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Allocation d'intrants variables entre les cultures :
Une approche de paramètres aléatoires variant dans le temps

Résumé

Dans cet article, nous proposons une approche permettant d'allouer, à chaque culture produite, les quantités d'intrants variables agrégées au niveau des exploitations que l'on peut observer dans les données de panel de comptabilité agricole. Notre approche permet simultanément (i) de contrôler l'hétérogénéité observée et inobservée des exploitations agricoles, (ii) de tenir compte du lien potentiel entre utilisation d'intrants et choix d'assolement des agriculteurs, et (iii) de garantir une certaine vraisemblance des valeurs estimées des utilisations d'intrants par culture. Il s'agit là d'éléments auxquels se confrontent généralement les économistes qui cherchent à estimer des modèles d'allocation d'intrants. Cette approche est basée sur un modèle d'allocation d'intrants dérivé d'identités comptables, où les utilisations d'intrants non observées par culture sont traitées comme des paramètres aléatoires variant dans le temps. Nous estimons notre modèle sur un échantillon de données comptables d'exploitations agricoles françaises, en nous appuyant sur une extension de l'algorithme SAEM (Stochastic Approximiation Expectation Maximization). Nos résultats démontrent de bonnes performances de notre approche pour allouer les intrants aux cultures, en particulier pour les cultures les plus fréquemment produites dans notre échantillon.

Mots-Clés : allocation d'intrants, choix de production agricole, modèles à paramètres aléatoires, algorithme SAEM

Classification JEL : Q12, C13, C15

Variable Inputs Allocation among Crops:
A Time-Varying Random Parameters Approach

Abstract

In this paper, we propose an approach to allocate input uses among crops produced by farmers, based on panel data that includes input use aggregated at the farm-level. Our proposed approach simultaneously allows for (i) controlling for observed and unobserved farm heterogeneity, (ii) accounting for the potential dependence of input uses on acreage decisions, and (iii) ensuring consistent values of input use estimates. These are significant issues commonly faced in the estimation of input allocation equations. The approach is based on a model of input allocation derived from accounting identities, where unobserved input uses per crop are treated as time-varying random parameters. We estimate our model on a sample of French farms' accounting data, by relying on an extension of the Stochastic Approximation of Expectation Maximization algorithm. Our results show good performance of our approach in accurately allocating input uses among crops, for the crops the most frequently produced in our data sample in particular.

Keywords: input allocation, crop production decisions, random parameter models, SAEM algorithm

JEL classification: Q12, C13, C15

1. Introduction

Getting information about production costs associated to each crop at the farm level is very important when analyzing multi-crop farms' behaviors. It is indeed very useful to investigate variable input uses decisions of farmers for policy purpose. Production costs per crop can also be used as explanatory variables in more complex models of production choice (Letort and Carpentier, 2010). However, information on these costs per crop is generally not provided in farm accountancy dataset, such as Farm Accountancy Data Network (FADN) data, available to agricultural economists. The information on variable input uses in these data only concerns aggregate use at the farm level, and adequate statistical and/or economic modeling are necessary to allocate this aggregate information among the different crops produced by the farms.

Different approaches have been proposed in the agricultural economics literature to overcome this issue. Carpentier and Letort (2012) distinguish two groups of approaches. The first group includes approaches that consider only variable input allocation equations, in which the input allocation coefficients are treated as unknown parameters to be estimated, these parameters being either fixed, parametric functions of exogenous variables, or random (Dixon, Batte and Sonka, 1984; Hornbaker et al., 1989; Just et al., 1990; Dixon and Hornbaker, 1992). The second group of approaches considers input allocation equations as a part of a system of equations that includes crop yield equations, acreage functions or production equations (Just et al., 1990; Chambers and Just, 1989; Letort and Carpentier, 2012). Even if the second group of approaches introduces a lot more economic information compared to the first group of approaches, the first one is the most widely used, owing to its ease of implementation using regression approaches (OLS, GLS, SUR), and to the satisfactory results it generally provide in terms of production cost predictions (Just et al., 1990). Estimating single variable input allocation equations however raises different issues that must be addressed to ensure the consistency of this approach. First, the use of standard regression approaches does not guarantee that estimated input uses lie in reasonable ranges. These approaches can, for instance, lead to negative estimates of input uses per crop. Secondly, because input uses vary across farms (and crops), the observed, but also unobserved, heterogeneity among farms and farmers have to be taken into account. Finally, input uses per crop depend on acreage choice decisions, which are also determined by unobserved farm characteristics. This can lead to estimation issues when one seeks to account for unobserved farm heterogeneity in input allocation equations.

Given the limited information generally available in observed data samples, a general way to overcome the issues concerning the magnitude of estimated input uses is to impose constraints on parameters or to introduce additional out-of-sample information. Approaches based on inequality-restricted regression estimation (Ray, 1985; Dixon and Hornbaker, 1992), on Bayesian estimation (Moxey et Tiffin, 1994; Heckelei et al., 2008) and on Generalized Maximum Entropy estimation (Léon et al., 1999) have been proposed to this end. Issues related to the presence of unobserved farm heterogeneity in input allocation equations have also been addressed in the literature (Dixon, Batte and Sonka 1984; Hornbaker et al., 1989; Dixon and Hornbaker 1992; Hallam et al., 1999). However, as pointed out by Lence and Miller (1998), Dixon and Hornbaker (1992) and Carpentier and Letort (2012), the random parameter (RP) approaches, generally used in that case, have to deal with issues related the dependence between variable input use and acreage choice decisions. Dixon and Hornbaker (1992) propose correlation tests without, however proposing a method allowing to take this correlation into account, while Carpentier and Letort (2012) propose an approach based on control functions, which requires a simultaneous estimation of input use and acreage choices equations. To our knowledge, the different approaches proposed in the literature to estimate single input allocation equations thus do not allow to simultaneously (i) control for unobserved farms and farmers heterogeneity, (ii) deal with the dependence of input uses per crop to acreage choices and (iii) guarantee consistent values of input use estimates.

Our main objective in this paper is to propose an approach addressing these three issues simultaneously. To do so, we consider a panel data model of input allocation derived from accounting identities. We use a random parameter specification to account for farm unobserved heterogeneity. The unobserved crop input uses are treated as time-varying random parameters, and we control for the potential correlation between crop input uses and acreage decisions by expressing these random parameters as functions of (time-constant and time-varying) exogenous variables containing acreage shares. To ensure that the estimated input uses per crop lie in reasonable ranges, we introduce additional information in the model through the distribution of random parameters. We notably assume this distribution to be lognormal to enforce the non-negativity of each estimated crop input use. This model is estimated, using an extension of Stochastic Approximation Expectation Maximization (SAEM) algorithm (Delyon et al., 1999), on a sample French farms' accounting data.

The rest of the paper is structured as follows. In section 2, we present our model of input use allocation. Our SAEM estimation approach is presented in section 3, and the empirical results in section 4. Finally, we conclude.

2. Random Parameter model of input use allocation

We consider a set of crops $\mathcal{C} \equiv \{1, 2, \ldots, C\}$ produced by a farmer i $(i = 1, \ldots, N)$ in period t. We denote by $s_{c,it}$ the acreage allocated to crop $c \in \mathcal{C}$ by farmer i in period t. In the following, we focus on one variable input used by farmer i to simplify the presentation of the model, given that the generalization to J inputs is straightforward. Let $\bar{x}_{it}$ denotes the quantity of input used at the farm level by farmer i at time t and $x_{c,it}$ denote the quantity of input used per unit of land of crop c.

The input allocation problem consists in recovering the input quantity $x_{c,it}$ for each crop such that:

$$\bar{x}_{it} = \sum_{c \in \mathcal{C}} s_{c,it} x_{c,it} = \mathbf{s}_{it} \mathbf{x}_{it}, \qquad (1)$$

with $\mathbf{x}_{it} \equiv (x_{c,it} : c \in \mathcal{C})$ and $\mathbf{s}_{it} \equiv (s_{c,it} : c \in \mathcal{C})$.

Including a (centered) measurement error term, u_{it}, in the input use equation at farm level, equation (1) becomes:

$$\bar{x}_{it} = \sum_{c \in \mathcal{C}} s_{c,it} x_{c,it} + u_{it} = \mathbf{s}'_{it} \mathbf{x}_{it} + u_{it} \text{ with } u_{it} \sim_{iid} \mathcal{N}(0, \sigma_0^2) \text{ and } E(u_{it} \mid \mathbf{s}_{it}) = \mathbf{0}. \qquad (2)$$

In equation (2), the time varying and farm specific input uses per crop, $x_{c,it}$, are unobserved. We consider these crop input uses as random parameters varying across farms and across time, the distribution of which we will seek to estimate. Following Train and Sonnier (2005), in order to impose the necessary non-negativity of $x_{c,it}$, we do not assume these random parameters to follow a standard normal distribution, but a transformation of normally distributed terms. The main advantage of this approach compared to non-negativity Ordinary Least Square and non-negative Random Coefficient Regression, presented in Dixion and Hornbaker (1992), is that it leaves the estimation problem unconstrained. Specifically, we consider here a log-transformation[1] of $x_{c,it}$ and assume that it can be decomposed into three components as presented in equation:

$$\ln(x_{c,it}) = \beta_{c,i} + \mathbf{z}'_{c,it}\boldsymbol{\delta}_{c,0} + \varepsilon_{c,it} . \quad (3)$$

The farm-specific parameter in equation (3), $\beta_{c,i}$, is assumed to vary randomly across farms according to a normal distribution, with $\boldsymbol{\beta}_i \equiv (\beta_{c,i} : c \in C) \sim_{iid} \mathcal{N}(\boldsymbol{\omega}_0, \boldsymbol{\Omega}_0)$. This parameter allows accounting for the variability of unobserved determinants – also called unobserved heterogeneity – of farmers' crop input choices (Woodridge, 2002; Arellano and Bonhomme, 2011). These determinants can be unobserved characteristics of farms (e.g., soil quality, spatial distribution of the plot, available material) and farmers (e.g., aptitudes, motivations) that do not vary over time. $\mathbf{z}_{c,it}$ contains a set of control variables that allows controlling for the effects, $\boldsymbol{\delta}_{c,0}$, of observed determinants of farmers' crop input choices. These control variables notably include year dummy variables[2] but can also integrate other characteristics, such as average crop yields observed at regional level as proxies of production conditions in each region. Finally, the error term of the crop input uses model in equation (3), $\varepsilon_{c,it}$, is assumed to be normally distributed, with $\boldsymbol{\varepsilon}_{it} \equiv (\varepsilon_{c,it} : c \in C) \sim_{iid} \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi}_0)$, and allows accounting for the effects of stochastic events not captured by $\beta_{c,i}$ or $\boldsymbol{\delta}_{c,0}$ (*e.g.*, weather events, pest infestation). We assume the covariance matrix $\boldsymbol{\Psi}_0$ to be diagonal, all correlations in crop input uses decisions being captured by $\boldsymbol{\beta}_i$ through the covariance matrix $\boldsymbol{\Omega}_0$, which is unrestricted.

We assume u_{it}, $\boldsymbol{\varepsilon}_{it}$ and $\boldsymbol{\beta}_i$ to be mutually independent. Then, since the stochastic events that can influence farmers' input use decisions during the cropping season, captured by $\boldsymbol{\varepsilon}_{it}$, are unknown at the time of acreage choices in the planting season, these random terms are

[1] It would also have been possible to use a transformation constraining crop input uses need to lie (in a probabilistic sense) between a low and upper bound by using a Johnson's (1949) SB distribution. Equation (3) would have then been expressed as: $x_{c,it} = l + (u - l)\left(\exp(\beta_{c,i} + \mathbf{z}'_{c,it}\boldsymbol{\delta}_{c,0} + \varepsilon_{c,it}) / (1 + \exp(\beta_{c,i} + \mathbf{z}'_{c,it}\boldsymbol{\delta}_{c,0} + \varepsilon_{c,it}))\right)$, where l and u are known lower and upper bounds respectively. We have chosen to constrain only crop input uses to be positive by using a log-transformation, firstly for the sake of model simplicity and secondly because we lack references for the upper and lower bounds to be defined. Furthermore, as shown by the estimation results presented in section 4, our estimated crop input uses actually lie within reasonable ranges, in contrast to what we obtain using a standard normal distribution, without log-transformation (these results are presented in Appendix B).

[2] For identification purpose the year dummy variables are normalized to 0 for the first time period

assumed to be independent of crop acreages choices: $E(\boldsymbol{\varepsilon}_{it} \mid \mathbf{s}_{it}, \mathbf{z}_{it}) = E(\boldsymbol{\varepsilon}_{it} \mid \mathbf{s}_{it}) = \mathbf{0}$. On the other hand, the assumption of independence between $\boldsymbol{\beta}_i$ and $\mathbf{s}_{it}$ may not be verified since $\boldsymbol{\beta}_i$ captures the impacts of unobserved farmers' characteristics that may also affect their acreage choice decisions. To account for this potential link between acreage choice and crop input use decisions and avoid some potential bias in the estimation of the model, we follow Mundlak (1978) and explicitly model the correlation between crop input uses and acreage shares by defining the $\beta_{c,i}$ parameters as functions of the average share of crop c in farm i's acreage:

$$\beta_{c,i} = \omega_{c,0} + \mathbf{m}'_{c,i}\boldsymbol{\varsigma}_{c,0} + e_{c,i}, \tag{4}$$

where $\mathbf{m}_{c,i}$ is a vector of time-constant variables that contain the average, over the considered time period, of the share of crop c in the acreage of farm i', $\bar{s}_{c,i}$. The variables in $\mathbf{m}_{c,i}$ are centered at their sample mean, so that $\boldsymbol{\omega}_0 \equiv (\omega_{c,0} : c \in C)$ effectively corresponds to the mean of random term $\beta_{c,i}$.

In a nutshell, our proposed input uses allocation model is defined by equations (5.a) to (5.b):

$$\bar{x}_{it} = \sum\nolimits_{c \in C} s_{c,it} x_{c,it} + u_{it} \tag{5.a}$$

$$\ln(x_{c,it}) = \beta_{c,i} + \mathbf{z}'_{c,it}\boldsymbol{\delta}_{c,0} + \varepsilon_{c,it}, \tag{5.b}$$

$$\beta_{c,i} = \omega_{c,0} + \mathbf{m}'_{c,i}\boldsymbol{\varsigma}_{c,0} + e_{c,i}, \tag{5.c}$$

where $u_{it} \sim_{iid} \mathcal{N}(0, \sigma_0^2)$, $\boldsymbol{\varepsilon}_{it} \sim_{iid} \mathcal{N}(\mathbf{0}, \boldsymbol{\Psi}_0)$, $\mathbf{e}_i = (e_{c,i} : c \in C) \sim_{iid} \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}_0)$ and $\mathbf{e}_i$, u_{it} and $\boldsymbol{\varepsilon}_{it}$ are assumed to be mutually independent and independent of $\mathbf{m}_{c,i}$ and $\mathbf{z}_{c,it}$.

3. Estimation appraoch

To simplify the presentation of the estimation procedure, we rewrite equation (5a) and (5.b) in the following compact form:

$$\bar{x}_{it} = \mathbf{s}'_{it}\mathbf{x}_{it} + u_{it}, \tag{6.a}$$

$$\boldsymbol{\mu}_{it} = \mathbf{ln}(\mathbf{x}_{it}) = \boldsymbol{\beta}_i + \mathbf{Z}_{it}\boldsymbol{\delta}_0 + \boldsymbol{\varepsilon}_{it}, \tag{6.b}$$

$$\boldsymbol{\beta}_i = \boldsymbol{\omega}_0 + \mathbf{M}_i\boldsymbol{\varsigma}_0 + \mathbf{e}_i, \tag{6.c}$$

where $\mathbf{ln}(\mathbf{x}_{it}) \equiv (\ln(x_{c,it}) : c \in \mathcal{C})$, $\boldsymbol{\beta}_i \equiv (\beta_{c,i} : c \in \mathcal{C})$, $\mathbf{M}_i$ is a block diagonal matrix where the c^{th} block element is the row vector $\mathbf{m}'_{c,i\cdot}$: $\mathbf{M}_i \equiv Blocdiag(\mathbf{m}'_{c,i\cdot} : c = 1,...,C)$, and $\mathbf{Z}_{it}$ is defined in a similar way: $\mathbf{Z}_{it} \equiv Blocdiag(\mathbf{z}'_{c,it} : j = 1,...,C)$.

Introducing the following notations: $\overline{\mathbf{x}}_{(i)} \equiv (\overline{x}_{i1}, \overline{x}_{i2}, ..., \overline{x}_{iT_{(i)}})'$, $\mathbf{s}_{(i)} = Blocdiag(\mathbf{s}'_{i1}, ..., \mathbf{s}'_{iT_{(i)}})$, $\boldsymbol{\mu}_{(i)} \equiv (\boldsymbol{\mu}'_{i1}, \boldsymbol{\mu}'_{i2}, ..., \boldsymbol{\mu}'_{iT_{(i)}})'$, $\mathbf{Z}_{(i)} \equiv (\mathbf{Z}'_{i1}, ..., \mathbf{Z}'_{iT_{(i)}})'$, and $\boldsymbol{\varepsilon}_{(i)} \equiv (\boldsymbol{\varepsilon}'_{i1}, \boldsymbol{\varepsilon}'_{i2}, ..., \boldsymbol{\varepsilon}'_{iT_{(i)}})'$, equations (6.a) and (6.b) can be further compacted and the model becomes:

$$\overline{\mathbf{x}}_{(i)} = \mathbf{s}_{(i)} \exp(\boldsymbol{\mu}_{(i)}) + \mathbf{u}_{(i)}, \quad (7.a)$$

$$\boldsymbol{\mu}_{(i)} = \boldsymbol{\iota}_{T_{(i)}} \otimes \boldsymbol{\beta}_i + \mathbf{Z}_{(i)} \boldsymbol{\delta}_0 + \boldsymbol{\varepsilon}_{(i)} \quad (7.b)$$

$$\boldsymbol{\beta}_i = \boldsymbol{\omega}_0 + \mathbf{M}_i \boldsymbol{\varsigma}_0 + \mathbf{e}_i \quad (7.c)$$

As already mentioned, the measurement error term in equation (7.a) follows a normal distribution:

$$\mathbf{u}_{(i)} \sim_{iid} \mathcal{N}(\mathbf{0}, \boldsymbol{\iota}_{T_{(i)}} \otimes \sigma_0^2), \quad (8.a)$$

Given the assumptions on the distribution of random terms $\boldsymbol{\varepsilon}_{(i)}$ in equation (7.b) and $\mathbf{e}_i$ in equation (7.c), we can deduce that $\boldsymbol{\mu}_{(i)}$ also follows a normal distribution such that:

$$\boldsymbol{\mu}_{(i)} \sim_{iid} \mathcal{N}(\boldsymbol{\iota}_{T_{(i)}} \otimes (\boldsymbol{\omega}_0 + \mathbf{M}_i \boldsymbol{\varsigma}_0) + \mathbf{Z}_{(i)} \boldsymbol{\delta}_0, \mathbf{G}_{(i)}), \text{ with } \mathbf{G}_{(i)} = \boldsymbol{\iota}_{T_{(i)}} \boldsymbol{\iota}'_{T_{(i)}} \otimes \boldsymbol{\Omega}_0 + \mathbf{I}_{T_{(i)}} \otimes \boldsymbol{\Psi}_0. \quad (8.b)$$

The model defined by equations (7.a)-(7.c) is thus fully parametric and can be estimated by relying on a Maximum Likelihood approach. The estimation procedure we use here is an extension of the Expectation Maximization (EM) algorithm proposed by Dempster et al (1977), which allows tackling several estimation issues inherent to structure of our model. In this procedure, $\mathbf{q} \equiv (\boldsymbol{\mu}_{(i)}, \boldsymbol{\beta}_i : i = 1,..., N)$ are treated as missing data and we seek to estimate the vector of parameters $\boldsymbol{\theta}_0 \equiv (\boldsymbol{\omega}_0, \boldsymbol{\varsigma}_0, \boldsymbol{\delta}_0, \sigma_0^2, \boldsymbol{\psi}_0, \boldsymbol{\Omega}_0)$. The complete data of the model is composed of the vector of observed variables, $\mathbf{y} = (\overline{\mathbf{x}}_{(i)}, \mathbf{s}_{(i)}, \mathbf{z}_{(i)}, \mathbf{M}_i : i = 1,..., N)$, and the vector of unobserved variables, $\mathbf{q}$. The log-likelihood function of complete data at $\boldsymbol{\theta} = (\boldsymbol{\omega}, \boldsymbol{\varsigma}, \boldsymbol{\delta}, \sigma^2, \boldsymbol{\Omega}, \boldsymbol{\Psi})$ is the sample log-likelihood function of the dependent and missing variables, $\bar{\mathbf{x}}$ and $\mathbf{q}$, given the exogenous variables of the model, $\mathbf{s}=(\mathbf{s}_{(i)}:i=1,...,N)$, $\mathbf{z}=(\mathbf{z}_{(i)}:i=1,...,N)$ and $\mathbf{M}\equiv(\mathbf{M}_i:i=1,...,N)$:

$$\ln\ell^c(\boldsymbol{\theta};\bar{\mathbf{x}},\boldsymbol{\beta},\boldsymbol{\mu}\,|\,\mathbf{s},\mathbf{z},\mathbf{M})=\sum\nolimits_{i=1}^{N}\ln f(\bar{\mathbf{x}}_{(i)},\boldsymbol{\beta}_i,\boldsymbol{\mu}_{(i)}\,|\,\mathbf{s}_{(i)},\mathbf{z}_{(i)},\mathbf{M}_i;\boldsymbol{\theta}) \tag{9.a}$$

where:

$$\begin{aligned}\ln f(\bar{\mathbf{x}}_{(i)},\boldsymbol{\beta}_i,\boldsymbol{\mu}_{(i)}\,|\,\mathbf{s}_{(i)},\mathbf{z}_{(i)},\mathbf{M}_i;\boldsymbol{\theta}) &= \sum\nolimits_{t=1}^{T_{(i)}}\ln\varphi(\bar{x}_{it}-\mathbf{s}_{it}'\mathbf{exp}(\boldsymbol{\mu}_{it});\sigma^2)\\ &\quad+\sum\nolimits_{t=1}^{T_{(i)}}\ln\varphi(\boldsymbol{\mu}_{it}-\boldsymbol{\beta}_i-\mathbf{Z}_{it}\boldsymbol{\delta};\boldsymbol{\Psi})\\ &\quad+\ln\varphi(\boldsymbol{\beta}_i-\boldsymbol{\omega}-\mathbf{M}_i\boldsymbol{\varsigma};\boldsymbol{\Omega}).\end{aligned} \tag{9.b}$$

The term $\varphi(\mathbf{A};\mathbf{B})$ denotes the probability density function at point $\mathbf{A}$ of the standard multivariate normal distribution with variance-covariance matrix **B.**

EM-type algorithms are iterative procedures which require computing, at each iteration, the expectation of the complete data log-likelihood function:

$$Q_i(\boldsymbol{\theta}\,|\,\boldsymbol{\theta}^{(n-1)})=\int\ln\ell^c(\boldsymbol{\theta};\bar{\mathbf{x}}_{(i)},\boldsymbol{\mu},\boldsymbol{\beta}\,|\,\mathbf{s}_i,\mathbf{z}_i)\,f(\boldsymbol{\mu},\boldsymbol{\beta}\,|\,\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})d(\boldsymbol{\mu},\boldsymbol{\beta}) \tag{10}$$

with $f(\boldsymbol{\mu},\boldsymbol{\beta}\,|\,\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})$, the probability density function of missing variables $(\boldsymbol{\mu}_{(i)},\boldsymbol{\beta}_i)$ given the observed variables $(\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i)$ and the current value of $\boldsymbol{\theta}$ at iteration (n), $\boldsymbol{\theta}^{(n-1)}$. The term $Q_i(\boldsymbol{\theta}\,|\,\boldsymbol{\theta}^{(n-1)})$ involving an integration over the missing variable $(\boldsymbol{\mu},\boldsymbol{\beta})$, it has no closed form expression. To simplify its computation, we consider the following factorization of $f(\boldsymbol{\mu},\boldsymbol{\beta}\,|\,\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})$:

$$f(\boldsymbol{\mu},\boldsymbol{\beta}\,|\,\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})=f(\boldsymbol{\beta}\,|\,\boldsymbol{\mu};\boldsymbol{\theta}^{(n-1)})\,f(\boldsymbol{\mu}\,|\,\bar{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)}), \tag{11}$$

which comes from the fact that $\boldsymbol{\mu}_i$ is a linear combination of $\boldsymbol{\beta}_i$.

Then, it can be shown quite readily that $\boldsymbol{\beta}\,|\,\boldsymbol{\mu};\boldsymbol{\theta}\sim_{iid}\mathcal{N}\left[\mathbf{m}_{\boldsymbol{\beta},i}(\boldsymbol{\rho},\boldsymbol{\theta}),\mathbf{V}_{\boldsymbol{\beta}}(\boldsymbol{\theta})\right]$ with

$$\begin{cases}\mathbf{m}_{\boldsymbol{\beta},i}(\boldsymbol{\mu};\boldsymbol{\theta})=\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta})\;\boldsymbol{\Psi}^{-1}\sum\nolimits_{t=1}^{T_{(i)}}(\boldsymbol{\mu}_{it}-\mathbf{Z}_{it}\boldsymbol{\delta})+\boldsymbol{\Omega}^{-1}(\boldsymbol{\omega}+\mathbf{M}_i\boldsymbol{\varsigma})\\ \mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta})=(\boldsymbol{\Omega}^{-1}+T_{(i)}\boldsymbol{\Psi}^{-1})^{-1}\end{cases} \tag{12}$$

Combining equations (11) and (12) to replace $f(\boldsymbol{\mu},\boldsymbol{\beta} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})$ in equation (10), we obtain:

$$Q_i(\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)}) = \int Q_i(\boldsymbol{\mu},\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)}) f(\boldsymbol{\mu} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)}) d\boldsymbol{\mu}\,, \tag{13}$$

where

$$\begin{aligned} Q_i(\boldsymbol{\mu},\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)}) &= \int \ln \ell^c(\boldsymbol{\theta};\overline{\mathbf{x}}_{(i)},\mathbf{q} \mid \mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i) f(\boldsymbol{\beta} \mid \boldsymbol{\mu}_i;\boldsymbol{\theta}^{(n)}) d\boldsymbol{\beta} \\ &= \sum_{t=1}^{T_{(i)}} \ln \varphi(\overline{x}_{it} - \mathbf{s}_{it}' \mathbf{exp}(\boldsymbol{\mu}_{it});\sigma^2) \\ &+ \sum_{t=1}^{T_{(i)}} \ln \varphi(\boldsymbol{\mu}_{it} - \mathbf{m}_{\boldsymbol{\beta},i}(\boldsymbol{\mu};\boldsymbol{\theta}^{(n-1)}) - \mathbf{Z}_{it}\boldsymbol{\delta};\boldsymbol{\Psi}) - \frac{1}{2} T_{(i)} \mathrm{tr}\left(\boldsymbol{\Psi}^{-1}\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})\right) \\ &+ \ln \varphi\left(\mathbf{m}_{\boldsymbol{\beta},i}(\boldsymbol{\mu};\boldsymbol{\theta}^{(n-1)}) - \boldsymbol{\omega} - \mathbf{M}_i\boldsymbol{\varsigma};\boldsymbol{\theta}^{(n-1)});\boldsymbol{\Omega}\right) - \frac{1}{2}\mathrm{tr}(\boldsymbol{\Omega}^{-1}\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})). \end{aligned} \tag{14}$$

$Q_i(\boldsymbol{\mu},\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)})$ can thus written as a sum of log of density functions of normal distributions (which belongs to the family of exponential distributions). Having now an explicit form for $Q_i(\boldsymbol{\mu},\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)})$, the computation of $Q_i(\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(n-1)})$ involves the integration over only the conditional distribution of random parameters vector $\boldsymbol{\mu}$, $f(\boldsymbol{\mu} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i;\boldsymbol{\theta}^{(n-1)})$, with:

$$\begin{aligned} f(\boldsymbol{\mu} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i;\boldsymbol{\theta}^{(n-1)}) \propto \varphi(\boldsymbol{\mu}_{(i)} - \boldsymbol{\iota}_{T_{(i)}} \otimes (\boldsymbol{\omega}^{(n-1)} + \mathbf{M}_i\boldsymbol{\varsigma}^{(n-1)}) - \mathbf{Z}_{(i)}\boldsymbol{\delta}^{(n-1)};\mathbf{G}_{(i)}^{(n-1)}) \\ \times \prod_{t=1}^{T_{(i)}} \varphi(\overline{x}_{it} - \mathbf{s}_{it}'\mathbf{h}^{-1}(\boldsymbol{\mu}_{it});\sigma^2) \end{aligned}. \tag{15}$$

This integration is done by stochastic approximation as proposed by Delyon et al. (1999) in their Stochastic Approximation EM algorithm (SAEM). At iteration (n) of the algorithm, given the observed data $\mathbf{y} = (\overline{\mathbf{x}}_{(i)},\mathbf{s}_{(i)},\mathbf{z}_{(i)},\mathbf{M}_i : i = 1,\ldots,N)$ and the current vector of parameters, $\boldsymbol{\theta}^{(n-1)}$, the considered SAEM algorithm consists in three steps, which are iterated until convergence to the maximum likelihood estimate:

- **Simulation Step (S-step):** Simulate $\left\{\tilde{\boldsymbol{\mu}}_{i,r}^{(n)} : r = 1,\ldots,R\right\}$ according to the probability density function $f(\boldsymbol{\mu} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i,\mathbf{M}_i;\boldsymbol{\theta}^{(n-1)})$ for $i = 1,\ldots,N$ (see Allassonnière and Chevallier (2021) for more detail on the simulation procedure)

- **Stochastic Approximation Step (SA-step):** update the following minimal sufficient statistics such that:[3]

$$\mathbf{s}_{1,i}^{(n)}=\mathbf{s}_{1,i}^{(n-1)}+\lambda_{(n)}\left(R^{-1}\sum_{r=1}^{R}\mathbf{m}_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)})-\mathbf{s}_{1,i}^{(n-1)}\right)\text{ for } i,$$

$$\mathbf{s}_{2}^{(n)}=\mathbf{s}_{2}^{(n-1)}+\lambda_{(n)}\left(R^{-1}\sum_{r=1}^{R}\sum_{i=1}^{N}\mathbf{m}_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)})\mathbf{m}'_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)})-\mathbf{s}_{1,i}^{(n-1)}\right)$$

$$\mathbf{s}_{3,it}^{(n)}=\mathbf{s}_{3,it}^{(n-1)}+\lambda_{(n)}\left(R^{-1}\sum_{r=1}^{R}(\tilde{\boldsymbol{\mu}}_{it,r}^{(n)}-\mathbf{m}_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)}))-\mathbf{s}_{3,it}^{(n-1)}\right)\text{, for } it,$$

$$\mathbf{s}_{4}^{(n)}=\mathbf{s}_{4}^{(n-1)}+\lambda_{(n)}\left(R^{-1}\sum_{r=1}^{R}\sum_{i=1}^{N}\sum_{t=1}^{T_{(i)}}(\tilde{\boldsymbol{\mu}}_{it,r}^{(n)}-\mathbf{m}_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)}))(\tilde{\boldsymbol{\mu}}_{it,r}^{(n)}-\mathbf{m}_{\boldsymbol{\beta}}(\tilde{\boldsymbol{\mu}}_{(i),r}^{(n)};\boldsymbol{\theta}^{(n-1)}))'-\mathbf{s}_{4}^{(n-1)}\right),$$

$$s_{5}^{(n)}=s_{5}^{(n-1)}+\lambda_{(n)}\left(R^{-1}\sum_{r=1}^{R}\sum_{i=1}^{N}\sum_{t=1}^{T_{(i)}}\left(\bar{\mathbf{x}}_{it}-\mathbf{s}'_{it}\mathbf{exp}(\tilde{\boldsymbol{\mu}}_{it,r}^{(n)})\right)\left(\bar{\mathbf{x}}_{it}-\mathbf{s}'_{it}\mathbf{exp}(\tilde{\boldsymbol{\mu}}_{it,r}^{(n)})\right)'-s_{5}^{(n-1)}\right)$$

- **Maximization Step (M-step):** update parameters $\boldsymbol{\theta}$ such that:

$$\boldsymbol{\omega}^{(n)}=N^{-1}\sum_{i=1}^{N}(\mathbf{s}_{1,i}^{(n)}-\bar{\mathbf{Z}}_{i}\boldsymbol{\varsigma}^{(n-1)}),$$

$$\boldsymbol{\varsigma}^{(n)}=\left(\sum_{i=1}^{N}\mathbf{M}'_{i}\boldsymbol{\Omega}_{(n-1)}^{-1}\mathbf{M}_{i}\right)^{-1}\sum_{i=1}^{N}\mathbf{M}'_{i}\boldsymbol{\Omega}_{(n-1)}^{-1}(\mathbf{s}_{1,i}^{(n)}-\boldsymbol{\omega}^{(n)})$$

$$\boldsymbol{\delta}^{(n)}=\left(\sum_{i=1}^{N}\sum_{t=1}^{T_{(i)}}\mathbf{Z}'_{it}\mathbf{Z}_{it}\right)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T_{(i)}}\mathbf{Z}'_{it}(\mathbf{s}_{3,it}^{(n)}-\mathbf{s}_{1,i}^{(n)})$$

$$\boldsymbol{\Omega}_{(n)}=N^{-1}\mathbf{s}_{2}^{(n)}+N^{-1}\sum_{i=1}^{N}\begin{pmatrix}(\boldsymbol{\omega}^{(n)}+\mathbf{M}_{i}\boldsymbol{\varsigma}^{(n)})(\boldsymbol{\omega}^{(n)}+\bar{\mathbf{Z}}_{i}\boldsymbol{\pi}^{(n)})'-\mathbf{s}_{1,i}^{(n)}(\boldsymbol{\omega}^{(n)}+\mathbf{M}_{i}\boldsymbol{\varsigma}^{(n)})'\\-(\boldsymbol{\omega}^{(n)}+\mathbf{M}_{i}\boldsymbol{\varsigma}^{(n)})\mathbf{s}'^{(n)}_{1,i}+\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})\end{pmatrix}$$

$$\boldsymbol{\Psi}_{(n)}=\operatorname{diag}\left(N_{tot}^{-1}\mathbf{s}_{4}^{(n)}+N_{tot}^{-1}\sum_{i=1}^{N}\begin{pmatrix}\sum_{t=1}^{T_{(i)}}\left(\mathbf{Z}_{it}\boldsymbol{\delta}^{(n)}(\mathbf{Z}_{it}\boldsymbol{\delta}^{(n)})'-\mathbf{s}_{3,it}^{(n)}(\mathbf{Z}_{it}\boldsymbol{\delta}^{(n)})'-(\mathbf{Z}_{it}\boldsymbol{\delta}^{(n)})(\mathbf{s}_{3,it}^{(n)})'\right)\\+T_{(i)}\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})\end{pmatrix}\right)$$

$$\sigma^{2(n)}=N_{tot}^{-1}s_{5}^{(n)}$$

where

$$\begin{cases}\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})=\left(\boldsymbol{\Omega}_{(n-1)}^{-1}+T_{(i)}\boldsymbol{\Psi}_{(n-1)}^{-1}\right)^{-1}\\ \mathbf{m}_{\boldsymbol{\beta},i}(\boldsymbol{\mu};\boldsymbol{\theta}^{(n-1)})=\mathbf{V}_{\boldsymbol{\beta},i}(\boldsymbol{\theta}^{(n-1)})\left((\boldsymbol{\Omega}^{(n-1)})^{-1}\sum_{t=1}^{T_{(i)}}(\boldsymbol{\mu}_{it}-\mathbf{Z}_{it}\boldsymbol{\delta}^{(n-1)})+\boldsymbol{\Omega}_{(n-1)}^{-1}(\boldsymbol{\omega}^{(n-1)}+\mathbf{M}_{i}\boldsymbol{\varsigma}^{(n-1)})\right)\end{cases}\tag{16}$$

---

[3] Differentiating $R^{-1}\sum_{i=1}^{N}\sum_{r=1}^{R}Q_{i}(\tilde{\boldsymbol{\mu}}_{i,r}^{(n)},\boldsymbol{\theta}\,|\,\boldsymbol{\theta}^{(n-1)})$ with respect to the parameters $\boldsymbol{\theta}$ allows us to obtain these statistics, which are sufficient for the SA-Step in our case because $Q_{i}(\boldsymbol{\mu},\boldsymbol{\theta}\,|\,\boldsymbol{\theta}^{(n-1)})$ belongs to the family of exponential distributions.

The sequence $(\lambda_{(n)})_{n\in\mathbb{N}}$ used in the SA-Step must be a decreasing positive sequence such that $\lambda_{(1)}=1$, $\sum_{n=1}^{+\infty}\lambda_{(n)}=+\infty$ and $\sum_{n=1}^{+\infty}\lambda_{(n)}^2<+\infty$. This sequence defines the step of the stochastic approximation and affects the speed of the convergence of the algorithm to the Maximum Likelihood. Kuhn and Lavielle (2005) propose to set $\lambda_{(n)}=1$ for the first n_1 iterations of the algorithm and then gradually reduce $\lambda_{(n)}$.

Once $\hat{\boldsymbol{\theta}}$, the estimates of $\boldsymbol{\theta}$, have been obtained using the considered SAEM algorithm, we derive their standard errors following Ruud (1991).

Finally, given $\hat{\boldsymbol{\theta}}$, we are able to recover the individual vector of parameters $\hat{\boldsymbol{\mu}}_{(i)}$ for each farmer *i* using a Maximum A Posteriori (MAP) approach[4], i.e. the estimate $\hat{\boldsymbol{\mu}}_{(i)}$ of $\boldsymbol{\mu}_{(i)}$ is given by:

$$\begin{aligned}\hat{\boldsymbol{\mu}}_{(i)} &= \arg\max_{\boldsymbol{\mu}_{(i)}} f(\boldsymbol{\mu}_{(i)} \mid \overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i;\hat{\boldsymbol{\theta}}) \\ &= \arg\max_{\boldsymbol{\mu}_{(i)}} \begin{pmatrix} \sum_{t=1}^{T_{(i)}} \ln\varphi(\overline{x}_{it}-\mathbf{s}_{it}'\mathbf{h}^{-1}(\boldsymbol{\mu}_{it});\hat{\sigma}^2) \\ +\ln\varphi(\boldsymbol{\mu}_{(i)}-\boldsymbol{\iota}_{T_{(i)}}\otimes(\hat{\boldsymbol{\omega}}+\mathbf{M}_i\hat{\boldsymbol{\varsigma}})-\mathbf{Z}_{(i)}\hat{\boldsymbol{\delta}};\hat{\mathbf{G}}_i) \end{pmatrix},\end{aligned} \tag{17}$$

which is computed using the optimizing function in Stan via the rstan R package. Using these individual estimated parameter, $\hat{\boldsymbol{\mu}}_{(i)}$, we can then "calibrate" the crop input uses for each farmer *i* at each time *t*, $\hat{\mathbf{x}}_{it}$, as:

$$\hat{\mathbf{x}}_{it}(\hat{\boldsymbol{\mu}}_{it},\overline{\mathbf{x}}_{(i)})=\mathbf{exp}(\hat{\boldsymbol{\mu}}_{it})+\boldsymbol{\iota}_{T_{(i)}}(\overline{x}_{it}-\mathbf{s}_{it}'\mathbf{exp}(\hat{\boldsymbol{\mu}}_{it})), \tag{18}$$

where $\boldsymbol{\iota}_{T_{(i)}}$ is the column vector of one with dimension $T_{(i)}$. The second term in the right hand side of equation (20) allows reallocating the estimated residue of equation (2), $\hat{u}_{it}$, to each crop equally for a given farmer in a given period, and improves the fit of the estimated model to the observed data.

[4] It is also possible to consider the mean of the conditional distribution of $\boldsymbol{\mu}_{(i)}$ given $\overline{\mathbf{x}}_{(i)},\mathbf{s}_i,\mathbf{z}_i;\hat{\boldsymbol{\theta}}$.

4. Empirical applications

4.1. Data

This section presents an application aimed at illustrating the empirical tractability of our modelling approach as well as to demonstrate its ability to predict variable input uses per crop. The model presented in section 2 is estimated, by relying on the estimation procedure presented in section 3, on a panel data sample of 1,043 French grain crop producers located in the North and North-East of France between 2008 and 2014, which has been extracted from data provided by an accounting agency located in the French territorial division La Marne. Each farmer is present in the sample for at least three consecutive years and the sample contains a total of 4,856 observations. We consider the 11 crops that cover more that 90% of the total farm acreage in this area (wheat, winter barley, spring barley, corn, sugar beets, alfalfa, peas, rapeseed, poppy seed, potatoes, starchy potatoes). For each observation (i.e. for each year of observation for each farm), we have information on crop acreages, crop yields and input expenditures at the farm level, as fond in standard farm accountancy dataset, such as FADN data, generally available to agricultural economists. One on the main advantage of our sample is that it also contains cost accounting data, which provide information on input expenditures per crop. These observed crop input uses will serve as benchmark data to validate our estimation results.

We focus here on the two main chemical inputs used by crop producers: fertilizers and pesticides. The first two columns of Table 1 report the average fertilizer and pesticide uses per crop observed in the sample. These are quantities, expressed in constant euros/ha, that are computed by dividing input expenditures by the fertilizer and pesticide price indices (base 1 in 2010) provided by the French department of Agriculture. Table 1 also reports the frequencies at which each crop is produced as well as their average share in farm acreage. One can notice that the crops that are the most frequently produced, i.e. those for which we the most observations in our sample, are also those that represent the largest share in farm acreage when they are produced. This is notably the case of wheat and rapeseed notably. On the other hand, some crops like potatoes are not frequently produced and represent a small share of farm acreage when they are produced. As will be shown later, these features will have significant impacts on the accuracy of our crop input use estimates.

Table 1: Descriptive statistics of the sample

	Average input uses per crop (constant euros/ha, base 1 in 2010)		Frequency of production (% of observations)	Average acreage shares (%)	
	Pesticides	Fertilizers		In the whole sample	When the crop is produced
Winter wheat	1.86	2.36	100	35	35
Spring barley	1.04	1.75	87	15	18
Winter barley	1.55	2.01	65	6	10
Corn	1.08	1.90	34	5	14
Sugar beet	2.56	2.82	81	12	15
Alfalfa	0.62	2.62	62	7	11
Peas	1.54	0.84	26	2	7
Winter rapeseed	2.02	2.28	92	16	17
Blue opium poppy	0.67	1.27	8	1	7
Potato	7.59	3.15	11	1	8
Starch potato	5.21	3.38	8	1	10

4.2. Estimation results

We estimate the input allocation model presented in equations (5.a)-(5.c) for pesticide and fertilizer uses which are allocated to the aforementioned 11 crops. This is done by using the `rinpallEst` function of the `WInputAll` R package[5] that we specifically developed based on the estimation procedure presented in Section 3. Time dummies are included as control variables, $\mathbf{z}_{c,it}$, in equation (5.b) and average crop acreage shares at the farm level (centered at the sample mean), are included as control variables, $\mathbf{m}_{c,i}$, in equation (5.c).[6] The `rinpallEst` function allows for customizing the execution of the SAEM algorithm and checking its convergence with various option. Here we use R=100 draws in the S-Step of the algorithm at each iteration. We perform 300 iterations of the algorithm in the first stage of estimation, where the algorithm explores the space parameters without memory, tries to escape local maxima and quickly reach the neighborhood of the maximum likelihood estimator. In the second stage of estimation, the smoothing stage, the algorithms converges without difficulties and the convergence is checked by plotting of the values of parameter estimates and the value of the complete data log-likelihood against iteration numbers.

[5] This package is available on the CRAN: https://cran.r-project.org/web/packages/winputall/index.html

[6] We tested different specifications by also introducing weather variables in $\mathbf{z}_{c,it}$ and second, third, and fourth powers of the average crop shares at farm level in $\mathbf{m}_{c,i}$ but this didn't have a significant impact on the estimation results.

Let's recall that crop input uses, $\mathbf{x}_{(i)}$, are treated as random parameters following a log-normal distribution such that $\ln\left(\mathbf{x}_{(i)}\right)=\boldsymbol{\mu}_{(i)} \sim_{iid} \mathcal{N}(\boldsymbol{\iota}_{T_{(i)}} \otimes(\boldsymbol{\omega}_0+\mathbf{M}_i\boldsymbol{\varsigma}_0)+\mathbf{Z}_{(i)}\boldsymbol{\delta}_0,\mathbf{G}_{(i)})$, with $\mathbf{G}_{(i)}=\boldsymbol{\iota}_{T_{(i)}}\boldsymbol{\iota}'_{T_{(i)}}\otimes\boldsymbol{\Omega}_0+\mathbf{I}_{T_{(i)}}\otimes\boldsymbol{\Psi}_0$, and that we estimate the parameters characterizing this distribution in order to recover the value of these parameters for each observation in our sample. The estimated values of the main parameters defining the expectation and variance of the distribution of pesticide and fertilizer uses per crop are reported in Table 2.a and Table 2.b respectively, the complete estimation results being presented in Appendix A. These results show that the model fits well the data, all the parameters being significantly estimated. The statistical significance of the estimated parameters characterizing the variance of crop input uses of particular interest since it demonstrates the importance of the unobserved determinants of crop input uses decisions related to farms' and farmers' heterogeneity.

Table 2.a: Estimates of parameters characterizing the distribution of input uses per crop for pesticides (standard errors in parenthesis)

	Parameters characterizing the estimated mean of $\hat{\mu}_{c,it}=\ln(\hat{x}_{c,it})$			Parameters characterizing the estimated variance of $\hat{\mu}_{c,it}=\ln(\hat{x}_{c,it})$	
	$\hat{\omega}_{0,c}$	$\hat{\varsigma}_{c,0}$	$\bar{\hat{\delta}}_{c,0}$ [a]	$\hat{\psi}_{0,cc}$	$\hat{\Omega}_{0,cc}$
Winter wheat	0.502	0.058	0.102	0.042	0.007
	(0.015)	(0.087)	(0.009)	(0.006)	(0.000)
Spring barley	-0.139	0.011	0.063	0.050	0.024
	(0.029)	(0.217)	(0.024)	(0.011)	(0.003)
Winter barley	0.474	0.018	0.012	0.089	0.019
	(0.035)	(0.340)	(0.022)	(0.021)	(0.003)
Corn	-0.045	-0.188	0.024	0.058	0.020
	(0.035)	(0.128)	(0.022)	(0.015)	(0.003)
Sugar beet	0.904	-0.761	0.038	0.067	0.006
	(0.024)	(0.136)	(0.013)	(0.013)	(0.000)
Alfalfa	-2.309	0.729	0.005	0.036	0.031
	(0.040)	(0.256)	(0.029)	(0.013)	(0.004)
Peas	0.260	3.519	0.007	0.073	0.019
	(0.037)	(0.149)	(0.024)	(0.019)	(0.003)
Winter rapeseed	0.690	0.411	0.0100	0.043	0.007
	(0.020)	(0.111)	(0.012)	(0.010)	(0.001)
Blue opium poppy	-0.249	1.120	-0.001	0.027	0.022
	(0.035)	(0.523)	(0.024)	(0.010)	(0.003)
Potato	1.841	0.420	0.017	0.017	0.015
	(0.021)	(0.231)	(0.018)	(0.006)	(0.002)
Starch potato	1.483	0.145	-0.021	0.022	0.021
	(0.028)	0.358)	(0.021)	(0.008)	(0.003)

[a] *$\bar{\hat{\delta}}_{c,0}$ corresponds to the average effect of the time dummy variables in $\mathbf{z}_{c,it}$:* $\bar{\hat{\delta}}_{c,0}=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T_i}\sum_{t=1}^{Ti}\mathbf{z}'_{c,it}\hat{\boldsymbol{\delta}}_{c,0}$.

Table 2.b: Estimates of parameters characterizing the distribution of input uses per crop for fertilizers (standard errors in parenthesis)

	Parameters characterizing the estimated mean of $\hat{\mu}_{c,it} = \ln(\hat{x}_{c,it})$			Parameters characterizing the estimated variance of $\hat{\mu}_{c,it} = \ln(\hat{x}_{c,it})$	
	$\hat{\omega}_{0,c}$	$\hat{\varsigma}_{c,0}$	$\bar{\hat{\delta}}_{c,0}$ [a]	$\hat{\psi}_{0,cc}$	$\hat{\Omega}_{0,cc}$
Winter wheat	0.530 (0.019)	-0.396 (0.056)	0.165 (0.018)	0.037 (0.008)	0.028 (0.005)
Spring barley	0.376 (0.031)	0.065 (0.239)	0.092 (0.035)	0.044 (0.018)	0.041 (0.010)
Winter barley	0.532 (0.043)	0.309 (0.289)	0.043 (0.038)	0.074 (0.027)	0.039 (0.011)
Corn	0.462 (0.044)	0.295 (0.114)	0.029 (0.034)	0.074 (0.028)	0.027 (0.006)
Sugar beet	0.956 (0.031)	-1.361 (0.105)	0.128 (0.029)	0.074 (0.019)	0.033 (0.007)
Alfalfa	0.967 (0.038)	-0.906 (0.356)	0.167 (0.039)	0.071 (0.031)	0.110 (0.018)
Peas	0.205 (0.067)	-3.861 (0.128)	-0.002 (0.059)	0.070 (0.041)	0.051 (0.016)
Winter rapeseed	0.783 (0.028)	-1.110 (0.092)	0.103 (0.028)	0.041 (0.014)	0.026 (0.007)
Blue opium poppy	0.446 (0.081)	-6.177 (0.188)	0.010 (0.053)	0.050 (0.032)	0.034 (0.009)
Potato	1.232 (0.047)	1.118 (0.207)	-0.021 (0.049)	0.062 (0.030)	0.059 (0.013)
Starch potato	0.990 (0.066)	-0.300 (0.606)	0.020 (0.052)	0.042 (0.029)	0.052 (0.013)

[a] $\bar{\hat{\delta}}_{c,0}$ *corresponds to the average effect of the time dummy variables in* $\mathbf{z}_{c,it}$: $\bar{\hat{\delta}}_{c,0} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T_i}\sum_{t=1}^{Ti}\mathbf{z}'_{c,it}\hat{\boldsymbol{\delta}}_{c,0}$.

As explained in section 3, once we have estimated the parameters characterizing the distribution of the random parameters $\boldsymbol{\mu}_{(i)}$, we can "statistically calibrate" those parameters for each farmer in our sample and thus obtain a set of "farmer-specific models" that can then be used to predict crop input uses $\hat{\mathbf{x}}_{it} = (\hat{x}_{c,it} : c \in C)$,with $\hat{\mathbf{x}}_{it} = \exp(\hat{\boldsymbol{\mu}}_{it})$, for each farmer *i* at each time period *t*. One interesting feature is that this procedure also allows us to calibrate potential input uses $\hat{\mathbf{x}}_{it}$ for crops that have not been grown by the considered farmer in the considered time period. By comparing the predicted crop input uses, $\hat{\mathbf{x}}_{it}$, to their corresponding observed values available in our data sample, we can compute some fitting criteria of the model, which are reported in Table 3.

Table 3: Fitting criteria

	Pesticides				Fertilizers			
	Mean of predicted values (st. dev.)	Mean of observed values (st. dev.)	AAD	Sim-R2 %	Mean of predicted values (st. dev.)	Mean of observed values (st. dev.)	AAD	Sim-R2 %
Winter wheat	1.90 (0.39)	1.86 (.041)	0.16	71.22	2.14 (0.65)	2.36 (0.59)	0.29	81.04
Spring barley	0.95 (0.22)	1.05 (0.29)	0.18	39.61	1.66 (0.43)	1.75 (0.48)	0.25	54.28
Winter barley	1.69 (0.33)	1.56 (0.42)	0.27	38.35	1.88 (0.48)	2.01 (0.54)	0.23	70.85
Corn	1.01 (0.23)	1.08 (0.37)	0.22	32.88	1.71 (0.41)	1.90 (0.53)	0.25	65.36
Sugar beet	2.64 (0.43)	2.56 (0.65)	0.33	53.70	3.17 (0.87)	2.82 (0.90)	0.54	47.05
Alfalfa	0.10 (0.15)	0.62 (0.28)	0.51	7.63	3.36 (1.03)	2.62 (1.01)	0.82	47.46
Peas	1.36 (0.32)	1.54 (0.46)	0.31	22.65	1.27 (0.30)	0.84 (0.42)	0.37	32.16
Winter rapeseed	2.25 (0.36)	2.03 (0.52)	0.33	56.72	2.51 (0.54)	2.28 (0.59)	0.33	62.12
Blue opium poppy	0.78 (0.15)	0.67 (0.31)	0.27	11.72	1.61 (0.24)	1.27 (0.41)	0.28	15.93
Potato	6.40 (0.35)	7.59 (1.59)	1.22	23.01	3.32 (0.55)	3.15 (0.85)	0.87	1.85
Starch potato	4.34 (0.39)	5.21 (1.20)	0.89	39.22	2.83 (0.49)	3.38 (0.99)	0.68	54.47
Whole model				82.00				80.00

First, as shown by the means and standard deviations of pesticide and fertilizer uses per crop computed on the observed data and those computed on the estimated values, our estimates appear to lie in reasonable ranges. This is also illustrated by the absolute average difference (AAD) between true and predicted values. However, one can notice a significant heterogeneity in AAD values among crops. These values are, in fact, much lower for wheat (AAD = 0.16 for an average observed value of 1.90), rapeseed (AAD = 0.33 for an average observed value of 2.25), and sugar beet (AAD = 0.33 for an average observed value of 53.70) than for other crops less frequently produced in our sample (see Table 1), such as potatoes (AAD = 1.22 for an average observed value of 6.40) and alfalfa (AAD = 0.51 for an average observed value of 7.53). These differences among crops in the performance of our model to recover crop input uses are confirmed by the Sim-R^2 criteria, also reported in Table 3. The

Sim-R^2 criterion measures the quality of the prediction of observed crop input uses by the estimated models. It is obtained by regressing the observed crop input use values on their predicted values. With global Sim-R^2 criteria around 80%, the estimated models appear to provide very good predictions of crop input uses globally. Yet, if we consider these criteria on a crop per crop basis, we observe significant differences in the quality of prediction, crop input uses being better predicted for the most frequently produced crops in our sample. The lower performances we observe for some crops are in fact due to a lack of information on these crops to in the data. In fact, since these crops are rarely produced by farmers and, when they are produced only represent a small share of their acreage, input uses for them weigh little in total input uses at farm level. Furthermore, because they contain a lot of zeros, their observed acreage shares lack variability which hinders the estimation of associated parameters.

These results are further illustrated by Figures 1-3, which display plots of our predicted input uses against their observed "true" counterparts for two frequently produced crops, winter wheat (Figure 1) and rapeseed (Figure 2), and one rarely produced crop, potatoes (Figure 3). Figure 1 demonstrate that we obtain reasonably good results when estimating fertilizer and pesticide input uses for winter wheat, which is produced by all sampled farmers and represents 35% of the arable crop acreage on average in our dataset. Rapeseed is produced by 92% of the sampled farms but its average acreage share does not exceed 16%. Figure 2 shows that the estimated fertilizer and pesticide uses for rapeseed are of a bit lower quality than those for wheat, but still remain in acceptable ranges. Finally, Figure 3 shows that our estimation approach fits relatively poorly the chemical input uses for potato production, which only concerns 11% of the sampled farms (for an average crop acreage of 2%).

Based on these results, our proposed approach appear to perform well in allocating crop input uses to crops produced by farmers, provided that sufficient information on these crop is provided in the data on which the allocation is performed, or, stated in another, for crops that represent sufficiently large shares of farm acreages in the data. A potential solution to overcome the issues encountered for crops that are rarely produced by farmer would be to aggregate those crops in an "other crop" category. [7]

[7] Some tests performed that we have on different datasets, including FADN data, suggest that considering a 5% threshold level as the maximum acreage share for crops to be aggregated in an "other crop" provides satisfactory results.

Figure 1: Observed versus Estimated pesticide and fertilizer uses for wheat (constant euros/ha, base 1 in 2010)

Pesticides **Fertilizers**

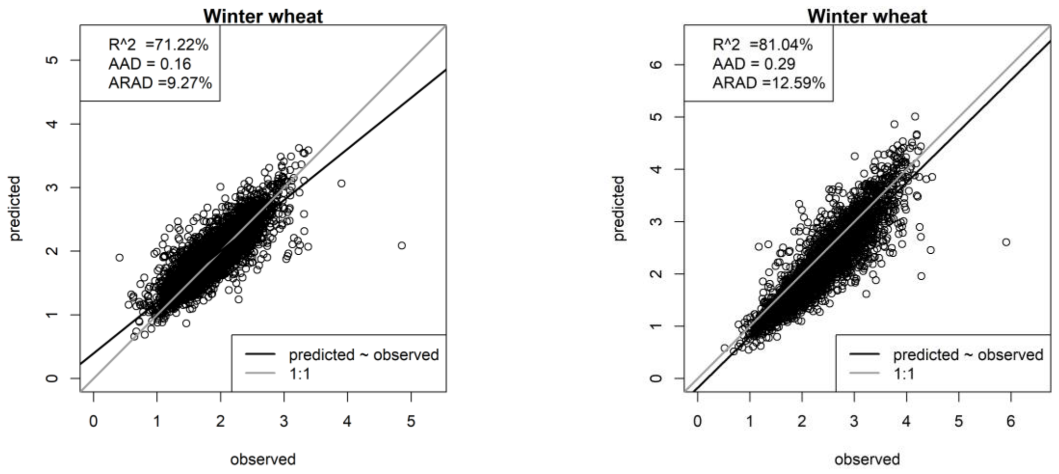


Figure 2: Observed versus Estimated pesticide and fertilizer uses for rapeseed (constant euros/ha, base 1 in 2010)

Pesticides **Fertilizers**

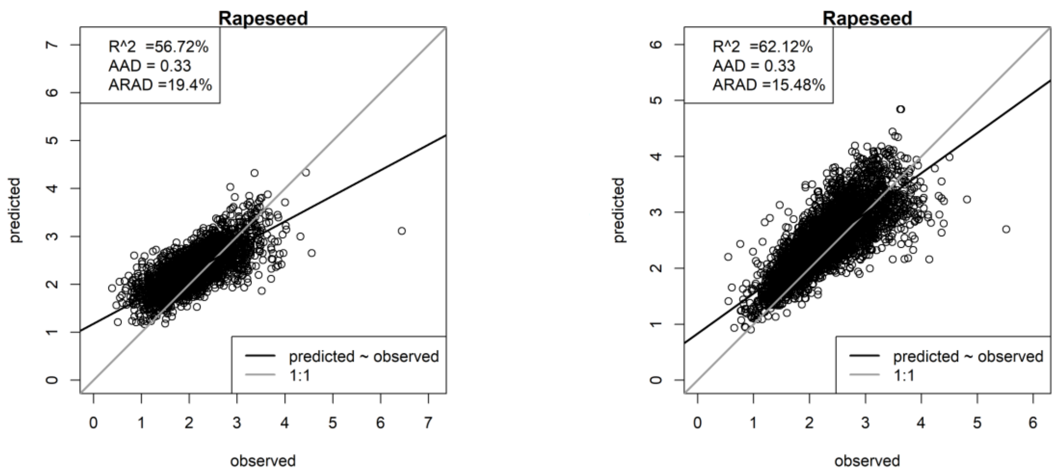
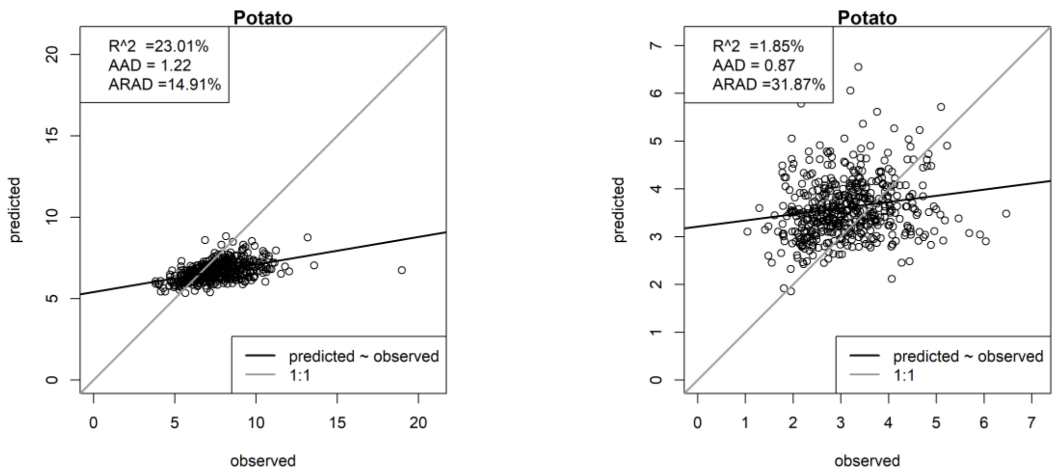


Figure 3: Observed versus Estimated pesticide and fertilizer uses for potato (constant euros/ha, base 1 in 2010)

Pesticides **Fertilizers**



5. Conclusion

We propose here a time-varying random parameter model of input allocation that allows accounting for observed and unobserved heterogeneity across farms and incorporating non-negativity constraints on crop input uses in flexible way. This model is estimated by using an approach based on an SAEM algorithm, which now available in the `WInpuAll` R package on the CRAN and can be apply to any farm accounting dataset, including FADN data.[8]

Our estimation results, obtained on a French cost accounting data sample are promising since estimated crop input uses are generally close to their true observed counterparts and that the accuracy of estimation decreases with the average share of the considered crops in farm acreage. This issue might be overcome by aggregating crops that only represent small shares of farm acreage into one single "other crops" category, even if it means losing some information. Another potential solution would be to use some prior information to guide the estimation. Ideally, additional information, such as average input use per crop at national or regional level, could be used as priors in the estimation procedure, but such information is

[8] We didn't mention it in the main body of the paper, as it wasn't relevant to our empirical application, but the `WInputAll` package also allows us to take into account the weights characterizing the representativeness of farms as provided in the FADN data.

rarely available. That being said, using priors, even if uninformative, to deal with the problem of rare events, can improve the quality of estimation and prediction of rare events, at least in the case of binary dependent variables (King and Zheng 2001). This is an area to be explored in future work.

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Appendix A Complete estimation results for lognormal specification: $\hat{\mu}_{c,it} = \ln(\hat{x}_{c,it})$

A.1 Estimation results for pesticides

Table A.1.1: Estimates of parameters $\hat{\delta}_{c,t,0}$ per crop per time period

		$\hat{\delta}_{c,t,0}$ Est.	Std. Err.		Est.	Std. Err.		Est.	Std. Err.
year 2008	Winter wheat	0.11	0.02	Sugar beet	0.02	0.02	Blue opium poppy	0.01	0.05
year 2009		0.08	0.02		-0.04	0.02		-0.01	0.05
year 2010		0.07	0.02		-0.02	0.03		0.01	0.05
year 2011		0.04	0.02		0.07	0.03		-0.07	0.05
year 2012		0.12	0.02		0.08	0.03		-0.04	0.05
year 2013		0.19	0.02		0.10	0.03		0.03	0.06
year 2014		0.25	0.02		0.17	0.03		0.09	0.06
year 2008	Spring barley	-0.01	0.04	Alfalfa	0.02	0.05	Potato	0.03	0.03
year 2009		0.08	0.04		0.11	0.05		0.03	0.03
year 2010		-0.06	0.05		-0.01	0.06		-0.02	0.04
year 2011		0.04	0.05		-0.06	0.06		0.01	0.04
year 2012		0.14	0.04		-0.01	0.06		0.08	0.04
year 2013		0.26	0.05		-0.04	0.06		-0.05	0.04
year 2014		0.10	0.06		-0.02	0.07		0.05	0.04
year 2008	Winter barley	-0.05	0.04	Peas	0.01	0.05	Starch potato	-0.07	0.04
year 2009		0.03	0.04		0.04	0.04		-0.11	0.04
year 2010		-0.01	0.04		-0.03	0.04		-0.11	0.04
year 2011		0.03	0.05		-0.03	0.05		0.01	0.05
year 2012		-0.02	0.05		0.06	0.05		0.08	0.04
year 2013		0.07	0.05		-0.02	0.06		0.07	0.05
year 2014		0.10	0.05		0.02	0.06		0.04	0.05
year 2008	Corn	0.01	0.04	Winter rapeseed	0.09	0.02			
year 2009		0.02	0.04		0.10	0.02			
year 2010		-0.05	0.04		0.06	0.03			
year 2011		-0.03	0.05		0.07	0.03			
year 2012		0.06	0.05		0.09	0.03			
year 2013		0.08	0.05		0.19	0.03			
year 2014		0.13	0.05		0.24	0.03			

Table A.1.2: Estimates of parameters characterizing the variance of error term u_{it} of the model

	$\hat{\sigma}^2$ Est.	Std. Err.
Parameters characterizing the estimated variance of u_{it}	0.040	0.0003

Table A.1.3: Estimates of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$

$\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.042	-0.001	0.011	0.026	0.000	-0.001	0.016	0.013	0.011	0.000	0.005
Spring barley	-0.001	0.050	0.027	0.013	0.021	-0.009	0.022	0.013	-0.006	-0.007	-0.002
Winter barley	0.011	0.027	0.089	0.032	0.034	-0.027	0.031	0.002	-0.014	-0.012	0.010
Corn	0.026	0.013	0.032	0.058	0.014	-0.007	0.021	0.013	0.004	-0.001	0.002
Sugar beet	0.000	0.021	0.034	0.014	0.067	-0.012	0.013	-0.014	-0.010	-0.005	0.009
Alfalfa	-0.001	-0.009	-0.027	-0.007	-0.012	0.036	-0.001	0.007	0.008	0.006	-0.006
Peas	0.016	0.022	0.031	0.021	0.013	-0.001	0.073	0.030	0.000	-0.010	-0.003
Winter rapeseed	0.013	0.013	0.002	0.013	-0.014	0.007	0.030	0.043	0.006	-0.003	-0.007
Blue opium poppy	0.011	-0.006	-0.014	0.004	-0.010	0.008	0.000	0.006	0.027	0.001	0.000
Potato	0.000	-0.007	-0.012	-0.001	-0.005	0.006	-0.010	-0.003	0.001	0.017	0.002
Starch potato	0.005	-0.002	0.010	0.002	0.009	-0.006	-0.003	-0.007	0.000	0.002	0.022

Table A.1.4: Estimates of Standard errors of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$

Standard errors of $\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.006	0.007	0.008	0.008	0.006	0.009	0.008	0.005	0.007	0.005	0.007
Spring barley	0.007	0.011	0.012	0.011	0.009	0.010	0.012	0.008	0.009	0.006	0.008
Winter barley	0.008	0.012	0.021	0.014	0.012	0.013	0.015	0.009	0.012	0.009	0.010
Corn	0.008	0.011	0.014	0.015	0.011	0.011	0.013	0.009	0.010	0.007	0.009
Sugar beet	0.006	0.009	0.012	0.011	0.013	0.011	0.011	0.008	0.009	0.006	0.008
Alfalfa	0.009	0.010	0.013	0.011	0.011	0.013	0.012	0.009	0.008	0.006	0.007
Peas	0.008	0.012	0.015	0.013	0.011	0.012	0.019	0.009	0.011	0.008	0.009
Winter rapeseed	0.005	0.008	0.009	0.009	0.008	0.009	0.009	0.008	0.007	0.006	0.007
Blue opium poppy	0.007	0.009	0.012	0.010	0.009	0.008	0.011	0.007	0.010	0.005	0.006
Potato	0.005	0.006	0.009	0.007	0.006	0.006	0.008	0.006	0.005	0.006	0.005
Starch potato	0.007	0.008	0.010	0.009	0.008	0.007	0.009	0.007	0.006	0.005	0.008

A.2 Estimation results for fertilizers

Table A.2.1: Estimates of parameters $\hat{\delta}_{c,t,0}$ per crop per time period

		$\hat{\delta}_{c,t,0}$							
		Est.	Std. Err.		Est.	Std. Err.		Est.	Std. Err.
year 2008	Winter wheat	-0.14	0.04	Sugar beet	-0.24	0.07	Blue opium poppy	0.03	0.11
year 2009		0.52	0.03		0.07	0.06		0.01	0.10
year 2010		0.14	0.04		0.32	0.05		0.05	0.11
year 2011		-0.02	0.05		0.18	0.06		-0.03	0.11
year 2012		0.17	0.04		0.25	0.06		-0.01	0.10
year 2013		0.31	0.04		0.31	0.07		0.04	0.11
year 2014		0.32	0.04		0.25	0.07		-0.03	0.12
year 2008	Spring barley	-0.04	0.07	Alfalfa	-0.07	0.08	Potato	-0.17	0.08
year 2009		0.40	0.05		0.55	0.05		-0.11	0.09
year 2010		-0.26	0.07		0.34	0.06		0.10	0.08
year 2011		0.17	0.07		-0.03	0.11		0.09	0.11
year 2012		0.15	0.06		0.19	0.08		-0.04	0.11
year 2013		0.18	0.08		0.14	0.10		0.05	0.10
year 2014		0.16	0.08		0.07	0.10		-0.03	0.12
year 2008	Winter barley	-0.21	0.08	Peas	-0.14	0.12	Starch potato	-0.09	0.11
year 2009		0.19	0.07		0.19	0.11		0.07	0.10
year 2010		-0.04	0.08		-0.15	0.12		-0.13	0.09
year 2011		-0.03	0.09		0.00	0.14		0.00	0.12
year 2012		0.06	0.09		0.07	0.13		-0.08	0.12
year 2013		0.17	0.09		-0.06	0.14		0.23	0.11
year 2014		0.30	0.08		0.09	0.13		0.25	0.12
year 2008	Corn	-0.10	0.08	Winter rapeseed	-0.05	0.06			
year 2009		0.24	0.06		0.19	0.05			
year 2010		-0.17	0.06		-0.01	0.06			
year 2011		-0.06	0.08		0.00	0.06			
year 2012		0.12	0.07		0.16	0.06			
year 2013		0.15	0.08		0.37	0.06			
year 2014		0.05	0.08		0.23	0.07			

Table A.2.2: Estimates of parameters characterizing the variance of error term u_{it} of the model

	$\hat{\sigma}^2$	
	Est.	Std. Err.
Parameters characterizing the estimated variance of u_{it}	0.048	0.001

Table A.2.3: Estimates of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$ for fertilizer

$\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.037	0.012	0.028	0.021	0.012	0.012	0.016	0.008	0.012	-0.026	0.014
Spring barley	0.012	0.044	0.019	0.020	0.016	0.002	0.012	0.011	0.001	-0.009	0.018
Winter barley	0.028	0.019	0.074	0.045	0.039	0.005	0.036	0.016	0.017	-0.026	0.019
Corn	0.021	0.020	0.045	0.074	0.028	0.008	0.028	0.028	0.018	-0.017	0.016
Sugar beet	0.012	0.016	0.039	0.028	0.074	0.005	0.041	0.016	0.015	-0.033	0.018
Alfalfa	0.012	0.002	0.005	0.008	0.005	0.071	0.001	0.006	0.006	0.000	0.010
Peas	0.016	0.012	0.036	0.028	0.041	0.001	0.070	0.019	0.020	-0.033	0.014
Winter rapeseed	0.008	0.011	0.016	0.028	0.016	0.006	0.019	0.041	0.009	-0.015	0.008
Blue opium poppy	0.012	0.001	0.017	0.018	0.015	0.006	0.020	0.009	0.050	-0.018	0.007
Potato	-0.026	-0.009	-0.026	-0.017	-0.033	0.000	-0.033	-0.015	-0.018	0.063	-0.014
Starch potato	0.014	0.018	0.019	0.016	0.018	0.010	0.014	0.008	0.007	-0.014	0.042

Table A.2.4: Estimates of standard errors of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$ for fertilizer

Standard errors of $\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.008	0.010	0.011	0.011	0.010	0.013	0.016	0.008	0.018	0.013	0.015
Spring barley	0.010	0.018	0.017	0.017	0.015	0.019	0.022	0.011	0.019	0.020	0.018
Winter barley	0.011	0.017	0.027	0.021	0.019	0.023	0.027	0.015	0.025	0.024	0.023
Corn	0.011	0.017	0.021	0.028	0.019	0.026	0.027	0.016	0.026	0.023	0.024
Sugar beet	0.010	0.015	0.019	0.019	0.019	0.018	0.023	0.013	0.024	0.021	0.021
Alfalfa	0.013	0.019	0.023	0.026	0.018	0.031	0.027	0.016	0.027	0.022	0.025
Peas	0.016	0.022	0.027	0.027	0.023	0.027	0.041	0.020	0.030	0.027	0.026
Winter rapeseed	0.008	0.011	0.015	0.016	0.013	0.016	0.020	0.014	0.018	0.016	0.017
Blue opium poppy	0.018	0.019	0.025	0.026	0.024	0.027	0.030	0.018	0.032	0.026	0.023
Potato	0.013	0.020	0.024	0.023	0.021	0.022	0.027	0.016	0.026	0.030	0.024
Starch potato	0.015	0.018	0.023	0.024	0.021	0.025	0.026	0.017	0.023	0.024	0.029

Appendix B Complete estimation results for normal specification: $\mu_{c,it} = x_{c,it}$

B.1 Estimation results for pesticides

Table B.1.1: Estimates of parameters characterizing the distribution of input uses per crop for pesticides (standard errors in parenthesis)

	Parameters characterizing the estimated mean of $\hat{\mu}_{c,it} = \hat{x}_{c,it}$			Parameters characterizing the estimated variance of $\hat{\mu}_{c,it} = \hat{x}_{c,it}$	
	$\hat{\omega}_{0,c}$	$\hat{\varsigma}_{c,0}$	$\bar{\hat{\delta}}_{c,0}$ [a]	$\hat{\psi}_{0,cc}$	$\hat{\Omega}_{0,cc}$
Winter wheat	1.706 (0.038)	0.190 (0.181)	0.182 (0.035)	0.177 (0.044)	0.079 (0.016)
Spring barley	0.942 (0.047)	-0.682 (0.238)	0.097 (0.049)	0.156 (0.066)	0.081 (0.036)
Winter barley	1.702 (0.079)	0.410 (0.870)	-0.011 (0.078)	0.419 (0.192)	0.285 (0.123)
Corn	1.039 (0.082)	-0.395 (0.317)	0.071 (0.060)	0.454 (0.142)	0.111 (0.051)
Sugar beet	2.781 (0.093)	-2.818 (0.474)	-0.018 (0.065)	1.216 (0.310)	0.196 (0.072)
Alfalfa	0.122 (0.051)	0.211 (0.134)	0.001 (0.038)	0.003 (0.006)	0.003 (0.002)
Peas	1.274 (0.124)	9.800 (0.800)	-0.018 (0.136)	1.003 (0.331)	0.415 (0.251)
Winter rapeseed	1.795 (0.067)	1.854 (0.330)	0.314 (0.058)	0.496 (0.133)	0.248 (0.073)
Blue opium poppy	0.849 (0.223)	-0.286 (4.791)	-0.055 (0.208)	0.186 (0.278)	0.172 (0.126)
Potato	7.042 (0.241)	-4.104 (1.778)	0.195 (0.222)	2.365 (1.808)	1.776 (0.765)
Starch potato	5.457 (0.275)	-4.903 (1.63)	-0.289 (0.194)	1.607 (1.667)	1.665 (0.687)

[a] $\bar{\hat{\delta}}_{c,0}$ *corresponds to the average effect of the time dummy variables in* $\mathbf{z}_{c,it}$: $\bar{\hat{\delta}}_{c,0} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T_i}\sum_{t=1}^{Ti}\mathbf{z}'_{c,it}\hat{\boldsymbol{\delta}}_{c,0}$

Table B.1.2: Estimates of parameters characterizing the variance of error term u_{it} of the model

	$\hat{\sigma}^2$	
	Est.	Std. Err.
Parameters characterizing the estimated variance of u_{it}	0.0045	0.0006

Table B.1.3: Estimates of parameters $\hat{\delta}_{c,t,0}$ per crop per time period

		$\hat{\delta}_{c,t,0}$							
		Est.	Std. Err.		Est.	Std. Err.		Est.	Std. Err.
year 2008	Winter wheat	0.25	0.07	Sugar beet	-0.08	0.14	Blue opium poppy	-0.10	0.39
year 2009		0.19	0.06		-0.28	0.13		0.10	0.36
year 2010		0.20	0.08		-0.26	0.15		-0.08	0.33
year 2011		0.00	0.08		0.19	0.17		0.03	0.43
year 2012		0.13	0.07		0.01	0.13		-0.11	0.48
year 2013		0.31	0.09		0.15	0.15		-0.15	0.47
year 2014		0.41	0.08		0.38	0.13		-0.20	0.51
year 2008	Spring barley	-0.01	0.09	Alfalfa	0.00	0.07	Potato	0.29	0.33
year 2009		0.16	0.10		0.00	0.07		0.19	0.34
year 2010		-0.04	0.12		0.00	0.08		-0.06	0.40
year 2011		-0.06	0.12		0.01	0.09		0.14	0.59
year 2012		0.26	0.09		0.00	0.08		0.91	0.49
year 2013		0.37	0.11		-0.01	0.09		0.01	0.50
year 2014		0.13	0.10		0.01	0.09		0.03	0.52
year 2008	Winter barley	-0.35	0.14	Peas	0.07	0.36	Starch potato	-0.83	0.42
year 2009		0.00	0.15		0.38	0.27		-1.12	0.36
year 2010		-0.05	0.17		-0.06	0.21		-0.96	0.42
year 2011		0.16	0.18		-0.05	0.29		-0.13	0.48
year 2012		-0.27	0.17		0.20	0.27		0.62	0.43
year 2013		0.24	0.16		-0.47	0.35		0.21	0.44
year 2014		0.42	0.17		-0.48	0.33		0.58	0.42
year 2008	Corn	-0.02	0.13	Winter rapeseed	0.32	0.12			
year 2009		-0.05	0.12		0.25	0.12			
year 2010		-0.10	0.15		0.15	0.13			
year 2011		-0.06	0.15		0.32	0.14			
year 2012		0.26	0.12		0.39	0.13			
year 2013		0.24	0.15		0.55	0.14			
year 2014		0.46	0.12		0.70	0.12			

Table B.1.4: Estimates of variance-covariance matrix $\hat{\psi}_0$ of $\mathbf{e}_i$ for pesticides

	$\hat{\psi}_0$										
	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.18	0.05	0.05	0.04	-0.11	0.00	0.00	-0.02	0.03	-0.20	-0.05
Spring barley	0.05	0.16	-0.04	-0.06	-0.23	0.00	0.02	0.05	0.02	-0.09	-0.16
Winter barley	0.05	-0.04	0.42	0.12	0.18	0.00	0.10	0.07	0.02	-0.10	0.05
Corn	0.04	-0.06	0.12	0.45	0.20	0.00	-0.23	-0.22	-0.01	0.02	0.02
Sugar beet	-0.11	-0.23	0.18	0.20	1.22	-0.01	-0.17	-0.25	-0.04	-0.37	0.12
Alfalfa	0.00	0.00	0.00	0.00	-0.01	0.00	0.01	0.01	0.00	0.00	0.00
Peas	0.00	0.02	0.10	-0.23	-0.17	0.01	1.00	0.41	0.10	-0.24	0.14
Winter rapeseed	-0.02	0.05	0.07	-0.22	-0.25	0.01	0.41	0.50	0.06	-0.11	-0.10
Blue opium poppy	0.03	0.02	0.02	-0.01	-0.04	0.00	0.10	0.06	0.19	-0.11	-0.06
Potato	-0.20	-0.09	-0.10	0.02	-0.37	0.00	-0.24	-0.11	-0.11	2.37	0.35
Starch potato	-0.05	-0.16	0.05	0.02	0.12	0.00	0.14	-0.10	-0.06	0.35	1.61

Table B.1.5: Estimates of standard errors of variance-covariance matrix $\hat{\psi}_0$ of $\mathbf{e}_i$ for pesticides

	Standard errors of $\hat{\psi}_0$										
	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.04	0.05	0.09	0.06	0.09	0.03	0.12	0.06	0.17	0.31	0.39
Spring barley	0.05	0.07	0.08	0.12	0.14	0.03	0.16	0.09	0.16	0.36	0.41
Winter barley	0.09	0.08	0.19	0.17	0.20	0.04	0.28	0.13	0.28	0.67	0.67
Corn	0.06	0.12	0.17	0.14	0.25	0.04	0.24	0.14	0.29	0.72	0.82
Sugar beet	0.09	0.14	0.20	0.25	0.31	0.07	0.35	0.16	0.40	0.76	0.89
Alfalfa	0.03	0.03	0.04	0.04	0.07	0.01	0.06	0.05	0.03	0.10	0.09
Peas	0.12	0.16	0.28	0.24	0.35	0.06	0.33	0.20	0.39	0.83	0.87
Winter rapeseed	0.06	0.09	0.13	0.14	0.16	0.05	0.20	0.13	0.29	0.53	0.61
Blue opium poppy	0.17	0.16	0.28	0.29	0.40	0.03	0.39	0.29	0.28	0.74	0.60
Potato	0.31	0.36	0.67	0.72	0.76	0.10	0.83	0.53	0.74	1.81	1.49
Starch potato	0.39	0.41	0.67	0.82	0.89	0.09	0.87	0.61	0.60	1.49	1.67

B.2 Estimation results for fertilizers

Table B.2.1: Estimates of parameters characterizing the distribution of input uses per crop for fertilizers (standard errors in parenthesis)

	Parameters characterizing the estimated mean of $\hat{\mu}_{c,it} = \hat{x}_{c,it}$			Parameters characterizing the estimated variance of $\hat{\mu}_{c,it} = \hat{x}_{c,it}$	
	$\hat{\omega}_{0,c}$	$\hat{\varsigma}_{c,0}$	$\bar{\hat{\delta}}_{c,0}$ [a]	$\hat{\psi}_{0,cc}$	$\hat{\Omega}_{0,cc}$
Winter wheat	1.877 (0.054)	-1.090 (0.147)	0.282 (0.054)	0.292 (0.101)	0.126 (0.036)
Spring barley	1.530 (0.074)	-0.388 (0.460)	0.175 (0.082)	0.177 (0.155)	0.262 (0.107)
Winter barley	1.666 (0.119)	0.268 (1.277)	0.275 (0.127)	0.847 (0.420)	0.962 (0.361)
Corn	1.597 (0.112)	0.491 (0.338)	0.086 (0.095)	0.492 (0.266)	0.211 (0.069)
Sugar beet	2.791 (0.144)	-4.126 (0.551)	0.385 (0.115)	1.423 (0.684)	0.541 (0.201)
Alfalfa	2.674 (0.183)	-5.329 (1.421)	0.789 (0.149)	2.325 (1.306)	2.187 (0.613)
Peas	0.788 (0.176)	-0.809 (2.622)	0.032 (0.212)	1.199 (0.774)	0.585 (0.412)
Winter rapeseed	2.308 (0.098)	-3.057 (0.354)	0.280 (0.095)	0.894 (0.326)	0.336 (0.161)
Blue opium poppy	1.809 (0.383)	-13.639 (1.689)	-0.006 (0.407)	1.002 (1.363)	0.748 (0.536)
Potato	3.618 (0.321)	5.579 (1.828)	-0.265 (0.380)	3.020 (2.789)	3.464 (1.531)
Starch potato	3.427 (0.380)	-6.550 (1.968)	0.333 (0.310)	1.896 (2.500)	1.955 (1.0798)

[a] $\bar{\hat{\delta}}_{c,0}$ *corresponds to the average effect of the time dummy variables in* $\mathbf{z}_{c,it}$: $\bar{\hat{\delta}}_{c,0} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T_i}\sum_{t=1}^{Ti}\mathbf{z}'_{c,it}\hat{\boldsymbol{\delta}}_{c,0}$

Table B.2.2: Estimates of parameters characterizing the variance of error term u_{it} of the model

	$\hat{\sigma}^2$	
	Est.	Std. Err.
Parameters characterizing the estimated variance of u_{it}	0.017	0.002

Table B.2.3: Estimates of parameters $\hat{\delta}_{c,t,0}$ per crop per time period

		$\hat{\delta}_{c,t,0}$							
		Est.	Std. Err.		Est.	Std. Err.		Est.	Std. Err.
year 2008	Winter wheat	-0.22	0.12	Sugar beet	-0.54	0.28	Blue opium poppy	-0.23	0.97
year 2009		1.02	0.08		0.24	0.18		0.36	0.52
year 2010		0.23	0.11		0.83	0.20		-0.03	0.67
year 2011		-0.14	0.12		0.54	0.29		0.25	0.77
year 2012		0.20	0.12		0.68	0.25		-0.14	1.15
year 2013		0.51	0.12		0.75	0.25		-0.03	0.96
year 2014		0.59	0.13		0.66	0.28		-0.31	1.00
year 2008	Spring barley	-0.06	0.17	Alfalfa	0.00	0.38	Potato	-1.24	0.59
year 2009		0.80	0.11		2.22	0.23		-0.97	0.42
year 2010		-0.47	0.16		1.24	0.27		0.73	0.53
year 2011		0.27	0.18		-0.10	0.40		0.37	1.05
year 2012		0.22	0.16		1.01	0.34		-0.93	0.83
year 2013		0.40	0.19		0.93	0.33		0.79	0.65
year 2014		0.27	0.20		0.55	0.35		-0.41	0.99
year 2008	Winter barley	-0.32	0.29	Peas	-0.66	0.57	Starch potato	-0.65	0.87
year 2009		0.73	0.18		0.42	0.35		0.89	0.42
year 2010		0.06	0.25		0.20	0.29		-0.06	0.49
year 2011		0.09	0.34		0.20	0.50		-0.10	0.78
year 2012		0.32	0.29		0.12	0.43		-0.02	0.70
year 2013		0.48	0.28		-0.29	0.58		1.31	0.76
year 2014		1.02	0.30		0.33	0.53		1.75	0.76
year 2008	Corn	-0.17	0.22	Winter rapeseed	-0.13	0.22			
year 2009		0.51	0.13		0.43	0.14			
year 2010		-0.27	0.16		0.09	0.18			
year 2011		-0.16	0.26		0.14	0.23			
year 2012		0.32	0.21		0.43	0.21			
year 2013		0.32	0.22		1.02	0.20			
year 2014		0.11	0.23		0.42	0.23			

Table B.2.4: Estimates of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$ for fertilizers

$\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.293	0.022	0.094	-0.039	-0.170	0.137	0.069	-0.299	0.149	-0.195	-0.075
Spring barley	0.022	0.177	0.072	-0.015	-0.021	-0.178	-0.056	-0.031	0.002	-0.055	0.067
Winter barley	0.094	0.072	0.848	0.248	0.265	-0.352	0.173	0.147	0.109	-0.679	0.143
Corn	-0.039	-0.015	0.248	0.493	0.120	0.003	0.070	0.251	-0.011	-0.428	0.046
Sugar beet	-0.170	-0.021	0.265	0.120	1.423	-0.576	0.209	0.584	0.223	-0.974	0.085
Alfalfa	0.137	-0.178	-0.352	0.003	-0.576	2.325	0.625	-0.311	-0.119	0.212	-0.171
Peas	0.069	-0.056	0.173	0.070	0.209	0.625	1.199	0.041	0.100	-0.337	0.128
Winter rapeseed	-0.299	-0.031	0.147	0.251	0.584	-0.311	0.041	0.894	-0.113	-0.517	0.006
Blue opium poppy	0.149	0.002	0.109	-0.011	0.223	-0.119	0.100	-0.113	1.002	-0.227	0.036
Potato	-0.195	-0.055	-0.679	-0.428	-0.974	0.212	-0.337	-0.517	-0.227	3.020	-0.121
Starch potato	-0.075	0.067	0.143	0.046	0.085	-0.171	0.128	0.006	0.036	-0.121	1.896

Table B.2.5: Estimates of standard errors of variance-covariance matrix $\hat{\boldsymbol{\psi}}_0$ of $\mathbf{e}_i$ for fertilizers

Standard errors of $\hat{\boldsymbol{\psi}}_0$

	Winter wheat	Spring barley	Winter barley	Corn	Sugar beet	Alfalfa	Peas	Winter rapeseed	Blue opium poppy	Potato	Starch potato
Winter wheat	0.101	0.103	0.167	0.141	0.227	0.314	0.258	0.146	0.475	0.510	0.618
Spring barley	0.103	0.155	0.213	0.181	0.263	0.365	0.313	0.176	0.474	0.603	0.610
Winter barley	0.167	0.213	0.420	0.300	0.488	0.680	0.495	0.285	0.971	1.161	1.147
Corn	0.141	0.181	0.300	0.266	0.481	0.706	0.472	0.302	0.739	0.932	1.040
Sugar beet	0.227	0.263	0.488	0.481	0.684	0.710	0.662	0.378	0.910	1.058	1.177
Alfalfa	0.314	0.365	0.680	0.706	0.710	1.306	0.761	0.540	1.282	1.226	1.648
Peas	0.258	0.313	0.495	0.472	0.662	0.761	0.774	0.449	0.931	1.353	1.343
Winter rapeseed	0.146	0.176	0.285	0.302	0.378	0.540	0.449	0.326	0.812	0.880	1.046
Blue opium poppy	0.475	0.474	0.971	0.739	0.910	1.282	0.931	0.812	1.363	1.771	1.585
Potato	0.510	0.603	1.161	0.932	1.058	1.226	1.353	0.880	1.771	2.789	2.268
Starch potato	0.618	0.610	1.147	1.040	1.177	1.648	1.343	1.046	1.585	2.268	2.500

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