Income sharing for common pool resources with uncertain productivity

Claudia Kelsall, Martin F Quaas, Nicolas Quérou

CEE-M
CENTER FOR ENVIRONMENTAL ECONOMICS - MONTPELLIER

Income sharing for common pool resources with uncertain productivity

Claudia Kelsall,
Martin F. Quaas
&
Nicolas Quérou

CEE-M Working Paper 2025-08

INRAE
CNRS
L'INSTITUT agro Montpellier

Income sharing for common pool resources with uncertain productivity *

C. Kelsall[a,b], M. F. Quaas[a], N. Quérou[b]

[a] Department of Economics, University of Leipzig and German Centre for Integrative Biodiversity Research (iDiv) Halle-Jena-Leipzig, Germany
[b] CEE-M, Univ. Montpellier, CNRS, INRAe, Institut Agro, Montpellier, France

June 23, 2025

Abstract

We address the interconnected issues of externalities in the use of a natural common-pool resource system and adverse effects of environmental uncertainty on risk-averse resource users. We set up a simple, analytical model, and derive conditions such that insurance by means of an income sharing mechanism mitigates externalities, and hence achieves a socially optimal outcome. For this to materialize, we show that the sharing rule must be sufficiently sensitive to individual contributions, and specifically show that a simple, proportional sharing rule can provide incentives for both, efficient contributions to the income pool and efficient use of the resource system.

Keywords : Resource Economics | Informal Insurance | Risk Aversion

JEL Codes : Q2; D81

*The authors acknowledge the ANR-DFG project CRaMoRes (Grant ANR-19-FRAL-0010-01 and QU 357/14-1). M. Quaas and C. Kelsall acknowledge funding from the German Federal Ministry of Research, Technology and Space under Grant 03F0976E. C. Kelsall acknowledges the support of labEx CeMEB, an ANR "Investissements d'avenir" program (ANR-10-LABX-04-01).

1 Introduction

The challenges of using common pool resources have been extensively researched (Ostrom, 1990), and yet the inefficiency of common-pool resource systems remains a pressing issue (Stavins, 2011). Theory predicts that individuals will over-use a common-pool resource under open access, as they do not internalize the opportunity costs imposed on other resource users. Uncertainty related to the productivity of the resource can alleviate this problem (Bramoullé and Treich, 2009), due to precautionary behavior. However, uncertain productivity of a natural resource can also increase resource extraction (Kelsall et al., 2023), and thus exacerbate the problem of resource overuse. Whilst cooperation is a potential solution to the commons problem, designing mechanisms to align individual incentives with the collective benefit remains difficult.

This paper studies the potential for an endogenous income sharing mechanism to induce cooperation and achieve efficient use of a common-pool resource with uncertain productivity. To do this, we leverage the fact that risk-averse individuals have an incentive to contribute to an income pool in order to spread risk. We focus on the use of a common-pool resource, which is particularly relevant in an environmental context. Fisheries and freshwater are prominent examples of common-pool resources where the difficulty to exclude beneficiaries from the use of the resource can lead to a tragedy of the commons and over-use (Cornes and Sandler, 1996; Quérou et al., 2022). Other relevant examples include climate mitigation, carbon offsets, green infrastructure (Kotchen, 2009), and biodiversity investments (Quaas and Baumgärtner, 2008).

It has been shown that sharing mechanisms can address the problem of efficient provision, thus solving the problem created by free-riding incentives (Deacon, 2012; Lefebvre et al., 2014) whilst also providing insurance by spreading risk (Tilman et al., 2018; Sethi et al., 2012). Significant analytical contributions from Kaffine and Costello (2011) and Tilman et al. (2018) examine an income pooling mechanism with exogenous participation and constant sharing rules where agents receive a fixed fraction of the pooled income. Kaffine and Costello (2011) develop a deterministic, dynamic model of income sharing in a spatially connected fishery and show that efficiency can only be achieved when all income is shared. When looking at endogenous cooperation, they side-step the question of free-riding on the contributions of others by adopting a Nash reversion strategy to punish defectors and find that efficiency can only be achieved if payoffs vary between patches. The authors admit that it may be difficult for individuals to agree on constant, heterogeneous payoffs due to spatial externalities because fish harvested in one patch may come from larvae produced in another patch due to dispersal. Tilman et al. (2018) develop a stochastic dynamic model, and find that income sharing mechanisms have the potential to provide insurance to individuals by spreading risk; however, free-riding incentives can lead to the breakdown of agreements.

In practice, revenue and risk sharing mechanisms are common in the management of wild fisheries (Mumford et al., 2009; Aceves-Bueno et al., 2017; Uchida and Wilen, 2005; Deacon,

2012) where co-management between governments and resource users through fishing cooperatives provides a foundation for successful coordination (Deacon, 2012; OECD and Uchida, 2010). However, empirical evidence of successful agreements tends to be over-represented in the literature. It only arises when free-riding incentives are diminished through strong social norms or costly external enforcement (Platteau and Seki, 2001). Traditionally, income sharing agreements impose constant, equal sharing rules (OECD and Uchida, 2010; Platteau and Seki, 2001), and whilst theory predicts this may lead to efficient outcomes if everyone agrees to share all of their income (Kaffine and Costello, 2011), in practice free-riding incentives lead to the collapse of many agreements (Sethi et al., 2012; Platteau and Seki, 2007). We develop an endogenous sharing mechanism that overcomes the challenges associated with free-riding and the collapse of many agreements, meaning that there is no reason to suggest income sharing cannot provide a solution to the management of other collective goods with inefficient incentive structures.

We build on the analytic work of Kaffine and Costello (2011) and Tilman et al. (2018) and study an endogenous income sharing mechanism. We develop a game-theoretic model of individual investments in a common-pool resource system, with uncertain resource productivity. Individuals retain a fraction of these uncertain benefits (retention), whereas the residual benefits are enjoyed by all other individuals (diffusion). In the absence of income sharing, individual investments are inefficient due to the combined effects of the diffusion externality and risk-averse behavior in the absence of insurance. In the presence of a diffusion externality, individuals disregard the benefits that spread to others, reducing investments. When these benefits are uncertain, risk-averse individuals further reduce their investment to self-insure against potential losses when financial insurance is unavailable. However, this self-insurance effect is weakened because the diffusion externality also causes them to ignore the risk that diffuses to others, diminishing the perceived impact of uncertainty. Thus, policy directly aimed at internalizing the diffusion externality may not lead to the expected increase in contributions if insurance is unavailable. In order to develop a policy to achieve efficient provision, the impact of risk-averse behavior must be considered.

In this paper, we develop a model where agents have access to an income sharing mechanism and can choose to contribute a proportion of their net benefits of resource investments to a mutual fund. Income is then pooled and redistributed, where payoffs from the fund are subject to a predetermined sharing rule. We aim to develop a sharing mechanism that induces cooperation, such that individuals compete to win a larger share of the pool. Contributing to the pool will provide insurance through a spread of risk and internalize the diffusion externality because individuals must now consider the social benefit of their contribution to the good. We develop a sharing rule, dependent on the proportion shared, that leads to efficient provision of the good. This rule leverages the fact that risk-averse individuals will want to contribute to the income pool to spread risk as insurance.

Our paper makes two contributions to the literature. The first is whether endogenous income sharing with a payoff rule can achieve efficient investments into a common-pool resource system. We provide an example of a simple sharing mechanism that achieves the socially optimal allocation, which overcomes the free-riding incentive associated with exogenous income sharing mechanisms typically studied in the literature.

Related to this, our second contribution regards contract design and the interaction between common-pool resource use, insurance, and moral hazard. Income sharing is an informal insurance mechanism, where the insurance dividend is analogous to the payoff from the pool. Using a payoff rule means agents are incentivized to change their behavior and influence their insurance dividend. In this sense, we use moral hazard to incentivize individuals to contribute to the pool.

Our results find that when an endogenous income sharing mechanism is available, the design of the sharing rule determines contributions. When the sharing rule is an exogenous constant, no income is shared due to free-riding. Using a 'competitive' sharing rule changes agents' behavior, inducing cooperation. When the sharing rule depends on the proportion of income shared, individuals are motivated to contribute to the pool. However, efficiency will only be achieved if the elasticity of the rule with respect to individual contributions is sufficiently high. We define a simple 'proportional sharing rule' which satisfies these conditions and thus conclude that income sharing may be a promising instrument for increasing collective good contributions. This rule leverages the fact that risk-averse agents demand insurance. Instead of free-riding, individuals are encouraged to contribute to the income pool to spread risk. By doing so, they must now consider the social benefit of their contribution to the collective good, which overcomes the diffusion externality and achieves efficient provision.

2 Model Framework

We consider a resource system with N individuals, who all have the same initial endowment x_0. Each individual $i = 1,...,N$ chooses the amount s_i, $0 \leq s_i \leq x_0$ they invest in resource reproduction, which is given by $z_i\, g(s_i)$. Reproduction is uncertain, as it depends on a stochastic shock z_i with unit mean and non-negative support. Expected resource productivity $g(s_i)$ is increasing with investment at a diminishing rate, $g'(s_i) > 0$ and $g''(s_i) < 0$. To assure interior solutions, we further impose the assumption $g'(0) = \infty$.

The resource system is interconnected: Of the resource reproduction, a fraction D accrues to the investor, whereas the remaining fraction $1-D$ goes to each of the other resource users $j \neq i$ in equal shares, such that everyone else receives a fraction $D_e := (1-D)/(N-1)$. As our leading example is a renewable resource that disperses in space, we call the fraction D the 'retention rate', and the fraction D_e the 'diffusion rate'.

This is a fairly general model of a common-pool resource system. In the case of a fishery, for example, each individual would represent the individual or group of individuals having the right to use the fishery resources, for example under a territorial user rights in fisheries, TURF, system, (Wilen et al., 2012). Whereas a fraction of the stock left after harvesting will stay in the TURF, there will often be some fraction of fish that diffuses to neighboring TURFs. In case of agro-biodiversity, an important common-pool resource is the diversity of pollinating insects (Quaas and Baumgärtner, 2008; Augeraud-Véron et al., 2019). Enhancing insect biodiversity provides pollination services on the owner's land, and also on the fields of the neighbors, as insects diffuse in space. For the case of naturally regenerating forests, as a third example, the seeds of the trees contribute to the recruitment of the owner's forest stands, and also on the stands of the neighbors (Tahvonen, 2015). In all these cases, the productivity of the resource stock is subject to considerable uncertainty, including environmental shocks on recruitment of fish or trees, as well as on the abundance of pollinating insects.

Individual i's net benefit from investing in the natural resource is

$$\pi_i = x_0 - s_i + z_i g(s_i) D + \sum_{j \neq i} z_j g(s_j) D_e. \tag{1}$$

In this expression, the first two terms, $x_0 - s_i$, are the residual endowment after investing, the second term is the benefit of own investment that accrues to the investor, and the last term is the total benefit from all others' resource investments. Although the shocks to resource productivity are independent and specific to each individual, the resource connectivity leads to dependence between investment decisions and productivity shocks.

To aid understanding of how resource connectivity and how the diffusion of risks influences decision-making, we distinguish two types of risk. *Internal risk* is the risk associated with an individual's own return on investment, and associated to the term $z_i\, g(s_i)\, D$. *External risk* is the risk associated with the returns on investments for everyone else, and associated to the term $\sum_{j \neq i}^{N} z_j\, g(s_j)\, D_e$. In line with this, we will call the shocks on the returns on investments for everyone else the *external shocks*, and the shock on ones own investment the *internal shock*.

Additionally, we consider an income sharing mechanism: Individuals contribute a fraction $\alpha_i \in [0,1]$ of their net benefit to an income pool. They decide on α_i ex-ante, before the risk is realized. After uncertainty has resolved, and net benefits materialized, each individual receives a dividend $\gamma_i := \gamma(\alpha_i)$ from the income pool. This dividend is non-decreasing, and concave, in the fraction of their net benefit they contributed: $\gamma'(\alpha_i) \geq 0$ and $\gamma''(\alpha_i) \leq 0$. Possible income sharing mechanisms differ in the way the dividend is specified. Our research question is how the specification of the dividend's dependency on the individuals' contributions α_i affects the efficiency of the common-pool resource system.

With the income sharing mechanism in place, individual i's income is given by

$$y_i = (1 - \alpha_i)\, \pi_i + \gamma(\alpha_i) \sum_{j=1}^{N} \alpha_j\, \pi_j, \tag{2}$$

where the net benefit π_i is given in equation (1).

Our main focus is on the Nash equilibrium, where individuals choose s_i and α_i to maximize expected utility,

$$\max_{s_i\,\alpha_i} \mathbb{E}\left[u(y_i)\right], \tag{3}$$

taking the other individual's decisions as given. The individual's Von Neumann-Morgenstern utility function is increasing and concave, $u'(y_i) > 0$ and $u''(y_i) < 0$, where $u''(y_i)/u'(y_i)$ is a measure of the individual's absolute risk aversion in the Arrow-Pratt sense.

We compare the Nash equilibrium – for different income sharing mechanisms, i.e. different specifications of the dividend $\gamma(\alpha_i)$ – to two benchmarks: One benchmark is the social optimum, which we define as the allocation that maximizes the sum of expected utilities,

$$\max_{\{s_i,\alpha_i\}} \sum_{i=1}^{N} \mathbb{E}\left[u(y_i)\right]. \tag{4}$$

The other benchmark is the Nash equilibrium without an income sharing mechanism in place.

3 Benchmark 1: Social optimum

As the first benchmark we consider the social optimum, the allocation that maximizes social welfare, equation (4). Due to symmetry, all individuals will invest the same amount s^*, contribute the same fraction α^* to the income pool, and receive equal shares $\gamma = 1/N$ from the pool. In the social optimum, every individual will invest the same amount, giving rise to incomes

$$y = x_0 - s + g(s)\left(z_i D + z_e\,(1-D) + \frac{\alpha}{N}\,(ND-1)\,(z_e - z_i)\right). \tag{5}$$

This expression for each individual's income is obtained by using symmetry in (1) and (2), and using $D_e = (1-D)/(N-1)$. In equation (5), we use z_i to denote the internal shock and z_e to denote the external shocks on the representative individual. As all external shocks have symmetric effects on the representative individual's income, there is no need to distinguish between external shocks.

The optimization problem (4) leads to two conditions, one for the socially optimal investment s^* and another one for the socially optimal fraction α^* contributed to the income pool. To determine the optimal α^*, consider the derivative of the representative individual's expected utility with respect to α:

$$\frac{\partial \mathbb{E}\left[u\left(y\right)\right]}{\partial \alpha} = g(s^*)\frac{ND-1}{N}\,\mathbb{E}\left[\left(z_e - z_i\right)u'\left(y\right)\right]. \tag{6}$$

This expression is zero if and only if $D = 1/N$, i.e. if only a fraction $1/N$ of the return remains in the hands of the investor. We conclude that in this case it does not matter how much to contribute to the income pool, as in this case the representative individual's income is independent of α. The reason is the diffusion of the resource productivity spreads risk and investments in exactly the same way as an income pool would do.

However, $D = 1/N$ is just a knife-edge case, and the more relevant cases are $D \neq 1/N$. In this case, the sign of the right-hand side of (6) determines the optimal fraction shared. We find that it is socially optimal to contribute all of the representative individual's net benefit to the income pool.

Proposition 1. *When $D \neq 1/N$, $\alpha^* = 1$.*

Proof. See Appendix A. □

The result is intuitive: full pooling of net benefits is socially optimal, as it provides a means to redistribute income in favor of those who face the relatively less favorable shock on investment, and thus minimizes income risk. The resulting income of each individual in the social optimum is obtained as

$$y^* = x_0 - s + \frac{z_i + (N-1)\,z_e}{N}\,g(s). \tag{7}$$

The shocks are spread out as far as possible and dispersal of the resource plays no role, as it is offset by full pooling. Given full sharing, $\alpha^* = 1$, we proceed to characterize socially optimal investment. The corresponding optimality condition, derived in Appendix B, can be written as

$$\mathbb{E}\left[u'(y^*)\right] = g'(s^*)\,\mathbb{E}\left[z_i\,u'(y^*)\right] \tag{8}$$

Optimal investment equates marginal costs – the expected utility foregone by giving up a unit of the sure endowment – with marginal benefit, which is the marginal resource productivity multiplied by the marginal expected utility of the income gained. The marginal utility gain depends on the productivity risk of the resource system, which is minimized by full income pooling.

We further conclude that a risk reduction has a positive effect on investments. Due to uncertainty, the expected marginal utility term on the right-hand side of equation (8) – a part of the expected marginal benefit of investment – is smaller than the expected marginal utility term on the left-hand side of the condition – the expected marginal cost of investment. Formally, we have the following proposition

Proposition 2.

$$\mathbb{E}\left[u'(y^*)\right] > \mathbb{E}\left[z_i\,u'(y^*)\right]. \tag{9}$$

Proof. See Appendix C. □

4 Benchmark 2: Nash equilibrium without income sharing

When no sharing mechanism exists, each individual's income equals the net benefit of investment, given by equation (1). In Nash equilibrium, each individual chooses their own investment, taking the others' investment decisions as given. Due to symmetry, all will choose the same investment $\hat{s}$, characterized by the condition (see Appendix B)

$$\mathbb{E}[u'(\hat{y}_0)] = D\,g'(\hat{s}_0)\,\mathbb{E}[z_i\,u'(\hat{y}_0)], \tag{10}$$

where the resulting income is

$$\hat{y}_0 = x_0 - \hat{s}_0 + g(\hat{s}_0)(z_i D + (N-1)\,z_e D_e), \tag{11}$$

given by equation (2) with $s = \hat{s}_0$ and $\alpha = 0$. Given the assumptions on utility and resource productivity, the symmetric Nash equilibrium is unique.

Condition (10) equates marginal costs of investment – the expected utility foregone by giving up a unit of the sure endowment – with the individual's own marginal benefit. It deviates from the social optimum, given in equation (8), in two ways. First, only the retained fraction of the return is considered, and thus the marginal benefit is discounted by a factor D equal to the internalized share D of the return on investment. Accordingly, the equilibrium investment will be smaller than the socially optimal investment. Moreover, as no income sharing is available, the full internal risk shows up in the condition. Individuals self-insure against the risk they face themselves, and do not consider the external risk for others.

The risk that the individual faces itself further reduces the investment. Kelsall et al. (2023) show this for the case without spillovers. This result generalizes to an interconnected resource system where investment benefits partially spill over to others. Similar as in the efficient case, we have that $\mathbb{E}\,[u'(\hat{y}_0)] > \mathbb{E}\,[z_i\,u'(\hat{y}_0)]$.

This shows that risk induces the individuals to reduce their investment in the non-cooperative setting. Proposition 2 has shown that this effect is present in the social optimum as well. If the resource system would not be interconnected, and each individual would fully retain the benefit of investment, i.e., if $D = 1$, the case without income sharing would correspond to the social optimum.

The effect that individuals reduce their investment into a resource with uncertain productivity can be interpreted as a form of self-insurance, as individuals act to reduce the size of a potential loss (Ehrlich and Becker, 1972). As this literature shows, access to other types of insurance will reduce self-insurance. We now turn to studying income sharing, which provides such insurance.

5 Endogenous Income Sharing

This section develops the key results on how the sharing mechanism affects the outcomes of the resource system. We consider the Nash equilibrium where individuals maximize their expected utility by choosing investment s_i and the fraction α_i they contribute to the income pool, taking as given the behavior of all other individuals, see equation (3).

Sharing mechanisms are characterized by the sharing rule that specifies the fraction $\gamma_i = \gamma(\alpha_i)$ an individual receives from the pool, depending on the fraction of their personal net benefit that they chose to contribute to the income pool. We obtain the following two conditions that determine the Nash equilibrium with endogenous income sharing, derived in Appendix D.

$$\mathbb{E}[u'(\hat{y})] = g'(\hat{s})\,\frac{(1-\hat{\alpha})ND+\hat{\alpha}}{(1-\hat{\alpha})N+\hat{\alpha}}\,\mathbb{E}[z_i\,u'(\hat{y})], \tag{12}$$

$$\frac{\hat{\alpha}\,\gamma_i'(\hat{\alpha})}{\gamma_i(\hat{\alpha})} = \frac{N-1}{N}\,\frac{(x_0-\hat{s})\mathbb{E}[u'(\hat{y})]+g(\hat{s})\,\left(D\,\mathbb{E}[z_i\,u'(\hat{y})]+(1-D)\,\mathbb{E}[z_e\,u'(\hat{y})]\right)}{(x_0-\hat{s})\mathbb{E}[u'(\hat{y})]+g(\hat{s})\,\left(\frac{1}{N}\,\mathbb{E}[z_i\,u'(\hat{y})]+\left(1-\frac{1}{N}\right)\mathbb{E}[z_e\,u'(\hat{y})]\right)}. \tag{13}$$

First we consider the condition for investment in Nash equilibrium, and how it depends on the equilibrium fraction $\hat{\alpha}$ of net benefit from investment shared. For $\hat{\alpha} = 0$, Condition (12) corresponds to Condition (8) derived for benchmark 2 without income sharing available.

The interesting case is obtained for full sharing, $\hat{\alpha} = 1$: In this case, Condition (12) becomes equivalent to Condition (8) for socially optimal investment.

One candidate for a sharing rule is that each individual receives a pre-specified fraction from the pool, independent of their contribution, i.e. γ_i is a given constant. This sharing rule is the one considered in most of the literature (Lefebvre et al., 2014; Tilman et al., 2018; Sethi et al., 2012; Kaffine and Costello, 2011). We find that for a such a constant sharing rule, individuals will choose to contribute nothing to the pool.

Proposition 3. *When the sharing rule is constant, $\gamma_i = \bar{\gamma}$, no income is shared and optimal investment is $\hat{s}_0$, given by* (10).

Proof. See Appendix E. □

If individuals always receive a constant proportion of the pooled income, then they will always choose to free-ride off the contributions of others, independent of their desire to spread risk. This is due to individuals free-riding to enjoy the benefits of the contributions of others. In order to induce cooperation, the sharing rule must depend on the proportion of income shared such that individuals are motivated to contribute to increase their pay-off.

In particular, individuals may have an incentive to contribute to the income pool if the fraction γ_i they receive is increasing with the fraction α of their net benefit they contribute. We specifically consider the elasticity at which γ_i increases with α,

$$\varepsilon := \frac{\alpha\,\gamma'(\alpha)}{\gamma(\alpha)}. \tag{14}$$

We find that this elasticity needs to be large enough to incentivize contributions. The following proposition provides conditions that must be satisfied for the sharing rule to incentivize positive contributions to the income pool.

Proposition 4. *1. There exists a lower bound, ε_0, to the elasticity of the sharing rule with respect to α such that individuals will contribute to the income pool whenever $\varepsilon > \varepsilon_0$.*

2. There exists another threshold value ε_1, with $\varepsilon_0 < \varepsilon_1 < 1 - \frac{1}{N}$, such that individuals will contribute all of their net benefits, $\hat{\alpha} = 1$, if $\varepsilon > \varepsilon_1$.

Proof. See Appendix E. □

This states that the sharing rule must depend on the proportion of income shared in order to induce contributions. Moreover, if the elasticity of the sharing rule is sufficiently high, individuals will contribute all of their net benefits to the income pool, which is the socially optimal contribution as stated in Proposition 1. In this case, all benefits of investment are shared, and no individual has an incentive to free-ride on others' investments. As a result, the levels of investment are socially optimal as well: Condition (12) becomes identical to the condition for socially optimal investment, equation (8). Given full income sharing, no individual has an incentive to free-ride on other's investments and reduce the own investment below the optimal level.

There is one particularly simple sharing rule that satisfies the condition $\varepsilon > \varepsilon_1$ from Proposition 4 and thus leads to socially optimal investments, namely the sharing rule $\gamma_i = \frac{\alpha_i}{\sum_j \alpha_j}$. This "proportional sharing rule" is used frequently in public good game experiments (Gallier et al., 2017), conflict games (Kugler et al., 2010), bankruptcy games (Thomson, 2015), and water sharing allocations (Lefebvre et al., 2014). In contrast to these previous deterministic applications, in the present context with uncertain productivity this rule leverages the fact that risk-averse managers demand insurance, and thus will contribute to the income pool in order to spread risk. This rule leads to full sharing, and investments characterized by Condition (8):

Corollary 1. *With the proportional sharing rule, $\gamma_i = \frac{\alpha_i}{\sum_{j=1}^N \alpha_j}$, individuals will share all income, $\hat{\alpha} = 1$, and invest the efficient amount $s_i = s^*$.*

Individuals have an incentive for full sharing and socially optimal investment, because the desire to spread risk drives cooperation between individuals. By sharing income to spread risk, individuals internalize the benefits that their investment have on others, in which they participate as all income is pooled.

6 Discussion

This paper analyzes under which conditions insurance by means of an income sharing mechanism can induce cooperation between individuals, and hence achieve efficient use of common pool resources. To address this question, we develop a tractable model of an interconnected resource system with uncertain productivity. Risk-averse individuals can invest in the resource and also have the option of contributing a fraction of their net benefits to an income pool, where the pay-off is determined by a sharing rule. The benchmark case of no income sharing highlights two aspects of an individual's investment decision leading to inefficient investments: the free-riding incentive and risk-mitigating behavior. Whilst previous literature has shown that income sharing can be a promising instrument to overcome free-riding (Kaffine and Costello, 2011) and can also provide insurance through spreading risk (Tilman et al., 2018), we have shown that only the connection of the two issues can lead to endogenous cooperation.

We have found that a constant sharing rule will lead to free-riding on the income pool and inefficiently small individual contributions. By contrast, a sharing rule that depends on the proportion of net benefits shared, risk-averse individuals will have an incentive to contribute. To understand the potential for income sharing as a policy mechanism, we compare the equilibrium strategy to the benchmark of socially efficient income pooling and investments. We show that full income pooling is efficient, as this minimizes the risk for the resource users. We further find that the social optimum can be achieved by decentralized management using a simple 'proportional sharing rule', where the individual pay-off from the pool depends on the fraction of net benefits shared by the individual, relative to the sum of the fractions shared by all individuals.

The result is driven by two effects, namely (a) the individual desire to spread risk, and (b) the interconnectedness of the resource system, given by the diffusion rate. When the spread of risk is uneven prior to sharing income, individuals will contribute all of their income to the pool. As a co-benefit, spreading risk internalizes the diffusion effects associated with investments in the resource, thus achieving efficient provision. The result that efficient provision can be achieved through sharing all income is analogous to the findings by Kaffine and Costello (2011), albeit achieved through a different mechanism.

We further find that for a resource system that resembles a pure public good, risk is already perfectly spread such that individuals are indifferent to contributing to the pool. This is because diffusion effects spread risk in a similar way to income sharing, influencing contributions through a natural insurance effect.

In addition to advancing the understanding of how risk-aversion impacts the outcomes of a common-pool resource system, our main contribution focuses on achieving socially optimal outcomes. We show that, even with a relatively simple pay-off rule, endogenous income sharing can overcome free-riding and provide insurance, thus implementing the socially optimal outcome. Whilst our results build on literature focusing on common pool resource extraction, we develop a general model of investments into a resource system with uncertain productivity and believe endogenous income sharing mechanisms have a much broader relevance to many collective goods such as green infrastructure, agriculture, and climate mitigation.

The findings of this paper also advance our understanding of the relationship between risk and common-pool resource use. Our model aligns with previous research showing that risk can exacerbate private under-investments in a common-pool resource system. By developing a tractable analytic model, we disentangle the interaction between risk and the diffusion externality, explaining how diffusion affects risk-averse behavior. This highlights how policy to address under-provision of collective goods should not be considered in isolation of risk. Moreover, a key challenge in insuring collective goods – particularly common-pool resources – is the influence of diffusion effects or resource mobility on decision making, providing further incentive to develop policy that considers risk and diffusion effects simultaneously.

We also found that self-insurance against risk exacerbates the problem of under-investment. Results from finance prove that access to financial insurance may crowd out self-insurance Ehrlich and Becker (1972). However, in our setting financial insurance alone would not overcome the diffusion externality. The endogenous income sharing mechanism we propose uses a sharing rule that leverages the demand insurance to induce cooperation and increase provision. By contributing to the pool, agents can diversify risk, which in turn crowds out self-insurance and internalizes diffusion effects, resulting in an overall increase of efficiency.

Appendices

Appendix A Proof of Proposition 1

. We need to determine the sign of $(ND-1)\mathbb{E}[(z_e - z_i)\, u'(y)]$. This is not trivial, as the representative individual's income y depends on both z_i and z_e.

To show that $(ND-1)\mathbb{E}[(z_e - z_i)\, u'(y)] > 0$ for all $D \neq 1/N$, it is useful to define the representative individual i's and another individual e's incomes, slightly rearranging (5) as:

$$y_i := x_0 - s + g(s)\left(z_i D + \frac{\sum_{j\neq i} z_j}{N-1}(1-D) + \frac{\alpha}{N}(1-ND)\left(z_i - \frac{\sum_{j\neq i} z_j}{N-1}\right)\right) \tag{15a}$$

$$y_e := x_0 - s + g(s)\left(z_e D + \frac{\sum_{j\neq e} z_j}{N-1}(1-D) + \frac{\alpha}{N}(1-ND)\left(z_e - \frac{\sum_{j\neq e} z_j}{N-1}\right)\right). \tag{15b}$$

With this notation, we have

$$y_i - y_e = g(s)\,(z_i - z_e)\,\frac{1}{N-1}\,(1-\alpha)\,(ND-1) \tag{16}$$

Condition (16) states that the income that experiences the more favorable shock has the higher, respectively lower, income if $D > 1/N$, respectively $D < 1/N$. We can accordingly rank marginal utilities,

$$u'(y_i) - u'(y_e) \lesseqgtr 0 \quad \Leftrightarrow \quad y_i - y_e \gtreqless 0 \quad \Leftrightarrow \quad (1-\alpha)\,(z_i - z_e)\,(ND-1) \gtreqless 0$$

We thus conclude

$$(ND-1)\mathbb{E}\left[(z_e - z_i)\, u'(y_i)\right] = \frac{1}{2}(ND-1)\mathbb{E}\left[\left(z_e u'(y_i) + z_i u'(y_e) - z_i u'(y_i) - z_e u'(y_e)\right)\right] \tag{17}$$

$$= \frac{1}{2}(ND-1)\mathbb{E}\left[(z_e - z_i)\left(u'(y_i) - u'(y_e)\right)\right] > 0, \tag{18}$$

whenever $z_e \neq z_i$ and if $1-\alpha > 0$.

Appendix B Derivation of Condition (8)

B.1 Socially optimal escapement

Given full income sharing (Proposition 1), the optimization problem 4 simplifies to

$$\max_{\{s_i\}} \mathbb{E}\left[u\left(\sum_{i=1}^{N}\left(x_0 - s_i + z_i g(s_i) D + \sum_{j\neq i} z_j g(s_j) D_e\right)\right)\right] \tag{19}$$

The first-order condition for a representative individual i reads

$$\mathbb{E}\left[u'(y_i)\left(-1+z_i\,g'(s_i)\,(D+(N-1)\,D_e)\right)\right]=0 \tag{20}$$

Using $D+(N-1)\,D_e=1$ and rearranging, we obtain condition (8).

B.2 Escapement in Nash equilibrium without income sharing

Without income sharing, individual i's optimization problem (3) simplifies to

$$\max_{\{s_i\}}\mathbb{E}\left[u\left(x_0-s_i+z_i\,g(s_i)\,D+\sum_{j\neq i}z_j\,g(s_j)\,D_e\right)\right] \tag{21}$$

As the individual takes all others' investments as given, the first-order condition for a representative individual i reads

$$\mathbb{E}\left[u'(y_i)\left(-1+z_i\,g'(s_i)\,D\right)\right]=0 \tag{22}$$

Rearranging, we obtain condition (10).

Appendix C Proof of Proposition 2

We want to show that $\mathbb{E}\,[(1-z_i)\,u'(y^*)]>0$. To this end, we first decompose this expression as

$$\mathbb{E}\left[(1-z_i)\,u'(y^*)\right]=\mathbb{E}\left[(1-z_i)\,u'(y^*)\big|\,z_i\geq 1\right]+\mathbb{E}\left[(1-z_i)\,u'(y^*)\big|\,z_i<1\right] \tag{23}$$

Using mean value theorem for integrals, there must be numbers $u'(\bar{y}_i)$ and $u'(\underline{y}_i)$ such that

$$\mathbb{E}\left[(1-z_i)\,u'(y^*)\right]=\mathbb{E}\,[(1-z_i)|\,z_i\geq 1]\,u'(\bar{y}_i)+\mathbb{E}\,[(1-z_i)|\,z_i<1]\,u'(\underline{y}_i) \tag{24}$$

Now using $\mathbb{E}\,[(1-z_i)|\,z_i\geq 1]=-\mathbb{E}\,[(1-z_i)|\,z_i<1]$, we can write

$$\mathbb{E}\left[(1-z_i)\,u'(y^*)\right]=\mathbb{E}\,[(1-z_i)|\,z_i\geq 1]\left(u'(\bar{y}_i)-u'(\underline{y}_i)\right) \tag{25}$$

As individual income is higher with a positive internal shock ($z_i>1$) than with a negative internal shock ($z_i<1$), it must be that $u'(\bar{y}_i)-u'(\underline{y}_i)<0$. The expression on the right-hand side of (25) is positive.

Appendix D Derivation of Nash equilibrium with income sharing

Due to symmetry of individuals, the solution is symmetric, $\hat{s}_i = \hat{s}_m = \hat{s}$ and $\hat{\alpha}_i = \hat{\alpha}_m = \hat{\alpha}$, and $\gamma_i = \gamma_m = \frac{1}{N}$. Given the sharing rule depends on the proportion of income shared, $\gamma(\alpha)$, the first order conditions for investment s and income shared α are, respectively:

$$\mathbb{E}\left[u'(\hat{y})\left((1-\hat{\alpha})\left(-1+z_i g'(\hat{s})D\right)+\hat{\alpha}\,\gamma(\hat{\alpha})\left(-1+g'(\hat{s})\,z_i\left(D+\sum_{m\neq i}^{N} D_e\right)\right)\right)\right]=0 \quad (26)$$

$$\mathbb{E}\Big[u'(\hat{y}_i)\Big(-(1-\gamma_i(\hat{\alpha}_i))\Big(x_0-\hat{s}_i+z_i\,g(\hat{s}_i)D+\sum_{m\neq i}^{N} z_m g(\hat{s}_m)D_e\Big)$$
$$+\gamma_i'(\hat{\alpha}_i)\Big(\sum_{m=1}^{N}\hat{\alpha}_m(x_0-\hat{s}_m)+\sum_{m=1}^{N} z_m\,g(\hat{s}_m)\Big)\Big)\Big]=0 \quad (27)$$

Condition (26) can be rewritten as

$$\mathbb{E}\Big[u'(\hat{y})\Big((1-\hat{\alpha})(-1+z_i g'(\hat{s})D)+\frac{\hat{\alpha}}{N}(-1+g'(\hat{s})z_i\Big)\Big]=0 \quad (28)$$

Using linearity of expectation this becomes:

$$-\mathbb{E}[u'(\hat{y})]\left(1-\hat{\alpha}\,\frac{N-1}{N}\right)+g'(\hat{s})\left(D-\hat{\alpha}\frac{ND-1}{N}\right)\mathbb{E}[z_i\,u'(\hat{y})]=0 \quad (29)$$

Rearranging, the condition becomes (12).

Similarly condition (27) can be rewritten as

$$\mathbb{E}\Big[u'(\hat{y})\Big(-\frac{N-1}{N}\Big(x_0-\hat{s}+g(\hat{s})(z_i D+D_e\sum_{m\neq i}^{N} z_m)\Big)+\gamma_i'(\hat{\alpha})\Big(N\hat{\alpha}(x_0-\hat{s})+g(\hat{s})\sum_{m=1}^{N} z_m)\Big)\Big)\Big]=0 \quad (30)$$

Using the linearity of expectation this becomes

$$-\frac{N-1}{N}\left((x_0-\hat{s})\mathbb{E}[u'(\hat{y})]+g(\hat{s})\left(\mathbb{E}[z_i\,u'(\hat{y})]D+D_e\sum_{m\neq i}^{N}\mathbb{E}[z_m\,u'(\hat{y})]\right)\right)$$
$$+\hat{\alpha}\,\gamma_i'(\hat{\alpha})N\left((x_0-\hat{s})\mathbb{E}[u'(\hat{y})]+g(\hat{s})\frac{1}{N}\sum_{m=1}^{N}\mathbb{E}[z_m u'(\hat{y})]\right)=0 \quad (31)$$

Given symmetry, using $D_e = (1-D)/(N-1)$ and that $\gamma = 1/N$ in equilibrium, this can be simplified to (13).

Appendix E Proof of Proposition 4

A positive $\hat{\alpha}$ will be optimal for each individual in Nash equilibrium, if the elasticity ε exceeds the expression on the right-hand side for sufficiently small α. We thus define the constant

$$\varepsilon_0 := \frac{N-1}{N} \frac{(x_0 - \hat{s}_0)\mathbb{E}[u'(\hat{y}_0)] + g(\hat{s}_0)\ (D\,\mathbb{E}[z_i\,u'(\hat{y}_0)] + (1-D)\,\mathbb{E}[z_e\,u'(\hat{y}_0)])}{(x_0 - \hat{s}_0)\mathbb{E}[u'(\hat{y}_0)] + g(\hat{s}_0)\ \left(\frac{1}{N}\,\mathbb{E}[z_i\,u'(\hat{y}_0)] + \left(1 - \frac{1}{N}\right)\,\mathbb{E}[z_e\,u'(\hat{y}_0)]\right)}, \tag{32}$$

where $\hat{s}_0$ is investment and $\hat{y}_0$ is income in the benchmark case of the Nash equilibrium without income sharing. We find that if $\varepsilon > \varepsilon_0$, then at least some income sharing will result in Nash equilibrium.

If the elasticity is everywhere large enough, then full sharing will be optimal. A sufficient condition is that the elasticity is everywhere larger than the elasticity evaluated at full income sharing,

$$\varepsilon_1 := \frac{N-1}{N} \frac{(x_0 - s^*)\mathbb{E}[u'(y^*)] + g(s^*)\ (D\,\mathbb{E}[z_i\,u'(y^*)] + (1-D)\,\mathbb{E}[z_e\,u'(y^*)])}{(x_1 - s^*)\mathbb{E}[u'(y^*)] + g(s^*)\ \left(\frac{1}{N}\,\mathbb{E}[z_i\,u'(y^*)] + \left(1 - \frac{1}{N}\right)\,\mathbb{E}[z_e\,u'(y^*)]\right)}. \tag{33}$$

Appendix F Proof of Corollary 1

Using the proportional sharing rule, $\frac{\alpha\,\gamma_i'(\alpha)}{\gamma_i(\alpha)} = 1 - \gamma = \frac{N-1}{N}$. Substituting this into condition (13) leads to:

$$1 = \frac{(x_0 - \hat{s})\mathbb{E}[u'(\hat{y})] + g(\hat{s})\ (D\,\mathbb{E}[z_i\,u'(\hat{y})] + (1-D)\,\mathbb{E}[z_e\,u'(\hat{y})])}{(x_0 - \hat{s})\mathbb{E}[u'(\hat{y})] + g(\hat{s})\ \left(\frac{1}{N}\,\mathbb{E}[z_i\,u'(\hat{y})] + \left(1 - \frac{1}{N}\right)\,\mathbb{E}[z_e\,u'(\hat{y})]\right)} \tag{34}$$

Yet, the right-hand side of this condition is always greater than one, as

$$D\,\mathbb{E}[z_i\,u'(\hat{y})] + (1-D)\,\mathbb{E}[z_e\,u'(\hat{y})] > \frac{1}{N}\,\mathbb{E}[z_i\,u'(\hat{y})] + \left(1 - \frac{1}{N}\right)\,\mathbb{E}[z_e\,u'(\hat{y})] \tag{35}$$

$$\Leftrightarrow \quad D\,\left(\mathbb{E}[z_i\,u'(\hat{y})] - \mathbb{E}[z_e\,u'(\hat{y})]\right) > \frac{1}{N}\,\left(\mathbb{E}[z_i\,u'(\hat{y})] - \mathbb{E}[z_e\,u'(\hat{y})]\right) \tag{36}$$

$$\Leftrightarrow \quad (ND-1)\,\left(\mathbb{E}[z_i\,u'(\hat{y})] - \mathbb{E}[z_e\,u'(\hat{y})]\right) > 0, \tag{37}$$

as we have shown in Appendix A.

References

Aceves-Bueno, E., Cornejo-Donoso, J., Miller, S.J., Gaines, S.D., 2017. Are territorial use rights in fisheries (turfs) sufficiently large? Marine Policy 78, 189–195.

Augeraud-Véron, E., Fabbri, G., Schubert, K., 2019. The Value of Biodiversity as an Insurance Device. American Journal of Agricultural Economics 101, 1068–1081.

Bramoullé, Y., Treich, N., 2009. Can uncertainty alleviate the commons problem? Journal of the European Economic Association 7, 1042–1067.

Cornes, R., Sandler, T., 1996. The Theory of Externalities, Public Goods, and Club Goods. Cambridge Books. Cambridge University Press.

Deacon, R.T., 2012. Fishery Management by Harvester Cooperatives. Review of Environmental Economics and Policy 6, 258–277.

Ehrlich, I., Becker, G.S., 1972. Market insurance, self-insurance, and self-protection. Journal of Political Economy 80, 623–648.

Gallier, C., Kesternich, M., Sturm, B., 2017. Voting for Burden Sharing Rules in Public Goods Games. Environmental and Resource Economics 67, 535–557.

Kaffine, D.T., Costello, C., 2011. Unitization of spatially connected renewable resources. The B.E. Journal of Economic Analysis & Policy 11.

Kelsall, C., Quaas, M.F., Quérou, N., 2023. Risk aversion in renewable resource harvesting. Journal of Environmental Economics and Management 121, 102855.

Kotchen, M.J., 2009. Voluntary Provision of Public Goods for Bads: A Theory of Environmental Offsets. The Economic Journal 119, 883–899.

Kugler, T., Rapoport, A., Pazy, A., 2010. Public good provision in inter-team conflicts: Effects of asymmetry and profit-sharing rule. Journal of Behavioral Decision Making 23, 421–438.

Lefebvre, M., Thoyer, S., Tidball, M., Willinger, M., 2014. Sharing Rules for Common-Pool Resources When Self-Insurance is Available. Environmental Modeling and Assessment 19, 111–125.

Mumford, J.D., Leach, A.W., Levontin, P., Kell, L.T., 2009. Insurance mechanisms to mediate economic risks in marine fisheries. ICES Journal of Marine Science 66, 950–959.

OECD, Uchida, H., 2010. Community-based management for sustainable fishery: Lessons from Japan. Technical Report. OECD. Paris.

Ostrom, E., 1990. Govering the commons: The evolution of institutions for collective action. Cambridge University Press, New York.

Platteau, J., Seki, E., 2001. Community Arrangements to Overcome Market Failures: Pooling Groups in Japanese Fisheries, pp. 344–402.

Platteau, J.P., Seki, E., 2007. Heterogeneity, social esteem and feasibility of collective action. Journal of Development Economics 83, 302–325.

Quaas, M.F., Baumgärtner, S., 2008. Natural vs. financial insurance in the management of public-good ecosystems. Ecological Economics 65, 397–406.

Quérou, N., Tomini, A., Costello, C., 2022. Limited-tenure concessions for collective goods. Journal of Economic Dynamics and Control 143, 104484.

Sethi, S.A., Dalton, M., Hilborn, R., 2012. Managing harvest risk with catch-pooling cooperatives. ICES Journal of Marine Science 69, 1038–1044.

Stavins, R., 2011. The problem of the commons: Still unsettled after 100 years. American Economic Review 101, 81–108.

Tahvonen, O., 2015. Economics of naturally regenerating, heterogeneous forests. Journal of the Association of Environmental and Resource Economists 2, 309–337.

Thomson, W., 2015. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update. Mathematical Social Sciences 74, 41–59.

Tilman, A.R., Levin, S., Watson, J.R., 2018. Revenue-sharing clubs provide economic insurance and incentives for sustainability in common-pool resource systems. Journal of Theoretical Biology 454, 205–214.

Uchida, H., Wilen, J., 2005. Harvester cooperative, pooling arrangements and market power .

Wilen, J.E., Cancino, J., Uchida, H., 2012. The economics of territorial use rights fisheries, or turfs. Review of Environmental Economics and Policy 6, 237–257.

CEE-M Working Papers[1] - 2025

WP 2025-01	**Katrin Erdlenbruch**, **Mabel Tidball** & Julia de Frutos Cachorro« Resource extraction and land-use choice in a two-player two-period game »
WP 2025-02	Gabriel Bayle & **Marc Willinger** « Efficiency of the Minimum Approval Mechanism with heterogeneous players »
WP 2025-03	Karine Constant, **Marion Davin** & **Emmanuelle Lavaine** « Does income inequality influence health vulnerability to pollution? Evidence from France »
WP 2025-04	Denis Claude & **Mabel Tidball** « Revisiting Stackelberg in his own light »
WP 2025-05	Gabriel Bayle, **Dimitri Dubois** & **Marc Willinger** « L'économie à l'ère de la science ouverte : un nouvel élan pour la reproductibilité »
WP 205-06	**Daniel Serra** & Adda Benslimane « Economic Modeling of 'Axiological Rationality': a Tedious Challenge.»
WP 205-07	**Nicolas Quérou** « Inter-group relocation policies »
WP 205-08	**Claudia Kelsall,** Martin F. Quaas & **Nicolas Quérou** « Income sharing for common pool resources with uncertain productivity »

[1] **CEE-M Working Papers / Contact** : laurent.garnier@inrae.fr

- RePEc https://ideas.repec.org/s/hal/wpceem.html
- HAL https://halshs.archives-ouvertes.fr/CEE-M-WP/