Global sensitivity analysis for models described by continuous time Markov chains with application to epidemic models

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Global sensitivity analysis for models described by continuous time Markov chains with application to epidemic models

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PhD expected duration: Oct. 2019 - Sep. 2022

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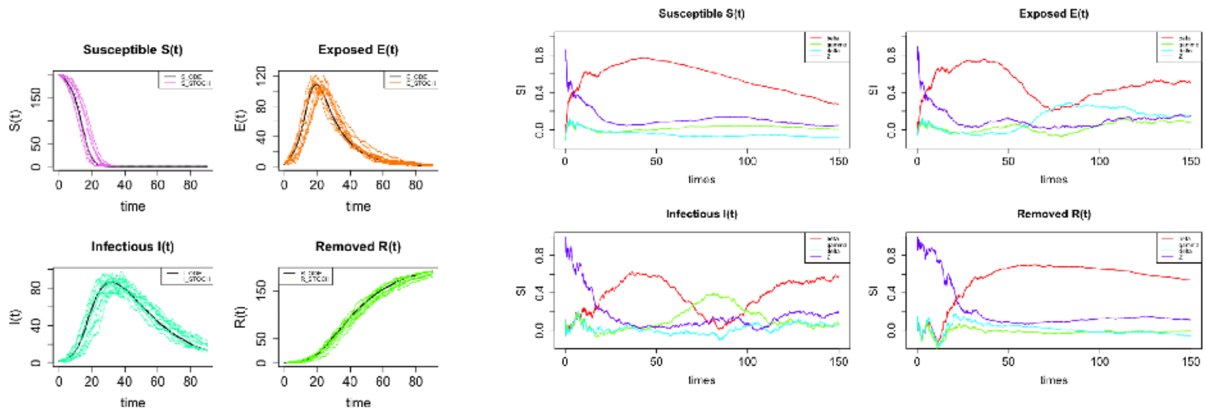
Abstract: Stochastic models are increasingly used in various fields (epidemiology, biology, physics etc) to describe different phenomena. They can provide insights into complex situations at large to small scales. Models can be described by stochastic processes, hence including an intrinsic randomness. Performing global sensitivity analysis (GSA) for such models is challenging, if the intrinsic randomness of the system is considered as a noise on specific quantities of interests (QoIs). The objective of our work is to propose a generic strategy to perform GSA on such stochastic systems. In this context, several studies have been led to point out the impact of the inputs on the stochastic QoIs. Global sensitivity analysis methods (Sobol', Polynomial chaos expansion, Karhunen-Loeve decomposition, ...) have been implemented for stochastic models with scalar QoIs [4], [7], models described by stochastic differential equations [2], Markov chain based models for chemical physics [8], etc. However, either these studies did not propose a generic framework Markov chain based models or they only deal with scalar QoIs.

In order to introduce a more generic framework, our strategy aims firstly at separating the two sources of variability, namely parameters uncertainty and intrinsic randomness and secondly at putting the model functional QoI Y under the form:

$$Y = f\left(X, Z\right), \tag{1}$$

where f is a function, X stands for the uncertain parameters and Z represents the intrinsic randomness such as X and Z are independent. For this purpose, two approaches are introduced: Sellke construction [9] and Kurtz representation [1],[5]. Depending on the class of models, Sellke construction allows to describe Z as a finite-dimensional vector with known distribution while under Kurtz representation, Z is a vector of independent Poisson processes with intensity 1. These two approaches allow not only to estimate the sensitivity indices of the input parameters but also to measure the influence of the intrinsic randomness on the global variability of the QoI under study. Furthermore, Sellke construction can be used to extend to the non-markovian framework.

From (1), we put ourselves in various frameworks of sensitivity analysis and implement different methods (global sensitivity indices [6], general metric space sensitivity indices [3], etc) by introducing the indices and their estimators and finally by carrying out the simulations. In order to illustrate the approaches we developed, they will be applied to some SIR-like models (fig. 1) which are usually used in stochastic modeling of epidemics.



(a) SEIR stochastic model outputs S,E,I,R

(b) Evolution in time for Sobol' indices of model parameters and intrinsic randomness for each model coordinate S, E, I, R

Figure 1: Example of the SEIR stochastic epidemic model

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Short biography – After a background in applied and fundamental mathematics, I obtained a master's degree in mathematics applied to life sciences at the Paris-Saclay University. I am currently pursuing a thesis thanks to an INRAE-Inria funding and my work aims at studying sensitivity analysis methods for stochastic models with functional outputs.