Interdependencies between intra and intergenerational equity in sustainable environmental resources management

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Interdépendances entre l'équité intra et intergénérationnelle dans la gestion durable des ressources environnementales

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Résumé

Cette thèse se propose de montrer l'intérêt de considérer simultanément l'équité intra et intergénérationnelle pour des questions liées à la gestion des ressources environnementales. Plus spécifiquement, la thèse examine les arbitrages entre ces deux dimensions de l'équité pour définir une distribution juste des ressources au cours du temps et au sein des générations. Les inégalités sont considérées à travers deux régions hétérogènes. Le premier chapitre se focalise sur le maintien du niveau maximal de bien-être au cours du temps, à travers le critère maximin, lorsque l'économie a une aversion aux inégalités intragénérationnelles. De manière contre-intuitive, la région la moins dotée en ressources paye un plus lourd tribut pour la durabilité globale. Le second chapitre étudie la croissance vers le niveau maximal soutenable de bien-être, la règle d'or. De la même manière, la région la moins dotée en ressources doit contribuer davantage à cette croissance, en limitant relativement plus sa consommation. Le troisième chapitre étudie les transferts qui doivent être opérés de la région relativement mieux lotie vers celle moins bien lotie. Le transfert doit être soit forfaitaire soit proportionnel à la consommation de la région contributrice, selon que l'objectif est de favoriser ou de limiter sa consommation. Dans tous les cas, la région la plus défavorisée reçoit un transfert compensatoire pour la contrainte qui lui est imposée.

Mots-clés : équité intragénérationnelle, équité intergénérationnelle, ressource naturelle, développement durable

Abstract

This dissertation proposes to show the interest of considering simultaneously intra and intergenerational equity for environmental resources management issues. More specifically, the dissertation examines the trade-offs between these two dimensions of equity to define an equitable allocation of resources over time and within generations. Inequalities between two heterogeneous regions are considered. The first chapter focuses on sustaining the highest level of welfare over time, through the maximin criterion, when the economy has an intragenerational inequality aversion. Counter-intuitively, the region with the lower resource stock pays a higher price for overall sustainability. The second chapter examines growth toward the maximum sustainable level of welfare, the golden rule. Similarly, the region with the lower resource stock shall contribute more to the growth, by limiting relatively more its consumption. The third chapter examines the transfers that shall be made from the well-off to the worse-off region. The transfer shall either be a lump-sum or proportional to the consumption of the contributing region, depending on whether the objective is to promote or to limit its consumption. In any case, the worst-off region receives a compensatory transfer for the constraint imposed on it.

Keywords: intragenerational equity, intergenerational equity, natural resource, sustainable development

Contents

General introduction

Natural resources have always been useful for mankind. And we cannot conceive a world without them. In all likelihood, our descendants will also need them. Then a natural balance between present and potential future claims arises. But balancing claims between contemporaries may also be important, even necessary. For example, individualistic water management from Israel, Jordan and Palestine led to a shrinking of the level of the Dead Sea in the last decades. In this region, thinking to a bequest of a good quality of water for posterity may be pointless if the countries do not reach an agreement. That being said, depending on the solution found (reducing extraction, conveying water from the Red Sea, etc.), consequences for future generations may be very different. In the light of this example, one can argue that a sustainability objective shall not overshadow the importance, or the requirement, of inter-individual issues. As aptly stated by the American philosopher John Rawls (1971, p. 137): "*questions of social justice arise between generations as well as within them, for example, the question of the appropriate rate of capital savings and of the conservation of natural resources and the environment of nature*".

0.1 Context

In economics, while one is first of all interested in *efficiency* – reaching goals relatively to means used, – one may also be interested in *equity*. But most of the time, compromises have to be made between these two dimensions. An example of this ancient literature is the book by Okun (1975), *Equality and Efficiency: The Big Tradeoff*. An opposition is usually made between *equity* and *equality*, but they have a common origin. The former comes from Latin *aequitatem*, and the latter from Latin *aequalitatem*, both formed from *aequus*, equal.[1] An equity definition has therefore to encompass a certain dimension of equality. But as would say Amartya Sen (1980), "*Equality of What?*". This is maybe why so much theories of equity, fairness and justice exist. It is maybe useful here to resort to a dictionary definition: "*equity is the quality of being fair and reasonable in a way that gives equal treatment to everyone*".[2] As a requirement, I will translate this definition in public decision making as treating equally every individual. Indeed, d'Aspremont and Gevers (1977, p. 202) argued that "*equity would seem to require that collective choice be unchanged when individuals exchange position*". More justifications will be given in the Section 0.2.3. Whatever, this still leaves the door open for many theories of justice. When equity is implemented, one expects a situation to be 'fair' or 'just'. For some authors, fairness would refer to procedures, while justice would refer to outcomes (Barry, 1989, p. 145). I will follow this distinction here. The idea of justice can be traced back to philosophers of the Greek Antiquity. But for Barry (1989), two modern philosophers are particularly notable in this domain: David Hume and John Rawls. For Hume (1738/1978, cited in Barry, 1989, pp. 148-9), in *A Treatise of Human Nature*, justice comes from a desire of 'mutual advantages' (a condition for property). And in his second theory, justice can be adopted if there is an impartial sympathy with all individuals (a condition to be a virtue). For Rawls (1971, cited in Barry, 1989, p. 152), in *A Theory of Justice*, mutual advantages are the circumstances of justice. And according to him, impartiality has not to be ascribed to an observer, but to the situation in which choices are made. Hume can be regarded as a descendant of Hobbes and a proto-utilitarian, while Rawls proclaimed himself to follow Kant's tradition (Barry, 1989, p. 148). While former are more interested in consequences of justice (*consequentialism*), latter are more interested in procedures of justice (*deontology*).[3] Whatever, one can easily agree that distributive justice – sharing goods and services between different individuals, living or not at the same time – is of interest only if we are in a world of competition for scarce resources. It is interesting to note that land was

[1] Source: `www.etymonline.com`, visited on May 2018.

[2] Source: `www.collinsdictionary.com/dictionary/english/equity`, visited on May 2018.

[3] This distinction is frequently used in theories of justice (Van Parijs, 1991, p. 257).

at the core of one of the first schools of thought in economics in the second half of the Eighteenth century: Physiocracy. Leaded by François Quesnay, they considered that only agricultural lands are productive since they 'create' something, contrary to industry and trade (see e.g. Abraham-Frois, 2001, pp. 4-10). But natural assets being not produced, they were not considered as being valuable by economists of this period. For example, the classical economist Jean-Baptiste Say (1803, p. 8) qualified natural wealths in his *Traité d'économie politique* as being "*goods that nature gives us freely and without measure*"[4]. He went even further in subsequent editions saying that "*everyone can enjoy [them] at his pleasure, without being obliged to acquire them, without fear of exhausting them, such as air, water, sunlight*"[5]. He was indeed right about some of them, nonetheless we know very well that some resources are, or may become, scarce (e.g. fish, fresh water, clean air). A clarification is therefore needed. Environmental resources may be defined as natural materials or components which have a current worth and are technologically and economically available (now or in a foreseeable future).[6] Those inexhaustible, like solar radiation, water, wind energy, are indeed in themselves of little interest from an economic perspective.[7] And among exhaustible ones, an important distinction has to be made between renewable and non-renewable resources. That said, to avoid any confusion, I will take exhaustible and non-renewable resources as synonyms. Renewable ones replenish themselves indefinitely, like timber, as long as consumption does not always overtake its renewal. Non-renewable ones do not replenish themselves, either due to their nature, like uranium, or because this process is not in the same time-frame than its consumption, like petroleum. Due to the importance of renewable and non-renewable resources in the production of goods and services of modern societies and their finiteness or threat of collapsing, their sustainability is

[4]Personal translation from: "*[les richesses naturelles sont] des biens que la nature nous accorde gratuitement et sans mesure*".

[5]Personal translation from: "*chacun peut [en] jouir à sa volonté, sans être obligé de les acquérir, sans crainte de les épuiser, tels que l'air, l'eau, la lumière du soleil*" (transcribed in Mouchot, 2006, p. 1155).

[6]*Environmental resources* have a broader meaning than *natural resources*, they include all natural assets such as clean air.

[7]As indicates the classic Lionel Robbins's (1932, p. 15) definition: "*Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses*".

now a major issue. For example, the second section of the Agenda 21 is entitled: *Conservation and Management of Resources for Development*. More recently, resources issues are at the heart of the Sustainable Developments Goals of the United Nations.[8] Indeed, for a few decades there seems to have been a consensus on having an evolution of our societies – sustainable development – toward a state that can be sustained – sustainability (Dovers and Handmer, 1993). But while everyone agrees on the target, means are still to be defined. One of the recent bases of this reasoning is the well-known definition of sustainable development given by the World Commission on Environment and Development (the "Brundtland report", 1987, p. 8): "*[it] is development that meets the needs of the present without compromising the ability of future generations to meet their own needs*". But its vagueness spawned to multiple divergent interpretations, especially in economics (for a survey of recent developments see Martinet, 2012). Nevertheless, if one adopts a "welfarist" view (Sen, 1977, 1979) – only utility matters – and therefore anthropocentric, it is hard to be against the Solow's (1991, p. 181) definition: "*[sustainability] is an obligation to conduct ourselves so that we leave to the future the option or the capacity to be as well off as we are*". This puts the emphasis on the well-being of the current generations in relation to that of the future ones, and therefore on equity between generations in sustainability issues. But why maintaining or increasing stocks of natural assets, if fewer and fewer individuals have access to them? Let us study the intragenerational dimension of equity before turning to its intergenerational counterpart.

0.2 Intragenerational equity

0.2.1 Historical background

Equity is an old concept. Philosophers and economists have for a long time tried to give their definition through their vision. Obviously, an exhaustive review of developments, even recent, is both impossible and unnecessary for the

[8]They include for example: water (6), energy (7), marine (14) and land resources (15). They include also inequalities (10) and justice (16) issues. Source: `https://sustainabledevelopment.un.org/sdgs`, visited on September 2018.

present purpose. I shall rather focus on main theories of distributive justice. Interested reader is advised to refer to classic reviews. They include: Barry (1989); Van Parijs (1991); Young (1994); Fleurbaey (1996); Roemer (1996); Arnsperger and Van Parijs (2003); Fleurbaey and Maniquet (2011). For the sake of the presentation, I follow the useful distinction between macro and micro justice. The former may be seen as defining a just society as a whole, while the latter is more concerned with equity in particular economic environments.

Macro Justice

The oldest found theory is the Aristotle's equity principle. According to him, goods shall be divided proportionally to individual differences (e.g. depending on contributions) (Young, 1994, pp. 64-5).

OUTCOME JUSTICE

Utilitarianism. For Barry (1989, pp. 173-8) the Hume's basic idea, that modification of property should be done without any conflict, leaded to two different traditions. The first one, following his theory of the origin of justice, was pursued by the philosopher David Gauthier and the economist James Buchanan, for whom changes in property require unanimity. The second tradition tried to find an impartial criterion. Indeed, Hume refrained himself to apply a case-by-case approach for a distribution. The notion of public utility was only used later on by Jeremy Bentham for such issues. But still, Bentham does not advocate any modification on the basis that an equal distribution of income maximizes overall utility. The full potential of this utilitarian criterion was showed by John Stuart Mill (1848/1965, cited in Barry, 1989, p. 175) in his *Principles of Political Economy*, for whom while production is a matter of natural laws, distribution of production is a matter of social decision. Bentham and J. S. Mill gave birth to the utilitarian tradition, which peaked with Pigou (1920/1932, cited in Barry, 1989, p. 176) in *The Economics of Welfare*. For him, the amount received by individuals shall be done according to their capacity for turning it into utility and not according to their contribution. Broadly speaking, utilitarianism is concerned with the maximization of total welfare.

Social Welfare Function. Different developments of welfare economics were interestingly synthesized into a single social welfare function (SWF) – mapping individuals and global welfare – by Abram Bergson (1938). This work was further developed and popularized by Samuelson (1947, sec. VIII), to become afterward so-called Bergson-Samuelson functions. Taking utilities as elements of such functions, this approach generalizes in a sense the utilitarian one. Indeed, the sum of utilities may be seen as a special case of SWF.

Social Choice. Social choice theory was born in 1951 with the Arrow's pioneering work (reprinted and commented in Arrow, 1963). Arrow asked the question whether a social decision could be based upon aggregated individual preferences. He showed that if one imposes some natural "conditions" – among which ordinal orderings, the irrelevance of independent alternatives,[9] at least three alternatives are considered and non-dictatorship – a social welfare function cannot be obtained (Arrow, 1963, p. 2). Further notable developments were made by Sen (1970). The historical origins of this theory can be tracked down in the Eighteenth century with works on voting by Borda and Condorcet (Arrow, 1963, pp. 93-4). For a review of this extensive literature, see Arrow et al. (2002, 2011).

Marxism. Marx was interested in explaining why wealth concentrate in the hands of owners of means of production. In a sense, there is exploitation when a certain ratio between income and work is not respected (Van Parijs, 1991, p. 106). More broadly, his vision gave birth to theories of economic exploitation, where there is a sharp distinction between labor-hiring class (exploiters) and labor-selling class (exploited). This holds true according to Marx until communism is reached, a society characterized by "*from each according to his ability, to each according to his need*" (cited in Roemer, 1982, pp. 265-6). For developments on the Marxist tradition, see e.g. Cohen (1978/2000), Roemer (1982) and Van Parijs (1993).

Equality of Rights.[10] Libertarianism advocates that justice shall be solely devoted to an equality of rights. State has no role to play except in defining property rights. An interesting insight for environmental resources was given by Nozick

[9]Fleurbaey and Maniquet (2011, p. 10) prefer to use the less normative expression "independence requirement".

[10]This theory could be put in the next subsection, but I follow here Fleurbaey (1996, p. 20) emphasizing that what is equalized is a result.

(1974). Based on a Locke's theory of appropriation, Nozick (1974, pp. 174-82) argued toward an appropriation of unowned objects by mixture with one's labor, as long as this does not worsen the well-being other individuals. Nozick called it the *Lockean proviso*. Arnsperger and Van Parijs (2003, pp. 33-6) classified these theories of appropriation between "right-libertarians" – who advocate the first-come first-served principle – and "left-libertarians" – equalizing shares of rent. But for some authors, justice should not be based on results but on possibilities given to individuals.

PROCEDURAL JUSTICE

Equality of Means. In one of the most noteworthy works of the field, Rawls (1971) detailed his vision of justice, providing an alternative to utilitarianism. In the tradition of the social contract theory, he supposed an original position of equality. In such a position individuals are supposed to ignore their identity, their preference, their abilities, etc., even their conception of the good. "*The principles of justice are chosen behind a veil of ignorance*" (p. 12). He argued that in such a situation two principles would emerge. (1) Equality of rights. (2) Social and economic inequalities have to (a) be to the benefit of the least advantaged members (the "difference principle") and (b) exist under an equality of opportunity (esp. pp. 60-7, 83). Rawls dealt with what he called "primary social goods",[11] but his criterion (2a) was also applied to utilities (e.g. Arrow, 1973b) and leaded to the well-known maximin criterion (Rawls, 1971, 1974; Arrow, 1973a,b; Solow, 1974).[12] In the same vein, Dworkin (1981a,b) supposed that individuals know their talents (including handicaps), but ignore their "economic rent". He argued that an imaginary insurance market would lead at an equality of resources (in a no-envy sense, explained hereinafter). But some authors prefer focus, rather than means given to individuals, on actual opportunities they have. The underlying idea is that, for some authors, individuals shall not be held responsible for difference in turning resources into actual opportunities (on this see Roemer, 1996, sec. 8.1-8.2, and Fleurbaey, 2008).

[11] They include rights, liberties, opportunities, powers, income and wealth (p. 92).

[12] This criterion was already present in economics for choice under uncertainty. For analogy between the Rawls' theory of justice and uncertainty, see Rawls (1971, pp. 152-61, 1974) and Dasgupta (1974).

Equality of Opportunity. An important contribution in this domain was made by Sen (1980). For him, neither utilities nor primary goods are fully satisfactory for elements of a theory of justice. He rather argued in favor of an equality of "basic *capabilities*", which may be understood as the ability to achieve combinations of "*functionings*". Such as nutritional requirement, clothes, participation in social life, etc. In a word, this approach focuses on overall personal opportunity of achievements (see Sen, 2009, pp. 231-5).

Micro Justice

Some rather different approaches, based on microeconomic theory, were followed by some authors to study more precisely equity of some resource allocations. They either resorted to bargaining or to general equilibrium theory.

Bargaining. Here agents bargain in order to reach an agreement on sharing a given bundle. If they fail they receive a predefined bundle, possibly empty (*threat point*). Classic solutions include those by Nash (1950) and by Kalai and Smorodinsky (1975). For a survey see Thomson (2001) and the references therein. But many authors disprove bargaining theory as being appropriate to address distributive justice issues. Both for an impartiality concern (Barry, 1989; Rawls, 1971; Sen, 1970) and for an informational basis concern (Roemer, 1986, 1990). See Fleurbaey (1996, pp. 195-202) for more details.

Competitive Equilibrium. Theories in here study the so-called equity in economic environment. Individuals begin with, for instance, an equal share of the total endowment and exchange freely. The final allocation is said to be equitable according to different criteria, without referring to comparisons of utilities. Two important criteria exist. The first one is the *no-envy* criterion (Tinbergen, 1953, cited in Thomson, 2011; Foley, 1967; Kolm, 1972): no agent strictly prefers any other agent's bundle to his/her own. The second one is the *egalitarian-equivalent* criterion (Pazner and Schmeidler, 1978): each agent is indifferent between a benchmark bundle and his/her own. These approaches are called *fair allocation theory*. A survey may be found in Thomson (2011).

These different theories are opposing each other concerning values to which they refer. Economists are used to transform these values into axioms, which may be understood as simple mathematical properties. Studying the axioms being satisfied is generally a convenient way to elect a social criterion.

0.2.2 Axiomatic approach

I voluntarily omit here mechanisms of bargaining since they do not correspond to the view of distributive justice standing in here. Reader interested by the axiomatic bargaining literature may refer to Thomson (2001).

General equilibrium from equal incomes. An axiomatization of free exchange from an equal bundle was proposed by Jaskold-Gabszewicz (1975). He resorted to coalitions, supposing that a large number of individuals are split into two coalitions of equal size. A coalition is said to be fair if individuals in there do not envy those from the other coalition. It can be shown that it is the unique mechanism characterized by the envy-free between coalitions. See also the references in Yaari and Bar-Hillel (1984, p. 4).

Utilitarianism. It advocates an allocation of resources in such a way that the sum of individuals utilities is the highest possible, whatever the distribution. In other words, a change is permitted if gains overtake losses. Of course, such variations of welfare need to be expressed in the same unit. It has been axiomatized with help of the theory of decision making under risk. Indeed, Vickrey (1945) coined an original justification of his theory linking justice and uncertainty, which was also developed by Harsanyi (1953) and Rawls (1971)[13] in a similar way.[14] Let us imagine that an individual has to choose between different states in which s/he has an equal probability to be anyone. Rationally, argued Vickrey (1945), s/he would maximize the sum of utilities.[15] Independently, Harsanyi (1953, 1955) used the same reasoning to justify utilitarianism.

[13]These contributions do not mention his work. A possible explanation is given by Arrow (1973a, p. 250n): "*Vickrey's 1945 statement has been overlooked by all subsequent writers, not surprisingly, since it received relatively little emphasis in a paper overtly devoted to a seemingly different subject. I read the paper before I was concerned with the theory of social choice; the implications for that theory were so easy to overlook that they did not occur to me at all when they would have been relevant*".

[14]A notable difference is that Vickrey-Harsanyi's observer does know the preferences of the population, while individuals in Rawls' original position ignore both their identity and their preferences.

[15]Obviously, with a given population, sum and arithmetical mean give the same information.

In particular, he showed that under the axiom of Bayesian rationality – maximizing the expected utility – strong Pareto[16] and symmetry, the SWF is utilitarian (Harsanyi, 1977). A classic and natural criticism is that it is insensitive to inequality: some may starve while some live in opulence.

Maximin and leximin. To meet this objection, another criterion of distributive justice emerged to concentrate on the worst-off individual. Society is assume here to maximize his/her situation to the extend possible. It is based on the vision of justice by Rawls (1971). Understood that utilities are of interest here rather than "primary goods" as in its original version. The potential indeterminacy of maximin for states giving the same result leaded Sen (1970, p. 138n)[17,18] to coin the lexicographic version – applying maximin at the upper rank if the one being concerned does not allow to decide between two states (it compares the worst-off, if equal, the second worst-off, and so on) – named thereafter leximin (Sen, 1980). Axiomatization of these criteria relies on a modification of traditional axioms of social choice, that allows in particular interpersonal comparisons of utility. It was proposed independently by Strasnick (1976) and Hammond (1976). Strasnick (1976) considered the following axioms. The "priority principle" – individual claims over primary goods are of equal importance, – binariness – two states are evaluated only on basis of preferences over these two states, – anonymity and neutrality – independence (respectively) to individual and alternative permutations, – unanimity – if a state is preferred to another in every partitions of the problem it is also preferred in the whole problem. Under these axioms, the social welfare satisfies the difference principle, i.e. is maximin. Hammond (1976) stated a similar theorem. He considered the following axioms. Unrestricted domain, independence of irrelevant alternatives, strong Pareto, an axiom of "weak equity" – based on Sen (1973, p. 18, 1974), this axiom tells basically that if two individuals have contradictory preferences but one is worse-off in every configurations, then the social choice must follow (or being indifferent to) the interest of

[16]An allocation of resources satisfies the strong Pareto property if no individual can be made better-off at the expense of any other one. The weak version: if from the initial situation every individual cannot be made better-off.

[17]This concept was based on previous Rawls' works and was mentioned in Rawls (1971, p. 83n).

[18]A similar concept was independently coined by Kolm (1972, pp. 18,115-6).

the latter – and an axiom of anonymity – independence of permutations in individual positions, inspired by the Suppes-Sen grading principles (Suppes, 1969; Sen, 1970, pp. 153-4). If these axioms are satisfied, he showed that the SWF is leximin. These results were restated in more general terms by Arrow (1977). But Arrow (1977) provided two reservations, apart from the necessary assumption on interpersonal comparisons of utilities. Only "extreme" individuals count in the decision process, those intermediate are ignored. And, more fundamentally, a change is not advocated if the worst-off loses a little while all others win a lot. Besides, d'Aspremont and Gevers (1977) pointed out that leximin can be seen as a come back of Arrow's dictator applied to rank.

Informational content of utility. d'Aspremont and Gevers (1977, 2002) provided an interesting synthesis of main results of social choice (Arrow, 1963; Sen, 1970) based on the informational content of utility. Indeed, two dimensions are crucial in this literature: to what extend utility is measurable (are the orderings, the levels or the variations meaningful)? And are utilities of individuals comparable? If we assume ordinality and non-comparability, a social welfare ordering of alternatives based upon individual indicators cannot be obtained in general, as showed by Arrow (1951/1963) in his pioneering work. At the opposite, the cardinal-comparable case leads directly to the result: in this case the "constitution" is given by the sum of utilities (Arrow, 1977, p. 227). Intermediary cases allow to distinguish, between others, utilitarianism and leximin.[19] Interestingly, they showed that we can separate these two criteria only upon their informational basis. Basically, if utilities are assumed to be cardinal and only welfare variations are meaningful, we should favor utilitarianism. While, if we assume ordinality and that only welfare levels are meaningful, we should favor leximin. Depending on the assumptions retained, drastically different theory of social welfare judgment may therefore emerge. One may also notice that the latter "*asks less of our judgments of welfare*" (Rawls, 1971, p. 92). But as Fleurbaey (1996, p. 71) suggested, we shall not base a conception of justice upon the availability of information, inequality aversion shall be based upon deeper ethical considerations.

[19]Taking the opposite problem, Deschamps and Gevers (1978) found a set of axioms leading both to utilitarianism and leximin.

0.2.3 Selected approach

My purpose is to study equity issues about environmental resources of a whole society. Such issues may of course be studied in particular environment, but at a macro scale, links between intra and intergenerational concerns are easier to understand. In their short but relevant review, Arnsperger and Van Parijs (2003) mentioned four main theories of social justice: utilitarianism, liberal egalitarianism, libertarianism and Marxism. Out of reaching exhaustiveness, a convenient way to deal with different theories may be found in classic SWF *à la* Bergson-Samuelson. We need first to assume a welfarist view. It can of course be argued than more objective elements, such as capabilities, may be relevant for a justice purpose. But conversely, it is hard to argue that utility has nothing to do with justice. Besides, the utility concept may be interpreted here in several ways: primary goods, resources (broad sense), opportunities, etc. The only restriction is its unidimensionality (Fleurbaey and Hammond, 2004, pp. 1189-90). For the sake of simplicity, I concentrate in here only on utility. That being said, let me argue why SWF are flexible enough to be interesting here. It is better understandable if one relies on inequality aversion. Indeed, society may be assumed to have a nil inequality aversion, that is to say only the sum of happiness counts. This is precisely the utilitarianism. At the opposite, society may be assumed to have an infinite inequality aversion. Such a feature is exactly what the Rawlsian liberal egalitarianism applied to utilities would suggest: one should focus on the worst-off individual, i.e. using the maximin criterion. Of course, all intermediary cases are conceivable. As it has been said, when natural assets are at hand, libertarianism relies on appropriation. Right-libertarians would simply recommend no transfer at all, while left-libertarians would recommend a transfer of resources such as to perfectly equalize the utilities (in a welfarist interpretation). Finally, Marxism could be interpreted in a way giving a share to individuals according to their needs. A minimalist view could be to set a minimal threshold on utility. Developments on needs may be found e.g. in Sen (1973, ch. 4).

The artificial distinction between, in a broad sense, social choice theory and fair allocation theory has been criticized for a long time by Fleurbaey and by Maniquet. For example, in Fleurbaey and Maniquet (2011), they try to gather social choice (social preferences), fair allocation (fairness issues), and public economics (policy recommendations). For this, they "*construct interpersonal comparisons on the basis of* ordinal noncomparable preferences *over bundles of resources*" (p. xvi). But while such recent developments are appealing and give insights for future researches, they are hardly applicable in simple frameworks. In particular, their criterion is not a function (inspired from envy-free and leximin). For this reason, I prefer resorting to more rudimentary SWF which will give an easier interpretable indicator for dynamic evolution.

Finally, let me have a word on interpersonal comparison of utility. In classic social choice theory, they are completely ruled out, and impossibilities come out. But as argued by Hammond (1976, p. 793), "*some notions of equity rest on interpersonal comparisons*". More recent works were able to avoid both interpersonal comparison of utility and impossibilities (see e.g. Fleurbaey and Maniquet, 2011) with the modification of some 'classic' axioms. Nonetheless, I am still convinced by the relevance of interpersonal comparison of utility to some extend. Even if utility represents generally an individualistic measure of welfare, it can be understood as seen by an ethical observer. Such an observer would be able to determine if some claims are legitimate or not. This is abstract, but in everyday life, we are generally able to say that healthy people are better-off than disabled, that workers are better-off than unemployed, etc. Of course, there are some exceptions and some preferences (e.g. expensive tastes) have not to be taken into account. Interested reader on interpersonal comparisons may refer to Fleurbaey and Hammond (2004) for more details and references. A short and nuanced summary may also be found in Fleurbaey and Maniquet (2011, pp. xvii-xviii).

Obviously, environmental and resources issues exist only if we care about future generations. The equity concern has therefore to be extended to the intergenerational dimension.

0.3 Intergenerational equity

0.3.1 Historical background

Intergenerational issues concerning sharing equitably capital and consumption goods across generations is not new. This was dealt for example by the literature on capital accumulation and growth. A well-known example is the Ramsey's (1928) pioneering work on optimal saving. But this issue clearly peaked with environmental concerns. Firstly with pollution concerns, with notably the Rachel Carson's (1962) book *Silent Spring*, and secondly with the warning of finiteness of some resources with the well-known report by the Club of Rome *The Limits to Growth* (Meadows et al., 1972). This has shifted the priority as mentioned by Arrow (1973a, p. 260): "*to what extent is one generation obligated to save, so as to increase the welfare of the next generation? The traditional economic problem has been the general act of investment in productive land, machines, and buildings which produce goods in the future; more recently, we have become especially concerned with preservation of undisturbed environments and natural resources*". Basically, the literature is interested in comparing individuals belonging to different generations, that is to say here living at a different date. A generation is generally summarized through one single individual. Generations may be successive or overlap each other. And the literature on sustainable development is mainly split into two different schools of thought: 'weak' and 'strong' sustainability. The first one concentrates on welfare and assumes a substitutability between human-made and natural capitals, while the second one stresses the necessity to maintain some stocks of natural assets. For more details, interested reader may refer to Neumayer (1999/2013) and Martinet (2012). Like in the previous section on intragenerational equity, the intergenerational equity may be understood with or without referring to welfare. Here also, I argue that it is hard to completely not refer to welfare when choice of development paths are at hand. But, that said, some specific natural assets may be preserved *per se* (e.g. the Great Barrier Reef). Concerning the debate on substitutability, as concluded Pezzey and Toman (2002, p. 213), it is at the end an empirical issue. But if we agree with Solow (1991, p, 182) that "*sustainability* [...] *is about the sharing* *of well-being between present people and future people"*, one still has to find ethical bases to make such a distribution.

0.3.2 Axiomatic approach

A criterion – a function of all instantaneous SWFs – is generally used for intertemporal decisions. A SWF is generally simply the utility of the representative agent of the generation. Time may be discrete or continuous and is generally infinite, in order to avoid the controversial issue of defining the last date, i.e. the last generation taken into account. I described in the intragenerational part axioms that a social ordering could satisfy, likewise axioms that an intertemporal criterion could satisfy can be described.

In a famous article, Koopmans (1960) showed that imposing some *a priori* desirable axioms on a trajectory of utility (or SWF) over an infinite horizon, strongly constrains the criterion. He considered the following properties: the utility is ordinal, continuous, satisfies sensitivity – one can distinguish two trajectories even if they differ only in their first element, – intertemporal complementarity – consumption on a given period does not impact the comparison between two alternatives on another period, – stationarity – comparison of two alternatives does not change when one goes forward in time – and if a best path and a worst path exist. All together, these requirements lead to elect a criterion exhibiting "impatience", i.e. a constant discount rate has to be used. This obliges to prefer the present over the future.

Among axioms that we might wish to be satisfied, the Pareto one seems unavoidable. As it is well-known, considering only the Pareto principle is not enough for an equity concern: a world with one agent having everything and another one having nothing is optimal. But conversely, a non-optimal situation cannot be selected since there would be unnecessary losses. Indeed, under its weak version, it says that a path always superior to another is preferred. Under its strong version, a path that is at least as good as another and strictly better than it on at least one date is preferred. Efficiency is generally understood as satisfying the strong Pareto axiom. Intergenerational equity may be understood as a matter of sharing welfare between generations. But in this view, some restrictions have to be imposed. As in the intratemporal perspective, it will be required here the social planner to be impartial between all generations, whatever their date of appearance. This ensure each generation to have the same weight. In other words and in a more abstract way, a criterion is required to be neutral to any number of permutations in the order of appearance of the generations. Here also, this idea is captured by the anonymity axiom. But in the vein of Koopmans (1960), Diamond (1965) stated an impossibility result, recalling the classic 'efficiency-equity' trade-off. Indeed, he showed that such a criterion cannot be both efficient and anonymous. For more details see Martinet (2012, pp. 46-7) and the references therein. Especially Asheim (2010), who proposed a critical review of the results about this dilemma and means to avoid it. Basically, one has either to work with incomplete binary relations of preference or to weaken the two concerned axioms.

Set of axioms can lead to construct criteria. And conversely, usual criteria can be analyzed in light of the axioms they satisfy. Intergenerational equity issues are indeed usually dealt with a criterion to optimize over time. Let us now turn to the presentation of the main ones. Constraints such as a non-declining utility (or SWF), keeping a minimal level of stock, etc. could also be added.

0.3.3 Intertemporal criteria

I detail here the main criteria used in the literature. More details may be found in Heal (1998) and Martinet (2012). For an overview of different approaches dealing with intergenerational equity, see Roemer and Suzumura (2007).

DISCOUNTED UTILITARIANISM. It consists of summing all future expected successive utilities (or SWF), weighted by a constant discount rate. That is to say, the farther a generation, the less its utility counts. Example: with a 1% yearly rate, 100 units in a hundred years is approximatively equivalent to 37 units today. This criterion comes from work on optimal growth theory. It was indeed first envisioned by Ramsey (1928), then adopted and enhanced notably by Samuelson (1937), Cass (1965) and Koopmans (1965). In parallel, it was applied to the field of sustainable growth in presence of exhaustible resources by notably Hotelling (1931) and Dasgupta and Heal (1974, 1979). This criterion has rapidly dominated the literature. But regarding intergenerational issues, this criterion has been criticized by economists themselves, especially for not giving importance to the far future, while environmental issues have to be managed on a long time horizon (Heal, 1997). The 'right' rate to apply for public policies is also controversial (Arrow et al., 1996, 2013), especially in the climate change debate (Stern, 2007; Nordhaus, 2007; Heal, 2009; Drupp et al., 2018). I think that Solow (1993, p. 165) summarized quite well this debate. "*You may wonder why I allow discounting at all. I wonder, too: no generation 'should' be favored over any other. The usual scholarly excuse – which relies on the idea that there is a small fixed probability that civilization will end during any little interval of time – sounds far-fetched. We can think of intergenerational discounting as a concession to human weakness or as a technical assumption of convenience (which it is). Luckily, very little of what I want to say depends on the rate of discount, which we can just imagine to be very small*".

Another type of criticism is its incompatibility with a sustainability objective, which can be approximately defined as sustaining an indicator over time (e.g. stocks, utility, etc.). For example, Pezzey and Toman (2002, pp. 176-7) stated that this criterion has nothing to do with sustainability, even if we add some constraints of sustainability. Indeed, it may lead consumption to approach zero in the classic Dasgupta-Heal-Solow model with manufactured capital and exhaustible resource (Dasgupta and Heal, 1974, 1979; Solow, 1974). From an axiomatic point of view, this criterion is efficient, but not anonymous.

Some authors still found interest in discounting and try to apply it only in some circumstances. For example, the *Sustainable Discounted Utilitarianism* (Asheim, 2010; Asheim et al., 2012) allows for discounting future utilities if and only if the future is better-off than the present. And the *Rank Discounted Utilitarianism* (Zuber and Asheim, 2012; Asheim, 2012) applies discounting according to the rank of the welfare of generations instead of their place in time (more weight on utility of worse-off generations). Finally, it should be mentioned that this equity-efficiency dilemma arises in infinite but not in finite time (Fleurbaey and Michel, 1999).[20]

[20] Supplementary difficulties arise with a growing population (Koopmans, 1965; Fleurbaey and Michel, 1999). Population is assumed constant here.

UNDISCOUNTED UTILITARIANISM. Ramsey (1928) was against discounting and coined a criterion to avoid it. By a trick, he managed to sum all future utilities with an equal weight. Technically, he minimized the difference between the maximal utility attainable, called "Bliss", and the actual utility, at every single date. But not avoiding the previously mentioned impossibility, this criterion is incomplete (all trajectories cannot be ranked). Indeed, if the trajectory does not converge fast enough, no optimum exist (Chakravarty, 1962). But it is both efficient and anonymous.

MAXIMIN. As in its intratemporal version, maximin focuses on the least well-off generation and tries to rise its welfare to the extend possible. Applied by Rawls (1971) to justice issues, it was popularized by Solow (1974) for capital accumulation problems. More precisely, to face the challenge of sustaining a utility with an exhaustible resource as an input of production. This criterion was criticized for intertemporal issues by Rawls (1971, sec. 44, 1974) himself. He qualified the difference principle to be inapplicable due to the unidirectionality of time: "*there is no way for later generations to improve the situation of the least fortunate first generation*" (Rawls, 1971, p. 291). But Solow (1974, p. 30) was in its own words "plus Rawlsien que le Ralws" and found nonetheless interest in maximin for intergenerational issues. Besides, and as in its intratemporal version, criticisms focus on its 'radical' feature: only the worst-off generation counts. In particular, if the current generation is poor, maximin, forbidding any sacrifice from this generation, lead the economy to stay in a 'poverty trap'. Partisans of maximin retort that maximin paths are efficient. Therefore, a growth can only come with investment, this implies consuming less for the current generation, even if this one is very poor (Cairns, 2011, p. 1352). This criterion satisfies finite anonymity (finite number of permutations) and the weak version of the Pareto axiom: all generations cannot be better-off in another configuration. The strong version is satisfied by the leximin (Asheim and Zuber, 2013).

THE GREEN GOLDEN RULE. The only objective here is to obtain the maximum utility level on the long run. The intermediary path to reach this final goal does not count. Inspired from the golden rule in capital accumulation theory (popularized by Phelps, 1961), this criterion was coined by Chichilnisky et al. (1995). It applies to environmental issues its basic idea: finding the savings rate giving the highest level of utility for the entire future. A natural criticism to this criterion is that it does not take into account current and near generations. Ironically, Chichilnisky (1996, p. 233) herself stated: "*I [do not] accept the romantic view which relishes the future without regards for the present*". As the undiscounted utilitarianism, this criterion is incomplete.

MIXED CRITERIA. To overcome the drawbacks of some criteria, some authors preferred to combining them. I present here the two main mixed criteria.

The Chichilnisky criterion. Wanting to avoid what she called the dictatorship of the present (e.g. discounted utilitarianism) and the dictatorship of the future (e.g. the green golden rule, GGR), Chichilnisky (1996) proposed a mixed criterion. It is a linear combination of discounted utilitarianism and the GGR. But it does not tell anything on the specific weighting of each element. Cairns (2011, p. 1350) pointed out also that in this criterion the limit of the trajectory directly matters, while it is generally only a mathematical device for lack of determination.

The Bentham–Rawls criterion. Alvarez-Cuadrado and Long (2009) proposed a weighted average of discounted utilitarianism and maximin. It avoids three dictatorships: of the present, of the future and of the least advantaged generation. Even if some models which had no solution with the previous criterion may now have a solution, it may be time-inconsistent (Martinet, 2012, p. 71) – the decision-maker will want to change of path at a future date (Strotz, 1955-6).

0.3.4 Selected approach

To be consistent with the intratemporal part and to have a broad enough approach to be flexible, I will here also be welfarist. Here also, if other elements may come into the equation,[21] dealing with equity without referring to well-being seems unsatisfying. As in the intra part, a flexible criterion would be interesting, to not consider only a particular view of intergenerational equity. As argued by Rawls (1971, p. 286): "*how the burden of capital accumulation and of raising the standard of civilization and culture is to be shared between genera-*

[21]For example, non-welfarist information may include rights and entitlements (Anand and Sen, 2000), birth dates and lengths of life (Blackorby et al., 2007) and standard of living (Silvestre, 2007).

tions seems to admit of no definite answer". Still, our restriction on equal weight in intragenerational will be applied also in the intergenerational dimension. Besides, a criterion inefficient cannot, of course, be elected. I shall argue here that the Ramsey's (1928) criterion has kept interest for my purpose. Indeed, and as in the intragenerational part, I will be able to deal with different intertemporal inequality aversion. From zero, when any level of utility are summed identically, to infinity, when only the least level count, which is the maximin.

More recent works tried to overcome the dilemma by sophisticating the axioms asked. For example, Asheim and Tungodden (2004) coined the "Hammond Equity for the Future" (HEF) axiom: a sacrifice of the current generation to a uniform gain of the future is allowed if the current generation stays better-off. It was concretely applied to usual models by Asheim and Mitra (2010) and Asheim et al. (2012). It is both a conceptual and a practical progress. But except that the HEF axiom can be seen to be *ad hoc*, the SWF used seems to be too sophisticated to be employed when one wants to deal with another dimension, namely intragenerational issues. Still, it makes interest in discounting in some specific configurations that needs to be discussed.[22]

Here also, utility comparison arises. But now, and contrary to the previous section, it is physically not possible to know with certainty the utility functions of future generations. Except modeling endogenous modifications in the preferences, the best we can do is projecting our utility function toward the future. But more fundamentally, according to the information we think we know, drastic differences in the criterion may appear. Indeed, discounted utilitarianism needs a cardinal common unit, while maximin needs only ordinal level comparability (Lauwers, 1997). Here also it can be argued that the 'availability' of information cannot be in itself a criterion of choice.

I described the two dimensions separately as well as the way they are understood here. Let me now review how the literature has studied them both together.

[22] Asheim and Zuber (2014, 2017) studied population ethics, but with no intragenerational inequalities.

0.4 Linking the two dimensions

The two equity dimensions – intragenerational and intergenerational – have for a long time been studied separately. Either for methodological reasons, such as Rawls (1971), or, and more generally, for reasons of parsimony such as Solow (1974). But when the intragenerational distribution affects the global capacity to produce well-being, we can find interest in studying them both together.

Anand and Joshi (1979) was one of the first studies to address this issue in an article devoted to employment and wage policy. They studied, in a two-sector model (industry and agriculture), the correction of a distortion coming from a minimum wage by a State having an income distribution constraint. They discussed in an extension (sec. II, iv) intertemporal considerations through reinvestment of a possible industrial surplus, but without formalizing it explicitly. We can also mention the empirical study by Stymne and Jackson (2000) incorporating intragenerational distribution of income into measures of welfare. Vojnovic and Darden (2013) analyzed a particular case of intragenerational injustice: racial segregation in Detroit. Whites, living more and more far from Blacks in downtown, exert long-term pressures on the environmental (agricultural lands degradation, forests, rising of travels by car, etc.). Isaac and Piacquadio (2015) addressed the two dimensions through an overlapping generation model. But they were mainly interested in studding tensions between efficiency and equity criteria from the competitive equilibrium literature (e.g. no-envy). In theses models, 'intergenerational' is understood between young and old people. The links between the two dimensions was directly addressed by Baumgärtner and colleagues. In particular, Baumgärtner et al. (2012) analyzed, by mean of what they call an "opportunity set", the efficiency of "couple of equity", these ones being delimited by a "justice possibility frontier". The shape of such a frontier is determined by technology, resources, institutions, etc. And it characterizes the link between theses two objectives: independence, facilitation and/or rivalry. Glotzbach and Baumgärtner (2012) analyzed this link in the context of ecosystem services[23] and argued that it depends upon several elements: quantity and quality of services, their substitutability with manufactured goods, technol-

[23]Glotzbach (2013) further justified the choice of ecosystem services as object of justice.

ogy, institutions, etc. Besides, readers interested by the interdisciplinary literature of this field may refer to Glotzbach (2012). Generally, their approaches are a progress for understanding the links between the two dimensions. We can regret, however, that little is said about the long run (models are generally on two periods), about how to measure equity (to construct sets) and about dealing with different visions (the maximin, both intra and intergenerational versions, is mainly used). Whatever, these works demonstrate that the interest of this question increased quite recently in the economics literature related to environmental issues. Yet, this issue was existing for a long time in the climate change literature. For example, Schelling (1992, sec. IV) argued that the best way for developing countries to fight against the negative effects of climate change is to continue to develop. That said, with the recent developments in the field, it would seem that the two objectives do not point toward the same direction. Indeed, Heal (2009) argued that on the one hand our preference for equality between generations and our preference for equality within each generation may be opposed. If one expects a rising consumption, one may afford a higher (consumption) discount rate, what does not encourage us to take preventive measures due to the important net cost to bear. On the other hand, due to the greater vulnerability of poor countries, a higher preference for equality between rich and poor countries would lead to take action more rapidly. On the same topic, Kverndokk et al. (2014) study a two-region model – North and South – that have to reduce greenhouse gases emissions. They show that inequality aversion leads generally optimal climate policy to make higher investments in clean capital in the North and in dirty capital in the South, allowing this latter to develop faster. Their work is consistent with the climate issues since present and future generations are expected to bear different costs. As well, there are disparities in vulnerability between North and South. For the intergenerational dimension, they used the classic discounted utilitarian criterion, which is questionable from an ethical point of view, even in the case when it may vary. Indeed, does a lower discount rate make the problem more just? If it is unavoidable, it can be answered by the positive. As argued by Rawls (1971, p. 4): "*an injustice is tolerable only when it is necessary to avoid an even greater*". Otherwise, the answer is not trivial. These two dimensions are also present around climate negotiations issues (World Bank, 1992, p. 165;

Lecocq and Hourcade, 2012; Piketty and Chancel, 2015; Pottier et al., 2017). Intragenerational considerations may also be found in social cost-benefit analysis of investment projects. For example, Fleurbaey and Zuber (2015) studied the impact of risk and inequality on the discount rate. The impacts of inequality within generations on the social discount rate were also studied by Gollier (2015), Budolfson et al. (2017) and Emmerling (2018).[24] Meya et al. (2018) was interested in the impacts of the intra and the intertemporal distribution of income on the willingness to pay for a public good. More generally, such considerations may be found in the inequality-environment nexus. For example, Laurent (2011) distinguished four types of environmental inequalities: unequal access to resources (including unequal exposure), unequal effects from policies, unequal responsibilities and unequal involvement in environmental policies. For more developments see Boyce (2002, 2013) and Chancel (2017).

0.5 Research questions

The emergence of anthropogenic environmental problems, like the famous example of the damage caused by DDT,[25] became the new threat to Western countries in the late 1960s. At the end of the 1980s, this pessimistic view of the environment became widespread around the world with problems such as the ozone hole, Chernobyl nuclear disaster and climate change. This new ecological wave has made us aware, both locally and internationally, that changes in socioeconomic structures have to be operated in order to avoid irreversible damage to our environment. In this debate, the term *sustainable development* has found some resonance after being defined for the first time in a 'politically acceptable' way by the World Commission on Environment and Development (1987). This definition, although vague, has the merit of appreciating present development in relation to its subsequent repercussions, that is to say to emphasize intergenerational equity. But elsewhere in the report we can read that "*even the narrow notion of physical sustainability implies a concern for social equity*

[24]For sub-regional inequalities in climate issues, see Anthoff and Emmerling (2016).

[25]DDT stands for Dichlorodiphenyltrichloroethane, a chemical compound widely used in the United States as an insecticide after the World War II (Carson, 1962).

between generations, a concern that must logically be extended to equity within each generation" (chap. 2, § 3). Sustainable development is therefore based, from its 'origin', on a dual principle of inter and intragenerational equity.

While the first dimension was extensively studied in the economic literature concerned with these questions, the second one has received much less attention (Stymne and Jackson, 2000). Yet, both seem inevitably linked when it comes to environmental problems, according to Haughton (1999, p. 234): "*the unjust society is unlikely to be sustainable in environmental or economic terms; the social tensions that are created undermine the recognition of reciprocal rights and obligations, leading to environmental degradation and ultimately to political breakdown*". For some authors, it is also curious to focus on the well-being of future generations, and therefore not yet born, even before that of the present generation (Solow, 1991; Schelling, 1995; Anand and Sen, 2000). Interestingly, Boyce (2013, p.13-4) raised two hypothetic channels by which inequalities and the environment are linked. "*The relatively wealthy and powerful tend to benefit disproportionately from the economic activities that generate environmental harm. The relatively poor and powerless tend to bear a disproportionate share of the environmental costs. Second, the total magnitude of environmental harm depends on the extent of inequality. Societies with wider inequalities of wealth and power will tend to have more environmental harm. Conversely, societies with relatively modest degrees of economic and political disparities will tend to have less environmental harm*".

On these bases, I propose to study the interconnection between intra and intergenerational equity through the sharing of environmental resources. In particular, can greater justice be conceived for those who benefit from the environment today to improve the capacity to benefit from it in the future? This refers, among other things, to the fact that equity within a generation would be a prerequisite to that between generations, as argued for example by Anand and Sen (2000) and Boyce (2013). Or, on the contrary, is there a necessary trade-off between the two dimensions? In the sense that greater concern for the future requires investment (in the broadest sense), while greater concern to the current deprived would imply greater consumption (Solow, 1991, pp. 185-6).[26] More precisely,

[26]Another argument is that "*when those who bear the costs are more powerful than the benefi-*

can one conceive that different visions of equity between individuals living in the same period impact the welfare distribution differently between individuals living at different dates? Conversely, does the requirement of sustainability affect the intragenerational distribution of resources? And if so, how to implement a redistribution in one dimension that does not negatively impact the other one? Or what type of trade-off a society has to make? I now turn to the structure of the dissertation.

0.6 Outline and results

In the Chapter 1, I study the impacts of an intergenerational equity constraint on intragenerational consumptions of renewable resources. The economy is composed of two regions, 'North' and 'South' for simplicity. The representative agent of each region has access to a renewable resource. But I assume the stock of South is relatively scarcer and marginally more productive. A social planner bases the decisions upon a SWF derived from utilities in North and in South. An equal treatment is guaranteed through its symmetry. But different substitutability between the two utilities are considered to capture different inequality aversion. For the sake of simplicity, individual utilities will be assumed to be linear. This has two advantages: it avoids utility comparisons and it considered the two regions being responsible for turning consumption into welfare. The intergenerational dimension is taken through the maximin criterion. Even if this choice can be criticized, it is certainly a good benchmark to evaluate the possibility given to future generations. It may also be viewed as a limit case of infinite intertemporal inequality aversion. I also show that the maximin is not only (weakly) anonymous, but it is also most of the time strongly efficient. More importantly, departures from its dictated trajectory are analyzed. A first result from this framework is that an economy can do better for the future when it dictates specific consumption paths to the regions than when resources are exploited in isolation. South shall under-consume (compared to its highest sustainable level of consumption) - its consumption increases, - while North shall over-consume - its consumption decreases. This comes from the fact that savings in South have

ciaries, we might expect the opposite: greater inequality yields less environmental harm" (Boyce, 2013, p. 15).

a better return. The Hartwick's rule still holds: keeping a constant SWF when the economy relies on several assets requires net investments to be nil. In other words, and out of a steady state, the decreasing of a consumption in a region has to be compensated by a increasing in the consumption in the other region. The two regions converge toward a steady state where their stock are equally productive at the margin. A second result concerns relations between inequality aversion and the highest sustainable welfare level. Globally, the lower the intratemporal inequality aversion, the higher intertemporal welfare. Intuitively, less inequality aversion makes substitutions of individual well-beings (consumptions here) easier and allows more possibility to 'manipulate' consumptions and investments to improve the intertemporal dimension. To better understand this feature I consider two limit cases. The first one corresponds to a classic utilitarian criterion, with a nil inequality aversion. In this case, substitution of individual well-beings are maximal: North consumes at the highest possible (the global maximin consumption) while South consumes nothing, to let its stock grow. The second case corresponds to the (intratemporal) maximin, with an infinite inequality aversion. Here, the region with the lowest production 'constrains' the intertemporal dimension since no substitution of well-being is allowed. Finally, I study intratemporal decisions that do not corresponds to what the intertemporal criterion dictates. I show, among other things, that consuming less than what the maximin allows may not necessarily lead to make future generations globally better-off. Marginal productivities of each stock have to be considered. Here "taking advantage" of physical inequalities lead to a better welfare but at a price of higher inequalities during the transition. In the first and second chapters, no transfer between North and South is allowed. This is studied in the third chapter. Maximin provided a good benchmark, but would the results be affected if society seeks to maximize the welfare of every future generations rather than that of the worst-off?

In the Chapter 2, I study how the two regions have to share a required sacrifice to grow toward the highest sustainable welfare, the golden rule. Following the approach of the first chapter, each region has access to a different resource. The intratemporal dimension is still dealt with a symmetric SWF. But the intertemporal dimension is now dealt with the undiscounted utilitarianism. This latter, additionally of being anonymous, allows a lot of special cases such as the maximin. The main result is that the sacrifice sharing depends upon three fundamental elements: the difference of marginal productivities of the two resources, the intra and the intertemporal inequality aversions. Except if resources are marginally equally productive or if there is an infinite intra or intergenerational inequality aversion, regional consumption growth rates are different. In particular, and for the same previously mentioned reason, South has to make generally a relatively higher sacrifice than North. Interestingly, when the intratemporal inequality aversion is nil, the problem reduces to two independent problems where each region reaches its golden rule in isolation. The (original) Ramsey rule still holds here: the farther we are from the golden rule, the more we have to invest. Eventually, I study an unequal treatment of both individuals and generations putting different weights on them. This modifies the evolution of the regional consumptions and of the welfare. But the initial rule of the evolution of 'sacrifice sharing' is unchanged. The main change is that introducing discounting reduces sacrifice. Especially, the economy grows toward the *modified* golden rule. On this basis I discuss the justification of discounting. On the one hand one may prefer to not discount the future, but on the other hand one may argue that discounting can prevent huge sacrifice to the benefit of future generations that are, in a productive world, better-off. It turns out that the Ramsey criterion is sufficiently 'malleable' to refuse such an argument. It also turns out that giving equal weights to South and North may be not enough to obtain appealing ethical outcomes. The first two chapters come to the same conclusion. Sustaining, or growing, welfare implies to 'take advantage' of natural differences of the regions. The better-off region shall thus make transfers to the worse-off one.

In the Chapter 3, I study a transfer from the North to the South. In particular, I assume North has a free access to its resource and works to extract a part. While South is constrained on its harvesting. I study then the 'net' utility of South above its constrained level. The intratemporal dimension is dealt as before with a symmetric SWF, while the intertemporal dimension is taken through a minimalist view to keep things simple, by considering a value function which is non-decreasing with the stock. I analyze the introduction of a transfer taken on the catch: a lump sum transfer and a proportional tax. The consequences, depending on the chosen transfer, are different. In each case, I build utility possibility frontier ('Pareto-frontier') to see the choices allowed. Afterward, maximizing the social welfare allows to determine the optimal allocation. The frontier from the lump-sum ('first-best') exhibits an intuitive trade-off: the higher the transfer, the higher the utility of South and the lower the utility of North. The frontier from the tax ('second-best') exhibits a less intuitive but well-known feature: a higher transfer makes the North still worse-off, but it makes the South better-off until a certain point from which on its utility decreases. This is known as a 'Laffer effect'. Not surprisingly, as the proportional tax creates a distortion (it changes the relative price of consumption compared to that of leisure), it leaves fewer possibilities of redistribution to the public decision-maker than the lump sum transfer. The intragenerational inequality aversion is allowed to vary. But even if the intragenerational inequality aversion tends toward infinity, a perfect equality of regional welfares may not arise. This is because the resource needs to be harvested and it is not freely available. It also raises 'interpersonal' comparison issues since very different results are obtained depending on who society considers to be better-off. As North is assumed to be better-off here. a higher aversion leads to increase the transfer. More importantly, lump-sum and tax have opposite consequences on the resource. A rising lump-sum leads North to compensate the transfer and at the end the global harvest rises. On the opposite, a rising tax rate leads North to work less and at the end the global harvest decreases. I take reaching the golden rule a benchmark of development for North. If the inequality aversion increases, the social planner shall use the right mechanism depending on the situation. When the current configuration leads to a steady-state stock lower than the golden-rule one, it has to implement a transfer through a tax. While when the current configuration leads to a steady-state stock higher than the golden-rule one, it has to implement a lump-sum transfer. If the right mechanism cannot be used, a necessary trade-off between the two dimensions has to be made. The trade-off disappears only in the very specific case where the optimal allocation is equality, whatever the inequality aversion. This model also raises the feature that a huge decreasing of consumption may lead to over-accumulation of the resource, and being 'too generous' toward the future.

Chapter 1

Sustaining welfare when intratemporal inequalities matter[1]

Abstract

Sustaining the highest possible level of welfare over time has different consequences on regions unequally endowed with renewable resources. Consider the North being endowed with a larger stock than the South. The stock of the North is then relatively less productive at the margin. It is showed that North has to over-consume – its marginal productivity grows – while South has to under-consume – its marginal productivity rises. The steady state is reached at an equality of the marginal productivities in the two regions. The higher the intragenerational

[1]The first chapter is largely inspired from Cairns, Robert D.; Del Campo, Stellio and Martinet, Vincent, *Sustainability of an Economy Relying on Two Reproducible Assets* (December 12, 2016). CESifo Working Paper Series No. 6314.
Abstract: *Evaluating the sustainability of a society requires a system of shadow or accounting values derived from the sustainability objective. As a first step toward the derivation of such shadow-values for a maximin objective, this paper studies an economy composed of two reproducible assets, each producing one of two consumption goods. The effect of the substitutability between goods in utility is studied by postulating, in turn, neoclassical diminishing marginal substitutability, perfect substitutability and perfect complementarity. The degree of substitutability has strong effects on the maximin solution, affecting the regularity or non-regularity of the program, and on the accounting values. This has important consequences for the computation of genuine savings and the sustainability prospects of future generations.*
I restate this model in this chapter. The calculus are identical. Only the interpretations and the results are reformulated. Nonetheless, one restriction is imposed: the social welfare (ex-utility) function is now symmetric.

inequality aversion, the lower the highest sustainable welfare. But as long as this inequality aversion is finite, South bears a higher cost than North to sustain the global economy. The next chapter will study this feature when one no longer wants to sustain welfare, but one wants to grow. The first two chapters ignore the possibility of transfer, which is addressed in the last chapter.

1.1 Introduction

The growing impacts of human activity on the environment have increased concern for sustainability and call for the definition of tools to assess it. To Solow (1993), sustainability means the ability to support a standard of living for the very long-run, and requires conserving a "*generalized capacity to produce economic well-being*", accounting for all components of human well-being, including the consumption of manufactured goods, the flow of services from the environment, etc. A growing body of work proposes metrics for sustainability accounting (Neumayer, 2013), among which genuine savings indicators are prominent. Genuine savings measures the evolution of the productive capacities of the economy through net investment in a comprehensive set of capital stocks. If the concerns for sustainability come from the hypothesis that society's current decisions are not sustainable, it can hardly be held that observed, market prices can be used for sustainability accounting. Most of the genuine savings literature is based on the maximization of a welfare function, which defines a value $V(X)$ for any economic state X (vector of capital stocks). *Shadow values* $\partial V(X)/\partial X_i$ are then used to compute genuine savings as $\sum_i \frac{\partial V(X)}{\partial X_i} \frac{\mathrm{d}X_i}{\mathrm{d}t}$ (Asheim, 2007; Dasgupta, 2009).[2] Genuine savings then measures the net investment in the *capacity* to produce the chosen measure of welfare.

But distributional issues happen alongside with the sustainability objective. Indeed, people living in rich countries and people living in poor countries are

[2] A notable exception is the work of Dasgupta, Mäler, and colleagues (Dasgupta and Mäler, 2000; Arrow et al., 2003) who use a general, possibly non-optimal resource allocation mechanism (*ram*) instead of maximizing welfare. As in the optimization models, the accounting price of each capital stock corresponds to the marginal contribution of that stock to the value (discounted utility in their case) associated to the trajectory determined by the *ram*. Integrating the dynamic path and computing the associated value as a function of all capital stocks can be done only for simple models with strong assumptions on the *ram*.

likely to face different challenges. This is well known in the climate change literature (Heal, 2009), but it is intimately linked with natural assets. Environmental problems are complex, uncertain and important, but ultimately ethical. A first step toward a better implementation of policies would be to consider directly distributional issues into a sustainability objective. Do we want a general capacity of the economy (including goods and services from the environment) to produce well-being if some individuals would be more and more deprived in the future? Still, defining and assessing sustainability remains an issue in itself.

The literature on genuine savings mostly adopts discounted utility as a measure of welfare. While it is the customary measure of intertemporal value in economics, discounted utility is criticized in the sustainability literature as being inequitable (Heal, 1998; Martinet, 2012). An alternative measure is the maximin value (Rawls, 1971), which is related to intergenerational equity (Solow, 1974) and defines the highest egalitarian and efficient path that could be implemented from current state in regular problems (Burmeister and Hammond, 1977). This criterion motivated Hartwick's work on nil net investment (Hartwick, 1977), which is the backbone of genuine savings measures. For non-regular problems, though, the highest egalitarian path may not be efficient. As the maximin criterion does not satisfy Pareto efficiency (Asheim and Zuber, 2013), an important stream of the literature, mainly axiomatic, has focused on the definition of alternative social welfare functions (SWFs) that encompass both economic efficiency and intergenerational equity (Chichilnisky, 1996; Alvarez-Cuadrado and Long, 2009; Asheim et al., 2012; Asheim and Zuber, 2013). This literature tries to overcome impossibility theorems stating that there is no SWF satisfying both the axiom of *strong anonymity* and the axiom of *strong Pareto efficiency*. A criterion relaxes either the axiom for efficiency (e.g., the maximin criterion is anonymous but does not satisfy strong Pareto efficiency) or the axiom for equity (such as Chichilnisky's (1996) criterion, which replaces anonymity by the axioms of *non-dictatorship of the present* and *non-dictatorship of the future*), or it has to be incomplete (such as overtaking criteria) or non-constructible.[3] While this literature raises interesting normative issues, most of the SWFs it produced are not as

[3]See Basu and Mitra (2003); Zame (2007); Asheim (2010); Lauwers (2012), among others.

readily implementable into a sustainability accounting system as maximin.[4]

I see at least three interests in studying maximin for intergenerational purposes. First, the maximin value has a clear, positive interpretation in terms of sustainability, as soon as one defines sustainability as the 'ability to sustain welfare'. This value is the highest level of welfare that can be sustained forever given the current state of the economy (Cairns and Long, 2006; Cairns, 2011, 2013; Fleurbaey, 2015a). It is my measure of '*intergenerational equity*' herein. A genuine savings indicator can be defined for maximin, the shadow-value of a stock being its marginal contribution to the maximin value. Net investment accounted using these shadow-values, for any given dynamic path – efficient or not, and whether or not maximin is the pursued social objective, – represents the evolution over time of the highest sustainable level of utility and is interpreted as a measure of sustainability improvement or decline (Cairns and Martinet, 2014). Computing net maximin investment is thus meaningful for sustainability accounting as it informs on the effect of current decisions on the ability to sustain utility that is bequeathed to future generations. Second, it guarantees a procedural equity since all generations are treated equally (finite anonymity). It seeks to maximize the welfare of the worst-off generation to the extend possible, irrespectively of its date of appearance. At the end, every generation can enjoy the maximin welfare, possibly constant over time. Third, if society is concerned with the effective distribution of welfare over time (consequentialist approach), it may have an intertemporal inequality aversion (IA). If one interprets IA through (the converse of) the elasticity of substitution (Atkinson, 1970), it may be shown that maximin corresponds to the limiting case of infinite intertemporal IA (d'Autume and Schubert, 2008a).

The possibility of developing a sustainability accounting system based on maximin values requires defining the various capital stocks' shadow-values. As with any other measure of welfare, the shadow-values are determined through

[4] A genuine savings indicator can be defined for any dynamic, forward-looking welfare function satisfying the property of *independent future* (Asheim, 2007). The *sustainable recursive social welfare functions* characterized by Asheim et al. (2012), and the particular case of *sustainable discounted utilitarianism* (Asheim and Mitra, 2010), satisfy this axiom, but the associated genuine savings indicators have not been studied yet, to the best of my knowledge. Dietz and Asheim (2012), however, implemented this criterion in the DICE integrated assessment model for the evaluation of climate policies, emphasizing its tractability.

differentiating the corresponding value function. The computation of maximin shadow-values for any actual economy, with all its various assets, consumption goods, production techniques, etc., is presently out of reach. A sensible way to proceed is to build up from the few solved maximin problems, and to try to gain a greater understanding of the economic issues involved, as it was done for discounted utility (Arrow et al., 2003). To tackle distributional considerations within each generation, I will consider that there are two distinct renewable resources available in two different regions, say 'North' and 'South'. Besides, the general idea of genuine savings was constructed on several assets (Hartwick, 1977). I postulate an economy with two regions that do not interact together. By solving the maximin problem for this economy, I provide both some insights of the interplay between intratemporal IA and the sustainable welfare and some insights for the future development of a system of accounting based on maximin shadow-values. For the sake of simplicity, I will assume that each regional indicator of well-being is linear with respect to the regional consumption. This avoids inter-regional comparisons of utility, but mainly it considers regions responsible for turning consumption into utility. Thus I will be interested in allocation of regional consumptions.

A key constituent of my examination is the question of elasticity of substitution (EOS) between consumption of North and consumption of South. This refers both to IA and substitutability issues. The 'welfarist' approach allows for dealing with two famous limiting cases, as well as all intermediary cases. Indeed, utilitarianism is concerned with the global welfare irrespectively of its inter-individual distribution. It is characterized by a nil IA (infinite EOS). At the opposite, intragenerational maximin is concerned with the worst-off individual. It corresponds to an infinite intratemporal IA (nil EOS). Interestingly, this echoes concerns with substitutability between different stocks, in North and South here. Neumayer (2013) stresses that substitutability in production as well as in welfare plays a central role in the study of sustainability. The influence of substitutability in production on the maximin solution has been emphasized since the work of Solow (1974) and Dasgupta and Heal (1979), who studied interactions between sectors in the form of a sector extracting a non-renewable resource used as an input to a manufacturing sector. Some authors (e.g. Asako, 1980; Stollery, 1998;

d'Autume and Schubert, 2008a; d'Autume et al., 2010) study maximin problems with two substitutes in welfare (utility), one of which is a decision variable (consumption) and the other a state variable (the ambient temperature or the stock of a non-renewable resource). Substitutability of consumption goods in welfare has received less scrutiny but is as important a question for sustainability as substitutability in production.[5]

Keeping the economy at a steady state by maintaining current capital stocks is a proposal that appeals to some proponents of sustainability (Daly, 1974; Neumayer, 2013). By solving the maximin problem for our economy, it is shown in Section 1.2 that, whenever the IA is finite (but non-zero), if regions are differently productive at the margin, it is possible to sustain welfare at a higher level than at that regional steady state. There are a higher consumption in the marginally less productive region (North) and a lower consumption in the more productive one (South). From a social welfare point of view, the depletion of the less productive stock is compensated for by investment in the more productive one, which echoes the Hartwick's nil net investment rule (Hartwick, 1977; Dixit et al., 1980; Cairns and Long, 2006). This investment pattern is driven by the shadow-values of the stocks, i.e., the sustainability accounting prices. What is maintained is a general capacity to sustain welfare, not levels of particular natural assets. This calls for transfers from North to South as discussed later on.

In single-sector models such as the Solow (1956) growth model with capital depreciation or the simple fishery, stocks that are beyond the golden-rule level or the maximum sustainable yield have negative marginal products and are redundant for maximin value (Solow, 1974; Asako, 1980). In such a non-regular case, an egalitarian path is not efficient (Burmeister and Hammond, 1977), and the maximin value and shadow-values are of little information. It is thus important to identify the conditions under which such non-regularity occurs. In our two-sector model, there is no such inefficiency when abundant stocks can be used in the investment pattern (in North) to build up a scarce resource (in South), and so have a positive sustainability accounting value. Stock redundancy, which is as-

[5] Substitutability of consumption goods in utility has been studied in the discounted utilitarian framework (Quaas et al., 2013; Baumgärtner et al., 2017) and shown to strongly influence optimal development paths and their ability to sustain utility.

sociated with nil sustainability accounting prices, arises only if *all* technologies have a single productivity peak, and is thus less likely to occur in a multi-region model with substitutability. Positive shadow-values directly indicate scarcity.

It is shown is the Section 1.3 that substitution/investment pattern is influenced by the degree of intragenerational IA between regional consumption goods in a subtle way, in interplay with the relative 'productivity' of the natural stocks. Moreover, in the limiting case of infinite IA, the region (South) with the less abundant resource limits sustainability and the more abundant one is redundant (North). Only the former has a positive accounting price for intergenerational equity.

The consequences of these results for sustainability accounting with maximin shadow-values are discussed in Section 1.4. In particular, two conditions for current decisions to improve the level of welfare that can be sustained are determined. First, current welfare has to be lower than the maximin value. Second, the resource thus freed-up must be invested in order to get a positive maximin net investment. Llavador et al. (2011) stressed that the year 2000 consumption in the USA was lower than the sustainable, maximin value. Such a lower welfare can be consistent with long-run growth as long as both investment decisions result in an increase of the maximin value and proper transfers toward developing countries are implemented.

Conclusions and prospects for future research are given in Section 1.5. Additional mathematical details and analyses (for special cases, in particular) are provided in the Appendix.

1.2 Two regions, two reproducible assets

In this section, the maximin solution in a two-region model is characterized: each region, North and South, having access to a distinct renewable resource. A social planner derives an instantaneous social welfare function (SWF) W from utilities of the regions. For the sake of simplicity, utilities are assumed to be linear: $u(c_i) = c_i,\ i = N,S$. This avoids utility comparisons and it considers region responsible for turning consumption into utility. I start with neoclassical assumptions on production. Then *single-peaked technologies* (SPT) are considered, introducing a source of *stock redundancy* which happens generally with renewable resources (e.g. a logistic growth).

1.2.1 A neoclassical benchmark

Consider an economy with two reproducible assets, X_N and X_S, produced by separate regions, North and South, according to technologies $F_i(X_i)$, which depend only on the stock X_i and are assumed to be twice continuously differentiable, strictly increasing ($F_i' > 0$)[6] and strictly concave ($F_i'' < 0$).[7] Production is either consumed (c_i) or 'invested' $\left(\dot{X}_i\right)$ and capital dynamics are

$$\dot{X}_i(t) \equiv \frac{\mathrm{d}X_i(t)}{\mathrm{d}t} = F_i(X_i(t)) - c_i(t) \ , \quad i = N, S \ . \tag{1.1}$$

The economy is composed of infinitely many generations of identical consumers, each living for an instant in continuous time. Consider an ordinal social ordering over the two consumptions, represented by a twice-differentiable, strictly quasi-concave and symmetric SWF $W(c_N, c_S)$, such that both goods have a positive marginal social welfare and are socially 'essential'.[8]

The maximin value m of a state (X_N, X_S) is the highest level of welfare that

[6]Derivatives of single-argument functions are denoted with primes. Partial derivatives of functions with several arguments are denoted with subscripts, e.g., $W_{c_i} \equiv \frac{\partial W(c_N, c_S)}{\partial c_i}$.

[7]This is a stylized, canonical model to start with. The general results of this subsection hold if one production function (say F_i) is weakly concave and the other one (say F_j) is strictly concave, as long as $F_i'(0) < F_j'(0)$. An example combining an AK-technology with a sector with capital decay is analyzed in the Appendix A.5. The assumption of positive marginal product is relaxed in Subsection 1.2.2, so that the model can represent the production function of a manufactured good with capital decay or the growth function of a renewable natural resource with a carrying capacity.

[8] Formally, it means $W_{c_i} > 0$ and $\lim_{c_i \to 0} W_{c_i}|_{W=w} = +\infty$. Strict quasi-concavity implies that marginal rates of substitution are decreasing, a property that is used below in the proof of Proposition 2. These hypotheses are relaxed in Section 1.3, where I study the limiting cases of intragenerational maximin and utilitarianism.

can be sustained forever from that state:

$$m(X_1,X_2) = \max_{w,c_N(\cdot),c_S(\cdot)} w \ , \tag{1.2}$$

$$\begin{aligned} \text{s.t. } & (X_N(0),X_S(0)) = (X_N,X_S) \ ; \\ & \dot{X}_i(t) = F_i(X_i(t)) - c_i(t), \ i = N,S, \ \text{ and} \\ & W(c_N(t),c_S(t)) \geq w \text{ for all } t \geq 0 \ . \end{aligned} \tag{1.3}$$

Herein, the term *value* refers to maximin value. Below, I omit the time argument in the expressions where no confusion is possible.

Differentiation of the maximin value with respect to time yields the net maximin investment (Cairns and Martinet, 2014, Lemma 1):

$$\frac{\mathrm{d}m(X_N,X_S)}{\mathrm{d}t} = \frac{\partial m(X_N,X_S)}{\partial X_N}\dot{X}_N + \frac{\partial m(X_N,X_S)}{\partial X_S}\dot{X}_S \ . \tag{1.4}$$

The links between the maximin problem and net investment have been studied since the work of Hartwick (1977), with recent contributions by Doyen and Martinet (2012) and Fleurbaey (2015a). The links between net maximin investment and sustainability accounting are studied in Section 1.4.

Before solving the maximin problem for this economy, let us establish the following lemmata.

Lemma 1 (Stationary fallback)**.** *For any state (X_N,X_S), the maximin value is at least equal to the welfare derived from consumption at the corresponding steady state:*

$$m(X_N,X_S) \geq W(F_N(X_N),F_S(X_S)) \ .$$

Proof of Lemma 1. The dynamic path $\dot{X}_i = 0$ driven by decisions $c_i = F_i(X_i)$ is feasible and yields the constant utility $W(F_N(X_N),F_S(X_S))$. This provides a lower bound for the maximin value. □

Lemma 1 relies on the fact that consuming the whole production, keeping the economy in a steady state, makes it possible to sustain $W(F_N(X_N),F_S(X_S))$. A dynamic path may, however, yield a higher sustainable utility.

Lemma 2 (Dynamic maximin path)**.** *If the maximin value of a state* (X_N, X_S) *is greater than the welfare derived at the steady state, i.e., if* $m(X_N, X_S) > W(F_N(X_N), F_S(X_S))$*, then, along the maximin path (i) the consumption of at least one good is greater than the production of the corresponding stock and (ii) that stock decreases.*

Proof of Lemma 2. This is a direct result from Lemma 1 and the dynamics. □

The existence of such a dynamic path means that keeping the economy at a steady state (Daly, 1974) is not the only sustainable option. Solving the maximin problem (1.2) may provide a superior path. To do so, I follow the direct approach to maximin proposed by Cairns and Long (2006). Taking the sustained utility level w as a control parameter and denoting the costate variables of the stocks by μ_i, the Hamiltonian writes:

$$\mathscr{H}(X,c,\mu) = \mu_N \dot{X}_N + \mu_S \dot{X}_S = \mu_N (F_N(X_N) - c_N) + \mu_S (F_S(X_S) - c_S) \ . \quad (1.5)$$

Denoting the multiplier associated with the constraint (1.3) by ω, the Lagrangian is

$$\mathscr{L}(X,c,\mu,w,\omega) = \mathscr{H}(X,c,\mu) + \omega (W(c_N,c_S) - w) \ . \quad (1.6)$$

In an interesting problem, both initial stocks are strictly positive, i.e., $X_i(0) > 0$ for $i = N,S$ (otherwise, one is back to the single sector problem). Under the condition that both goods are essential to consumption, and given Lemma 1, one can say that consumption of both goods is positive ($c_i > 0, \ i = N,S$) at any time along a maximin path. The necessary conditions are, for $i = N,S$, and for any time t:

$$\frac{\partial \mathscr{L}}{\partial c_i} = 0 \ ; \quad (1.7)$$

$$\frac{\partial \mathscr{L}}{\partial X_i} = -\dot{\mu}_i \ ; \quad (1.8)$$

$$\int_0^\infty \left(-\frac{\partial \mathscr{L}}{\partial w} \right) \mathrm{d}s = \int_0^\infty \omega(s) \, \mathrm{d}s < \infty \ ; \quad (1.9)$$

$$\lim_{t\to\infty} \mu_i X_i = 0 \ ; \quad (1.10)$$

$$\lim_{t\to\infty} \mathscr{H}(X,c,\mu) = 0 \ ; \quad (1.11)$$

$$(\omega, \mu_N, \mu_S) >> (0,0,0) \ ; \quad (1.12)$$

along with the usual complementary slackness conditions

$$W(c_N, c_S) - w \geq 0\ , \quad \omega \geq 0\ , \quad \omega\,(W(c_N, c_S) - w) = 0\ . \tag{1.13}$$

Whenever equality is efficient, the maximin path corresponds to an egalitarian and efficient path, with welfare equal to the constant maximin value. The solution is said to be *regular* (Burmeister and Hammond, 1977) and corresponds to a (strong) Pareto allocation of utility among generations. The variable ω measures the shadow cost of the equity constraint, i.e., how much would be gained in value if the constraint was locally relaxed. This is the opportunity cost of meeting the constraint at the current state.[9]

Cairns and Long (2006, Proposition 1) show that along a maximin path, the μ_i are the shadow-values of each stock at current state, i.e., $\mu_i = \frac{\partial m(X_N, X_S)}{\partial X_i}$, and that the Hamiltonian, which thus represents net investment at maximin shadow-values (eq. 1.4), is nil:

$$\mathscr{H}(X, c, \mu) = \mu_N \dot{X}_N + \mu_S \dot{X}_S = \dot{m} = 0\ . \tag{1.14}$$

Eq. (1.14) is related to Hartwick's rule (Hartwick, 1977; Dixit et al., 1980; Withagen and Asheim, 1998). The maximin value is constant over time and, as welfare is equal to the maximin value ($W(c_N, c_S) = m(X_N, X_S)$), so is welfare.

The optimality conditions above can be given economic meanings in a regular problem, when $(\mu_N, \mu_S, \omega) \neq (0, 0, 0)$.[10]

[9] Maximizing the minimal utility over time can be perceived as successively raising the level of the least well-off to the extent possible (Solow, 1974). The end result of this sequence of redistributions can be an equalization of welfare. Intergenerational equality may be the outcome of the maximin problem but is not its objective. In this context, Cairns and Long (2006) interpret the multiplier ω as a shadow-value or cost of equity, which provides information on the difficulty of satisfying the minimal utility constraint at time t. Equity does not necessarily mean equality, however. Even if a maximin solution exists, a redistribution to achieve equality is not always possible or efficient. When it is not efficient to distribute well being equally over time, the solution is *non-regular*, as discussed below in Subsection 1.2.2.

[10] Note that, for simple fishery models, the constraint qualification is not satisfied (cf. Cairns and Long, 2006, pp. 279 and 291-295). In the case of maximin, the usual necessary conditions may not be necessary. In particular, it may be that $\omega = 0$ (that the problem is not regular). One has to solve the problem and check that the solution "makes sense" and in particular does not violate feasibility constraints. This will be the case in our analysis of cases departing from the neoclassical benchmark in Subsection 1.2.2.

From eq. (1.7), the shadow-value of stock X_i is equal to the marginal utility of consumption c_i weighted by the shadow-value of equity

$$\mu_i = \omega W_{c_i}\ , \quad i = N, S\ . \tag{1.15}$$

As long as $\omega > 0$, the relative shadow-value is equal to the marginal social rate of substitution in consumption:

$$\frac{\mu_N}{\mu_S} = \frac{W_{c_N}}{W_{c_S}}\ . \tag{1.16}$$

From eq. (1.8), $\dot{\mu}_i = -\mu_i F_i'(X_i)$, so that each shadow-value decreases at a rate equal to the current marginal product of the corresponding stock:

$$-\frac{\dot{\mu}_i}{\mu_i} = F_i'(X_i)\ , \quad i = N, S\ . \tag{1.17}$$

This depreciation rate is the cost of postponing an investment over a short period of time (see Dorfman, 1969, p. 821). The lower a stock, the higher its marginal product and the more costly in terms of maximin value it is to postpone investment in the stock.

The relative shadow-value $\frac{\mu_N}{\mu_S}$ decreases at a rate equal to the current difference between the stocks' marginal products:

$$\frac{1}{\mu_N/\mu_S}\frac{\mathrm{d}(\mu_N/\mu_S)}{\mathrm{d}t} = \frac{\dot{\mu}_N}{\mu_N} - \frac{\dot{\mu}_S}{\mu_S} = -\left(F_N'(X_N) - F_S'(X_S)\right). \tag{1.18}$$

Taking the logarithmic derivative of eq. (1.15) gives $\frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{\omega}}{\omega} + \frac{\dot{W}_{c_i}}{W_{c_i}}$. Substituting $\frac{\dot{\mu}_i}{\mu_i}$ by $-F_i'(X_i)$, we obtain, for $i = N, S$,

$$-\frac{\dot{\omega}}{\omega} = F_i'(X_i) + \frac{\dot{W}_{c_i}}{W_{c_i}}\ . \tag{1.19}$$

The shadow-value of equity decreases at a rate equal to the sum of a stock's marginal product and the rate of change of the marginal utility of consumption for the associated good. Eq. (1.19) is analogous to the 'Keynes-Ramsey rule'. The rate $\rho \equiv -\frac{\dot{\omega}}{\omega}$ has features of a utility discount rate and ω can be interpreted as a virtual discount factor along the maximin path (Cairns and Long, 2006). A maximin path thus has analogies to a discounted-utility path with this discount factor.[11] The shadow-value ω, however, is endogenous, and so is the virtual discount rate ρ, which is unlikely to be constant (except at a steady state), unlike in a discounted-utilitarian problem.[12]

The following Proposition characterizes optimal, regular steady states. Since conditions (1.15)-(1.19) remain valid for n stocks, this condition is general to an economy with any number of separate regions.[13]

Proposition 1 (Steady state)**.** *In an optimal steady state* $(X_N^\star, X_S^\star)$ *the marginal products of all stocks are equal:*

$$F_N'(X_N^\star) = F_S'(X_S^\star) = \rho^\star \ . \tag{1.20}$$

Proof of Proposition 1. At a steady state $(X_N^\star, X_S^\star)$, $\dot{X}_i = 0$, i.e. $c_i = F_i(X_i)$, then $\dot{c}_i = 0$ and thus $\dot{W}_{c_i} = 0$, $i = N, S$. It follows from eq. (1.19) that $-\frac{\dot{\omega}}{\omega} = F_N'(X_N^\star) = F_S'(X_S^\star)$. □

For any of the states satisfying the marginal productivity condition of Proposition 1, the stationary path yielding utility $W(F_N(X_N^\star), F_S(X_S^\star))$ is egalitarian and efficient, and corresponds to the maximin solution from that state. At such steady states, the virtual discount rate $\rho^\star$ is endogenously set equal to the marginal productivity of all capital stocks so that there is no possibility of arbitrage by investing in or depleting the stocks.

Any state satisfying the condition $F_N'(X_N^\star) = F_S'(X_S^\star)$ is an optimal steady state. This is different from the optimality condition for an optimal steady state in a discounted utility problem with constant discount rate δ, which is fully determined by the exogenous discount rate through the condition $F_N'(X_N^\star) = F_S'(X_S^\star) = \delta$. In the discounted utility framework, the no-arbitrage condition of stationarity is verified only for that particular steady state, and the optimal trajectory would converge to that state whatever the initial state of the economy. One shall see

[11] If the marginal welfare W_{c_i} is the shadow current price of consumption for good i, by eq. (1.15), the shadow-values μ_i are analogous to present-value prices.

[12] E.g., the discount rate in Withagen and Asheim (1998) varies over time.

[13] Weitzman (1976) originally used such a model with separate sectors to establish the formal links between national accounting and welfare in the discounted utility framework.

that in the maximin framework trajectories converge to different steady states, depending on the maximin value of their initial state.

When the economy is not at an optimal steady state, it follows a dynamic path characterized by nil net investment (eq. 1.14) and the following conditions.

Proposition 2 (Transition path)**.** *Along an optimal maximin path, when $F_i'(X_i) > F_j'(X_j)$, consumption and investment levels are such that:*

- *There should be a positive investment in the region with the higher marginal productivity of capital (with $c_i < F_i(X_i)$ and $\dot{X}_i > 0$). Its marginal product decreases and its consumption increases.*
- *The stock with the lower marginal product is reduced (with $c_j > F_j(X_j)$ and $\dot{X}_j < 0$). Its marginal product increases and its consumption decreases.*
- *The maximin path leads to an optimal steady state, either in finite time or asymptotically, with the marginal products converging to equality.*

Proof of Proposition 2. Since both stocks have to satisfy condition (1.19), there is the equality $F_N'(X_N) + \frac{\dot{W}_{c_N}}{W_{c_N}} = F_S'(X_S) + \frac{\dot{W}_{c_S}}{W_{c_S}}$, which gives us, for $F_S'(X_S) > F_N'(X_N)$:

$$F_S' - F_N' = \dot{c}_S \left(\frac{W_{c_N c_S}}{W_{c_N}} - \frac{W_{c_S c_S}}{W_{c_S}} \right) - \dot{c}_N \left(\frac{W_{c_S c_N}}{W_{c_S}} - \frac{W_{c_N c_N}}{W_{c_N}} \right) > 0 \, . \tag{1.21}$$

Under strict quasi-concavity, marginal rates of substitution are decreasing. It can be shown that the expressions in both parenthesis in eq. (1.21) are positive,[14] which allows us to rearrange the inequality as follows:

$$\dot{c}_S > \dot{c}_N \left(\frac{W_{c_S c_N}}{W_{c_S}} - \frac{W_{c_N c_N}}{W_{c_N}} \right) / \left(\frac{W_{c_N c_S}}{W_{c_N}} - \frac{W_{c_S c_S}}{W_{c_S}} \right) \, . \tag{1.22}$$

Levels of consumption cannot both increase or decrease at the same time along the maximin path, where utility is constant over time. As such, $\dot{c}_N$ and $\dot{c}_S$ must

[14] $\frac{\partial \mathrm{MRS}_{c_N/c_S}}{\partial c_S} < 0 \Leftrightarrow \frac{\partial (W_{c_S}/W_{c_N})}{\partial c_S} < 0 \Leftrightarrow W_{c_S c_S} W_{c_N} - W_{c_S} W_{c_N c_S} < 0 \Leftrightarrow \frac{W_{c_N c_S}}{W_{c_N}} - \frac{W_{c_S c_S}}{W_{c_S}} > 0$ and $\frac{\partial \mathrm{MRS}_{c_N/c_S}}{\partial c_N} > 0 \Leftrightarrow \frac{\partial (W_{c_S}/W_{c_N})}{\partial c_N} > 0 \Leftrightarrow W_{c_S c_N} W_{c_N} - W_{c_S} W_{c_N c_N} > 0 \Leftrightarrow \frac{W_{c_S c_N}}{W_{c_S}} - \frac{W_{c_N c_N}}{W_{c_N}} > 0.$

be of opposite sign. Under condition (1.22), $\dot{c}_N > 0$ would imply $\dot{c}_S > 0$. One must thus have $\dot{c}_N < 0$ and $\dot{c}_S > 0$.

From eq. (1.18), we know that when $F_S'(X_S) > F_N'(X_N)$, one has $\frac{\dot{\mu}_S}{\mu_S} - \frac{\dot{\mu}_N}{\mu_N} = -F_S'(X_S) + F_N'(X_N) < 0$. The relative price $\frac{\mu_S}{\mu_N}$ decreases. As the optimal Hamiltonian is nil (eq. (1.14)), we have $-\frac{\mathrm{d}X_S}{\mathrm{d}X_N} = \frac{\mu_S}{\mu_N}$; the tangents (in absolute value) to the paths in the state map (X_S, X_N) have to decrease as well, implying $\dot{X}_S = F_S(X_S) - c_S > 0$ and $\dot{X}_N = F_N(X_N) - c_N < 0$. Therefore, $F_N'(X_N)$ rises while $F_S'(X_S)$ decreases.

Formally, we can study the stability of the steady states. To do so, sum up the necessary conditions into the following dynamic equations, with $\pi \equiv \frac{\mu_N}{\mu_S}$;

$$\begin{cases} \dot{X}_i & = F_i(X_i) - c_i \ , i = N, S \ ; \\ \dot{\pi} & = \pi \left(F_S'(X_S) - F_N'(X_N) \right) \ . \end{cases}$$

Steady states are characterized by $c_i = F_i(X_i^\star)$, $i = N, S$, and $F_N'(X_N^\star) = F_S'(X_S^\star)$. It may be shown (see the Appendix A.3) that the Jacobian matrix of the linearized system, evaluated at the steady states, has eigenvalues with opposite sign. Therefore, a saddle-point steady state exists on the locus $F_N'(X_N^\star) = F_S'(X_S^\star) > 0$. □

The transition path to a steady state is such that welfare is sustained at the maximin level (i.e., $W(c_N(t), c_S(t)) = m(X_N(t), X_S(t))$) through substitution of the less productive stock for the more productive one. There is a *dis*investment in the stock with the lower marginal product (North), compensated for by a positive investment in the stock with the higher marginal product (South), so that maximin net investment is nil and the maximin value is constant over time. The two stocks evolve in opposite directions so long as the marginal products are unequal, toward an optimal steady state. The steady state reached depends on the initial state, in the sense that a trajectory starting for an arbitrary state $(X_N(0), X_S(0))$ with maximin value $m(X_N(0), X_S(0))$ will converge to the steady state $(X_N^\star, X_S^\star)$ satisfying $W(F_N(X_N^\star), F_S(X_S^\star)) = m(X_N(0), X_S(0)) = m(X_N(t), X_S(t))$, as well as the optimality condition of Proposition 1: $F_N'(X_N^\star) = F_S'(X_S^\star)$. The maximin trajectories are characterized by the shape of the associated iso-value curves as follows.

Proposition 3 (Iso-value curves)**.** *Along a maximin path with constant welfare w, the current state* (X_N, X_S) *and optimal controls* (c_N, c_S) *satisfy*

$$\frac{\dot{X}_S}{\dot{X}_N} = \frac{\dot{c}_S}{\dot{c}_N} = -\frac{\mu_N}{\mu_S} \quad \Leftrightarrow \quad \left.\frac{\mathrm{d}X_S}{\mathrm{d}X_N}\right|_{m(X_N,X_S)=w} = \left.\frac{\mathrm{d}c_S}{\mathrm{d}c_N}\right|_{W(c_N,c_S)=w} . \tag{1.23}$$

Proof of Proposition 3. Along the optimal path, for any state $(X_N(t), X_S(t))$, the partial derivatives of the maximin value equal the shadow-price of the stocks, i.e., $\frac{\partial m}{\partial X_i} = \mu_i(t)$ (Cairns and Long, 2006). From condition (1.15), one gets $\frac{\partial m}{\partial X_N} / \frac{\partial m}{\partial X_S} = \frac{\mu_N}{\mu_S} = \frac{W_{c_N}}{W_{c_S}}$. At the optimum, the global marginal rate of transformation equals the social marginal rate of substitution. Along the optimal path, $\mu_N \dot{X}_N + \mu_S \dot{X}_S = 0$ (eq. 1.14). By eq. (1.15), apart from a steady state:

$$\frac{\dot{X}_S}{\dot{X}_N} = -\frac{\mu_N}{\mu_S} = -\frac{W_{c_N}}{W_{c_S}} < 0 . \tag{1.24}$$

When welfare is constant, $\frac{\mathrm{d}W(c_N,c_S)}{\mathrm{d}t} = \dot{c}_N W_{c_N} + \dot{c}_S W_{c_S} = 0$ and, apart from a steady state:

$$\frac{\dot{c}_S}{\dot{c}_N} = -\frac{W_{c_N}}{W_{c_S}} < 0 . \tag{1.25}$$

Combining conditions (1.24) and (1.25), one gets

$$\frac{\dot{X}_S}{\dot{X}_N} = \frac{\dot{c}_S}{\dot{c}_N} \quad \Leftrightarrow \quad \frac{\mathrm{d}X_S}{\mathrm{d}X_N} = \frac{\mathrm{d}c_S}{\mathrm{d}c_N} . \tag{1.26}$$

At the steady state, i.e. when $\dot{X}_i = 0$, for $i = N, S$, $\frac{\mathrm{d}c_i}{\mathrm{d}X_i} = F_i'(X_i^\star)$. As $F_N'(X_N^\star) = F_S'(X_S^\star)$, $\frac{\mathrm{d}c_S}{\mathrm{d}c_N} = \frac{\mathrm{d}X_S}{\mathrm{d}X_N}$. Therefore, eq. (1.26) is also satisfied at the steady state. □

A maximin trajectory thus follows an iso-value curve toward a steady state at which the marginal products of the two stocks are equal. A graphical representation in Fig. 1.1 below illustrates the paths of consumption and stock levels starting at an arbitrary point A in the state space (X_S, X_N) and obeying eq. (1.23).

The conditions derived in Propositions 1-3 are general, for any strictly concave, strictly increasing SWF, and strictly concave production functions. Deriving further general results is difficult. In particular, determining explicitly the

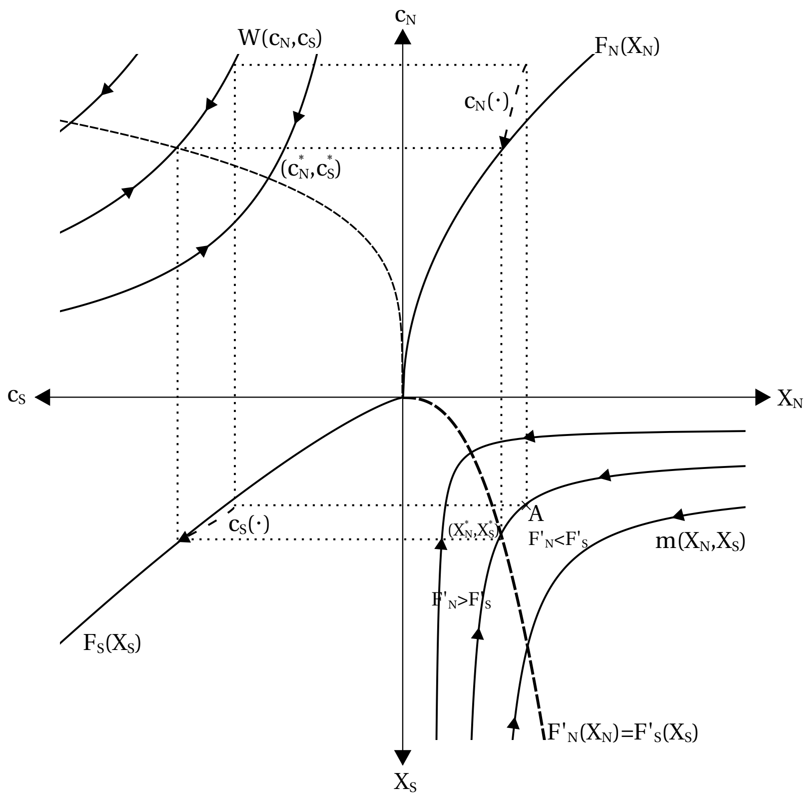


Figure 1.1: Graphical representation for the general case

steady state reached from an arbitrary initial state would require integrating the trajectory implicitly characterized by Proposition 3. This requires specifying all functional forms and solving completely the particular maximin problem. The results would then be case-specific. A closed-form solution can be obtained for some problems, but for other problems, numerical approaches may be needed. A full treatment for a particular case, with closed-form solutions is given in the Appendix A.5 as an illustration.

The implicit results of Proposition 3 allow us to provide general interpretations, however. By equation (1.23), the slopes of the indifference curve in the decision map and of the path of the state variables are equal. Therefore, the maximin paths are convex to the origin. The relative shadow-value of the stocks governs the slopes $\frac{\mathrm{d}c_N}{\mathrm{d}c_S}$ and $\frac{\mathrm{d}X_N}{\mathrm{d}X_S}$. In particular, it defines the relative value of the two stocks in terms of maximin investment. In Section 1.4 is examined how this result can be used to set up a sustainable accounting system based on maximin values.

Graphical representation Fig. 1.1 is a plot of the solution, with power functions used to represent technologies and symmetric Cobb-Douglas SWF. It is a four-quadrant graph in which the east axis represents X_N, the south axis X_S, the north axis c_N and the west axis c_S. The upper-right quadrant represents production $F_N(X_N)$ and the lower-left quadrant production $F_S(X_S)$. The upper-left quadrant plots indifference curves in the consumption map (c_S, c_N), and the lower-right quadrant is the state map (X_S, X_N) in which state trajectories can be drawn as well as iso-value curves.

The dashed curve starting at $(0,0)$ in the state map corresponds to optimal steady states satisfying $F_N'(X_N) = F_S'(X_S)$.[15] The corresponding optimal steady state consumption levels are represented by the dashed curve starting at $(0,0)$ in the consumption map, which makes it possible to relate the steady states to their maximin value on the indifference curves.

For any state north-east of the steady states curve (e.g., for state A on the figure), stock X_N is less productive at the margin ($F_N'(X_N) < F_S'(X_S)$). Along the maximin path, consumption of stock N exceeds its production, while consumption of stock S is lower than its production. The trajectory goes south-west along the iso-value curve. Consumption levels and states converge toward the corresponding steady state. A similar pattern occurs south-west of the equilibrium line. For any state, maximin shadow-values are positive.

1.2.2 Single-peakedness: A source of stock redundancy

In the neoclassical benchmark with increasing and concave technologies, the solution of the maximin problem is characterized by Propositions 1-3 and positive shadow-values (μ_N, μ_S, ω). Many possible technologies, however, attain a maximum for a finite level of the state variable. In the Solow (1956) growth model with capital decay at a constant rate, output reaches a maximum, called the

[15] This curve starts from $(0,0)$ because the two production functions have the same, infinite marginal product at zero in this example. For different technologies, the curve could start from a point $(\underline{X}_N, 0)$ satisfying $F_N'(\underline{X}_N) = F_S'(0)$.

Golden-Rule (GR) level, so that marginal net product turns negative for a large capital stock. In the study of environmental and resource economics, to which sustainability is intimately related, the simple fishery has a maximum or GR level called the maximum sustainable yield (MSY), and from that point productivity turns downward, reaching a growth rate of zero at what is called the environmental carrying capacity.[16] Such technologies have interesting features for the study of sustainability, and have been studied in one-sector models (Asheim and Ekeland, 2016).

Let us now consider SPT, they are characterized by $F_i(0) = 0 = F_i(\bar{X}_i)$, where $\bar{X}_i$ stands for the carrying capacity, and take a maximum at $F_i\left(X_i^{GR}\right)$, reached at the golden-rule stock.[17]

From a maximin point of view, a central property in a one-sector model is that a stock is *redundant* if it is beyond the golden-rule level (Asako, 1980), resulting in non-regularity (Burmeister and Hammond, 1977). For such non-regular paths, welfare can be larger than the maximin value without affecting it: the shadow cost of equity ω is nil (Cairns and Martinet, 2014), as well as the shadow-values of stocks (eq. (1.15)). There is no opportunity cost of satisfying the minimal consumption constraint. Non-regularities in maximin problems have been found in the models by Solow (1974), Asako (1980), and Cairns and Tian (2010), and are discussed in Doyen and Martinet (2012). Cairns and Martinet (2014) stress its consequences for maximin shadow-values, and thus for sustainability accounting. As non-regularity emerges even for simple problems and is a concern for accounting purposes, it is important to determine if, and under what conditions, non-regularity occurs in our two-region model.

These features are characterized with the following definition.

Definition 1 (Single-peaked technology)**.** *A natural renewal $F(X)$ is single-peaked if there exists X^{GR} such that $F'(X) > 0$ for $X < X^{GR}$ and $F'(X) < 0$ for $X > X^{GR}$.*

[16]As I do not include stock dependent harvesting costs on the one hand, and consider concave production functions, the model may not be suitable for some resources. Here again, the purpose is to study a stylized economy.

[17]In practice, the domain of X_i is $[0, \bar{X}_i]$, but technically it is possible to admit that, somehow, historically, X_i is bounded and greater than $\bar{X}_i$. This possibility is, however, not pursued herein.

By differentiability, X^{GR} is implicitly defined by the condition $F'\left(X^{GR}\right)=0$. Stock X^{GR} is the stock which yields the highest production level. A SPT is a source of non-regularity in a maximin problem, because production is bounded from above by level $F(X^{GR})$, and so is the sustainable utility in the single-sector case (Cairns and Martinet, 2014).

Stock redundancy can occur in our two-region economy too.

Proposition 4 (Bounded maximin value)**.** *If each natural renewal has a single peak, the maximin value is bounded from above by*
$m^{GR}=W\left(F_S\left(X_S^{GR}\right),F_N\left(X_N^{GR}\right)\right)$.

Proof of Proposition 4. Consider two SPTs with $F_i'(X_i^{GR})=0$, $i=N,S$, and the value $m^{GR}=W\left(F_S\left(X_S^{GR}\right),F_N\left(X_N^{GR}\right)\right)$. Assume that the utility level $m^{GR}+\varepsilon$ for some $\varepsilon>0$ is sustainable. A maximin path sustaining $m^{GR}+\varepsilon$ would have no steady state, as none can sustain this level. Such a dynamic path cannot have a consumption decreasing to zero and the other increasing to infinity either, as production is bounded from above. The maximin path would either correspond to a limit cycle or to a back-and-forth along a curve. Along a limit cycle, there would be a part of the cycle where the two stocks increase at the same time. This requires $c_i<F_i(X_i)\leq F_i\left(X_i^{GR}\right)$ for both regions, which would imply $W(c_N,c_S)\leq W(F_S(X_S^{GR}),F_N(X_N^{GR}))\leq m^{GR}+\varepsilon$, a contradiction. For a back-and-forth, at switching times, both stocks are at a steady state. Here again, $m^{GR}+\varepsilon$ cannot be sustained. The highest sustainable level of utility is m^{GR}. Any path sustaining this level converges to state (X_S^{GR},X_N^{GR}). □

Proposition 4 generalizes to two dimensions (and can be extended to more than two dimensions) the idea of GR. It also generalizes to two dimensions the non-regularity associated with the GR in an economy with a single asset. It does not mean, however, that capital above the production peak is necessarily redundant. For states such that $m(X_N,X_S)<m^{GR}$, a stock above the peak can be used intensively as a substitute for a more productive resource to build it up, just as in the neoclassical benchmark. Capital is not redundant. On the other hand, one stock may be below the productive peak and yet the other stock may be so abundant that $m(X_N,X_S)=m^{GR}$. Redundancy occurs when capital stocks are more than sufficient to sustain the highest possible maximin value m^{GR}.

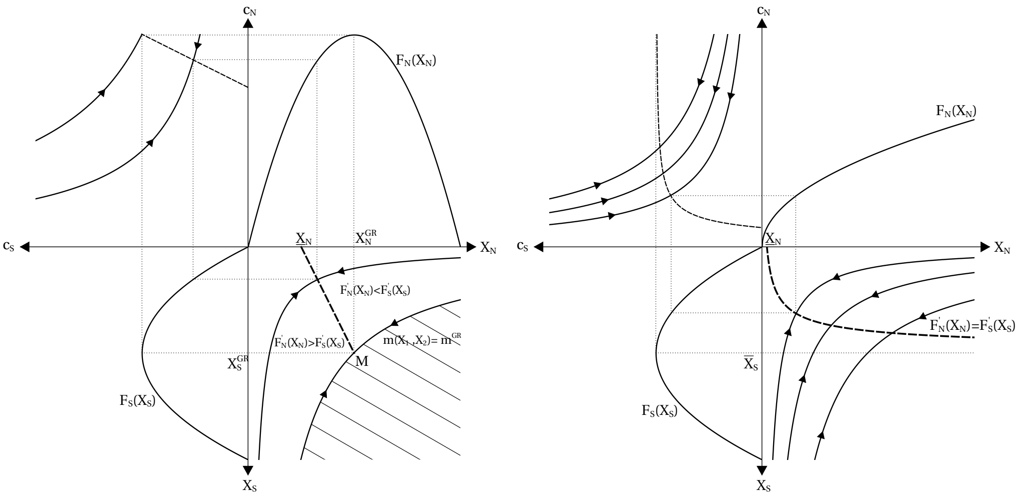
Corollary 1 (Stock redundancy)**.** *The iso-value curve $m(X_N, X_S) = m^{GR}$ delimits an area of redundant stocks with maximin value m^{GR} and shadow-values of zero.*

The iso-value curve $m(X_N, X_S) = m^{GR}$ is the edge of a plateau of the maximin value function, represented as the hatched area in Fig. 1.2a. This figure is similar to Fig. 1.1, except that the production functions are single-peaked.[18] Without loss of generality, let us assume that the stocks are indexed such that there is an $\underline{X}_N \geq 0$ where $F_N'(\underline{X}_N) = F_S'(0)$. Optimal steady states are along the line $\underline{X}_N M$. Any state in the hatched area has maximin value $m(X_N^{GR}, X_S^{GR})$. Any maximin path starting from a state in this region is non-regular in the sense that welfare can be larger than the maximin value m^{GR} for some time, until a point on the iso-value curve $m(X_N, X_S) = m^{GR}$ is reached, after which utility is constant at $W(c_N, c_S) = m^{GR}$. In this case, both stocks are redundant and have nil shadow-values, even a stock below its sector productive peak. Along the iso-value curve $m(X_N, X_S) = m^{GR}$, which is the boundary separating the area of redundant stocks from the states with positive shadow-values, the shadow-prices are also zero. Such a frontier is known as the *as-good-as-golden locus* (Phelps and Riley, 1978).

Proposition 4 and Corollary 1 depend on the fact that both technologies have a production peak. If one technology does not, so that $F_i'(X_i) > 0$ for all X_i (whether or not the production is bounded from above), there is neither a upper bound on the maximin value nor stock redundancy. When one region is always productive at the margin, the asset of another, single-peaked region is never redundant if it can be used as a substitute for the other region to grow, as illustrated in Fig. 1.2b. All capital stocks have positive shadow-values. This is a useful result for building a sustainability accounting system in a world with substitutable assets.

An example combining an AK-technology with a region with capital decay is provided in the Appendix A.5 and fully characterized.

[18] Quadratic growth functions are used to represent the technologies. As a consequence, the steady states curve is a straight line as the marginal products $F_i'(X_i)$ are proportional to the stocks X_i, $i = N, S$, giving a linear relationship between $X_N^\star$ and $X_S^\star$, for which $F_N'(X_N^\star) = F_S'(X_S^\star)$.



(a) Two single-peaked technologies (b) One single-peaked technology

Figure 1.2: Graphical representation for single-peaked technologies

1.3 Inequality aversion and sustainability

Proposition 3 characterizes the shape of the iso-value curves, which is related to the shape of the social indifference curves and thus to the inequality aversion of the two regions. In this section, the interplay between inequality aversion and sustainability is discussed, focusing on the neoclassical benchmark and then describing the limiting cases of utilitarianism and intragenerational maximin. Recall that the higher the inequality aversion (IA), the lower the substitutability of consumptions.

The degree of substitutability of consumptions influences the maximin value in a subtle way. Its effect depends on the shape of the welfare function of course but also, and perhaps less intuitively, on that of the production functions. For the sake of simplicity, let us consider only (symmetric) Constant Elasticity of Substitution (CES) SWFs in the following discussion, to focus on the role of technology.

One is at comparing the maximin value of any economic state in two different substitutability contexts. As welfare has not been assumed to be cardinally meaningful so far, different maximin values are compared through the corresponding optimal steady states, ranking them according to their relative position on the optimal steady states curve (the farther from the origin, the better). I start with a simplified case in which the two technologies are the same (i.e., $F_N \equiv F_S$) and then examine how the results change when the two technologies are different.

Similar technologies with CES welfare. Assume for now that $F_N \equiv F_S$. In this case, optimal steady states (satisfying $F'_N(X^\star_N) = F'_S(X^\star_S)$) fall on the line $X_N = X_S$. The corresponding steady state consumption levels are on the line $c_N = c_S$.[19] Consider two different sets of social indifference curves having different degrees of substitutability, representing different degrees of IA. For symmetric CES SWFs, the indifference curves of both sets are tangent along the equal consumption line. According to Proposition 3, the iso-value curves are also tangent in the state map along the optimal steady states line $X_N = X_S$, the ones with the lower degree of substitutability having greater curvature and thus lying south-east of the corresponding ones for the higher degree of substitutability. This pattern is illustrated in Fig. 1.3. Technologies are represented by the same power function, and the two sets of preferences by symmetric CES SWFs with different elasticity parameters.

For any initial endowments $(X_S(0), X_N(0))$ away from the steady state (e.g., state A on the figure), a lower degree of inequality aversion (higher degree of substitutability) implies higher steady state levels of the stocks and higher steady state levels of consumption. As the relative shadow-value μ_N/μ_S is closer to unity with higher substitutability, compensating a given reduction of a resource stock requires a lower investment in the other resource stock. In this case, the effect of IA on sustainability is unambiguous. The lower the degree of IA, the 'better' for sustainability (apart from the steady states, where IA plays no role). The farther from the optimal steady states curve, the stronger the effect of IA on sustainability.

The degree of substitutability strongly influences the optimal steady state of the economy in the maximin framework. In the discounted-utilitarian frame-

[19] This configuration allows to rank all iso-value maximin curves through their steady state. Furthermore, at the steady states $c_N = c_S$. Thus, they can be related to an *equally distributed equivalent* level of consumption (Atkinson, 1970).

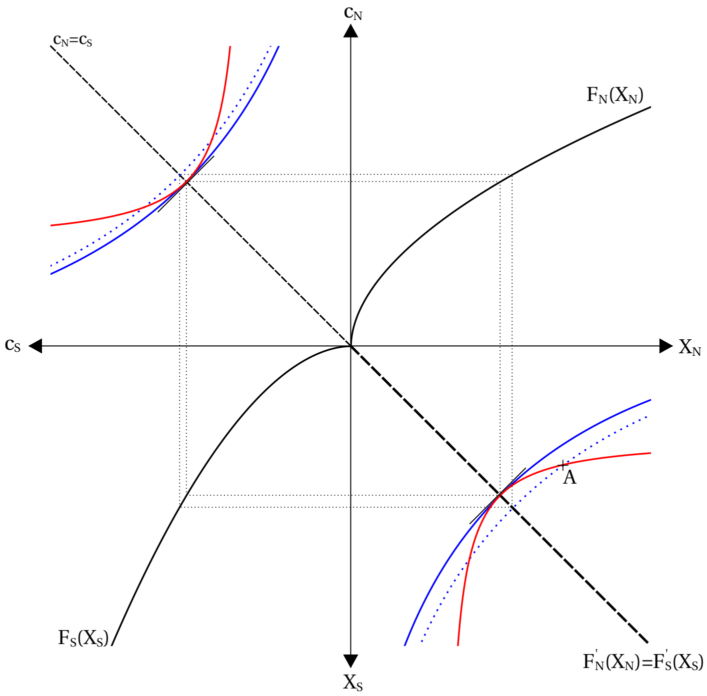


Figure 1.3: Effect of inequality aversion on sustainability with identical technologies

work, on the other hand, modifying substitutability does not change the optimal steady state, but only the transition path to it (Baumgärtner et al., 2017).

Different technologies and CES welfare. For $F_N \neq F_S$, the findings just above are modified because consumption levels at the optimal steady states are not points of tangency of indifference curves for the different elasticities of substitution. Fig. 1.4 is helpful to illustrate and discuss the effects of the non-tangency. It is drawn with different power functions for production and symmetric CES SWFs.

Consider an initial state with relatively low capital stocks, for which the optimal steady states is such that $F_N(X_N^\star) > F_S(X_S^\star)$, and thus $c_N^\star > c_S^\star$. On the associated iso-value curves, away from the optimal steady state, for states such that $F_N'(X_N) < F_S'(X_S)$, stock S is more productive and is built up along the maximin path. This entails the substitution of consumption of S by a higher consumption

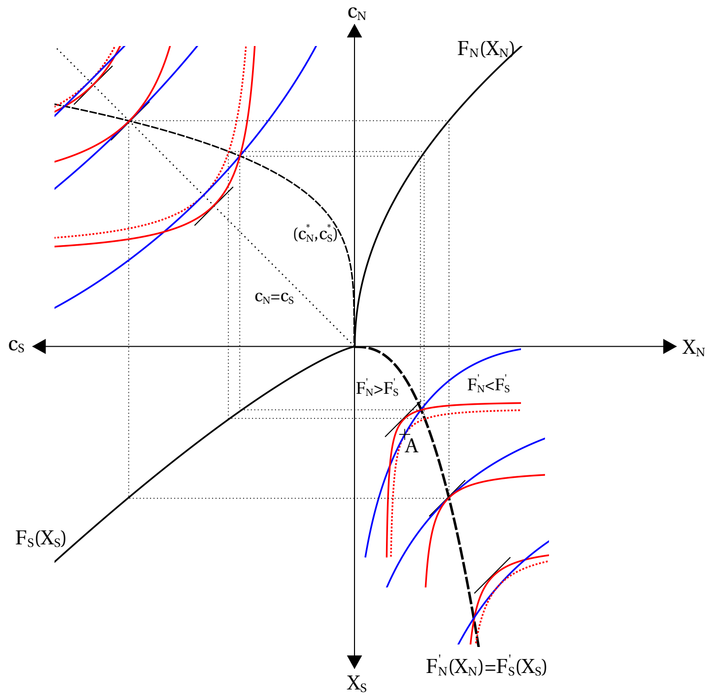


Figure 1.4: Effect of inequality aversion on sustainability with different technologies

in N, and we have $c_N(t) > c_S(t)$ all along the path (including at equilibrium). As substitutability of good S by good N is improved by a higher elasticity of substitution for such consumption levels, a lower IA increases the level of the sustainable utility.

On the contrary, for states such that $F'_N(X_N) > F'_S(X_S)$, the effect of a higher elasticity of substitution is ambiguous. The maximin path entails a higher consumption of good S to build up the capital stock N, with possibly an initial situation with $c_N < c_S$, but eventually $c_N > c_S$ as the economy gets closer to the steady state. In this case, iso-value curves for the two levels of substitutability are not tangent at a steady state but cross twice. When the state is far from the optimal steady state (i.e., when stock S is large and stock N is small) substitution improves sustainability. When the two stocks are close to the steady state (roughly speaking, if the state is north-east of the crossing point), a higher elasticity of substitution implies reduced sustainability. The substitutability of good

N by good S is improved by a higher elasticity only for $c_N < c_S$, and not all along the maximin path. As a consequence, a higher steady state can be reached from Point A in Fig. 1.4 when the elasticity is lower.

Even though this result may seem surprising, it is not unheard of in the maximin literature. The same type of ambiguous effect of substitutability on sustainability occurs for substitutability in production in the Dasgupta-Heal-Solow model (Solow, 1974; Dasgupta and Heal, 1979; Martinet and Doyen, 2007). Martinet (2012, pp. 145-6) shows that, for low capital stocks, a higher elasticity between inputs reduces the level of utility that can be sustained from a given state, whereas for larger capital stocks a higher elasticity increases sustainability.

The previous analysis examines the effect of the degree of (marginal) substitutability on maximin value within the class of 'moderate' inequality aversion. However, the literature has for a while been considering to polar cases: utilitarianism, with a nil IA, and the intragenerational maximin, with an infinite IA. I now examine the two limiting cases of the spectrum of substitutability of regional consumptions.[20]

Utilitarianism. Consider the case in which the social planner is indifferent to inequalities between consumptions in North and consumption in South. The SWF is then assumed to be linear in the consumption of each good: $W(c_N, c_S) = a_N c_N + a_S c_S,\ a_N = a_S = \frac{1}{2}$.[21]

Just as in the neoclassical benchmark, interior optimal steady states have equal marginal products $\left(F_N'(X_N^\star) = F_S'(X_S^\star) > 0\right)$. Apart from these steady states, maximin paths follow a dynamics in which the less productive resource (North) is used up and the more productive resource (South) is built up. However, with perfect substitutability, substitution in welfare is complete: the less productive

[20]Interestingly, these special cases are of interest of (classic) sustainability literature. Indeed in the discounted-utility framework, the degree of substitutability between goods affects the social discount rate and is central to the debate between proponents of weak and strong sustainability, as discussed in Traeger (2011) and Drupp (2018). That debate is, however, more often presented in terms of the limiting cases of perfect substitutability and perfect complementarity (Neumayer, 2013). Non-substitutability between manufactured and environmental goods may make sense (Baumgärtner et al., 2006), especially in the very-long run (Gerlagh and van der Zwaan, 2002).

[21]I keep a_N and a_S for generality. Illustrations and additional results are provided in the Appendices A.1 and A.4.

stock is consumed exclusively, to let the more productive stock grow as fast as possible. When the steady state is reached, the levels of consumption jump to $c_i = F_i(X_i^\star)$, $i = N, S$.

Contrary to the benchmark neoclassical case, on the path toward the steady state, the relative shadow-value is different from the marginal rate of substitution, with $\frac{\mu_N}{\mu_S} > \frac{a_N}{a_S}$. The equality $\frac{\mu_N}{\mu_S} = \frac{a_N}{a_S}$ holds only at a steady state. The more productive resource is relatively more valued in maximin terms than in welfare terms. The relative value used to account for investment is larger than the relative welfare from consumption. The shadow-prices provide the signal for the more productive resource to be conserved and its entire product to be invested along the maximin path. The iso-value curves are convex to the origin in the state map and do not have the same shape as the linear indifference curves.

Moreover, under utilitarianism, exhaustion of the resource stock with the lower marginal product may be optimal (see Lemma 4 in the Appendix). This result is endogenous to the initial stocks. It occurs when the two marginal products can not be equalized, if the stock with the lower marginal product is still relatively unproductive when it declines toward zero whereas the marginal product of the other stock does not fall too much as it is built up. This result is in striking contrast to the result of Quaas et al. (2013) in the discounted-utilitarian framework, where exhaustion of the stock with the *higher* marginal product may be optimal when resources are *complements*.[22] This difference is due to the inequitable treatment of generations under discounting.

Intragenerational maximin. Let us now consider a case where the social planner is infinitively averse to inequalities between regions. No one substitution in

[22]Quaas et al. (2013) study a model with a manufactured good and two renewable natural resources in the discounted-utilitarian framework. They investigate the resilience of this economy to a one-time shock. Solving the post-shock optimum, they show that when a stock is low (and thus has a higher marginal product), building it up is optimal only if the two resources are substitutable enough. The transition requires limiting the consumption of the more productive stock to build it up. If the resources are substitutes, there is only a limited effect on utility. On the other hand, if the resources are complements, building up the stock of the scarce resource has a high utility cost. It may even be optimal to exhaust this stock if the discount rate is high. However, Quaas et al. (2013) assume identical technologies. The discussion in this section has shown that this is not an innocuous assumption for the study of the effects of substitutability on sustainability.

consumptions is tolerated.[23] As the social planner focuses only on the worse-off region, the SWF is of the form $W(c_N, c_S) = \min\{a_N c_N, a_S c_S\}$, $a_N = a_S = \frac{1}{2}$. In this case, total, not marginal, products drive the solution. The maximin value depends on the level of production that can be maintained in each region[24] and is given by $m(X_N, X_S) = \min\{a_N F_N(X_N), a_S F_S(X_S)\}$. The production in one region limits sustainable welfare, reflecting the scarcity of the corresponding resource.

Complementarity among consumption goods implies the redundancy of a stock whenever it produces more (in welfare terms) than the other. If stocks are such that $a_i F_i(X_i) > a_j F_j(X_j)$, increasing stock i does not increase the maximin value, and this stock has a nil shadow-value. It is redundant, even if it is below its production peak, and even if there is no production peak with the technology. Only the capital of the limiting sector has a positive shadow-value $\mu_j = a_j F_j'(X_j)$.[25] Maximin value $m(X_N, X_S)$ is still a non-decreasing function.[26]

It was stated in Subsection 1.2.2 that a lower IA (more substitutability) limits stock redundancy due to SPT (as long as one of the region has a strictly positive marginal product). On the contrary, whatever the type of technology (single-peaked or not), an infinite IA (complementarity) induces redundancy. In the world depicted by the proponents of strong sustainability (Neumayer, 2013), in a maximin accounting system, only the limiting resource would have a positive shadow-price. At the margin, increasing that resource increases the sustainable welfare.

[23] An issue raised by environmentalists and ecologists is the possibility that some natural assets may not have substitutes in well-being (Neumayer, 2013).

[24] In the case of SPT, a region may limit the long-run consumption. In that case, it is not the current level of production that matters, but the level of production that can be sustained, i.e., the current level of production $F_i(X_i)$ if $X_i < X_i^{GR}$, or $F_i\left(X_i^{GR}\right)$ if $X_i \geq X_i^{GR}$.

[25] For the steady state satisfying $a_i F_i(X_i) = a_j F_j(X_j)$, both shadow-values are zero because increasing one stock in isolation does not change the maximin value.

[26] There is an upper bound for the maximin value if one technology, say j, is single-peaked: $m^{GR} = a_j F_j\left(X_j^{GR}\right)$. Any state with this maximin value is characterized by stock redundancy, with nil shadow-values.

1.4 Accounting for changes in sustainability

The maximin path in this two-region economy was characterized. It can now be used as a benchmark to assess the sustainability of current decisions, which may not correspond to maximin decisions. From previous results, one can examine the consequences of consumption choices on the evolution of the sustainable level of welfare, measured by the maximin value and its evolution. This examination provides insights for sustainability accounting for non-maximin paths.

Cairns and Martinet (2014) described the interplay among consumption, the maximin value and changes in sustainability. Net investment at maximin shadow-values is a measure of these changes. For any economic state, it is possible to define the consumption levels resulting in a positive maximin investment and an increasing of the level of welfare that can be sustained, i.e., *sustainability improvement*. I consider the neoclassical benchmark case of subsection 1.2.1 in the following discussion.[27]

For a given economic state (X_N, X_S), denote by $(c_N^\star, c_S^\star)$ the maximin consumption levels. These decisions, which satisfy $W(c_N^\star, c_S^\star) = m(X_N, X_S)$, can be used as a reference point. To do so, consider the indifference curve $W(c_N, c_S) = m(X_N, X_S)$. At $(c_N^\star, c_S^\star)$, one has $\frac{W_{c_N}}{W_{c_S}} = \frac{\mu_N}{\mu_S}$ (eq. 1.16). From the definition of net investment (eq. 1.4), let us derive the condition for non-negative net investment, for given levels of the stocks and shadow-values:

$$\begin{aligned} & \dot{m} = \mu_N[F_N(X_N) - c_N] + \mu_S[F_S(X_S) - c_S] \geq 0 \\ \Leftrightarrow \quad & c_S \leq \frac{\mu_N}{\mu_S}[F_N(X_N) - c_N] + F_S(X_S) \ . \end{aligned}$$

When $\dot{m} = 0$, there is a linear relationship between c_N and c_S.

Fig. 1.5 depicts the possible consumption decisions along with their consequences for changes in sustainability. In the consumption map, the line $\dot{m} = 0$ is tangent to the indifference curve $W(c_N, c_S) = m(X_N, X_S)$ at $(c_N^\star, c_S^\star)$. Three areas of interest are defined by the two curves.

Area 1 corresponds to consumption decisions with a sustainable welfare

[27] A similar analysis can be performed for the cases of utilitarianism and intragenerational maximin. The corresponding figures are provided in the Appendix A.2.

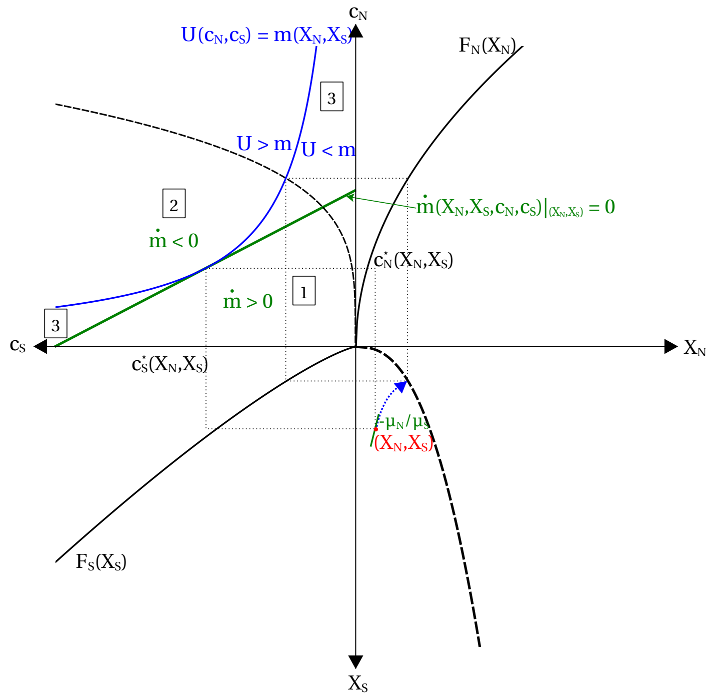


Figure 1.5: Welfare, maximin and sustainability improvement

$W(c_N, c_S) < m(X_N, X_S)$ and positive net maximin investment, i.e., to sustainability improvement. Area 2 corresponds to decisions with a unsustainable welfare $W(c_N, c_S) > m(X_N, X_S)$, implying sustainability decline ($\dot{m} < 0$). In the two parts of Area 3, consumption decisions induce a sustainability decline in spite of the fact that welfare is lower than the maximin value. With different decisions, the same welfare could have been compatible with sustainability improvement. These areas illustrate how reducing welfare below the maximin level improves sustainability only if there is an investment that increases the maximin value.

Changes in sustainability can be measured by net maximin investment along any path. Therefore, maximin shadow-values can be used as sustainability accounting prices. Basing an accounting system on shadow-values requires that these values be well defined and computable. The present analysis stresses that, as for any optimization problem, finding shadow-values is a challenging task. The task for a maximin problem, however, is likely no more difficult than for other objectives.[28] The challenge can be met with proper numerical tools. Given the theoretical characterization of the maximin solution in this paper, and given the recursive structure of the maximin objective, a Bellman algorithm could be used to compute approximate maximin values and shadow-values. Because there are strong links between maximin and viability (Doyen and Martinet, 2012), the numerical tools for set-valued analysis could also be used to find maximin values and shadow-prices.

We have found that substitutability in utility is important for the properties of the maximin solution and accounting values, as is substitutability in production (Solow, 1974; Hartwick, 1977; Mitra et al., 2013). One task is to estimate the substitutability among the different goods in the economy. For some sectors (manufactured goods and services), substitutability at the margin is a reasonable assumption and the elasticities of substitution estimated in the macroeconomic literature can provide a starting point. For environmental resources, the task is harder. Drupp (2018) surveys the empirical estimates of the substitutability between manufactured goods and ecosystem services. He relates this substitutability to the income elasticity of the willingness-to-pay for environmental goods, and emphasizes the variability of the substitutability parameter. Most studies find a relatively high substitutability. However, the degree of substitutability may change as the environment becomes scarcer (Baumgärtner et al., 2017; Drupp, 2018). At some point, or for some resources or ecosystem services, substitution may not be possible. In an extreme case of complementarity, our results imply that only limiting resources have a maximin shadow-value, and improving sustainability requires building up these particular stocks, possibly environmental resources, at the cost of reducing current utility below the maximin level.

Last, when all technologies are single-peaked, stock redundancy may occur. In that case, all maximin shadow-values are zero. Society is faced with surplus stocks (from the maximin point of view) and sustainability accounting is trivial.

[28]The fishery problem is quite difficult to solve in the discounted utilitarian framework, and is comparatively simpler in a maximin setting. The Dasgupta-Heal-Solow model was solved immediately for a maximin problem (Solow, 1974), but not until much later for a discounted-utilitarian problem (Benchekroun and Withagen, 2011).

1.5 Conclusion

Considering heterogeneous assets, one renewable resource in North and one in South here, allowed to give insights both for sustainability accounting purposes and for intragenerational inequality considerations. Indeed, sustaining welfare is the very objective of a maximin problem. Even if maximin is not chosen as a social objective and society does not aim at following a maximin path from its current stage of development, the maximin value is an indicator of the highest welfare level that could be sustained given current economic and environmental endowments (Cairns and Martinet, 2014; Fleurbaey, 2015a). Maximin shadow-values can be used *along any trajectory* to compute maximin net investment, a particular genuine savings indicator which measures the evolution of the capacity of the economy to sustain welfare. These shadow-values have to be derived from the resolution of the maximin problem.

Maximin has been applied to only a handful of problems. In Cairns et al. (2016), we have characterized the maximin solution for an additional class of problems, corresponding to economies with two separate resources. In this chapter, I restated this problem in my dissertation formulation, in which each resource is available in a distinct region of which consumptions enter in a social welfare function. Maximin calls for a dynamic investment pattern that outperforms the static option of maintaining current productive stocks. Whenever a resource in a region renews itself more at the margin than the other, positive investment in this region would be made possible through substitution in consumption and a decline of the less productive capital stock, according to Hartwick's rule of nil net investment (Hartwick, 1977; Dixit et al., 1980; Solow, 1993). The maximin path ultimately leads to some optimal steady state, however, which depends on the initial state of the economy and its maximin value. In the limiting case of nil inequality aversion (utilitarianism), a less productive stock can be exhausted.

Maximin has been criticized as a social objective at possibly maintaining a poor economy in poverty. If growing out of poverty is pursued, it must be within sustainable limits. Computing the evolution of the maximin value informs us on the effect of current consumption and investment decisions on the level of sustainable welfare. This accounting has to be done with maximin shadow-values.

The degree of inequality aversion between the regions influences the relative shadow-values of the stocks and thus the value of the investments. In the limiting case of infinite inequality aversion, only the limiting capital stock, possibly an environmental asset in a poor region, has a positive accounting price for sustainability. Investment in this region is required to improve sustainability and make growth possible. In the presence of a single-peaked technology (e.g. a logistic growth of a renewable resource), if the other technology is everywhere productive at the margin (e.g. a manufactured capital), both asset stocks have positive accounting prices. If all technologies are single-peaked, capital stocks may be redundant, and accounting prices for sustainability nil. In all cases, maximin accounting prices are well-defined and provide the relevant information about the relative marginal values of different stocks for net investment in the capacity to sustain welfare.

Concerning the intragenerational inequalities, four messages may be retained. Consider the South is endowed with a relatively less abundant resource than the North, and has then a relatively more marginally productive stock. Firstly, out of the steady state, a maximin policy leads North to consume more than its natural renewal (to deplete the stock and make its stock productivity increase) and South to consume less than its natural renewal (to make its stock level increase). Inequalities increase in the short-term but decrease over time to reach an equality of marginal renewals, except if there is an infinite inequality aversion (third point below). Secondly, a less averse economy is likely to reach a higher sustainable level of welfare. The intuition is that one can take more advantages of disparities between regions to make consumption-investment trade-offs more ‘efficient’. Conversely, a more inequality aversion limits ‘sustainability’. Thirdly, the two special cases - infinite and nil intragenerational inequality aversion - highlight the previous result by exacerbating the mechanisms. Utilitarianism, with a nil inequality aversion, indicates to North to increase its consumption while South shall consume nothing. At the opposite, the intratemporal maximin, with an infinite inequality aversion, allows no substitution between regional consumptions, then the lower regional (absolute) renewal, in South, ‘constraints’ the intergenerational welfare at its current level. Fourthly, deviations from the maximin policy were studied. It has been showed that reducing consumptions is not enough, such sacrifices have to be properly coordinated according to shadow-prices of resources.

At this stage, two questions remain unanswered. If an economy is endowed with low resource stocks, maximin may stick it at a low welfare level. If growth comes at the price of reducing consumptions, how do they have to be made between North and South? Besides, it has been showed that sustaining welfare asks a larger sacrifice to South compared to North. Naturally, this calls for implementing transfers in oder to improve the general capacity to produce economic well-being. Theses question are addressed respectively by the next and by the last chapters. Chapter 2 will endorse the well-known critics – acknowledged by Solow (1974) himself – against maximin: if initial stocks are low, it perpetuates ‘poverty’. Growth is then analyzed is a similar model. Chapter 3 will tackle the issue of transfers from North to South, which are ignored in the first two chapters. Chapters 2 and 3 will echo arguments stated by Rawls (1971, p. 286). “*If for theoretical purposes one thinks of the ideal society as one whose economy is in a steady state of growth (possibly zero), and which is at the same time just, then the savings problem is to choose a principle for sharing the burdens of getting to that growth path (or to such a path if there is more than one), and of maintaining the justice of the necessary arrangements once this is achieved*”.

A.1 Illustrations for the limiting cases

A.1.1 Illustration: Utilitarianism

Let $W(c_N, c_S) = a_N c_N + a_S c_S$. A full mathematical treatment, which is quite similar to that of the neoclassical benchmark, is provided in the Appendix A.4.

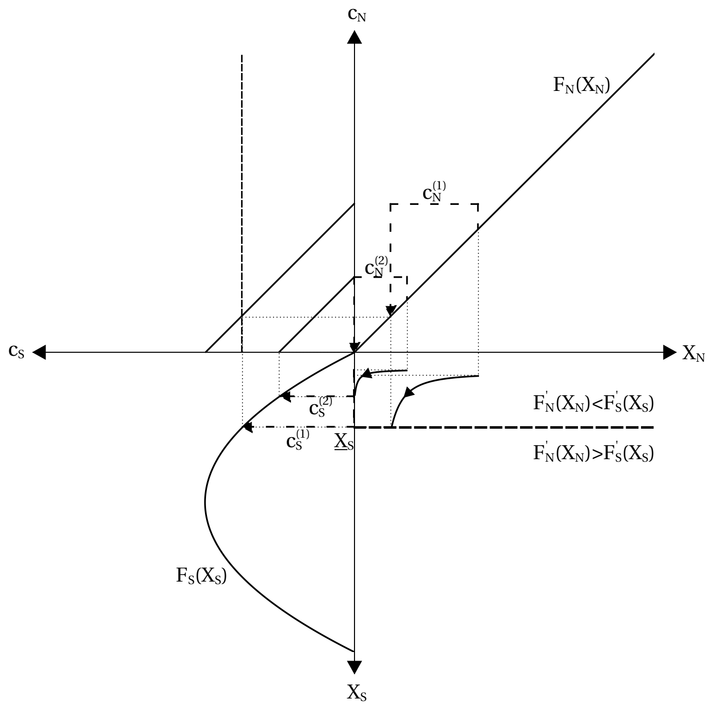


Figure A.6: Utilitarianism: Nil inequality aversion

In this case, the iso-value curves are convex to the origin in the plane (X_N, X_S), with $-\frac{\mathrm{d}X_S}{\mathrm{d}X_N} = \frac{\mu_N}{\mu_S} \geq \frac{a_N}{a_S}$. Exhaustion of the less productive stock may be optimal. Let $F_i'(0) > F_j'(0)$, and denote by $\underline{X}_i > 0$ the stock level such that $F_i'(\underline{X}_i) = F_j'(0)$. A maximin path starting from any state (X_i, X_j) such that $F_i'(X_i) > F_j'(X_j)$ and $m(X_i, X_j) \leq a_i F_i(\underline{X}_i)$ exhausts asset X_j. At exhaustion, the steady state must satisfy $F_i'(X_i^\star) \geq F_j'(0)$. The results for this case are illustrated in Fig. A.6, with a renewable natural resource with logistic growth ($F_N(X_N) = rX_N(1 - X_N)$) and an AK technology ($F_S(X_S) = AX_S$). In the Appendix A.5, I provide a full resolution of this case, computing an explicit solution for the maximin value and shadow-values.

The interpretation is the same as that of Fig. 1.1, except that one has corner solutions for the consumption. A path exhausting resource X_S illustrates Lemma 4, which is formulated in the Appendix A.1.1.

A.1.2 Illustration: Intragenerational maximin

Let $W(c_N, c_S) = \min\{a_N c_N, a_S c_S\}$, whith $a_N = a_S = \frac{1}{2}$.

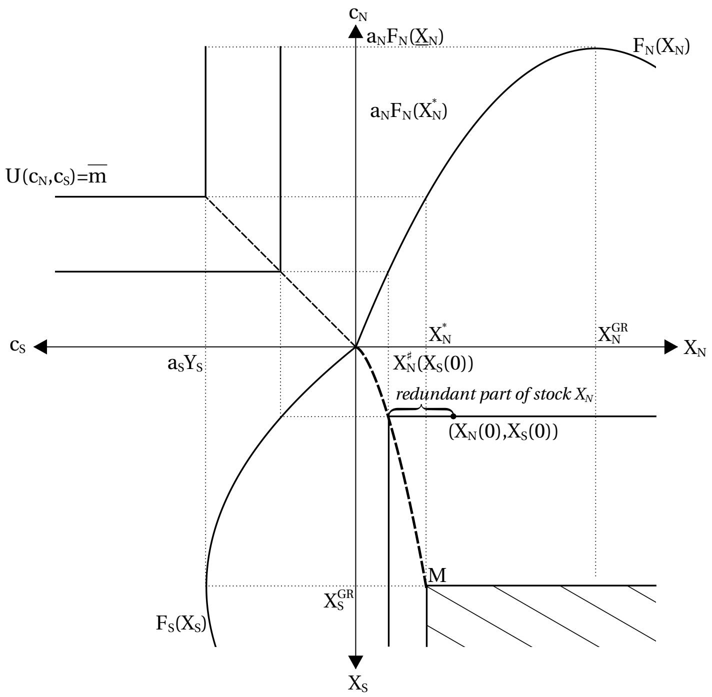
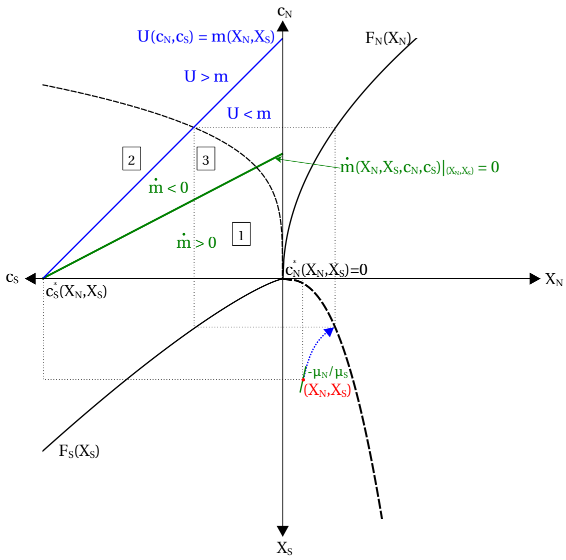


Figure A.7: Intragenerational maximin: Infinite inequality aversion

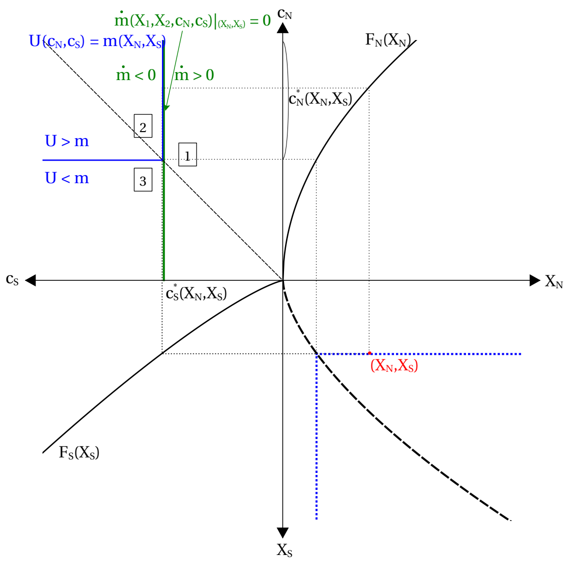
The results for this case are illustrated in Fig. A.7, for two SPTs with $a_N F_N\left(X_N^{GR}\right) > a_S F_S\left(X_S^{GR}\right)$. The curve $0M$, from $(0,0)$ to $\left(X_N^{\star}, X_S^{GR}\right)$ where $X_N^{\star} = a_S/a_N F_S(X_S^{GR})$, represents the states for which $a_N F_N(X_N) = a_S F_S(X_S)$. For these states, both resources limit utility and there is no redundancy of one of the stocks, but the shadow-values are zero. The maximin solution is to stay at the steady state (X_N, X_S). The corresponding maximin value is given by $W(c_N, c_S) = a_N F_N(X_N) = a_S F_S(X_S)$. It increases along $0M$, from 0 at $(0,0)$ to

$m^{GR} = m(X_N^\star, X_S^{GR})$. Shadow values of the stocks are zero on $0M$ but off the line $0M$ one shadow-value is positive and the other zero. For states east of the curve $0M$, resource S is limiting. Its shadow-value is positive. A part of stock X_N is redundant and maximin value is given by the point on the curve directly west. Stock X_S has a positive marginal maximin value. For states south of the curve, resource N is limiting, with a similar interpretation. Iso-value curves are given by perpendicular lines starting from any state on $0M$. The hatched area south-east of point $M = \left(X_N^\star, X_S^{GR}\right)$ corresponds to states in which both stocks are redundant. From such states, one can temporarily increase the utility above the maximin value, along a non-regular maximin path. Both stocks' shadow-values are nil.

A.2 Sustainability improvement: Graphical illustrations for the limiting cases



(a) Utilitarianism



(b) Intragenerational maximin

Figure A.8: Welfare, maximin and changes in sustainability for the limiting cases

A.3 Stability of the steady state

Let me sum up the necessary conditions into three main equations to get the following dynamic system, with $\pi \equiv \frac{\mu_N}{\mu_S}$;

$$\left\{\begin{array}{rcl} \dot{X}_N & = & F_N(X_N) - c_N \; ; \\ \dot{X}_S & = & F_S(X_S) - c_S \; ; \\ \dot{\pi} & = & \pi\left(F'_S(X_S) - F'_N(X_N)\right) \; . \end{array}\right. \tag{A.27}$$

Steady states are characterized by

$$\left\{\begin{array}{rcl} c^\star_N & = & F_N(X^\star_N) \; ; \\ c^\star_S & = & F_S(X^\star_S) \; ; \\ F'_N(X^\star_N) & = & F'_S(X^\star_S) \; . \end{array}\right. \tag{A.28}$$

Consider the Jacobian matrix of the linearized system, evaluated at the steady states[29]

$$J^\star = \begin{pmatrix} F'_N(X^\star_N) & 0 & -\frac{\partial c_N(\pi)}{\partial \pi} \\ 0 & F'_N(X^\star_N) & -\frac{\partial c_S(\pi)}{\partial \pi} \\ -\pi F''_N(X^\star_N) & \pi F''_S(X^\star_S) & 0 \end{pmatrix} \tag{A.29}$$

Let us compute the roots of the characteristic polynomial $\mathscr{P}(\lambda) = \det(J^\star - \lambda I_3)$:

$$\begin{aligned} & \begin{vmatrix} F'_N(X^\star_N) - \lambda & 0 & -\frac{\partial c_N(\pi)}{\partial \pi} \\ 0 & F'_N(X^\star_N) - \lambda & -\frac{\partial c_S(\pi)}{\partial \pi} \\ -\pi F''_N(X^\star_N) & \pi F''_S(X^\star_S) & -\lambda \end{vmatrix} = 0 \\ \Leftrightarrow \quad & \left(F'_N - \lambda\right) \begin{vmatrix} F'_N - \lambda & -\frac{\partial c_S(\pi)}{\partial \pi} \\ \pi F''_S & -\lambda \end{vmatrix} - \pi F''_N \begin{vmatrix} 0 & -\frac{\partial c_N(\pi)}{\partial \pi} \\ F'_N - \lambda & -\frac{\partial c_S(\pi)}{\partial \pi} \end{vmatrix} = 0 \\ \Leftrightarrow \quad & \left(F'_N - \lambda\right)\left(-\left(F'_N - \lambda\right)\lambda + \pi F''_S \frac{\partial c_S}{\partial \pi}\right) - \pi F''_N \left(F'_N - \lambda\right) \frac{\partial c_N}{\partial \pi} = 0 \\ \Leftrightarrow \quad & \left(F'_N - \lambda\right)\left(-\left(F'_N - \lambda\right)\lambda - \pi \frac{\partial c_N}{\partial \pi} F''_N + \pi \frac{\partial c_S}{\partial \pi} F''_S\right) = 0 \; . \end{aligned} \tag{A.30}$$

The first eigenvalue is $\lambda_N = F'_N$. Also, due to the strict convexity of indifference

[29] We use the equality $F'_N(X^\star_N) = F'_S(X^\star_S)$ and express the Jacobian with respect to $F'_N(X^\star_N)$ only.

curves (recall that at the optimum, $\pi = \frac{W_{c_i}}{W_{c_j}}$), let $\alpha_N \equiv -\frac{\partial c_N}{\partial \pi} > 0$ and $\alpha_S \equiv \frac{\partial c_S}{\partial \pi} > 0$. Let $\Gamma \equiv -\pi\left(\alpha_N F_N'' + \alpha_S F_S''\right) > 0$. We can reduce eq. (A.30) to $\lambda^2 - F_N'\lambda - \Gamma = 0$. Eigenvalues are then $\lambda_N = F_N' > 0$, $\lambda_S = \frac{F_N' - \sqrt{(F_N')^2 + 4\Gamma}}{2} < 0$, and $\lambda_3 = \frac{F_N' + \sqrt{(F_N')^2 + 4\Gamma}}{2} > 0$.[30] The steady state is a saddle-point.

A.4 Utilitarianism: Mathematical details and proofs

Consider the case $W(c_N, c_S) = a_N c_N + a_S c_S$. The Lagrangean associated with the maximin problem is linear in the decisions, which implies corner solutions for the controls.

$$\mathscr{L}(X, c, \mu, \tilde{W}^{GR}, \omega) = \mu_N\left(F_N(X_N) - c_N\right) + \mu_S\left(F_S(X_S) - c_S\right) + \omega\left(a_N c_N + a_S c_S - \tilde{W}^{GR}\right). \tag{A.31}$$

By Lemma 1, $c_N = c_S = 0$ cannot be solution of the problem. The necessary conditions (1.7) are, for $i = N, S$;

$$\frac{\partial \mathscr{L}}{\partial c_i} = -\mu_i + a_i \omega \leq 0\ ,\ c_i \geq 0\ ,\ c_i \frac{\partial \mathscr{L}}{\partial c_i} = 0\ . \tag{A.32}$$

The other conditions are unchanged. The complementary slackness conditions are

$$\omega \geq 0\ , \quad (a_N c_N + a_S c_S - u) \geq 0\ , \quad \omega\left(a_N c_N + a_S c_S - u\right) = 0\ .$$

From conditions (1.8), we get for $i = N, S$

$$-\frac{\partial \mathscr{L}}{\partial X_i} = -\mu_i F_i'(X_i) = \dot{\mu}_i, \quad \Leftrightarrow \quad \frac{\dot{\mu}_i}{\mu_i} = -F_i'(X_i)\ . \tag{A.33}$$

For a regular path, $(\mu_N, \mu_S, \omega) \neq (0,0,0)$, $\omega > 0$ and thus $W(c_N, c_S) = m(X_N, X_S)$. Suppose that both consumption levels are strictly positive, i.e., $c_i > 0, i = N, S$. Eq (A.32) then implies that $\frac{\partial \mathscr{L}}{\partial c_i} = 0$ and thus that $\omega = \frac{\mu_N}{a_N} = \frac{\mu_S}{a_S}$. Taking the time derivative of ω, one gets $\frac{\dot{\omega}}{\omega} = \frac{\dot{\mu}_N}{\mu_N} = \frac{\dot{\mu}_S}{\mu_S}$. Moreover, from eq. (A.33), one gets the conditions $\frac{\dot{\mu}_i}{\mu_i} = -F_i'(X_i)$, $i = N, S$. Combining these conditions, I can state that an internal solution, with $c_N > 0$ and $c_S > 0$ is possible only for states (X_N, X_S)

[30]Note that strict concavity of production functions rule out nil eigenvalues.

such that $F'_N(X_N) = F'_S(X_S)$. This is dynamically possible only for a steady state, with $c_N = F_N(X_N^\star)$ and $c_S = F_S(X_S^\star)$.

Apart from a steady state, the dynamics is as follows.

Lemma 3 (Utilitarianism: Transition path)**.** *In an economy with a nil inequality aversion between two regions producing each one a reproducible asset, for any state such that $F'_i(X_i) > F'_j(X_j)$, the constant consumption path with $c_j = \frac{m(X_N,X_S)}{a_j} > F_j(X_j)$ and $c_i = 0$ is an optimal maximin path. Stock X_j decreases while stock X_i increases.*

Proof of Lemma 3. Consider a state $(X_i, X_j) >> (0,0)$ such that $F'_i(X_i) > F'_j(X_j)$ and $F'_i(X_i) > 0$. This last condition ensures that the more productive resource is below its production peak X_i^{GR}.[31] I demonstrate that, under these conditions, stock X_j is consumed alone while stock X_i builds up as long as the previous inequality holds by proving that the opposite is not possible.

Assume that $c_i > 0$ and $c_j = 0$. Let us denote the maximin value by m. Along such a maximin path, one would have $W(c_N, c_S) = a_i c_i = m$, which is constant. Therefore, c_i would is. By Lemmata 1 and 2, $c_i > F_i(X_i)$. Therefore, $\frac{\mathrm{d}X_i}{\mathrm{d}t} = F_i(X_i) - c_i < 0$. Also, $\frac{\mathrm{d}^2 X_i}{(\mathrm{d}t)^2} = F'_i(X_i)\frac{\mathrm{d}X_i}{\mathrm{d}t} - \frac{\mathrm{d}c_i}{\mathrm{d}t} = F'_i(X_i)\frac{\mathrm{d}X_i}{\mathrm{d}t} < 0$. Therefore, stock X_i is exhausted in finite time (τ). After that time, utility is derived only from the sustained consumption of stock X_j. At time τ, stock X_j would have increased to some level $X_j^\star \equiv X_j(\tau)$ such that $F_j(X_j^\star) = m/a_j$, allowing consumption c_j to sustain exactly the maximin utility. The steady state is $(0, X_j^\star)$.

Let me make a step backward and examine the states through which the path goes just prior to exhaustion. The dynamics before exhaustion is

$$\dot{X}_i = F_i(X_i) - c_i \quad \Leftrightarrow \quad \mathrm{d}X_i = (F_i(X_i) - m/a_i)\,\mathrm{d}t\ ,$$
$$\dot{X}_j = F_j(X_j) \quad \Leftrightarrow \quad \mathrm{d}X_j = F_j(X_j)\mathrm{d}t\ .$$

Consider an infinitesimal time lapse $\mathrm{d}t$. At time $\tau - \mathrm{d}t$, stock X_i is equal to $\tilde{X}_i = \mathrm{d}X_i = \frac{m}{a_i}\mathrm{d}t$. Stock X_j is equal to $\tilde{X}_j = X_j^\star - \mathrm{d}X_j = X_j^\star - \frac{m}{a_j}\mathrm{d}t$.

[31] If both marginal products are negative, both stocks are above their golden-rule value X^{GR} and one is in a non-regular case in which the two stocks decrease and converge to a steady state at $\left(X_i^{GR}, X_j^{GR}\right)$. Lemma 3 is relevant only if one stock is below its golden-rule level.

By Lemma 1, we know that the maximin value at time $\tau - \mathrm{d}t$ is greater than or equal to the equilibrium utility of state $(\tilde{X}_i, \tilde{X}_j)$. Let us denote this utility level by $\tilde{W} = W\left(\tilde{X}_i, \tilde{X}_j\right) = a_i F_i(\frac{m}{a_i}\mathrm{d}t) + a_j F_j(X_j^\star - \frac{m}{a_j}\mathrm{d}t)$. We have $m(\tau - \mathrm{d}t) \geq \tilde{W}$. By subtracting $m(\tau)$ from both sides of the equation, we obtain the following.

$$\begin{aligned}
m(\tau - \mathrm{d}t) - m(\tau) &\geq \tilde{W} - m(\tau)\,; \\
&\geq a_i F_i\left(\frac{m}{a_i}\mathrm{d}t\right) + a_j F_j\left(X_j^\star - \frac{m}{a_j}\mathrm{d}t\right) - a_j F_j(X_j^\star)\,; \\
&\geq a_i\left(F_i\left(0 + \frac{m}{a_i}\mathrm{d}t\right) - F_i(0)\right) + a_j\left(F_j\left(X_j^\star - \frac{m}{a_j}\mathrm{d}t\right) - F_j(X_j^\star)\right)\,; \\
&\geq m\mathrm{d}t\frac{F_i\left(0 + \frac{m}{a_i}\mathrm{d}t\right) - F_i(0)}{\frac{m}{a_i}\mathrm{d}t} - m\mathrm{d}t\frac{F_j\left(X_j^\star - \frac{m}{a_j}\mathrm{d}t\right) - F_j(X_j^\star)}{-\frac{m}{a_j}\mathrm{d}t}\,.
\end{aligned}$$

Let us write $\varepsilon_i = \frac{m}{a_i}\mathrm{d}t$ and $\varepsilon_j = -\frac{m}{a_j}\mathrm{d}t$, as well as $\tilde{\tau} = \tau - \mathrm{d}t$ (and thus $\tau = \tilde{\tau} + \mathrm{d}t$). We get

$$\begin{aligned}
m(\tilde{\tau}) - m(\tilde{\tau} + \mathrm{d}t) &\geq m\mathrm{d}t\frac{F_i(0+\varepsilon_i) - F_i(0)}{\varepsilon_i} - m\mathrm{d}t\frac{F_j(X_j^\star + \varepsilon_j) - F_j(X_j^\star)}{\varepsilon_j}\,; \\
\Leftrightarrow \quad \frac{1}{m}\left(\frac{m(\tilde{\tau}+\mathrm{d}t) - m(\tilde{\tau})}{\mathrm{d}t}\right) &\leq \frac{F_j(X_j^\star + \varepsilon_j) - F_j(X_j^\star)}{\varepsilon_j} - \frac{F_i(0+\varepsilon_i) - F_i(0)}{\varepsilon_i}\,.
\end{aligned}$$

By taking the limits $\varepsilon_i, \varepsilon_j, \mathrm{d}t \to 0$, we obtain

$$\frac{\dot{m}}{m} \leq F_j' - F_i' < 0\,.$$

As the maximin value cannot decrease along a maximin path, we get a contradiction. We thus can say that if $F_i'(X_i) > F_j'(X_j)$, $c_i = 0$ and $c_j > 0$.

By regularity, $m = W(c_N, c_S) = a_N c_N + a_S c_S$. Thus, $c_j = \frac{m(X_N, X_S)}{a_j}$. □

Exhaustion of the less productive stock may be optimal. Let $F_i'(0) > F_j'(0)$, and denote by $\underline{X}_i > 0$ the stock level such that $F_i'(\underline{X}_i) = F_j'(0)$.

Lemma 4 (Intragenerational maximin: Optimal exhaustion)**.** *In an economy with an infinite inequality aversion between two regions producing each one a reproducible asset, a maximin path starting from any state (X_i, X_j) such that $F_i'(X_i) > F_j'(X_j)$ and $m(X_i, X_j) \leq a_i F_i(\underline{X}_i)$ exhausts asset X_j.*

Proof of Lemma 4. According to Lemma 3, if $F_i'(X_i) > F_j'(X_j)$, $c_i = 0$ and $c_j = \frac{m}{a_j}$. Assume that there is a steady state such that $W(\cdot) = m \leq a_i F_i(\underline{X}_i)$, then this steady state must satisfy $X_j^\star = 0$ and $X_i^\star \leq \underline{X}_i$. Otherwise, it would satisfy $F_i'(X_i) = F_j'(X_j)$ and we would have $W \geq a_i F_i(\underline{X}_i)$. Given that $X_i \leq \underline{X}_i$, we have $F_i'(X_i) > F_j'(X_j)$ and we exhaust X_j. □

From the necessary conditions, one has $\omega = \frac{\mu_j}{a_j}$ and $\frac{\dot{\omega}}{\omega} = \frac{\dot{\mu}_j}{\mu_j} = -F_j'(X_j)$. One also has $-\mu_i + a_i\omega \leq 0 \Leftrightarrow \mu_i \geq \frac{a_i}{a_j}\mu_j \Leftrightarrow \frac{\mu_i}{\mu_j} \geq \frac{a_i}{a_j}$. Given that $\mu_i = m_{X_i}$ and $a_i = W_{c_i}$, $i = N, S$, one gets that the marginal rate of transformation of maximin value is greater than the marginal rate of substitution of consumption.

Let me examine how this relative price evolves over time, recall $\pi \equiv \frac{\mu_i}{\mu_j}$. Given conditions (A.33), I obtain

$$\frac{\dot{\pi}}{\pi} = \frac{\dot{\mu}_i}{\mu_i} - \frac{\dot{\mu}_j}{\mu_j} = -F_i'(X_i) + F_j'(X_j) < 0 \,. \tag{A.34}$$

The relative price of stock j rises. Since one harvests only the stock j, $F_j'(X_j)$ increases and $F_i'(X_i)$ decreases until the marginal levels of growth are equal.[32] In other words, if one begins with a resource relatively abundant and with a low relative (shadow) price, one has to harvest only that resource at a level that keeps utility constant. Meanwhile the other one grows until the steady state at which both resources have the same marginal product.

Lemma 5 (Utilitarianism: Iso-value curves)**.** *The iso-value curves are convex to the origin in the plane (X_N, X_S), with $-\frac{\mathrm{d}X_S}{\mathrm{d}X_N} = \frac{\mu_N}{\mu_S} \geq \frac{a_N}{a_S}$.*

Proof of Lemma 5. From condition (1.26), the optimal path is characterized by

$$-\frac{\mathrm{d}X_S}{\mathrm{d}X_N} = \frac{\mu_N}{\mu_S} \,, \tag{A.35}$$

but contrary to the general case, here the SWF and the maximin value do not have the same shape. □

[32] One cannot pass from $\frac{\mu_i}{\mu_j} > \frac{a_i}{a_j}$ to $\frac{\mu_i}{\mu_j} < \frac{a_i}{a_j}$ since the equality is a steady state of the system.

A.5 An example: Utilitarianism with AK-technology and a renewable resource

I provide an example which illustrates the results of the utilitarianism case, with an explicit solution for the maximin value and shadow-values in a simple economic model with a renewable natural resource with logistic growth ($F_N(X_N) = rX_N(1-X_N)$) and an AK technology ($F_S(X_S) = AX_S$). The respective dynamics are $\dot{X}_N = rX_N(1-X_N) - c_N$ and $\dot{X}_S = AX_S - c_S$, with $A, r > 0$. Welfare is $W(c_N, c_S) = a_N c_N + a_S c_S$.

I assume that $r > A$, so that low resource stocks are more productive than the manufactured capital stock, and the natural resource is not exhausted. I give the maximin value for states (X_N, X_S) satisfying $F_N'(X_N) \geq F_S'(X_S)$, i.e., for $X_N(0) \leq \underline{X}_N \equiv \frac{1}{2}\left(1-\frac{A}{r}\right) > 0$.

Along the maximin path, the capital stock X_S is consumed to let the natural resource X_N grow (recovery). For paths that do not exhaust the manufactured capital stock, i.e, trajectories reaching a steady state with $X_S^\star > 0$ and $X_N^\star = \underline{X}_N$, the maximin value function is given by $m(X_N, X_S) = a_N \Omega \left(\frac{1}{X_N}-1\right)^{-\frac{A}{r}} + a_S A X_S$, with $\Omega = \frac{r}{4}\left(1+\frac{A}{r}\right)^{1+\frac{A}{r}}\left(1-\frac{A}{r}\right)^{1-\frac{A}{r}}$. Associated shadow-prices are $\mu_N = a_N \frac{A}{r}\Omega(1-X_N)^{-1-\frac{A}{r}} X_N^{-1+\frac{A}{r}}$ and $\mu_S = a_S A$.

For states (X_N, X_S) such that $m(X_N, X_S) \leq a_N F_N(\underline{X}_N)$, maximin paths exhaust the manufactured capital stock and the maximin value function is implicitly given by $m = \frac{a_N r\left(1-\frac{a_S A X_S}{m}\right)^{\frac{r}{A}}\left(\frac{1}{X_N}-1\right)}{\left(1+\left(1-\frac{a_S A X_S}{m}\right)^{\frac{r}{A}}\left(\frac{1}{X_N}-1\right)\right)^2}$.

The mathematical details are as follows.

I have a linear social welfare function $W(c_N, c_S) = a_N c_N + a_S c_S$, a renewable resource with the renewal $F_N(X_N) = rX_N(1-X_N)$), and a manufactured capital with the production function $F_S(X_S) = AX_S$. The steady state condition for a non-exhaustion, $F_i' = F_j'$, is $\underline{X}_N = \frac{1}{2}\left(1-\frac{A}{r}\right)$. I note $X_i(0) = X_{i0}, i = N, S$.

I have two types of paths, according to the steady states characterized by:

1. $X_S^\star > 0, X_N^\star = \underline{X}_N$;
2. $X_S^\star = 0, X_N^\star \leq \underline{X}_N$ (manufactured capital is exhausted).

A.5.1 Case 1

Let m be the maximin value. I know that in such a case; $c_N = 0$ and $c_S = \frac{m}{a_S}$, i.e. $\dot{X}_N = rX_N(1-X_N)$ and $\dot{X}_S = AX_S - \frac{m}{a_S}$. It may be checked that the solution of theses equations is of the form $X_N(t) = \frac{X_{10}}{X_{10}+\mathrm{e}^{-rt}(1-X_{10})}$ and $X_S(t) = \left(X_{20} - \frac{m}{a_S A}\right)\mathrm{e}^{At} + \frac{m}{a_S A}$.

From X_N, one may obtain the inverse function $t^\star = -\frac{1}{r}\ln\left(\frac{1-X_N}{X_N}\frac{X_{10}}{1-X_{10}}\right)$ and compute it at the steady state: $t^\star = -\frac{1}{r}\ln\left(\frac{1+\frac{A}{r}}{\left(1-\frac{A}{r}\right)\left(\frac{1}{X_{10}}-1\right)}\right)$, which may serve to compute $X_S^\star = X_S(t^\star)$ as

$$X_S^\star = \left(X_{20} - \frac{m}{a_S A}\right)\left(\frac{1+\frac{A}{r}}{\left(1-\frac{A}{r}\right)\left(\frac{1}{X_{10}}-1\right)}\right)^{-\frac{A}{r}} + \frac{m}{a_S A}\,. \tag{A.36}$$

Besides, at the steady state, one has $c_N^\star = F_N(X_N^\star) = r\frac{1}{2}\left(1-\frac{A}{r}\right)\left(1-\frac{1}{2}\left(1-\frac{A}{r}\right)\right) = \frac{r}{4}\left(1-\frac{A^2}{r^2}\right)$. Then, one may compute the maximin value at the steady state

$$m^\star = a_N c_N^\star + a_S c_S^\star = a_N \frac{r}{4}\left(1-\frac{A^2}{r^2}\right) + a_S A X_S^\star\,. \tag{A.37}$$

I substitute (A.36) into (A.37) to get the maximin value function

$$m(X_{10},X_{20}) = a_N\Omega\left(\frac{1}{X_{10}}-1\right)^{-\frac{A}{r}} + a_S A X_{20}\,,\ \text{with } \Omega = \frac{r}{4}\left(1+\frac{A}{r}\right)^{1+\frac{A}{r}}\left(1-\frac{A}{r}\right)^{1-\frac{A}{r}}\,.$$

Knowing that shadow-prices are partial derivatives of the maximin value function, I have

$$\mu_N = \frac{\partial m(X_{10},X_{20})}{\partial X_{10}} = a_N\frac{A}{r}\Omega\,(1-X_{10})^{-1-\frac{A}{r}}\,X_{10}^{-1+\frac{A}{r}}\,;$$
$$\mu_S = \frac{\partial m(X_{10},X_{20})}{\partial X_{20}} = a_S A\,.$$

A.5.2 Case 2

I want to find paths that end up with $X_S^\star = 0$.

$$X_S^\star = 0 \quad \Leftrightarrow \quad \left(X_{20} - \frac{m}{a_S A} \right) \mathrm{e}^{At^\star} + \frac{m}{a_S A} = 0 \quad \Rightarrow \quad t^\star = \frac{1}{A} \ln \left(\frac{m}{m - a_S A X_{20}} \right) .$$

The corresponding resource stock is

$$X_N^\star = \frac{X_{10}}{X_{10} + \mathrm{e}^{-\frac{r}{A} \ln\left(\frac{m}{m - a_S A X_{20}}\right)} (1 - X_{10})} \quad \Leftrightarrow \quad X_N^\star = \frac{1}{1 + \left(1 - \frac{a_S A X_{20}}{m}\right)^{\frac{r}{A}} \left(\frac{1}{X_{10}} - 1\right)} .$$

Besides, I have $m = a_N F_N(X_N^\star)$, which becomes

$$m = a_N r \frac{1}{1 + \left(1 - \frac{a_S A X_{20}}{m}\right)^{\frac{r}{A}} \left(\frac{1}{X_{10}} - 1\right)} \left(1 - \frac{1}{1 + \left(1 - \frac{a_S A X_{20}}{m}\right)^{\frac{r}{A}} \left(\frac{1}{X_{10}} - 1\right)} \right) ;$$

$$\Leftrightarrow \quad m = \frac{a_N r \left(1 - \frac{a_S A X_{20}}{m}\right)^{\frac{r}{A}} \left(\frac{1}{X_{10}} - 1\right)}{\left(1 + \left(1 - \frac{a_S A X_{20}}{m}\right)^{\frac{r}{A}} \left(\frac{1}{X_{10}} - 1\right)\right)^2} .$$

Even in this simple framework, if the manufactured capital is exhausted, maximin value function and associated shadow-prices are hard to find.

Fig. A.6 in the Appendix plots a non-exhaustion case (1) and an exhaustion one (2).[33]

[33] The dashed lines represent consumption paths, denoted by exponents $^{(1)}$ and $^{(2)}$.

Chapter 2

Distributional considerations during growth toward the golden rule

Abstract

In a productive economy, savings made by a generation are expected to benefit more than proportionally to future generations. But how such a sacrifice for growth should be shared among heterogeneous regions? The unequal 'burden-sharing' between North and South depicted in the previous chapter is less significant, but still present, when the economy grows toward the highest sustainable level of welfare. Here, contrary to the previous chapter, the intragenerational inequality aversion does not impact the final steady state. But the intra and intergenerational inequality aversions do impact the dynamic. In particular, the higher the inequality aversions, the more North and South share an equal burden to grow. This chapter leads to the same conclusion than the previous one: transfers from North to South shall be implemented for a sustainable development. This will be addressed in the last chapter.

2.1 Introduction

Growth rather than decline is surely an unanimous criterion of economic development. At least as long as this process is sustainable. But for making a production (then implicitly an asset) to grow, we shall not consume our entire income, i.e. "*the amount which [people] can consume without impoverishing themselves*" (Hicks, 1939/1946, p. 172).[1] For a natural resource, we shall not harvest the entire renewal. In a word, we need savings. In an intergenerational perspective, as conceived by Ramsey (1928), this can be viewed as a sacrifice of the current generation to the benefit of future ones. But how should this sacrifice be shared among individuals living in the same generation? Growth theory already answered this question for an aggregate capital and a representative agent through the optimal savings timing. But the question of sharing sacrifice comes to interest once there is an heterogeneity. As argued by Schelling (1995): "*it is this willingness to model all humankind as a single agent that makes optimization models attractive, feasible, and inappropriate*".

The one-sector Solow-Swan (1956) neoclassical growth model was extended to the two-sector case by Uzawa (1961, 1963), but the savings rate is considered as exogenous. The Ramsey model of optimal saving was extended to the multiple goods-sectors case by Samuelson and Solow (1956). But their approach was not very much followed in the literature. A notable exception that does not collapse to the single sector case was provided by Pitchford (1977). But the two sectors, each in one region of the economy, produce one single good, which does not allow to consider inequality in consumptions. Rather, two-sector economy was mainly analyzed through the discounted version of the Ramsey model (Ramsey, 1928; Cass, 1965; Koopmans, 1965) (RCK hereafter). This was the case to deal with, for instance, physical capital and exhaustible resources (Dasgupta and Heal, 1974) or physical and human capital (Lucas, 1988).[2] On the

[1] Hicksian income refers more precisely to "*the maximum value which [a man] can consume during a week, and still expect to be as well off at the end of the week as he was at the beginning. Thus, when a person saves, he plans to be better off in the future; when he lives beyond his income, he plans to be worse off*" (p. 172). This idea can be tracked back to Fisher (1906) and Lindahl (1933) (cited in Heal, 1998, p. 156).

[2] In a sense, a two-sector RCK model was earlier proposed by Srinivasan (1964) and Uzawa (1964), but in an non-utilitarian approach.

other hand, heterogeneity of agents was analyzed both in the Solow-Swan model (Stiglitz, 1969) and in the RCK model, through different individual discount rates (Ramsey, 1928; Becker, 1980; Becker and Foias, 1987) or through different initial endowments of wealth (Chatterjee, 1994; Caselli and Ventura, 2000)[3]. Heterogeneity of both sectors and agents in the RCK model was proposed by Becker and Tsyganov (2002), but there are one capital good and one production good that can be aggregated into one single production. Endress et al. (2014) combined intergenerational equity with individual impatience in an overlapping generations model, but without referring to inequality aversion.

More broadly, both intra and intergenerational considerations of sharing resources can be found in the growth-inequality literature. Even if trends of long-term economic development and allocation of wealth was analyzed by the classical economists in the Nineteenth century, it was first extensively analyzed with use of data by Kuznets (Kuznets and Jenks, 1953; Kuznets, 1955). According to him, inequalities rise, then decrease, in the process of development. This topic was further analyzed by Atkinson and Harrison (1978), and current works refute the Kuznets hypothesis (e.g. Piketty, 2015). For a review of the growth-inequality relationship see Aghion et al. (1999) and Garcìa-Peñalosa (2017).

Though linked, the question here is different in the sense that I am interested in linking ethical judgments with outcomes of the allocation, both between individuals living at the same time and between different generation living, by definition, at different dates. This is the so-called intra and intergenerational equity relationship (Isaac and Piacquadio, 2015, Kverndokk et al., 2014, and the references in there). Equity can be taken as a requirement of equal treatment. Here, all individuals (and all generations between them) will have the same weight in the public decision[4].

On the intergenerational dimension, the intertemporal social welfare criterion has to satisfy finite anonymity: a finite number of permutations in the time of appearance of two generations shall not change the result. This is supported by a

[3]See also Garcìa-Peñalosa and Turnovsky (2012) and the references in there.

[4]There is not a unique accepted definition of the word ‘equity’. Our requirement here is in line with a dictionary definition: “equity is the quality of being fair and reasonable in a way that gives equal treatment to everyone.” (source: `https://www.collinsdictionary.com/dictionary/english/equity`, visited on May 2018).

long tradition in economics and philosophy against discounting future welfares. For Sidgwick (1874/1962, p. 414) “*the time at which a man exists cannot affect the value of his happiness from a universal point of view*”. Ramsey (1928, p. 543) qualified discounting as “*a practice which is ethically indefensible and arises merely from the weakness of the imagination*”. Pigou (1920, p. 25) argued that “*our telescopic faculty is defective*”, and suggested that “*the State should protect the interests of the future* in some degree *against the effects of our irrational discounting and of our preference for ourselves over our descendants* (Pigou, 1932, p. 29). For Harrod (1948, p. 40), the pure time preference is a “*polite expression for rapacity and the conquest of reason by passion*”. For Rawls (1971, p. 287), “*from a moral point of view there are no grounds for discounting future well-being on the basis of pure time preference*”. Finally, according to Solow (1993, p. 165) “*no generation ‘should’ be favored over any other*”. But as shown by Koopmans (1960), if a (social) criterion of infinite utility stream satisfies some[5] *a priori* desirable axioms, it has to exhibit ‘impatience’, i.e. discounting. Following his approach, Diamond (1965) stated a classic impossibility result: such a criterion cannot both be efficient (strong Pareto) and treat all generations equally. For a critical survey of related results see Asheim (2010). The Ramsey (1928) approach satisfies both ‘efficiency’ and ‘equity’ but is incomplete.

The intragenerational equity can be understood in several ways, see for example Fleurbaey and Maniquet (2011). But, far from reaching exhaustiveness, a *symmetric* social welfare function *à la* Bergson (1938)-Samuelson (1947) can represent different equity views of a society.

Here I study savings – understood as the difference between the sustainable individual maximal consumption and the actual consumption – made by different individuals when society wishes to reach the highest sustainable social well-being: the golden rule (Ayong Le Kama, 2001). This may be interpreted as ‘generosity’ toward future possible attainable welfare (Gerlagh, 2017). I model two regions, North and South, each one has access to a different resource stock, in the spirit of Samuelson and Solow (1956). The intratemporal social preferences are represented through a social welfare function (SWF) with a constant elasticity of substitution (CES). The parameter of elasticity may be interpreted

[5] See the introduction of the dissertation.

as a parameter of inequality aversion (Atkinson, 1970). Indeed, such a function encompasses two famous special cases: utilitarianism (elasticity goes to infinity) and the symmetric minimum, sometimes called "Rawlsian" (elasticity goes to zero). Symmetrically, the intertemporal social preferences are represented through an intertemporal SWF with a constant intertemporal elasticity of substitution (CIES). More precisely, I minimize the difference between the welfare targeted and the actual welfare, in the spirit of Ramsey (1928). Here also, the parameter of intertemporal elasticity may be interpreted as a parameter of intertemporal inequality aversion (IA). As in its instantaneous counterpart, such a function can tend to a nil IA case (elasticity goes to infinity) or to an infinite IA case (elasticity goes to zero). Interestingly, this latter case corresponds to the maximin (d'Autume and Schubert, 2008a), which has resonance with assessing sustainability (Cairns and Martinet, 2014; Fleurbaey, 2015a).

I show that when society has to make an intergenerational sacrifice, three elements guide its sharing. The marginal productivity gap, the intragenerational inequality aversion and the intergenerational inequality aversion. The former indicates that the region having access to a relatively more productive asset (North) may afford a relative higher consumption. Paradoxically, the region having access to the relatively less productive asset (South) has to make a higher sacrifice. This comes from the fact that the South has a higher marginal productivity. The impact of the productivity gap on the consumption growth rates of the region is weighted by the two inequality aversions (IAs). The role played by the IAs can be ambiguous according to their level. But generally, the more society is willing to substitute welfare of each region, the more one can take advantage of their differences, then the more is the difference in terms of sacrifice. At the opposite, when society exhibits an infinite intragenerational inequality aversion, growth rates of North and South are equal, whatever the productivity gap. And if society exhibits an infinite intergenerational inequality aversion, growth rates of North and South are equal (at the limit) to zero, since no sacrifice for the future is tolerated.

The next Section introduces the model: first in isolation, regions are collectively considered afterward. Section 2.3 exhibits links with the green accounting literature. I allow for an unequal treatment of regions and generations in Section 2.4. I discuss the relevance of discounting as a tool to prevent huge sacrifices in Section 2.5. Section 2.6 concludes.

2.2 Maximizing intergenerational welfare

I study an economy composed of two regions – North and South – with an infinite number of generations, each one consuming c_i of the asset $i = N,S$. I first study how each region in isolation can grow efficiently toward their highest sustainable level of utility, their golden rule (GR) u_i^{GR}. Then I consider a global social planner that derives an instantaneous social welfare function (SWF) from the utilities of the regions. And I study how this global economy can grow efficiently toward the global golden rule W^{GR}.

2.2.1 Framework

Each region is endowed with a stock of renewable resource X_i. The natural renewals F_i is assumed to be increasing, strictly concave and such that $F_i(0) = 0 = F_i(\bar{X}_i)$, where $\bar{X}_i$ stands for the carrying capacity.[6] They reach a maximum at the golden-rule stock X_i^{GR}. The dynamics of the stock is then given by

$$\frac{\mathrm{d}X_i(t)}{\mathrm{d}t} \equiv \dot{X}_i(t) = F_i(X_i(t)) - c_i(t) , \quad i = N,S . \tag{2.1}$$

Two approaches are possible to handle the welfare part. (1) Considering a constant elasticity of substitution (CES) SWF of consumptions (or utilities) from North and from South. And taking an increasing transformation of such a SWF to obtain a constant *intertemporal* elasticity of substitutions (CIES) SWF. (2) Considering a CIES utility function in each region with the same elasticity parameter and representing them through a CES SWF. I represent them formally below.

Let θ be the intratemporal elasticity of substitution, with $\theta > 0$, $\theta \neq 1$. And let σ be the intertemporal elasticity of substitution, $\sigma > 0$, $\sigma \neq 1$. For the sake of

[6] I recognize that the dynamics of some resources cannot be represented by such functions. This is a stylized feature.

simplicity, let us introduce the parameters $\eta \equiv \frac{\theta-1}{\theta}$ and $\nu \equiv \frac{\sigma-1}{\sigma}$, with $\eta, \nu < 1$ and $\eta, \nu \neq 0$. Those elasticities (θ and σ) can be interpreted as parameters of inequality aversion (Atkinson, 1970): from 0 (infinite inequality aversion) to infinity (nil inequality aversion). Let me also put regional weights $a_N, a_S > 0$ such that $a_N + a_S = 1$.[7] But for now consider only the case $a_N = a_S = \frac{1}{2}$. In mathematical terms, the two versions of the SWF are[8]

$$W^{(1)}(c_N, c_S) = \frac{\left(\left(a_N \cdot c_N^{\eta} + a_S \cdot c_S^{\eta}\right)^{\frac{1}{\eta}}\right)^{\nu}}{\nu}; \tag{2.2}$$

$$W^{(2)}(c_N, c_S) = \left(a_N \left(\frac{c_N^{\nu}}{\nu}\right)^{\eta} + a_S \left(\frac{c_S^{\nu}}{\nu}\right)^{\eta}\right)^{\frac{1}{\eta}}. \tag{2.3}$$

The first version allows to disentangle intertemporal from intratemporal distributional issues. Indeed the intertemporal substitution applies on each instantaneous aggregated SWF.[9] On the opposite, the second version allows to compare intertemporal decisions for each region and for the whole economy. With $\eta \to 1$ ($\theta \to \infty$) the global problem tends to be the sum of each regional ones. For this reason, I will follow the second approach (actually, it can easily be shown that the two approaches are formally equivalent[10]). Regional utilities are strictly concave but identical: $u_i(c_i) = \frac{c_i^{\nu}}{\nu}$. Here again, this captures the idea of responsibility for turning consumption into utility.

Let me now solve the intertemporal problem for our two regions considered separately. Afterward they will be considered collectively to allow for comparisons.

[7]The condition $a_N + a_S = 1$ (resp. the -1 in its intertemporal counterpart) is there only for having the Cobb-Douglass (resp. the logarithm) as a special case in the limit.

[8]To simply calculus, I omit the -1 usually put in CIES functions.

[9]See d'Autume and Schubert (2008a) and d'Autume et al. (2010) for models adopting this approach.

[10]Consider the following variable change: $\eta' = \nu\eta$. It comes $W^{(2)}(c_N, c_S) = \left(a_N \left(\frac{c_N^{\nu}}{\nu}\right)^{\frac{\eta'}{\nu}} + a_S \left(\frac{c_S^{\nu}}{\nu}\right)^{\frac{\eta'}{\nu}}\right)^{\frac{\nu}{\eta'}} = \frac{\left(a_N \cdot c_N^{\eta'} + a_S \cdot c_S^{\eta'}\right)^{\frac{\nu}{\eta'}}}{\nu}$.

2.2.2 Two regions considered separately

At the beginning, the regions are endowed with the stocks $X_i(0) = X_{i0}$. It is useful to compute the maximin consumption for each region in isolation. One directly obtains it by

$$c_i^m = \begin{cases} F_i(X_{i0}) & \text{if} \quad X_{i0} \le X_i^{GR} \,; \\ F_i\left(X_{i0}^{GR}\right) & \text{if} \quad X_{i0} > X_i^{GR} \,. \end{cases} \tag{2.4}$$

If a region is endowed with a low stock (lower than the golden-rule stock), it can at best consume its regional production. Growth toward a higher consumption can come only at the price of a sacrifice, $c_i < F_i(X_i)$, over any period of time. On the contrary, if a region is endowed with an abundant stock (higher than the golden-rule stock), the GR $u_i^{GR} = u_i\left(F_i\left(X_i^{GR}\right)\right)$, is attainable at no cost. Indeed a path with dissavings, $c_i > F_i(X_i)$, is possible as long as the stock, when decreasing, does not overshoot the golden-rule stock. I am particularly interested here in the first scenario because reaching the GR calls for sharing sacrifice between generations. To do so, let us maximize the welfare of all future generations. To get a solvable problem, I resort to the classic Ramsey's (1928) device. Recall $u_i(c_i) = \frac{c_i^\nu}{\nu}$. The regional intertemporal welfares are given by[11]

$$v(X_{i0}) = \max_{c_i} \quad \int_0^\infty \left(u_i(c_i) - u_i^{GR}\right) \mathrm{d}t \,; \tag{2.5}$$

$$\text{subject to} \quad \dot{X}_i = F_i(X_i) - c_i \,; \tag{2.6}$$

$$X_{i0} \text{ given} \,. \tag{2.7}$$

I assume the utilities evolve fast enough such that the integrals are finite (discussion on convergence may be found in Chakravarty, 1962, and Chiang, 1992, pp. 99-101). The Hamiltonians are

$$\mathscr{H}_i(X_i, c_i, \psi_i) = u_i(c_i) - u_i^{GR} + \psi_i(F_i(X_i) - c_i) \,. \tag{2.8}$$

[11] As a by-product of this analysis, the 'abundance' case – when the initial stock is higher than the golden-rule stock – is also handled: the objective maximize the current utility, net of its lower limit.

The necessary conditions are

$$\frac{\partial \mathscr{H}_i(X_i, c_i, \psi_i)}{\partial c_i} = 0 \quad \Leftrightarrow \quad \psi_i = c_i^{\nu-1} \; ; \tag{2.9}$$

$$-\frac{\partial \mathscr{H}_i(X_i, c_i, \psi_i)}{\partial X_i} = \dot{\psi}_i \quad \Leftrightarrow \quad -\frac{\dot{\psi}_i}{\psi_i} = F_i'(X_i) \; ; \tag{2.10}$$

$$\lim_{t \to \infty} \mathscr{H}_i(X_i, c_i, \psi_i) = 0 \; . \tag{2.11}$$

As usual, the shadow-price of a stock equals its marginal value in terms of utility (eq. (2.9)). Here, as there is no time preference, it always pays to save ($\dot{\psi}_i < 0 \Leftrightarrow \dot{c}_i > 0$) as long as the marginal return on capital is positive (eq. (2.10)). From the equations (2.9) and (2.10), one obtains

$$(1-\nu)\frac{\dot{c}_i}{c_i} = F_i'(X_i) \quad \Leftrightarrow \quad \frac{\dot{c}_i}{c_i} = \sigma F_i'(X_i) \; . \tag{2.12}$$

This is the so-called ‘Keynes-Ramsey rule’ without discount factor. The consumption growth rates depend positively on the marginal productivity, since it pays to save for growth, and positively on the intertemporal elasticity of substitution (IES), since a higher value of this parameter represents a higher willingness to substitute current for future welfares. Society saves more today in order to have a higher utility in the future. At the limit, if the parameter tends to infinity, the sacrifice at the benefit of future generations is maximal. At the other limit, the growth rate becomes constant, i.e. the sacrifice tends to be nil, when the IES approaches zero. No substitutions between current and future welfares are tolerated and one approaches the individual maximin consumptions c_i^m.[12]

As a preliminary for comparison with the collective problem, let me simply remark that the eq. (2.12) hold for North and South. One can therefore write

$$\frac{\dot{c}_S}{c_S} - \frac{\dot{c}_N}{c_N} = \sigma \left(F_S'(X_S) - F_N'(X_N) \right) \; . \tag{2.13}$$

The difference in regional growth rates equals the (marginal) productivity gap weighted by the IES.

[12] For a proof of a CIES approaching the maximin at a zero IES, see d’Autume and Schubert (2008a). See also d’Autume et al. (2010) for the undiscounted case.

With the transversality condition, the unique individual steady states are characterized by (the proof of the stability is omitted since this case is very classic)

$$c_i^\star = F_i\left(X_i^{GR}\right)\ ,\ F_S'\left(X_S^{GR}\right) = 0 = F_N'\left(X_N^{GR}\right)\ ,\ u_i(c_i^\star) = u_i^{GR}\ ,\ i = N,S\ . \quad (2.14)$$

Integrating both sides of the equation (2.13) and using the final consumptions, it comes[13]

$$\frac{c_N(t)}{c_S(t)} = \frac{c_N^{GR}}{c_S^{GR}} \mathrm{e}^{\sigma \int_t^\infty \left(F_S'(X_{Sr}) - F_N'(X_{Nr})\right)\mathrm{d}r}\ . \quad (2.15)$$

I will come back to this equation for the collective problem.

2.2.3 Two regions considered collectively

I now consider the problem of a 'world' social planner. At the beginning, the economy is still endowed with stocks (X_{N0}, X_{S0}). The maximin solution of this problem was given in the Chapter 1. A useful reference path is the *as-good-as-golden locus* (Phelps and Riley, 1978) obtained with the maximin framework: all stocks from which the welfare level is exactly that of the GR. It generalizes the GR concept to several dimensions. As in the previous subsection, I will mainly be interested in initial stocks lower than such a locus (this corresponds to the *regular part* of Chapter 1). The GR is now given by $W^{GR} = W\left(c_N^{GR}, c_S^{GR}\right)$, with $c_i^{GR} = F_i\left(X_i^{GR}\right)$. Recall $W(c_N, c_S) = \left(a_N \left(\frac{c_N^\nu}{\nu}\right)^\eta + a_S\left(\frac{c_S^\nu}{\nu}\right)^\eta\right)^{\frac{1}{\eta}}$. Let us compute

[13] $\int_t^\infty \left(\frac{\dot c_S}{c_S} - \frac{\dot c_N}{c_N}\right)\mathrm{d}r = \int_t^\infty \sigma\left(F_S'(X_{Sr}) - F_N'(X_{Nr})\right)\mathrm{d}r + a \Leftrightarrow \left[-\ln\left(\frac{c_N}{c_S}\right)\right]_t^\infty = \sigma\int_t^\infty \left(F_S'(X_{Sr}) - F_N'(X_{Nr})\right)\mathrm{d}r + a \Leftrightarrow \ln\left(\frac{c_N(t)}{c_S(t)}\right) = \ln\left(\frac{c_N^{GR}}{c_S^{GR}}\right) + \sigma\int_t^\infty \left(F_S'(X_{Sr}) - F_N'(X_{Nr})\right)\mathrm{d}r$.

the global intertemporal welfare in the same manner than previously[14]

$$V(X_{N0},X_{S0}) = \max_{c_N,c_S} \int_0^\infty \left(W(c_N,c_S) - W^{GR}\right) \mathrm{d}t \; ; \tag{2.16}$$

$$\text{subject to} \quad \dot{X}_N = F_N(X_N) - c_N \; ; \tag{2.17}$$

$$\dot{X}_S = F_S(X_S) - c_S \; ; \tag{2.18}$$

$$(X_{N0},X_{S0}) \text{ given} \; . \tag{2.19}$$

Here also, I assume that the welfare evolves fast enough such that the integral is finite. The Hamiltonian is

$$\mathscr{H}(X_i,c_i,\psi_i) = W(c_N,c_S) - W^{GR} + \psi_N(F_N(X_N) - c_N) + \psi_S(F_S(X_S) - c_S) \; . \tag{2.20}$$

The necessary conditions are

$$\frac{\partial \mathscr{H}(X_i,c_i,\psi_i)}{\partial c_i} = 0 \quad \Leftrightarrow \quad \psi_i = a_i W^{1-\eta} \cdot \frac{c_i^{\eta\nu-1}}{\nu^{\eta-1}} \; ; \tag{2.21}$$

$$-\frac{\partial \mathscr{H}(X_i,c_i,\psi_i)}{\partial X_i} = \dot{\psi}_i \quad \Leftrightarrow \quad -\frac{\dot{\psi}_i}{\psi_i} = F_i'(X_i) \; ; \tag{2.22}$$

$$\lim_{t\to\infty} \mathscr{H}(X_i,c_i,\psi_i) = 0 \; . \tag{2.23}$$

From the equations (2.21) and (2.22), it comes

$$-(1-\eta)\frac{\dot{W}}{W} + (1-\eta\nu)\frac{\dot{c}_i}{c_i} = F_i'(X_i) \quad \Leftrightarrow \quad \frac{\dot{c}_i}{c_i} = \frac{\theta\sigma}{\theta+\sigma-1}\left(F_i'(X_i) + \frac{1}{\theta}\frac{\dot{W}}{W}\right) \; . \tag{2.24}$$

The (apparent) consumption growth rates still depend on marginal productivity, but also on elasticities of substitutions in a subtle way. Let me first study the consumption growth rates when each elasticity reaches its limits. There are four cases.[15] Note $\theta \equiv \frac{1}{1-\eta}$ and $\sigma \equiv \frac{1}{1-\nu}$.

L1. Intragenerational utilitarianism: $\theta \to \infty$, $\frac{\dot{c}_i}{c_i} \to \sigma F_i'(X_i)$.

L2. Intragenerational maximin: $\theta \to 0$, $\frac{\dot{c}_i}{c_i} \to \frac{\sigma}{\sigma-1}\frac{\dot{W}}{W}$.

[14] Here also, as a by-product of this analysis, the ‘abundance’ case is also handled.

[15] The names of theses four cases are a misuse of language since the general case approaches the one mentioned.

L3. Intergenerational utilitarianism: $\sigma \to \infty$, $\frac{\dot{c}_i}{c_i} \to \theta F_i'(X_i) + \frac{\dot{W}}{W}$.

L4. Intergenerational maximin:[16] $\sigma \to 0$, $\frac{\dot{c}_i}{c_i} \to 0$.

When the intragenerational IA is nil (case L1), the collective problem corresponds to the problem of two separated regions (see the eq. (2.12)). And only the intergenerational IA matters. In the same way, when the intergeneration IA is nil (L3), only the intragenerational IA matters. Interestingly, when the IA becomes infinite in one dimension (L2 or L4), the consumption growth rates do not longer depend on productivities, and are therefore equal. Intuitively, as no ‘substitution of well-being’ is tolerated with the maximin, there is no growth rates inequality. In the intragenerational maximin (L2), growth rates depend equally on the intergenerational dimension. When the intergenerational IA is infinite (L4), growth rates approach zero, and the sacrifice is very low. For the opposite reason, when IA become nil in the two dimensions ($\theta, \sigma \to \infty$), growth rates becomes infinite and then ‘close’. I now turn the study of the difference in growth rates.

Let us equalize the common term in the eq. (2.24), for $i = N, S$, to obtain

$$\frac{\dot{c}_S}{c_S} - \frac{\dot{c}_N}{c_N} = \kappa \left(F_S'(X_S) - F_N'(X_N) \right) \ , \quad \text{with } \kappa \equiv \frac{\theta\sigma}{\theta + \sigma - 1} \ . \tag{2.25}$$

Relatives growth of individual consumptions depend on the difference of marginal productivities, weighted by the intra and the intergenerational inequality aversions. Without loss of generality, I will only consider the case of a marginally more productive stock in the South, $F_S'(X_S) \geq F_N'(X_N)$, to capture the idea of a relative less abundant stock.

It may firstly be noted that the two elasticities have a different impact on the regional consumption paths (eq. (2.24)), but exactly the same on the difference of consumption growth rates (eq. (2.25)). Let me differentiate κ in order to study the relationship between the two dimensions. One has

$$-\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}\right|_{\kappa} > 0 \quad \Leftrightarrow \quad \frac{1-\sigma}{1-\theta} > 0 \ . \tag{2.26}$$

[16]This case shall not be compared with the framework presented in the Chapter 1. It is valid only at the limit and the economy has to grow toward the GR (to insure convergence of the objective).

Then, as far as the evolution of the difference in consumptions is concerned, the two IA are 'substitutes' if they are both sufficiently high ($\theta, \sigma < 1$) or both sufficiently low ($\theta, \sigma > 1$). And 'complements' in all other cases.

To better understand these features, I now study the sign of κ and make simple comparative statics.

- $\kappa > 0 \;\Leftrightarrow\; \theta + \sigma > 1$;
- $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} > 0 \;\Leftrightarrow\; \sigma > 1$ and $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} > 0 \;\Leftrightarrow\; \theta > 1$.

Five cases have to be distinguished, as it is shown in Fig. 2.1.

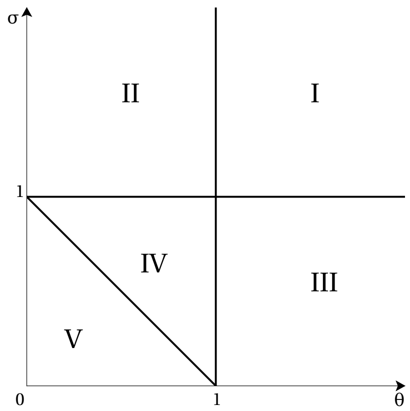


Figure 2.1: Possibilities for inequality aversions

Let us sum up the signs of level and variations of κ in each situation.

I. $\kappa > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} > 0$.

II. $\kappa > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} < 0$.

III. $\kappa > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} < 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} > 0$.

IV. $\kappa > 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} < 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} < 0$.

V. $\kappa < 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\theta} < 0$; $\frac{\mathrm{d}\kappa}{\mathrm{d}\sigma} < 0$.

Regarding sign, if intra and intergenerational IAs are low (I to IV), the difference in regional consumption rates is positive (see the eq. (2.25)). That means that everything else being equal, the higher the marginal productivity of the stock available in the South, the higher its consumption growth rate. Since the South has access to a more productive stock, a reduction of its consumption leads to a better return. South has therefore to make a relative higher sacrifice. This could seem surprising: taking into account intragenerational concerns in a classic accumulation model leads to rise the difference of optimal development between unequal regions. Rather, it seems rational to take advantage of initial differences, but of course this calls for redistribution mechanism both for ethical and for practical reasons of implementing such a decision rule. Interestingly, if intra and intergenerational IAs are sufficiently high (V), the previous reasoning is reverse and North has to make a relative higher sacrifice. In any case, when marginal productivities converge toward each other (approaching the GR), the regional consumption growth rates converge too. The levels of consumption converge as well, as explained below.

Regarding variations, if intra and intergenerational IAs are sufficiently low (I), the lower the IA, the higher the difference in regional consumption growth rates (see the eq. (2.25)). On the opposite, and counter-intuitively, if both IA are high (IV and V), the lower the IA, the lower the difference in regional consumption growth rates. There are two intermediary cases when the IA is low in one dimension but high in the other dimension (II and III). They correspond to the 'complementarity' cases displayed in the eq. (2.26).

Let me now have a word on the comparison between regions in isolation and regions collectively considered. Of course, regional consumption growth rates differ in the two problem (compare eq. (2.12) and (2.24)). But as far as the difference in growth rates in concerned, the problem reduces in comparing σ (eq. (2.13)) with κ (eq. (2.25)). It is not hard to see that $\kappa > \sigma$ if and only if $\sigma < 1$ and $\theta + \sigma > 1$. In other words, the collective problem lead to a higher difference in consumption growth rates than the individual problem only in cases III and IV. They correspond to a case where both the intragenerational IA is relatively low (intuitively, a low IA lead to high differences) and the intergenerational IA is high (counter-intuitively). Let me now go back to the study of the limits, but for κ only.

L1. Intragenerational utilitarianism: $\theta \to \infty$, $\kappa \to \sigma$: if the intragenerational IA is nil, the difference of consumption growth rates depends only on the intertemporal IA and on the productivity gap (as in the problem of two separated regions).

L2. Intragenerational maximin: $\theta \to 0$, $\kappa \to 0$: if the intragenerational IA is infinite, the regional consumptions grow at the same rate.

L3. Intergenerational utilitarianism: $\sigma \to \infty$, $\kappa \to \theta$: if the intergenerational IA is nil, the difference of consumption growth rates depends only on the intratemporal IA and on the productivity gap.

L4. Intergenerational maximin:[17] $\sigma \to 0$, $\kappa \to 0$: if the intergenerational IA is infinite, the regional consumptions grow at the same rate.

L1 and L3 corresponds to cases when the 'distribution of well-being' does not count in one dimension. Then the relative sacrifice depends only on the IA of the other dimension. Cases L2 and L4 lead to the same result of an equal evolution of the consumption growth rates, but for different reasons. In the case L2, the infinite intragenerational IA leads the growth rates to be as close as possible. While in the case L4, one refrains oneself as much as possible to substitute a current (lower) consumption for a future (higher) consumption. This leads the growth rates to be as small as possible, and therefore to be close. In other words, if the IA is infinite in at least one dimension consumptions evolve at the same rate. If the IA is nil in one dimension, the difference in growth rate depends only on the IA of the other dimension. And finally, if the IAs are nil in the two dimensions, the social welfare is nearly only supported by the consumption of the North (low sacrifice) while the South let its stock grow as fast as possible (huge sacrifice). Consumptions approach a corner solution.[18] In this last case, the productivity gap is fully exploited. It calls even more clearly for a transfer from the North to the South.

[17] This case shall not be compared with the framework presented in the Chapter 1. Firstly, because it is valid only at the limit. Secondly, because the approach followed here lead to a CES of two regional maximin paths. And thirdly, because the economy grows toward the GR.

[18] This limit is easily computable with the notation $\kappa \equiv \frac{1}{1-\eta\nu}$. When $\eta, \nu \to 1$, $\kappa \to \infty$.

Finally, it shall be noted that in case of equal marginal productivities (e.g. if natural renewal are identical), the IAs play no role and the consumptions grow at the same rate toward their golden-rule levels.

With the transversality condition, the unique steady state is characterized by (the stability is shown in the Appendix A.1)

$$c_i^{\star\star} = F_i\left(X_i^{GR}\right) \;,\; F_S'\left(X_S^{GR}\right) = 0 = F_N'\left(X_N^{GR}\right) \;,\; W(c_N^{\star\star}, c_S^{\star\star}) = W^{GR} \;. \quad (2.27)$$

Integrating both sides of the equation (2.25) and using the final consumptions, it comes (as for the eq. (2.15))

$$\frac{c_N(t)}{c_S(t)} = \frac{c_N^{GR}}{c_S^{GR}} \mathrm{e}^{\kappa \int_t^\infty \left(F_S'(X_{Sr}) - F_N'(X_{Nr})\right) \mathrm{d}r} \;. \quad (2.28)$$

This equation helps understanding the evolution of the consumptions through their ratio. Let me concentrate on cases where $\kappa > 0$ (i.e. $\theta + \sigma > 1$). As long as the stock of the South is more productive than that of the North, the current ratio is higher than the final ratio and converges toward it. It shows, more directly, how a more productive stock leads to a higher sacrifice. But to visualize this feature even more easily, I turn to a graphical representation.

2.2.4 Graphical representation

As in the Chapter 1, Fig. 2.2 is a four-quadrant plot that represents the different elements. The upper-right quadrant plots the evolution of the consumption and the stock of the North. Symmetrically, the lower-left plots these elements for the South. In such a way, the upper-left quadrant plots the consumption paths. And the lower-right quadrant plots the stock paths.

The upper-right and lower-left quadrants are, individually, similar to a one-sector Ramsey model.[19] They depict a saddle point: consumption has to increase (resp. decrease) if the initial stock is lower (resp. higher) than the golden-rule

[19] It is direct from the eq. (2.21) and (2.22), noticing that $\psi_i > 0$ is a decreasing function of c_i: $\frac{\partial \psi_i}{\partial c_i} = \frac{\partial^2 W(\cdot)}{(\partial c_i)^2} < 0$.

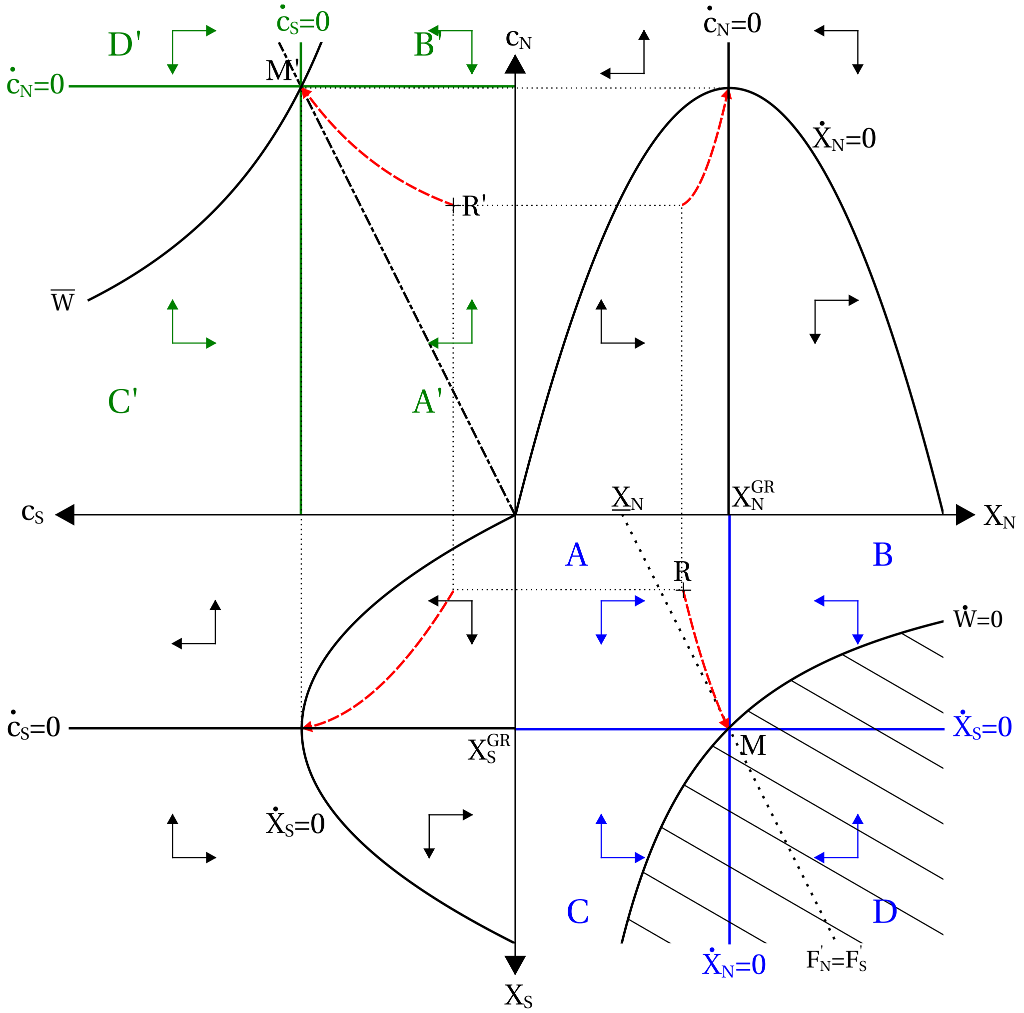


Figure 2.2: Global evolutions of the consumptions and the stocks

stock. For the sake of the representation, I use the transversality condition (reaching the GR) to project the $\dot{c}_i = 0$ curves in the upper-left quadrant and the $\dot{X}_i = 0$ curves in the lower-right one. The upper-left and lower-right quadrants represent then only optimal paths, ending up at the steady state M (corresponding to M'). The paths never cross the $F_N' = F_S'$ locus[20] ($\underline{X}_N M$ and $0M'$), otherwise one would have a steady state that does not satisfy the transversality condition.[21] I conjecture that paths never cross the curve passing through M (M') depicting $\dot{W} = 0$

[20]It is a straight line because I plotted symmetric production functions.

[21]Approaching the steady state (possibly asymptotically), stock and consumption paths have

($W = W^{GR}$).[22] As mentioned before, this curve represents the as-good-as-golden locus. The hatched area depicts stocks from which the social welfare decreases toward its steady-state value. More generally, no cycle occurs (in consumptions or states) since paths cannot cross the 'critical loci'.

An example of path is plotted from R' to M'. It is very similar to a classic path in a Ramsey model. But we can now better visualize the impact of κ on the consumption ratio, exposed in the eq. (2.28). The lower κ, the closer the current ratio to the final ratio. The consumptions path gets closer and closer to the straight line $0M'$. For example, if the IAs are initially low ($\theta, \sigma > 1$), the higher the inequality aversions, the lower the actual inequalities during the convergence. But the previous explanations showed that this intuitive feature does not always hold.

Finally, let me have a word on the 'regular' part (the non-hatched area) of the area B. From initial stocks in this part, the North has an abundant stock $X_{N0} > X_N^{GR}$ while the South has a scarce stock $X_{S0} < X_S^{GR}$. Here North shall overshoot its production to decrease toward the GR, while South shall make classic savings to grow. Clearly, here, we could take a part of the 'manna' in North to compensate the sacrifice in South. But how regional sacrifices contribute to the improvement of the welfare? This is depicted by Green accounting tools.

2.3 Accounting considerations

During the transition toward the GR, savings are made. One expects therefore an indicator of sustainability to be increasing. What I interpreted as 'sacrifice' at any given date is given by the difference between the production $F_i(X_i)$ and the current consumption c_i, i.e. by the distance between the current and the sustainable consumption, for $X_i < X_i^{GR}$. But this difference was already measured by the evolution of the stocks since $\dot{X}_i = F_i(X_i(t)) - c_i(t)$. In other words, the instantaneous sacrifice made by a region is represented by $\dot{X}_i$ and its total instantaneous sacrifice by $X_i^{GR} - X_{i0}$. And, as well-known, the marginal impact of a

the same tangent (by L'Hospital's rule): $\lim_{t\to\infty} \frac{\mathrm{d}X_N}{\mathrm{d}X_S} = \lim_{t\to\infty} \frac{F_N'(X_N)\dot{X}_N - \dot{c}_N}{F_S'(X_S)\dot{X}_S - \dot{c}_S} = \frac{\mathrm{d}c_N}{\mathrm{d}c_S} = \frac{c_N^{GR}}{c_S^{GR}}$.

[22] A discussion on this point can be found in Samuelson and Solow (1956).

stock on the value function V is given by shadow-prices ψ_i. I study this formally.

In autonomous problems, the optimal Hamiltonian is constant (see e.g. Chiang, 1992, p. 190). The transversality condition (2.23) implies therefore $\mathscr{H}^{\text{opt}} = 0$. The equation (2.20) can then be rewritten as

$$\underbrace{\psi_N \dot{X}_N + \psi_S \dot{X}_S}_{\text{Net investment}} = \underbrace{W^{GR} - W}_{\text{Welfare gap}} \, . \tag{2.29}$$

Along a rising path, $W < W^{GR}$, net investment (NI) is positive and is linked to the 'welfare distance'. The farther the economy is from the target, the more it has to invest. Each region makes a contribution of $\psi_i \dot{X}_i$. And investment tends toward zero as W approaches W^{GR}. Once at the GR is reached (possibly asymptotically) one obtains, in a sense, the Hartwick's rule (Hartwick, 1977; Dixit et al., 1980; Withagen and Asheim, 1998).[23] Actually, this feature was already found by Ramsey (1928) himself – it is the 'genuine' Keynes-Ramsey rule – and was restated with exhaustible resource and capital accumulation by d'Autume and Schubert (2008b). Here, positive, nil and negative investments could be possible according to the initial states.

Let me now turn to the final analysis when a unequal treatment is considered.

2.4 Intra and intergenerational inequity

From a purely positivist view, one may consider treating both regions and generations unequally. Actually, Ramsey (1928) himself considered discounting at the end of his paper, while he strongly rejected it on an ethical ground at the beginning. In the same vein, Koopmans (1965) used a discount rate to compare discounted and undiscounted versions. At any time, the discount rate may be nil to come back to the previous case of undiscounted welfare.

Let me follow this approach by considering discounting as well. What represents a higher weight on current and near welfares than futures ones. What Heal (2009, p. 5) called as "*the rate of intergenerational discrimination*". By the same token, let me also attach different weights on regions to see if changes hap-

[23]This always holds for the as-good-as-golden locus.

pen. These intra and intergenerational weights may be interpreted as a measure of (procedural) 'inequity'.

Let a_N and a_N be now elements of $(0,1)$, still with $a_N + a_S = 1$. And let us introduce a positive discount rate δ. I study only the collective problem, but at any time, recall that letting $\theta \to \infty$ allows to approach individual ones.

The intertemporal welfare is now given by

$$V^d(X_{N0}, X_{S0}) = \max_{c_N, c_S} \int_0^{\infty} \mathrm{e}^{-\delta t} \left(W(c_N, c_S) - W^{GR} \right) \mathrm{d}t \ ; \tag{2.30}$$

subject to the same constraints .

The eq. (2.24) becomes

$$\frac{\dot{c}_i}{c_i} = \frac{\theta\sigma}{\theta + \sigma - 1} \left(F_i'(X_i) + \frac{1}{\theta} \frac{\dot{W}}{W} - \delta \right) . \tag{2.31}$$

In comparison with the situation without discounting, consumption growth rates are lower. Intuitively, if future generations count less, we less need to save in order to increase their welfares. And less investments lead to a lower growth. Apparently, intragenerational weights do not play a role here. Indeed, they impact W but they have the same impact on the evolution of the consumptions (same $\frac{\dot{W}}{W}$ for both). Actually, they do impact the levels of optimal consumptions. To see this let us rewrite the green accounting equation.

$$\psi_N \dot{X}_N + \psi_S \dot{X}_S = \mathrm{e}^{-\delta t} \left(W^{GR} - W \right) . \tag{2.32}$$

As ψ_i depends positively on a_i, the 'sacrifice sharing' depends well on those weights. Everything else being equal, a higher weight a_i means a lower $\dot{X}_i$, then a lower sacrifice.

Interestingly, when the IA is infinite in at least one dimension (L2 or L4), intergenerational weights (discounting) disappear in the eq. (2.31).[24] They become "immaterial" (d'Autume and Schubert, 2008b). This feature is well-known in a static framework: when the elasticity of substitution tends to zero, a CES func-

[24]When $\theta \to 0$ discounting disappear but intragenerational weights do still play a (undifferentiated) role on consumption growth rates through W (L2). But when $\sigma \to 0$, no weight matter (L4).

tion tends to the *symmetric* minimum.[25] This common feature both in intra and in intergenerational dimensions is not so surprising. Weights (attached to individuals or generations) guide the priority of resources. But, in parallel, the elasticity of substitution tells to what extend society is willing to substitute the well-being of an individual (or generation) for that of another one. When the elasticity tends to zero, substitution are no longer tolerated. Therefore, whatever the attached weights, the absolute priority is given to the worst-off. Only in this case, weights do not matter. This is even clearer when $\theta \to \infty$ ($\eta \to 1$):

$$\frac{\dot{c}_i}{c_i} = \sigma \left(F_i'(X_i) - \delta \right) \ . \tag{2.33}$$

In the classic so-called 'Keynes-Ramsey rule', a nil IES 'erases' the discount rate. Let us now merge the two equations (2.31), for $i = N, S$. One still has

$$\frac{\dot{c}_S}{c_S} - \frac{\dot{c}_N}{c_N} = \kappa \left(F_S'(X_S) - F_N'(X_N) \right) \ . \tag{2.34}$$

Therefore, the evolution of the inequality between the consumption growth rate in the North and in the South is robust to both intra and intergenerational 'inequity'. But as said before, the levels of consumption are impacted. In particular, the steady state is now characterized by a lower welfare, the modified golden rule. It is reached when

$$F_S'(X_S) = F_N'(X_N) = \delta \ . \tag{2.35}$$

That means that the sacrifice shall stop when the marginal productivities are both equal to the pure rate of time preference. Taken in another way, the discount rate can be chosen so as to meet any states on the $F_S' = F_N'$ locus, and then limits the global sacrifice. This brings me to my last discussion.

[25] A proof is provided in the Appendix A.1 of Chapter 3, p. 129.

2.5 Discussion: Equity, inequality aversion and discount rate

The Ramsey criterion has been criticized for leading to a significant sacrifice of every generation at the benefit of subsequent ones (Arrow, 1999). As argued by Rawls (1971, p. 287): "*the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for later ones that are far better off. But this calculus of advantages, which balances the losses of some against benefits to others, appears even less justified in the case of generations than among contemporaries*". But as Rawls (1971, sec. 44) refrained himself to apply either discounting or the difference principle criterion ('maximin') in an intertemporal perspective, he gave no precise recommendations about the optimal savings rate.[26] And he admitted that "*it is not possible, at present anyway, to define precise limits on what the savings rate should be*" (p. 286). To find a solution, some authors prefer to adopt a consequentialist approach and to discuss the resulting allocation of well-being of a particular criterion. For example, Zuber and Asheim (2012) allow for weights according to the rank of generational well-being. This was built up on the "Hammond Equity for the Future" axiom (Asheim and Tungodden, 2004), which captures the following idea: a sacrifice of the present for the future is (weakly) desirable if the present stays better-off than the future. In this view, discounting comes from the expression of inequality aversion if future generation are better-off (Asheim and Mitra, 2010; Asheim, 2012; Zuber and Asheim, 2012; Asheim and Ekeland, 2016). I see several potential problems with the view of 'equitable discounting'.

The intratemporal part allows to better understand the distinct role of weights and of the intragenerational IA. Undesirable consequences from unequal weights can indeed be reduced with a higher inequality aversion. And when IA is infinite, weights disappear. But why changing weights rather than the IA? In the present

[26] It is sometimes mentioned that Rawls (1971) acknowledged the interest of discounting to avoid sacrifices. But one shall add that he was in contradiction with the primitive problem. He wrote: "*introducing time preference may be an improvement in such cases; but I believe that its being invoked in this way is an indication that we have started from an incorrect conception. [. . .] It is introduced in a purely ad hoc way to moderate the consequences of the utility criterion*" (p. 298).

framework, sacrifices may also be reduced by two ways: with a higher intertemporal IA or with a discount rate. The first option (consider $\theta > 1$ for simplicity) leads to make regional consumption paths closer when growing toward the GR. The second option leads to a situation where sacrifice is less needed. And then each generation has to make a lower sacrifice. Both options may be desirable, but in light of this framework it seems odd to justify discounting by the IA. By the way, such a practice would be intolerable in the intragenerational dimension. Also, I find odd that the IA be dependent upon the situation. How to justify that such a change in the IA arises merely because the situation changed? I think that linking directly the IA with the specific situation can lead to arbitrary decisions. The underlying justification that future can benefit from current sacrifice, more than proportionally, does not seem to justify *per se* a departure from impartiality (Fleurbaey and Michel, 1999, p. 723). Besides, if equity requires that something is equalized, I do not see precisely what is equalized with discounting. That said, resulting allocation of well-being may be worth discussing, but one needs, I think, another vocable.

Giving the priority to the worst-off individuals because they are worse-off is advocated by proponents of *prioritarianism* (Parfit, 1995; Fleurbaey, 2015b). In this view, weights should be inversely related to the relative initial well-being. Here, in the intertemporal dimension, it would justify discounting for increasing welfare paths. In the intratemporal dimension, it would also recommend to give more weight to the South as long as it is less well-off than the North.

I propose to argue that the undiscounted utilitarian criterion is sufficiently malleable to handle different intergenerational inequality outcomes (Fleurbaey and Michel, 1994, 1999; Asheim and Buchholz, 2007). Which comes back to asking, ultimately, what is the appropriate shape of the welfare function (Schelling, 1995). But this malleability comes at a price when intratemporal issues are taken into account. Even if the difference in relative sacrifices reduces over time, the inequality may be important during the transition path. Giving the same weight to the regions turned out to be insufficient.

2.6 Conclusion

This simple framework enables us to have straightforward insights. When regions have access to unequally productive renewable resources, the sacrifice to attain the highest sustainable level of welfare – the golden rule – is not evenly shared. Basically, the region which is endowed with a lower stock, South, has to make a higher sacrifice than the other one, North. The intuition behind this startling result is that the resource of the South, being lower than that of the North, is marginally more productive. The same sacrifice leads then to a higher return in the South than in the North. The difference in regional growth rates depends generally positively on the productivity gap, and negatively on the intratemporal and the intertemporal inequality aversions. The sacrifice for reaching the golden rule is evenly shared *only* in three cases. (1) If the marginal productivities are equal, because one cannot take advantage of differences of the two regions. (2) If the intratemporal inequality aversion is infinite, because one does not want to take advantage of differences of the two regions. (3) If the intertemporal inequality aversion is infinite, because one does not want to make sacrifice. The (original) Keynes-Ramsey rule holds in this framework: the farther the economy is from the golden rule, the more it has to invest. At the end, unequal weights put on regions and on generations were considered. I came to two conclusions. (i) Discounting cannot be advocated on the sole basis of the willingness to limit sacrifices. (ii) Intratemporal weights do not play a fundamental role in the relative regional growth rates.

The main conclusion of this model, based on the most efficient way to share sacrifice in order to grow toward the golden rule, can be difficult to hear in the real world. Both from an ethical and a pragmatic view, such a policy can only be implemented with transfers from the North to the South. This is subject of the next chapter. The current and the next chapters together will echo results from the literature on growth and inequality. Especially, Aghion et al. (1999, p. 1616) stated that "*redistribution can foster growth. However, the growth process is unlikely to leave inequality unchanged*".

A.1 Stability of the steady state

Let me sum up the necessary conditions into three main equations to get the following dynamic system, with $Y \equiv \ln\left(\frac{c_S}{c_N}\right)$;

$$\begin{cases} \dot{X}_N &= F_N(X_N) - c_N \,; \\ \dot{X}_S &= F_S(X_S) - c_S \,; \\ \dot{Y} &= \kappa\left(F'_S(X_S) - F'_N(X_N)\right) \,. \end{cases} \tag{A.36}$$

Steady states are characterized by (using the transversality condition)

$$\begin{cases} c_N^{GR} &= F_N\left(X_N^{GR}\right) \,; \\ c_S^{GR} &= F_S\left(X_S^{GR}\right) \,; \\ F'_N\left(X_N^{GR}\right) &= F'_S\left(X_S^{GR}\right) \,. \end{cases} \tag{A.37}$$

Consider the Jacobian matrix of the linearized system, evaluated at the steady states[27]

$$J^\star = \begin{pmatrix} F'_N & 0 & \alpha_N \\ 0 & F'_N & -\alpha_S \\ -\kappa F''_N & \kappa F''_S & 0 \end{pmatrix} \,; \tag{A.38}$$

with $\alpha_N \equiv -\frac{\partial c_N}{\partial Y}\Big|_{c_N^{GR}, c_S^{GR}} = \frac{\left(c_S^{GR}\right)^2}{c_N^{GR}} > 0$ and $\alpha_S \equiv \frac{\partial c_S}{\partial Y}\Big|_{c_N^{GR}, c_S^{GR}} = c_S^{GR} > 0$.

Let us compute the roots of the characteristic polynomial $\mathscr{P}(\lambda) = \det\left(J^{GR} - \lambda I_3\right)$:

$$\left(F'_N - \lambda\right)\left(-\left(F'_N - \lambda\right)\lambda + \kappa\alpha_N F''_N + \kappa\alpha_S F''_S\right) = 0 \,. \tag{A.39}$$

The first eigenvalue is $\lambda_N = F'_N$. Let $\Gamma \equiv -\kappa\left(\alpha_N F''_N + \alpha_S F''_S\right) > 0$. I can reduce the eq. (A.39) to $\lambda^2 - F'_N\lambda - \Gamma = 0$. Eigenvalues are then $\lambda_N = F'_N > 0$, $\lambda_S = \frac{F'_N - \sqrt{(F'_N)^2 + 4\Gamma}}{2} < 0$, and $\lambda_3 = \frac{F'_N + \sqrt{(F'_N)^2 + 4\Gamma}}{2} > 0$.[28] The steady state is a (classic) saddle-point.

[27] We use the equality $F'_N = F'_S$ and express the Jacobian with respect to F'_N only.

[28] Notice that strict concavity of production functions rule out nil eigenvalues.

Chapter 3

Transfers of resources for a sustainable development

Abstract

The first chapter highlighted a significant disparity during a sustainable development path: North shall over-consume while South shall under-consume. The second chapter highlighted a similar, but less significant, disparity during a growing path: South shall relatively save more because its stock allows a better return. They came to the same conclusion: implementing transfers for ethical and practical reasons. Then this chapter studies the stylized case where North has a free access to its stock while South has a limited access to its stock. It does not solve an intertemporal problem but rather concentrates on policy implications of mechanisms of transfers. It turns out that either a tax has to be used to reduce the consumption of North, or a lump-sum to increase it, according to the sacrifice asked to North.

3.1 Introduction

Emphasis has been put on futurity in the sustainability debate. Economists translated this question on how weighting the future compared to the present in the evaluation of different development paths. Among these paths, some can be considered as fair if generations that arise at different dates count equally. This view is often summarized under the vocable *intergenerational equity*. But if society cares about the 'difference of well-being' between two generations, it cares also about that of two individuals from the same generation. When they count equally, one can talk about *intragenerational equity*[1]. The economics literature has generally separated these two equity dimensions, but there are several arguments for dealing with them together. First, it can be argued that an unjust society is likely to be unsustainable, either on the political side, with revolution, or on the environmental side, through degradation (World Commission on Environment and Development, 1987, ch. 2. § 4; Haughton, 1999). Second, it seems curious to attach more importance to future generations, thus unborn, than to the current one (Solow, 1991; Schelling, 1995; Anand and Sen, 2000). Finally, from a policy view, one may wonder if intra and intergenerational concerns can be designed independently. On the contrary, one should be interested in how they interact to formulate consistent policies.

The interactions between these two dimensions are actually ever-present in economics. An example can be that of a fiscal policy that aims to reduce the public debt. But alongside that debt there is environmental debt, which can reduce the possibility of development for future generations. In this respect, three major dimensions can be taken into account: the climate change (with the question of burden-sharing between generations and into each one of them, between expected losers and winers), the exhaustion of non-renewable resources (but it requires an understanding of the industrial processes, especially to what extend these resources can be substitutes for manufactured capital) and the management

[1]There is not a unique accepted definition of the word 'equity'. I take here the requirement of equal weights in decision making. Which is in line with a common definition: "Equity is the quality of being fair and reasonable in a way that gives equal treatment to everyone". (source: https://www.collinsdictionary.com/dictionary/english/equity, visited on May 2018).

of renewable resource stocks. I am interested here in renewable resources.

The linkages between the two equity dimensions seem not to have been extensively studied in the economics literature, and in the environmental and resources economics literature in particular. Nonetheless, some authors have highlighted this question in the climate change debate for some time. For example, Schelling (1992) argued that the best way for developing countries to fight against the negative effects of climate change is to continue to develop. Heal (2009), on the opposite, explains that a preference for equality between generations and the preference for equality within each one of them may be opposed. If one expects consumption to grow, one can further discount the future, but that does not incite us to take preventive measures against the negative effects of climate change. Conversely, as developing countries are more vulnerable, more concerns about them would incite us to act more quickly. Kverndokk et al. (2014) proposed a model to analyze the burden-sharing between North and South in the reduction of greenhouse gas emissions through clean and dirty investments. These two dimensions are also present in the explanation of climate negotiations (e.g. Lecocq and Hourcade, 2012). Baumgärtner et al. (2012) proposed a framework to summarize the possible links between the two equity dimensions: independence, facilitation and/or rivalry. Those features are detailed in the context of ecosystem services by Glotzbach and Baumgärtner (2012).

As renewable resources are not produced, the choice of the welfare criterion is of all importance. More precisely this may allow for expressing the implicit values of stocks of natural resources. Associated *shadow-prices* are essential to compute *genuine savings* (Hamilton, 1994; Pearce et al., 1996; Asheim, 2007). Expenditures that enhance the environment are seen as savings and depletion of natural resources and environmental degradations as dissavings. It generalizes the traditional concept of savings, and its positivity indicates that welfare, however defined, is currently non-declining. Renewable resources have to be managed on the long run, but compared to non-renewable ones: they are generally directly consumed, they may have amenities and one can have ‘win-win’ solutions. Further, some can be ‘essential’ for life. For all of these reasons, the regulation can be justified. Here, renewable resources are viewed as a parable to link the two dimensions. Some current redistributions can be decided, but they will inevitably have consequences on redistribution between now and the future.

I am not aware of any work that builds analytically the welfare possibility between heterogeneous agents that depends on a renewable resource and has consequences on future generations. The purpose of this study is to analyze the intragenerational and the intergenerational equity trade-off with a renewable resource. The question I am asking is what are the consequences of equity concerns on current and future welfares, when some individuals are authorized to have a full access to their renewable resource, while other individuals have only a limited access to their stock, to let it grow. And when the social planner orders a redistribution. And does the type of redistribution matters in such a context? More precisely, I showed in Chapter 1 that the intertemporal maximin criterion leads the region with the scarce resource – South – to under-consume (to let its stock grow) and the region with the relative abundant resource – North – to over-consume (to compensate the under-consumption of South in the global welfare). This may even be more radical in the case of a nil intragenerational inequality aversion: South consumes nothing. I showed in Chapter 2 that this feature is still present, in some sense, when one wants to grow toward the golden rule. Both North and South have to make a sacrifice, but the South has to make a relatively higher sacrifice (for the same above-mentioned reason). In this context, it appears natural to study a transfer from North to South both for ethical reasons and for a plausible implementation of development paths suggested. In this chapter I deviate from a pure optimal trajectory framework (as analyzed in the first two chapters). The present purpose is to study qualitative policies and possible implementations. The intergenerational equity may be represented in several ways (for a short review, see Asheim, 2010). But to be consistent with my previously mentioned definition of equity, I shall adopt the anonymity axiom: the outcome is invariant to (finite) permutations in the date of appearance of generations. This still leaves the door open for different criteria such as maximin (Solow, 1974) or the Ramsey (1928) criterion. Here, I will only adopt a minimalist view: the intertemporal value function is non-decreasing with respect to the stock. More precisely, the benchmark objective for the North will be to reach the golden rule.

The intragenerational equity may also be expressed in different ways. The main theories of distributive justice at a society scale are: utilitarianism, liberal egalitarianism, libertarianism and Marxism (Arnsperger and Van Parijs, 2003). I assume here a welfarist view: a state is only assessed through utilities of individuals. Society is assumed to have an inequality aversion (possibly nil). That is to say, it restrains the substitution of the well-being of one agent for the well-being of another. This approach is not as restrictive as it could seem since it allows to deal with different theories of justice as special cases. Indeed, the utilitarianism assumes a nil inequality aversion, only the total of utility matters (Vickrey, 1945; Harsanyi, 1953, 1955, 1977). The liberal egalitarianism can take a maximin form, popularized by Rawls (1971), if utility are considered as "primary goods". Here, inequality aversion is infinite, no substitution is possible between the individual utility. 'Right-libertarians' would promote no transfer and 'left-libertarians' would promote a transfer such as to perfectly equalize utility. The Marxist approach would determine thresholds of utility representing 'needs'. According to the intergenerational view, the intragenerational fulfillment may be constrained. Studying both together allows for determining all possible choices of policy and to estimate opportunity costs.

I use a social objective that allows to deal with the two main doctrines (utilitarianism and maximin) as well as all intermediary cases. Besides, I build sets of possibilities for the utilities. They indicate to what extend one can take from one agent to give to another agent. Then, efficient allocations will be represented by Pareto frontiers. From an allocation situated on such a frontier, one cannot increase any more the utility of one agent without decreasing that of another. Afterward, I will be able to choose between those efficient allocations using the social criterion. To respond to my issue I need, as said, two different regions, North and South. The North has access to a resource and works to extract a part of it. On the contrary, the South has also access to a resource, but shall extract less than what it originally would want. I introduce a redistribution mechanism from North to South. Compared to previous chapter, it is necessary here to introduce labor in order to explicitly take into account effort and to analyze if a transfer is a disincentive to harvest. Again, the present purpose is more qualitative and 'policy-oriented'. The utility of North depends on individual leisure time and on available consumption. The net utility of South depends only on the amount received. I analyze the utility allocation possibilities offered by two redistribution mechanisms. The first one is a lump-sum transfer: whatever the effort of North, South obtains the same amount of the resource caught. The second one is a proportional tax. Whatever the effort of North, South obtains a given proportion of the resource caught. I introduce a renewable resource which varies according to the catch. As the resource may affect the potential catch, it impacts the utility possibility sets. First, I will see how the optimal transfer (lump-sum and tax) evolves according to a change in the inequality aversion. Second, I will analyze the consequences of the evolution of the resource on the welfare of the regions in an intergenerational perspective.

My contribution is to have stated the conditions underlying the construction of well-known utility possibility frontiers in the context of two heterogeneous agents. When the transfer is absolute (through a lump-sum), this does not pose any particular difficulties as long as leisure and consumption are normal goods. When the transfer is relative (proportional tax), it depends on the labor supply of the North in reaction to the tax, which is given by its disposition to substitute leisure for consumption. In particular, if the effort decreases when taxed, the utility possibility set is bounded by a 'Laffer-like curve': the amount received from a proportional tax is first increasing, then decreasing, with respect to its rate (see Laffer, 2004). In any case, the higher the inequality aversion the higher the transfer. But regarding the redistribution of a renewable resource over time, an increasing of the intragenerational inequality aversion does not 'worsen' the intergenerational dimension in two situations. If the consumption has to decrease, it has to be through a tax: discouragement effect. While if it has to increase, it has to be through a lump-sum: North gets less while harvesting more.

The intragenerational dimension is built upon a framework proposed by Mas-Collel et al. (1995, pp. 823-4), borrowed itself from Atkinson (1973). I extend their example in two dimensions. First, I state clearly the conditions under their results, especially the construction of utility possibility frontiers. And second, I introduce a resource to take into account the intergenerational concern in a Gordon-Schaefer model (Clark, 1990). The welfare analysis is based upon social welfare functions, in the spirit of Bergson (1938) and Samuelson (1947), where the elasticity of substitution is interpreted as a measure of the relative inequality aversion (Atkinson, 1970).

The next Section presents the model. I solve it and I present some welfare analyses. Section 3.3 exhibits the links between the two equity dimensions. Section 3.4 concludes.

3.2 Two regions, one harvestable resource

3.2.1 Framework

I consider an economy with two heterogeneous regions, North and South. Each representative agent from each region lives one instant in that continuous framework. But as I will not solve a dynamic program, I will only study the *potential* consequences of current decisions on the future. I assume the following policy in order to implement a 'sustainable' development. North can access freely to its stock, but South is constrained to let its stock grow. A social planner implements a mechanism of transfer from the North to the South. I consider two options: the first one is a lump-sum transfer and the second one a distorting tax. To make the problem interesting, I explicitly formalize effort.

As only the stock of North matters here, I simply note $X_N(t) \equiv X(t)$. Its law of evolution is given by the gap between renewal $F(X_t)$ and harvesting $H(l_N, X_t)$, with l_i the leisure time of the region $i = N, S$ (available times are normalized to unity). Formally, the dynamics reads: (the time index will be dropped since no confusion may arise)

$$\dot{X}(t) \equiv \frac{\mathrm{d}X_t}{\mathrm{d}t} = F(X_t) - H(l_N, X_t) \tag{3.1}$$

The function F describes a "bell curve" ($F(0) = 0 = F(\bar{X})$ and reaches a maximum for X^{GR}, the golden-rule stock. This formulation can represent resource issues as well as capital accumulation (Asheim and Ekeland, 2016). $H(l_N, X)$ is the production (catch-effort) function of the North. I assume it depends linearly on labor (or leisure), and it is (weakly) convex with respect to the resource: $H(l_N, X) = (1 - l_N)h(X)$, where $h(X)$ represents the catchability.[2] The production is bounded between zero (no work) and $h(X)$ (no leisure). The utilities

[2] I assume $\lim_{X\to 0} F'(X) > \lim_{X\to 0} \frac{\partial H(\cdot)}{\partial X}$. Otherwise, the stock is asymptotically exhausted.

depend on the leisure time and the effective consumption: $u_i(l_i, c_i)$. The utility functions are assumed to be interpersonally comparable, strictly concave and homothetic[3]. The assumption of interpersonal comparison may need comments. To explain it, I think that the utility function has to be differently understood. Here, it is not only an individualistic measure of well-being, but an objective evaluation of 'legitimate requests'. It can be obtained by revealed preferences for example. Further, I think that society is able to say whom is worse-off between to types of individuals (regions here). Here, it can be justifiable since natural resources may be inclusive of "primary goods". The assumption of comparison allows also for comparing utilitarianism and maximin (d'Aspremont and Gevers, 1977).

As one needs to reduce the 'original' consumption of South, I impose its consumption to be lower than $c_S^\#$. Then its work time has to be lower than $1 - l_S^\#$. Suppose that individually the South works $1 - l_S^\star$ hours. To make the problem interesting, let us assume that South shall work less that what it would want, then $l_S^\star < l_S^\#$. Without transfer, the South has therefore the utility level $u_S^\# = u_S(l_S^\#, c_S^\#)$. Let us note T the transfer. The utilities read:

$$u_N(l_N, c_N) \quad \text{and} \quad u_S(T) = u_S\left(l_S^\#, c_S^\# + T\right) \,. \tag{3.2}$$

Besides, each good is assumed to be essential[4]. Let us note the global consumption $c \equiv c_N + c_S$. I assume no loss, so that the whole production is consumed: $c = H(l_N, X)$. By construction, the transfer amounts to $T = c - c_N \geq 0$.

To represent the inequality aversion in a simple way, I use an ordinal social welfare function (SWF) with a constant elasticity of substitution (CES) between regional consumptions. It is given by

$$W(u_N, u_S) = \left(a_N \cdot u_N^\eta + a_S \cdot u_S^\eta\right)^{\frac{1}{\eta}} \,, \quad \text{with } a_N + a_S = 1. \tag{3.3}$$

Consider only the case $a_N = a_S = \frac{1}{2}$ for now. The elasticity of substitution is given by $\theta \equiv \frac{1}{1-\eta}$ (η inferior to unity but non-zero). A decrease of the elastic-

[3]Formally, homothecy means $\left.\frac{\mathrm{d}c_i}{\mathrm{d}l_i}\right|_{u(c_i,l_i)} = \left.\frac{\mathrm{d}c_i}{\mathrm{d}l_i}\right|_{u(\beta c_i,\beta l_i)}$, with $\beta > 0$.

[4]Formally, $\lim_{c_i \to 0} \left.\frac{\partial u_i(\cdot)}{\partial c_i}\right|_{u_i} = \lim_{l_i \to 0} \left.\frac{\partial u_i(\cdot)}{\partial l_i}\right|_{u_i} = \infty$, $i = N, S$.

ity of substitution represents an increase of the inequality aversion.[5] It is non-paternalist (only utility matters), paretian, symmetric and concave. It is also homothetic, which satisfies the relative invariance with respect to proportional shifts (see Atkinson, 1970).

The intergenerational dimension is dealt with through an intertemporal SWF $f(W(u_{N1}, u_{S1}), W(u_{N2}, u_{S2}), \dots)$. I do not solve an intertemporal problem here, I simply assume the intertemporal welfare to be increasing with respect to the intratemporal welfare.

3.2.2 Utility possibility frontier with lump-sum transfer

The North has a free access to its resource while the South has a limited access to its resource. If one wants to implement a mechanism of transfer, a first step can be to determine the constraint set of the social planner.[6] In this perspective, the social planner can virtually take an amount from what the North harvests, to give it to the South. For each amount of what would be transfered, one can determine the optimal utility of each region. By continuity, one can construct a frontier, which gives the maximum of utility of the North, given that of the South (or vice versa). That frontier bounds the utility possibility set (UPS).[7] From any state inside the set, one could make every region better-off, but once one is on the boundary, one cannot increase the utility of a region without decreasing the utility of the other one. This is so called 'first-best Pareto frontier'.

$$\max_{l_N, c_N} \quad u_N(l_N, c_N) \; ; \tag{3.4}$$

$$\text{subject to} \quad c_N = c - T \; . \tag{3.5}$$

As leisure time of the North l_N is the only decision variable, I will express, for simplicity, the problem and solve it with respect to leisure time. The transfer being absolute, the North can consume what it harvests minus a constant transfer:

[5]The cases of symmetric minimum and pure utilitarianism (mean) are obtained in the limit when, respectively, θ tends to zero or to infinity (a proof is retranscribed in the Appendix A.1).

[6]Non-welfarist approaches might not need such sets, but as they necessary lie in such sets, there is no loss of generality.

[7]I ignore special cases of linear bounds, for which Pareto frontier and bounds may differ.

$c_N = c - T$. The budget constraint can thus be rewritten as the consumption minus the transfer: $c_N = (1 - l_N)h(X) - T$.

For a given utility of South (i.e. a given transfer), North makes a labor-leisure trade-off so as to maximize its utility. As the utility function of North is assumed to be homothetic, for a given consumption per leisure time, the marginal rate of substitution (MRS) is constant. It will be convenient to express this ratio as a function of the optimal MRS. Let Ω be such a function. It is an increasing function due to the strict convexity of indifference curves. The following Proposition characterizes the first-best frontier.

Proposition 5 (First-best frontier)**.** *The maximal utility $u_N(l_N, c_N)$ subject to a constant transfer T represents a frontier in the (u_N, u_S) map, on which the following holds:*

- $u_N^\star = u_N\left(\frac{h(X)-T}{h(X)+\Omega(h(X))}, \frac{\Omega(h(X))(h(X)-T)}{h(X)+\Omega(h(X))}\right)$ *and* $u_S^\star = u_S(T)$
- $\frac{\mathrm{d}u_N^\star}{\mathrm{d}T} < 0$ *and* $\frac{\mathrm{d}u_S^\star}{\mathrm{d}T} > 0$

Proof of Proposition 5. Let me substitute the budget constraint with the available consumption in the utility, so as to maximize it in l_N.

$$\max_{l_N} \quad u_N(l_N, (1 - l_N)h(X) - T) \tag{3.6}$$

A necessary condition to maximize the utility is that the marginal productivity of labor is equal to the MRS of leisure for consumption: $h(X) = \frac{\partial u_N(\cdot)/\partial l_N}{\partial u_N(\cdot)/\partial c_N}$.

As the utility function is homothetic, I can explicit the optimal consumption of North as a function of the leisure time, so as to obtain the expansion path.

$$\frac{c_N}{l_N} = \Omega(h(X)) \quad \Leftrightarrow \quad c_N = l_N \cdot \Omega(h(X)) \ . \tag{3.7}$$

Substituting it into the budget constraint to get $l_N^\star$, and substituting $l_N^\star$ into the expansion path to get $c_N^\star$.

$$l_N^\star = \frac{h(X) - T}{h(X) + \Omega(\cdot)} \quad \text{and} \quad c_N^\star = \frac{\Omega(\cdot)(h(X) - T)}{h(X) + \Omega(\cdot)} \ . \tag{3.8}$$

Trivially $c_S^\star = T$.

For the shape, one can directly see that $l_N^\star$ and $c_N^\star$ are decreasing with respect to T. Therefore: $\frac{\mathrm{d}u_N^\star}{\mathrm{d}T} < 0$. Trivially: $\frac{\mathrm{d}u_S^\star}{\mathrm{d}T} > 0$. Hence $\frac{\mathrm{d}u_S^\star}{\mathrm{d}u_N^\star} < 0$. □

The frontier is parametrized by the amount transfered T. Without surprise, the higher the transfer the lower the utility of North and the higher the utility of South. At the limits, $u_N^\star$ tends to zero when the transfer tends to its highest level $h(X)$. And $u_N^\star$ is maximal if there is no transfer. Hence, the first-best possibility frontier is strictly downward-sloping in (u_N, u_S). Let me give an example of such a frontier.

Example Consider the following functional forms:

- $F(X) = rX\left(1 - \frac{X}{\bar{X}}\right)$ and $h(X) = qX$, thus $\dot{X} = rX\left(1 - \frac{X}{\bar{X}}\right) - q(1 - l_N)X$. With $r > 0$ the rate of intrinsic growth of the resource, $\bar{X}$ its carrying capacity and $q > 0$ a parameter of efficiency.
- $u_N = (\alpha l_N{}^\rho + (1-\alpha) c_N{}^\rho)^{\frac{1}{\rho}}$ and $u_S = c_S{}^\gamma$. $\quad 0 < \alpha < 1,\ 0 < \gamma \leq 1,\ \rho < 1,$ $\rho \neq 0$. $\xi \equiv \frac{1}{1-\rho}$.

The optimal utilities are:

$$u_N^\star = \left(\alpha \left(\frac{qX - T}{qX + \left(\frac{1-\alpha}{\alpha} qX\right)^\xi} \right)^\rho + (1-\alpha) \left(\frac{qX - T}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-\xi} (qX)^{1-\xi}} \right)^\rho \right)^{\frac{1}{\rho}};$$
$$\text{and} \quad u_S^\star = T^\gamma . \tag{3.9}$$

In this framework, an explicit first-best frontier is easy to obtain taking the inverse function of $u_S^\star(T)$ and substituting it into $u_N^\star(T)$. A graphical illustration in the (l_N, c_N) map is plotted in Fig. 3.1.[8] An illustration of the same frontier in the (u_N, u_S) will be given in the subsection 3.2.4. Three levels of transfers are plotted in Fig. 3.1. First, a situation with no transfer is represented at M. The consumption of South is nil, $c_S^{\star 1} = 0$, and the consumption of North is maximal and is equal to the global consumption $c_N^\star = c^{\star 1}$. Then, the higher the tax rate, the higher the consumption of South, $c_S^{\star 1} < c_S^{\star 2} < c_S^{\star 3}$, and the lower the consumption

[8] Unless otherwise stated, all the numerical values used are: $\alpha = 0.5$; $\rho = 0.5$; $\gamma = 0.5$; $r = 1$; $q = 0.8$; $X = 5$. Values of the transfer are successively $T = 0; 1.5; 3$.

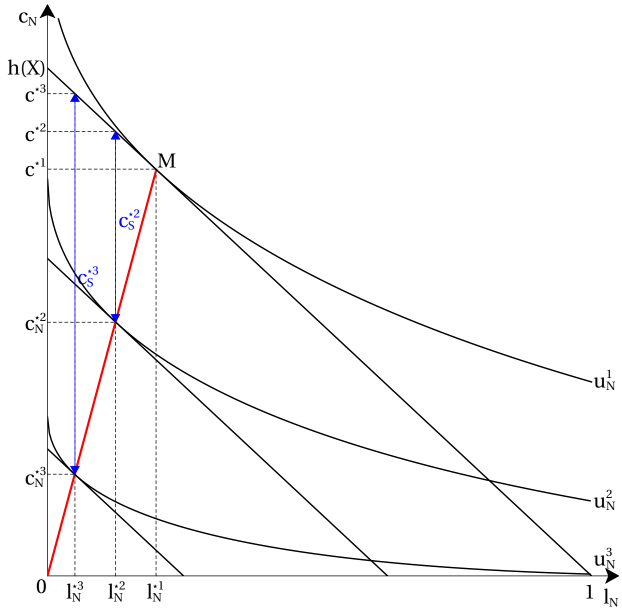


Figure 3.1: Construction of the utility possibility frontier with lump-sum transfers

of North, $c_N^{\star 1} > c_N^{\star 2} > c_N^{\star 3}$. The straight line $0M$ depicts a 'contraction path'. Note that the global consumption rises with respect to the transfer, $c^{\star 1} < c^{\star 2} < c^{\star 3}$.

3.2.3 Utility possibility frontier with tax

I study now the case where a social planner cannot transfer an absolute amount from North to South. In climate change issues, agreeing on a carbon tax is certainly easier than agreeing on transfers from developed to developing countries for example. The transfer is decentralized through a proportional tax, at the rate τ. This problem can be related to a second-best approach. This case is comparable to the previous one, but relatively different in its implications since the (implicit) relative prices change. Here, for each tax rate, one can determine optimal utilities so as to construct the 'second-best Pareto frontier'. This frontier bounds the utility possibility set as before.

$$\max_{l_N, c_N} \quad u_N\left(l_N, c_N\right) \; ; \tag{3.10}$$

$$\text{subject to} \quad c_N = (1-\tau)c \; . \tag{3.11}$$

I will explicit every variable as a function of the tax rate to solve the problem in that variable. The transfer being proportional, North can consume what it harvests minus the taxed part: $c_N = (1-\tau)c$. The receiver obtains $c_S = \tau c$. The budget constraint can thus be rewritten as the maximal possible consumption net of the tax: $c_N = (1-\tau)(1-l_N)h(X)$.

For a given tax rate, North works so as to maximize its utility. Let $\xi^{\star\star}$ be the elasticity of substitution of leisure for consumption of the North evaluated at the second-best optimum. I state the following proposition.

Proposition 6 (Second-Best Frontier)**.** *The maximal utilities of the North and of the South expressed as a function of a constant tax rate,* $0 \leq \tau \leq 1$*, represents a frontier in the* (u_N, u_S) *map, on which the following holds:*

- $u_N^{\star\star} = u_N\left(\frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X)}, \frac{\Omega((1-\tau)h(X))(1-\tau)h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X))}\right)$ *and* $u_S^{\star\star} = u_S\left(\frac{\Omega((1-\tau)h(X))\tau h(X)}{(1-\tau)h(X)+\Omega((1-\tau)h(X))}\right)$
- $\frac{\mathrm{d}u_N^{\star\star}}{\mathrm{d}\tau} < 0$ *and*
  $\begin{cases} \text{– } \textit{if } \xi^{\star\star} \leq 1, & \frac{\mathrm{d}u_S^{\star\star}}{\mathrm{d}\tau} > 0 \quad \textit{always} \\ \text{– } \textit{if } \xi^{\star\star} > 1, & \frac{\mathrm{d}u_S^{\star\star}}{\mathrm{d}\tau} > 0 \quad \textit{if} \quad \tau < \frac{1}{1+l_N^{\star\star}(\xi^{\star\star}-1)} < 1 \; . \end{cases}$

Proof of Proposition 6. Consider a given tax rate $\tau \geq 0$. The North makes a labor-leisure trade-off so as to maximize its utility. I substitute, here also, the budget constraint with the consumption in the utility, so as to maximize it with respect to the leisure time.

$$\max_{l_N} \quad u_N\left(l_N, (1-\tau)(1-l_N)h(X)\right) \; . \tag{3.12}$$

A necessary condition to maximize the utility is that the *net* marginal productivity of labor is equal to the MRS of leisure for consumption: $(1-\tau)h(X) = \frac{\partial u_N/\partial l_N}{\partial u_N/\partial c_N}$. As the utility function is homothetic, I can express the optimal consumption as a function of leisure and of the tax rate.

$$\frac{c_N}{l_N} = \Omega\left((1-\tau)h(X)\right) \quad \Leftrightarrow \quad c_N = l_N \cdot \Omega\left((1-\tau)h(X)\right) . \tag{3.13}$$

Using the same stages than with the absolute transfer:

$$l_N^{\star\star} = \frac{(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)} \quad \text{and} \quad c_N^{\star\star} = \frac{\Omega(\cdot)(1-\tau)h(X)}{(1-\tau)h(X)+\Omega(\cdot)} . \tag{3.14}$$

Recall that $c_S = \tau c$ and $c_N = (1-\tau)c$. Hence $c_S = \frac{\tau}{1-\tau}c_N$. It comes

$$c_S^{\star\star} = \frac{\Omega(\cdot)\tau h(X)}{(1-\tau)h(X)+\Omega(\cdot)} . \tag{3.15}$$

Let me now study the shape. For the North, since its constraint set is diminishing with respect to the tax rate, the utility of the North mechanically reduces.

$$\frac{\mathrm{d}u_N^{\star\star}}{\mathrm{d}\tau} < 0 . \tag{3.16}$$

For the South, with $c_S = \tau c$,

$$\frac{\mathrm{d}u_S^{\star\star}}{\mathrm{d}\tau} = \frac{\partial u_S^{\star\star}}{\partial c_S}\frac{\mathrm{d}c_S^{\star\star}}{\mathrm{d}\tau} = \underbrace{\frac{\partial u_S^{\star\star}}{\partial c_S}}_{>0}\left(\underbrace{c}_{>0} - \underbrace{\tau h(X)}_{>0}\underbrace{\frac{\mathrm{d}l_N^{\star\star}}{\mathrm{d}\tau}}_{?}\right) . \tag{3.17}$$

It is straightforward to see that if $\frac{\mathrm{d}l_N^{\star\star}}{\mathrm{d}\tau} \leq 0$ then $\frac{\mathrm{d}u_S^{\star\star}}{\mathrm{d}\tau} > 0$. But, otherwise, one cannot directly conclude. Unfortunately, the analysis of the sign of $\frac{\mathrm{d}c_S^{\star\star}}{\mathrm{d}\tau}$ is meaningless.[9] Rather, I study $\frac{\mathrm{d}c_S}{\mathrm{d}\tau}$ as a function of c_N. One has $c_S = \frac{\tau}{1-\tau}c_N = \frac{\tau}{1-\tau}l_N \cdot$

[9] $\frac{\mathrm{d}c_S^{\star\star}}{\mathrm{d}\tau} > 0 \Leftrightarrow \tau < \frac{h(X)+\Omega((1-\tau)h(X))}{\xi^{\star\star}h(X)}$. As $\xi = \frac{\mathrm{d}\Omega(\cdot)}{\mathrm{dMRS}}\frac{\mathrm{MRS}}{\Omega(\cdot)}$, at the optimum $\xi^{\star\star} = \frac{\Omega'(\cdot)}{\Omega(\cdot)}(1-\tau)h(X)$.

$\Omega((1-\tau)h(X))$. One obtains

$$\frac{\mathrm{d}c_S}{\mathrm{d}\tau} > 0 \quad \Leftrightarrow \quad 1 + (1-\tau)\frac{\mathrm{d}l_N}{\mathrm{d}\tau}\frac{\tau}{l_N} - \tau\xi > 0\,. \tag{3.18}$$

Let me now compute this inequality with the optimal leisure time $l_N^{\star\star}$.

$$\left.\frac{\mathrm{d}c_S}{\mathrm{d}\tau}\right|_{l_N^{\star\star}} > 0 \quad \Leftrightarrow \quad \tau < \frac{1}{1 + l_N^{\star\star}(\xi^{\star\star} - 1)}\,. \tag{3.19}$$

It is always positive. It can directly be seen that the right-hand side (RHS) is lower than one if and only if the elasticity of substitution is higher than one (the case $\xi = 1$ is obtain at the limit). To summarize:

$\text{sign}\left(\frac{\mathrm{d}u_S^{\star\star}}{\mathrm{d}\tau}\right)$	RHS ≥ 1	RHS < 1
$\xi^{\star\star} \leq 1$	+	Impossible
$\xi^{\star\star} > 1$	Impossible	+/-

□

Here, the frontier is parametrized by the tax rate. Without surprise the tax is always negative for the North. The tax reducing its budget set, the utility reached is lower. But for the South the result is not straightforward. If North works more when the tax rate increases, the amount received by South increases, because one taxes more a higher basis. If North works less, the effect is *a priori* ambiguous, because one taxes more a lower basis. Finally, if North is indifferent, South is also better-off, because one taxes more a constant basis.

Intuitively, the reaction of North to the tax depends on its elasticity of substitution of leisure for consumption. In particular, if it considers leisure and consumption as quite complementary ($\xi \leq 1$), the more North is taxed, the more it works (or does not react to), and then South is getting better and better as the tax rate grows. On the opposite, if they are quite substitutable ($\xi > 1$), the amount received by South increases if and only if the tax rate is not too high. Unfortunately, an explicit value of the threshold cannot be obtained.[10] Nonetheless, let me note $\bar{\tau}$ the threshold resulting of the inequality $\tau < \frac{1}{1+l_N^{\star\star}(\tau)(\xi^{\star\star}(\tau)-1)}$.

[10] As shown later, even with explicit functional forms.

The right-hand side being given by the optimal leisure time and the elasticity of substitution (evaluated at the optimum if not constant). Notice that this is close to the concept of the Laffer curve (see Laffer, 2004), with South 'playing role' of the State. South takes advantage of a higher tax until a certain point ($\bar{\tau}$), from which on, a higher rate reduces the amount perceived.

A non-common movement between the tax rate and and the labor supply is conceivable. But it is hard to argue that for any level of the tax rate, the higher the tax rate the higher (or constant) the labor supply ($\xi \leq 1$). I will rather assume in the next subsections that North always works less when the tax rate increases ($\xi > 1$).[11]

To sum up, under the assumption of an increasing labor supply, the utility of North is always decreasing with respect to the tax rate while the utility of South increases until a threshold and decreases afterward. Thus, this describes a 'bell curve' frontier. But actually only the decreasing part matters, since the increasing one represents states from which both regions can be better-off. These states are Pareto-dominated, and thus not of interest from a welfare point of view.[12]

Example Consider the same previous restrictions. The optimal utilities are:

$$u_N^{\star\star} = \left(\alpha \left(\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{\xi} ((1-\tau)qX)^{\xi-1}} \right)^{\rho} \right.$$

$$\left. + (1-\alpha) \left(\frac{1}{((1-\tau)qX)^{-1} + \left(\frac{1-\alpha}{\alpha}(1-\tau)qX\right)^{-\xi}} \right)^{\rho} \right)^{\frac{1}{\rho}} ;$$

$$\text{and} \quad u_S^{\star\star} = \left(\frac{\tau}{(qX)^{-1} + \left(\frac{1-\alpha}{\alpha}qX\right)^{-\xi} (1-\tau)^{1-\xi}} \right)^{\gamma} . \tag{3.20}$$

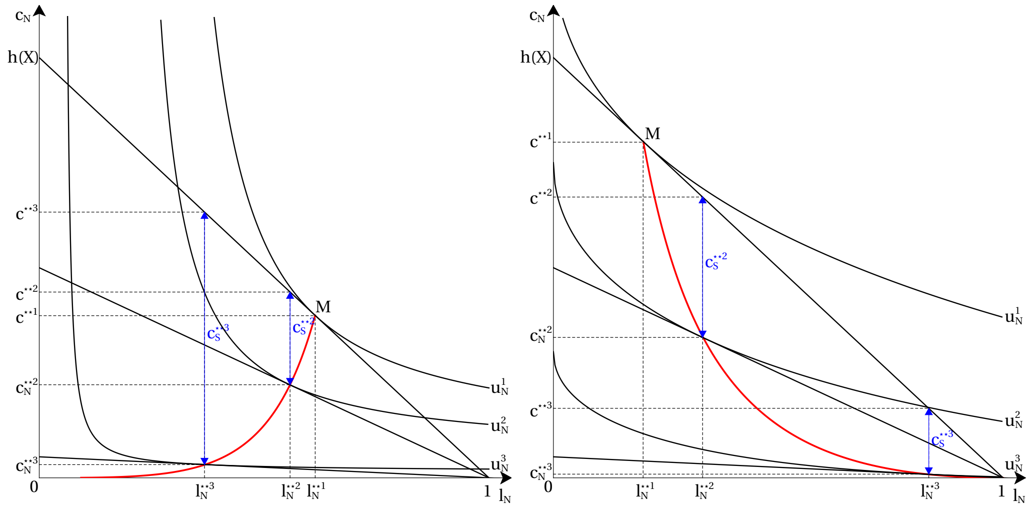
$$\text{Inequality of the threshold: } 1 - \xi\tau + \left(\frac{1-\alpha}{\alpha}\right)^{\xi} (qX)^{\xi-1}(1-\tau)^{\xi} > 0 \, . \tag{3.21}$$

In this framework, an explicit second-best frontier is not easy to obtain. A

[11]Considering both cases is also unnecessary since, qualitatively, the second-best approach is very close to the first-best one for $\xi \leq 1$.

[12]Nonetheless, this could be used to fuel the debate on high-tax rate countries, but this is left for future research.

graphical illustration of the two cases is plotted in Fig. 3.2.[13] As said, only the case $\xi > 1$ will be consider afterward. The corresponding frontier in utility map will be given in the subsection 3.2.4.



(a) Low elasticity of substitution ($\xi < 1$) (b) High elasticity of substitution ($\xi > 1$)

Figure 3.2: Construction of the utility possibility frontier with tax

Three levels of tax rate are plotted in Fig. 3.2a and Fig. 3.2b. First, a situation with no transfer is represented at M. In the two cases, the consumption of South is nil, $c_S^{\star\star 1} = 0$, and the consumption of North is maximal and is equal to the global consumption, $c_N^{\star\star} = c^{\star\star 1} = 0$.[14] When the tax rate rise, the cases diverge. Fig. 3.2a plots a case similar to the first-best case. The higher the tax rate, the higher the consumption of South and the lower the consumption of North. The curve $0M$ depicts a 'contraction path'. And the global consumption rises with respect to the tax. On the opposite, Fig. 3.2b plots a situation very different than the first-best case. The consumption of South is increasing with respect to the tax rate for low rates, $0 = c_S^{\star\star 1} < c_S^{\star\star 2}$, and decreasing for high rates, $c_S^{\star\star 2} > c_S^{\star\star 3}$.[15] Of course, the consumption of North decreases always with respect to the tax

[13] For the low and the high elasticity cases, values are respectively: $\rho_1 = -0.5$ and $\rho_2 = 0.5$. Values of the tax rate are successively $\tau = 0; 0.5; 0,95$.

[14] The two plots have the same normalization. The difference of $c^{\star\star 1}$ in the two cases come from the difference of elasticity of substitution ξ.

[15] In this example, the threshold is $\bar{\tau} = \frac{5-\sqrt{5}}{4} \approx 0.7$.

rate. But in this case, the global consumption decreases with respect to the tax.

Let me now compare the different implications of the two instruments, lump-sum transfer and tax.

3.2.4 Comparison of the two frontiers

Let me begin with the following proposition.

Proposition 7 (Comparison of the frontiers)**.** *The second-best utility possibility frontier lies below the first-best utility possibility frontier.*

Proof of the Proposition 7. For a given utility of North, let us seek the highest amounts one can transfer to South, whatever the mechanism used. Let $\tilde{c}_N(l_N)$ be the image of an indifference curve of North. And let us maximize the gap between the production and that curve.

$$\max_{l_N} \quad c_S = (1 - l_N)h(X) - \tilde{c}_N(l_N) \ . \tag{3.22}$$

A necessary condition is the equalization of the MRS with the marginal productivity: $\text{MRS} = h(X)$. As this is done with the lump-sum transfer, this mechanism is then the most favorable for the South. For a given utility of the North, the first-best frontier corresponds well to the highest utility for the South. Hence, the utility possibility set with the lump-sum transfer contains the set with the tax. When no transfer occurs, since net and gross productivity are equivalent, the two frontiers coincide. □

For a given utility of the North, the tax is always less favorable for the South than the lump-sum transfer. Indeed, the tax on the production discourages the North who works less and then harvests less. And if the tax rate is higher than the threshold $\bar{\tau}$, increasing its rate leads to worsen their both situations. As these states are not of interest in my framework, I do not consider high tax rates. Fig 3.3 plots the two frontiers.[16]

The difference between the two frontiers, depicted by the gray area in the figure, represents the inefficiency of the tax. This illustrates a well-known fea-

[16] With the same previously mentioned numerical values. Especially $\rho = 0.5$.

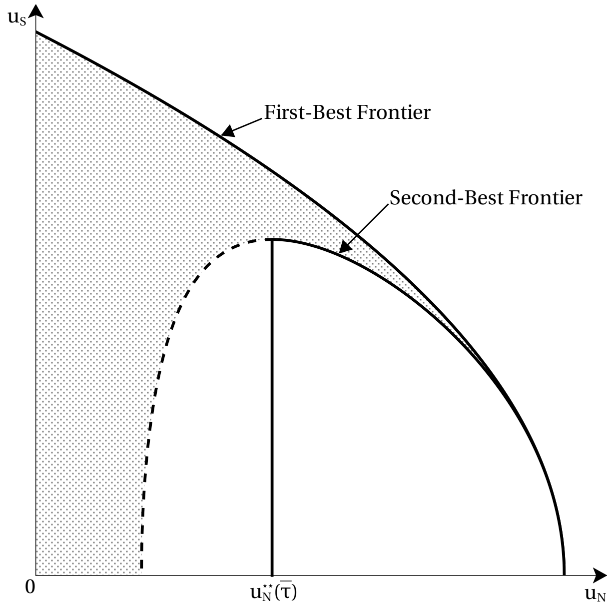


Figure 3.3: Comparison of the first-best and the second-best frontiers

ture. Tax impacting choices, the labor-leisure trade-off here, they impact the effort and therefore the global quantity to share. From a social point of view, lump-sum transfers seem preferable over taxes, as they allow more choices.

As the impact of the tax depends on the reaction of North, one may expect the difference between the two frontiers to be dependent on preferences of North. Let us verify this with an example.

Example I only need the restriction of CES utility function of the North here.

The function that links the MRS and the optimal consumption-leisure ratio is of the form: $\Omega(z) = \left(\frac{1-\alpha}{\alpha} z\right)^{\xi}$. The difference between the budget constraint (a straight line) and the indifference curve (strictly convex) is strictly concave in (l_N, c_N). We know that the optimum is reached at the first-best optimal consumption-leisure ratio $\Omega(h(X))$. So, for a given utility of the North, the higher is the difference between the first-best and the second-best optimal ratios, the higher is the distance between the two frontiers. Let the difference between the two optimal ratios be noted D:

$$D(\xi) \equiv \Omega(h(X)) - \Omega((1-\tau)(h(X)) = \left(\frac{1-\alpha}{\alpha}h(X)\right)^{\xi}\left(1-(1-\tau)^{\xi}\right).$$

For $0 < \tau < 1$, it can easily be shown that $\frac{\mathrm{d}D}{\mathrm{d}\xi} \geq 0$ as long as $\frac{1-\alpha}{\alpha}qX \geq 1$.[17] We then may expect the previously mentioned intuitive feature to hold most of the time, but not always. Besides, note that $\lim_{\xi\to 0} D(\xi) = 0$.

To summarize, the higher the (constant) elasticity of substitution, the higher (generally) the gap between the two instruments. At the limit, if leisure and consumption are perfectly complementary, the instrument used does no longer matters since the two frontiers merge together.[18] This latter result is not surprising since in this case the (implicit) relative prices do not matter for the choice of the North. Only in this very specific case, lump-sum and tax are equivalent.

Those frontiers represent the efficient possible allocations. Let us now find the optimal ones from a social welfare point of view.

3.2.5 Welfare analysis

I am now interested in finding the optimal allocation of utilities, which indicates implicitly the optimal (absolute or proportional) transfer. For that let us consider society changes its inequality aversion, for whatever reasons. In this perspective, I will see the consequence of a change in the inequality aversion (IA) on the optimal allocation. Let us recall that a higher IA corresponds to a lower elasticity of substitution ($\mathrm{d}\theta < 0$). The welfare analysis is qualitatively the same with both mechanisms. I present it only with the lump-sum.

In general terms, one has to seek the maximal welfare subject to the fact that utilities are elements of the utility possibility set. Here, as maximization problem has already been solved, we can maximize directly the welfare through

[17] If not, this holds only if τ is under a threshold implicitly given by $(1-\tau)^2\ln(r(1-\tau)) < \ln(r)$. With my numerical values, the condition always holds since $\frac{1-\alpha}{\alpha}h(X) = 4 \geq 1$.

[18] I exclude the perfect substitutability case to avoid unrealistic corner solutions (i.e. no work or no leisure).

the amount transfered. The economy will always be situated on the frontier.

$$\max_{T} \quad W^{\star}(T) = \left(a_N \cdot u_N^{\star}(T)^{\eta} + a_S \cdot u_S^{\star}(T)^{\eta}\right)^{\frac{1}{\eta}} \ . \tag{3.23}$$

Let me define the marginal social rate of substitution (MSRS) as the willingness of society to increase marginally the utility of the South taking from the utility of the North, keeping its global welfare equal. In the same vein, let me define the marginal social rate of 'transformation' (MSRT) the marginal utility gain of South for an infinitesimal loss of North.[19] Let $\lambda := \frac{u_S}{u_N}$ be the utility ratio.

Proposition 8 (Optimal allocation)**.** *An optimal allocation of the problem* (3.23) *satisfies the property that the utility ratio is a function of the MSRT: $\lambda^{\star} = MSRT^{\star\theta}$. It increases with respect to the inequality aversion if and only if it is lower than one: $\frac{d\lambda^{\star}}{-d\theta} \geq 0 \Leftrightarrow \lambda^{\star} \leq 1$.*

Proof of the Proposition 8. The first-order condition gives:

$$\frac{\partial W^{\star}(T)}{\partial T} = 0 \quad \Leftrightarrow \quad \frac{u_S^{\star}}{u_N^{\star}} = \left(\frac{a_S}{a_N}\right)^{\theta} \left(\frac{\frac{\partial u_S^{\star}}{\partial T}}{-\frac{\partial u_N^{\star}}{\partial T}}\right)^{\theta} \ . \tag{3.24}$$

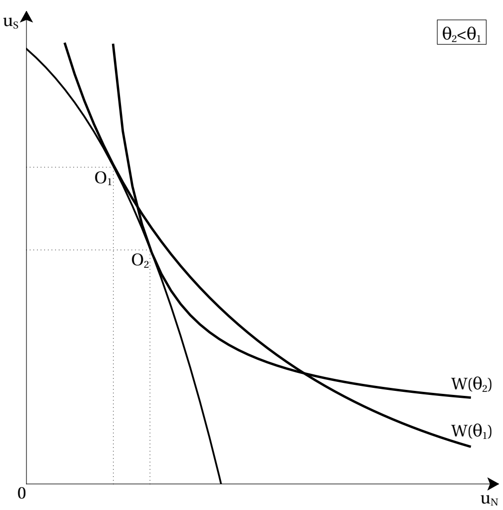
$$\frac{d\lambda^{\star}}{-d\theta} \geq 0 \ \Leftrightarrow \ \mathrm{MSRT}^{\star\theta} \ln\left(\mathrm{MSRT}^{\star}\right) \leq 0 \ \Leftrightarrow \ \mathrm{MSRT}^{\star} \leq 1 \ \Leftrightarrow \ \lambda^{\star} \leq 1 \ . \tag{3.25}$$

□

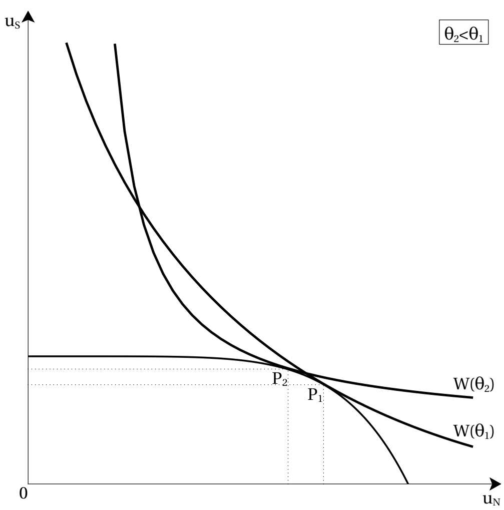
The previous proposition depicts two interesting features. First, it links the optimal utility ratio with two fundamental elements: one absolute, the intratemporal IA, and one relative to the situation, the MSRT. It shows in a simple way how the optimal transfer depends upon both on will and on constraints. For example, a pure equality situation ($\lambda^{\star} = 1$) is possible only in two situations. Either if the gain of South is locally the same than the loss of North ($\mathrm{MSRT}^{\star} = 1$). Or if the intratemporal IA is infinite ($\theta \to 0$). But note that in this last case, it holds

[19]Formally, $\mathrm{MSRS} := \left| \left.\frac{du_S}{du_N}\right|_W \right| = \frac{\frac{\partial W}{\partial u_N}}{\frac{\partial W}{\partial u_S}} > 0$. $\mathrm{MSRT} := \left| \frac{du_S^{\star}(T)}{du_N^{\star}(T)} \right| = \frac{\frac{\partial u_S^{\star}}{\partial T}}{-\frac{\partial u_N^{\star}}{\partial T}} > 0$. At the optimum: $\mathrm{MSRS}^{\star} = \mathrm{MSRT}^{\star}$.

only at the limit. Which means that a perfect equality may never arise. I will come back to this point later on. Second, the proposition tells us that a higher IA benefit to South (higher $\lambda^\star$) only if South is worse-off ($\lambda^\star < 1$).[20] That is to say, if the South is better-off, a higher IA *reduces* the optimal transfer. This could seem surprising: how does a higher IA can imply to less redistribute toward a region that is constrained on its consumption? Actually, this is quite logical. For example if one considers the South to have a higher utility compared to North (for the same consumption and leisure levels), the original transfer (before the variation of IA) may be too'heavy' for North. Augmenting the IA allow a higher utility of North and a lower of South. Note that this feature is not possible in the intertemporal dimension since the transfer is possible in only one dimension. Fig. 3.4 represents the two situations: one where the South is originally better-off (3.4a) and one where the North is originally better-off (3.4b).[21] Each figure plots a frontier with two social indifference curves, depending on IA. If South



(a) South is well-off



(b) North is well-off

Figure 3.4: Different inequality aversions with two types of frontier

is relatively better-off in the original situation, the higher the IA (the elasticity of substitution pass from θ_1 to θ_2) the lower the utility ratio (from O_1 to O_2).

[20] In the very specific case where $\lambda^\star = 1$, the IA has no influence on λ.
[21] It is a stylized feature, no meaningful numerical values are used here.

This implies to decrease the optimal transfer. On the contrary, if North better-off in the original situation, the higher the IA the higher the corresponding utility ratio (from P_1 to P_2). This implies to increase the optimal transfer from North to South. Whatever the initial situation, the higher the IA, the more one goes toward an egalitarian situation. But depending on the shape of the frontier, a perfect equality may never arise.[22] To be consistent with my original issue – where South is limited on its consumption – I will only consider the case where North is originally better-off and has to make a transfer positively linked with the IA (Fig. 3.4b). But the counter intuitive case (Fig. 3.4a) is actually useful for shedding light on the measure of utility.[23]

Let me open a parenthesis before turning to future consequences of a variation of intratemporal inequality aversion. The eq. (3.24) shows clearly the role played by individual weights. For example, the higher a_S, the higher the utility of South and the lower the utility of North. As argued in the previous chapter, weights can offset undesired consequences (e.g. a too high transfer). But here also, we see that better to address such issues through the inequality aversion parameter. Note that, like in the intertemporal dimension, when the IA is infinite, weights do not matter (Appendix A.1).

3.3 Intra and intergenerational considerations

Sustainability can be measured through net investment, once one adopts a value function (Asheim, 2007). Without a particular formulation of such a function, one can assume it to be non-declining with respect to the resource stock. Formally, the evolution of the value function over time is (for an autonomous problem)

$$\dot{V}(X) = \underbrace{\frac{\partial V(X)}{\partial X}}_{\mu} \dot{X} \; . \tag{3.26}$$

[22]Rawls (1974, p. 145) was perfectly aware of this feature when he wrote "*historically [the maximin criterion] has attracted little attention, and yet it is a natural focal point between strict equality and the principle of average utility*".

[23]A figure illustrating the impact of utility of South on the frontiers is plotted in Fig. A.7, p. 131.

The value function is increasing over time if genuine savings is positive (shadow-price μ times the evolution of the stock). But this is an indicator and cannot directly be used to measure intergenerational equity. I will thus simply focus on the instantaneous welfare, but verifying that the value function does not decline along a path.[24]

I will analyze firstly how the inequality aversion impacts the current global consumption. Secondly, I will see how the evolution of the resource, in turn, transforms the welfare possibilities.

3.3.1 Inequality aversion and the current consumption

Let me recall that, with the previous restrictions, the higher the IA, the higher the transfer. Then the utility of North decreases and the utility of South increases. But the consequences of an higher IA on the current global consumption is not the same according to the redistribution mechanism used.

With a lump-sum, North 'compensate' the transfer by working more. As the harvesting increases with work, global consumption increases (as depicted in Fig. 3.1). With a tax, on the opposite, North is 'discourage' to work. As the harvesting decreases with leisure, global consumption decreases (as depicted in Fig. 3.2b). In the specific case of equal utilities, the inequality aversion has no influence on the redistribution, it has then no influence on the global consumption too.

3.3.2 Evolution of the resource and the possibilities for futures welfares

The redistribution influences the current consumption which has consequences on the evolution of the resource stock. And reciprocally, the evolution of the resource stock has consequences on the possibilities of consumption and then on possibilities of redistributions. If the resource stock varies, I have to determine beforehand to whom will go that supplement (variations of X on $\lambda^{\star}$). This should

[24]In the formulation of the Chapter 1: if the maximin would be pursued, the net-investments rule would dictate that investment in South compensate the disinvestment in North: $\mu_S \dot{X}_S = -\mu_N \dot{X}_N$.

be done according to the social welfare function and the shape of the frontier. But, to begin, let us study the deformation of the frontier due to the variation of the resource stock.

Proposition 9 (Variations of the frontiers)**.** *For $X' > X$, the utility possibility frontier associated with X lies strictly below the utility possibility frontiers associated with X'.*

Proof of the Proposition 9. By construction, $\frac{\mathrm{d}c}{\mathrm{d}X} > 0$. With lump-sum: $c_N = c - T$ and $c_S = T$. Everything else begin equal, the utility of the North increases with respect to the stock and the utility of the South remains constant. With tax: $c_N = (1-\tau)c$ and $c_S = \tau c$. Everything else begin equal, both utilities increases. □

As expected, the frontiers shift outward with an increase of the resource stock. An illustration in given in Fig. 3.5. First and second-best frontiers are plotted with two values of the resource stock. Blue and solid line corresponds to the benchmark. Red and dashed line corresponds to a higher stock level.[25] As

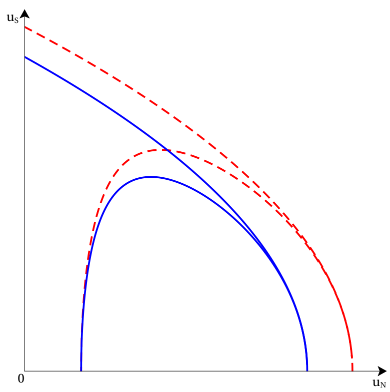


Figure 3.5: Sensitivity of the frontiers to the resource stock

the constraint set is augmenting, the welfare is unambiguously increasing with respect to the resource stock.[26] I can then link the evolution of the welfare with the evolution of the resource. Hence, a variation of the inequality aversion has

[25]Numerical values: $X = 5$ for the blue and solid line curves and $X = 6$ for the red and dashed line ones.

[26]I have not been able to prove that the optimal utilities are effectively increasing. I suppose they are.

a direct effect on the current generation through the transfer and an indirect one on future generations through the evolution of the stock.

3.3.3 Interactions between the two dimensions

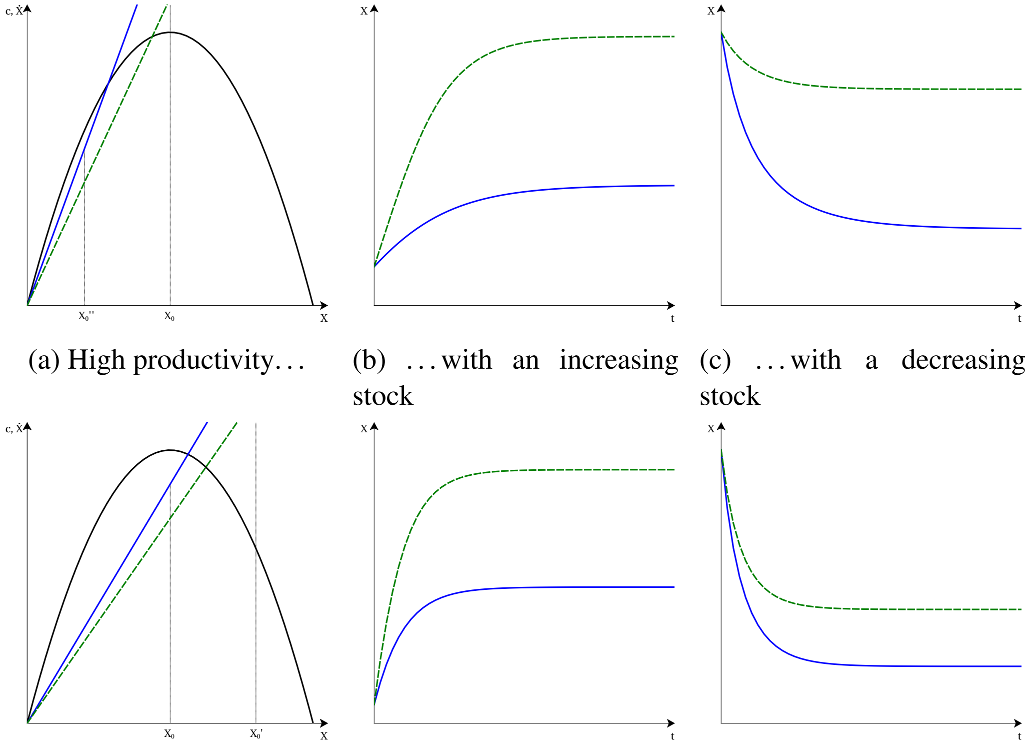
Let us recall that an increasing inequality aversion implies a rising global consumption with a lump-sum transfer, but a declining global consumption with a proportional tax. The intergenerational concern should be based on the evolution of welfare along time. More precisely, we should analyze the modification of the welfare path after a change in the inequality aversion. But as we know that the welfare is positively linked with the resource stock (Proposition 9), we can focus on the resources paths. Four cases have to be distinguished depending on, on the one hand, whether the resource stock is rising or declining, and one the other hand, whether the steady-state stock level is higher or lower than the golden-rule one[27]. The steady-state stock level depends on the work productivity of the North. I consider a work productivity as being relatively low if the steady-state stock level if superior to X^{GR}, and vice versa.

Fig. 3.6 plots the different situations, with a logistic growth and a linear catch-effort curve.[28] Blue and solid line curves plot a benchmark situation. Green and dashed line ones plot a situation with a lower consumption. I plotted a high productivity situation (a), and corresponding paths with a rising stock (b) and a decreasing one (c). Symmetrically, I plotted a low productivity situation (d), with a rising stock (e) and a decreasing one (f). In all the situations, a higher consumption (from green to blue) impacts negatively the resource – either the growth is lower or the decreasing is higher – and it converges to a lower steady state.

Interestingly, Fig. 3.6b can be related to Chapter 2 and Fig. 3.6c can be related to Chapter 1. In Fig. 3.6b, passing from blue to green represents a higher sacrifice in the North, the stock grows faster and toward a higher steady state. In Fig. 3.6c, passing from blue to green represents a lower disinvestment in the North – to compensate a lower need of investment in the South, – the stock declines slower

[27]By definition, a steady state $X^{ss} > 0$ satisfies $\dot{X} = 0 \Leftrightarrow F(X^{ss}) = (1 - l_N^\star)h(X^{ss})$.

[28]It is a stylized feature, no meaningful numerical values are used here.



(a) High productivity. . .
(b) . . . with an increasing stock
(c) . . . with a decreasing stock
(d) Low productivity. . .
(e) . . . with an increasing stock
(f) . . . with a decreasing stock

Figure 3.6: Resource paths as indicators of intergenerational welfare

and toward a higher steady state. In this last case, an increasing consumption in the North may be sustainable (from green to blue in Fig. 3.6c) if the supplement is transfered to the South, to grow out from poverty for example.

Which instrument the social planer of this economy shall use when the IA comes to increase? Let us take the golden rule as a reference target for the North. If the North has a relatively high productivity, the social planer shall seek to decrease its consumption. This is made possible by a tax proportional to its production. The tax, reducing the effort, reduces the harvesting and then makes the resource stock grow toward a higher level. On the contrary, if the North has a relatively low productivity, the social planer shall seek to increase its consumption. This is made possible by a lump-sum transfer. The transfer encourages work to compensate the withdrawn amount. More effort means more harvesting, and thus the resource stock grow toward a lower level. A last feature is that a huge decreasing of consumption may be such that one passes from a high-work productivity case to a low-work productivity case, i.e. the steady-state stock passes from below to above the golden-rule one. In that case, intragenerational decisions can be seen as being too 'generous' toward the future and lead to over-accumulation.

These results illustrate, in a sense, the independence, facilitations and rivalries between the two dimensions guessed by Baumgärtner et al. (2012). Here, except in a very specific case of perfect equality, either the two dimensions facilitate each other – using the right mechanism – or trade-offs have to me made – when using the right mechanism is not possible.

3.4 Conclusion

I depicted a world where one region – North – has a free access to its resource while another – South – has a limited access to its resource. This is a parable of a relative higher sacrifice asked to the South since its lower stock is marginally more productive. In such a world, society wants to implement a transfer of a part of the harvest from the North to the South. I studied the effects of an absolute transfer and of a relative one based on the resource caught. I constructed utility possibility frontiers in each case. They represent the necessary trade-offs a society has to make when it wants to enhance the well-being of one region at the expense of another. Not surprisingly, the utility possibility set with a tax is lower than the one with a lump-sum transfer. The inefficiency of the tax depends on the preferences of the North. Besides, the social criterion allowed for find the social optimum, and I was particularly interested in its variation due to a change in the intratemporal inequality aversion. This depends on the initial situation and on the mechanism of transfers used. In particular, I found that society may afford a higher inequality aversion 'without worsening' the future only using the right mechanism. If the North is relatively productive, the transfer shall be made through a tax. And if the North is relatively unproductive, the transfer shall be made through a lump-sum. In any case, the higher intratemporal inequality aversion leads to a convergence between the welfare of the North and the welfare of the South.

A.1 Special cases of CES functions

It could seem to be a pointless exercise to demonstrate a very well-known result. But, to my knowledge, a rigorous demonstration of the minimum is not present in the literature.

Proof. Let me consider a continuously differentiable function with a constant elasticity of substitution $\frac{1}{1-\rho}$ such that: $f(x_N,\ldots,x_n) = \left(\alpha_N x_N^\rho + \cdots + \alpha_n x_n^\rho\right)^{\frac{1}{\rho}}$. With $\sum_{i=1}^{i=n} \alpha_i = 1$, $\alpha_i > 0\ \forall i$ and $\rho < 1, \rho \neq 0$.

- Case 1: the elasticity tends to positive infinity. Trivially, if $\theta \to \infty$ ($\rho \to 1$), the CES function tends to the perfect substitutes function.

$$\lim_{\rho \to 1} f(x_N,\ldots,x_n) = \alpha_N x_N + \cdots + \alpha_n x_n \,. \tag{A.27}$$

- Case 2: the elasticity tends to zero.[29] Let me rewrite the CES function as:

$$f(x_N,\ldots,x_n) = x_k \left(\alpha_N \left(\frac{x_N}{x_k}\right)^\rho + \cdots + \alpha_k + \cdots + \alpha_n \left(\frac{x_n}{x_k}\right)^\rho\right)^{\frac{1}{\rho}} . \tag{A.28}$$

Let $\min\{x_N,\ldots,x_n\} = x_k$. Then

$$\lim_{\rho \to -\infty} \left(\frac{x_i}{x_k}\right)^\rho = 0,\ \forall i \neq k \,. \tag{A.29}$$

Thus

$$\lim_{\rho \to -\infty} x_k \left(\alpha_N \left(\frac{x_N}{x_k}\right)^\rho + \cdots + \alpha_k + \cdots + \alpha_n \left(\frac{x_n}{x_k}\right)^\rho\right)^{\frac{1}{\rho}} = x_k \,. \tag{A.30}$$

As x_k can be any good,

$$\lim_{\rho \to -\infty} f(x_N,\ldots,x_n) = \min\{x_N,\ldots,x_n\} \,. \tag{A.31}$$

[29] I am grateful to Jean-Baptiste Michau for having provided to me this proof. I also thank Théo Benonnier for having informed me of its existence.

- Case 3: for exhaustiveness, let the elasticity tend to one. Let me take the logarithm of $f(\cdot)$:

$$\ln\left(f(x_N,\ldots,x_n)\right) = \frac{\ln\left(\alpha_N x_N^{\rho} + \cdots + \alpha_n x_n^{\rho}\right)}{\rho} \ . \tag{A.32}$$

Let $g_N(\rho)$ and $g_S(\rho)$ be equivalent, respectively, to the numerator and to the denominator. As $\lim_{\rho\to 0}\, g_N(\rho) = \lim_{\rho\to 0}\, g_S(\rho) = 0$, by the Hospital's rule:

$$\lim_{\rho\to 0}\frac{g_N(\rho)}{g_S(\rho)} = \lim_{\rho\to 0}\frac{g_N'(\rho)}{g_S'(\rho)} \ , \tag{A.33}$$

and as

$$\frac{g_N'(\rho)}{g_S'(\rho)} = \frac{\alpha_N x_N^{\rho}\ln(x_N) + \cdots + \alpha_n x_n^{\rho}\ln(x_n)}{\alpha_N x_N^{\rho} + \cdots + \alpha_n x_n^{\rho}} \ , \tag{A.34}$$

one gets

$$\lim_{\rho\to 0}\ln\left(f(x_N,\ldots,x_n)\right) = \alpha_N\ln(x_N) + \cdots + \alpha_n\ln(x_n) \ . \tag{A.35}$$

Finally,

$$\lim_{\rho\to 0} f(x_N,\ldots,x_n) = x_N^{\alpha_N}\ldots x_n^{\alpha_n} \ . \tag{A.36}$$

□

A.2 Illustration: Sensitivity to the utility of the South

Let me keep all the numerical values used in the examples until now, and make γ passing from 0.5 to 0.2. Fig. A.7 represent in an orthonormal plot the first-best and the second-best frontiers for those two values of gamma. The blue and solid line ones with $\gamma = 0.5$. The red and dashed line ones with $\gamma = 0.2$.

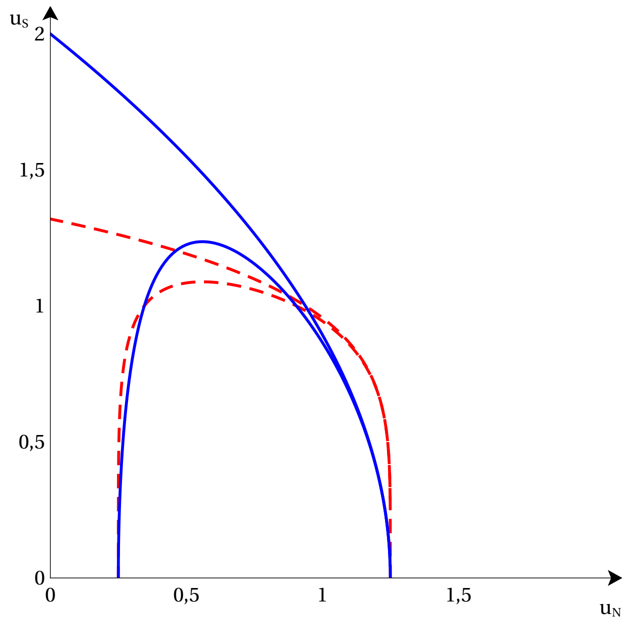


Figure A.7: Sensitivity of the frontiers to the utility of the South

General conclusion

Although universal, we saw that the notion of equity remains plural. This dissertation focused on studying how interact freedom of choice between sharing resources among different individuals and sharing it among different generations. The interactions depend both on ethical considerations and on physical constraints. Mathematical modeling choices play also, necessarily, a determinant role in this relationship. Let me summarize the results before analyzing limits and prospects of the dissertation.

3.2.1 Summary

Throughout the dissertation, I considered a world composed of two regions, North and South. Each one is endowed with a stock of a renewable resource. But it was assumed that the one of the South was less abundant than the one of North, and thus grow marginally more rapidly. Over an infinite horizon, a welfarist social planner 'embodied' social ethical considerations. In particular, I gave the same weight to each individual as well as to each generation. The difficult issue of measuring utility was handled in three stages, with an aim to obtain exploitable conclusions. In the first chapter, utility functions were assumed to be identical and linear. In the second chapter, they were assumed to be identical and strictly concave. And in the third chapter, they were assumed to be different and strictly concave.

I began, in Chapter 1, by studying the objective of sustaining the highest level of welfare over time. This is made possible by a well-known criterion, the maximin. As its name suggests, it seeks to maximize the welfare of the worst-off generation. This leads, most of the time, to a constant intertemporal welfare. As each generation has the ability to benefit of this level of welfare, it has been argued that it has strong resonance with sustainability objective. But even though the welfare remains constant, regional consumption levels do not stay still. North over-consumes (compared to the individual maximin consumption), then its stock becomes marginally more and more productive. On the contrary, South under-consumes, then its stock becomes marginally less and less productive. This illustrates a convergence between North and South until stocks become equally productive. Hence, inequalities reduce over time, but it requires to worsen them during the transition (compared to a situation where each region is considered in isolation). The inequality aversion plays, reciprocally, a role on the optimal welfare. Generally, the higher the inequality aversion, the lower the sustainable welfare level. This comes from the fact that when inequalities are less tolerated, the economy can less 'take advantage' of unequal productivities. Two limiting cases of intratemporal social welfare function were studied. The first one is the intratemporal maximin, with an infinite inequality aversion. In this case, the sustainable welfare is at its lowest level, given by the production of the absolutely less productive stock (the one of the South). The second one is the utilitarianism, with a nil inequality aversion. In this case, the sustainable welfare is only supported by the consumption of North. Substitution between consumption of the North and of the South is complete, and the South shall consume nothing during the transition. In this last case, transfers become vital. But maximin has been criticized because an economy may be locked to a low welfare level. Decisions that would lead to an increase of the maximin welfare were also considered. They imply savings from the current generation. But it was showed that such 'sacrifices' have to be coordinated to lead effectively to an improvement of the welfare level.

Chapter 2 studied more directly such an objective, through the Ramsey criterion. The social planner maximizes the intertemporal social welfare in order to reach the highest sustainable level of welfare, the golden rule. In a world of scarcity, growth requires savings. Here both North and South under-consume, and their stocks become marginally less and less productive to be equal to zero at the steady state. In a sense, North and South converge, but the relative sacrifice is generally not evenly shared. The difference in consumption growth rates (indicating a high savings rate) depends on three elements. The marginal productivity gap (physical constraint) and the intra and intergenerational inequality aversions (ethical considerations). The relationship in not always unambiguous. But we can retain that the more the stock of the South is productive and the lower are the inequality aversions, the higher will be the relative sacrifice of South. The sacrifice is negatively related to each inequality aversion for different reasons. The less intratemporal inequalities are tolerated, the less the economy can 'take advantage' of the unequal productivities (as in the first chapter). But the less intertemporal inequalities are tolerated, the less we can substitute current for future welfares, and then the less we make sacrifices. Of course, sacrifices are evenly shared if marginal productivities are, at each time, equal. Unequal weights were eventually considered to study their consequences on allocations. Neither intra nor intertemporal (discounting) weights impact determinately the results. They impact the evolution of consumptions, but not their relative growth. That said, introducing a discount rate reduces the sacrifice asked to each generation. Consequently, the economy grows toward a lower steady state, the modified golden rule. This allowed me to discuss discount rate as a tool to limit sacrifice. I came globally to two conclusions. Firstly, in a productive economy the discount rate cannot be advocated on the sole basis of (intertemporal) inequality aversion. Secondly, giving an equal weight to South and North turned out to be not enough. Transfers from North to South have to be implemented both for ethical and pragmatic reasons.

Chapter 3 dealt with transfers. The North is assumed to have a free access to its stock while the South has a limited access to its stock. More precisely, a constraint is imposed on the harvesting of the South, which has a benchmark utility level. Transfers from North to South are implemented to compensate the lower utility level due to the constraint. Two mechanisms were studied. The first one is a lump-sum: a given amount of available consumption in North is transfered. The second one is a tax: a given proportion of available consumption in North is transfered. This leaded to two different utility possibility sets, bonded respectively by a 'first-best' and a 'second-best' frontiers. The first-best one exhibits an intuitive feature: making South better-off makes North worse-off. The second one exhibits a counter-intuitive but well-known feature, called

'Laffer effect'. The higher the tax rate the higher the utility of South, until a threshold from which on the utility decreases. It corresponds to the point where discouragement of effort of the North, consequentially to a rising of the tax rate, outweighs the gain from the tax. The second-best frontier lies below the first-best one, which represents the well-known 'inefficiency' of a tax. Maximizing a social welfare function allows to choose the optimal allocation of utility, and implicitly the optimal (absolute or proportional) transfer. In particular, the higher the inequality aversion, the higher the transfer. Besides, and without surprise, I showed that frontiers are shifting outward with an increasing resource. This allowed me to study the evolution of welfare through the evolution of the resource. I took the golden rule as a benchmark target. If the current steady-state stock level is higher than the golden-rule one, I argued that a higher inequality aversion has to be implemented through a higher lump-sum. The intuition is that the effort being positively linked with the transfered amount, the North compensates by working more. On the opposite, if the current steady-state stock level is lower than the golden-rule one, I argued that a higher inequality aversion has to be implemented through a higher tax. The intuition is that the effort being negatively linked with the transfered amount, the North is discouraged and works less. In any case, one gets closer to the 'benchmark sacrifice' leading to the golden rule. The main message of the dissertation is that intragenerational and intergenerational equity considerations may, ethically, be independent. But accounting for real-world constraints conditions the relationship. In particular growing, or sustaining, welfare over generations in a productive economy implies to constrain countries with relatively scarce resources, which shall be compensated by countries with relatively abundant ones.

3.2.2 Limits and prospects

This work has of course its limitations. Both by the choices made to represent the issue and by the voluntary omissions of some elements for the sake of parsimony.

On choices, I want to raise three potentially controversial points. First, I used a welfarist approach, with a constant elasticity of substitution to represent inequality aversion. As results suggest, it is a fairly malleable approach. But I shall acknowledge that some results might strongly depend on those assumptions. Second, I only considered an infinite horizon. This is in line with most of the approaches in the sustainable development literature. Considering a finite horizon allows to escape from the classic equity-efficiency dilemma. But it solves a problem and creates another one, the date of the horizon. That said, I acknowledge that it is more consistent to study a finite horizon than justifying discounting in an infinite-horizon framework on the basis of 'myopia' of the social planer. Third, I concentrate more on procedural justice than on outcome justice. My restriction was indeed to treat everyone equally. But the welfarist approach is in essence interested in consequences of decisions on actual well-being. Actually, the welfarist approach is sufficiently malleable to encompass both considerations. The true question being what and how individual welfare index are measured. I tried to show how some results are sensitive to the functional form of the utility chosen. I propose to argue that focusing only on deontological aspects without looking at consequences can turn into 'monomania'. But at the opposite, focusing only on consequences can turn into 'arbitrariness'.

On omissions, I want to raise seven important elements that could have been relevant. First, the non-renewable resources. As we know, they have attracted most of the attention in sustainability issues. But their finiteness became recently less problematic than greenhouse gases they release during production and consumption. Whatever, the ultimate aim is to find a substitute, which is a technological issue (e.g. solar energy and breeder reactors). Second, climate change. It is maybe the more determinant environmental issue. But as renewable assets (e.g. wetlands) play also a role in the carbon cycle, I hope the present framework give also some insights in this literature. I recommend also North-South transfers, but for another reason. While the climate change literature has focused on inequality in responsibility and vulnerability, the present work has focused on inequality in endowment of natural assets. Third, technological progress. As progress makes constraints less binding, it impacts outcomes of distributional choices. It also played a role on justification of discounting.[30] Fourth, I omitted

[30] *"Sustainability is not always compatible with discounting the well-being of future generations if there is no continuing technological progress. But I will slide over this potential contra-*

uncertainty. As most of environmental issues are inherently uncertain, a deterministic framework can only be seen as a benchmark. This also applies to the fifth point, population growth. It is surely one of the main issues the world will have to address in the near future. But it deserves a work apart. Sixth, I also did not deal with within-country inequalities. Those seem to be of all importance, especially for vulnerability and resilience issues. Seventh, and lastly, I also omitted altruism. Knowing if altruist preferences shall be considered in equity issues is a problem in itself. Here, as I considered symmetrically the two dimensions, I then did not take into account non-egoistic individual preferences.

This work, as many others in the field, is not normative. I rather tried to formulate conditional recommendations. A criterion of social evaluation can never be universal. But shedding light on the underlying inequality aversion is maybe the best way to promote informed discussions, both on ethical premises and on formulation of quantifiable policy objectives.

diction because discount rates should be small and, after all, there is technological progress" (Solow, 1993, p. 168).

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