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Hossein Gholamian, Maksym Loginov, Geneviève Gésan-Guiziou. A multiparameter model for local filtrate flux and solids concentration distribution in cross-flow membrane filtration of colloidal suspensions. Euromembrane 2024, Sep 2024, Prague, Czech Republic. hal-04694170

HAL Id: hal-04694170

<https://hal.inrae.fr/hal-04694170v1>

Submitted on 11 Sep 2024

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A multiparameter model for local filtrate flux and solids concentration distribution in cross-flow membrane filtration of colloidal suspensions



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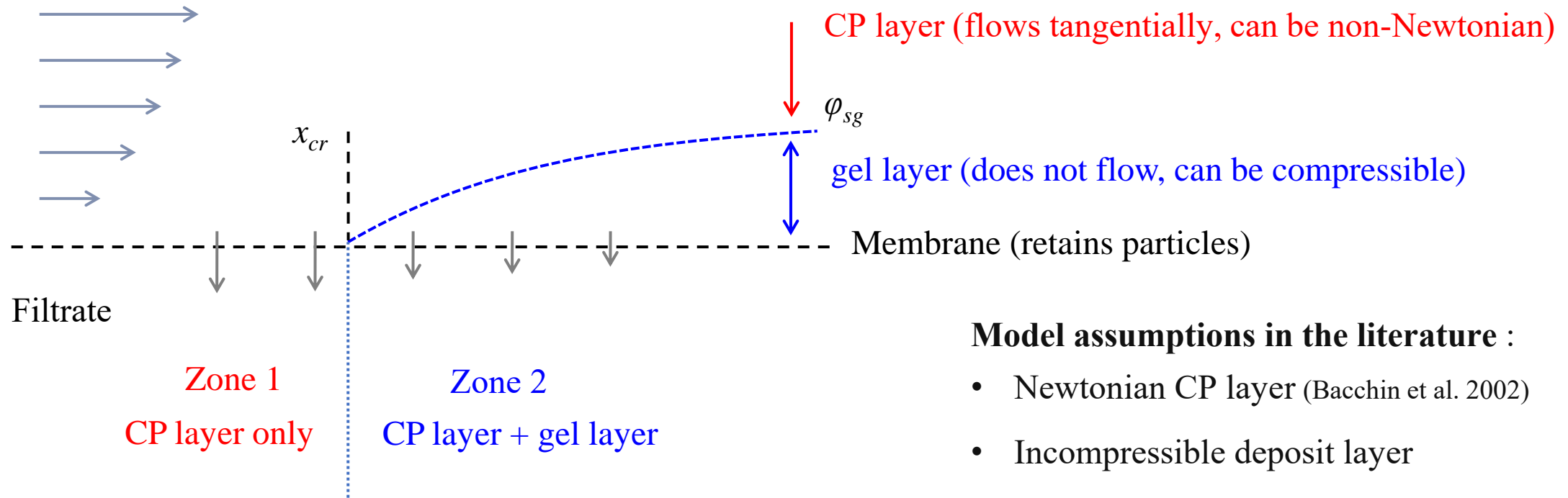
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Model

Bulk (colloidal suspension, laminar tangential flow)



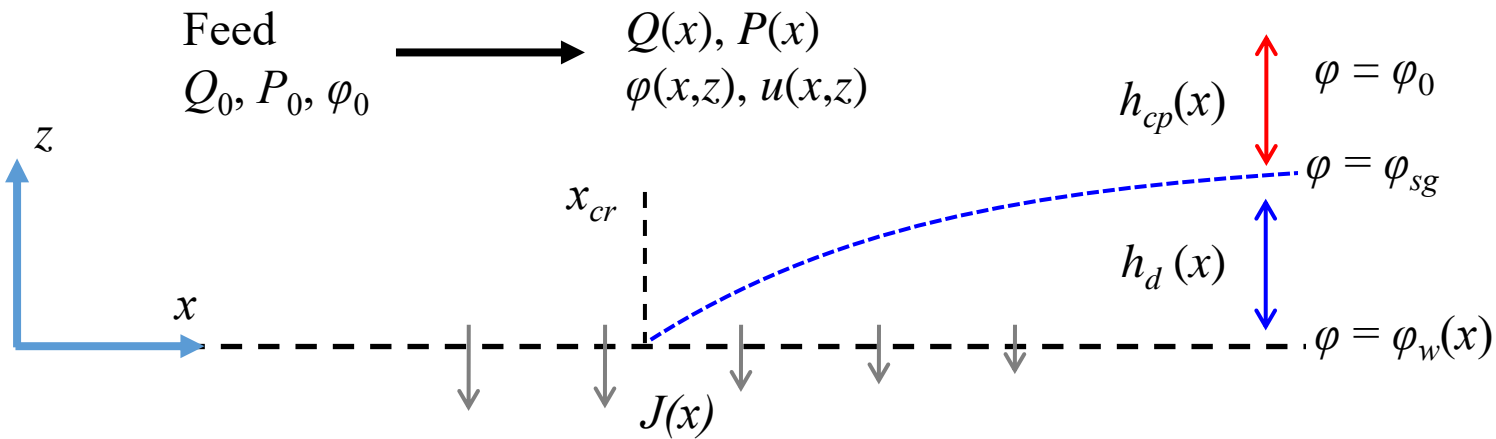
Model assumptions in the literature :

- Newtonian CP layer (Bacchin et al. 2002)
- Incompressible deposit layer
- Concentration-independent permeability

Casein micelles (CM): concentration-dependent permeability and compressibility, complex rheological properties (Bouchoux et al. (2014))

Objective: to extend the model of (Bacchin et al. 2002) in order to account material properties and to find out how these properties impact the filtration kinetics, $J(x)$.

System of equations



Eq. for flow in the CP layer under the applied shear stress

$$u = f(\dot{\gamma}, \tau)$$

Darcy Eq. for filtrate flow across the CP layer, the deposit, and the membrane

$$\mu_f J(x) = -k(x,z) \frac{d\pi(x,z)}{dz}$$

Mass balance Eq. that relates filtrate flux with solids flux in the CP layer

$$Q_0 \phi_0 = 2\pi R \int_{h_d(x)}^{h_d(x) + h_{cp}(x)} u(x,z) (\phi(x,z) - \phi_0) dz + Q(x) \phi_0$$

- | | | | |
|----------|----------------------------|--------|-----------------------------|
| h_{CP} | thickness of CP layer | u | cross flow velocity |
| h_d | thickness of deposit layer | x | distance along the membrane |
| P | pressure | z | distance from the membrane |
| Q | volumetric cross flow rate | ϕ | particle volume fraction |

- | | |
|------|--------------------------------|
| 0 | in feed |
| sg | in point of sol-gel transition |
| w | on the membrane surface |

Main model equations

From Eq. III and Eq. I:

$$\frac{d\varphi_w}{dx} = \frac{\frac{\varphi_0}{\mu_f R_m^3} \left[P_0 - \frac{2\tau}{R} x - \pi(\varphi_w) \right]^4 - \frac{4\tau}{R} M(\varphi_w, \tau)}{\left[P_0 - \frac{2\tau}{R} x - \pi(\varphi_w) \right] \frac{dM(\varphi_w, \tau)}{d\varphi_w} + 2M(\varphi_w, \tau) \frac{d\pi(\varphi_w)}{d\varphi_w}} \quad \text{Eq. IV}$$

Where:

$$M(\varphi_w, \tau) = \int_{\varphi_0}^{\varphi_w} (\varphi - \varphi_0) k(\varphi) \frac{d\pi(\varphi)}{d\varphi} \left[\int_{\varphi}^{\varphi_w} \dot{\gamma}(\varphi, \tau) k(\varphi) \frac{d\pi(\varphi)}{d\varphi} d\varphi \right] d\varphi$$

NEW

Modified Darcy Eq. from Eq. II:

$$\text{Eq.V} \quad J(x) = \frac{P(x) - \pi(\varphi_w(x))}{\mu_f R_m} \quad \text{Where: } P(x) = P_0 - \frac{2\tau}{R} x$$

Once M is known, we could obtain:

- $\varphi_w(x)$ by Eq. IV
- $J(x)$ by Eq. V

$\varphi_w(x)$ **local particle concentration on membrane wall**
 φ_0 particle concentration in bulk
 τ shear stress
 R radius of membrane
 P_0 pressure at the entrance to filter channel
 M **Filterability (function of material properties of filtered material and of wall shear stress)**

$\dot{\gamma}$ shear rate
 k permeability of concentrated particle
 π osmotic pressure (compressibility) of particles

μ_f permeate viscosity
 R_m membrane resistance

$J(x)$ **local filtrate flux**
 $P(x)$ local pressure

Flux calculations for 2 zones

$$\varphi_w < \varphi_{sg} \rightarrow M(\varphi_w, \tau) = \int_{\varphi_0}^{\varphi_w} (\varphi - \varphi_0) k(\varphi) \frac{d\pi(\varphi)}{d\varphi} \left[\int_{\varphi}^{\varphi_w} \dot{\gamma}(\varphi, \tau) k(\varphi) \frac{d\pi(\varphi)}{d\varphi} d\varphi \right] d\varphi$$

$M(\varphi_w, \tau)$ is the material properties-dependent ($\dot{\gamma}$, k and π) function of concentration (φ).

$$J(x) = \frac{P(x) - \pi(\varphi_w(x))}{\mu_f R_m}$$

Eq.V

$$\varphi_w \geq \varphi_{sg} \rightarrow \dot{\gamma} = 0 \rightarrow M(\varphi_{sg}, \tau) = \int_{\varphi_0}^{\varphi_{sg}} (\varphi - \varphi_0) k(\varphi) \frac{\partial \pi(\varphi)}{\partial \varphi} \left[\int_{\varphi_0}^{\varphi_{sg}} \dot{\gamma}(\varphi, \tau) k(\varphi) \frac{\partial \pi(\varphi)}{\partial \varphi} d\varphi \right] d\varphi \rightarrow M(\varphi_{sg}, \tau) \text{ would be a constant.}$$

$$J(x) = \left[J^3(x_{cr1}) + \frac{3 \varphi_0 \mu_f^2}{2 M(\varphi_{sg}, \tau)} (x - x_{cr1}) \right]^{-\frac{1}{3}} \quad \text{Eq.VI}$$

M is independent of gel properties ($\dot{\gamma}$, k and π).



Gel properties do not impact the filtration kinetics ($J(x)$).

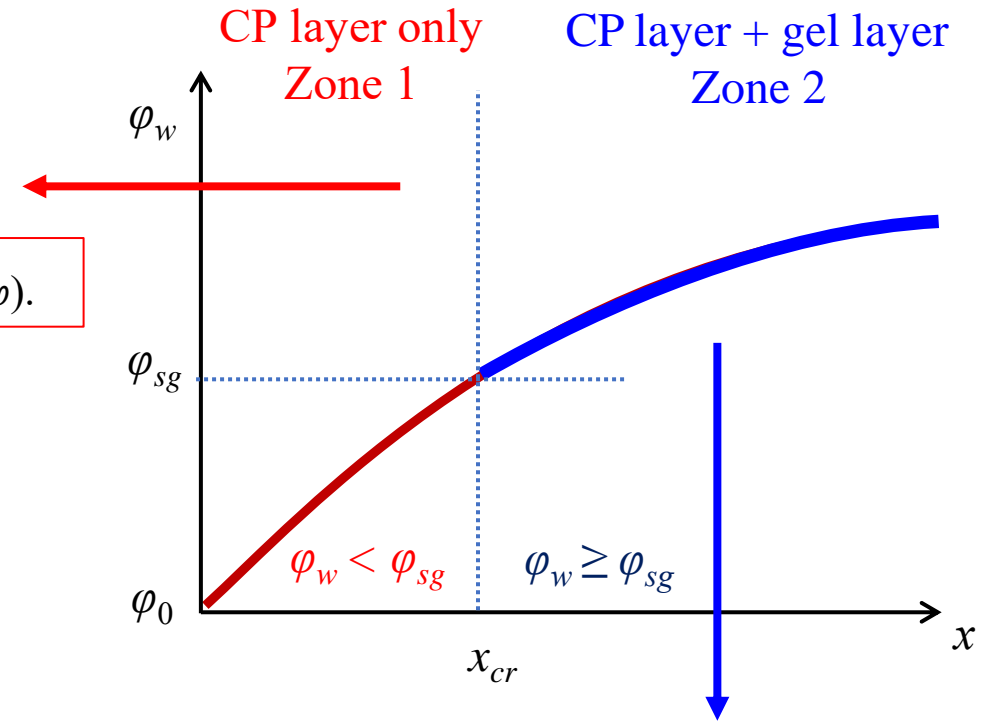


Illustration with the case of casein micelles filtration : definition of Function M

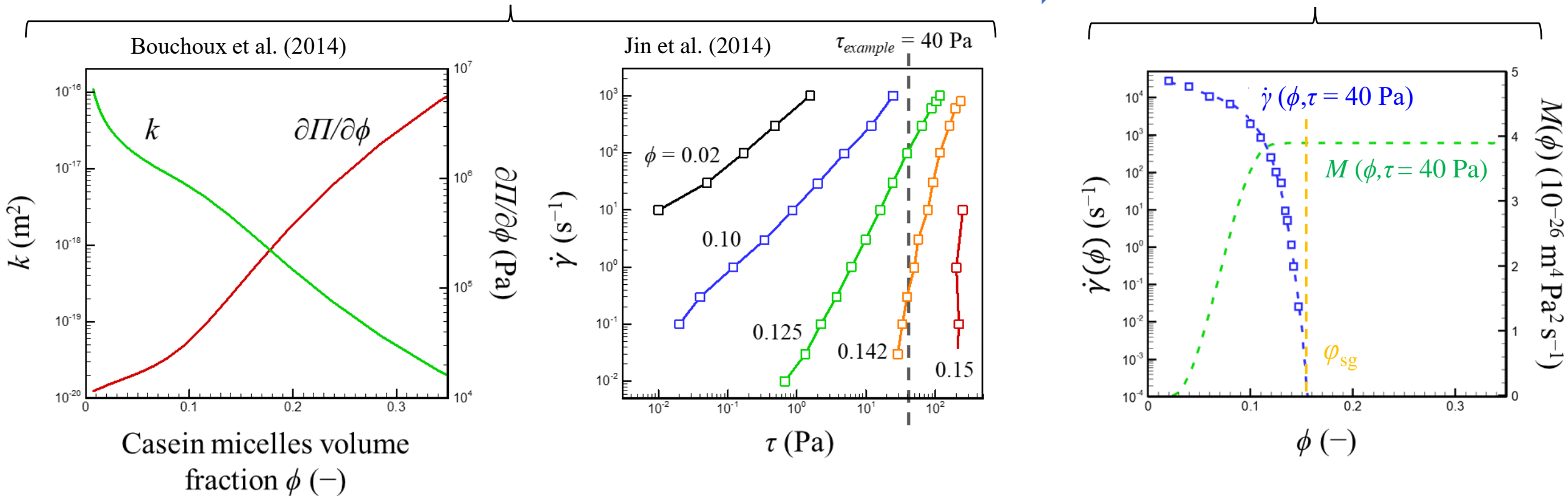
$M =$ function of $\left\{ \begin{array}{l} \dot{\gamma} = f(\phi, \tau) \\ k = f(\phi) \\ \pi = f(\phi) \end{array} \right.$

Shear rate at given shear stress
 Permeability
 Osmotic pressure

Literature data for aqueous casein micelles dispersions:



$\dot{\gamma}$ and M (for $\tau_{example} = 40$ Pa):



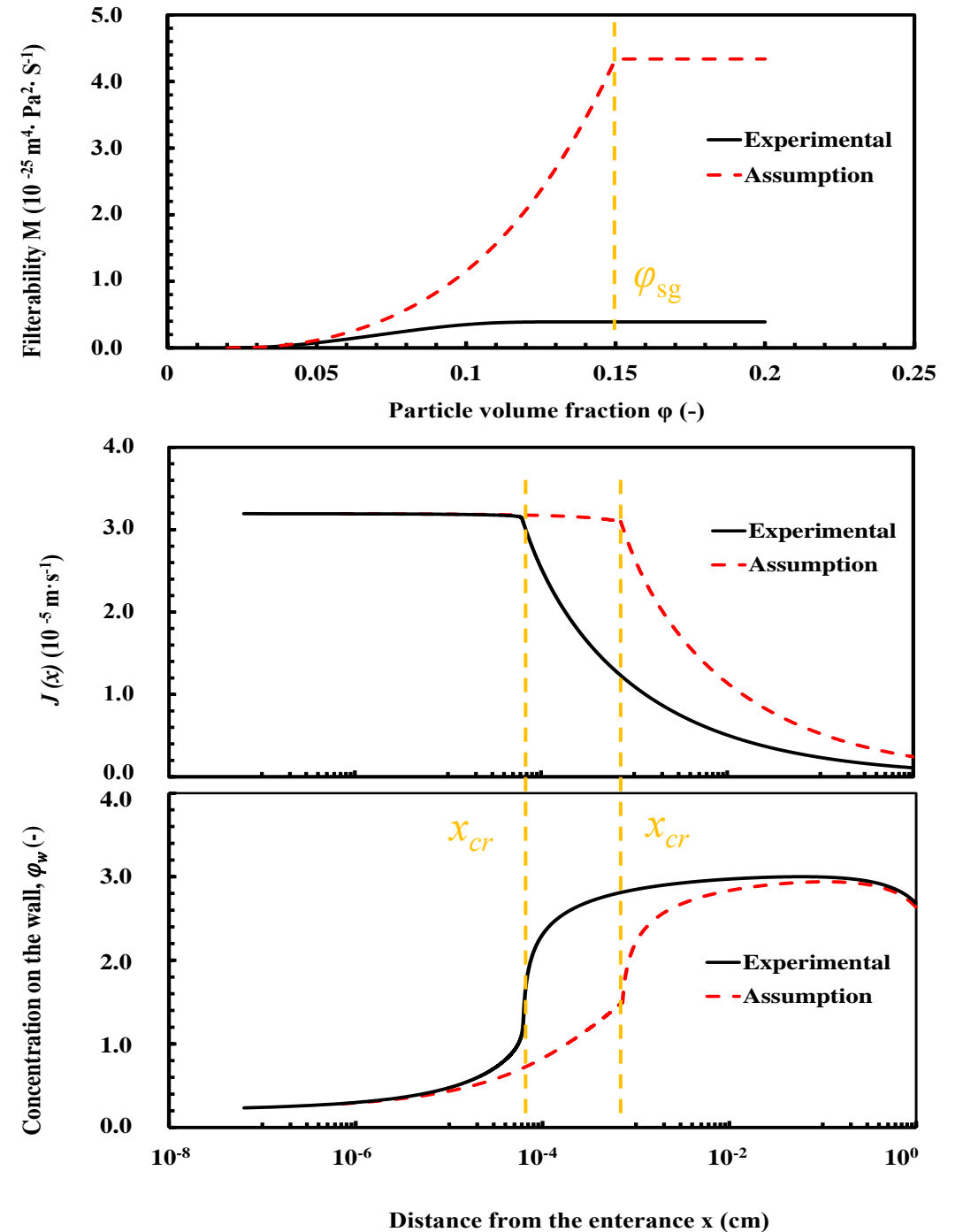
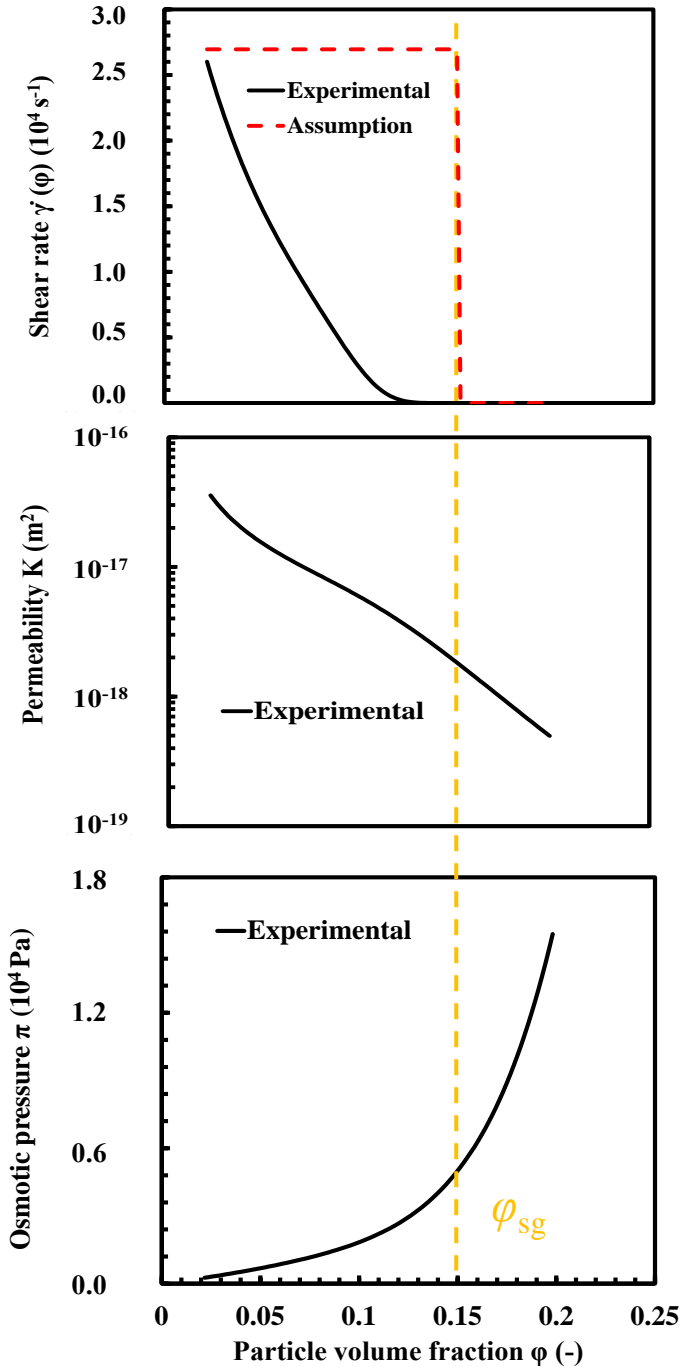
Assumption 1:

$$\dot{\gamma} = f(\varphi_0 = \text{const})$$

$$k = f(\varphi(x))$$

$$\pi = f(\varphi(x))$$

Concentration dependency of **Rheological behaviour** of CM is crucial for correct modeling of $\varphi_w(x)$ and $J(x)$.



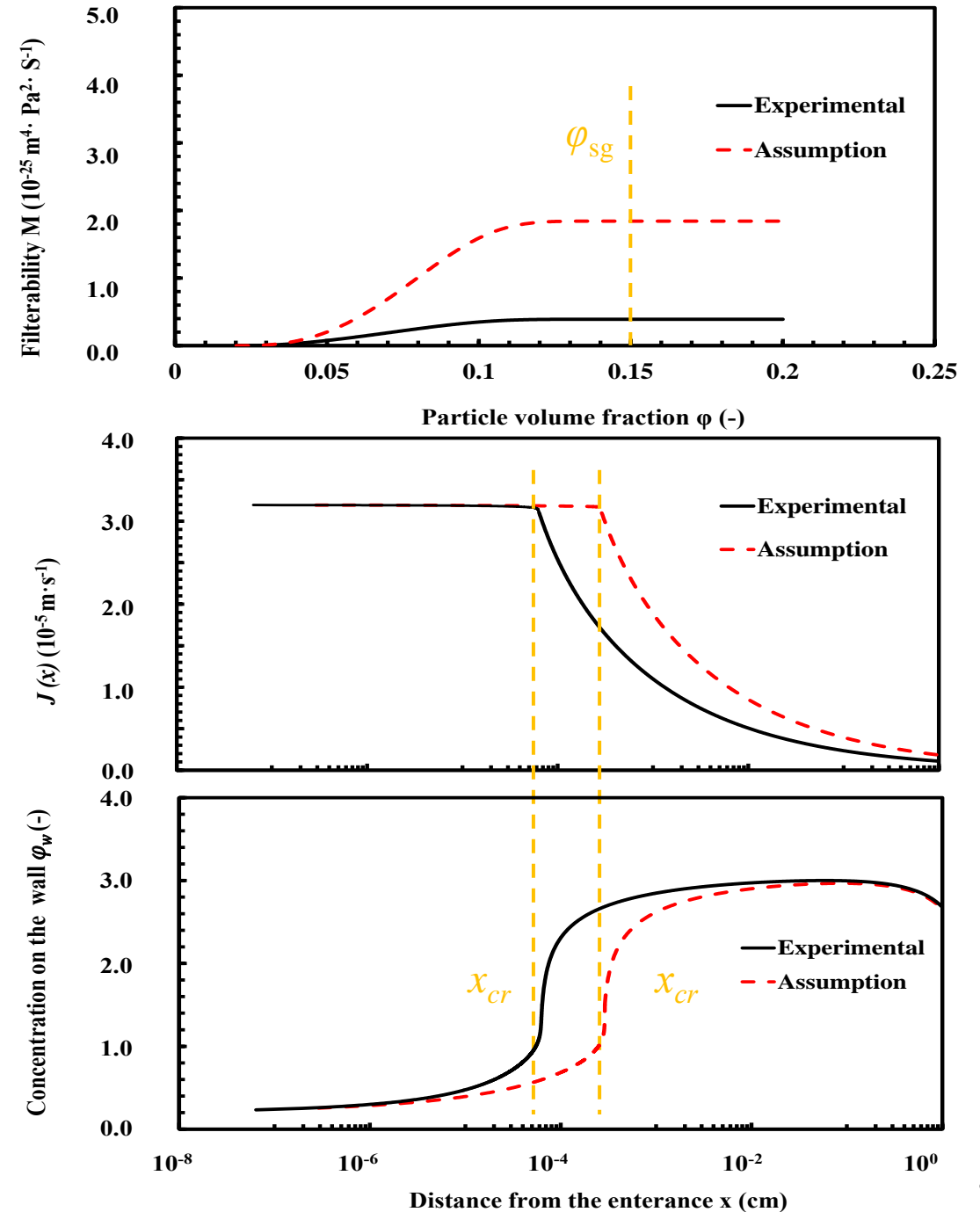
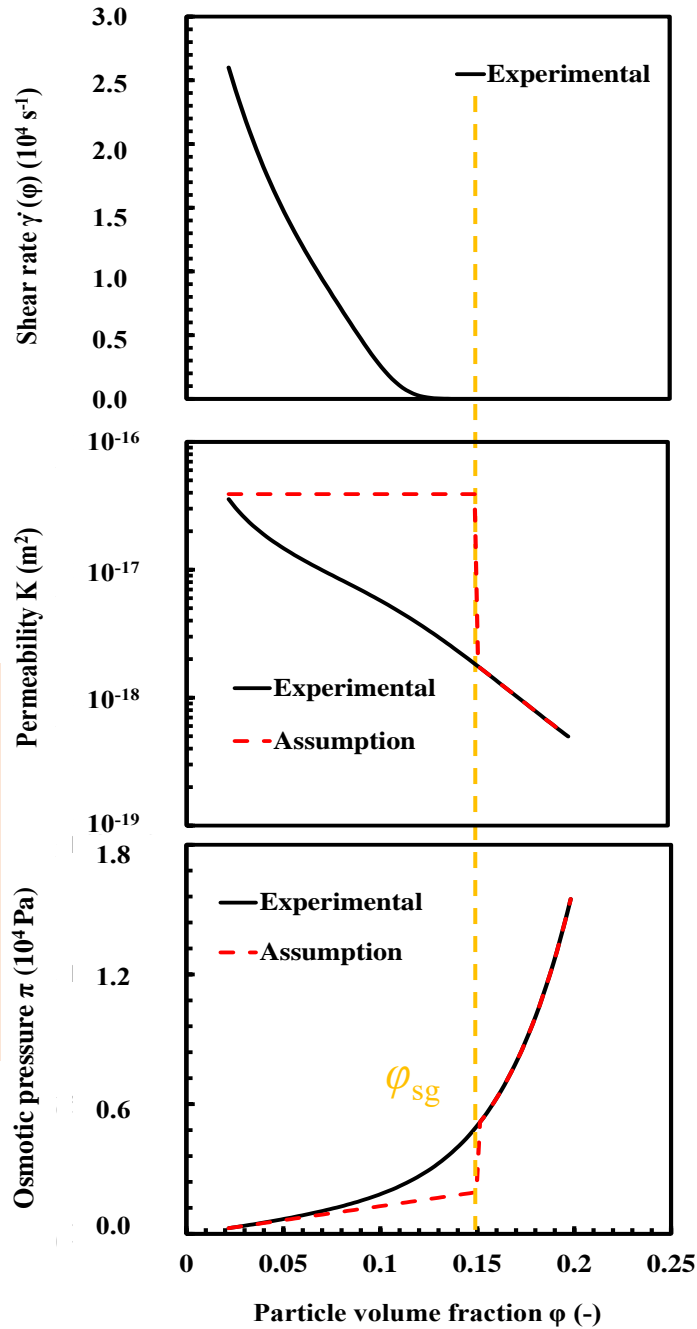
Assumption 2:

$$\dot{\gamma} = f(\varphi(x))$$

$$k = f(\varphi_0 = \text{const})$$

$$\pi = f(\varphi_0 = \text{const})$$

Concentration dependency of permeability (k) and compressibility (π) of CM is crucial for correct modeling of $\varphi_w(x)$ and $J(x)$.



Summary

Model advantages:

- * Local compressibility and permeability of CP layer may vary with local particle concentration.

(one can input any dependency of **permeability** and **compressibility** on particle concentration)

- * Model accounts for non-Newtonian nature of CP layer.

(one can input any dependency of **Rheological behaviour** on particle concentration and shear stress)

- * Model is **simple**.

(only one differential equation is solved, numerically)

Conclusion:

- * CP layer properties (compressibility, fluidity and permeability) defines filtration kinetics, $J(x)$, even in zone 2.

- * Deposit layer properties (compressibility, fluidity and permeability) do not impact filtration kinetics, $J(x)$, even in zone 2.



Perspectives

- * extend for solutes transmission (e.g. partial rejection of solutes by deposit in two-component suspension)
- * extend for turbulent cross-flow

