A multiparameter model for local filtrate flux and solids concentration distribution in cross-flow membrane filtration of colloidal suspensions

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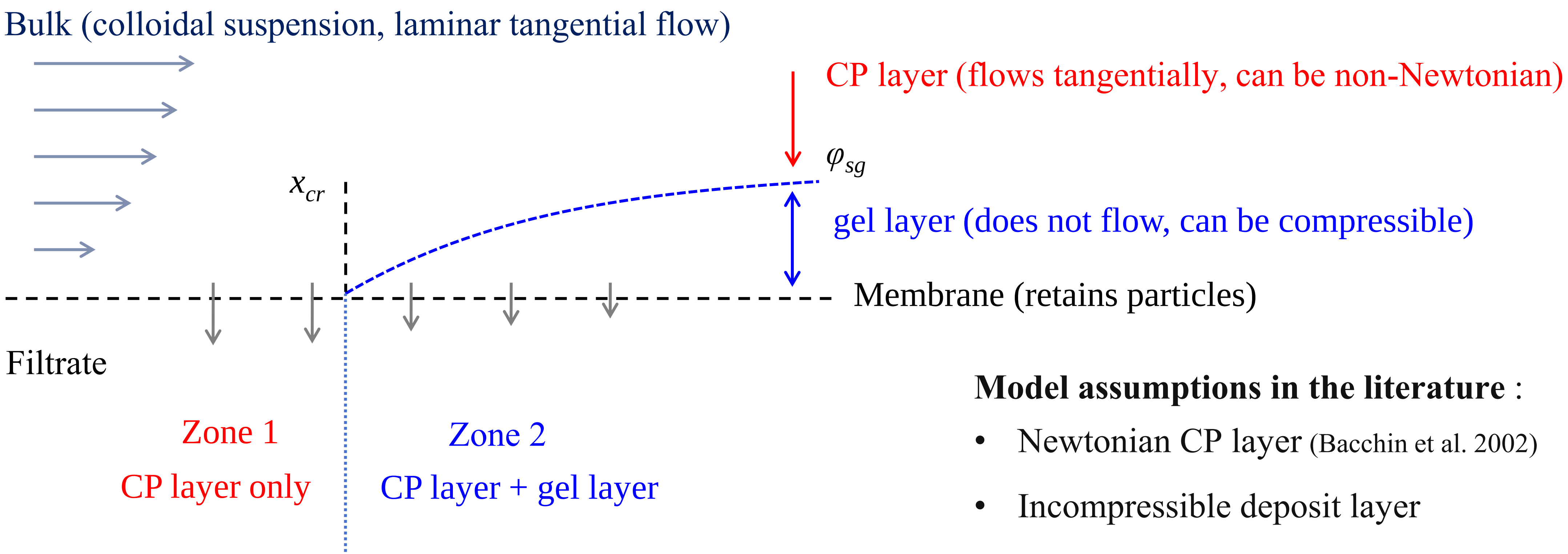
Model



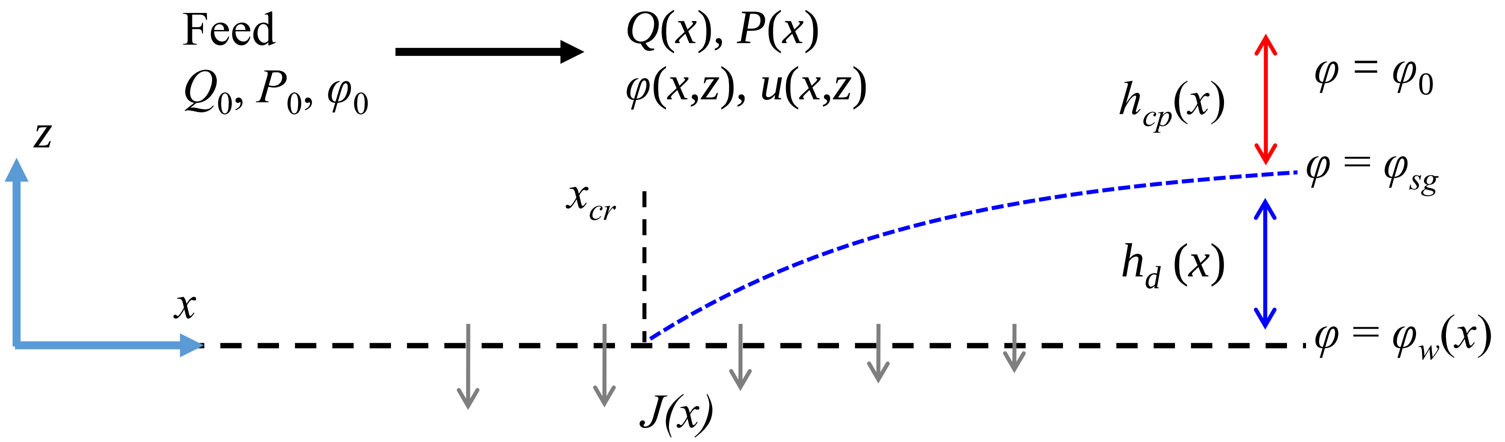
Model assumptions in the literature :

- Newtonian CP layer (Bacchin et al. 2002)
- Incompressible deposit layer
- Concentration-independent permeability

Casein micelles (CM): concentration-dependent permeability and compressibility, complex rheological properties (Bouchoux et al. (2014))

Objective: to extend the model of (Bacchin et al. 2002) in order to account material properties and to find out how these properties impact the filtration kinetics, *J(x)*.

System of equations



Eq. for flow in the CP layer under the applied shear stress **Eq.I**

$$u = f(\dot{\gamma}, \tau)$$

Darcy Eq. for filtrate flow across the CP layer, the deposit, and the membrane **Eq.II**

$$\mu_f J(x) = -\,k(x,z)\frac{\mathrm{d}\pi(x,z)}{\mathrm{d}z}$$

Mass balance Eq. that relates filtrate flux with solids flux in the CP layer **Eq.III**

$$Q_0\varphi_0 = 2\pi R \int_{h_d(x)}^{h_d(x)+h_{cp}(x)} u(x,z)\,(\varphi(x,z) - \varphi_0)\mathrm{d}z + Q(x)\varphi_0$$

h_{CP}	thickness of CP layer	u	cross flow velocity	0	in feed
h_d	thickness of deposit layer	x	distance along the membrane	sg	in point of sol-gel transition
P	pressure	z	distance from the membrane	w	on the membrane surface
Q	volumetric cross flow rate	φ	particle volume fraction		

Main model equations

From Eq. III and Eq. I:

Eq. IV

$$\frac{d\varphi_w}{dx} = \frac{\frac{\varphi_0}{\mu_f R_m^3}\left[P_0 - \frac{2\tau}{R}x - \pi(\varphi_w)\right]^4 - \frac{4\tau}{R}M(\varphi_w,\tau)}{\left[P_0 - \frac{2\tau}{R}x - \pi(\varphi_w)\right]\frac{dM(\varphi_w,\tau)}{d\varphi_w} + 2M(\varphi_w,\tau)\frac{d\pi(\varphi_w)}{d\varphi_w}}$$

Where:

$$M(\varphi_w,\tau) = \int_{\varphi_0}^{\varphi_w}(\varphi-\varphi_0)k(\varphi)\frac{d\pi(\varphi)}{d\varphi}\left[\int_{\varphi}^{\varphi_w}\dot{\gamma}(\varphi,\tau)k(\varphi)\frac{d\pi(\varphi)}{d\varphi}d\varphi\right]d\varphi$$

NEW

Modified Darcy Eq. from Eq. II:

Eq.V

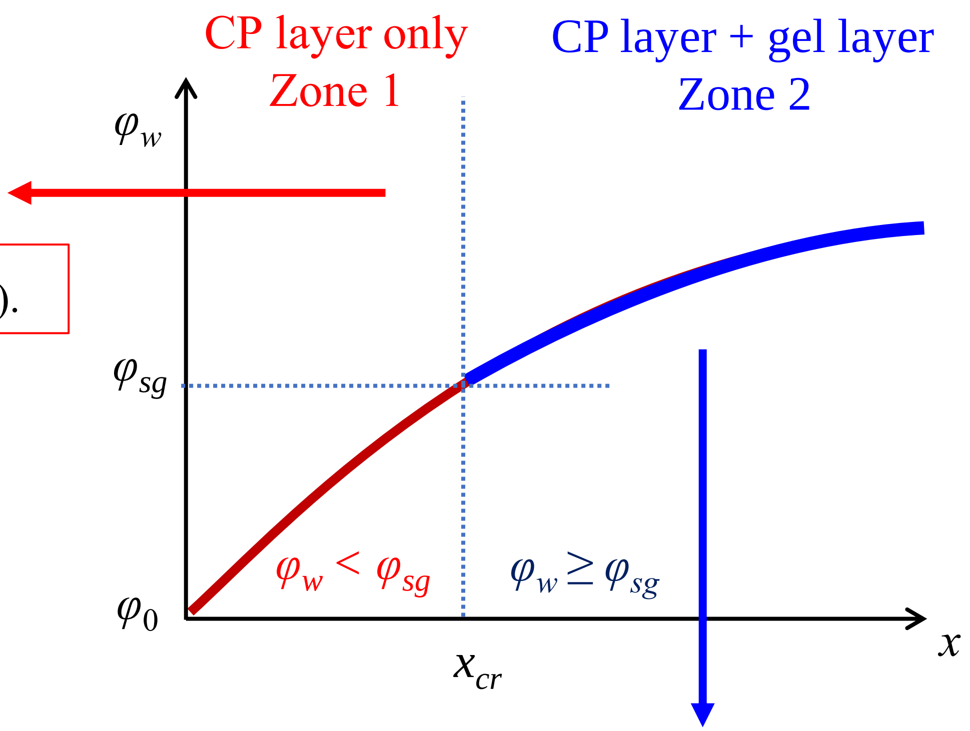
$$J(x) = \frac{P(x) - \pi(\varphi_w(x))}{\mu_f R_m}$$ Where: $P(x) = P_0 - \frac{2\tau}{R}x$

Once M is known, we could obtain:
- $\varphi_w(x)$ by Eq. IV
- $J(x)$ by Eq. V

$\varphi_w(x)$	**local particle concentration on membrane wall**
φ_0	particle concentration in bulk
τ	shear stress
R	radius of membrane
P_0	pressure at the entrance to filter channel
M	**Filterability (function of material properties of filtered material and of wall shear stress)**
$\dot{\gamma}$	shear rate
k	permeability of concentrated particle
π	osmotic pressure (compressibility) of particles
μ_f	permeate viscosity
R_m	membrane resistance
$J(x)$	**local filtrate flux**
$P(x)$	local pressure

Flux calculations for 2 zones

$$\varphi_w < \varphi_{sg} \;\rightarrow\; M(\varphi_w,\tau) = \int_{\varphi_0}^{\varphi_w} (\varphi-\varphi_0)k(\varphi)\frac{d\pi(\varphi)}{d\varphi}\left[\int_{\varphi}^{\varphi_w} \dot{\gamma}(\varphi,\tau)k(\varphi)\frac{d\pi(\varphi)}{d\varphi}d\varphi\right]d\varphi$$



$M(\varphi_w,\tau)$ is the material properties-dependent ($\dot{\gamma}$, k and π) function of concentration (φ).

$$J(x) = \frac{P(x) - \pi(\varphi_w(x))}{\mu_f R_m}$$ Eq.V

$$\varphi_w \geq \varphi_{sg} \;\rightarrow\; \dot{\gamma}=0 \;\rightarrow\; M(\varphi_{sg},\tau) = \int_{\varphi_0}^{\varphi_{sg}} (\varphi-\varphi_0)\, k(\varphi) \frac{\partial\pi(\varphi)}{\partial\varphi}\left[\int_{\varphi_0}^{\varphi_{sg}} \dot{\gamma}(\varphi,\tau)\, k(\varphi)\frac{\partial\pi(\varphi)}{\partial\varphi} d\varphi\right] d\varphi \;\rightarrow$$

$M(\varphi_{sg},\tau)$ would be a constant.

$$J(x) = \left[J^{-3}(x_{cr1}) + \frac{3\,\varphi_0\,\mu_f^2}{2\,M(\varphi_{sg},\tau)}(x - x_{cr1})\right]^{-\frac{1}{3}}$$ Eq.VI

M is independent of gel properties ($\dot{\gamma}$, k and π). ⇨ Gel properties do not impact the filtration kinetics ($J(x)$).

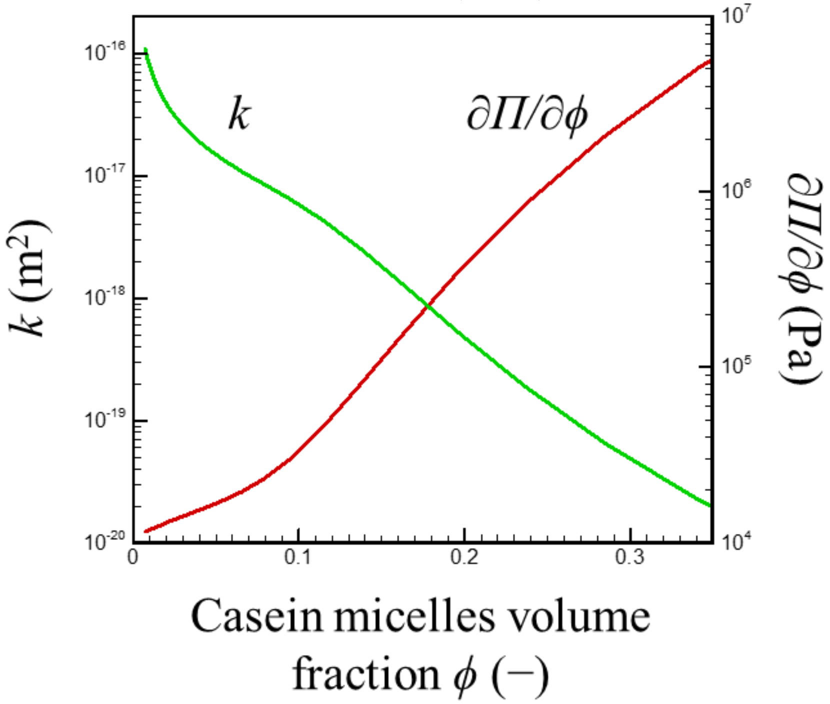
Illustration with the case of casein micelles filtration : definition of Function *M*

M= function of
- $\dot{\gamma}= f(\varphi,\tau)$ Shear rate at given shear stress
- $k= f(\varphi)$ Permeability
- $\pi= f(\varphi)$ Osmotic pressure

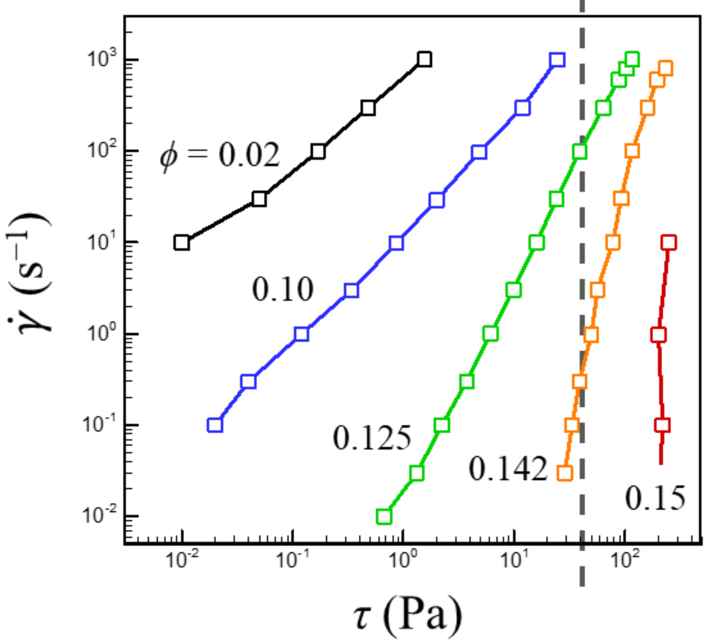
Literature data for aqueous casein micelles dispersions:

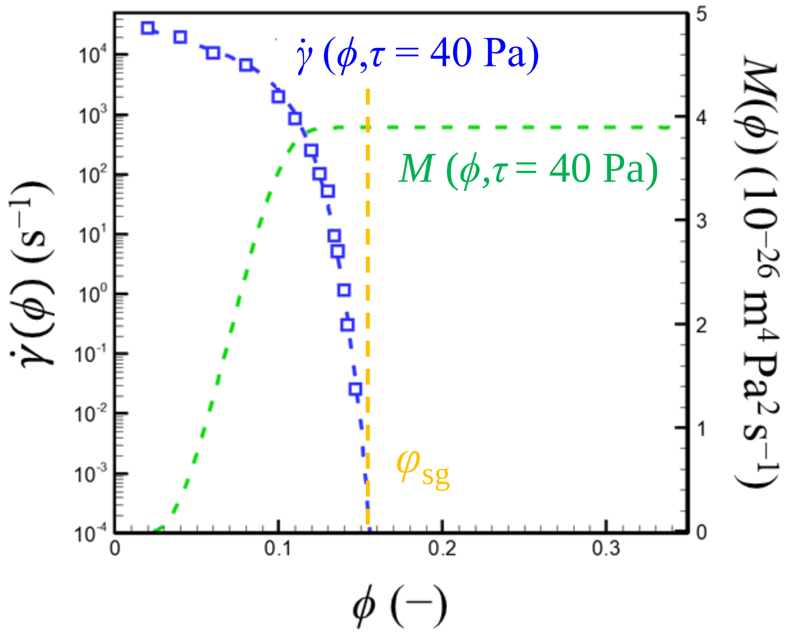
$\dot{\gamma}$ and M (for $\tau_{example}$ = 40 Pa) :

Bouchoux et al. (2014)



Jin et al. (2014)





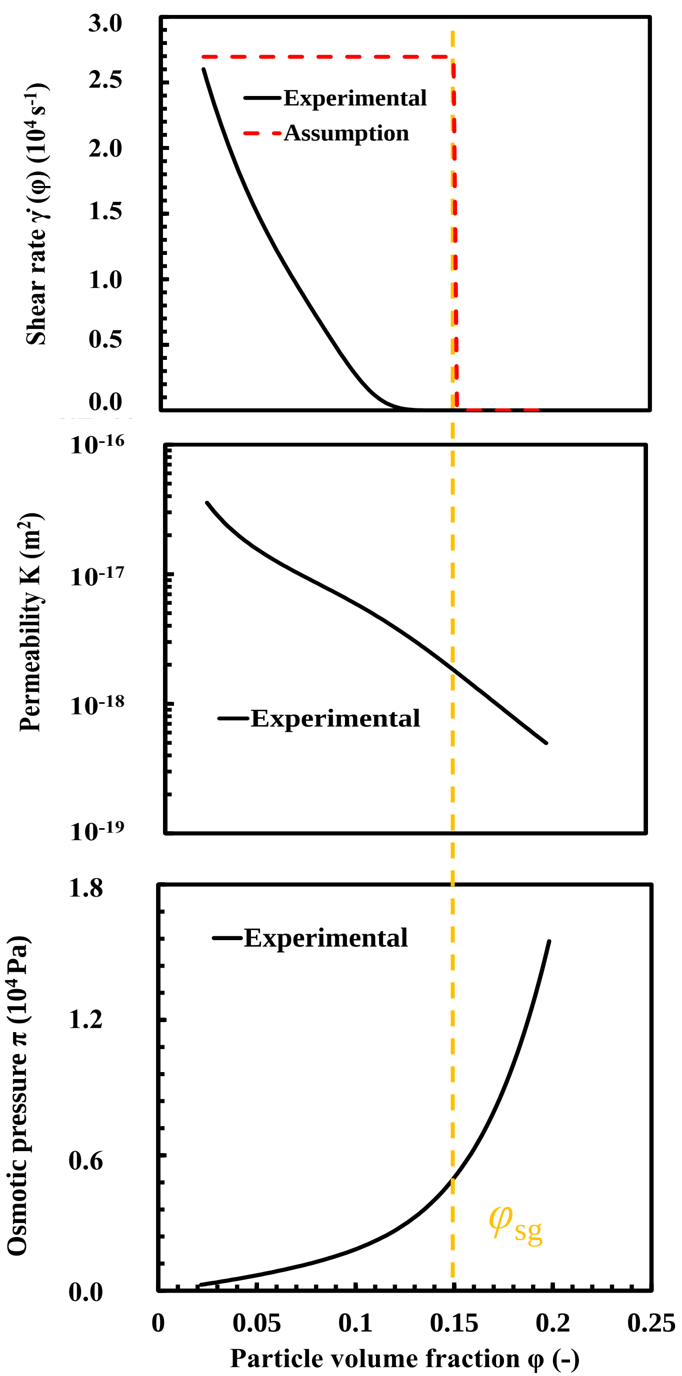
Assumption 1:

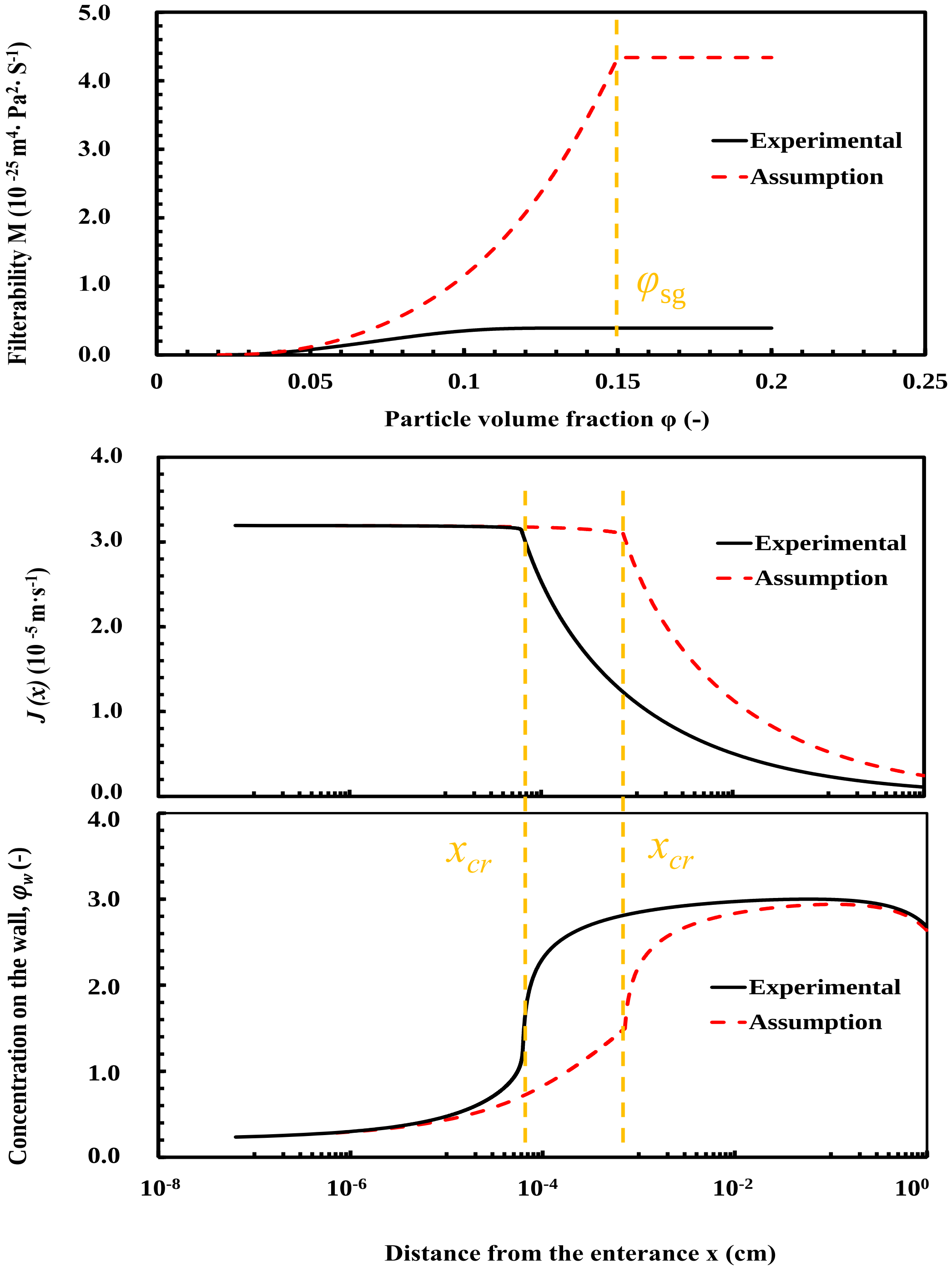
$\dot{\gamma} = f(\varphi_0 = \text{const})$

$k = f(\varphi(x))$

$\pi = f(\varphi(x))$

Concentration dependency of Rheological behaviour of CM is crucial for correct modeling of $\varphi_w(x)$ and $J(x)$.





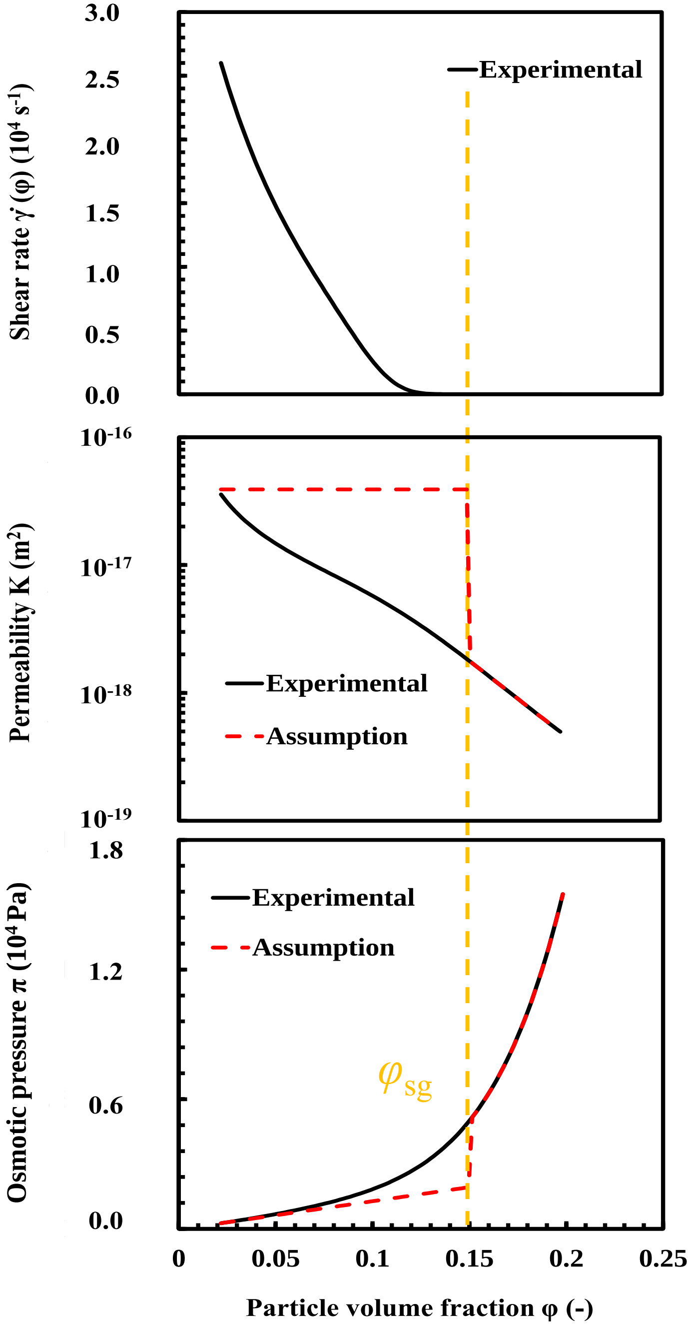
Assumption 2:

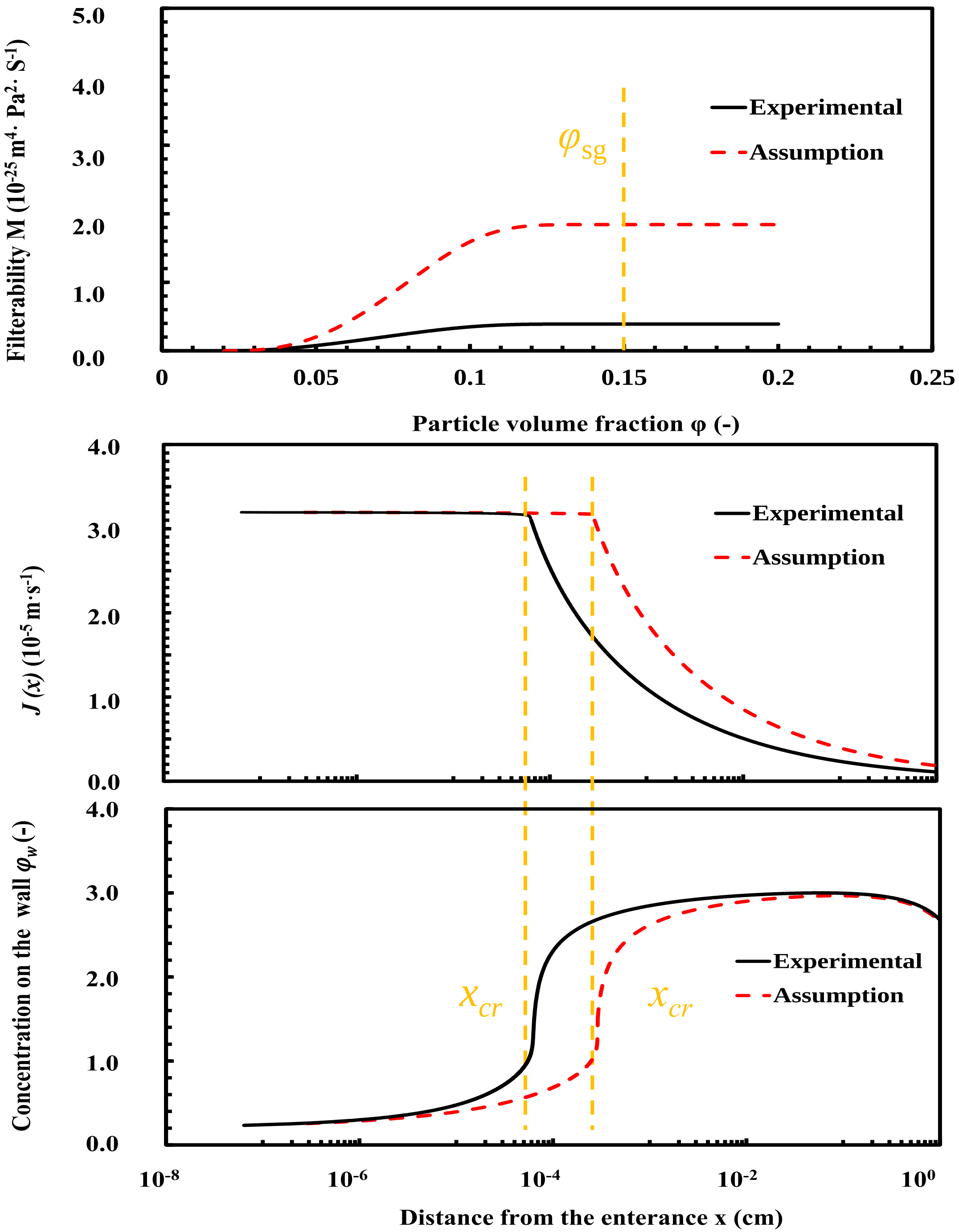
$\dot{\gamma} = f(\varphi(x))$

$k = f(\varphi_0 = \text{const})$

$\pi = f(\varphi_0 = \text{const})$

Concentration dependency of permeability (k) and compressibility (π) of CM is crucial for correct modeling of $\varphi_w(x)$ and $J(x)$.





Summary

Model advantages:

* Local compressibility and permeability of CP layer may vary with local particle concentration.

(one can input any dependency of permeability and compressibility on particle concentration)

* Model accounts for non-Newtonian nature of CP layer.

(one can input any dependency of Rheological behaviour on particle concentration and shear stress)

* Model is **simple.**

(only one differential equation is solved, numerically)

Conclusion:

* CP layer properties (compressibility, fluidity and permeability) defines filtration kinetics, *J(x)*, even in zone 2.

* Deposit layer properties (compressibility, fluidity and permeability) do not impact filtration kinetics, *J(x)*, even in zone 2.

Perspectives

* extend for solutes transmission (e.g. partial rejection of solutes by deposit in two-component suspension)
* extend for turbulent cross-flow

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