

Calibrating a hydrological model robustly to rain perturbations with stochastic surrogates

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Calibrating a hydrological model robustly to rain perturbations with stochastic surrogates

Katarina Radišić¹² Claire Lauvernet¹,Arthur Vidard²

¹INRAE, RiverLy, Lyon-Villeurbanne ²Univ. Grenoble-Alpes, Inria, CNRS, Grenoble-INP, LJK



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Context

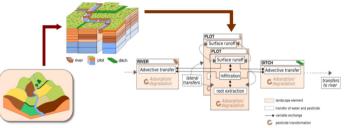


Figure: PESHMELBA^a, a process-based, spatially distributed water and pesticide transfer model, representing dynamical behavior of pesticides in agricultural catchments. Semi-conceptual, semi-physically based model, highly non-linear.

^aEmilie Rouzies et al. (June 2019). "From agricultural catchment to management scenarios: A modular tool to assess effects of landscape features on water and pesticide behavior". en. In: *Science of The Total Environment* 671, pp. 1144–1160.

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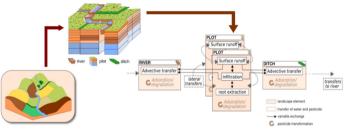


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Not all model parameters can be measured directly

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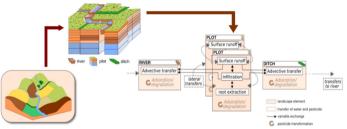


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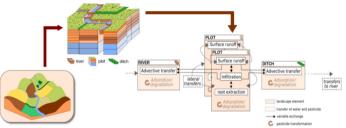


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 Impact of external uncertainties on the calibration results

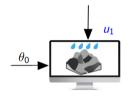






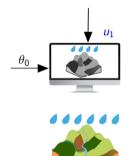
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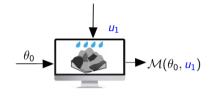






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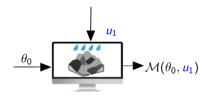


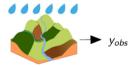




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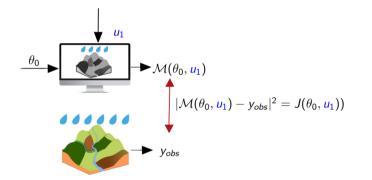






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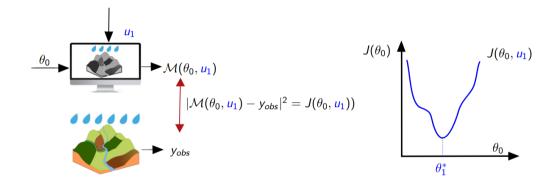




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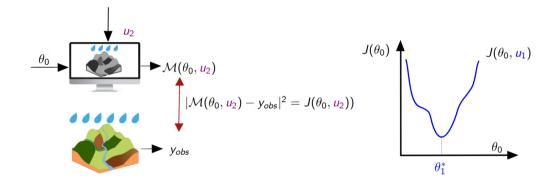




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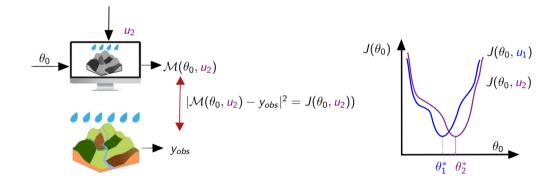




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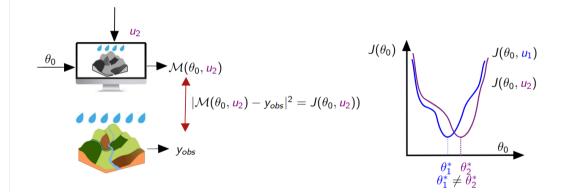
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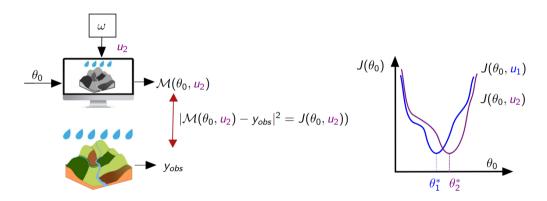




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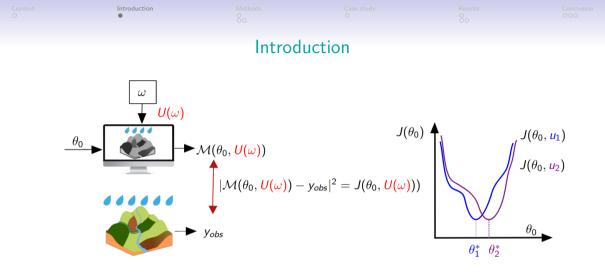
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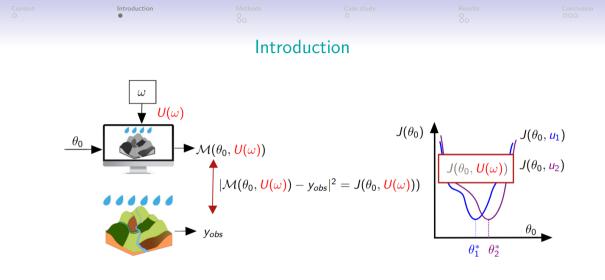


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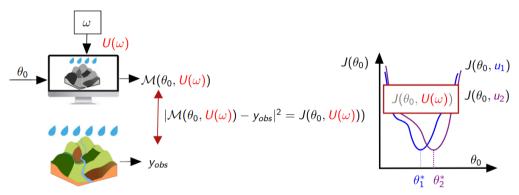


Methods

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Introduction



What does it mean to find a robust minimizer ?



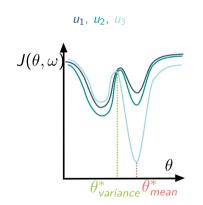
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Conclusion

Methods: Robust estimators



1.
$$\theta_{\mathbb{E}}^* = \underset{\theta}{\operatorname{argmin}} \mathbb{E}_{U}[J(\theta, U)],$$

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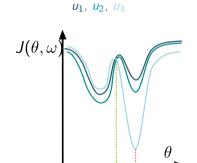
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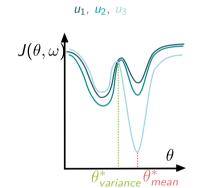
 $\theta^*_{variance} \theta^*_{mean}$



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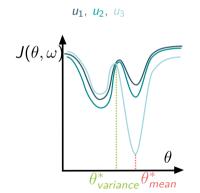
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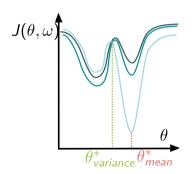


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• How to estimate the robust parameters from a limited number of model simulations?



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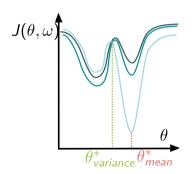


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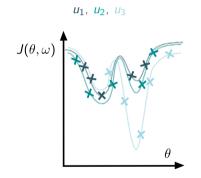
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\longrightarrow Stochastic metamodel^a

^aNora Lüthen, Stefano Marelli, and Bruno Sudret (Mar. 2023). "A spectral surrogate model for stochastic simulators computed from trajectory samples". en. In: *Computer Methods in Applied Mechanics and Engineering* 406, p. 115875.

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Methods: Stochastic metamodel

• Polynomial chaos expansion (useful for models with sparse observations)





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- $\bullet \longrightarrow$ ok, for when a rain realization is fixed





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$$J(oldsymbol{ heta}) = \sum_{lpha \in \mathbb{N}^K} oldsymbol{c}_lpha \psi_lpha(oldsymbol{ heta}) pprox \sum_{lpha \in \mathcal{A}_q^{K,
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- PCE on each rain realization, then Principal component analysis on the PCE coefficients
- representation of the impact of the variability of the rain on the cost function through a random variable in the latent space

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- PCE on each rain realization, then Principal component analysis on the PCE coefficients
- representation of the impact of the variability of the rain on the cost function through a random variable in the latent space
- ullet the mean and variance are analytically available and independent of the distribution of Z

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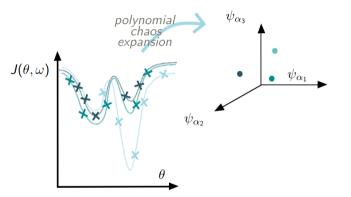
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Methods: Stochastic metamodel



$$\hat{J}(\boldsymbol{\theta}, U(\omega)) = \hat{\mu}(\boldsymbol{\theta}) + \sum_{k=1}^{K} \sqrt{\lambda_k} Z_k(\omega) (\sum_{\alpha \in \mathcal{A}} b_{\alpha}^{(k)} \psi_{\alpha}(\boldsymbol{\theta}))$$

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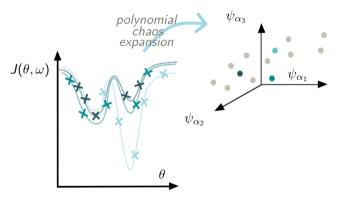
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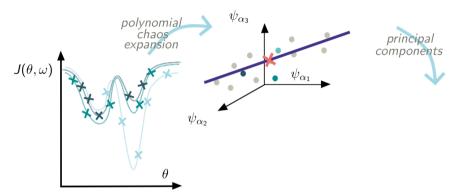
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Methods: Stochastic metamodel



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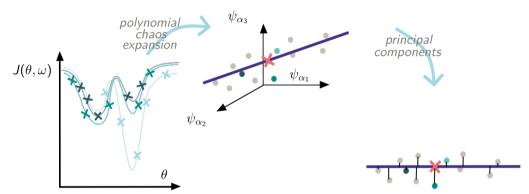
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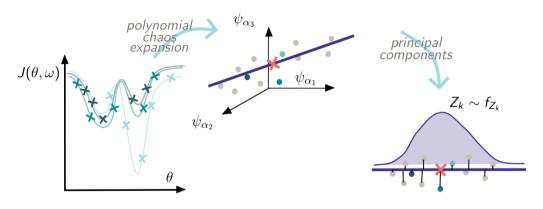
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Methods: Stochastic metamodel



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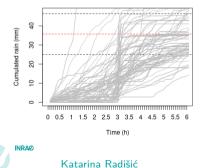
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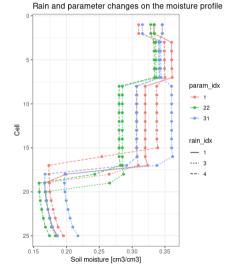
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$$egin{aligned} J(oldsymbol{ heta},u) &= (\mathcal{M}(oldsymbol{ heta},u) - y_{true})^2 \ oldsymbol{ heta} &\in \mathbb{R}^6 \ y_{true} \in \mathbb{R}^{25} \end{aligned}$$



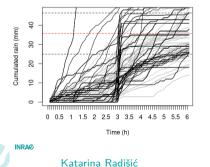


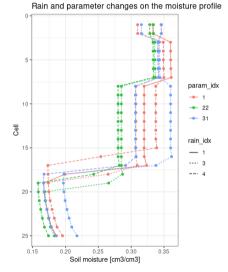




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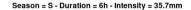


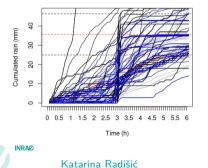


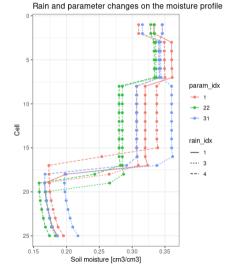




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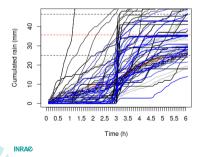






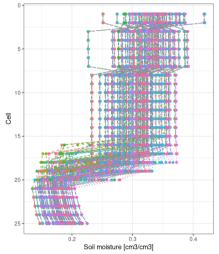
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Rain and parameter changes on the moisture profile



Introdi

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Results: Conditional minimization

- PCE constructed conditionally to each train rain realization.
- Determination coefficients $R^2 > 0.95$
- Minimization BFGS conditionally to each rain realization.

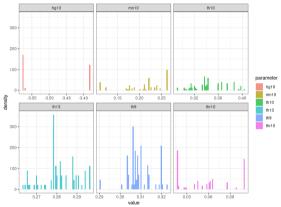


Figure: Histogram of conditional minimizers to each train rain realization.

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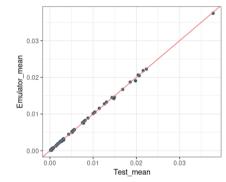


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Results: Validation of the emulator



 $\hat{J}(\boldsymbol{\theta}, U(\omega)) = \hat{\mu}(\boldsymbol{\theta}) + \sum_{k=1}^{K} \sqrt{\lambda_k} Z_k(\omega) (\sum_{\alpha \in \mathcal{A}} b_{\alpha}^{(k)} \psi_{\alpha}(\boldsymbol{\theta}))$

• The emulator mean is a good estimator of the true mean.

Figure: Comparison of the train and the test means $\mathbb{E}_U[J(\theta, U)]$.

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Results: Validation of the emulator

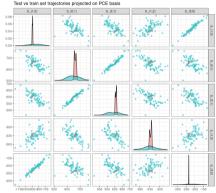


Figure: Comparison of the **train** and the **test** trajectories of PESHMELBA simulations, projected on the PCE basis.

$$(m{ heta},U(\omega))=\hat{\mu}(m{ heta})+\sum_{k=1}^{K}\sqrt{\lambda_k}Z_k(\omega)(\sum_{lpha\in\mathcal{A}}b^{(k)}_{lpha}\psi_{lpha}(m{ heta}))$$

- The emulator mean is a good estimator of the true mean.
- The train set and the test set do not present the same variabilities, thus the emulator does not reproduce correctly the impact of rain perturbations on the cost function.

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Results: Validation of the emulator

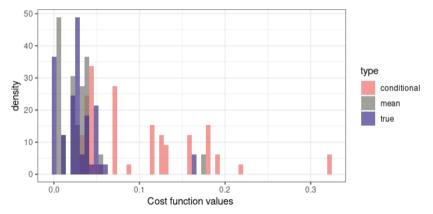


Figure: Cost function evaluated at the test rains, the simulations performed with the minimizer of the mean have a lower mean and variance than the one obtained with the minimizer conditioned on one rain realization.

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Conclusion

- The PCE approximations on each conditioned rains have a determination coefficient $R^2 > 0.95$ and the approximation of the metamodel mean is good.
- The minimizer of the mean has lower cost function on unseen rains than a randomly chosen conditional minimizer.
- The variance of the metamodel is not a good estimate What next ?
 - $\rightarrow\,$ work on the estimation of the variance
 - \rightarrow study the Pareto-optimal minimizers
 - $\rightarrow\,$ augment rain perturbations
 - $\rightarrow\,$ minimize mean and variance with Gaussian Process based Efficient Global Optimization, compare the precision and number of model evaluations needed.
 - $\rightarrow\,$ observe another output, for example pesticide quantities in the river, interaction will be added.
 - $\rightarrow\,$ up the domain to the catchment scale

INRAØ

Katarina Radišić

Context O			Conclusion ○●○

Thank you for your attention !



Methods 00 Case study O Resul

Conclusion

