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Group-level cooperation

Nicolas Quérou



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Group-level cooperation ^{*}

Nicolas Quérou[†]

Abstract

We analyze the existence, coordination and efficiency effects of group-level cooperation in games featuring heterogeneous strategic spillovers and externalities within and between groups of agents. Conclusions depend on whether out-group interactions feature strategic substitutes or complements. Under high out-group strategic spillovers, group-level cooperation triggers either equilibrium existence or coordination issues. When these effects are not too high, a unique group-level cooperation equilibrium exists and its characterization depends on the in-group externality level. With respect to efficiency, cases are characterized under which this initiative is self-enforcing. Conditions are also provided under which it backfires: the cooperating group members would be actually better off if they could ex-ante commit to not cooperate, even if it would be costly to do so.

Key words: Externalities, group-level cooperation, strategic interactions, spillover effects.

JEL classification: C72, D21, D60, D70, H40.

1 Introduction

There are plenty of economic and social settings that involve groups or communities. Examples range from settings in which communities provide collective goods to members that may harm outsiders, teams interacting within organizations, groups of owners managing common-pool resources, to issues of local public goods provision.

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This contribution introduces a model where agents are organized in separate groups and simultaneously choose continuous activity levels. Members of a given group are affected by the other group members' activity levels (out-group externalities) and a given group member's marginal returns are either negatively affected or enhanced when other group members' exert higher activity levels (out-group strategic substitutability or complementarity). This framework allows for potentially group-specific out-group strategic spillovers. With regard to in-group members, we also consider potentially heterogeneous strategic spillover and externality effects.

As such, it generalizes specifications used in the literature on social effects (such as Belhaj and Deroian (2019) for instance), models used extensively in problems of common-pool resource management (Walker et al. 2000), other classical frameworks such as investment games featuring either in- and out-group strategic complementarities and positive externalities or in- and out-group strategic substitutes and negative externalities, and even local public goods problems that may be modeled as games of in- and out-group strategic substitutes and positive externalities.¹

We introduce group-level cooperation: the agents of one given group cooperate with each other, while there is non-cooperation both between groups and within the other groups. This is one of the simplest forms of cooperation that can be introduced in the present framework.² We compare this setting to the benchmark in which there is full non-cooperation. More precisely, we analyze its existence, coordination, distributional and welfare effects. These effects depend on whether the setting features out-group strategic complements or substitutes.

We first show that, under high out-group strategic spillovers, group-level cooperation face either equilibrium existence or coordination issues. When the in-group externality is low, one equilibrium exists in which the non-cooperating group members exert a positive effort while the cooperating group members contribute nothing as this group internalizes the in-group externality. A second equilibrium exists in this case under out-group strategic complementarity: all group members exert

¹See Section 2 for a discussion of other relevant examples.

²One way to understand this form of cooperation is to interpret it as an institution: the whole point is then to analyze its performances. There are multiple reasons that might explain the potential existence of such an institution. One is that groups are heterogeneous in terms of the strength of their identities: the group with the strongest identity would be the one most likely to cooperate (Kramer and Brewer 1984). A similar type of reasoning might be based on the notion of group membership (McAdams 1995), cohesiveness (Esteban and Ray 2011), collectivism (Batson 1998), or parochialism (Romano et al. 2017). This may be due to the fact that the members of a given group confuse self-interest and group-interest (Baron 2001).

a positive activity level. When the in-group externality is high, there is no equilibrium outcome as the only candidate would be an interior equilibrium, which exists only if the level of out-group strategic spillovers is not too high. In contrast, under out-group strategic substitutability, for an intermediate range of the in-group externality level, there is no equilibrium outcome as the only candidate would be the interior equilibrium, which would in turn require that the level of out-group strategic spillovers be not too high.

Second, we characterize the conditions under which a unique Nash equilibrium exists in both settings. The out-group strategic spillover effects must lie below a threshold value, and the characterization also depends on the in-group externality level. Intuitively, as the cooperating group members internalize the in-group externality effect via cooperation, it is optimal for these agents to be inactive if and only if this externality level is low enough. In this case, the other group members' activity levels remain positive under a regularity assumption, and it does not depend on the in-group externality effect as they do not internalize it. In contrast, when the in-group externality effect is high enough, the cooperating group members exert a positive activity level that increases as the in-group externality increases. This high activity level induces the non-cooperating group members to be inactive under out-group strategic substitutability and to exert a positive activity effort level under out-group strategic complementarity. A final case exists under out-group strategic substitutability: for intermediate levels of the in-group externality, both group members are induced to remain active.

Adopting a utilitarian perspective, we then analyze the conditions under which group-level cooperation might be self-enforcing: in other words, conditions under which it results in a Pareto improvement compared to the benchmark setting.³ Under out-group strategic complementarity, the first case corresponds to situations where externality effects are high. In such a case both settings result in interior solutions and, when the out-group externality is high enough, the marginal effect of the in-group externality is positive on the cooperating group members' difference in equilibrium payoffs. Thus, for high enough levels of in-group externality these agents are better off under group-level cooperation. Regarding the non-cooperating group members, due to out-group strategic complementarity their equilibrium activity levels get higher under group-level cooperation for high enough in-group externality. Thus the standalone returns from activity and the direct in-group externality effect are positively affected. Moreover, as the cooperating group

³In this case, group-level cooperation can be considered as a group-based solution to the inefficiency problems driven by decentralization. Other group-based approaches proposed in the literature involve, among others, group incentives (Nalbantian and Schotter 1997) or coordination (Loeper 2011).

members also increase their activity levels (compared to the benchmark) for high enough in-group externality, there is a resulting positive out-group externality effect for the non-cooperating group members (provided the out-group externality is high enough). All together, for these members the effect of group-level cooperation is also positive. Second, under low out-group externality, Pareto improvements obtain except for an intermediate range of in-group externality levels.

In contrast, under out-group strategic substitutability, the conclusions are more involved. When the in-group externality is low, group-level cooperation is never Pareto improving when out-group strategic spillovers are sufficiently asymmetric. Otherwise, it is Pareto improving if and only if the out-group externality is intermediate. This is also a necessary condition when the in-group externality level is intermediate. Pareto improvements then depend on the interplay between out-group strategic spillover and in-group externality effects. Finally, when the in-group externality is high and the out-group strategic spillover resulting from the cooperating group is the highest, group-level cooperation is necessarily Pareto improving when externalities are negative (positive) under in-group strategic substitutability (complementarity). In this last case Pareto improvements are likely for investment games or common-pool resource management settings when they feature negative externalities, while they are unlikely in local public goods as these settings feature positive externalities.

Finally we show that, in several cases, group-level cooperation may backfire: the cooperating group members would actually be better off if they could ex-ante commit to not cooperate, even if it would be costly to do so. Under out-group strategic complementarity, this is for instance the case when the out-group externality level is high and the in-group externality level is low, which can be explained as follows. Both group members are active in the decentralized benchmark. Also, the non-cooperating group members are always active under group-level cooperation. As such, when comparing the cooperating group members' payoffs, the resulting out-group externality effect depends on the difference in the non-cooperating group members' equilibrium activity levels. When the level of in-group externality lies below a threshold value, the cooperating group members decrease their activity levels and do not contribute. This induces the non-cooperating group members to decrease their activity levels compared to the decentralized benchmark under out-group strategic complementarity. Since the non-cooperating group members' activity levels do not depend on the in-group externality, the cooperating group members are worse off under group-level cooperation if and only if the out-group externality lies above a threshold value.

Under out-group strategic substitutability, even though this setting is more involved, we can summarize the main conclusions as follows. Specifically, the cooperating group members are worse off under group-level cooperation when the levels of in- and out-group externality are simultaneously either low, intermediate, or high. For instance, for low levels of the in-group externality, when both out-group strategic spillover effects are not too strong, the decentralized benchmark results in an interior equilibrium outcome. Under group-level cooperation, the cooperating group members internalize the (low) in-group externality effect, which induces them to not contribute. Due to out-group strategic substitutability the other group members are in turn induced to increase their activity levels compared to the benchmark. So, the comparison between the cooperating group members' equilibrium payoffs is driven by the difference between the net out-group externality effect and the sum of their standalone returns from activity and in-group externality effect in the benchmark. Thus, these agents are worse off if and only if the level of out-group externality is low enough. This implies among other conclusions that, when the magnitude of externalities is strong enough, group-level cooperation can backfire in common-pool resource management settings (when they feature negative externalities) or local public good problems.

The present institution, in which one out of several groups cooperate while there is non-cooperation both between groups and within other groups, can be related to an existing literature on the performance of partial cooperation. A notable part of this literature focuses on the problem of contributing to a public good, whether in a static (Buchholz et al. 1998) or a dynamic setting (Colombo et al. 2022) within the framework of international environmental agreements, or on the effects of partial mergers, again in a static (Salant et al. 1983) or a dynamic setting (Benckroun and Gaudet 2015) within the framework of common-pool resource management. With respect to most of these contributions, we introduce a fairly generic form of preferences that extends forms widely used in several important types of problems, as explained above. Moreover, most of these contributions consider symmetric frameworks in which all features are homogeneous. Apart from the potentially negative effect of partial cooperation, the focus of these contributions is quite different from the present paper.

Another related contribution is Beaudry et al. (2000), who analyze whether it is harmful to allow for partial cooperation, that is, whether cooperating agents might be worse off under partial cooperation compared to a full non-cooperation benchmark. They account for a generic form of preferences, and derive conditions related to the nature of strategic spillovers. Their focus is on a more specific form of partial cooperation: they consider only non-cooperation between groups. There

is only one group that cooperates within our setting, which allows for a more generic analysis. They also consider a symmetric framework in which all features of the problem are homogeneous. We introduce a setting in which externalities and strategic spillovers are heterogeneous. This allows to analyze the distributional effects and to understand the diverse effects of asymmetry or inequality in such a problem, consistently with contributions such as Gangadharan et al. (2017) or Camera et al. (2023). We highlight some potential inefficiency effects of asymmetry on group-level cooperation (Hubner et al. 2024). Finally, they rely on specific starting assumptions: the necessary and sufficient optimal conditions characterize a unique and interior equilibrium. This rules out interesting questions, that we do address here. We characterize conditions under which group-level cooperation may actually trigger either equilibrium existence or coordination issues. Second, allowing for corner solutions highlights new interesting cases when analyzing the efficiency effects of the institution.

The remainder of the paper is organized as follows. The model and assumptions are presented in Section 2. The analysis of existence and coordination issues raised by group-level cooperation is provided in Section 3, together with the characterizations of the decentralized benchmark and group-level cooperation outcomes. The group and global welfare effects are analyzed in Section 4. Section 5 concludes. The proofs of all results are provided in an Appendix at the end of the paper.

2 Model and assumptions

We focus on a simple setting featuring heterogeneous strategic spillovers and externalities between groups. So we consider two groups, A and B (of size $N_A = N_B = N$), which are subject to simultaneous in- and out-group effects. For a given group i , each member's activity level imposes in-group strategic spillover and externality effects on other members of this same group modeled by, respectively, parameters δ_{in} and e_{in} . Moreover, each group i 's member imposes out-group strategic spillover and externality effects on group j 's members modeled by, respectively, parameters δ_{ji} and e_{out} . For a given group (say, A) the payoff of a given agent $l \in \{1, \dots, N_A\}$ is thus specified as follows:

$$\Pi_l^A = ax_{lA} - \frac{b}{2}(x_{lA})^2 - \delta_{in}x_{lA}X_{-lA} - \delta_{BA}x_{lA}X_B + e_{in}X_{-lA} + e_{out}X_B, \quad (1)$$

where $x_{lA} \geq 0$ denotes this agent's activity level, $X_{-lA} = \sum_{j \in A, j \neq l} x_{jA}$ (respectively, X_B) the aggregate activity level of other group A members (respectively, the aggregate activity level in group B), and a and b are positive parameters specifying the standalone returns from activity. We will consider that this game setting

exhibits either out-group strategic complementarity or out-group strategic substitutability: either $\delta_{ij} < 0$ or $\delta_{ij} > 0$ for $i, j = A, B, i \neq j$. We will also consider the two cases where $\delta_{in} < 0$ or $\delta_{in} > 0$ might hold. In- and out-group externalities may be positive or negative. This model features (qualitatively) heterogeneous strategic spillovers and externalities, unlike the usual specifications used in the literature on social effects (such as Baetz (2015) or Belhaj and Deroian (2019) for instance). It also extends models used extensively in problems of common-pool resource management (Walker et al. 2000), other classical frameworks such as investment games featuring either in- and out-group strategic complementarities and positive externalities (Ballester et al. 2006) or in- and out-group strategic substitutes and negative externalities (Goyal and Moraga-Gonzalez 2001), and local public goods problems modeled as games of in- and out-group strategic substitutes and positive externalities (see Galeotti et al. (2020) for a related discussion). Other relevant examples might be found in, among others, settings where peer decisions have positive spillovers to members of the same group and negative spillovers to other groups:

- social activities exhibiting forms of limited morality behavior applying to an agent's own group (Tabellini 2010)
- activities in which a decision benefiting an agent's own group negatively affects the performance and the payoffs of other groups (see Markussen et al. (2014) for related examples)
- problems of provision of collective goods having heterogeneous qualitative effects on the different groups involved.

In the decentralized case agents from both groups act non-cooperatively with respect to in- as well as out-group members. In the group-level cooperation setting, we assume that group- A members cooperate with each other, while they act non-cooperatively with respect to group- B members, who act non-cooperatively with respect to in-group as well as out-group members. As such, under group-level cooperation group- A members act in order to maximize the aggregate payoffs in group A , while group- B members act in order to maximize their individual payoffs.

Under decentralization, for a given agent l in group A , the corresponding optimality condition is:

$$a - bx_{lA} - \delta_{in}X_{-lA} - \delta_{BA}X_B + \lambda_{lA} = 0, \quad (2)$$

where $\lambda_{lA} \geq 0$ denotes the lagrangian parameter related to the non-negativity constraint, and a similar type of condition holds for any agent in group B .

By contrast, under group-level cooperation, the corresponding optimality condition for this same group- A member is:

$$a - bx_{lA} - 2\delta_{in}X_{-lA} - \delta_{BA}X_B + (N - 1)e_{in} + \lambda_{lA}^{gc} = 0, \quad (3)$$

where $\lambda_{lA}^{gc} \geq 0$ denotes the lagrangian parameter related to the non-negativity constraint. For any agent j in group B the optimality condition is then:

$$a - bx_{jB} - \delta_{in}X_{-jB} - \delta_{AB}X_A + \lambda_{jB}^{gc} = 0, \quad (4)$$

where $\lambda_{jB}^{gc} \geq 0$ denotes the lagrangian parameter related to the non-negativity constraint. With respect to group- A members, by relying on in-group cooperation they internalize the in-group strategic spillover and externality effects, as reflected by the different terms related to the effect of the in-group strategic spillover and the new effect of in-group externality. The optimality condition of any group- B member is identical to the one in the decentralized case.

3 Existence and coordination issues

The first result characterizes the conditions under which the decentralized benchmark and, more interestingly, group-level cooperation, face the following issues: (i) an equilibrium may fail to exist or (ii) multiple equilibria may coexist, raising coordination issues.

Proposition 1. *Assume that $b + 2(N - 1)\delta_{in} > 0$ holds under in-group strategic complementarity and that $b > 2\delta_{in}$ holds under in-group strategic substitutability. We have the following conclusions:*

- *When $b + (N - 1)\delta_{in} - N\delta_{AB} \leq 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} \leq 0$ hold, the decentralized benchmark admits multiple equilibria.*
- *When the following condition holds*

$$[b + (N - 1)\delta_{in}] [b + 2(N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} \leq 0$$

the case of out-group strategic complementarity results in multiple group-level cooperation equilibria (no equilibrium) when in-group externality e_{in} is low (high). Under out-group strategic substitutability, there is no group-level cooperation equilibrium outcome for intermediate levels of e_{in} .

The first point necessarily refers to the decentralized case under out-group strategic substitutability by assumption. When out-group strategic spillover effects are high enough, that is, both inequality are satisfied weakly, two corner

equilibria co-exist where either group A - or group- B members exert positive efforts while the other group contributes nothing. When both inequality are strict, an additional equilibrium exists where both group members exert positive efforts.

The second point refers to the case of group-level cooperation. The inequality relates to cases where the out-group strategic spillover effect is high enough. The optimality condition (3) highlights that one of the main features of group-level cooperation is that group- A members internalize the in-group externality effect. The conclusions depend on the nature of out-group spillover effects.

Under out-group strategic complementarity, when the in-group externality is low, two equilibria co-exist: one in which group- B members exert a positive effort while group- A members contribute nothing; the other corresponds to an interior equilibrium. At the corner equilibrium, the activity level exerted by group- B members does not depend on the in-group externality, as these agents do not internalize it. In contrast, at the interior equilibrium, the group- A members' equilibrium activity level depends on e_{in} as cooperating agents internalize this effect, while the group- B members' activity level also depends on the in-group externality effect due to out-group strategic interactions. There does not exist a corner equilibrium in which group- A members exert a positive effort while group- B members contribute nothing, as this would require a too strong in-group complementarity effect, which would imply that group- A members' maximization problem would be ill-defined. When the in-group externality is high, there is no equilibrium outcome as the only candidate would be an interior equilibrium, which exists only if the level of out-group strategic complementarities is not too high.

In contrast, under out-group strategic substitutability, for an intermediate range of the in-group externality effect the problem is similar to the previous case when the in-group externality was high. There is no equilibrium outcome as the only candidate would be the interior equilibrium, which would in turn require the out-group strategic substitutability effect be not too high.

Before moving on to the characterizations of the equilibrium outcomes, an important implication of Proposition 1 is that group-level cooperation can trigger stability (equilibrium existence) or coordination issues when neither problem would emerge under the decentralized benchmark. Also, due to the above conclusions we assume that the following condition holds in the remainder of the paper:

$$[b + (N - 1)\delta_{in}][b + 2(N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} > 0 \quad (5)$$

We also rule out the case where $b + (N - 1)\delta_{in} - N\delta_{AB} \leq 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} \leq 0$ are satisfied simultaneously. As implied by Proposition 1 these con-

ditions ensure the existence and uniqueness of the equilibrium outcomes in the benchmark and group-level cooperation settings. These are important properties since the remainder of the analysis will focus on the welfare comparisons between the decentralized and group-level cooperation settings. The next result characterizes the decentralized equilibrium outcomes, which depends on whether the game setting features out-group strategic complementarity or substitutability:

Proposition 2. *Assume that $b + (N - 1)\delta_{in} > 0$ holds under in-group strategic complementarity and that $b > \delta_{in}$ holds under in-group strategic substitutability. The decentralized equilibrium outcome $(\hat{x}_{1A}, \dots, \hat{x}_{NA})$ and $(\hat{x}_{1B}, \dots, \hat{x}_{NB})$ is uniquely characterized as follows, for any $i \in A$ and $j \in B$:*

- $\hat{x}_{jB} = \hat{x}_B = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_A = \hat{x}_{iA}$ if and only if $b+(N-1)\delta_{in} - N\delta_{AB} > 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} \leq 0$ hold.
- $\hat{x}_{iA} = \hat{x}_A = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_B = \hat{x}_{jB}$ if and only if $b+(N-1)\delta_{in} - N\delta_{AB} \leq 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} > 0$ hold.
- $\hat{x}_{iA} = \hat{x}_A > 0$ and $\hat{x}_{jB} = \hat{x}_B > 0$ for any $i \in A$ and $j \in B$

$$\hat{x}_A = \frac{b + (N - 1)\delta_{in} - N\delta_{BA}}{[b + (N - 1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}} a \quad \hat{x}_B = \frac{b + (N - 1)\delta_{in} - N\delta_{AB}}{[b + (N - 1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}} a$$

if and only if $b + (N - 1)\delta_{in} - N\delta_{AB} > 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} > 0$ hold.

Under out-group strategic complementarity, the only feasible case is the third one.

The proof of Proposition 2 is provided in the Appendix. First, it is easily checked that there is no equilibrium where all agents choose a zero activity level. The condition $b > \delta_{in}$ rules out the possibility that, within a given group, some members might be inactive.

Moreover, an equilibrium where agents in one group (say A) choose a positive activity level, while the agents choose a zero activity level in the other group, requires that the spillover imposed on group B by group A be strong enough. This is the case when condition $b + (N - 1)\delta_{in} - N\delta_{AB} \leq 0$ is satisfied. Moreover, under in-group strategic complementarity, the in-group strategic spillover effect must not be too strong and $b + (N - 1)\delta_{in} > 0$ must hold.

Finally, an interior equilibrium outcome would exist in two cases: either the out-group strategic spillover effects are low enough for both groups, or they are both strong enough. The second case is actually unfeasible as it would require that

$b + (N - 1)\delta_{in} - N\delta_{AB} < 0$ and $b + (N - 1)\delta_{in} - N\delta_{BA} < 0$ be satisfied simultaneously, which is ruled out by assumptions. We now move on to the characterization of the group-level cooperation outcome:

Proposition 3. *Assume that $b + 2(N - 1)\delta_{in} > 0$ holds under in-group strategic complementarity and that $b > 2\delta_{in}$ holds under in-group strategic substitutability. The group-level cooperation outcome $(x_{1A}^{gc}, \dots, x_{NA}^{gc})$ and $(x_{1B}^{gc}, \dots, x_{NB}^{gc})$ is characterized as follows, for $i \in A$ and $j \in B$:*

1. *Under out-group strategic complementarity:*

- When $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ we have

$$x_{jB}^{gc} = x_B^{gc} = \frac{a}{b + (N - 1)\delta_{in}} > 0 = x_A^{gc} = x_{iA}^{gc}$$

- When $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ then $x_{iA}^{gc} = x_A^{gc} > 0$ and $x_{jB}^{gc} = x_B^{gc} > 0$

$$x_A^{gc} = \frac{[b + (N - 1)\delta_{in} - N\delta_{BA}]a + (N - 1)e_{in}[b + (N - 1)\delta_{in}]}{[b + 2(N - 1)\delta_{in}][b + (N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}},$$

$$x_B^{gc} = \frac{[b + 2(N - 1)\delta_{in} - N\delta_{AB}]a - (N - 1)e_{in}N\delta_{AB}}{[b + 2(N - 1)\delta_{in}][b + (N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

2. *Under out-group strategic substitutability:*

- When $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ we have

$$x_{jB}^{gc} = x_B^{gc} = \frac{a}{b + (N - 1)\delta_{in}} > 0 = x_A^{gc} = x_{iA}^{gc}$$

- When $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a < e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ we have $x_{iA}^{gc} = x_A^{gc} > 0$ and $x_{jB}^{gc} = x_B^{gc} > 0$

$$x_A^{gc} = \frac{[b + (N - 1)\delta_{in} - N\delta_{BA}]a + (N - 1)e_{in}[b + (N - 1)\delta_{in}]}{[b + 2(N - 1)\delta_{in}][b + (N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}},$$

$$x_B^{gc} = \frac{[b + 2(N - 1)\delta_{in} - N\delta_{AB}]a - (N - 1)e_{in}N\delta_{AB}}{[b + 2(N - 1)\delta_{in}][b + (N - 1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

- When $e_{in} \geq \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ we have $x_{iA}^{gc} = x_A^{gc} = \frac{a+(N-1)e_{in}}{b+2(N-1)\delta_{in}} > 0 = x_B^{gc} = x_{jB}^{gc}$

The proof of Proposition 3 is provided in the Appendix. Conditions stated are necessary and sufficient for existence and uniqueness. Direct calculations yield that $b + 2(N - 1)\delta_{in} > 0$ (under in-group strategic complementarity) and $b > 2\delta_{in}$ (under in-group strategic substitutability) are necessary and sufficient conditions to ensure the maximization program of the cooperating group A is well defined and has a unique solution. Compared to the decentralized benchmark setting, the differences can be highlighted by relying on the optimality conditions. For any $i \in A$ and $j \in B$:

$$a - bx_{iA} - 2\delta_{in}X_{-iA} - \delta_{BA}X_B + (N - 1)e_{in} + \lambda_{iA}^{gc} = 0$$

Compared to the decentralized benchmark, there are two additional effects resulting from cooperation within group A. First, the in-group strategic spillover effect is highlighted by term $-2\delta_{in}X_{-iA}$ as agent i also accounts for the effect of her own activity level on the other group members. Second, there is an additional term $(N - 1)e_{in}$ as the in-group externality effect is now internalized.

It is easily checked that there is no equilibrium in which all agents choose a zero activity level. The condition $b > 2\delta_{in}$ rules out the possibility that, within a given group, some members might be inactive.

Intuitively, as group-A members internalize the in-group externality effect via cooperation, it is optimal for these agents to be inactive if and only if this externality level is low enough. In this case, group-B members do not internalize the in-group externality effect, as such their activity level remains positive as $b + (N - 1)\delta_{in} > 0$ is satisfied by assumption, and it does not depend on e_{in} .

In contrast, when the in-group externality effect is strong enough, group-A members exert a positive activity level that increases as e_{in} increases, since the in-group externality effect is internalized. This high activity level induces the non-cooperating group members to be inactive under out-group strategic substitutability. This induces the non-cooperating group members to remain active under out-group strategic complementarity.

Under out-group strategic substitutability, the remaining case is that of intermediate levels of the in-group externality. Within this range both group members are induced to remain active, and the equilibrium is characterized by positive activity levels in both groups. The group-A members' activity level depends on e_{in} as the in-group externality is internalized, while the group-B members' activity level also depends on e_{in} due to out-group strategic interactions.

Under out-group strategic complementarity, due to the unfeasibility of the case

of a corner solution in which group-B members do not contribute, there are only two cases. Either the in-group externality is low and group-A members are inactive as explained above, or it is high and both group members are induced to remain active. In this second case, the group-A members' activity level depends on e_{in} as this effect is internalized, while the group-B members' activity level depends on e_{in} due to out-group strategic interactions.

In the next section we will rely on the characterizations provided in Propositions 2 and 3 in order to analyze the conditions under which group-level cooperation has positive or negative welfare effects.

4 Group and global welfare effects

4.1 Out-group strategic complementarity

We focus on out-group strategic complementarity ($\delta_{ij} < 0$ for $i, j = A, B, i \neq j$) and the characterization of two cases. The first one provides the conditions under which group-level cooperation is Pareto improving with respect to the decentralized benchmark. As such, these conditions ensure that this initiative would be grounded in efficiency and self-enforcing. The second case provides the conditions under which group-level cooperation backfires: that is, group-A members are worse off compared to the decentralized benchmark. Under such conditions, we derive the important conclusion that group-A members would actually be better off if they could ex-ante commit to not cooperate, even if it would be costly to do so. We assume the conditions related to the case of out-group strategic complementarity under which the conclusions of Propositions 2 and 3 are valid.

Proposition 4. *Under out-group strategic complementarity, we obtain the following conclusions:*

1. *Group-level cooperation is Pareto improving if and only if one of the following cases is satisfied:*

- e_{out} is low and either e_{in} is low or $e_{in} \geq \bar{e}_{in} = \frac{\delta_{in}[b+(N-1)\delta_{in}-N\delta_{BA}]}{[b+(N-1)\delta_{in}]^2-N^2\delta_{AB}\delta_{BA}}a$
- e_{out} is high and $e_{in} \geq \bar{e}_{in}$

2. *Group-A members are worse off under group-level cooperation if and only if*

- *either e_{out} is high and e_{in} is low*
- *or e_{out} is low and e_{in} is intermediate.*

The mechanism at work concerning the second point is as follows. We know from Proposition 2 that, under out-group strategic complementarity, both group members are active in the decentralized benchmark. Also, according to Proposition 3 group-B members are always active under group-level cooperation. As such, when comparing the group-A members' payoffs in the group-level cooperation and decentralized settings, the resulting out-group externality effect is given by $Ne_{out}(x_B^{gc} - \hat{x}_B)$. It is thus important to understand the effect of group-level cooperation on group-B members' equilibrium activity levels.

First, when the in-group externality effect lies below a threshold value, group-A members decrease their equilibrium activity levels and do not contribute. This induces the non-cooperating group members to decrease their activity levels compared to the decentralized benchmark under out-group strategic complementarity. Thus we have $x_B^{gc} < \hat{x}_B$ and, since group-B members' equilibrium activity levels do not depend on the in-group externality, we directly conclude that group-A members are worse off under group-level cooperation if and only if e_{out} is high enough.

Second, when the in-group externality effect lies above but close to the threshold value, both group members still choose activity levels that are lower compared to the decentralized benchmark. As group-A members internalize the in-group externality effect, they increase their activity levels as e_{in} gets higher, which in turn induces the non-cooperating group members to increase their activity levels due to out-group strategic complementarity. Both group members eventually increase their activity levels above the benchmark. Up to this threshold value on the in-group externality we still have $x_i^{gc} < \hat{x}_i$ ($i = A, B$) and the marginal effect of e_{in} on the difference in group-A members' equilibrium payoffs can be decomposed as follows. A first term relates to an indirect and positive effect via the group-A members' equilibrium activity level. A second term relates to an indirect and positive effect via the group-B members' activity level. The last term relates to a direct and negative effect on the in-group externality faced by group-A members (in the benchmark). The second effect is related to the out-group externality faced by group-B members in the case of group-level cooperation: when the out-group externality is strong enough, the marginal effect of e_{in} is then positive. In such a case, when $x_B^{gc} < \hat{x}_B$ holds, that is, when the in-group externality is lower than a threshold value, then the difference in group-A members' equilibrium payoffs is negative, and they are worse off under group-level cooperation.

All together, group-A members are worse off under group-level cooperation when e_{out} is high and e_{in} is low. In contrast, when the out-group externality lies below a threshold value, for low enough levels of in-group externality the negative effect of

the out-group externality on the difference in the group-A members' equilibrium payoffs is offset by the positive effects driven by changes in equilibrium activity levels and by the internalization of the in-group externality. In this case, the difference in group-A members' equilibrium payoffs is positive when the in-group externality is low, and it is always positive for high enough values of e_{in} . Thus, group-A members are worse off under group-level cooperation when e_{out} is low and e_{in} takes on intermediate values.

With respect to group-B members, we already know that group-level cooperation induces the cooperating group members to decrease their activity levels when the out-group and in-group externality effects lie below threshold values. In this case, as the out-group externality term in the difference between the non-cooperating group members' equilibrium payoffs is given by $Ne_{out}(x_A^{gc} - \hat{x}_A)$ with $x_A^{gc} - \hat{x}_A < 0$ group-level cooperation also positively affects group-B members. The other cases follow from mechanisms similar to those described above.

There are several interesting implications resulting from this result. The second point in Proposition 4 does not imply that in- and out-group externalities have necessarily opposite signs. Indeed, a similar conclusion to the results obtained in Beaudry et al. (2000), relying on a notion of strategic spillovers as defined by Cooper and John (1988), would here require that e_{in} and e_{out} have opposite signs. This does not hold here. For instance, we know that group-A members are worse off under group-level cooperation when e_{out} is high and e_{in} is low. Relying on the expressions of the thresholds on the values of in- and out-group externalities characterized in the proof of Proposition 4, we obtain that:

- The threshold on the value of e_{out} is negative (positive) when the out-group strategic spillovers are strong (weak) enough
- The threshold on the value of e_{in} is negative (positive) under in-group strategic complementarity (substitutability).

Among other conclusions, this implies that, when the out-group strategic spillovers are strong enough, group-level cooperation backfires under in-group strategic substitutability for cases involving positive (or moderately negative) out-group and negative in-group externalities.

4.2 Out-group strategic substitutability

We now move on to the case of out-group strategic substitutability, and we focus again on the characterizations of Pareto improving cases and of conditions under which the cooperating group members are made worse off compared to the status

quo. We assume the conditions related to the case of out-group strategic substitutability under which the conclusions of Propositions 2 and 3 are valid. The first result focuses on the case where in-group externalities are low.

Proposition 5. *Assume that $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. Then we have*

1. *When δ_{BA} is high relative to δ_{AB} :*

$$N\delta_{AB} < b + (N - 1)\delta_{in} < N\delta_{BA}$$

group-level cooperation has no effect.

2. *When δ_{AB} is high relative to δ_{BA} :*

$$N\delta_{BA} < b + (N - 1)\delta_{in} < N\delta_{AB}$$

group-level cooperation is never Pareto improving. Moreover, group-A members are worse off under group-level cooperation if and only if e_{out} is low.

3. *When $b + (N - 1)\delta_{in} - N\delta_{ij} > 0$ for $i, j = A, B$ and $i \neq j$ assume furthermore that $\delta_{AB} = \delta_{BA} = \delta_{out}$ holds. Group-level cooperation is Pareto improving if and only if e_{out} is intermediate. In contrast, group-A members are negatively affected if and only if e_{out} is low.*

When the in-group externality e_{in} is low, the conclusions about the welfare implications of group-level cooperation are fairly negative. First, if the out-group spillover effect imposed by group B is sufficiently strong, the decentralized benchmark is such that group-B members exert a positive activity level while group-A members do not contribute. Under group-level cooperation, group-A members do not contribute as they internalize a low in-group externality effect. This makes the group-B members' incentives to contribute unchanged compared to the benchmark case: group-level cooperation results in the same equilibrium outcome. If the process of within-group cooperation is costly by itself, this would imply that the net welfare effect of this institution is negative.

Second, if the out-group spillover effect imposed by the cooperating group is sufficiently strong, the decentralized benchmark is such that group-A members exert a positive activity level while group-B members do not contribute. Yet, the outcome under group-level cooperation is unchanged: members from the cooperating group do not contribute while group-B members exert positive activity levels. Clearly, the problem faced by group-A members in the benchmark and by group-B members under group-level cooperation is then the same, and results in the same equilibrium activity levels. As such, the comparison of the group-A members' equilibrium

payoffs is driven by the difference between the out-group externality effect under group-level cooperation and the payoffs from the decentralized benchmark. These members are made worse off when the first effect is smaller than the second one: that is, when e_{out} is low enough. Conversely, they are better off when e_{out} lies above a threshold.

In contrast, the comparison between group-B members' equilibrium payoffs is driven by the difference between the payoffs under group-level cooperation and the out-group externality effect in the decentralized benchmark. These members are made better off when the first effect is larger than the second one: that is, when e_{out} is low enough. Since the role of group-A and group-B members are reversed under the two settings, the out-group externality level cannot simultaneously satisfy the constraints that would make both groups better off. In this case, when the out-group externality is low, we obtain a negative conclusion: group-level cooperation negatively affects both groups and global welfare.

Finally, when both out-group strategic spillover effects are not too strong, the decentralized benchmark results in an interior equilibrium outcome. Under group-level cooperation, group-A members internalize the (low) in-group externality effect, which induces them to not contribute. Due to out-group strategic substitutability group-B members are in turn induced to increase their activity levels compared to the benchmark. The comparison between the cooperating group members' equilibrium payoffs $\pi_A^{gc} - \hat{\pi}_A$ is thus driven by the difference between the net out-group externality effect $Ne_{out}(x_B^{gc} - \hat{x}_B)$ and the sum of their standalone returns from activity and in-group externality effect in the benchmark. We conclude that these agents are made worse off (better off) if and only if the out-group externality effect is low (high) enough. Finally, the comparison between group-B members' equilibrium payoffs is driven by the difference between two effects:

- the change in both the standalone returns from activity and the in-group externality effect;
- the out-group externality resulting from group A in the decentralized case, since this group does not contribute under group-level cooperation.

Group-B members are thus made better off if and only if the second effect is not too high: that is, if and only if the out-group externality effect is low enough. All together, group-level cooperation is Pareto improving if and only if e_{out} is neither too low nor too high. Since the in-group externality effect is here low and negative by assumption, the net change in the in-group externality effect ensures that the set of Pareto improving cases is non-degenerate.

The first case in Proposition 5 corresponds to a positive threshold value on the in-group externality effect. It implies that group-level cooperation has no effect in any situation characterized by negative out-group externality effects and markedly stronger out-group strategic spillover resulting from the non-cooperating group. If the cooperation process is actually costly, it would then be worth making group-A members ex-ante commit to not-cooperate. Such cases of out-group strategic substitutability and negative externality correspond, among other examples, to cases of common-pool resource management (see Walker et al. (2000)).

The second and third cases correspond to a negative threshold value on the in-group externality effect. In both cases group-level cooperation may be harmful to the cooperating group members for low out-group externality levels. The sign of the corresponding threshold value depends on the net effect between these members' (positive) standalone returns from activity and the (negative) in-group externality effect under the decentralized benchmark. When the in-group externality effect gets arbitrarily high, the threshold value on the out-group externality effect is negative. When this externality gets low, the sign of the threshold value is positive when the out-group spillover effect resulting from group B is high enough. The conclusions of these cases thus apply to situations characterized by out-group strategic substitutability and negative in-group externality effects, but with potential qualitatively heterogeneous out-group externality effects.

A final conclusion is that, in cases where the in-group externality effect is low, group-level cooperation has at best no effect and can be potentially harmful under sufficient asymmetry in the out-group strategic spillovers. It can harm the non-cooperating group but can also negatively affect all involved parties. The only case where it might be Pareto improving is when out-group strategic spillovers are not too strong and the out-group externality level is intermediate.

The next result focuses on the case where the in-group externality is intermediate. We only focus on the characterization of cases where group-A members are negatively affected by group-level cooperation, when the decentralized equilibrium outcome is interior and the situation is homogeneous ($\delta_{AB} = \delta_{BA} = \delta_{out} > 0$). This simplifies the description of the conclusions corresponding to this case.

Proposition 6. *Assume that $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a < e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ hold. Assume also $\delta_{AB} = \delta_{BA} = \delta_{out} > 0$ and $b+(N-1)\delta_{in} - N\delta_{out} > 0$ is satisfied. Then there exist $e_{out}^t < e_{out}^+$ ($e_{out}^+ > 0$) together with $\bar{e}_{in} \in \mathbf{R}$ such that group-A members are worse off under group-level cooperation if and only if one of the following cases is satisfied:*

1. e_{in} and e_{out} are low: $e_{in} \leq \bar{e}_{in}$ and $e_{out} \leq e_{out}^t$.

2. e_{in} and e_{out} are intermediate, with $e_{out}^t < e_{out} < e_{out}^+$.

3. e_{in} and e_{out} are high: $e_{in} \geq \bar{e}_{in}$ and $e_{out} \geq e_{out}^+ > 0$.

The difference in group A members' equilibrium payoffs is made of the difference in the standalone returns from activity depending on whether there is group-level cooperation or not, the difference in out-group externalities, and the in-group externality effect (which is not internalized in group B).

The marginal effect of e_{in} on the difference in group A members' equilibrium payoffs is thus made of three terms: a positive indirect effect on the standalone returns from activity via the change in cooperating group members' equilibrium activity level, an indirect effect on the out-group externality effect via the change in the non-cooperating group members' equilibrium activity level, which sign is given by $e_{out}(x_B^{gc} - \hat{x}_B)$ and a direct negative effect on the in-group externality term.

For low levels of the out-group externality, this marginal effect is positive which, combined with the fact that the difference in equilibrium payoffs is zero when $e_{in} = \bar{e}_{in}$ (as equilibrium activity levels are equal under both scenarios), implies that the difference in equilibrium payoffs is negative when $e_{in} \leq \bar{e}_{in}$ holds.

For intermediate levels of the out-group externality, the marginal effect of e_{in} is now negative up to a threshold value which may be either smaller or higher than \bar{e}_{in} . This allows to conclude that group A members are worse off under group-level cooperation for an intermediate range of in-group externality levels.

Finally, for high out-group externality levels, the marginal effect of e_{in} is negative. This is mostly driven by the impact on the difference in the out-group externality, which is negative when the non-cooperating group members' equilibrium activity level becomes lower compared to the benchmark case, that is, when the in-group externality level is high enough ($e_{in} \geq \bar{e}_{in}$).

The proof of Proposition 6 highlights that $\bar{e}_{in} > 0$ (< 0) under in-group strategic substitutability (complementarity). Moreover, e_{out}^t is positive if and only if the out-group strategic spillover effects are high. This allows to derive certain implications from Proposition 6. For instance, under in-group strategic substitutability, for high enough levels of the out-group strategic spillover, group-level cooperation always backfires under either negative or sufficiently high positive externalities. Under in-group strategic complementarity, for low enough levels of the out-group strategic spillover, the cooperating group members are always worse off under positive in- and out-group externalities.

We abstract from providing the characterization of the cases where group-level cooperation is Pareto improving as it is quite involved. The main features of this characterization are as follows:⁴

- The out-group externality level is necessarily intermediate
- Whether the set of Pareto-improving cases is degenerate depends on the interplay between the out-group strategic spillover and in-group externality

We conclude the analysis with the case where the in-group externality effect is high. We here focus on one case in which group-level cooperation has positive effects on group and global welfare.

Proposition 7. *Assume $e_{in} \geq \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ and that δ_{AB} is high relative to δ_{BA} , that is $N\delta_{BA} < b + (N-1)\delta_{in} \leq N\delta_{AB}$ hold. Then group-level cooperation always makes group-A members better off compared to the status quo. Moreover, it is Pareto improving if and only if e_{in} lies below (above) a threshold value and the out-group externality is negative (positive).*

We know from Propositions 2 and 3 that, when the level of the in-group externality is high and the out-group strategic spillover resulting from group A is highest, the decentralized and group-level cooperation settings result in equilibrium outcomes in which group-B members do not contribute. Specifically, we have $x_A^{gc} = \frac{a+(N-1)e_{in}}{b+2(N-1)\delta_{in}}$ while $\hat{x}_A = \frac{a}{b+(N-1)\delta_{in}}$ and $x_B^{gc} = \hat{x}_B = 0$ are satisfied. Here the out-group externality effect plays no role, and group-A members' optimal activity level increases as the in-group externality effect increases, as they internalize this effect under group-level cooperation.

Intuitively, for not too high levels of this externality, the out-group strategic spillover is the dominant effect, which results in lower activity levels compared to the decentralized benchmark due to strategic substitutability (as group-B members still do not contribute). As the in-group externality level increases, it eventually outweighs the out-group strategic spillover effect. Equilibrium activity levels eventually get higher compared to the decentralized benchmark. Thus, for not too high levels of the in-group externality, the negative difference in standalone returns from activity is lower than the difference in externality-related effects, and group-A members benefit from group-level cooperation. Once the in-group externality effect gets large enough, the cooperating group members eventually increase their activity levels above those in the decentralized benchmark. Past this point, the marginal effect of an increase in the in-group externality is positive, and the

⁴Details are available from the author upon request.

difference in group-A members' equilibrium payoffs becomes increasingly larger.

Regarding the non-cooperating group members, as they do not contribute under both settings, the difference in payoffs is due to the difference in out-group externalities. We know that group-A members' equilibrium activity level is lower under group-level cooperation for not too strong in-group externality and eventually gets higher. As such, group-B members are positively affected when group-A members' activity levels are lower (higher) compared to the decentralized benchmark under negative (positive) out-group externality.

The actual threshold value on the in-group externality effect is $e_{in}^i = \frac{\delta_{in}}{b+(N-1)\delta_{in}}a$ which is negative (positive) under in-group strategic complementarity (substitutability). The conclusion in Proposition 7 implies several things:

- Under in-group strategic complementarity and negative out-group externality, Pareto improvements require that the in-group externality be negative.
- Under in-group strategic substitutability and positive out-group externality, Pareto improvements require that the in-group externality be positive.
- The remaining cases for Pareto improvements allow for positive and negative in-group externality effect.

We focus on one case in the statement of Proposition 7 because, in all remaining cases, the conclusions are similar to those obtained when in-group externalities are intermediate (Proposition 6). Specifically, Pareto improvements obtain if and only if the out-group externality is intermediate; group-A members are worse off under group-level cooperation when e_{out} is positive and high.

5 Conclusion

We analyze the existence, coordination and efficiency effects of group-level cooperation in settings featuring heterogeneous strategic spillovers and externalities within and between groups of agents. Conclusions depend on whether out-group interactions feature strategic substitutes or complements. Under high out-group strategic spillovers, group-level cooperation can trigger either equilibrium existence or coordination issues. When these effects are not too high, a unique group-level cooperation equilibrium exists and depends on the in-group externality level.

With respect to efficiency, cases are characterized under which this initiative is self-enforcing. Under out-group strategic complementarity, the first case corresponds

to high externality effects. Second, under low out-group externality, Pareto improvements obtain except for an intermediate range of in-group externality levels. In contrast, the conclusions are more involved under out-group strategic substitutability. When the in-group externality effect is low, group-level cooperation is never Pareto improving when out-group strategic spillovers are sufficiently asymmetric. Otherwise, it is Pareto improving if and only if the out-group externality level is intermediate. This is also a necessary condition when the in-group externality level is intermediate: Pareto improvements then depend on the interplay between out-group strategic spillover and in-group externality effects. Finally, when the in-group externality level is high and the out-group strategic spillover resulting from the cooperating group is highest, group-level cooperation is Pareto improving when externalities are negative (positive) under in-group strategic substitutability (complementarity).

Conditions are then provided under which it backfires: the cooperating group members would be better off if they could ex-ante commit to not cooperate, even if it would be costly to do so. Under out-group strategic complementarity, this is so when the out-group externality level is high and the in-group externality level is low. Under out-group strategic substitutability, group-level cooperation backfires when externality effects are simultaneously either low, intermediate, or high.

The goal of this work was to assess the features of group-level cooperation. Even though the model considered is characterized by heterogeneous strategic spillovers and externalities, we abstract from several issues. For instance, accounting for other regarding preferences (Lange and Vogt 2003), considering dynamic settings where activity levels have a more or less persistent effect (Battaglini et al. 2014), or analyzing the dynamics of group-level cooperation (Wang et al. 2026) would constitute interesting extensions that deserve future research.

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Appendix

Proof of Proposition 1

We refer to the steps in the proof of Propositions 2 and 3.

- When $b+(N-1)\delta_{in}-N\delta_{AB} < 0$ and $b+(N-1)\delta_{in}-N\delta_{BA} < 0$ are satisfied simultaneously (which necessarily relates to the case of out-group strategic substitutability), the case of the decentralized benchmark leads immediately to the conclusion that, as $[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA} < 0$ necessarily holds, three equilibria co-exist: (1) $\forall i \in A$ and $\forall j \in B$ $\hat{x}_{iA} = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_{jB}$ (2) $\forall i \in A$ and $\forall j \in B$ $\hat{x}_{jB} = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_{iA}$ and (3)

$\forall i \in A$ and $\forall j \in B$ $\hat{x}_{iA} = \hat{x}_A > 0$ and $\hat{x}_{jB} = \hat{x}_B > 0$ with

$$\hat{x}_A = \frac{b + (N-1)\delta_{in} - N\delta_{BA}}{[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a \quad \hat{x}_B = \frac{b + (N-1)\delta_{in} - N\delta_{AB}}{[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a$$

- When the following condition holds

$$[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} \leq 0$$

we have that $\frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a < -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ for the case of out-group strategic complementarity and the proof of Proposition 3 yields the following conclusions. First, when $e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ two equilibria co-exist, namely (1) $\forall i \in A$ and $\forall j \in B$ $x_{jB}^{gc} = \frac{a}{b+(N-1)\delta_{in}} > 0 = x_{iA}^{gc}$ and (2) $\forall i \in A$ and $\forall j \in B$ $x_{iA}^{gc} = x_A^{gc} > 0$ and $x_{jB}^{gc} = x_B^{gc} > 0$ with

$$x_A^{gc} = \frac{[b + (N-1)\delta_{in} - N\delta_{BA}]a + (N-1)e_{in}[b + (N-1)\delta_{in}]}{[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

$$x_B^{gc} = \frac{[b + 2(N-1)\delta_{in} - N\delta_{AB}]a - N(N-1)\delta_{AB}e_{in}}{[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

Second, when $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ there is no equilibrium outcome as the only candidate would be the interior equilibrium, which exists only if $[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} > 0$ holds.

- we have that $\frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ for the case of out-group strategic substitutability and the proof of Proposition 3 yields the following conclusion. When $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a < e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ there is no equilibrium outcome as the only candidate would be the interior equilibrium, which exists only if

$$[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} > 0$$

Proof of Proposition 2

By assumptions we know that $b \geq \delta_{in} \geq 0$ and $[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA} > 0$ are satisfied. The optimality conditions are necessary and sufficient, and given by, for any $i \in A$ and $j \in B$:

$$a - bx_{iA} - \delta_{in}X_{-iA} - \delta_{BA}X_B + \lambda_{iA} = 0$$

and

$$a - bx_{jB} - \delta_{in}X_{-jB} - \delta_{AB}X_A + \lambda_{jB} = 0,$$

where $\lambda_{iA} \geq 0$ and $\lambda_{jB} \geq 0$ are the lagrangian parameters associated to each optimality condition. First, it is easily checked that there is no equilibrium where no agent in each group chooses a positive activity level. Second, there cannot be an equilibrium outcome for which $\lambda_{iA} > 0$ and $x_{lA} > 0$ simultaneously (and the same holds for group B). Otherwise, we would have:

$$a - \delta_{in}X_{-iA} - \delta_{BA}X_B = a - \delta_{in}X_A - \delta_{BA}X_B < 0$$

while

$$a - \delta_{in}X_{-lA} - \delta_{BA}X_B = bx_{lA} > 0$$

The second condition yields

$$a - \delta_{in}X_A - \delta_{BA}X_B = (b - \delta_{in})x_{lA}$$

which is non-negative when $b \geq \delta_{in}$ holds. This is a contradiction.

Now, if there is one equilibrium such that $x_{iA}^N > 0$ for any $i \in A$ while $\lambda_{jB} > 0$ for any $j \in B$, then one has:

$$a - \delta_{AB}X_A^N < 0$$

while

$$a - \delta_{in}X_A^N = (b - \delta_{in})x_{iA}^N = (b - \delta_{in})x_{lA}^N,$$

which in turn implies that $x_{iA}^N = x_{lA}^N = x_A^N$ for any i and $l \in A$. Rewriting, we obtain:

$$x_A^N = \frac{a}{b + (N-1)\delta_{in}} > 0$$

and

$$a \frac{b + (N-1)\delta_{in} - N\delta_{AB}}{b + (N-1)\delta_{in}} < 0$$

The expression of x_A^N implies that $x_A^N > 0$ if and only if $b + (N-1)\delta_{in} > 0$ holds. The second condition is then satisfied if and only if $b + (N-1)\delta_{in} - N\delta_{AB} < 0$ holds.

The symmetric case for group B leads to the following conditions:

$$x_B^N = \frac{a}{b + (N-1)\delta_{in}} > 0$$

and

$$a \frac{b + (N-1)\delta_{in} - N\delta_{BA}}{b + (N-1)\delta_{in}} < 0$$

The expression of x_B^N implies that $x_B^N > 0$ if and only if $b + (N-1)\delta_{in} > 0$ holds. The second condition is then satisfied if and only if $b + (N-1)\delta_{in} - N\delta_{BA} < 0$ holds.

In the final case one must have $\lambda_{iA} = 0 = \lambda_{jB}$ for all $i \in A$ and $j \in B$, and more specifically $x_{iA}^N = x_{lA}^N = x_A^N > 0$ for any i and $l \in A$, and the same holds within group B : $x_{jB}^N = x_{kB}^N = x_B^N > 0$ for any j and $k \in B$. Now, coming back to the optimality conditions and solving for x_A^N and x_B^N , we obtain:

$$x_A^N = \frac{b + (N-1)\delta_{in} - N\delta_{BA}}{[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}} a \quad x_B^N = \frac{b + (N-1)\delta_{in} - N\delta_{AB}}{[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}} a$$

Due to the assumptions, this equilibrium outcome is interior (x_A^N and $x_B^N > 0$ hold) if and only if either $b + (N-1)\delta_{in} - N\delta_{AB} > 0$ and $b + (N-1)\delta_{in} - N\delta_{BA} > 0$ hold or $b + (N-1)\delta_{in} - N\delta_{AB} < 0$ and $b + (N-1)\delta_{in} - N\delta_{BA} < 0$ are satisfied. If there is in-group strategic complementarity the second case is unfeasible as it implies

$$[b + (N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA} < 0$$

which in turn implies $[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} < 0$ as $\delta_{in} < 0$ holds. This is ruled out by our assumptions. If there is in-group strategic substitutability, the second case does not result in a unique equilibrium outcome: indeed, the previous cases immediately imply that $x_A^N = \frac{a}{b + (N-1)\delta_{in}} > 0 = x_B^N$ and $x_B^N = \frac{a}{b + (N-1)\delta_{in}} > 0 = x_A^N$ would also coexist in this second case. This concludes the proof.

Proof of Proposition 3

Direct calculations yield that $b + 2(N - 1)\delta_{in} > 0$ (under in-group strategic complementarity) and $b > 2\delta_{in}$ (under in-group strategic substitutability) are necessary and sufficient conditions to ensure that the maximization program of the cooperating group A is well defined and has a unique solution. The optimality conditions are then necessary and sufficient, and given by, for any $i \in A$ and $j \in B$:

$$a - bx_{iA} - 2\delta_{in}X_{-iA} - \delta_{BA}X_B + (N - 1)e_{in} + \lambda_{iA}^{gc} = 0$$

and

$$a - bx_{jB} - \delta_{in}X_{-jB} - \delta_{AB}X_A + \lambda_{jB}^{gc} = 0,$$

where $\lambda_{iA}^{gc} \geq 0$ and $\lambda_{jB}^{gc} \geq 0$ are the lagrangian parameters associated to each optimality condition. First, it is easily checked that there is no equilibrium where no agent in each group chooses a positive activity level. Second, there cannot be an equilibrium outcome for which $\lambda_{iA}^{gc} > 0$ and $x_{lA} > 0$ simultaneously (and the same holds for group B). Otherwise, we would have:

$$a - 2\delta_{in}X_{-iA} - \delta_{BA}X_B + (N - 1)e_{in} = a - 2\delta_{in}X_A - \delta_{BA}X_B + (N - 1)e_{in} < 0$$

while

$$a - 2\delta_{in}X_{-lA} - \delta_{BA}X_B + (N - 1)e_{in} = bx_{lA} > 0$$

The second condition yields

$$a - 2\delta_{in}X_A - \delta_{BA}X_B + (N - 1)e_{in} = (b - 2\delta_{in})x_{lA}$$

which is non-negative when $b \geq 2\delta_{in}$ holds. This is a contradiction. Now, if there is one equilibrium such that $x_{iA} > 0$ for any $i \in A$ while $\lambda_{jB}^{gc} > 0$ for any $j \in B$, then one has:

$$a - \delta_{AB}X_A < 0$$

while

$$a - 2\delta_{in}X_A = (b - 2\delta_{in})x_{iA} = (b - 2\delta_{in})x_{lA},$$

which in turn implies that $x_{iA}^{gc} = x_{lA}^{gc} = x_A^{gc}$ for any i and $l \in A$. Rewriting, we obtain $a + (N - 1)e_{in} = [b + 2(N - 1)\delta_{in}]x_A^{gc}$ or

$$x_A^{gc} = \frac{a + (N - 1)e_{in}}{b + 2(N - 1)\delta_{in}}$$

together with

$$a < N\delta_{AB} \frac{a + (N - 1)e_{in}}{b + 2(N - 1)\delta_{in}}$$

Rewriting this inequality, we obtain:

$$[b + 2(N - 1)\delta_{in} - N\delta_{AB}]a < N(N - 1)\delta_{AB}e_{in}$$

Under out-group strategic complementarity, this is equivalent to

$$e_{in} < \frac{b + 2(N - 1)\delta_{in} - N\delta_{AB}}{N(N - 1)\delta_{AB}}a$$

while under out-group strategic substitutability, this is equivalent to

$$e_{in} > \frac{b + 2(N - 1)\delta_{in} - N\delta_{AB}}{N(N - 1)\delta_{AB}}a$$

The equilibrium outcome $x_A^{gc} = \frac{a+(N-1)e_{in}}{b+2(N-1)\delta_{in}} > 0 = x_B^{gc}$ also requires that $a + (N-1)e_{in} > 0$ or $e_{in} > -\frac{a}{N-1}$ holds. As such, under out-group strategic complementarity, the existence of this equilibrium outcome requires that $\frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a > -\frac{a}{N-1}$ holds which, since $\delta_{AB} < 0$ holds, can be rewritten as

$$b + 2(N-1)\delta_{in} - N\delta_{AB} < -N\delta_{AB}$$

or $b + 2(N-1)\delta_{in} < 0$ holds, which is a contradiction. Thus, this case is feasible under out-group strategic substitutability only.

Third, if there is one equilibrium such that $x_{jB} > 0$ for any $j \in B$ while $\lambda_{iA}^{gc} > 0$ for any $i \in A$, then one has:

$$a - \delta_{BA}X_B + (N-1)e_{in} < 0$$

while

$$a - bx_{jB} - \delta_{in}X_{-jB} = 0,$$

This condition implies that $x_{jB}^{gc} = x_{kB}^{gc} = x_B^{gc}$ for any j and $k \in B$. Rewriting, we obtain $a = [b + (N-1)\delta_{in}]x_B^{gc}$ or

$$x_B^{gc} = \frac{a}{b + (N-1)\delta_{in}}$$

together with

$$a - N\delta_{BA}\frac{a}{b + (N-1)\delta_{in}} + (N-1)e_{in} < 0$$

Rewriting this inequality, we obtain:

$$e_{in} < -\frac{b + (N-1)\delta_{in} - N\delta_{BA}}{(N-1)[b + (N-1)\delta_{in}]}a$$

In this case we have $x_B^{gc} = \frac{a}{b+(N-1)\delta_{in}} > 0 = x_A^{gc}$.

In the final case one must have $\lambda_{iA}^{gc} = 0 = \lambda_{jB}^{gc}$ for all $i \in A$ and $j \in B$, and more specifically $x_{iA}^N = x_{lA}^N = x_A^N > 0$ for any i and $l \in A$, and the same holds within group B : $x_{jB}^N = x_{kB}^N = x_B^N > 0$ for any j and $k \in B$. Now, coming back to the optimality conditions and solving for x_A^N and x_B^N , we obtain:

$$x_A^{gc} = \frac{[b + (N-1)\delta_{in} - N\delta_{BA}]a + (N-1)e_{in}[b + (N-1)\delta_{in}]}{[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

$$x_B^{gc} = \frac{[b + 2(N-1)\delta_{in} - N\delta_{AB}]a - N(N-1)\delta_{AB}e_{in}}{[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}}$$

As $[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA} > 0$ by assumption $x_A^{gc} > 0$ if and only if $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. Now $x_B^{gc} > 0$ if and only if $e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ holds under out-group strategic substitutability, or $e_{in} > \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ under out-group strategic complementarity. All together, we obtain that this equilibrium outcome is interior if and only if

$$-\frac{b + (N-1)\delta_{in} - N\delta_{BA}}{(N-1)[b + (N-1)\delta_{in}]}a < e_{in} < \frac{b + 2(N-1)\delta_{in} - N\delta_{AB}}{N(N-1)\delta_{AB}}a$$

under out-group strategic substitutability or

$$e_{in} > \max\left\{-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a, \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a\right\} = -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$$

under out-group strategic complementarity. Summarizing, we obtain under out-group strategic complementarity:

- $x_B^{gc} > 0 = x_A^{gc}$ if and only if $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds
- $x_A^{gc} > 0$ and $x_B^{gc} > 0$ if and only if $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds

Under out-group strategic substitutability:

- $x_B^{gc} > 0 = x_A^{gc}$ if and only if $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds
- $x_A^{gc} > 0$ and $x_B^{gc} > 0$ if and only if $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a < e_{in} < \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ hold
- $x_A^{gc} > 0 = x_B^{gc}$ if and only if $e_{in} \geq \frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}}a$ holds

This concludes the proof.

Proof of Proposition 4

From Propositions 2 and 3 we know that $x_B^{gc} = \frac{a}{b+(N-1)\delta_{in}} > 0 = x_A^{gc}$ if and only if $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ while the group-level cooperation outcome is interior if and only if $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. Since $\delta_{ij} < 0$ for $i \neq j$ we conclude that $b+(N-1)\delta_{in}-N\delta_{ij} > 0$ for $i, j = A, B$ with $i \neq j$. As such, the decentralized equilibrium outcome is interior under out-group strategic complementarity. We deduce that

$$\Pi_A^{gc} - \Pi_A^N = Ne_{out}(x_B^{gc} - \hat{x}_B) - \hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]$$

$$\Pi_B^{gc} - \Pi_B^N = (x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right] - Ne_{out}\hat{x}_A$$

when $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. Then, as $x_B^{gc} - \hat{x}_B < 0$ we deduce immediately that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if e_{out} is high:

$$e_{out} \geq -\frac{\hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]}{N(\hat{x}_B - x_B^{gc})}$$

Now, group-level cooperation will in turn be Pareto improving if and only if $\Pi_A^{gc} - \Pi_A^N > 0$ and $\Pi_B^{gc} - \Pi_B^N > 0$ hold: we already know that $\Pi_A^{gc} - \Pi_A^N > 0$ if and only if e_{out} is low, and we obtain that $\Pi_B^{gc} - \Pi_B^N > 0$ if and only if

$$e_{out} < \frac{(x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right]}{N\hat{x}_A}$$

We conclude that, when $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds group-level cooperation is Pareto improving if and only if e_{out} is low.

We now consider the case where $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. The equilibrium outcomes are interior in both settings, and we have:

$$\begin{aligned}\Pi_A^{gc} - \Pi_A^N &= \frac{b}{2} [(x_A^{gc})^2 - (\hat{x}_A)^2] + (N-1) [\delta_{in}(x_A^{gc})^2 - e_{in}\hat{x}_A] + Ne_{out}(x_B^{gc} - \hat{x}_B) \\ \Pi_B^{gc} - \Pi_B^N &= (x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right] + Ne_{out}(x_A^{gc} - \hat{x}_A)\end{aligned}$$

We obtain quickly that $x_A^{gc} \geq \hat{x}_A$ and $x_B^{gc} \geq \hat{x}_B$ are equivalent to $e_{in} \geq \bar{e}_{in} = \frac{\delta_{in}[b+(N-1)\delta_{in}-N\delta_{BA}]}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a$. Denoting $D = [b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}] - N^2\delta_{AB}\delta_{BA}$ and $\phi_A(e_{in})$ the difference $\Pi_A^{gc} - \Pi_A^N$ and differentiating with respect to e_{in} :

$$\phi'_A(e_{in}) = \frac{(N-1)[b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}]}{D}x_A^{gc} - Ne_{out}\frac{N(N-1)\delta_{AB}}{D} - (N-1)\hat{x}_A$$

Differentiating a second time we quickly deduce that ϕ''_A is positive and thus ϕ'_A increases as e_{in} increases. Using the fact that x_A^{gc} gets close to zero as e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ we obtain that, as e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ then $\phi'_A(e_{in})$ gets close to

$$-\frac{Ne_{out}\delta_{AB}}{D} - \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a$$

As $-N\delta_{AB} > 0$ we conclude that this expression is positive if and only if

$$e_{out} \geq \frac{D}{-N\delta_{AB}} \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a = \frac{D}{-N\delta_{AB}}\hat{x}_A$$

In this case we conclude that ϕ'_A is positive for any $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ and as such ϕ_A increases as e_{in} increases. It is easily checked that $\phi_A(\bar{e}_{in}) = 0$ and we conclude that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if $e_{in} \leq \bar{e}_{in}$ holds.

When $e_{out} < \frac{D}{-N\delta_{AB}}\hat{x}_A$ holds we know that ϕ_A first decreases, then increases, as e_{in} increases. Again, as x_A^{gc} gets close to zero and x_B^{gc} gets close to $\frac{a}{b+(N-1)\delta_{in}}$ when e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ we obtain that $\phi_A(e_{in})$ gets close to

$$\begin{aligned}Ne_{out}N\delta_{AB} \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{[b+(N-1)\delta_{in}]\{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}}a \\ - [b+(N-1)\delta_{in}-N\delta_{BA}]^2 \frac{[b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}] + 2N^2\delta_{AB}\delta_{BA}}{[b+(N-1)\delta_{in}]\{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}}a^2\end{aligned}$$

This expression is a decreasing function of e_{out} and it is thus positive if and only if

$$e_{out} < \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{-N^2\delta_{AB}} \frac{[b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}] - 2N^2\delta_{AB}\delta_{BA}}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a$$

It is easily checked that this threshold value is smaller than $\frac{D}{-N\delta_{AB}}\hat{x}_A$ and this implies that (i) $\phi_A(e_{in})$ is non-positive for e_{in} higher but close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ when

$$\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{-N^2\delta_{AB}} \frac{[b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}] - 2N^2\delta_{AB}\delta_{BA}}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}}a \leq e_{out} < \frac{D}{-N\delta_{AB}}\hat{x}_A$$

and (ii) it is positive when $e_{out} < \frac{b+(N-1)\delta_{in}-N\delta_{BA}}{-N^2\delta_{AB}} \frac{[b+(N-1)\delta_{in}][b+2(N-1)\delta_{in}]-2N^2\delta_{AB}\delta_{BA}}{[b+(N-1)\delta_{in}]^2-N^2\delta_{AB}\delta_{BA}} a$ holds. As $\phi_A(\bar{e}_{in}) = 0$ we thus conclude that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if $e_{in} \leq \bar{e}_{in}$ in (i), while there exists in (ii) a threshold on the value of e_{in} (higher than $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ but lower than \bar{e}_{in}) such that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if e_{in} lies in between this threshold value and \bar{e}_{in} . Combining the case $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ with (i) we conclude that group-A members are worse off under group-level cooperation when e_{out} and e_{in} are low. Then case (ii) implies that the only other case where $\Pi_A^{gc} - \Pi_A^N \leq 0$ corresponds to e_{out} low and e_{in} intermediate. This concludes the case of the second point in Proposition 4.

Coming back to the first point in Proposition 4, it remains to consider the case where $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ holds. Denoting $\phi_B(e_{in})$ the difference $\Pi_B^{gc} - \Pi_B^N$ and differentiating with respect to e_{in} :

$$\phi'_B(e_{in}) = \frac{-N(N-1)\delta_{AB}}{D} [bx_B^{gc} + (N-1)e_{in}] + Ne_{out} \frac{(N-1)[b+(N-1)\delta_{in}]}{D} + (N-1)(x_B^{gc} - \hat{x}_B)$$

Differentiating a second time we quickly deduce that ϕ''_B is positive and thus ϕ'_B increases as e_{in} increases. Using the fact that x_B^{gc} gets close to $\frac{a}{b+(N-1)\delta_{in}}$ as e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ we obtain that, as e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ then $\phi'_B(e_{in})$ gets close to

$$N\delta_{AB} \frac{F}{D[b+(N-1)\delta_{in}]\{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}} a + \frac{Ne_{out}[b+(N-1)\delta_{in}]}{D}$$

where

$$F = \{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\} \{b+2(N-1)\delta_{in} - 2N\delta_{BA}\} + (N-1)\delta_{in}[b+(N-1)\delta_{in}][b+(N-1)\delta_{in} - N\delta_{BA}]$$

As $N\delta_{AB} < 0$ we conclude that this expression is positive if and only if

$$e_{out} \geq -\delta_{AB} \frac{F}{[b+(N-1)\delta_{in}]^2 \{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}} a$$

In this case we conclude that ϕ'_B is positive for any $e_{in} > -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ and as such ϕ_B increases as e_{in} increases. It is easily checked that $\phi_B(\bar{e}_{in}) = 0$ and we conclude that $\Pi_B^{gc} - \Pi_B^N \geq 0$ if and only if $e_{in} \geq \bar{e}_{in}$ holds.

When e_{out} is lower than this threshold value, we know that ϕ_B first decreases, then increases, as e_{in} increases. Again, as x_A^{gc} gets close to zero and x_B^{gc} gets close to $\frac{a}{b+(N-1)\delta_{in}}$ when e_{in} gets close to $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]} a$ we obtain that $\phi_B(e_{in})$ gets close to

$$\frac{N\delta_{AB}[b+(N-1)\delta_{in} - N\delta_{BA}]}{[b+(N-1)\delta_{in}]\{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}} aG - Ne_{out} \frac{b+(N-1)\delta_{in} - N\delta_{BA}}{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}} a$$

with

$$G = \frac{b[b+(N-1)\delta_{in} - N\delta_{AB}]}{2\{[b+(N-1)\delta_{in}]^2 - N^2\delta_{AB}\delta_{BA}\}} a - \frac{b+2(N-1)\delta_{in} - 2N\delta_{BA}}{2[b+(N-1)\delta_{in}]} a$$

This expression is a decreasing function of e_{out} and it is thus positive if and only if e_{out} lies below a threshold value. If this does not hold, then we conclude as before that $\Pi_B^{gc} - \Pi_B^N \geq 0$ if and only if $e_{in} \geq \bar{e}_{in}$ holds. When this condition is eventually satisfied (for e_{out} low enough) then there exists a threshold value on e_{in} (higher than $-\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ and lower than \bar{e}_{in}) such that $\Pi_B^{gc} - \Pi_B^N \geq 0$ if and only if e_{in} is either lower than this threshold value or $e_{in} \geq \bar{e}_{in}$ holds.

All together, we conclude that group-level cooperation simultaneously satisfies $\Pi_A^{gc} - \Pi_A^N \geq 0$ and $\Pi_B^{gc} - \Pi_B^N \geq 0$ when:

- e_{out} is high and $e_{in} \geq \bar{e}_{in}$
- e_{out} is low and either e_{in} is low (below a threshold value strictly lower than \bar{e}_{in}) or $e_{in} \geq \bar{e}_{in}$

This concludes the proof of the first point in Proposition 4.

Proof of Proposition 5

We proceed with the proofs of the three points as follows:

1. From Propositions 2 and 3 we know that $x_B^{gc} = \frac{a}{b+(N-1)\delta_{in}} > 0 = x_A^{gc}$ when $e_{in} \leq -\frac{b+(N-1)\delta_{in}-N\delta_{BA}}{(N-1)[b+(N-1)\delta_{in}]}a$ holds. We also know that the decentralized equilibrium outcome is given by $\hat{x}_B = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_A$ when $b+(N-1)\delta_{in}-N\delta_{AB} > 0$ and $b+(N-1)\delta_{in}-N\delta_{BA} < 0$ hold. As such, when δ_{BA} is high enough, group-level cooperation and the status quo are outcome equivalent.
2. Again, we know from Proposition 2 that the decentralized equilibrium outcome is given by $\hat{x}_A = \frac{a}{b+(N-1)\delta_{in}} > 0 = \hat{x}_B$ when $b+(N-1)\delta_{in}-N\delta_{AB} < 0$ and $b+(N-1)\delta_{in}-N\delta_{BA} > 0$ hold. We thus have

$$\begin{aligned}\Pi_A^{gc} - \Pi_A^N &= Ne_{out}x_B^{gc} - \hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right] \\ \Pi_B^{gc} - \Pi_B^N &= x_B^{gc} \left[\frac{b}{2}x_B^{gc} + (N-1)e_{in} \right] - Ne_{out}\hat{x}_A\end{aligned}$$

We deduce immediately that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if e_{out} is low:

$$e_{out} \leq \frac{\hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]}{Nx_B^{gc}}$$

Keeping in mind that here $\hat{x}_A = x_B^{gc} = x$ we conclude that group-level cooperation is Pareto improving if and only if e_{out} is high and $\Pi_B^{gc} - \Pi_B^N > 0$ which is equivalent to e_{out} being low:

$$\frac{x \left[\frac{b}{2}x + (N-1)e_{in} \right]}{Nx} \leq e_{out} < \frac{x \left[\frac{b}{2}x + (N-1)e_{in} \right]}{Nx}$$

It is easily checked that this condition cannot hold: group-level cooperation is not Pareto improving.

3. Finally, if $b+(N-1)\delta_{in}-N\delta_{ij} > 0$ for $i, j = A, B$ with $i \neq j$ then the decentralized equilibrium outcome is interior, and we obtain

$$\Pi_A^{gc} - \Pi_A^N = Ne_{out}(x_B^{gc} - \hat{x}_B) - \hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]$$

$$\Pi_B^{gc} - \Pi_B^N = (x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right] - Ne_{out}\hat{x}_A$$

We obtain $x_B^{gc} - \hat{x}_B > 0$ and we then deduce that $\Pi_A^{gc} - \Pi_A^N \leq 0$ if and only if e_{out} is low:

$$e_{out} \leq \frac{\hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]}{N(x_B^{gc} - \hat{x}_B)}$$

Now group-level cooperation is Pareto improving if and only if e_{out} is high and $\Pi_B^{gc} - \Pi_B^N > 0$ which is equivalent to e_{out} being low:

$$\frac{\hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]}{N(x_B^{gc} - \hat{x}_B)} \leq e_{out} < \frac{(x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right]}{N\hat{x}_A} \quad (6)$$

This condition is non degenerate if and only if $\frac{\hat{x}_A \left[\frac{b}{2}\hat{x}_A + (N-1)e_{in} \right]}{N(x_B^{gc} - \hat{x}_B)} < \frac{(x_B^{gc} - \hat{x}_B) \left[\frac{b}{2}(\hat{x}_B + x_B^{gc}) + (N-1)e_{in} \right]}{N\hat{x}_A}$ holds. Assuming $\delta_{AB} = \delta_{BA} = \delta_{out}$ we have $\hat{x}_A = \hat{x}_B = \frac{a}{b+(N-1)\delta_{in}+N\delta_{out}} = x$ and this inequality can be rewritten as

$$x^2 \left[\frac{b}{2}x + (N-1)e_{in} \right] < (x_B^{gc} - x)^2 \left[\frac{b}{2}(x + x_B^{gc}) + (N-1)e_{in} \right]$$

or

$$\frac{b}{2} [x^3 - (x_B^{gc} - x)^2(x + x_B^{gc})] + (N-1)e_{in} [x^2 - (x_B^{gc} - x)^2] < 0$$

The inequality can be rewritten as

$$x_B^{gc} \left[-\frac{b}{2} [(x_B^{gc})^2 - x^2 - x_B^{gc}x] + (N-1)e_{in} (2x - x_B^{gc}) \right] < 0$$

As $x_B^{gc} > 0$ this inequality is equivalent to

$$\phi(e_{in}) = -\frac{b}{2} [(x_B^{gc})^2 - x^2 - x_B^{gc}x] + (N-1)e_{in} (2x - x_B^{gc}) < 0$$

Differentiating ϕ with respect to e_{in} , we obtain:

$$\phi'(e_{in}) = (N-1)(2x - x_B^{gc}) = (N-1)a \frac{b + (N-1)\delta_{in} - N\delta_{out}}{[b + (N-1)\delta_{in}][b + (N-1)\delta_{in} + N\delta_{out}]} > 0$$

It is easily checked that $\phi(e_{in}) \rightarrow -\infty$ as e_{in} gets arbitrarily small. Then we have

$$\phi\left(-\frac{b + (N-1)\delta_{in} - N\delta_{out}}{(N-1)[b + (N-1)\delta_{in}]}a\right) = \frac{f(N\delta_{out})}{2[b + (N-1)\delta_{in}]^2[b + (N-1)\delta_{in} + N\delta_{out}]^2}$$

with

$$f(N\delta_{out}) = -[b + (N-1)\delta_{in}]^2[b + 2(N-1)\delta_{in}] + N\delta_{out}[b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] \\ + (N\delta_{out})^2[b + 2(N-1)\delta_{in} - 2N\delta_{out}]$$

Differentiating, we obtain:

$$f'(N\delta_{out}) = [b + (N-1)\delta_{in}][b + 2(N-1)\delta_{in}] + 2N\delta_{out}[b + 2(N-1)\delta_{in}] - 6(N\delta_{out})^2$$

and

$$f''(N\delta_{out}) = 2[b + 2(N - 1)\delta_{in}] - 12N\delta_{out}$$

Because $f''(0) > 0$ and $f''(b + (N - 1)\delta_{in}) < 0$ (for $\delta_{in} > 0$) or

$$f''(\sqrt{[b + (N - 1)\delta_{in}][b + 2(N - 1)\delta_{in}]}) < 0$$

for $\delta_{in} < 0$ and we conclude that f' increases, then decreases, as $N\delta_{out}$ increases. The same step leads to the conclusion that f increases, then decreases, as $N\delta_{out}$ increases. Thus we know that f reaches its maximum for δ_{out}^s that satisfies $f'(N\delta_{out}^s) = 0$ and we use this condition to rewrite and simplify the expression as

$$f(\delta_{out}^s) = -[b + (N - 1)\delta_{in}][b + 2(N - 1)\delta_{in}]\left[\frac{17}{18}b + \frac{8}{9}(N - 1)\delta_{in}\right] + N\delta_{out}^s[b + 2(N - 1)\delta_{in}]\left[\frac{7}{9}b + \frac{8}{9}(N - 1)\delta_{in}\right]$$

Since we know that $N\delta_{out}^s < b + (N - 1)\delta_{in}$ we conclude that $f(N\delta_{out}^s) < 0$ and for any $\delta_{out} > 0$ we have that $f(N\delta_{out}) < 0$ is satisfied. Thus $\phi\left(-\frac{b + (N - 1)\delta_{in} - N\delta_{out}}{(N - 1)[b + (N - 1)\delta_{in}]}a\right) < 0$ and we deduce that $\phi(e_{in}) < 0$ and condition (6) is non-degenerate. We conclude that group-level cooperation is Pareto improving for intermediate levels of out-group externalities.

Proof of Proposition 6

Under the assumptions of this Proposition we know that both equilibrium outcomes are interior and that $\hat{x}_A = \hat{x}_B = \frac{a}{b + (N - 1)\delta_{in} + N\delta_{out}} = \hat{x}$, we thus have:

$$\Pi_A^{gc} - \Pi_A^N = \frac{b + 2(N - 1)\delta_{in}}{2}(x_A^{gc})^2 + Ne_{out}(x_B^{gc} - \hat{x}) - \left[\frac{b}{2}(\hat{x})^2 + (N - 1)e_{in}\hat{x}\right] \quad (7)$$

We rely on expression (7) of the difference in equilibrium payoffs. Again, denoting $D = [b + (N - 1)\delta_{in}][b + 2(N - 1)\delta_{in}] - (N\delta_{out})^2$ and $\Pi_A^{gc} - \Pi_A^N = \phi_A(e_{in})$ and differentiating with respect to e_{in}

$$\phi'_A(e_{in}) = [b + 2(N - 1)\delta_{in}]\frac{(N - 1)[b + (N - 1)\delta_{in}]}{D}x_A^{gc} - Ne_{out}\frac{N(N - 1)\delta_{out}}{D} - (N - 1)\hat{x}$$

The second derivative is positive, which implies that ϕ'_A increases as e_{in} increases. Now we obtain after direct calculations that the sign of $\phi'_A\left(-\frac{b + (N - 1)\delta_{in} - N\delta_{out}}{(N - 1)[b + (N - 1)\delta_{in}]}a\right)$ is given by that of $-N^2\delta_{out}e_{out} - D\hat{x}$ and so $\phi'_A\left(-\frac{b + (N - 1)\delta_{in} - N\delta_{out}}{(N - 1)[b + (N - 1)\delta_{in}]}a\right) \geq 0$ when $e_{out} \leq -\hat{x}\frac{D}{N^2\delta_{out}}$ holds, which in turn implies that ϕ'_A is positive and ϕ_A increases as e_{in} increases. We then obtain quickly that $\phi_A(\bar{e}_{in}) = 0$ with $\bar{e}_{in} = \frac{\delta_{in}}{b + (N - 1)\delta_{in} + N\delta_{out}}a$ and we first conclude that

$$\phi_A(e_{in}) = \Pi_A^{gc} - \Pi_A^N \leq 0 \iff e_{in} \leq \bar{e}_{in}$$

Now, when $e_{out} \geq -\hat{x}\frac{D}{N^2\delta_{out}}$ holds we obtain that the sign of $\phi'_A(\bar{e}_{in})$ is given by that of $(N\delta_{out})^2\hat{x} - N^2\delta_{out}e_{out}$ and as such when $e_{out} \in \left[-\hat{x}\frac{D}{N^2\delta_{out}}, \delta_{out}\hat{x}\right]$ we know that $\phi'_A(\bar{e}_{in}) > 0$ is satisfied. This in turn implies that there exists $e_{in}^o \in \left[-\frac{b + (N - 1)\delta_{in} - N\delta_{out}}{(N - 1)[b + (N - 1)\delta_{in}]}a, \bar{e}_{in}\right]$ such that ϕ_A decreases up to e_{in}^o then increases. Straightforward calculations enable to conclude that $\phi_A\left(-\frac{b + (N - 1)\delta_{in} - N\delta_{out}}{(N - 1)[b + (N - 1)\delta_{in}]}a\right) \geq 0$ if and only if

$$e_{out} \geq e_{out}^t = a\frac{b[b + (N - 1)\delta_{in}] - 2\{[b + (N - 1)\delta_{in}]^2 - (N\delta_{out})^2\}}{2N^2[b + (N - 1)\delta_{in} + N\delta_{out}]}$$

Moreover $e_{out}^t \in] -\hat{x} \frac{D}{N^2 \delta_{out}}, \delta_{out} \hat{x}[$ which implies that for $e_{out} \in] -\hat{x} \frac{D}{N^2 \delta_{out}}, e_{out}^t[$ we can conclude that $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \leq \bar{e}_{in}$ holds.

For $e_{out} \in]e_{out}^t, \delta_{out} \hat{x}[$ we conclude that there exists $\hat{e}_{in} < \bar{e}_{in}$ such that $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \in]\hat{e}_{in}, \bar{e}_{in}[$ is satisfied. For $e_{out} > \delta_{out} \hat{x}$ we obtain that

$$\phi'_A \left(\frac{b + 2(N-1)\delta_{in} - N\delta_{out}}{N(N-1)\delta_{out}} a \right) \geq 0 \iff e_{out} \leq e_{out}^s$$

with

$$e_{out}^s = \frac{[b + 2(N-1)\delta_{in}][b + (N-1)\delta_{in}]^2 + (N\delta_{out})^3}{N(N\delta_{out})^2[b + (N-1)\delta_{in} + N\delta_{out}]} a > \delta_{out} \hat{x}$$

When $e_{out} \in]\delta_{out} \hat{x}, e_{out}^s[$ we know there exists $e_{in}^g > \bar{e}_{in}$ such that ϕ_A decreases up to e_{in}^g then increases. Since $\phi_A(\bar{e}_{in}) = 0$ we deduce that ϕ_A is non-negative for $e_{in} \leq \bar{e}_{in}$ and negative when $e_{in} \in]\bar{e}_{in}, \hat{e}_{in}[$ holds. Direct calculations then lead to the conclusion that $\phi_A \left(\frac{b + 2(N-1)\delta_{in} - N\delta_{out}}{N(N-1)\delta_{out}} a \right) \geq 0$ if and only if $e_{out} \leq e_{out}^+$ with

$$e_{out}^+ = \frac{P}{2N(N\delta_{out})^2[b + (N-1)\delta_{in} + N\delta_{out}]} a$$

where

$$P = b[b + (N-1)\delta_{in}][b + (N-1)\delta_{in} + 2N\delta_{out}] + 2(N-1)\delta_{in}[b + (N-1)\delta_{in} + N\delta_{out}]^2 - 2N\delta_{out}[b + 2(N-1)\delta_{in} - N\delta_{out}][b + (N-1)\delta_{in} + N\delta_{out}]$$

We have $e_{out}^+ \in]\delta_{out} \hat{x}, e_{out}^s[$ and when $e_{out} \in]\delta_{out} \hat{x}, e_{out}^+[$ we conclude that there exists $e_{in}^* > \hat{e}_{in}$ such that $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \in]\bar{e}_{in}, e_{in}^*[$ holds.

For $e_{out} \in]e_{out}^+, e_{out}^s[$ we have $\phi_A \left(\frac{b + 2(N-1)\delta_{in} - N\delta_{out}}{N(N-1)\delta_{out}} a \right) \leq 0$ and we conclude that $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \geq \bar{e}_{in}$ holds. Finally, when $e_{out} \geq e_{out}^s$ we know that $\phi'_A \left(\frac{b + 2(N-1)\delta_{in} - N\delta_{out}}{N(N-1)\delta_{out}} a \right) \leq 0$ and this implies that ϕ_A decreases as e_{in} increases. Since $\phi_A(\bar{e}_{in}) = 0$ we obtain that $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \geq \bar{e}_{in}$ holds.

To summarize the conclusions, we obtain:

- When $e_{out} \leq e_{out}^t$ we have $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \leq \bar{e}_{in}$
- When $e_{out} \in]e_{out}^t, e_{out}^+[$ we have $\phi_A(e_{in}) \leq 0$ if and only if e_{in} is intermediate
- When $e_{out} \geq e_{out}^+$ we have $\phi_A(e_{in}) \leq 0$ if and only if $e_{in} \geq \bar{e}_{in}$

This concludes the proof.

Proof of Proposition 7

Under the assumptions of Proposition 7 we know that $x_A^{gc} = \frac{a + (N-1)e_{in}}{b + 2(N-1)\delta_{in}} > 0 = x_B^{gc}$ while $\hat{x}_A = \frac{a}{b + (N-1)\delta_{in}} > 0 = \hat{x}_B$ hold. We deduce that

$$\Pi_A^{gc} = \frac{b + 2(N-1)\delta_{in}}{2} (x_A^{gc})^2 \quad \Pi_B^{gc} = N e_{out} x_A^{gc}$$

and

$$\Pi_A^N = \hat{x}_A \left[\frac{b}{2} \hat{x}_A + (N-1)e_{in} \right] \quad \Pi_B^N = N e_{out} \hat{x}_A$$

We thus obtain

$$\pi_A^{gc} - \pi_A^N = \frac{b + 2(N-1)\delta_{in}}{2} (x_A^{gc})^2 - \hat{x}_A \left[\frac{b}{2} \hat{x}_A + (N-1)e_{in} \right] = \phi_A(e_{in})$$

Differentiating with respect to e_{in} :

$$\phi'_A(e_{in}) = [b + 2(N-1)\delta_{in}] \frac{N-1}{b + 2(N-1)\delta_{in}} x_A^{gc} - (N-1)\hat{x}_A = (N-1)(x_A^{gc} - \hat{x}_A)$$

Using the expressions of x_A^{gc} and \hat{x}_A we have

$$\hat{x}_A^{gc} \leq \hat{x}_A \iff e_{in} \leq e_{in}^i = \frac{\delta_{in}}{b + (N-1)\delta_{in}} a$$

Since $\frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}} a < a \frac{\delta_{in}}{b+(N-1)\delta_{in}}$ (due to $b + (N-1)\delta_{in} - N\delta_{AB} < 0$) we conclude that $\phi'_A(e_{in}) \leq 0$ if and only if $\frac{b+2(N-1)\delta_{in}-N\delta_{AB}}{N(N-1)\delta_{AB}} a \leq e_{in} \leq e_{in}^i$ hold. Thus ϕ_A decreases then increases as e_{in} increases. We notice that $x_A^{gc} = \hat{x}_A = \frac{a}{b+(N-1)\delta_{in}}$ when $e_{in} = e_{in}^i$ and thus

$$\phi_A(e_{in}^i) = \hat{x}_A \left[\frac{b + 2(N-1)\delta_{in}}{2} \hat{x}_A - \frac{b}{2} \hat{x}_A - (N-1)e_{in}^i \right] = \hat{x}_A(N-1) [\delta_{in}\hat{x}_A - e_{in}^i] = 0$$

This implies that $\pi_A^{gc} - \pi_A^N \geq 0$ and this concludes the first part of the proof.

Now, regarding the second part of the proof, we have

$$\pi_B^{gc} - \pi_B^N = N e_{out} (x_A^{gc} - \hat{x}_A)$$

Group-level cooperation will thus be Pareto improving when either $e_{out} < 0$ (if and only if $x_A^{gc} \leq \hat{x}_A$ so $e_{in} \leq e_{in}^i$) or $e_{out} > 0$ (if and only if $x_A^{gc} > \hat{x}_A$ so $e_{in} > e_{in}^i$).

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