Mathematical modeling and control of nematode impact on banana production

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1. Context

Banana: major staple food in the tropics

Burrowing nematodes (*Radopholus similis*)

- 60% of global crop losses
- Obligate root endoparasites < 1mm
- Life cycle: 20-25 days

Control

- Chemical nematicides
- Soil sanitation & vitroplants
- Tolerant or resistant banana varieties
- Biostimulants (to enhance plant defense)

A: Jesus, Sustain Dev 2014; B: M. MacClure, Univ. Arizona; C: Zhang, EJPP 2012

D. Coyne

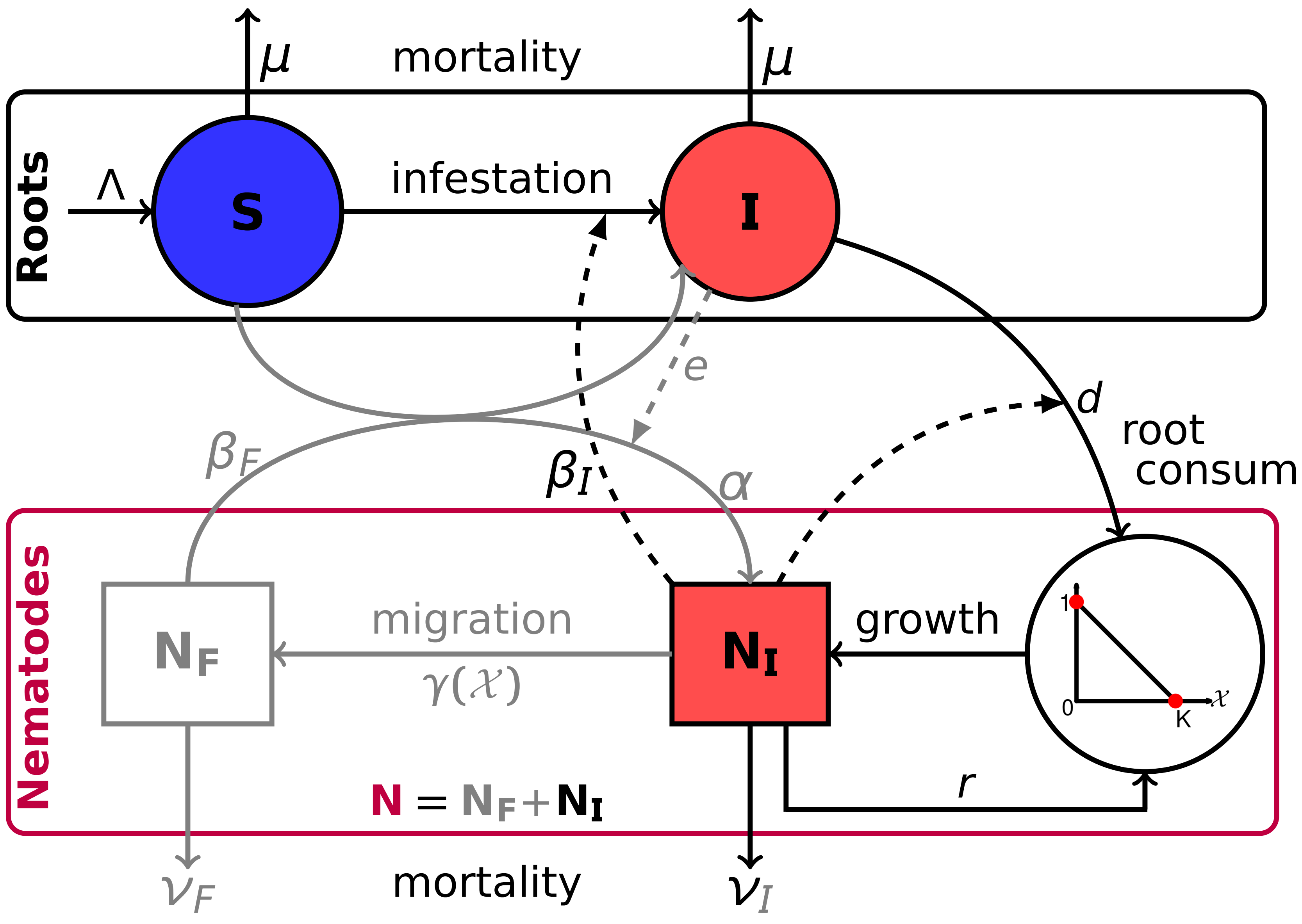
2. Research questions

- ▶ **How do nematodes affect banana root growth?**
- ▶ **How to control nematodes to optimize profit?**

Approach

- Epidemiological model
- Optimal control theory

3. Epidemiological model



State variables

S: healthy roots
I: infected roots
N_F: free nematodes
N_I: infesting nematodes

Originality: $\mathcal{X} = \frac{N_I}{I}$
"variable density"

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - (\beta_F N_F + (1-u)\beta_I N_I)S - \mu S \\ \dfrac{dI}{dt} = (\beta_F N_F + (1-u)\beta_I N_I)S - \mu I - dN_I\dfrac{I}{a+I} \\ \dfrac{dN_F}{dt} = -\alpha\beta_F(S+eI)N_F + \left(\gamma + \gamma\dfrac{N_I}{KI}\right)N_I - \nu_F N_F \\ \dfrac{dN_I}{dt} = \alpha\beta_F(S+eI)N_F - \left(\gamma + \gamma\dfrac{N_I}{KI}\right)N_I + \left(r + \rho d\dfrac{I}{a+I}\right)N_I\left(1-\dfrac{N_I}{KI}\right) - \nu_I N_I \end{cases}$$

$u \equiv$ control

Full model
⇕
Reduced model
(Tikhonov's theorem)

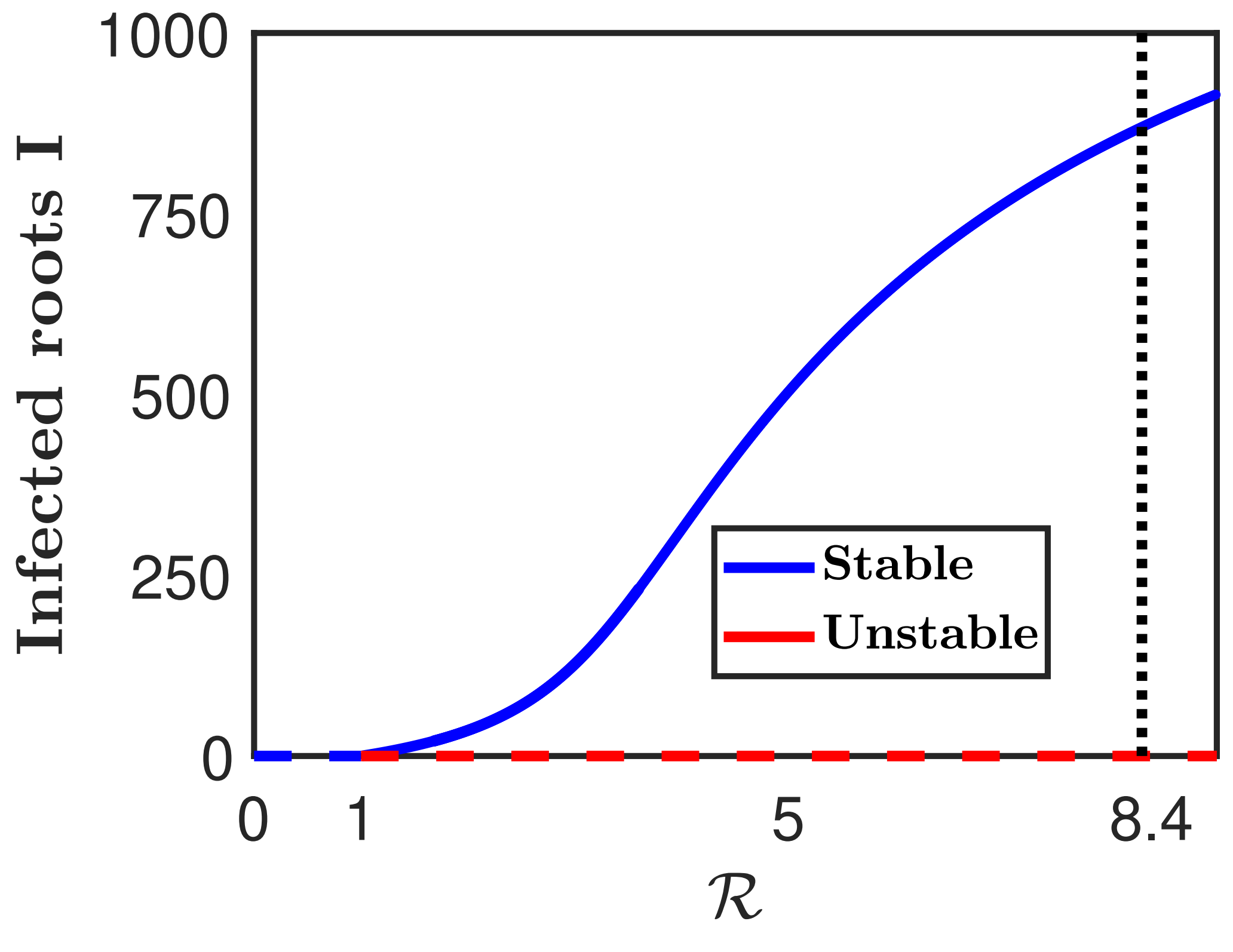
4. Equilibria and stability

Reproduction number:

$$\mathcal{R} = \frac{\Lambda\beta K}{\mu^2}\left(1 - \frac{\nu_I}{r}\right)$$

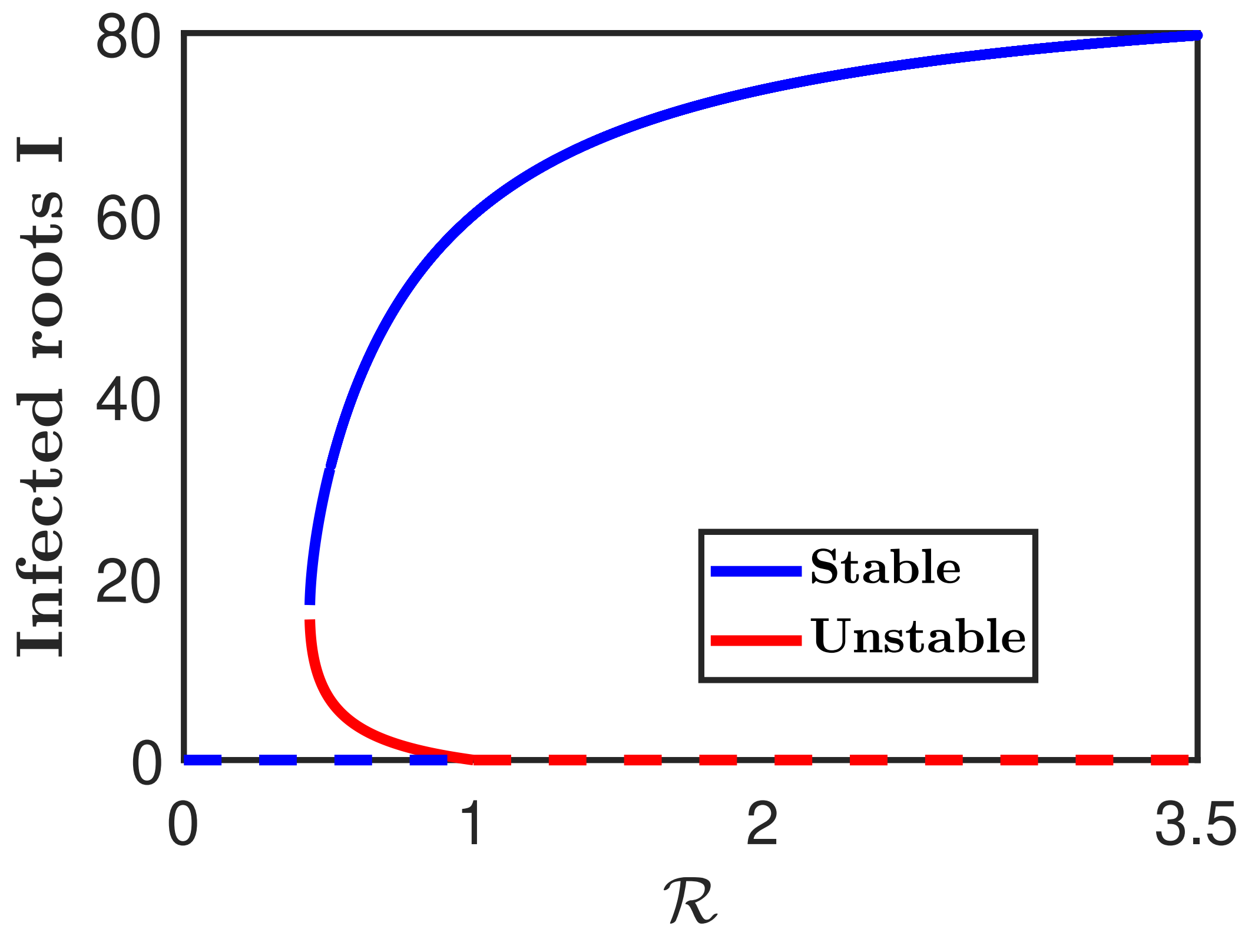
Case 1: forward bifurcation

$\mathcal{R} > 1 \Rightarrow$ existence and stability of a unique endemic equilibrium
(+ unstable pest-free situation)



Case 2: backward bifurcation

$\mathcal{R} < 1$ does not ensure pest eradication

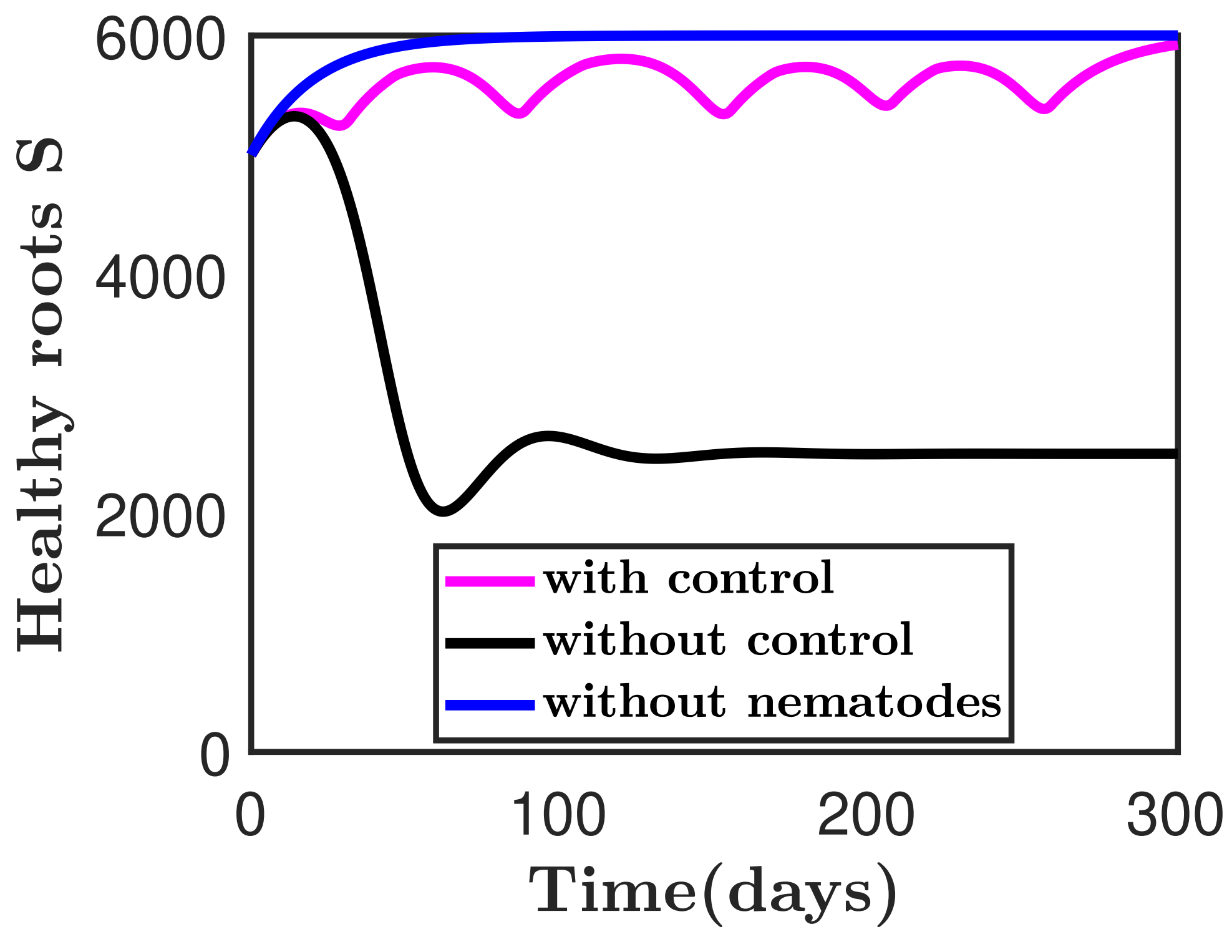


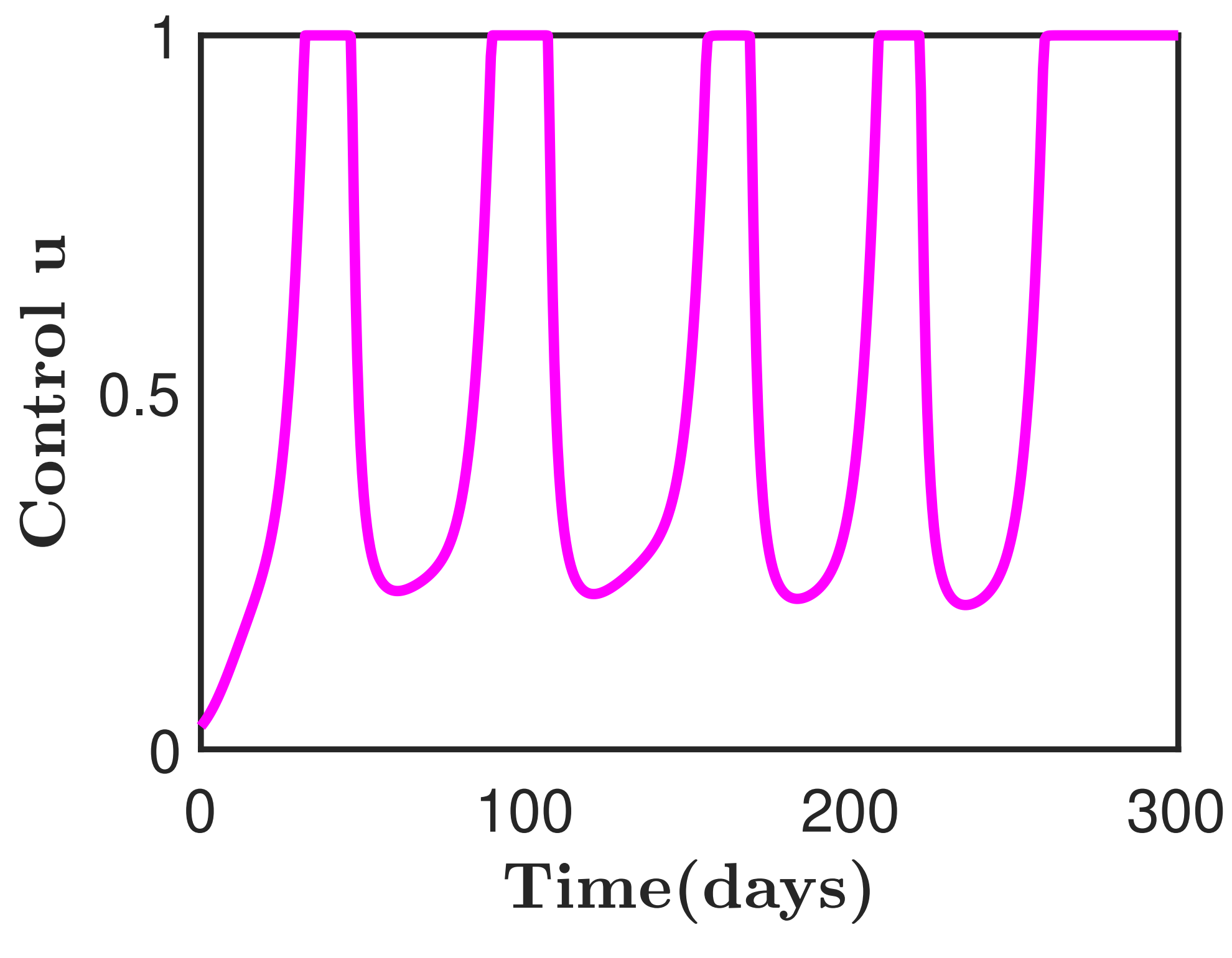
5. Optimal control

Problem: maximize profit (yield − control costs) while minimizing infestation at the end of the cropping season, to ensure reasonable yield for the next season

$$\min_u \mathcal{J}(u) = \int_0^{t_f} (\underbrace{u^2(t)}_{\text{costs}} - \underbrace{B_1 S(t)}_{\text{yield}})\,dt + \underbrace{B_2 I(t_f)}_{\text{penalty}}$$

→ Bang(1)-singular-bang(1) "oscillating" optimal control (forward-backward sweep)





→ Yield (proxy: $\int_t S$) with optimal control almost at pest-free level