Review of reduced-order models for online protection of water distribution networks

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Abstract: This paper presents a review of reduced-order models (ROMs) and digital twins, with a primary focus on their application to water distribution networks (WDNs). Initially, we concentrate on the physical modeling of WDNs. Following this, we introduce relevant programming, specifically addressing WDNs and solving equations for extended-period simulations. The paper then explores various ROM methods outlined in existing literature. Lastly, we highlight recent initiatives in the implementation of digital twins and approaches aimed at reducing uncertainty.

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Keywords: Reduced-order model (ROM); Digital twin; Water distribution network.

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1. Introduction

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The protection of WDNs is becoming increasingly important due to climate change and its impacts, such as water scarcity. Consequently, WDN modeling has become a critical issue to protect them against threats and detect anomalies in real time. Given the complexity of common urban water networks, it is costly to use hydraulic solvers to address optimization problems. To overcome this challenge, we are seeking to develop reduced models and integrate them with real-time network data in order to enhance the security and resilience of water infrastructure.

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In this review article, initially, we describe the physical modeling of WDNs and introduce some hydraulic simulation software's, and then we present some ROM methods. Finally, we highlight recent efforts to deploy digital twins.

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2. Mathematical modeling

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The hydraulic model under interest in this review is based on the Saint-Venant equations, assuming that the flow is one-dimensional and incompressible. This model is made of two equations: the first expresses the conservation of mass, the second the conservation of momentum. The flow is incompressible, as we are assuming a rigid water column, with negligible pressure and volume variations. The system can be written in the following form:

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$$
\begin{cases}
\dfrac{\partial q}{\partial x} = 0 \\
\dfrac{\partial q}{\partial t} + gA\,\dfrac{\partial h}{\partial x} = -gA\,\xi_x(q)
\end{cases}
\tag{1}
$$

Where h: piezometric head, q: flow rate, A: cross-sectional area of the pipe, g: gravity acceleration, c: pressure wave velocity, $\xi_x(q)$: the head loss function.

3. Tools

- PORTEAU: It is a modeling software developed by the French National Research Institute for Agriculture, Food and the Environment (INRAE) for managing, sizing WDNs and simulating the behavior of pressurized water distribution or transportation networks [1]. PORTEAU is a robust piece of software, although it lacks real-time engine functionality.

- EPANET: It is another hydraulic simulation software developed by the US Environmental Protection Agency (EPA). An extension of EPANET called EPANET RTX, includes real-time analysis to the modeling of WDNs [2].

Given Python's top position in the PYPL rankings, a Python interface "OOPNET" [3] has been developed to read, manipulate and simulate EPANET input files. The modular nature of OOPNET's object-oriented programming means that users can incorporate new features, test and evaluate them without having to rewrite all the code, making integration and development easier.

4. Methods

4.1. WDN simplification and model reduction:

This approach aims to reduce the dimension of a system of nonlinear equations describing WDNs. This can be summarized in the following steps: Formulate the complete nonlinear model, linearize this model, reduce the linear model using the Gauss elimination procedure, and retrieve a reduced nonlinear model from the reduced linear model. An extension of this algorithm has been proposed to incorporate energy aspects and to fulfil the requirements of the online optimisation strategy for energy management and leakage control in WDNs [4].

4.2. Graph decomposition

- FCPA (Forest-Core Partitioning Algorithm): This method shows that the linear (treed) components of the system is associated to the graph's forest and the non-linear (looped) portion to the core. Indeed, it isolates the forest from the core facilitating thereby the separate resolution of linear and non-linear components of the problem using appropriate methods [5].

- GMPA (Graph Matrix Partitioning Algorithm): This approach is based on FCPA, where it initially separates the forest from the core. Following this, it reduces the dimension of the core by identifying supernodes and superlinks. Finally, the resolution of non-linear components is carried out on this subgraph, which exclusively contains the supernodes and superlinks [6]. **Figure 1** shows the decomposition of the graph using the GMPA algorithm and the interface provided by OOPNET.
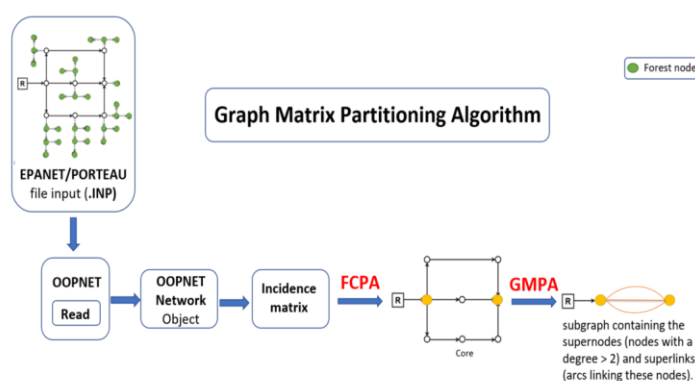
Figure 1. This illustration shows the implementation of the GMPA algorithm [6] through the OOPNET interface. This interface reads an .INP file and converts it into an object that can be manipulated by the Python libraries.

- Simplified model for pressure control: The objective of this approach is to establish a reduced model for estimating pressure measurements in a multiple-input WDN, which is divided into two sets of vertices, one for the inputs and the other for the other vertices. This method is based on input pressure adjustment for a target node pressure value according to total network demand [7].

4.3. Complex system analysis and metamodels

- Proper Orthogonal Decomposition methods (POD): Braun [8] proposed a first application of the projection-based POD method to the hydraulic equations of a WDN, and showed that POD is efficient because it preserves the physical information of the hydraulic equations through the Galerkin projection. In addition, the possible dimensional reduction in the degrees of freedom of the systems results in an increase in performance for solving the non-linear set of equations. However, the gain in execution time is small, as the matrices concerned lose their sparsity.

- Graph neural network metamodel: Machine and deep learning approaches to estimate water network behavior are based on the collection of data that will be processed using techniques such as auto-encoders to extract the most important data, thereby significantly reducing the dimensionality of the system. Zanfei et al [9] proposed a metamodel based on a graphical learning approach to calculate pressure and flow and to support the calibration of the water network model, which directly deals with uncertainty by transferring it from the available data to the metamodel.

- Complex Network Analysis approach (CNA): is one of the methods used for order reduction based on identifying parts of the network that have less impact on overall performance. By analysing some network properties, we can identify pipes or nodes that can be aggregated or neglected without compromising the accuracy of the model. Therefore, CNA can also be applied as an optimisation procedure for real and very large WDS with a reduced computational effort compared to evolutionary optimisation methods [10].

4. Digital Twin

The need for protection, risk assessment and remote control has given rise to a great deal of research, which has led to the digital representation of systems such as WDNs, providing a detailed view of the situation, using measurements and observations from

equipment including sensors, controllers and actuators that are reported and assimilated in real time in order to observe and control pressures, flows and water quality; predict hydraulic behaviour by simulating scenarios; and optimise the operations. As with any numerical application, digital twins can involve uncertainties, whether related to the input data or to the results of the calculations. To validate them, it is essential to quantify these uncertainties and analyse their propagation.

5. Conclusion

The idea behind this review is to develop a digital twin of WDN, built on a reduced-order model, which is connected to online data, and with a feedback loop to control and adapt the system for sustainable and resilient decision-making. These approaches are designed to reduce both the computational time and cost required.

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