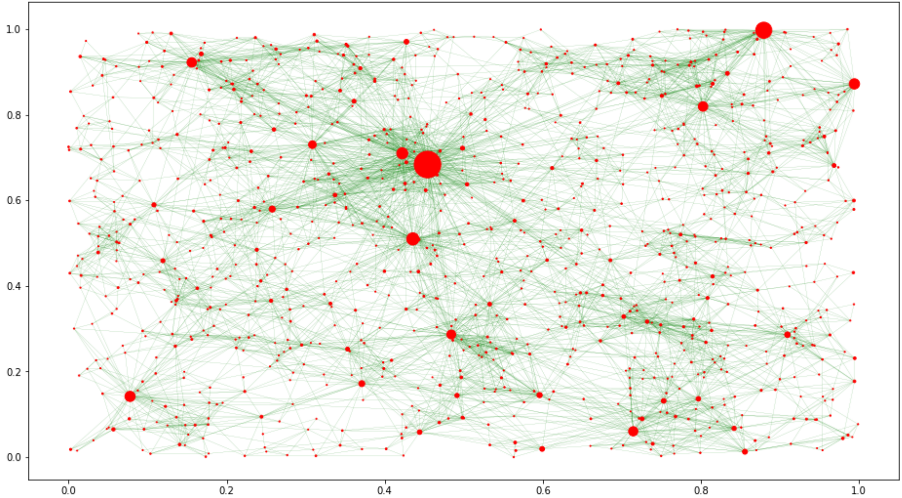
Improved simulation scheme, adapted from [Bringmann et al. 2018] :

- Decompose space into cells
- For (x, y) in adjacent cells, sample naively : $\mathcal{B}(p_{x,y})$
- For distant cells A, B, compute upper bound $\bar{p}$ for the connection probabilities $(p_{x,y},\ x \in V_A,\ y \in V_B)$
- Select potential edges with probability $\bar{p}$
- Keep each potential edge (x, y) with probability $p_{x,y} / \bar{p}$

For well-chosen cell sizes, complexity $\mathcal{O}(|V|)$



Example of a simulation

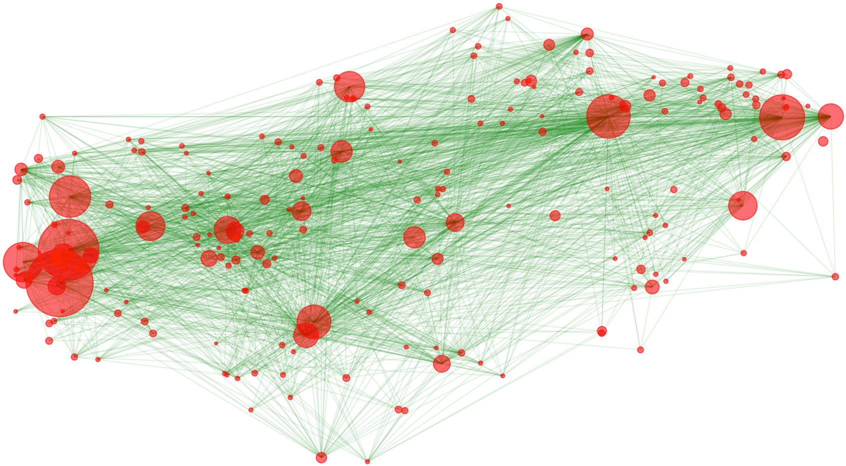


Model calibration

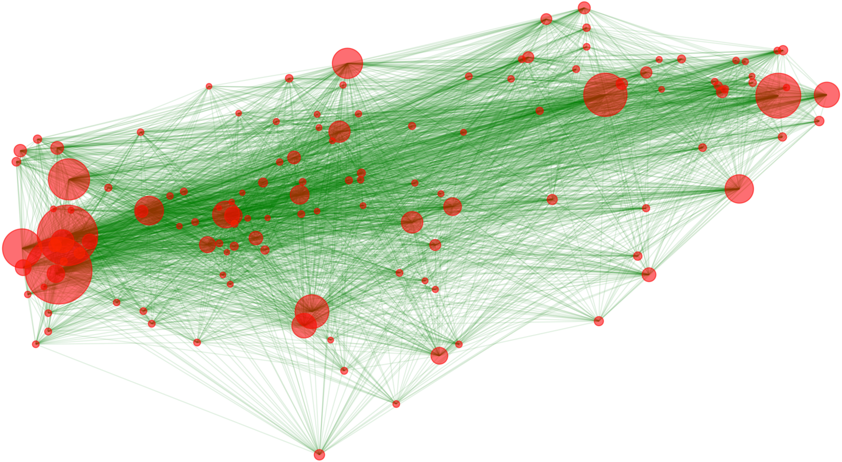
Use BDNI for node data :

- Locations (x_i, y_i) of holdings at commune level (1,208 in Brittany), randomised within commune boundaries
- Use ***trading volume*** V_i (number of animals entering and leaving a given holding over 1 year) as weights, which has power-law distribution with $\tau \sim 2.06$
- Best fit obtained for $\alpha = 1$ (away from $\alpha > 2$, $\gamma > 1$ range considered by Dalmau-Salvi)
    - Mean clustering 0.262 (true value 0.279)
    - Mean exponent of degree distribution 1.30 (true value 1.55)
    - Distance distribution qualitatively reproduced, but low variance

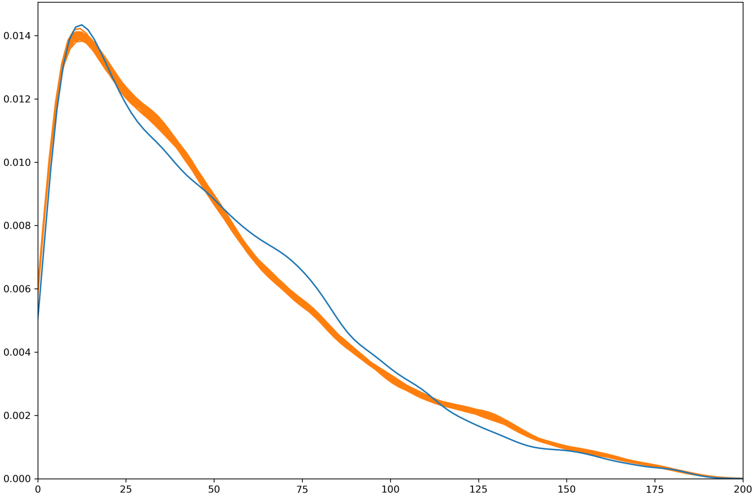
2015 observed network



Simulated network



Distance distribution



Parameter estimation

Hierarchical structure of the SFP model :

$$\mathbb{P}(W_x > s) = s^{-\tau}$$

$$\mathbb{P}(x \leftrightarrow y | W_x, W_y) = 1 - \exp\left(-\lambda \frac{W_x W_y}{\|x - y\|^{\alpha}}\right)$$

- Use HMM theory to infer latent parameter τ from observation of absence/presence of edges
- Maximum-likelihood estimation of τ, given α, λ, using SAEM-MCMC algorithm
- Estimation of α and λ computationally very intensive (use of ***minibatch*** technology [Kuhn et al. 2020])
- Joint inference out of reach at the moment

Conclusion

Scale-free percolation is a flexible model for spatially embedded trade networks :

- Model structure reflects inhomogeneity typical of these networks
- Key statistical features can be recovered
- Can be used for data analysis or forward simulation

Future work :

- Improve parameter estimation
- Incorporate high-resolution geospatial data to improve model fit at short scales
- Understand mechanistic drivers behind spatial structure
- Formulate a temporal version of the model

Thank you for your attention